Using genetic algorithms to improve the thermodynamic efficiency of gas turbines designed by traditional methods

Jose M. Chaquet[a,*], Enrique J. Carmona[b], Roque Corral[a]

[a] Technology and Methods Dep., Industria de TurboPropulsores S.A., Avda. de Castilla 2, 28830, San Fernando de Henares, Madrid, Spain

[b] Dpto. de Inteligencia Artificial, Escuela Técnica Superior de Ingeniería Informática, Universidad Nacional de Educación a Distancia, Madrid, Spain

Abstract

A method for optimizing the thermodynamic efficiency of aeronautical gas turbines designed by classical methods is presented. This method is based in the transformation of the original constrained optimization problem into a new constrained free optimization problem which is solved by a genetic algorithm. Basically, a set of geometric, aerodynamic and acoustic noise constraints must be fulfilled during the optimization process. As a case study, the thermodynamic efficiency of an already optimized by traditional methods real aeronautical low pressure turbine design of 13 rows has been successfully improved, increasing the turbine efficiency by 0.047% and reducing the total number of airfoils by 1.61%. In addition, experimental evidence of a strong correlation between the total number of airfoils and the turbine efficiency has been observed. This result would allow us to use the total number of airfoils as a cheap substitute of the turbine efficiency for a coarse optimization at the first design steps.

Keywords: Gas turbine, Thermodynamic efficiency, Genetic algorithm, Throughflow, Number of airfoils

1. Introduction

The *Low Pressure Turbine* (LPT) module has a major contribution in gas turbine engines with one third to the total weight and up to 20% to the total cost [12]. The LPT design process is a very challenging task. A lot of different constraints must be taken into account and usually the final decision on the particular optimum configuration needs a trade-off among different requirements.

The extraction of work from the fluid is done by means of several aerodynamic surfaces called *airfoils* that are placed in an annular way to form *rows*. A turbine *stage* consists of two consecutive rows, called *stator* and *rotor*. Stator airfoils are called *vanes*, whereas rotor airfoils are called *blades*. The stator is attached to the casing and directs the flow towards the rotor, whereas the rotor transmits the power to the turbine *shaft*. The number of airfoils in a row is called *NumberOff*.

In the literature related to this area, few works have been focused on the importance of the *NumberOffs*. For instance, work [21] investigates the influence of blade height and blade number on

*Corresponding author. Tel.: +34 91 207 9147

Email addresses: jose.chaquet@itp.es (Jose M. Chaquet), ecarmona@dia.uned.es (Enrique J. Carmona), roque.corral@itp.es (Roque Corral)

the performance of low head axial flow turbines for micro-hydro applications. The study concluded that the influence of blade number is higher than the blade height and that the choice of blade number should be carefully made. Other work where the number of airfoils is optimized can be seen in [13]. In that contribution the objectives were to simultaneously minimize the total pressure loss, maximize the total aerodynamic loading and minimize the number of airfoils for a turbomachinery cascade. Several constraints were taken into account, as fixed mass flow rate, fixed axial chord, fixed inlet and exit flow angles, etc.

Regarding the optimization process, the usual approach modifies the airfoil shapes, but without modifying the *NumberOffs*. For instance in [8] we can see the aerodynamic optimization of highly loaded turbine cascade blades for heavy duty gas turbine applications. The main target was the reduction of the total pressure losses, which is equivalent to increase the thermodynamic efficiency. Other works have been reported where airfoil shape optimization is performed not only for gas turbine engine airfoils [13, 17] but as well for Micro-Air-Vehicle airfoils [6], propellers [7], wind turbine blades [11] and steam turbine airfoils [19].

In order to obtain performance variables such as thermodynamic efficiency, pressure losses, power, etc, the Navier-Stokes equations must be solved in the fluid domains. This is done with Computational Fluid Dynamics (CFD) solvers. One specific type of CFD solver for turbomachinery applications is the so called *throughflow* code [20, 22], which computes the flow variables along all the rows. Navier-Stokes equations are circumferentially averaged solving the 2D flow field over a meridional plane. Using *throughflow* models in the optimization process is appealing because they can produce a fast design avoiding expensive full 3D CFD simulations [9, 23, 18]. *Throughflow* models have been used not only for airfoil optimization, but as well for other geometry optimization as we can see in [14], where entropy minimization (or efficiency maximization) optimizes hub and shroud geometries and inlet and exit flow-field for each blade row of a two-stage axial flow gas turbine. Although *throughflow* codes could give accurate solutions, in some cases *throughflow* and 3D CFD solutions must be linked [5].

Several techniques have been used for Aerodynamic optimization, such as gradient-based schemes. These methods require knowledge about the aerodynamic derivatives for each parameter, which are normally expensive to compute. Furthermore, gradient-based methods cannot be applied to problems where there are discontinuities in the design space because the derivatives in these regions are not defined [15]. In such cases Genetic Algorithms (GAs) have demonstrated good performance. Multidisciplinary airfoil optimization (aerodynamic and acoustic) can be seen in [16] where a parallel GA was used to generate a family of aerodynamically efficient, low-noise rotor blade designs which represent the Pareto optimal set. The multiple objectives of that work were to maximize lift-to-drag of a rotor airfoil shape and to minimize an overall noise measure, including the effects of loading and thickness noise of the airfoil. More recently [6], a GA for optimizing the shape of low Reynolds number airfoils for generating maximum lift for Micro-Air-Vehicle (MAV) applications has been developed where the GA computational efficiency has been significantly enhanced with an artificial neural network (ANN). The authors showed that the combined GA/ANN optimization technique is capable of finding globally optimal airfoils accurately and efficiently. Another example of coupling a GA with an ANN can be seen in [11], where wind-turbine blades are optimized for generating maximum lift to drag ratio.

Regarding thermal system engineering, several recent works have shown that GAs can be successfully employed in the optimization of real power generation systems based on gas turbines. In this way, [4] presents the simulation and multiobjective optimization of a gas turbine power plant with preheater, [1] reports the optimization of a combined heat and power plant for cogeneration purposes, [2] makes a thermodynamic and exergoeconomic modeling and optimization of a gas turbine plant, and [3] gives an exergy, exergoeconomic and environmental analysis and optimization of several combined cycle power plants. Among other important parameters, the gas turbine isentropic efficiency is investigated as design parameter. It can be appreciated that improving the design of several key components (as the compressor, turbine or combustion chamber) important savings can be obtained in the complete cycle, not only from the thermodynamic point of view, but as well from other dimensions such as the environmental impact or the monetary costs. These complex systems usually require a trade-off between different requirements, so multiobjective optimization must be employed. Evolutionary algorithms such as GAs have proved to be very efficient for solving this type of problems.

This paper presents a method for optimizing the thermodynamic efficiency of a turbine while a set of geometric, acoustic and aerodynamic restrictions are fulfilled. The optimization problem at hand involves seeking a solution which maximizes the turbine efficiency fulfilling at the same time several constraints. In this context, the function to optimize is not guaranteed to be continuous. Therefore, it is not recommendable to use gradient-based methods. Moreover, we are interested in finding global optimum in a problem which naturally is multimodal due to the high dimensionality and complex relationship among the control variables. The large size of the search space makes direct methods such as exhaustive or random search impracticable [10]. Therefore the use of Evolutionary Computation (EC) paradigms is more suitable to that type of problems: a smooth continuous and derivable optimization fitness function is not required, and the use of a population of candidate solutions facilitates the global optimum finding. The ability of evolutionary algorithms to maintain a population of potential solutions not only provides a means of escaping from one local optimum; it also provides a means to manage large and discontinuous search spaces. As it will be shown in the next section, our initial constraint satisfaction problem can be reduced to optimize a real valued function of integer variables. This fact makes the use of a Genetic Algorithm (GA) more appropriated than other EC paradigms such as Evolutionary Strategies, more suitable for functions of real variables.

Unlike other approaches in the literature, the main control parameter used in this work for optimizing the efficiency is the number of airfoils for each row. This approach is applied to a turbine that has already been designed using a classical methodology. Fluid variables are computed using a *throughflow* solver. The approach adopted to solve the optimization problem uses a Genetic Algorithm (GA). The particular optimization performed considers the fluid variables as constant, so *throughflow* models are not updated until the GA run is finished. This approach saves computational time and makes the algorithm more robust. Since the GA geometry modification affects the flow field, there must be an iterative process between the *throughflow* and the GA. Convergence is achieved in only 2 or 3 global iterations in the given case studies.

In summary, the main contributions of the present work are the following:

- Improving the thermodynamic efficiency of an already designed by traditional methods aeronautical LPT using an evolutionary algorithm.

- Demonstrating that, for a given number of airfoils, it is possible to fulfill all the geometric constraints with a correct election of other design parameters such as chords and gaps between rows.

- Showing that a GA can successfully deal this type of constrained optimization problems.

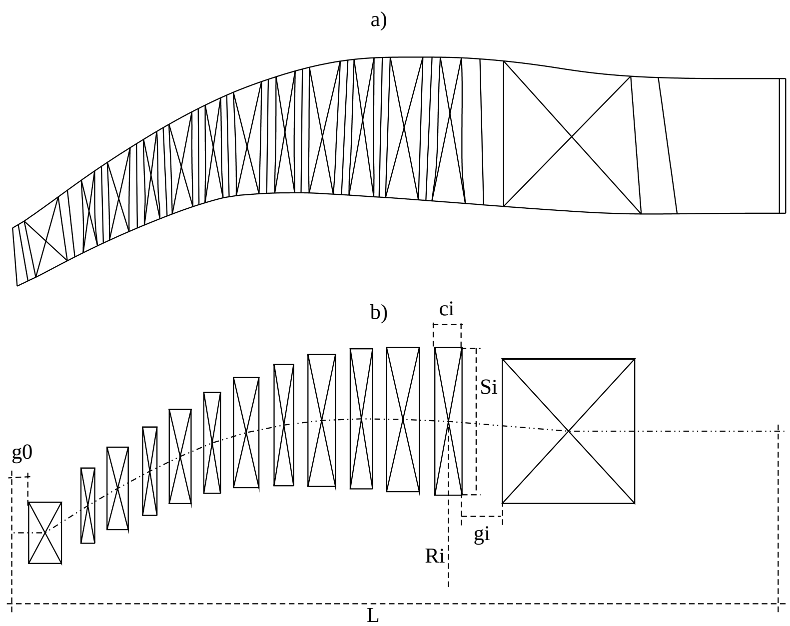

Figure 1: Real geometry (a) and simplified one (b)

- Showing the importance of reducing the total number of airfoils to improve the turbine efficiency, giving some possible recommendations for speed-up in the first steps of the design process.

In this article, first a description of the problem to solve is presented (section 2). Then the GA approach (section 3) and how the algorithm interacts with other design tools (section 4) are described. The results obtained in the optimization of a real 13 row aeronautical gas turbine are presented for two different sets of constraints (section 5). Finally, some conclusions and future works are given (section 6).

2. Problem description

The problem consists of optimizing the thermodynamic efficiency of a LPT for a given flow-path (Fig. 1a) and aerodynamic exit angles. The optimization process has to fulfill a set of aerodynamic, acoustic and geometric restrictions that may be reduced to a set of explicit analytical expressions. As a consequence, the restrictions are extremely fast to evaluate.

In order to parametrize the problem, simplified geometry will be used to approximate each row to a rectangle (Fig. 1b). For a turbine of M number of rows, each row is defined with only 5 parameters: NumberOff (N_i), gap (g_i), chord (c_i), span (S_i) and mean radius (R_i) where i goes from 1 to M. The chord c_i for a given row i is defined as the axial length between the leading and the trailing edges, and the span S_i is the length of the row in radial direction. The mean radius is the distance between the middle point of the row to the turbine axis. There must also be one global variable L, which is the total axial length of the turbine. The turbine inner and outer annuli are supposed to be optimized in an outer loop and in this work are kept constant. Therefore the mean radius and the span of all the rows are constant. NumberOffs, gaps and chords will be modified in order to find optimum feasible configurations. The gap for row i is the distance between the trailing edge of row i and the leading edge of the next row $i+1$, or the exit station for the last row. The initial gap g_0, is defined as the distance between the inlet station and the leading edge of the first row (Fig. 1b) and can be computed using the following expression

$$g_0 = L - \sum_{i=1}^{M} (c_i + g_i) \,. \tag{1}$$

2.1. Constraint description

The geometric constraints are defined with the following parameters for each row: maximum aspect ratio (MA_i), minimum pitch to chord ratio (mPC_i), maximum pitch to chord ratio (MPC_i), minimum gap (mG_i), minimum gap to chord ratio (mGC_i), maximum gap to chord ratio (MGC_i), maximum NumberOff (MN_i) and the NumberOff for each package (P_i). The maximum aspect ratio should be limited by structural and flutter considerations. The pitch to chord ratio is limited in order to maintain Zweiffel coefficients bounded. Gaps are bounded in order to prevent mechanical interferences and by noise restrictions. The package parameter forces the NumberOff to be a multiple of P_i. For the inlet gap g_0, two constraints are given for bounding it between a minimum and a maximum value: mG_0 and MG_0.

It is well known that one way of reducing the generation of noise associated with pure tones is to force the NumberOffs ratio for two consecutive rows to lie within some specific intervals [12]. When the NumberOff ratio fulfills this conditions, the acoustic wave amplitudes decrease with the axial distance, the stage is said to be *cut-off* and the perturbations do not propagate outside the turbine. The cut-off condition also depends on the flow variables, but in our problem these are assumed to remain constant. Noise constraints are given by four parameters: α_i, β_i, γ_i and δ_i. These parameters define two intervals $[\alpha_i, \beta_i]$ and $[\gamma_i, \delta_i]$ where the NumberOff ratio of row i and row $i+1$ must be located. Always $0 \le \alpha_i \le \beta_i \le 1 \le \gamma_i \le \delta_i$. When both ranges are used, the configuration is called *Mixed* cut-off. For the *Direct* cut-off mode, the $[\alpha_i, \beta_i]$ interval is chosen for even rows and the $[\gamma_i, \delta_i]$ for odd rows, therefore there will be more vanes than blades. The opposite is chosen for the *Reverse* cut-off mode.

Putting everything together, the mathematical problem consists of finding the M positive integer numbers N_i and the M positive real numbers c_i and g_i which fulfill the following constraints for $i \in [1, M]$

$$\frac{S_i}{c_i} \le MA_i \,, \tag{2}$$

$$mPC_i \le \frac{2 \cdot \pi \cdot R_i}{N_i \cdot c_i} \le MPC_i \,, \tag{3}$$

$$mG_i \le g_i \,, \tag{4}$$

$$mGC_i \le \frac{g_i}{c_i} \le MGC_i \,, \tag{5}$$

$$N_i \le MN_i \,, \tag{6}$$

$$N_i \% P_i = 0 \,, \tag{7}$$

$$mG_0 \le g_0 \le MG_0 \,, \tag{8}$$

$$if(i \neq M \,\&\, Mixed)\ \frac{N_i}{N_{i+1}} \in [\alpha_i, \beta_i] \cup [\gamma_i, \delta_i]\ , \tag{9}$$

$$if \left\{ \begin{array}{c} i \neq M \,\&\, (Direct \,\&\, i\%2 = 0) \\ or \\ (Reverse \,\&\, i\%2 = 1) \end{array} \right\} \frac{N_i}{N_{i+1}} \in [\alpha_i, \beta_i]\ , \tag{10}$$

$$if \left\{ \begin{array}{c} i \neq M \,\&\, (Direct \,\&\, i\%2 = 1) \\ or \\ (Reverse \,\&\, i\%2 = 0) \end{array} \right\} \frac{N_i}{N_{i+1}} \in [\gamma_i, \delta_i]\ . \tag{11}$$

In equation (7) the symbol % means the remainder of the integer division. In equation (8) the first gap g_0 is obtained using the expression 1.

2.2. Degrees of Freedom Reduction

Taking into account the three parameters for each row (N_i, c_i and g_i), there are $3M$ Degrees of Freedom (DoF). In [10] it was shown that the problem may be reduced to that of finding the M DoF associated with the number of airfoils for each individual row. This reduction is very desirable because the computational cost is notably reduced. Nevertheless the DoF reduction must to fulfill the condition that any optimal solution is lost.

This section describes the outline of the DoF reduction. A detailed description can be obtained from the paper by the same authors [10] where an easier problem was solved using the same set of constraints and optimizing the total number of airfoils instead of the thermodynamic efficiency. The reduction consists of choosing the minimum feasible gap and chord for a given row. Smaller gaps and chords give more room to other rows. On the one hand, the effect of the gap on the global turbine efficiency is negligible if the minimum gap constraint, equation (4), is fulfilled. On the other hand, a minimum chord for the same NumberOff gives as well more room to other rows.

For a given set of NumberOffs N_i, the optimum chords and gaps are chosen with the following expressions:

$$c_i \equiv c_i(N_i) = \max\left(c_{min,i}, \frac{2 \cdot \pi \cdot R_i}{N_i \cdot MPC_i}\right)\ , \tag{12}$$

$$g_i \equiv g_i(c_i) = \max\left(mG_i, mGC_i \cdot c_i\right)\ . \tag{13}$$

where $c_{min,i}$ is the minimum feasible chord:

$$c_{min,i} = \max\left(\frac{S_i}{MA_i}, \frac{mG_i}{MGC_i}, \frac{2 \cdot \pi \cdot R_i}{MN_i \cdot MPC_i}\right)\ . \tag{14}$$

The range of N_i to be explored is given by the following expressions, where $floor_{P_i}()$ function is the largest integer value not greater than the argument and multiple of P_i, and function $ceil_{P_i}()$ is the smallest integer value not less than the argument and multiple of P_i:

$$N_{max,i} = floor_{P_i}\left(\min\left(MN_i, \frac{2 \cdot \pi \cdot R_i}{mPC_i \cdot c_{min,i}}\right)\right)\ , \tag{15}$$

$$N_{min,i} = ceil_{P_i}\left(\min\left(N_{max,i}, \frac{2 \cdot \pi \cdot R_i}{MPC_i \cdot c_{min,i}}\right)\right)\ . \tag{16}$$

3. Genetic Algorithm

Once it has been demonstrated that each design configuration is determined by a set of NumberOffs, an exhaustive search could be performed computing the thermodynamic efficiency and checking the constraints for all the possible solutions. Using expressions (15) and (16), the number of configurations to be explored will be

$$\prod_{i=1}^{M} \left(N_{max,i} - N_{min,i}\right)/P_i \ . \tag{17}$$

As will be shown in section 5, huge numbers appear in real problems.

Owing to the multiple restrictions, it is difficult to define a continuous and derivable optimization function. Therefore methods based on the gradient of the optimization function are not recommended. On the other hand a Genetic Algorithm (GA) could be used because of the characteristics of the problem. First of all, the formalism of a GA easily transforms a *Constrained Optimization Problem* (COP) into a *Free Optimization Problem* (FOP). Secondly, the optimization function is not necessary to be continuous. Thirdly, the solution codification is easily done using a numeric vector.

3.1. Individual representation

The first step for defining a GA is to link the *real* world to the GA world. Objects forming possible solutions within the original problem context are referred to as *phenotypes*, while their encoding are called *genotypes*. In our problem, the phenotypes are vectors of natural numbers with the NumberOff for each row. Each NumberOff can only change in the range given by (15) and (16). The encoding of each genotype is a vector of natural numbers n_i for each NumberOff in a range given by

$$n_i \in \left[0, \frac{N_{max,i} - N_{min,i}}{P_i}\right] \ . \tag{18}$$

The number of natural numbers n_i will be M, one for each row. The way of decoding the genotype into the phenotype consists of obtaining the NumberOff N_i associated with each *gene* n_i:

$$N_i = N_{min,i} + n_i \cdot P_i \ . \tag{19}$$

Knowing each N_i, gaps and chords are obtained using the expressions (12) and (13). Equation (13) does not take into account the constraint (8). A repairing process may be necessary if, on obtaining the phenotype, g_0 does not meet that constraint. If $mG_0 \leq g_0 \leq MG_0$ the fixing is not necessary. On the other hand, if $g_0 \leq mG_0$ the solution cannot be repaired and the individual receives a high penalty in its fitness. If $g_0 > MG_0$ a repairing process is needed. The repairing process is done in the phenotypic space and this consists of distributing the amount $\triangle g = g_0 - MG_0$ among the rest of the gaps maintaining the constraints $g_i \leq MGC_i \cdot c_i$.

3.2. Fitness function

The role of the *fitness* function F is to represent the requirements to be optimized. In this work it is defined in such a way that our initial COP is transform into an FOP.

A penalty function F_C is defined to deal with the constraints. Negative values are used for unfeasible individuals and zero value for feasible ones. With the representation adopted, all the constraints are satisfied except (8), (9), (10) and (11). The penalty function F_C is computed as

$$F_C = \begin{cases} 1 - \exp\left(\lambda \frac{mG_0 - g_0}{L}\right) + \sum_{i=1}^{M} F_i & if\ g_0 < mG_0 \\ \sum_{i=1}^{M} F_i & if\ mG_0 \le g_0 \le MG_0 \\ 1 - \exp\left(\lambda \frac{g_0 - MG_0}{L}\right) + \sum_{i=1}^{M} F_i & if\ MG_0 < g_0 \end{cases}, \tag{20}$$

where g_0 is computed using expression (1). The value of constant parameter λ is used to modulate the exponential decreasing in the unfeasible regions. Its value is taken experimentally and does not have a pronounced effect on the performance of the algorithm. F_i deals with the noise restrictions depending on the cut-off mode. For instance, for Mixed cut-off mode:

$$F_i = \begin{cases} 1 - \exp\left[\lambda\left(\alpha_i - \frac{N_i}{N_{i+1}}\right)\right] & if \quad \frac{N_i}{N_{i+1}} < \alpha_i \\ 0 & if \quad \alpha_i \le \frac{N_i}{N_{i+1}} \le \beta_i \\ 1 - \exp\left[\lambda\left(\frac{N_i}{N_{i+1}} - \beta_i\right)\right] & if \quad \beta_i < \frac{N_i}{N_{i+1}} \le \frac{\beta_i+\gamma_i}{2} \\ 1 - \exp\left[\lambda\left(\gamma_i - \frac{N_i}{N_{i+1}}\right)\right] & if \quad \frac{\beta_i+\gamma_i}{2} < \frac{N_i}{N_{i+1}} < \gamma_i \\ 0 & if \quad \gamma_i \le \frac{N_i}{N_{i+1}} \le \delta_i \\ 1 - \exp\left[\lambda\left(\frac{N_i}{N_{i+1}} - \delta_i\right)\right] & if \quad \delta_i < \frac{N_i}{N_{i+1}} \end{cases}. \tag{21}$$

The fitness function solves the initial target problem: optimizing the thermodynamic turbine efficiency among the feasible individuals:

$$F_\eta = \begin{cases} F_C & if\ F_C < 0 \\ \eta & if\ F_C \ge 0 \end{cases}, \tag{22}$$

where η is the thermodynamic turbine efficiency whose value is calculated as it will be described in section 4.1.

3.3. Genetic Operators

The parent selection mechanism implemented here is the *tournament method*, i.e. k individuals with replacement are chosen randomly from the population and the final individual chosen will be the best of these k in terms of their fitness value.

Once the parents have been selected, there is a *recombination* probability p_r that determines whether the offspring of two parents are just a copy of the parents or a real recombination is produced. Single point crossover is used, i.e. a single crossover point on both parent chromosomes is selected. All genes beyond that point in either parent genotype are swapped between the two parents.

Another parameter that controls the algorithm is the *mutation* probability p_m. After performing the crossover of the parents, the offspring is mutated. The mutation is done in each gene adding or subtracting a random number with constant distribution between 0 and 10% of the total possibilities for that gene given by expression (18). Modular arithmetic is used in order to maintain the genes inside their boundaries.

A *generational* model is used, so for each generation all parents are replaced by their offspring. *Elitism* was implemented swapping the worst individual for the best individual of the previous generation after the mutation operator was applied.

The initialization was done by taking a random representation of possible solutions from the design space and carrying out fitness evaluations on all the individuals.

4. GA interaction with other design tools

Several tools are involved in the traditional design process of a gas turbine. The purpose of this work is not to present these tools. However this section will describe briefly two of them due to their interactions with the GA: *efficiency estimator* module and *throughflow* code.

4.1. Efficiency estimator

This module computes the thermodynamic efficiency of a turbine design and, therefore, it can calculate the fitness function of each individual using Eq. (22). Turbine isentropic efficiency is used, i. e. other engine components as fan, compressor or combustion chamber are not considered. Detailed geometry and the complete fluid solution in the meridional plane are needed for the efficiency evaluation. All this information is not contained in the individual representation managed by the GA. Therefore, extra data must be provided.

Since the mean radius and spans for each row are constant, it is assumed that the fluid solution does not change for all the possible individuals in the population. This hypothesis allows not to update the fluid solution with CFD simulations, which would need high computational effort. The extra data provided to the GA for computing efficiencies are associated with a specific individual called *reference individual*, whose defining parameters are $\widehat{N}_i$, $\widehat{g}_i$ and $\widehat{c}_i$ for each row i. The input data to the *Efficiency estimator* can be classified into two sets. One set consists of pure fluid variables such as velocities, temperatures, pressures, Mach Numbers, etc. The other set consists of variables associated with geometry such as axial and radial coordinates, pitch, lift coefficients, Reynolds Numbers, chords, etc. When the efficiency of an individual is needed, the internal data of the *efficiency estimator* of the first set of variables are maintained constant, but the second set is updated using the geometry increments with the reference individual. For instance, the pitch for a particular radial position j of row i is defined as

$$pitch_i^j = \frac{2\pi r_i^j}{N_i} \,. \tag{23}$$

Therefore, the new pitch for a given individual can be computed with the pitch of the reference individual as

$$pitch_i^j = \widehat{pitch}_i^j \cdot \frac{\widehat{N}_i}{N_i} \,. \tag{24}$$

Similar expressions such as (24) can be obtained for the other variables of the second set of the *efficiency estimator* module.

4.2. Throughflow

A ThroughFlow is a specific CFD code for turbomachinery design which computes the flow variables along all the rows over the meridional plane. Navier-Stokes equations are circumferentially averaged and the axisymmetric flow field is obtained. Several formulations have been developed [20]. The code used in this work corresponds to the classical formulation developed by Wu [22], which consists of solving the equations on steady, inviscid flow in a relative reference frame.

Fig. 2 shows the new proposed method for the optimization of a *throughflow* model. The *throughflow* uses the *efficiency estimator*, so it can compute the turbine efficiency using the fluid solution. The GA communicates with the *throughflow* by two interfaces called *export* and *import* which have been specifically designed to carry out our implementation. In the export process,

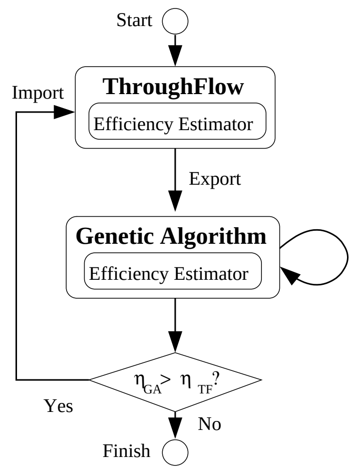

Figure 2: GA relations with *throughflow* and *efficiency estimator*.

Property	**Value**
Inlet Mass Flow	50.8 kg/s
Inlet Total Pressure	243 kPa
Inlet Total Temperature	943 K
Inlet Angle	18^o
Pressure ratio	5.78
Axial Length	1.225 m
Power	18.302 MW
Total NumberOff	1486

Table 1: Turbine main properties

all the data needed by the GA is generated, particularly all the fluid variables of the reference individual for computing the efficiency. After the run of the GA, the efficiency of best individual in the last generation η_{GA} is compared with the efficiency of the *throughflow* model η_{TF}. If $\eta_{GA} > \eta_{TF}$ the *throughflow* model is modified with the new geometry computed by the GA using the *import* interface. NumberOff, gaps and chords of the *throughflow* model are modified accordingly. After a run of the *throughflow* model the loop is repeated until $\eta_{GA} \leq \eta_{TF}$. If this condition is fulfilled, the GA cannot find a better individual than the reference one, so the iterative loop is finished.

5. Case Studies

The methodology described in the previous sections was applied to the optimization of an aeronautical LPT consisting of 6 stages and an *Outlet Guide Vane* (OGV), which gave a total of 13 rows (Fig. 1). The initial LPT was designed following a conventional methodology. In Table 1 the main properties of the turbine are shown.

Two sets of constraints were applied. The first one, called *Case A*, are not realistic but try to increase the number of possible feasible solutions in order to check the performance of the GA when the solution space is big. On the other hand, in *Case B* the restrictions imposed were the same as

Property	**Value**
Population	$5 \cdot 10^4$
Generations	50
Tournament parameter	5
Recombination probability	0.8
Mutation probability	0.01

Table 2: GA parameters

Constrain	**Value**
MA_i	$\widehat{S}_i/\widehat{c}_i$
mPC_i	$0.98 \cdot 2\pi\widehat{R}_i/\widehat{N}_i\widehat{c}_i$
MPC_i	$1.02 \cdot 2\pi\widehat{R}_i/\widehat{N}_i\widehat{c}_i$
mG_i	$0.95\widehat{g}_i$
mGC_i	0.4
MGC_i	$\max\left(0.75, \widehat{g}_i/\widehat{c}_i\right)$
MN_i	200
P_i	$\begin{cases} 1 & if \quad OGV \\ 2 & if \quad rotor \\ 5 & if \quad stator\&\widehat{N}_i\%5=0 \\ 6 & if \quad statot\&\widehat{N}_i\%6=0 \\ 7 & default \end{cases}$
$\left[\alpha_i, \beta_i\right], \left[\gamma_i, \delta_i\right]$	$\left[0.6, 0.8\right], \left[1.3, 1.6\right]$
mG_0	$0.8\widehat{g}_0$
MG_0	$1.2\widehat{g}_0$
Cut-off mode	Mixed

Table 3: Constraints for Case A

those used by the design team, so the case study must be considered as real. In both cases, the control parameters of the GA were chosen by trial and error. The best results were obtained for the parameters shown in table 2. All runs were performed using a 2.40 GHz Intel Core Duo machine with a 4 GB of RAM memory and a Linux openSUSE 10.3 operative system.

5.1. Case A

As already highlighted, the constraints are artificially chosen in order to increase the number of possible feasible solutions. Using the hat symbol ($\widehat{\ }$) to refer to the values in the initial *throughflow* model, which coincides with the first reference individual, the constraints are given in Table 3.

Multiplying the possibilities for each row given by expression (17), $4.6 \cdot 10^{18}$ possible configurations are obtained. If we use an exhaustive search and consider that each configuration was evaluated in 10^{-6} seconds, the computing time would be 146235 years. So an exhaustive search cannot be used in this case.

Owing the stochastic nature of the GA, several runs have been performed with different random generator seeds in order to study the dispersion of the results. The same optimum solution is obtained in all the runs, providing evidence about the robustness of the method. The average time needed for each run was 12 minutes, and the number of different feasible individuals in the last

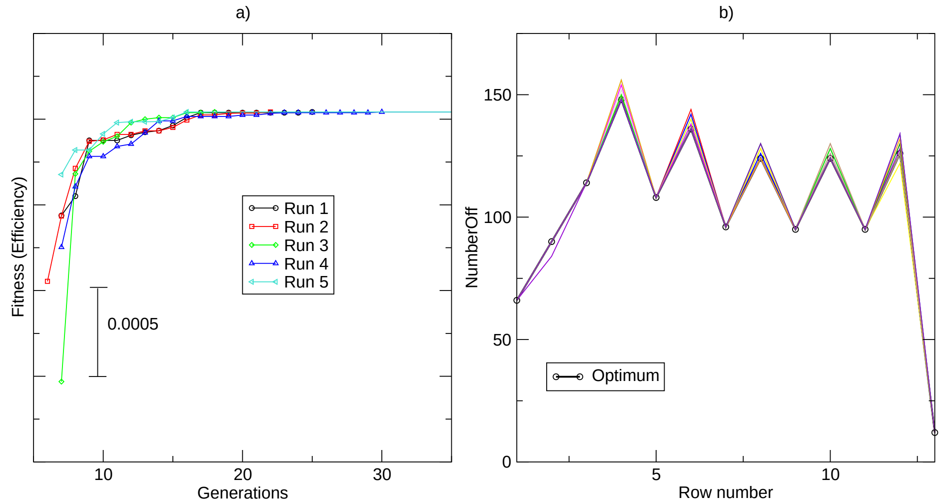

Figure 3: Case A results: best individual fitness versus generation number for 5 runs (a) and NumberOffs for each row for all the feasible individuals in the last generation in one run (b).

	Throughflow		**GA**	
Iteration	$\eta - \eta_0$	$\sum N_i$	$\eta - \eta_0$	$\sum N_i$
1	0	1486	0.0039236	1334
2	0.003253	1334	0.0034802	1329
3	0.003408	1329	0.003408	1329

Table 4: Iterative process between *throughflow* and GA for Case A. Efficiencies are expressed relative to the initial one η_0.

generation was around 30. Nevertheless, the optimal individual found in the last generation did not change. In Fig. 3a the best individual fitness versus generation number for 5 runs is plotted. The evolving process involves two phases: an initial stage ($F_\eta < 0$) for satisfying the constraints, and a second stage ($F_\eta \geq 0$) for maximizing the turbine efficienccy once the constraints are satisfied. Fitness is only plotted if there is at least one feasible individual in the population. Only 7 to 8 generations are needed to have at least one individual that meets all the restrictions. In Fig. 3b the number of airfoils for each row for all the feasible individuals in the last generation for one run is shown. All the configurations are very similar due to the typical decrease of diversity in the standard GA in last generations. We can observe that all the feasible configurations are pure Mixed cut-off ($N_1 < N_2 < N_3 < N_4$).

Following the iterative process shown in Fig. 2, only three iterations are needed to obtain the optimal configuration as shown in Table 4. For each iteration, the thermodynamic efficiency and the total number of airfoils are given for the *throughflow* model and for the best individual in the last generation of the GA. We can check that the total number of airfoils in *throughflow* models is the same as in the GA at the previous iteration. The iterative process is stopped when the GA does not change the input data from the *throughflow*. Comparing the first and last iteration we can see that the GA has increased efficiency a 0.36% and reduced the total number of airfoils by 10.56%.

In Fig. 4 the new geometry in meridional plane obtained by the GA is shown. Notice that the

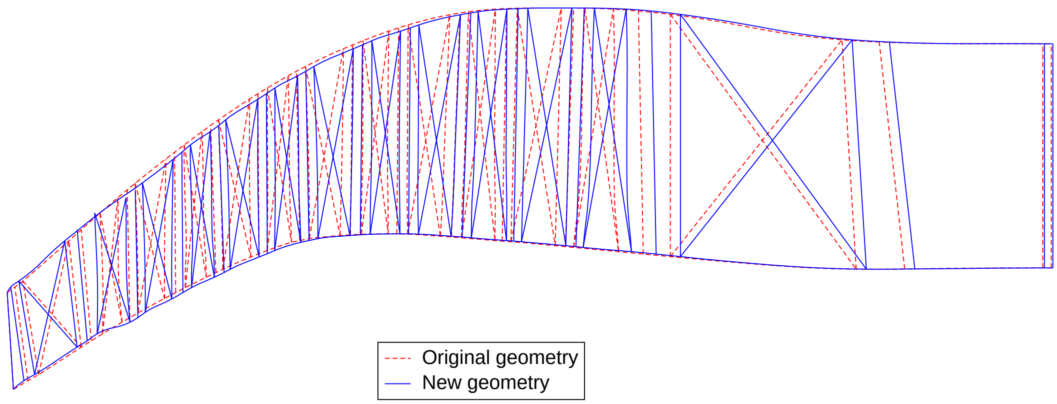

Figure 4: Optimized *throughflow* geometry (continuous line) compared with the original one (dashed line) for Case A

algorithm has changed the chord and the gap of some rows in order to fulfill the constraints.

5.2. Case B

The constraints imposed for Case B were the same as those used by the design team, so the case study must be considered as real (Table 5). Only the Reverse cut-off condition is considered feasible. Owing to the mechanical restrictions, the first and last row will not be modified.

Constrain	Value
MA_i	$\begin{cases} 2.128 & if \quad i=1 \\ 7.142 & if \quad i=2,3,..,12 \\ 1.244 & if \quad i=13 \end{cases}$
mPC_i	$0.98 \cdot 2\pi \widehat{R}_i / \widehat{N}_i \widehat{c}_i$
MPC_i	$1.01 \cdot 2\pi \widehat{R}_i / \widehat{N}_i \widehat{c}_i$
mG_i	$0.95 \widehat{g}_i$
MN_i	$\begin{cases} 200 & if \quad i \neq 13 \\ 20 & if \quad i=13 \end{cases}$
mG_0	$\widehat{g}_0$
MG_0	$1.0001 \widehat{g}_0$

Table 5: Aerodynamic and Geometric constraints for Case B

The number of blades in each package for rotors are $P_i = 2$, whereas $P_1 = 66$, $P_3 = P_5 = P_7 = 6$, $P_9 = P_{11} = 5$ and $P_{12} = 12$. The feasible noise intervals are given in Table 6. Any noise constraints are imposed for rows 1, 12 and 13.

As in Case A, exhaustive search is not feasible because the number of possible configurations to be explored ($2.3 \cdot 10^{18}$) is too high. The average time needed for each run was 10 minutes, and the number of different feasible individuals in the last generation was around 30. As in Case A, the optimal individual found in the last generation did not change for the 5 runs performed. In Fig. 5a the best individual fitness versus generation number for 5 runs is shown. The constraints

Row	α_i	β_i	γ_i	δ_i
2	0.685	0.86	1.37	1.73
3	0.6	0.79	1.21	1.58
4	0.7	0.85	1.4	1.69
5	0.61	0.77	1.23	1.54
6	0.695	0.83	1.39	1.66
7	0.62	0.76	1.24	1.52
8	0.69	0.82	1.38	1.63
9	0.62	0.75	1.25	1.5
10	0.675	0.8	1.35	1.61
11	0.63	0.75	1.25	1.5

Table 6: Noise Constraints for Case B

	Throughflow		**GA**	
Iteration	$\eta - \eta_0$	$\sum N_i$	$\eta - \eta_0$	$\sum N_i$
1	0	1486	0.000504	1462
2	0.000444	1462	0.000444	1462

Table 7: Iterative process between *throughflow* and GA for Case B

used make the problem harder and more than 10 generations are needed to find the first feasible individual versus the 7 to 8 generations in Case A.

Following the iterative process shown in Fig. 2, only two iterations are needed fto obtain the optimal configuration as shown in Table 7. Turbine efficiency increased by 0.047% and the total number of airfoils were reduced to 1.61%. In Fig. 6 the new geometry in the meridional plane obtained by the GA is shown. Notice that the algorithm has changed the chord and the gap of some rows in order to fulfill the constraints.

5.3. Efficiency-NumberOff correlation

A strong correlation was observed between the total number of airfoils $\sum N_i$ and turbine efficiency η. In order to study this correlation, the GA was run for both cases A and B using the penalty function F_C as the fitness function. In a post-processing step, the efficiency of all the feasible individuals in the last generation was computed.

In Fig. 7 all ($\sum N_i$, η) pairs are plotted. The original configuration and the optimum solution found for both cases are also shown. As can be observed, all the pairs are distributed along a straight line with negative slope. We can also observed that the solution space for Case A is bigger than for Case B. 66248 solutions were found for Case A, compared with 4059 for Case B. The difference between both cases can be seen too in the range of efficiency and total NumberOff where pairs are distributed.

5.4. Comparative study

It is not straightforward to make a comparative study between our results and other previous approaches because the number of airfoils normally is maintained constant. On the other hand, fitness function and number and type of constraints do not match. Furthermore, in this study an

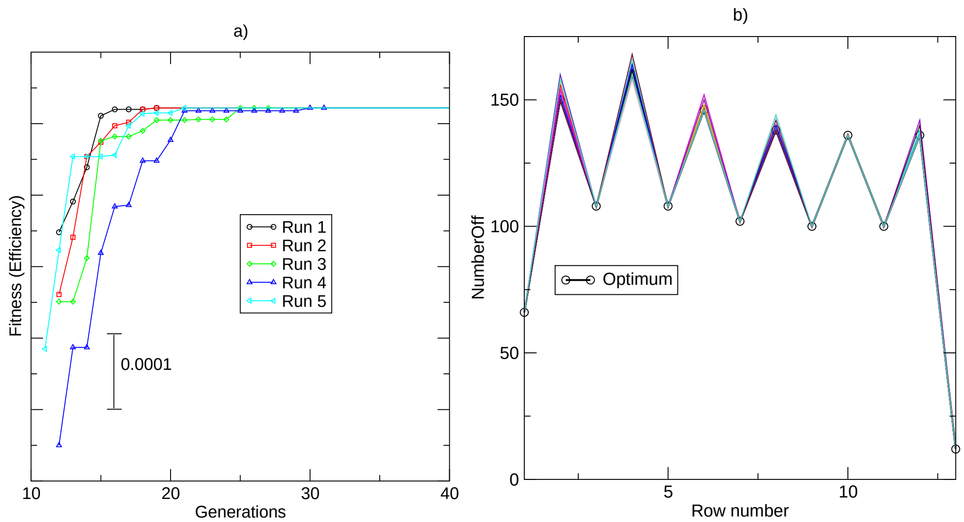

Figure 5: Case B results: best individual fitness versus generation number for 5 runs (a) and NumberOffs for each row for all the feasible individuals in the last generation in one run (b).

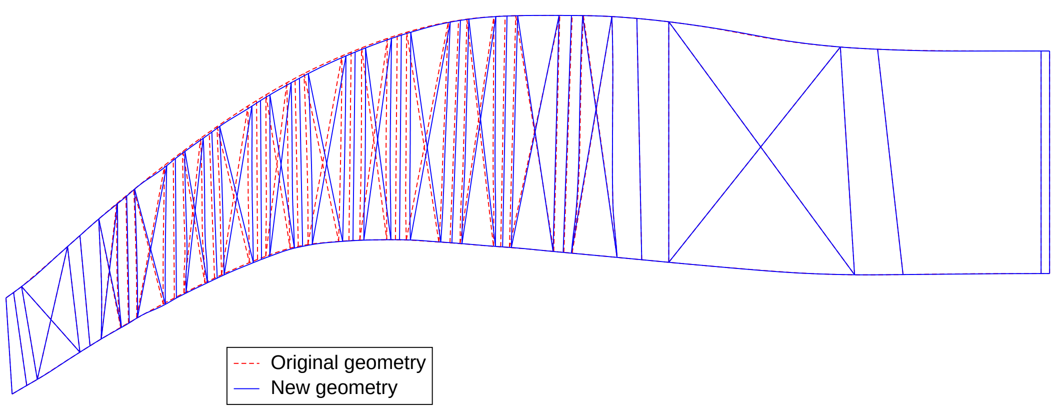

Figure 6: Optimized *throughflow* geometry (continuous line) compared with the original one (dashed line) for Case B

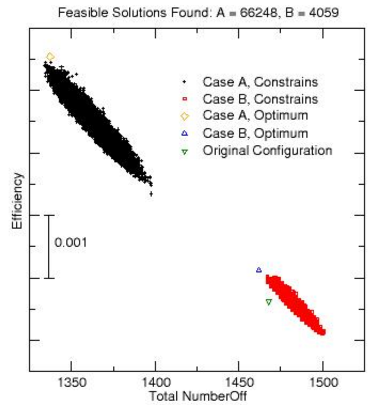

Figure 7: Efficiency-Total NumberOff pairs

already optimized design by traditional methods is used as initial point, meanwhile some other works have more room for improvements.

The work presented in [8] optimizes a unique airfoil shape by means of 15 parameters. In our study, 13 rows are optimized simultaneously with 39 parameters. The number of constraints are also very different, going from two relative complex constraints (smooth acceleration on the suction and pressure surface and an efficient cooling of the blade) in [8] to around one hundred simple constraints in the present contribution. A maximal reduction of the total pressure losses coefficient (which is directly related with the efficiency) by about 20% was achieved in [8], meanwhile in the present work the maximum increment in the global efficiency is around 1.61%. This discrepancy can be explained by the fact that in the present contribution the initial design corresponds to an already optimized turbine with traditional methods.

The work presented in [13] tries to reduce the number of airfoils in a turbomachinery cascade among other targets. A maximum reduction respect to the initial design of 2% of the number of blades was obtained. In the present work, the reduction in the total *NumberOff* is 10.56% for Case A and 1.61% for Case B.

In [17] an axial turbine rotor cascade shape optimization with unsteady passing wakes was performed to obtain improved aerodynamic performance. The objective function was defined either as minimization of total pressure loss or as maximization of lift, while the mass flow rate was fixed during the optimization. The design variables were geometric parameters characterizing airfoil leading edge, camber, stagger angle, and inter-row axial spacing. The optimization results indicated that only minor improvements were possible in the unsteady rotor/stator aerodynamics by varying these geometric parameters. These results are similar to what has been obtained for our realistic Case B.

6. Conclusions and Future Work

A GA has been applied to improve the thermodynamic efficiency optimization of an LPT gas turbine fulfilling a set of restrictions. The turbine model used as input to the GA corresponds to the final design of a turbine based on a standard design methodology. Two sets of restrictions, called Case A and Case B, were used to measure the algorithm performances. The first one is less realistic and checks the performance of the GA when the solution space is large. The second one is the same as the one used by the design team to obtain the turbine model used as input. In Case A, the algorithm increased efficiency by 0.36% and reduced the total number of airfoils by 10.56%. In Case B, these values dropped to 0.047% for efficiency and 1.61% for the total number of airfoils. The hypothesis of maintaining the fluid properties constant during the GA runs requires less computational effort and only two or three global loops with the *throughflow* code are needed to achieve the convergence. Experimental evidence of a strong correlation between turbine efficiency and the total NumberOff was observed.

As a future work new fitness functions could be implemented to perform the optimization of other parameters such as the total turbine weight. Multi-objective optimization could be performed using *Pareto Front* techniques. Other constraints could be considered as well. The DoF reduction would have to be modified to adjust to the new constraints. For instance, a new noise constraint could be considered imposing not only a cut-off mode with the appropriate N_i/N_{i+1} intervals, but also a minimum *cut-off decay*, i.e. the acoustic tones produced by the turbine decay exponentially with the distance at least with a specified rate. Fluid properties must be known for cut-off decay computations, so a similar technique to that implemented for efficiency could be used.

7. Acknowledgements

The authors wish to thank Industria de Turbopropulsores S. A., for allowing the publication of this paper and for its support during the project.

References

References

[1] P. Ahmadi and I. Dincer. Exergoenvironmental analysis and optimization of a cogeneration plant system using multimodal genetic algorithm (mga). *Energy*, 35:5161–5172, 2010.

[2] P. Ahmadi and I. Dincer. Thermodynamic and exergoenvironmental analyses, and multi-objective optimization of a gas turbine power plant. *Applied Thermal Engineering*, 31:2529–2540, 2011.

[3] P. Ahmadi, I. Dincer, and M. A. Rosen. Exergy, exergoeconomic and environmental analyses and evolutionary algorithm based multi-objective optimization of combined cycle power plants. *Energy*, 10:5886–5898, 2011.

[4] H. Barzegar Avval, P. Ahmadi, A. R. Ghaffarizadeh, and M. H. Saidi. Thermo-economic-environmental multiobjective optimization of a gas turbine power plant with preheater using evolutionary algorithm. *International Journal of Energy Research*, 35:389–403, 2011.

[5] K. Becker, M. Lawerenz, C. Voss, and R. Moenig. Multi-objective optimization in axial compressor design using a linked cfd-solver. In *Proceddings of ASME Turbo Expo 2008: Power for Land, Sea, and Air GT2008-51131*, pages 2533–2542, 2008.

[6] M. Bellman, J. Straccia, B. Morgan, K. Maschmeyer, and R. Agarwal. Improving genetic algorithm efficiency with an artificial neural network for optimization of low reynolds number airfoils. In *47th AIAA Aerospace Sciences Meeting Including The New Horizons Forum and Aerospace Exposition, 5-8 January, Orlando, Florida*, 2009.

[7] C. Burger, R. Hartfield, and J. Burkhalter. Performance and noise optimization of a propeller using the vortex lattice method and a genetic algorithm. In *48th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, 23-26 April, Honolulu, Hawaii*, 2007.

[8] P. Cardamone. Aerodynamic optimisation of highly loaded turbine cascade blades for heavy duty gas turbine applications. Master's thesis, Bundeswehr Munchen, Munich, 2006.

[9] M. Casey, F. Gersbach, and C. Robinson. An optimization technique for radial compressor impellers. In *Proceddings of ASME Turbo Expo 2008: Power for Land, Sea, and Air GT2008-50561*, pages 2401–2411, 2008.

[10] J. M. Chaquet, E. J. Carmona, and R. Corral. Optimizing the number of airfoils in turbine design using genetic algorithms. In *Proceddings of 23rd IEA/AIE, Cordoba, Spain*, 2010.

[11] X. Chen and R. Agarwal. Optimization of flatback airfoils for wind turbine blades using a genetic algorithm with an artificial neural network. In *48th AIAA Aerospace Sciences Meeting Including the New Horizons Forum and Aerospace Exposition, 4-7 January, Orlando, Florida*, 2010.

[12] P. de la Calzada. Aerothermodynamic design of low pressure turbines. In *Aeroengine design: from state of the art turbofans towards innovative architecture. Von Karman Institute Lecture Series LS-2008-04, Brussels, Belgium*, 2008.

[13] B. H. Dennis, G. S. Dulikravich, and Z.-X. Han. Constrained optimization of turbomachinery airfoil cascade shapes using a navier-stokes solver and a genetic/sqp algorithm. *AIAA Journal of Propulsion and Power*, 17(5):201–210, 2001.

[14] G. S. Dulikravich, T. J. Martin, B. H. Dennis, and N. F. Foster. Multidisciplinaryhybrid constrained ga optimization. In *Evolutionary Algorithms in Engineering and Computer Science: Recent Advances and Industrial Applications, EUROGEN'99, Finland, May 30 - June 3*, pages 231–260, 1999.

[15] P. Gage and I. Kroo. A role of genetic algorithms in a preliminary design envioroment. In *AIAA paper No. 93-3933. Aircraft Design, Systems and Operations Meeting, Monterey, CA, USA*, 1993.

[16] B. R. Jones, W. A. Crossley, and A. S. Lyrintzis. Aerodynamic and aeroacoustic optimization of airfoils via a parallel genetic algorithm. In *7th AIAI/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization*, 1998.

[17] E.-S. Lee, G. S. Dulikravich, and B. H. Dennis. Rotor cascade shape optimization with unsteady passing wakes using implicit dual time stepping and genetic algorithm. In *Proceedings of the 9th International Symposium on Transport Phenomena and Dynamics of Rotating Machinery (ISROMAC-9), Honolulu, HI, February 10-14*, 2002.

[18] M. V. Petrovic, G. S. Dulikravich, and T. J. Martin. Maximizing multistage turbine efficiency by optimizing hub and shroud shapes and inlet and exit conditions of each blade row. *International Journal of Turbo & Jet-Engines*, 17:267–278, 2000.

[19] X. Qin, L. Chen, F. Sun, and C. Wu. Optimization for a steam turbine stage efficiency using a genetic algorithm. *Applied Thermal Engineering*, 23:2307–2316, 2003.

[20] J.-F. Simon. Contribution to throughflow modelling for axial flow turbomachines. Master's thesis, University of Liege, Liege, Belgium, 2007.

[21] P. Singh and F. Nestmann. Experimental investigation of the influence of blade height and blade number on the performance of low head axial flow turbines. *Renewable Energy*, 36:272–281, 2011.

[22] C.-H. Wu. A general theory of three-dimensional flow in subsonic or supersonic turbomachines of axial, radial and mixed flow types. In *National Advisory Committee for Aeronautics Technical Note 2604*, 1952.

[23] H. Wu, Q. Li, and S. Zhou. Optimization of highly loaded fan rotor based on throughflow model. In *Proceddings of ASME Turbo Expo 2007: Power for Land, Sea, and Air GT2007-27603*, pages 1321–1331, 2007.