2nd International Congress on Maritime Technological Innovations and Research

Proceedings

New Maritime Impulses in the presence of a new century

Edited by
F.Piniella - A. Bocanegra (UCA)
J.Olivella - R. Rdguez-Martos (UPC)

Servicio de Publicaciones - Universidad de Cadiz

2nd International Congress on Maritime Technological Innovations and Research.

Organized by

Universidad de Cádiz
Facultad de Ciencias Náuticas

Under the patronage of
FOROSUR - Southern Maritime & Industrial Forum (Cádiz)

In cooperation with

Universitat Politècnica de Catalunya
Departament de Ciéncia i Enginyería Nàutiques

The International Congress on Maritime Technological Innovations and Research was born as an initiative from the Department of Nautical Sciences and Engineering of the Universitat Politècnica de Catalunya (UPC) to the development of shipping a challenging action over the past few years. Today, this Conference is intended to improve important aspects such as maritime training, new technologies and R&D programs, together with management systems, among others. The 2nd Congress will serve as a forum for cooperation among the different national and international institutions involved in Maritime Education.

Proceedings edited by:

Francisco Piniella
Ana Bocanegra
Juan Olivella
Ricardo Rdguez. -Martos

Servicio de Publicaciones - Universidad de Cádiz 2000
ISBN 84-7786-669-4 (Paperback)
ISBN 84-7786-670-8 (CD-Rom)

INTERVAL MODELLING OF HIGH SPEED CRAFT FOR ROBUST CONTROL

J. Aranda*, J. M. De la Cruz**, J. M. Díaz*, P. Ruipérez*

*Dept. de Informática y Automática. UNED. Madrid, Spain. **Dept. de Arquitectura de Computadores y Automática. U. Complutense. Madrid,

Abstract

The interval transfer functions from wave height to pitch and heave movement described in this job is interpreted as a family of transfer functions whose coefficients are bounded by some know intervals and centred at nominal values. The nominal mo del is obtained by a non-linear least square algorithm applied in the frequency domain. Low frequency constrained for pitch and heave was considered. Once the nominal model was obtained, then the tightest intervals around each coefficient of the nominal transfer functions was created while satisfying the membership and frequency response requirements. Different model validation tests was made (magnitude and phase plots, frequency envelope of the interval model, simulations). These tests show that the uncertainty model obtained is a valid interval model and it can be used for robust control design.

Introduction

The main problem for the development of high speed craft is concerned with the passenger's comfort and the safety of the vehicles. The vertical acceleration associated with roll, pitch and heave motion is the cause of motion sickness. The roll control is the most attractive candidate for control since increasing roll damping can be obtained more easily. However, shipbuilders are also interested in increasing pitch and heave damping. In order to solve the problem antipiching devices and pitch control methods must be considered. Previously, models for the vertical ship dynamic must be developed for the design, evaluation and verification of the results.

The number of published investigations about ship modelling is immense. For example, a non-lineal model in 6 degrees of freedom is shown in [1], [2] and [3] This model is theoretical and it is obtained from the equations of the rigid solid partially immersed in water.

Obtaining a very accurate mathematical model of a system is usually impossible and very costly. It also often increases the complexity of the control algorithm. A trend in the area of system identification is to try to model the system uncertainties to fit the available analysis and design tools of robust control [4].

The interval functions described in this paper [5] are interpreted as a family of transfer functions from wave height to pitch and heave movement whose coefficients are bounded by some know intervals and centred at nominal values. The nominal model is obtained by a non-linear least square algorithm applied in the frequency domain. Once the nominal model is obtained, then the tightest intervals around each coefficient of the nominal transfer functions are created while satisfying the membership and frequency response requirements.

Identification methodology

The method described in this paper follows the steps of classical identification diagram [6], [7] and [8]. A model test was carried out in the towing tank of CEHIPAR [9] (Madrid, Spain). The model was free to move in heave direction and pitch angle. The wave surface elevation was measured at 68.75 m. forward from model bow. Different regular and irregular waves and ship speed were tested. A set of simulated data have been generated by the program PRECAL [9] (which uses a geometrical model of the ship to predict her dynamic behaviour), reproducing the same conditions of the experiments with regular waves.

Two transfer functions are identified (see Figure 1) :

- $G_P(s)$: transfer function from wave height (metres) to pitch movement (degrees).
- $G_H(s)$: transfer function from wave height (metres) to heave movement (metres).

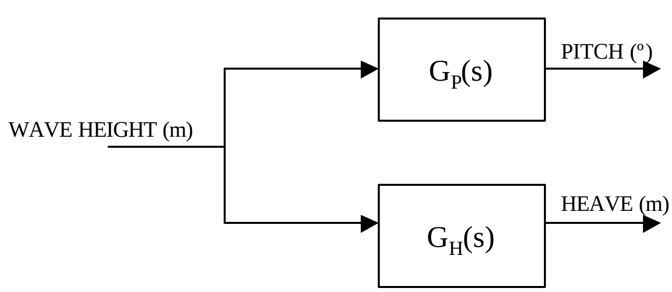

Figure 1: Blocks diagram of the identified system

The identification is made in the frequency domain and uses the simulated data of magnitude and phase obtained by the program PRECAL [9] in the encounter frequency ω_{ei} (i=1,2,...,25) for the transfer functions $G_P(j\omega_{ei})$ and $G_H(j\omega_{ei})$.

$$G_P(jw_{ei}) = \mathrm{Re}(G_P(jw_{ei})) + j\,\mathrm{Im}(G_P(jw_{ei}))$$
$$G_H(jw_{ei}) = \mathrm{Re}(G_H(jw_{ei})) + j\,\mathrm{Im}(G_H(jw_{ei}))$$

In general, the estimated transfer functions $\hat{G}_P(s)$ and $\hat{G}_H(s)$ can be written in the following form:

$$\hat{G}(s) = \frac{x_{n+m+1}s^m + x_{n+m}s^{m-1} + \ldots.. + x_{n+1}}{s^n + x_n s^{n-1} + \ldots. + x_1} \qquad (1)$$

where m is the number of zeros and n is the total number of poles. The parameter vector is:

$$\vec{P} = (x_1, x_2, \ldots, x_n, x_{n+1}, \ldots, x_{n+m+1})$$

The estimation of the parameter vector $\vec{P}$ is made by a non-linear least squares procedure that uses the following cost function [7]:

$$K(\vec{P}) = \sum_{k=1}^{N} \left| \left(\mathrm{Re}(G(jw_{ek})) - \mathrm{Re}(\hat{G}(jw_{ek}))\right) + j\left(\mathrm{Im}(G(jw_{ek})) - \mathrm{Im}(\hat{G}(jw_{ek}))\right) \right|^2 \qquad (2)$$

A number of considerations need to be made based in a priori knowledge of the ship dynamics. So, there are three constraints in the identification process of the models:

- The models must be stables.
- The gain of $G_P(s)$ must be tend to zero in low encounter frequencies.
- The gain of $G_H(s)$ must be tend to one in low encounter frequencies

The solution to this non-linear least squares problem with constrains is described in [8] and it uses tools described in [9].

Interval modelling

Bhattacharyya [4] describes a method to obtain the family of linear time invariants systems $\vec{G}(s)$ by letting the transfer function coefficients lie in intervals around those of the nominal G(s). This method is adapted to our problem. Let

$$y(jw_{ei}) = D(jw_{ei})u(jw_{ei}) \;\; i = 1, 2, ..., N$$

where $\omega_{e1},\omega_{e2},...,\omega_{eN}$ are the test encounter frequencies and the complex number $u(j\omega_{ei})$, $y(j\omega_{ei})$ denote in phasor notation the input-output pair at the frequency ω_{ei} generated from an identification experiment. Suppose that $G^I(s)$ is the transfer function of a linear time-invariant system which is such that $G^I(j\omega_e)$ is closest to $D(j\omega_e)$ in some norm sense. In general it is not possible to find a single rational function $G^I(s)$ for which $G^I(j\omega_{ei})= D(j\omega_{ei})$ and the more realistic identification problem is to fact identify an entire family $\vec{G}(s)$ of transfer functions which is capable of validating the data in the sense that for each point $D(j\omega_{ei})$ there exists some transfer function $G_i \in \vec{G}(s)$ with the property that $G^I(j\omega_{ei})= D(j\omega_{ei})$.

Let the nominal transfer function $G^I(s)$, which has been identified by a non-linear least squares procedure explained in the previus section, and the transfer function G(s) with the form:

$$G(s) = \frac{\hat{x}_{n+m+1}s^m + \hat{x}_{n+m}s^{m-1} + ...+ \hat{x}_{n+1}}{s^n + \hat{x}_n s^{n-1} + ...+ \hat{x}_1} \qquad (3)$$

The family of linear time-invariant systems $\vec{G}(s)$ is defined by:

$$\bar{G}(s)=\left\{G(s):\hat{x}_i\in[x_i-w_{x_i}\cdot\boldsymbol{e}^{-}_{x_i},x_i+w_{x_i}\cdot\boldsymbol{e}^{+}_{x_i}]\quad\forall i\right\}\qquad(4)$$

where w_{xi} are to be regarded as *weigths* chosen apriori whereas the ε's are to be regarded as *dilation parameters* to be determined by the identification algorithm and the data $D(j\omega_{ei})$

Weight selection

Suppose the test data consists of N data points obtained at corresponding frequencies,

$$D(jw_e)=\{D(jw_{ei})=a_i+jb_i\ ,i=1,2,...N\}$$

the l^{th} model is defined as :

$$G_l(jw_e)=\begin{cases}D(jw_{ei}) & i=l\\ G^I(jw_{ei}) & i=1,2,.....,l-1,l+1,....,N\end{cases}\qquad(5)$$

The model $G_l(j\omega_{ei})$ is identical to the nominal identified model $G^I(j\omega_{ei})$ with the l^{th} data point replaced by the l^{th} component of the test data $D(j\omega)$. Now the l^{th} identified model $G_l^I(s)$ is constructed, which is identified from the l^{th} data set $G_l(j\omega)$. Let

$$G_l^{\ I}(s)=\frac{x^l{}_{n+m+1}s^m+......+x_{n+1}{}^l}{s^n+x_n{}^l s^{n-1}......+x_1{}^l}\qquad(6)$$

The models $G_l^I(s)$ must be identified with the same method used to identify the nominal model $G^I(j\omega)$. The weight vector $\vec{w}$ is:

$$\vec{w}=\left[\frac{1}{N}\sum_{l=1}^{N}\left|x_1-x_1{}^l\right|,......,\frac{1}{N}\sum_{l=1}^{N}\left|x_{n+m+1}-x^l_{n+m+1}\right|\right]\qquad(7)$$
$$\vec{w}=[w_{x_1},......,w_{x_n},w_{x_{n+1}},......,w_{x_{n+m+1}}]$$

The weight selection is important because inappropriate selection of weights may results in an unneccesarily large family.

Computation of the intervals of the transfer function coefficients

Recall the nominal system given in (1) and substitute s=jω_{ei}, then we have

$$G(jw_{ei}) = \frac{\left(\hat{x}_{n+1} - w_{ei}{}^{2}\hat{x}_{n+3} + ...\right) + j\cdot\left(w_{ei}\hat{x}_{n+2} - w_{ei}{}^{3}\hat{x}_{n+4} + ...\right)}{\left(\hat{x}_{1} - w_{ei}{}^{2}\hat{x}_{3} + ...\right) + j\cdot\left(w_{ei}\hat{x}_{2} - w_{ei}{}^{3}\hat{x}_{4} + ...\right)}$$

$$G(jw_{ei}) = \frac{n1 + j\cdot n2}{d1 + j\cdot d2}$$

if G(jω_{ei}) is made equal to the data set D(jω_{e}) for a particular encounter frequency ω_{ei}, we have

$$D(jw_{ei}) = a_i + jb_i = \frac{n1 + j\cdot n2}{d1 + j\cdot d2}$$

Operating, we obtain the next pair of equations:

$$F_1\left(a_i, b_i, x_1^i, ..., x^i{}_{n+m+1}\right) = (a_i d1 - b_i d2) - n1 = 0$$
$$F_2\left(a_i, b_i, x_1^i, ..., x^i{}_{n+m+1}\right) = (b_i d1 + a_i d2) - n2 = 0$$

Recall $\hat{x}_i$ for all i is defined by:

$$\hat{x}_i = x_i + w_{x_i} e^{\,l}_{\,x_i} \quad \begin{cases} i = 1,....,n+m+1 \\ l = 1,......,N \end{cases} \qquad (8)$$

If we rewrite this in terms of a linear matrix equations, we have

$$A \cdot \vec{x} + A \cdot W \cdot \vec{e}\ ^{l}_{x} = -E$$
$$A \cdot W \cdot \vec{e}\ ^{l}_{x} = -B - E$$

where

$$A = \begin{bmatrix} a_i & -b_i w_{ei} & a_i w_{ei}{}^2 & -b_i w_{ei}{}^3 & . & -1 & 0 & w_{ei}{}^2 & 0 & w_{ei}{}^4 & . & . \\ b_i & a_i w_{ei} & -b_i w_{ei}{}^2 & a_i w_{ei}{}^3 & . & 0 & w_{ei} & 0 & w_{ei}{}^3 & 0 & . & . \end{bmatrix}$$

$$E = \begin{bmatrix} k_1 w_{ei}{}^n \\ k_2 w_{ei}{}^n \end{bmatrix}$$

$$k_1 = \begin{pmatrix} a_i \;\; si \; n = 0,4,8,.... \\ -b_i \;\; si \; n = 1,5,9,.... \\ -a_i \;\; si \; n = 2,6,10,.... \\ b_i \;\; si \; n = 3,7,11,.... \end{pmatrix} \qquad k_2 = \begin{pmatrix} b_i \;\; si \; n = 0,4,8,.... \\ a_i \;\; si \; n = 1,5,9,.... \\ -b_i \;\; si \; n = 2,6,10,.... \\ -a_i \;\; si \; n = 3,7,11,.... \end{pmatrix}$$

$$W = \begin{bmatrix} w_{x_1} & & & 0 \\ & . & & \\ & & . & \\ 0 & & & w_{x_{n+m+1}} \end{bmatrix}$$

$$\vec{e}\ ^{l}_{x} = \begin{bmatrix} e_{x_1}{}^{l} & . & . & . & . & . & . & e^{l}{}_{x_{n+m+1}} \end{bmatrix}^T$$

$$\vec{x} = [x_1, ..., x_{n+m+1}]^T$$

$$B = A \cdot \vec{x}$$

$\vec{e}\ ^{l}_{x}$ is the vector of the dilation parameters obtained for the encounter frecuency ω_{el}. Here it is assumed without loss of generality that $A(\omega_{ei},\alpha_i,\beta_i)$ has full rank. Then the minimun norm solution $\vec{e}\ ^{l}_{x}$ can be computed as:

$$\vec{e}\ ^{l}_{x} = -W^{-1}\left(A^T A\right)^{-1} A^T (B + E) \qquad (9)$$

After finding $\vec{e}_x^{\,l}$ for all l=1,...,N, the dilation parameters of the intervals of the transfer function coefficients are determined as follows:

$$e_{x_k}^{\ -} = \min_l \{0, e_{x_i}^{\ l}\} \quad e_{x_k}^{\ +} = \max_l \{0, e_{x_i}^{\ l}\} \qquad (10)$$

Results

Tables 1 and 2 show, respectively, the model structure order (where m is the number of zeros, n is the total number of poles and nps is the number of simple poles) for heave and pitch movement. For each ship speed is showed different structures, so, we can compare the cost function and mean square error when the model structure is reduced.

Table 1: Model structure for heave movement

Ship speed (knots)	**Model Structure (m,n,nps)**	**Value of the cost function**	**Mean square error (m^2)**
20	(4,6,2)	0.0383	0.014333
	(3,5,1)	0.0692	0.014062
	(2,3,1)	0.0696	0.013793
30	(4,6,2)	0.0385	0.011061
	(3,5,1)	0.1012	0.011476
	(2,3,1)	0.2381	0.017047
40	(4,6,2)	0.0471	0.011186
	(3,5,1)	0.1045	0.011323
	(2,3,1)	0.4510	0.01246

Table 2: Model structure for pitch movement

Ship speed (knots)	**Model Structure (m,n,nps)**	**Value of the cost function**	**Mean square error $((^o)^2)$**
20	(4,6,2)	0.1213	0.10562
	(3,5,1)	0.1228	0.10518
30	(4,6,2)	0.0938	0.099554
	(3,5,1)	0.0946	0.099763
40	(4,6,2)	0.0942	0.12141
	(3,5,1)	0.0989	0.12256

The model interval was obtained for each of model structure show in Table 1 and Table 2. For example, the model structure (4,6,2) for heave movement with a ship speed of 20 knots has the transfer function:

$$G_H(s) = \frac{115.8s^4 - 78.38s^3 + 831.2s^2 - 557.7s + 1325}{s^6 + 39.47s^5 + 471.6s^4 + 1688s^3 + 2690s^2 + 2459s + 1325}$$

Table 3 shows the model interval of $G_H(s)$. Figure 2 shows the Bode plot of $G_H(s)$ and data obtained by PRECAL program.

Table 3: Model interval of $G_H(s)$

x	Lower Interval	Nominal value	Upper Interval
x_1	1309.3	1325	1388
x_2	2411	2459	2479
x_3	2624	2690	2832.4
x_4	1649.3	1688	1793.6
x_5	331.88	471.6	548.86
x_6	33.488	39.47	42.611
x_8	-557.88	-557.7	-556.55
x_9	830.22	831.2	832.12
x_{10}	-83.187	-78.38	-77.61
x_{11}	90.822	115.8	128.37

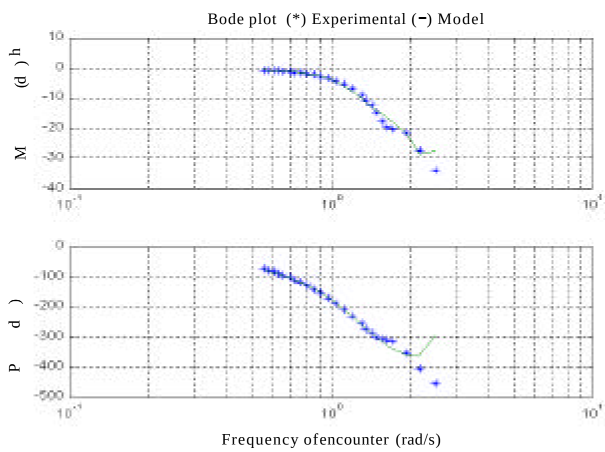

Figure 2 : Bode plot of $G_H(s)$ and data of PRECAL program

Figure 3 shows the output of $G_H(s)$ and the measured output in the CEHIPAR when the input was irregular waves at 20 knots and SSN=5.

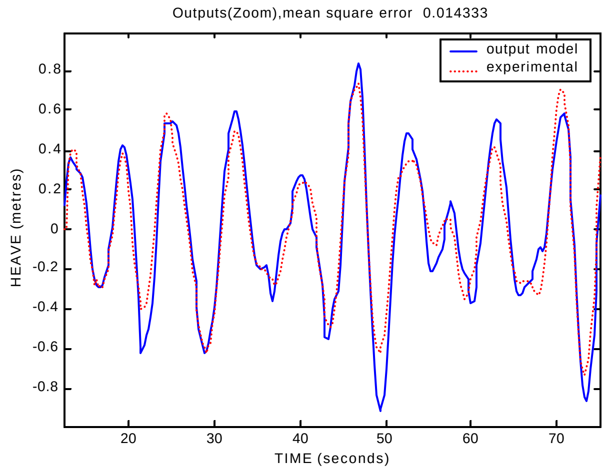

Figure 3: Simulation of $G_H(s)$ and real data at 20 knots and sea state SSN=5.

The model structure (4,6,2) for pitch movement with a ship speed of 20 knots has the transfer function:

$$G_P(s) = \frac{-1.19s^4 - 10.85s^3 + 14.98s^2 - 50.62s}{s^6 + 19.27s^5 + 50.86s^4 + 87.04s^3 + 113s^2 + 71.95s + 40.93}$$

Table 4 shows the model interval of $G_p(s)$. Figure 4 shows the Bode plot of $G_p(s)$ and data obtained by PRECAL program.

Table 4: Model interval of $G_p(s)$

x	Lower Interval	Nominal value	Upper Interval
x_1	40.6239	40.9301	42.1549
x_2	69.8317	71.9466	72.2921
x_3	111.9658	113.0089	113.4570
x_4	86.6317	87.0402	88.3162
x_5	50.1333	50.8560	52.1965
x_6	17.4087	19.2750	19.6824
x_8	-51.6473	-50.6223	-50.3770
x_9	14.4200	14.9826	15.0770
x_{10}	-11.6252	-10.8545	-8.9156
x_{11}	-1.3675	-1.1900	0.5597

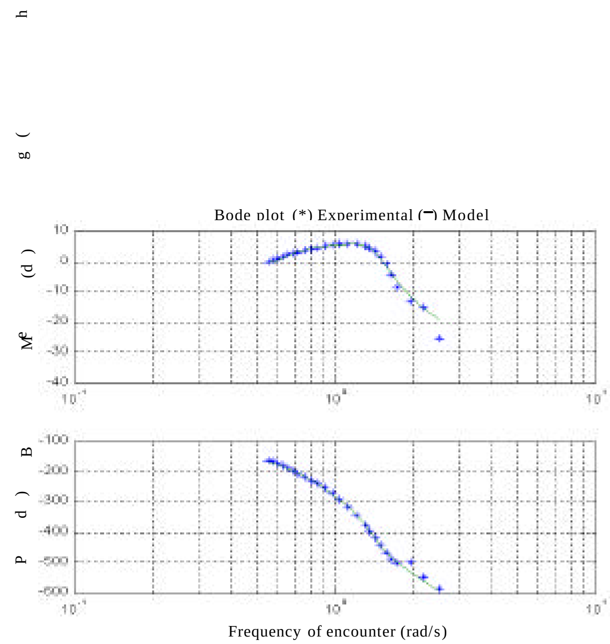

Figure 4: Bode plot of $G_P(s)$ and data of PRECAL program

Figure 5 shows the output of $G_P(s)$ and the measured output in the CEHIPAR when the input was irregular waves at 20 knots and SSN=5.

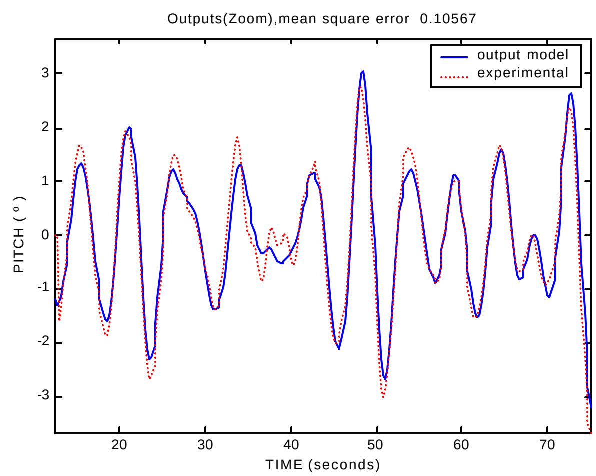

Figure 5: Simulation of $G_P(s)$ and real data at 20 knots and sea state SSN=5

Conclusions

In this paper has been showed the continuos linear models for vertical dynamics of a high speed craft identified by a non-lineal least square algorithm applied in the frequency domain

Once the nominal model was obtained, then the tightest intervals around each coefficient of the nominal transfer functions was created while satisfying the membership and frequency response requirements. Different model validation tests was made.

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Acknowledgements

This development was supported by CICYT of Spain under contract TAP97-0607-C03-02.