Robust GPC-QFT approach using Linear Matrix Inequalities

José Luis Guzmán, Teodoro Álamo, Manuel Berenguel, Sebastián Dormido, and Eduardo F. Camacho

***Abstract*— As well-known in the Model Predictive Control (MPC) community there is a need for looking computationally affordable robust predictive control algorithms that are suitable for on-line implementation. A new control approach mixing Generalized Predictive Control (GPC) and Quantitavie Feedback Theory (QFT) is presented for controlling a family of linear uncertain plants. A cascade structure is proposed, combining an inner loop containing a QFT controller with an outer loop where a GPC controller provides adequate references for the inner loop considering input saturations. The idea consists in translating a QFT design to state space and then using Linear Matrix Inequalities (LMI) to obtain a state-vector feedback in such a way that the input reference for the inner loop is calculated in order to satisfy robust tracking problems considering input saturations. Therefore, the proposed solution results in solving a set of constraints being formed by several LMI and Bilineal Matrix Inequalities (BMI), where the aim is to regulate to a fixed reference value.**

***Keywords*—-predictive control, robust control, tracking control, constrained control.**

I. INTRODUCTION

MPC is a family of control techniques that optimize a given criterion by using a model to predict system evolution and compute a sequence of future control actions. Therefore, the performance and robustness of this kind of controllers depend on how well a model is able to capture the dynamics of a plant. A mathematical model can have different degrees of complexity, but invariably in a realistic situation a model cannot exactly emulate a physical process, and the problems of stability and performance in a system mostly manifest themselves from this model-plant uncertainty [4]. A large number of works about robustness have been developed in the GPC framework. Most results obtained for the unconstrained case are based on using tuning guidelines to increase the robustness, taking the T-*polynomial* as design element, or using the Youla parametrization to robustify the system. The Small Gain Theorem (SGT) is the tool used to study the robust stability. Several tuning guidelines have been proposed in [11], [3] and [26] to augment the robustness at expense of poor performance. In [30], [26], [35], [27] some rules are introduced to the selection of the T-polynomial in order to reduce the effect of model uncertainties. As pointed out, other formulations are found based on the Youla parametrization, firstly used by [21], [22], and later by [2], [1] and [31]. In most of these works, the computational load of the algorithms is low, but the uncertainties are represented as unmodelled dynamics while the SGT is used to study the stability, obtaining too conservative results. In addition to the above results, one of the most widely studied robust stability methods for predictive control is the well-known *min-max* approach [24]. Nowadays, this approach has a great interest in the predictive control community [25], [33], [5], [6], [29] and it was applied to GPC in [9] and [10]. Its main drawbacks are that the control performance may be too conservative in some cases (additive uncertainty), and its computational load, although during the last years it has been shown that MPC can be considered as a multiparametric quadratic or linear programming problem, and that MPC solution turns out to be a relatively easy-to-implement piecewise affine (PWA) controller [5], [6], [29].

In this work, a new approach to increase the robustness of the GPC algorithm is proposed related with the underlying idea of the feedback linearization techniques [23], [19], and feedback stabilizing laws [10], where an inner loop stabilizes and linearizes the system so that a *linear* GPC algorithm can be used transparently in order to control the linearized system in the unconstrained case, or a constrained GPC when input and output amplitude constraints are active. In this case, instead of focusing on linearization issues, the inner loop is included to decrease uncertainty in presence of parametric uncertainties in the plant to control, so that the QFT technique [16], [34] is selected to reduce them and permit the use of a nominal GPC in the outer loop. Thanks to QFT, the uncertainties are taken into account in a systematic way to obtain results without conservatism. Together, the QFT and the GPC algorithms provide a new approach that is less sensitive to process uncertainties and has low computational load.

On the other hand, it is well-known that the QFT technique has several difficulties to manage the constraints, being necessary to reformulate the original problem for this purpose [28], [17]. So, the GPC-QFT approach [14] is used to take into account input constraints in the QFT loop thanks to the GPC abilities to include constraints in a systematic way. The proposed ideas have also some similarities to the reference governor [7], [32] approaches, which have been proposed in several works as suboptimal solutions to predictive control.

The GPC-QFT approach was developed in previous works using transfer function formulations and obtaining good computational efficiency and promising simulation results

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Teodoro Álamo and Eduardo F. Camacho with Department of Ingeniería de Sistemas y Automática, University of Seville, C. de los Descubrimientos, s/n. Pabellón Pza. de América, 41092, Seville, Spain. alamo@cartuja.us.es, eduardo@esi.us.es

José Luis Guzmán and Manuel Berenguel are with the Department of Lenguajes y Computación, University of Almería, Ctra. Sacramento s/n, 04120, Almería, Spain. joguzman@ual.es, beren@ual.es

Sebastián Dormido is with the Department of Informática y Automática, UNED, C/. Juan del Rosal 16, 28040, Madrid, Spain. sdormido@dia.uned.es

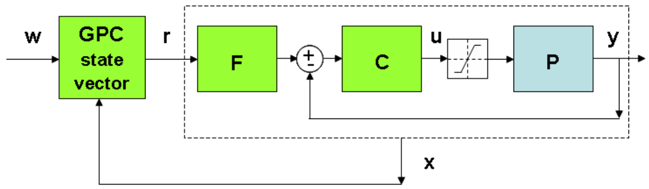

Fig. 1. Control system scheme for LMI-based approach

[13], [14]. However, the algorithm lacks of a rigorous theoretical study to guarantee robust stability. The use of transfer function formulation with uncertain parameters has a great acceptance in the industrial environment, but it is difficult to formalize the results in presence of constraints (for example, to ensure constrained robust stability). LMI have been proposed by several authors as solution to solve constrained robust predictive control algorithms in polynomial time using state space representations [20], where constrained robust stability can explicitly be ensured. Thus, a possible solution to prove robust stability in the GPC-QFT approach when constraints are active, could be to translate the problem into a state space representation, and then using LMI to obtain a state-vector feedback in such a way that the input reference to the inner loop is calculated in order to satisfy robust tracking problems considering input saturation (see Figure 1). This work describes the different steps necessary to obtain such solution and how the problem with input saturation in the inner loop can be solved using LMI-based solutions.

The final solution consists in solving a set of constraints being formed by several LMI and BMI, where a Branch and Bound algorithm has been developed in order to solve the bilinear terms. Notice that the algorithm is implemented for tracking problems where the aim is to regulate to a fixed reference value and not to the origin, and also input constraints are presented in the inner loop of the system.

II. QFT DESIGN

Most control techniques require the use of a plant model during the design phase in order to tune the controller parameters. The mathematical models are an approximation of real systems and contain imperfections by several reasons: use of low-order descriptions, unmodelled dynamics, obtaining linear models for a specific operating point (working with poor performance outside of this working point), etc.

The robust control technique which considers more exactly the uncertainties is QFT. It is a methodology to design robust controllers based on frequency domain, and was developed by Prof. Isaac Horowitz [15], [16]. This technique allows designing robust controllers which fulfil some minimum quantitative specifications considering the presence of uncertainties in the plant model and the existence of perturbations. With this theory, Horowitz showed that the final aim of any control design must be to obtain an open-loop transfer function with the suitable bandwidth in order to sensitize the plant and reduce the perturbations. The Nichols chart is used to achieve a desired robust design over the specified region of plant uncertainty where the aim is to design a compensator $C(s)$ and a prefilter $F(s)$ (if it is necessary), so that performance and stability specifications are achieved for the family of plants $\Pi(s)$ describing a plant $P(s)$. In the case of this work, the family $\Pi(s)$ is represented using parametric uncertainties

$$\Pi(s) = \Big\{ P(s) = \kappa \frac{\prod_{i=1}^{n}(s+\ell_i)\prod_{j=1}^{m}(s^2+2\beta_j\omega_{0j}+\omega_{0j}^2)}{s^N\prod_{r=1}^{a}(s+\zeta_r)\prod_{s=1}^{b}(s^2+2\beta_s\omega_{0s}+\omega_{0s}^2)} : \quad (1)$$
$$\kappa \in [\kappa_{min},\ \kappa_{max}],\ \ell_i \in [\ell_{i,min},\ \ell_{i,max}],\ \zeta_r \in [\zeta_{r,min},\ \zeta_{r,max}],$$
$$\beta_j \in [\beta_{j,min},\ \beta_{j,max}],\ \omega_{0j} \in [\omega_{0j,min},\ \omega_{0j,max}],$$
$$\beta_s \in [\beta_{s,min},\ \beta_{s,max}],\ \omega_{0s} \in [\omega_{0s,min},\ \omega_{0s,max}],$$
$$n+m < a+b+N \Big\}$$

As commented in the first section, in order to perform a robust design in GPC, the T-polynomial or the min-max approaches are mainly used. However, in both of them conservatism and computational load (in the second case) are usual. The approach presented in this paper tries to reduce the effect of the uncertainties in a systematic way without conservatism in order to utilize the original formulation of GPC without including additional terms. The control structure is shown in Figure 1, where a QFT controller with two d.o.f. is placed in an inner loop to reduce the effect of modelling errors presented in a plant with parametric uncertainties, and a nominal GPC can be used to control the inner loop as the effects of these modelling errors are highly reduced [14].

So, the first steps to define the inner loop are the following [14]:

1) *Plant*. The plant must be represented as a family of plants with parametric uncertainties (see equation (1)).
2) *Specifications*. The desired requirements for the QFT design are defined. For a design to be used in GPC, it will be sufficient to establish the *tracking* and *stability* specifications [16]. By tracking specification the effect of the uncertainties will be reduced. It is only necessary to impose the minimum and maximum values for the magnitude of the closed-loop system in all frequencies

$$M_l \le \Big(\Big|\frac{C(j\omega)P(j\omega)}{1+C(j\omega)P(j\omega)}\Big|\Big) \le M_u \quad (2)$$

With respect to the stability specification, the desired gain (GM) and phase (PM) margins are established, translating these margins to magnitude limits (M_s) in frequency domain according to

$$GM = 1+\frac{1}{M_s} \quad PM = 180-\frac{180}{\pi}\arccos(\frac{0.5}{M_s^2}-1) \quad (3)$$

Providing the following condition for stability issues

$$\Big|\frac{C(j\omega)P(j\omega)}{1+C(j\omega)P(j\omega)}\Big| \le M_s \quad (4)$$

So, the effect of the uncertainties is reduced and the robust stability is ensured.

3) *QFT controller*. Now, the inner controller $C(s)$ and the prefilter $F(s)$ are designed in the frequency domain using QFT to reach the above specifications.
   Remark 1: The design of the prefilter $F(s)$ from QFT could be omitted centering the design on ensuring robust stability and moving the prefilter effect to GPC. However, the use of the prefilter allows reaching robust tracking specifications softening the set-point signal and thus obtaining less aggressive control signals.
4) *Discrete model for GPC*. The plant used by GPC is $G(s) = F(s)\frac{C(s)P(s)}{1+C(s)P(s)}$, so that the discrete nominal model $G_0(z)$ can be obtained with the appropriate sample time. The sample time will be chosen based on the bandwidth of the nominal model. Notice that the nominal model in GPC can be different from the nominal model used in the QFT design.

III. STATE SPACE REPRESENTATION OF THE INNER LOOP

Once the uncertainties have been reduced using QFT, the inner loop is translated to state space representation in order to applied LMI-based solutions.

A. Plant, prefilter and controller representations

GPC is based on CARIMA model, where the following plant representation is considered

$$A(z^{-1})y(t) = B(z^{-1})u(t-1) + T(z^{-1})\frac{\varepsilon(t)}{\Delta} \tag{5}$$

where $\Delta = 1 - z^{-1}$ and the delay is included into the $B(z^{-1})$ polynomial.

The polynomials $A(z^{-1})$ and $B(z^{-1})$ can be rewritten as

$$A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-1} + \ldots + a_n z^{-n} \tag{6}$$

$$B(z^{-1}) = b_0 + b_1 z^{-1} + b_2 z^{-1} + \ldots + b_m z^{-m} \tag{7}$$

$$T(z^{-1}) = 1 + t_1 z^{-1} + t_2 z^{-1} + \ldots + t_r z^{-r} \tag{8}$$

where the coefficients $a_1, \ldots, a_n$, $b_0, \ldots, b_m$, depend multilineally on the parametric vector $\phi = [K_z, c_1, \ldots, c_m, p_1, \ldots, p_n]^\top$. That is, each coefficient depends affinely on each element of vector ϕ. In order to express explicitly the dependence on the vector ϕ, the coefficients can be expressed as $b_0(\phi), \ldots, b_m(\phi)$, $a_1(\phi), \ldots, a_n(\phi)$. It is assumed that the coefficients of the $T(z^{-1})$ polynomial depends multilineally on a bounded parametric vector ϕ_T and $\varepsilon(t)$ is bounded for all $t > 0$, that is, $\|\varepsilon(t)\|_\infty < \varepsilon_{max}$, $\forall t$.

The plant dynamics can be represented by a state-space representation, where the proposed state depends on the current output, and the past outputs and inputs in the following way

$$x_p(t) = [y(t)\ y(t-1)\ \ldots\ y(t-n+1)\ u(t-1)\ \ldots\ u(t-m)]^\top \tag{9}$$

This state selection has the advantage that the state $x_p(t)$ is always accessible, that is, the value of $x_p(t)$ is known since it is always possible to access to the output $y(t)$ and input $u(t)$ signals. So, the state space representation is given by

$$\begin{aligned} x_p^+ &= A_p(\tilde{\phi})x_p + B_p(\tilde{\phi})u + E_p(\tilde{\phi}) \\ y &= C_p x_p \end{aligned} \tag{10}$$

where x_p denotes the state vector, u the system input, and x_p^+ the next state for the state x_p ($x_p(t+1)$). In this representation, $\tilde{\phi}$ is a parametric vector containing ϕ, ϕ_T and ε. Furthermore, it can be assumed that $\tilde{\phi}$ can only take values within a convex set (typically an hyperrectangle). Finally, notice that $A_p(\tilde{\phi})$, $B_p(\tilde{\phi})$ and $E_p(\tilde{\phi})$ depends multilineally on parametric vector $\tilde{\phi}$.

Assume available state space descriptions for the prefilter $F(z^{-1})$ and controller $C(z^{-1})$. Denoting x_F as the state vector of the filter $F(z^{-1})$, r the filter input and r_F the filter output, it is supposed that matrixes A_F, B_F, C_F, and D_F describe the filter dynamics as follows

$$\begin{aligned} x_F^+ &= A_F x_F + B_F r \\ r_F &= C_F x_F + D_F r \end{aligned} \tag{11}$$

In the same way, x_C denotes the state vector for the controller $C(z^{-1})$ and u the controller output. The matrices A_C, B_C, C_C and D_C describe the controller dynamics as follows

$$\begin{aligned} x_C^+ &= A_C x_C + B_C(r_F - y) \\ u &= C_C x_C + D_C(r_F - y) \end{aligned} \tag{12}$$

Note that the input to the controller is given by the filter output r_F minus the plant output y, and the plant is subject to uncertainties and disturbances as discussed above.

B. Inner loop representation. QFT loop.

As commented previously, the goal is to design a robust predictive controller considering input saturation in the inner loop. Therefore, the state space representation of the inner loop must be developed including the saturation.

The input saturation in the inner loop is given by

$$\sigma_p(u) = \begin{cases} U_{min} & \text{if } u < U_{min} \\ u & \text{if } U_{min} \le u \le U_{max} \\ U_{max} & \text{if } u > U_{max} \end{cases} \tag{13}$$

where nonsymmetric saturation can be presented.

Firstly, the saturation is redefined in order to use a symmetric representation to facilitate the calculation. Therefore, the saturation is obtained as

$$\sigma_p(u) = L_s \sigma(\frac{1}{L_s}(u - u_c)) + u_c \tag{14}$$

where

$$\sigma(u) = \begin{cases} -1 & if \quad u < -1 \\ u & if \quad -1 \le u \le 1 \\ 1 & if \quad u > 1 \end{cases} \tag{15}$$

$$u_c = \frac{U_{max} + U_{min}}{2}, \ L_s = \frac{U_{max} - U_{min}}{2}$$

Then, the plant representation (10) is modified to consider input saturation in the following way

$$\begin{aligned} x_p^+ &= A_p x_p + B_p(L_s\sigma(\frac{1}{L_s}(u-u_c))+u_c)+E_p \quad (16)\\ y &= C_p x_p \end{aligned}$$

where $A_p = A_p(\tilde{\phi})$, $B_p = B_p(\tilde{\phi})$, and $E_p = E_p(\tilde{\phi})$ will be considered from now on for the sake of simplifications. The proposed extended vector x including the inner loop dynamics is defined as

$$x = \begin{bmatrix} x_p & x_C & x_F \end{bmatrix}^\top \quad (17)$$

Then, the full system described by the plant, prefilter, and controller has r as input (prefilter input), and y as output (plant output). In this way and after some algebraic manipulations the closed-loop state representation for the inner loop is described as

$$\begin{aligned} x^+ &= Ax + B_u\sigma\Big(\frac{C_u}{L_s}x + \frac{D_u r - u_c}{L_s}\Big) + E + B_r r \quad (18)\\ y &= C_y x \end{aligned}$$

where

$$A = \begin{bmatrix} A_p & 0 & 0 \\ -B_C C_p & A_C & B_C C_F \\ 0 & 0 & A_F \end{bmatrix},\ B_u = \begin{bmatrix} L_s B_p \\ 0 \\ 0 \end{bmatrix}$$

$$E = \begin{bmatrix} E_p + B_p u_c \\ 0 \\ 0 \end{bmatrix}, B_r = \begin{bmatrix} 0 \\ B_C D_F \\ B_F \end{bmatrix}, C_y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

$$C_u = \begin{bmatrix} -D_C C_p & C_C & D_C C_F \end{bmatrix}, D_u = D_C D_F$$

IV. TRACKING PROBLEM. PRELIMINARIES IDEAS

Most of the results obtained for constrained MPC using LMI have been proposed to regulate the system to the origin. In this way, the results obtained in [20] can be used to calculate a control law $r = K_s x$ for the plant (18) considering the system free of disturbances ($E(\tilde{\phi}) = 0$) and regulating to the origin. Notice that $x = 0$ is an equilibrium for the system and for all value of $\tilde{\phi}$. Therefore, K_s can be calculated ensuring robust stability and in such a way that the control law $r = K_s x$ regulates to the origin for all posible initial conditions and any value of $\tilde{\phi}$ [20], [8]. However, one of the objectives in the GPC-QFT approach is to make the output y reach the reference value w. Therefore, the problem formulation must be oriented to this objective. This chapter presents some preliminary ideas based on the extensions proposed in [20] for set-point tracking.

Firstly, it is necessary to notice that due to the dependence on the parametric vector $\tilde{\phi}$, it is imposible to find static values for x and r (x_e and r_e) such that the system finds an unique equilibrium for all values of the parametric vector $\tilde{\phi}$. In this work in order to address this problem, the following control law is proposed

$$r = r_e + K_s(x - x_e) \quad (19)$$

where x_e, r_e, and K_s will be obtained in such a way that the performance of the closed-loop system is enhanced and the system evolution is ensured to be in an invariant ellipsoid containing the problem initial conditions.

Substituting r in equation (18) by the desired control law $r = r_e + K_s(x - x_e)$ and after some basic manipulations it is obtained that

$$\begin{aligned} \bar{x}^+ &= A_{Bc}\bar{x} + B_{Bc}\sigma(\frac{K_u\bar{x} + d_u}{L_s}) + E_{Bc} \quad (20)\\ y &= C_y\bar{x} + C_y x_e \end{aligned}$$

where

$$A_{Bc} = (A + B_r K_s),\ B_{Bc} = B_u,\ E_{Bc} = E + Ax_e + B_r r_e - x_e$$

$$K_u = C_u + D_u K_s,\ d_u = C_u x_e + D_u r_e - u_c$$

Define $\wp(P_s,\rho) = \bar{x}^\top P_s \bar{x} \le \rho$ as an ellipsoid where $\bar{x}_0 \in \wp(P_s,\rho)$ with $\bar{x}_0 = x_0 - x_e$. In this way, the tracking problem ensuring constrained robust stability for the system (20) will be solved using LMI and performing the following objectives:

1) Firstly, the decision variables x_e, r_e, K_s, P_s, and ρ are calculated in such a way that the ellipsoid $\wp(P_s,\rho)$ is invariant containing the system initial conditions $\bar{x}_0$ and using the control law $r = r_e + K_s(x - x_e)$.
2) After that, new constrains will be included in order to fulfill a certain performance criteria.

These objectives will be addressed in next sections, but how to take into account the saturation term presented in (20) will be addressed before.

A. Linear Difference Inclusion of the saturation function

Notice that due to the input saturation, a nonlinear term appears in the system dynamics, $\sigma(\frac{K_u\bar{x}+d_u}{L_s})$. This nonlinear term can be approximated using the *Lineal Difference Inclusion* (LDI) results obtained in [18] and [12] where it is shown that, if $b \in \mathbb{R}$ satisfies $|b| \le 1$ then

$$\sigma(a) \ \in \ Co\{a, b\},\ \forall a \in \mathbb{R}$$

being Co the convex hull. In particular, if $|H_s\bar{x} + h| \le 1\ \forall \bar{x} \in \wp(P_s,\rho)$ then

$$\sigma(\frac{K_u\bar{x}+d_u}{L_s}) \ \in \ Co\{\frac{K_u\bar{x}+d_u}{L_s}, H_s\bar{x}+h\},\ \ \forall(\frac{K_u\bar{x}+d_u}{L_s}) \in \mathbb{R},$$
$$\forall \bar{x} \in \wp(P_s,\rho)$$

Therefore and as will be shown in next sections, each objective commented above will be translated to analyze if it satisfies the extremes of the convex hull

$$\bar{x}^+ = A_{Bc}\bar{x} + B_{Bc}(\frac{K_u\bar{x}+d_u}{L_s}) + E_{Bc} \quad (21)$$

$$\bar{x}^+ = A_{Bc}\bar{x} + B_{Bc}(H_s\bar{x} + h) + E_{Bc} \quad (22)$$

On the other hand, the inequality $|H_s\bar{x} + h| \le 1$ must be considered. This inequality can be translated to a LMI in order to be included in the final optimization problem.

The inequality can be expressed as two inequalities in the following way

$$H_s\bar{x}+h \leq 1 \;\Rightarrow\; H_s\bar{x} \leq 1-h, \quad \forall \bar{x} \in \wp(P_s,\rho) \tag{23}$$

$$H_s\bar{x}+h \geq -1 \;\Rightarrow\; H_s\bar{x} \geq -1-h, \quad \forall \bar{x} \in \wp(P_s,\rho) \tag{24}$$

where these inequalities must be satisfied in the ellipsoid $\wp(P_s,\rho)$. In the next section, this ellipsoid will be forced to be invariant containing the system initial conditions.

Considering the previous inequalities (23), (24) and using the $S-procedure$ (Farkas lemma [8]), it is equivalent to study the existence of $\lambda_2 \geq 0$ such that the following inequalities are fulfilled:

$$\begin{bmatrix} 1-h-\lambda_2\rho & V \\ V^\top & 4\lambda_2 W \end{bmatrix} > 0 \tag{25}$$

$$\begin{bmatrix} 1+h-\lambda_2\rho & V \\ V^\top & 4\lambda_2 W \end{bmatrix} > 0 \tag{26}$$

Property 1: Suppose that there exits $\lambda_2 \geq 0$ such that LMI (25) and (26) are fulfilled, then:

$$\sigma(\frac{K_u\bar{x}+d_u}{L_s}) \;\in\; Co\{\frac{K_u\bar{x}+d_u}{L_s}, H_s\bar{x}+h\}, \; \forall \bar{x} \in \wp(P_s,\rho),$$

where $P=W^{-1}$ and $H=W^{-1}V$.

B. Robust Invariant Ellipsoid

As commented previously, one of the objectives is to calculate the decision variables x_e, r_e, K_s, P_s, and ρ in such a way that the ellipsoid $\wp(P_s,\rho)$ is invariant including the system initial conditions and using the control law $r = r_e + K_s(x-x_e)$. Therefore, in order to ensure the ellipsoid being invariant the following inequality must be fulfilled

$$(\bar{x}^+)^\top P_s(\bar{x}^+) \leq \rho, \quad \forall \bar{x} \in \wp(P_s,\rho) \tag{27}$$

This problem can be reformulated using $S-procedure$ as follows:

$$(\bar{x}^+)^\top P_s(\bar{x}^+) - \lambda_1\bar{x}^\top P_s\bar{x} + \rho(\lambda_1-1) \leq 0, \quad \forall \bar{x}, \; \lambda_1 \geq 0 \tag{28}$$

Lets consider the following property:

Property 2: Suppose that $P_s > 0$, then

$$z^\top P_s z \geq v^\top P_s v + 2v^\top P_s(z-v) = -v^\top P_s v + 2v^\top P_s z$$

and

$$z^\top P_s z = \max_v\{-v^\top P_s v + 2v^\top P_s z\} \tag{29}$$

Therefore, using the previous property and the closed-loop system dynamics (20), the inequality (28) results

$$\begin{aligned} -v^\top P_s v + 2v^\top P_s(A_{Bc}\bar{x} + B_{Bc}\sigma(\frac{K_u\bar{x}+d_u}{L_s}) + E_{Bc}) - \\ -\lambda_1\bar{x}^\top P_s\bar{x} + \rho(\lambda_1-1) \leq 0 \end{aligned} \tag{30}$$

where this inequality must be satisfied $\forall \bar{x}$ and $\forall v$.

Notice that in order to address this problem and demonstrate that the system evolution belongs to an invariant ellipsoid, it is necessary to obtain a LDI of the saturation term as shown in the previous section (see Property 1). Therefore, the inequality (31) must be satisfied for the extremes of the convex hull, $\frac{K_u\bar{x}+d_u}{L_s}$ and $H_s\bar{x}+h$, resulting the following inequalities:

$$\begin{bmatrix} \rho(1-\lambda_1) & * & * \\ 0 & \lambda_1 W & * \\ A_{x_e}x_e + B_{r_e}r_e + E_e & A_w W + B_y Y & W \end{bmatrix} > 0 \tag{31}$$

$$\begin{bmatrix} \rho(1-\lambda_1) & * & * \\ 0 & \lambda_1 W & * \\ A_{nlxe}x_e + B_{nlre}r_e + B_h h + E_{nle} & A_{nlW}W + B_{nlY}Y + B_v V & W \end{bmatrix} > 0 \tag{32}$$

where $A_{x_e} = A - I + (\frac{1}{L_s})B_u C_u$, $B_{r_e} = B_r + (\frac{1}{L_s})B_u D_u$, $E_e = E - (\frac{1}{L_s})B_u u_c$, $A_w = A + \frac{1}{L_s}B_u C_u$, $B_y = B_r + \frac{1}{L_s}B_u D_u$, $Y = K_s W$, $A_{nlxe} = (A-I)$, $B_{nlre} = B_r$, $B_h = B_u$, $E_{nle} = E$, $A_{nlW} = A$, $B_{nlY} = B_r$, $B_v = B_u$, $Y = K_s W$, and $V = H_s W$.

Property 3: Suppose that there exists $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$ such that the LMI (25), (26), (31) and (32) are fulfilled. Then, $\wp(P_s,\rho)$ is an invariant ellipsoid using the control law $r = r_e + K_s(x-x_e)$ and containing the system initial conditions, where $P = W^{-1}$ and $K = W^{-1}Y$.

Remark 2: Notice that the previous LMI depends multilineally on the parametric vector $\tilde{\phi} \in \Phi$ due to the dependence of $A_p = A_p(\tilde{\phi})$, $B_p = B_p(\tilde{\phi})$, and $E_p = E_p(\tilde{\phi})$. Then, properties 7.1 and 7.3 must be satisfied for all extreme plants of the hyperrectangle Φ.

C. Including Performance Inequality

Consider the representation of system (20) for the instant time k

$$\begin{aligned} \bar{x}_k^+ &= A_{Bc}\bar{x}_k + B_{Bc}\sigma(\frac{K_u\bar{x}_k+d_u}{L_s}) + E_{Bc} \\ y_k &= C_y\bar{x}_k + C_y x_e \end{aligned} \tag{33}$$

and suppose the following equality

$$w = C_y x_e \tag{34}$$

For an initial condition x_0 and the reference w, it is desired to calculate the system input r_k by the law $r_k = r_e + K_s(x-x_e)$ such that the the following functional is minimized

$$J = \sum_{k=0}^{N}(y_k-w)^\top Q(y_k-w) + \bar{x}^\top K_u^\top R_u K_u\bar{x} \tag{35}$$

where $Q > 0$ and $R_u > 0$ are symmetric matrices positive semi-defined.

From the equality (34), it results that the functional J can be rewritten as

$$J = \sum_{k=0}^{N}\bar{x}_k^\top C_y^\top Q C_y\bar{x}_k + \bar{x}^\top K_u^\top R_u K_u\bar{x} \tag{36}$$

Defining $L_J(\bar{x}_k) = \bar{x}_k^\top C_y^\top Q C_y\bar{x}_k + \bar{x}^\top K_u^\top R_u K_u\bar{x}$, it results that

$$J = \sum_{k=0}^{N} L_J(\bar{x}_k) \tag{37}$$

In the following property a strategy is defined for a correct selection of K_s, x_e and r_e in order to fulfill the performance criteria (35).

Property 4: Suppose that

$$\bar{x}_{k+1}^{\top} P_s \bar{x}_{k+1} - \bar{x}_k^{\top} P_s \bar{x}_k \leq -L_J(\bar{x}_k) + \gamma, \quad \forall \tilde{\phi} \in \Phi, \, \forall \bar{x}$$

and that an initial condition is equal to x_0. Suppose also that the control law $r_k = r_e + K_s(x_k - x_e)$ is applied to the system, then

$$J \leq \bar{x}_0^{\top} P_s \bar{x}_0 + N\gamma$$

where $\bar{x}_0 = x_0 - x_e$.

Proof: The assumption of the property leads to

$$\bar{x}_{k+1}^{\top} P_s \bar{x}_{k+1} - \bar{x}_k^{\top} P_s \bar{x}_k \leq -L_J(\bar{x}_k) + \gamma, \quad \forall \tilde{\phi} \in \Phi, \, \forall k \geq 0$$

Therefore,

$$\begin{array}{rcl} \bar{x}_1^{\top} P_s \bar{x}_1 - \bar{x}_0^{\top} P_s \bar{x}_0 & \leq & -L_J(\bar{x}_0) + \gamma \\ \bar{x}_2^{\top} P_s \bar{x}_2 - \bar{x}_1^{\top} P_s \bar{x}_1 & \leq & -L_J(\bar{x}_1) + \gamma \\ & \vdots & \\ \bar{x}_N^{\top} P_s \bar{x}_N - \bar{x}_{N-1}^{\top} P_s \bar{x}_{N-1} & \leq & -L_J(\bar{x}_{N-1}) + \gamma \\ \bar{x}_{N+1}^{\top} P_s \bar{x}_{N+1} - \bar{x}_N^{\top} P_s \bar{x}_N & \leq & -L_J(\bar{x}_N) + \gamma \end{array}$$

If the previous inequalities are added, it is obtained that

$$\bar{x}_{N+1}^{\top} P_s \bar{x}_{N+1} - \bar{x}_0^{\top} P_s \bar{x}_0 \leq -J + N\gamma$$

$$J \leq \bar{x}_0^{\top} P_s \bar{x}_0 + N\gamma$$

■

So, from the previous property the following optimization problem can be proposed

$$\begin{array}{ll} \min\limits_{P_s, K_s, x_e, r_e, \gamma} & \bar{x}_0^{\top} P_s \bar{x}_0 + N\gamma \\ s.a. & (\bar{x}^{+})^{\top} P_s (\bar{x}^{+}) - \bar{x} P_s \bar{x} < \\ & < -\bar{x}^{\top} C_y^{\top} Q C_y \bar{x} - \bar{x}^{\top} K_u^{\top} R_u K_u \bar{x} + \gamma, \quad \forall \tilde{\phi} \in \Phi, \, \forall \bar{x} \end{array} \quad (38)$$

in order to calculate the control law that minimizes an upper limit of the functional.

Then, the problem (38) can be reformulated as

$$\begin{array}{ll} \min\limits_{P_s, K_s, x_e, r_e, \gamma, \alpha_s} & \alpha_s \\ s.a. & \bar{x}_0^{\top} P_s \bar{x}_0 + N\gamma < \alpha_s \\ & (\bar{x}^{+})^{\top} P_s (\bar{x}^{+}) - \bar{x}^{\top} P_s \bar{x} < -\bar{x}^{\top} C_y^{\top} Q C_y \bar{x} - \bar{x}^{\top} K_u^{\top} R_u K_u \bar{x} + \gamma \end{array} \quad (39)$$

The problem inequalities will be translated to LMI form in order to address the optimization problem. Firstly, the upper inequality is considered

$$\bar{x}_0^{\top} P_s \bar{x}_0 + N\gamma < \alpha_s \quad (40)$$

This can be easily expressed as a LMI using the *Schur* complement in the form

$$\begin{bmatrix} \alpha_s - N\gamma & \bar{x}(0)^{\top} \\ \bar{x}(0) & W \end{bmatrix} \geq 0 \quad (41)$$

On the other hand, and remembering the presence of the saturation term in (33), the another inequality

$$(\bar{x}^{+})^{\top} P_s (\bar{x}^{+}) - \bar{x}^{\top} P_s \bar{x} < -\bar{x}^{\top} C_y^{\top} Q C_y \bar{x} - \bar{x}^{\top} K_u^{\top} R_u K_u \bar{x} + \gamma \quad (42)$$

must be satisfied for two extreme vertices of the LDI, $\frac{K_s \bar{x} + d_u}{L_s}$ and $H_s \bar{x} + h$, in the same way that for the invariant ellipsoid. So, using the Property 1 on the previous inequality and after several manipulations the two following inequalities are obtained:

$$\begin{bmatrix} \gamma & * & * & * & * \\ 0 & W & * & * & * \\ A_{xe} x_e + B_{re} r_e + E_e & A_w W + B_y Y & W & * & * \\ 0 & Q^{1/2} C_y W & 0 & I & * \\ 0 & R_W W + R_Y Y & 0 & 0 & I \end{bmatrix} > 0 \quad (43)$$

$$\begin{bmatrix} \gamma & * & * & * & * \\ 0 & W & * & * & * \\ A_{nlxe} x_e + B_{nlre} r_e + B_h h + E_{nle} & A_{nlW} W + B_{nlY} Y + B_v V & W & * & * \\ 0 & Q^{1/2} C_y W & 0 & I & * \\ 0 & R_v V & 0 & 0 & I \end{bmatrix} > 0 \quad (44)$$

where $A_{x_e} = A - I + (\frac{1}{L_s}) B_u C_u$, $B_{r_e} = B_r + (\frac{1}{L_s}) B_u D_u$, $E_e = E - (\frac{1}{L_s}) B_u u_c$, $A_w = A + \frac{1}{L_s} B_u C_u$, $B_y = B_r + \frac{1}{L_s} B_u D_u$, $Y = K_s W$ and $R_w W + R_y Y = R_u^{1/2} C_u W + R_u^{1/2} D_u K_s W$.

Finally, the equality (34), which was supposed before, must be included in the optimization problem. Therefore, the optimization problem has been reformulated to minimize the value of α_s subject to a set of LMI. The following section describes the final optimization problem and the different obtained LMI.

D. Final Optimization Problem

Notice that in section III, it was considered that $A_p = A_p(\tilde{\phi})$, $B_p = B_p(\tilde{\phi})$, and $E_p = E_p(\tilde{\phi})$ for simplification reasons. That is, it is necessary to remind that the matrices of the plant depend multilineally on the parametric vector $\tilde{\phi}$. In this way, the previous LMI that were formulated for the nominal case, must be satisfied for all extreme values of the hyperrectangle Φ. Hence, the final problem can be formulated to calculate the decision variables x_e, r_e, K_s, P_s, ρ, H_s, and h, in such a way that using the control law $r = r_e + K_s(x - x_e)$, $\wp(P_s, \rho)$ is an invariant ellipsoid and the system fulfills the performance criteria given by J (35). The final optimization problem is given by

$$\begin{array}{ll} \min\limits_{P_s, K_s, x_e, r_e, \gamma, \alpha_s} & \alpha_s \\ s.a. & \bar{x}_0^{\top} P_s \bar{x}_0 + N\gamma < \alpha_s \\ & (\bar{x}^{+})^{\top} P_s (\bar{x}^{+}) - \bar{x}^{\top} P_s \bar{x} < -\bar{x}^{\top} C_y^{\top} Q C_y \bar{x} - \bar{x}^{\top} K_u^{\top} R_u K_u \bar{x} + \gamma \end{array} \quad (45)$$

Then, considering the results obtained in previous section, a conservative way to solve the optimization problem consists in solving the following constraints

$$w = C_y x_e \quad (46)$$

$$\begin{bmatrix} \alpha_s - N\gamma & \bar{x}(0)^{\top} \\ \bar{x}(0) & W \end{bmatrix} > 0 \quad (47)$$

$$\begin{bmatrix} \rho & \bar{x}(0)^\top \\ \bar{x}(0) & W \end{bmatrix} > 0 \quad (48)$$

$$\begin{bmatrix} \gamma & * & * & * & * \\ 0 & W & * & * & * \\ A_{xe}(\tilde{\phi})x_e + B_{re}(\tilde{\phi})r_e + E_e(\tilde{\phi}) & A_w(\tilde{\phi})W + B_y(\tilde{\phi})Y & W & * & * \\ 0 & Q^{1/2}C_y(\tilde{\phi})W & 0 & I & * \\ 0 & R_W(\tilde{\phi})W + R_Y Y & 0 & 0 & I \end{bmatrix} > 0 \quad (49)$$

$$\begin{bmatrix} \rho(1-\lambda_1) & * & * \\ 0 & \lambda_1 W & * \\ A_{xe}(\tilde{\phi})x_e + B_{re}(\tilde{\phi})r_e + E_e(\tilde{\phi}) & A_w(\tilde{\phi})W + B_Y Y & W \end{bmatrix} > 0 \quad (50)$$

$$\begin{bmatrix} \gamma & * & * & * & * \\ 0 & W & * & * & * \\ \begin{array}{c} A_{nlxe}(\tilde{\phi})x_e + B_{nlre}(\tilde{\phi})r_e + \\ + B_h(\tilde{\phi})h + E_{nle}(\tilde{\phi}) \end{array} & \begin{array}{c} A_{nlW}(\tilde{\phi})W + B_{nlY}Y + \\ + B_v(\tilde{\phi})V \end{array} & W & * & * \\ 0 & Q^{1/2}C_y(\tilde{\phi})W & 0 & I & * \\ 0 & R_v V & 0 & 0 & I \end{bmatrix} > 0 \quad (51)$$

$$\begin{bmatrix} \rho(1-\lambda_1) & * & * \\ 0 & \lambda_1 W & * \\ \begin{array}{c} A_{nlxe}(\tilde{\phi})x_e + B_{nlre}(\tilde{\phi})r_e + \\ + B_h(\tilde{\phi})h + E_{nle}(\tilde{\phi}) \end{array} & \begin{array}{c} A_{nlW}(\tilde{\phi})W + B_{nlY}Y + \\ + B_v(\tilde{\phi})V \end{array} & W \end{bmatrix} > 0 \quad (52)$$

$$\begin{bmatrix} 1-h-\lambda_2\rho & V \\ V^\top & 4\lambda_2 W \end{bmatrix} \geq 0 \quad (53)$$

$$\begin{bmatrix} 1+h-\lambda_2\rho & V \\ V^\top & 4\lambda_2 W \end{bmatrix} \geq 0 \quad (54)$$

where it is necessary to incorporate constrains for each extreme value of the hypercube Φ. Also, as observed from the resulting constraints, some of them are BMI (Bilineal Matrix Inequalities) containing different bilineal terms $\rho(1-\lambda_1)$, $\lambda_1 W$, $\lambda_2\rho$), and $4\lambda_2 W$. So, in order to obtain an stable MPC controller with good performance, it is necessary to choose λ_1 and λ_2 in a convenient way.

Property 5: Suppose that there exist $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$ such that the constraints (46), (47), (48), (49), (50), (51), (52), (53), and (54) are feasible for every extreme of the hypercube Φ. Then, there exits a control law $r = r_e + K_s(x - x_e)$ providing that $\wp(P_s, \rho)$ is an invariant ellipsoid and the system fulfills the performance criteria given by J (35), where $P = W^{-1}$ and $K = W^{-1}Y$.

A typical *Branch & Bound* algorithm has been used in order to find the optimal solution [14].

V. NUMERICAL EXAMPLES

An integrator example is presented in order to test the proposed optimization problem. The plant was defined by

$$P(s) = \frac{K_p}{s}, \quad K_p \in [1, 10]$$

and the prefilter and controller designed from QFT by

$$C(s) = \frac{0.2267s + 13.84}{0.0002331s^2 + 0.05145s + 1} \quad F(s) = \frac{1}{0.1761s + 1}$$

Considering the sample time $T_m = 0.01$ s, $N = 20$, $R_u = 1$, $Q = 1$, $w = 1$, and $x_0 = [0\ 0\ 0\ 0]^\top$, the Branch and Bound algorithm found an optimal solution for $\lambda_1 = 0.993275$ and $\lambda_2 = 0.005826$ obtaining

$$K_s = \begin{bmatrix} 22.89834 & -0.32026 & -1.64460 & -1.3475 \end{bmatrix}$$

Figure 2 shows the results of applying the obtained solution to the example considering all plants of the family. It can be seen how the system reaches the proposed reference $w = 1$ obtaining good performance.

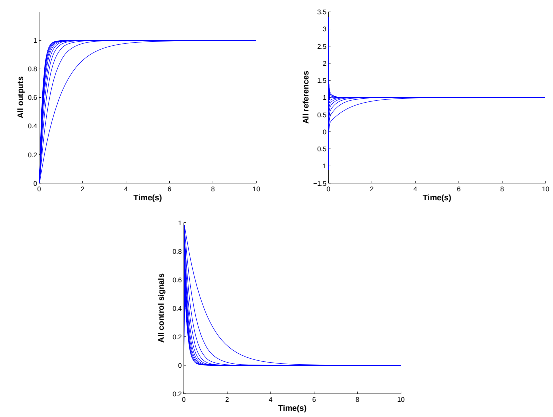

Fig. 2. Integrator example using LMI-based approach with $x_0 = [0\ 0\ 0\ 0]^\top$.

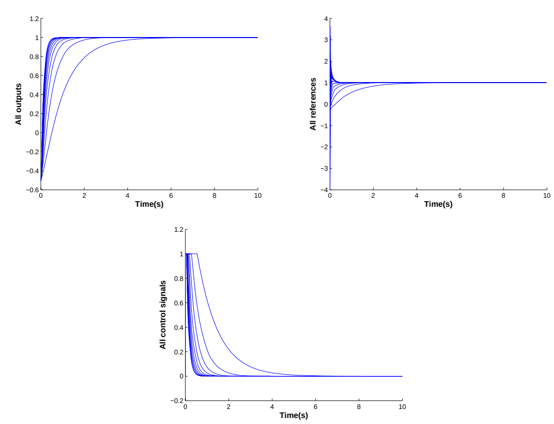

Fig. 3. Integrator example using LMI-based approach with $x_0 = [-0.5\ 0\ 0\ 0]^\top$.

The initial condition of the system is changed in order to lead the control into saturation. Then, the same design parameters are used considering $x_0 = [-0.5\ 0\ 0\ 0]^\top$. In this case, the obtained solution is given by $\lambda_1 = 0.9905853$ and $\lambda_2 = 0.0015258$ being

$$K_s = \begin{bmatrix} 23.41569 & -1.38874 & -1.23738 & -1.44615 \end{bmatrix}$$

Figure 3 shows the results where it can be observed how the system goes into saturation.

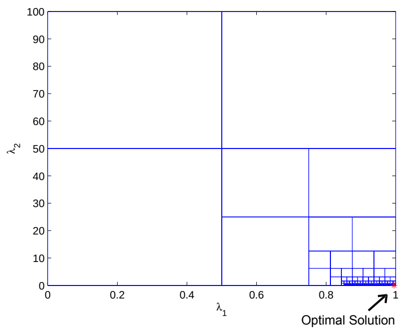

Fig. 4. Search space division by the Branch and Bound algorithm.

On the other hand, the Branch and Bound algorithm has presented a good behavior in finding optimal values for λ_1 and λ_2. This fact can be observed from Figure 4 where it is shown how, for the previous example, the algorithm divides correctly the search space in order to find optimal values.

VI. CONCLUSIONS

A mixed GPC-QFT approach has been presented where the aim is to design a predictive control algorithm augmenting the robustness in presence of uncertainties. A LMI-based approach has been proposed in order to obtain a state-vector feedback in such a way that the input reference to the inner loop is calculated in order to satisfy robust tracking problems considering input saturation. The proposed solution consists in solving a set of constraints being formed by several LMI and BMI, where a Branch and Bound algorithm has been developed in order to solve the bilinear terms. Notice that the algorithm is implemented for tracking problems where the aim is to regulate to a fixed reference value and not to the origin, and also input constraints are presented in the inner loop of the system.

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