# Technical efficiency in education from value-added measures: An application to Spanish primary schools.

Esther López-Martín
Departamento MIDE II, Universidad Nacional de Educación a Distancia (UNED)

José Luis Gaviria
Departamento MIDE, Universidad Complutense de Madrid (UCM)

**Abstract:** This paper presents a model to assess technical efficiency that incorporates value-added measures in schools as an output of the educational system. By using value-added measures, estimated from multilevel models, accurate, reliable and objective indices of the students' progress over time can be obtained, isolating the effect of variables that influence the results achieved by schools, but outside the control of the education managers. The model proposed to measure efficiency is composed of two different steps: first the schools' value added under nonlinear growth models is estimated and, then, the technical efficiency is calculated after incorporating value-added measures as indicators of their productivity. The objective of applying this model is to assess the technical efficiency of primary schools in the Madrid Region (Spain). In turn, the efficiency scores obtained here are compared with those resulting from applying other models in which efficiency levels are determined by pass rates or students overall performance at a given time, in other words, those in which outputs traditionally considered in efficiency studies conducted in non-university educational settings are introduced. In spite of there being some agreement among the estimates obtained by the different models, the differences found are statistically significant.

**Keywords:** Value added, Linear hierarchical models, Linear growth, quadratic growth, Evaluation of technical efficiency, Data Envelopment Analysis (DEA)

**Resumen:** Este artículo presenta un modelo de evaluación de la eficiencia técnica que incorpora el valor añadido de las escuelas como output del sistema educativo. La utilización de medidas de valor añadido, estimadas a partir de modelos multinivel, permite obtener índices precisos, fiables y objetivos del progreso de los alumnos a lo largo del tiempo, aislando el efecto de las variables que influyen en los resultados que consiguen los centros educativos, pero que están fuera del control de los gestores educativos. El modelo de medida de la eficiencia propuesto consta de dos fases diferenciadas; primero, se calcula el valor añadido de las escuelas y, a continuación, se estima la eficiencia técnica incorporado las medidas de valor añadido como indicadores de la productividad de las mismas. El modelo presentado es aplicado con el objetivo de evaluar la eficiencia técnica de las escuelas de educación primaria de la Comunidad de Madrid (Spain). A su vez, se comparan las puntuaciones de eficiencia obtenidas con las resultantes de aplicar otros modelos en los que el nivel de eficiencia lo determinan las tasas de promoción o el rendimiento bruto de los alumnos en un momento determinado, es decir, en los que se introducen los outputs tradicionalmente considerados en los estudios de eficiencia llevados a cabo dentro de los niveles educativos no universitarios. A pesar de observase cierta coincidencia entre las estimaciones proporcionadas por los diferentes modelos, las diferencias son estadísticamente significativas.

**Palabras clave**: Valor Añadido, Modelos Jerárquicos Lineales, Crecimiento Lineal, Crecimiento Cuadrático, Evaluación de la Eficiencia Técnica, Análisis Envolvente de Datos (DEA).

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**Corresponding author:** Esther López-Martín. Departamento MIDE II - UNED. C/ Juan del Rosal, 14 – 28040 Madrid (Spain). E-mail: estherlopez@edu.uned.es

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## Introduction

Educational efficiency refers to the relationship between inputs and investments that occurs in the educational system and the outcomes obtained (De La Orden et al., 1997; Lockheed & Hanushek, 1994). From this systematic conception, a series of mathematical equations are derived that can be used to estimate the changes that take place in the *outputs* when the *inputs* have been modified, in order to determine whether resources have been allocated efficiently. This allocation of resources is considered to be efficient, in a pareto sense, if the allocation of resources in any other way will diminish the outcomes for all or any of the individuals involved in the process.

As in other productive sectors, in the educational setting the three main measures of efficiency developed by Farrell (1957) can be considered: technical, allocative, and economic or global. Technical efficiency refers to the use of resources in the educational process in a technologically efficient manner. Allocative efficiency refers to the capacity of centers of education to use *inputs* in optimum proportions taking into account their costs, in other words, this type of efficiency involves choosing technically-efficient combinations of *inputs* that can produce the maximum amount of outcomes at the least possible cost. Finally, economic or global efficiency combines technical efficiency and allocative efficiency, such that a school will be economically efficient when it uses technical and allocative resources (Worthington, 2001). In addition to these three types of efficiency, Levin (1997) proposes applying the concept of X-efficiency within the educational sector. This author builds on the foundations laid by Leibenstein (1966, 1978a, 1978b), according to which the way in which individuals interact within an organization, the motivation of, or effort made by, the staff are all factors that can influence their efficiency. In spite of being able to identify these different types of efficiency, most studies conducted in the public sector in general and, more specifically, in the educational sector, have focused on studying technical efficiency. This is, mainly, owing to the difficulty of determining the costs of the factors involved in production (Worthington, 2001).

A review of the studies that assess efficiency in the non-university educational system has, also, revealed this tendency. These works have mainly focused on analyzing the technical dimension and have taken schools as the unit of reference (Agasisti, 2013; Agasisti, Bonomi, & Sibiano, 2014; Bradley, Johnes, & Millington, 2001; Conroy & Arguea, 2008; Cordero, 2006; Cordero, Pedraja, & Salinas, 2005; Cordero-Ferrera, Pedraja-Chaparro, & Salinas-Jimenez, 2008; Gómez, Buendía, Solana, & García, 2003; Haelermans & De Witte, 2012; Hernández & Fuentes, 2003; Kirjavainen & Loikkanen, 1998; Mancebón & Brandrés, 1999; Mancebón, Calero, Choi, & Ximénez-de-Embún, 2012; Mancebón & Muñíz, 2008; Muñiz; 2001; Podinovski, Ismail, Bouzdine-Chameeva, & Zhang, 2014; Ruggiero, 1996; Santín, 2003). Nonetheless, it is also possible to find studies in which the analytical units are the students (Perelman & Santín, 2011; Waldo, 2007), or even, the educational systems of countries (Afonso & St Aubyn, 2006; Aristovnik, 2013; Giménez, Prior, & Thieme, 2007).

By analyzing the resources that are usually introduced in the efficiency analysis, a distinction is usually made between the variables controlled by the education managers (discretional inputs) and those that cannot be controlled by the centers of education (non-discretional inputs or environmental variables). The main inputs considered in the former group correspond to: the number of teachers or the student-teacher ratio

(Agasisti, 2013; Agasisti et al., 2014; Cordero et al., 2005; Cordero-Ferrera, et al. 2008; Gómez et al., 2003; Mancebón & Bandrés, 1999; Muñiz, 2001; Podinovski et al., 2014; Ruggiero, 1996), the experience of the teaching staff (Conroy & Arguea, 2008; Kirjavainrn & Loikkanen, 1998), their level of education (Bradley, et al., 2001; Kirjavainen & Loikkanen, 1998; Ruggiero, 1996), number of teaching hours (Afonso & St. Aubyn, 2006; Hernández & Fuentes, 2003; Giménez, et al., 2007: Kirjavainen & Loikkanen, 1998) and the expenditure per student (Agasisti et al., 2014; Cordero, et al., 2005; Cordero, 2006; Cordero-Ferrera, et al., 2008; Gómez et al., 2003; Haelermans & De Witte, 2012; Mancebón & Brandrés, 1999; Muñíz, 2001). As non-controllable inputs, the studies cited tend to include indicators of the students' individual or family characteristics.

Regarding educational outcomes, the main ones considered are measures of the students' academic performance (direct marks or pass rates), obtained in national (Agasisti et al., 2014; Bradley, et al., 2001; Conroy & Arguea, 2008; Haelermans & De Witte, 2012; Kirjavainen & Loikkanen, 1998; Podinovski et al., 2014; Ruggiero, 1996; Waldo, 2007) and international (Afonso & St.Aubyn, 2006; Agasisti, 2013; Aristovnik, 2013; Giménez, et al. 2007). evaluations. In the case of studies conducted in Spain, a large proportion of these have used the marks obtained in the public university entrance exam (Cordero, 2006; Cordero, et al., 2005; Cordero-Ferrera, et al., 2008; Gómez, et al., 2003; Hernández & Fuentes, 2003; Mancebón & Bandrés, 1999; Mancebón & Muñiz, 2008; Muñiz, 2001), although in the last few years some studies have used the marks obtained by Spanish students in international evaluations in which they have participated, such as TIMSS (Santín, 2003) or PISA (Mancebón et al., 2012; Perelman & Santín, 2011).

### *The limitations of evaluations of efficiency in education*

Evaluation of efficiency in education is not free from obstacles. There are a series of limitations integral to characteristics of the education system, which must be considered, in addition to the decision-making process. These difficulties mainly affect the selection of inputs and outputs, measurement of the education product and the incorporation of non-discretionary inputs.

In relation to the former, in the school setting there is no general agreement about how variables within the productive process are related. This unawareness of the exact function of educational production (Hanushek, 1986), can affect decisions about the *inputs* and *outputs* that are introduced in the analysis and, consequently, about the estimates of efficiency made. In association with this, it is important to refer to the intangible and multidimensional nature of the educative product (Johnes, 2015) and, hence, to the impossibility of defining a single and, universally valid, concept to reflect the production of schools (Cordero, Muñíz, & Pedraja, 2006). In addition to the fact that the educational product is not materialized in quantifiable physical units, it varies greatly, and can be reflected in a greater development of academic, personal or social competences (Lockheed & Hanushek, 1994).

As can be observed from the previous section, in practice, studies conducted in an educational setting have considered partial measures of the outcomes of the educational system. A review of the literature reveals a tendency to include indicators of the schools' performance which, generally, correspond to measures of academic performance or pass rates. In spite of having an informative value, there are some limitations to these indicators that can give rise to biased results about the efficiency of schools. On the other hand, the students' performance refers to the accumulation of knowledge throughout their lives whereas, in efficiency studies inputs inform about the resources used over a specific time interval (Lockheed & Hanushek, 1994). Hence, when academic performance is included as an educational output it is important not only to consider the final result, but also the level of knowledge that the students presented at the start of the period relating to the inputs (Seijas, 2004). Moreover, not taking into account previous performance can be associated with another limitation, that of ignoring the different composition of the groups within and between schools. In the specific case of studies that take into account qualifications obtained by students in the public university entrance exam, there is the added bias that this exam is only done by students who have passed the academic year and want to study at university. The students are, therefore, subjected to a double selection.

Finally, another distinguishing characteristic of any educational system is the multitude of uncontrollable variables that influence the education process. Educational outcomes do not only depend on the input variables of the process, but are conditioned by another group of factors such as the students' personal or family traits, or the sociocultural and economic process in which this productive process is developed. These uncontrollable *inputs* could refer to several layers of grouping, such as students, classes, schools or, even, the neighborhoods or districts in which the schools are located. Studies aimed at evaluating efficiency in the past few years have realized the need to control the effect of these variables, and several of these have included uncontrollable factors in some stage of the analysis. These have mainly been conducted using data envelopment analysis, which include single-step models, where exogenous variables are included in a single Data Envelopment Analysis (DEA), or multi-step models that estimate the efficiency of units and, afterwards, correct this score by considering the effect of non-discretional variables (Cordero-Ferrera, et al., 2008).

**Objective**

The purpose of this article is to present a model to evaluate technical efficiency that incorporates the value added in schools as an output of the educational system. The use of value-added measures, estimated from multilevel models, will enable accurate, reliable and objective indices of the efficiency of schools to be obtained, thus solving three of the previously mentioned limitations:

- *Accumulated measure of knowledge acquired by students*. Value added in education informs about the school's contribution to the growth of students over a given period so that, in a second stage, the level of progress achieved can be related to the resources used to attain it.

- *Influence of uncontrollable inputs*. Contextualized value-added measures facilitate the control of students' personal and family traits, and other contextual factors, on school productivity, ensuring that these uncontrollable factors do not influence the measure of efficiency obtained.

- *Differences existing in the composition of groups between and within schools.* The application of linear hierarchical models to estimate value added respects the nested nature of educational data by acknowledging different sources of variation (Martínez, 2009; Martínez, Gaviria, & Castro, 2009) and, consequently, guarantees that the effects that can cause the results, the way students are assigned to the schools, are taken into account.

In addition to this main goal, another two complementary objectives have also been pursued: a) to apply the proposed model to evaluate the technical efficiency of schools of Primary Education in the Madrid Region (Spain), and b) to compare the scores obtained with those resulting from using other models that introduce pass rates or students' performance in a given model, in other words, outcomes traditionally considered in efficiency studies conducted in non-university education centers.

## Proposal for a model to assess the technical efficiency from value-added measures in education.

The model proposed in this work to assess efficiency is composed of two different steps: calculation of the value added and estimation of the technical efficiency. These two steps are described below.

### *Calculation of the value-added measures*

Value-added models take the form of a set of statistical techniques that aim to isolate the school's contribution to the academic growth of the students, by using marks obtained by the individuals over several years (McCaffrey, Lockwood, Koretz, & Hamilton, 2003; Martínez et al., 2009). The numerical estimates provided by these models can be used to develop, control, and assess the school and other aspects of the educational system (OECD, 2008), as they provide information about the students' growth or improved performance that is exclusively due to the influence of variables associated with the school.

Out of the alternative methodologies available to estimate value added in education, this work proposes using multilevel models. One of the reasons behind this choice is that these procedures respect the nested structure of educational data and can distinguish which part of the performance occurs at each level (Martínez, 2009; Martínez et al., 2009), `yielding more accurate estimates of the uncertainty to be attached to the estimates of schools VA′ (OECD, 2008, p. 142).

By considering a multi-level design in which students' performance is measured at different moments in time, growth can be represented by a three-level hierarchical model in which aggregation levels correspond to: time (Level 1), the student (Level 2), and the school (Level 3). If this growth is modeled as a polynomial of degree P (Raudenbush & Xiao-Feng, 2001; Raudenbush & Bryk, 2002; Maas & Snijders, 2003; Raudenbush, 2004), at the Level 1, performance of student *i* at school *j* at time *t* is equal to:

$$y_{tij} = \pi_{0ij} + \pi_{1ij}(t - t_0) + \pi_{2ij}(t - t_0)^2 + \cdots \pi_{Pij}(t - t_0)^P + \varepsilon_{tij} \quad (1)$$

Where $\pi_{0ij}$ is the initial performance (time $t_0$) of student *i* at school *j*, $\pi_{1ij}$, $\pi_{2ij}$,…,$\pi_{Pij}$ are the growth parameters associated with the time predictor $(t - t_0)$, and $\varepsilon_{tij}$ is the random error.

The fixed parameters in the Level 1 equation become dependent variables in the Level 2 equation, so Level 2 equations reflect how the initial performance and the growth rates can vary across individuals:

$$\begin{aligned} \pi_{0ij} &= \beta_{0j} + \mu_{0ij} \\ \pi_{1ij} &= \beta_{1j} + \mu_{1ij} \\ \pi_{2ij} &= \beta_{2j} + \mu_{2ij} \\ &\vdots \\ \pi_{Pij} &= \beta_{Pj} + \mu_{Pij} \end{aligned} \qquad (2)$$

Where $\beta_{0j}$ is the initial performance of the schools *j* at the moment $t_0$, $\beta_{1j}$, $\beta_{2j}$,…, $\beta_{Pj}$ are growth rates (linear, quadratic,…,) expected for the school *j* due to time effect, $\mu_{0ij}$ is the difference between the initial performance of the student *i* at the school *j* and the initial performance of his school, and $\mu_{1ij}$, $\mu_{2ij}$,…, $\mu_{Pij}$ are the differential growth of the student *i* at the school *j* in relation to the expected growth for his school.

Finally, the Level 3 equations reflect the variation among schools:

$$\begin{aligned} \beta_{0j} &= \beta_{00} + \nu_{0j} \\ \beta_{0j} &= \beta_{10} + \nu_{1j} \\ \beta_{2j} &= \beta_{20} + \nu_{2j} \\ &\vdots \\ \beta_{Pj} &= \beta_{P0} + \nu_{Pj} \end{aligned} \qquad (3)$$

Where $\beta_{00}$ is the initial average performance of all the schools at the time $t_0$, $\beta_{10}$, $\beta_{20}$,…, $\beta_{P0}$ are the growth rates (linear, quadratic,…,) expected for all schools due to the time effect, $\nu_{0j}$ is the difference between initial performance of the school *j* and the average performance of all schools, and $\nu_{1j}$, $\nu_{2j}$,…, $\nu_{pj}$ represent the differential growth of the school *j* in relation to the growth expected for all schools. The means of different random terms are zero, and the variances are constant.

The final equation is:

$$y_{tij} = \beta_{00} + \sum_{l=1}^{P} \beta_{l0} (t - t_0)^l + \nu_{0j} + \sum_{l=1}^{P} \nu_{lj} (t - t_0)^l + \qquad (4)$$

$$+\mu_{0ij} + \sum_{l=1}^{P} \mu_{lij} (t - t_0)^l + \varepsilon_{tij}$$

From this equation, more complex models can be formulated, introducing characteristics of the students or schools as covariates that can influence the progress of students over time. These value-added models are called contextualized models and enable the outcomes to be adjusted taking into account the subjects' socioeconomic and demographic variables, so that between-school comparisons can be as objective as possible. If in equation 4 we introduce *n* personal and family characteristics $(X_{1ij}, X_{2ij}, \dots, X_{nij})$ and *m* contextual factors $(Z_{1j}, Z_{2j}, \dots, Z_{mj})$, the resulting equation is:

$$y_{tij} = \beta_{00} + \sum_{k=1}^{n} \beta_{0k} X_{kij} + \sum_{s=n+1}^{n+m} \beta_{0s} Z_{sj} + \sum_{l=1}^{P} \beta_{l0} (t - t_0)^l +$$

$$+ \sum_{k=1}^{n} \sum_{l=1}^{P} \beta_{lk} (t - t_0)^l X_{kij} + \sum_{s=n+1}^{n+m} \sum_{l=1}^{P} \beta_{ls} (t - t_0)^l Z_{sj} \quad (5)$$

$$+ \sum_{k=0}^{n} v_{0kj} + \sum_{l=1}^{P} v_{l0j} (t - t_0)^l + \sum_{k=1}^{n} \sum_{l=1}^{P} v_{lkj} (t - t_0)^l X_{kij} + \mu_{0ij}$$

$$+ \sum_{l=1}^{P} \mu_{lij} (t - t_0)^l + \varepsilon_{tij}$$

where $\beta_{0k}$ and $\beta_{0s}$ show the effect of individual and contextual factors on initial performance for all schools; $\beta_{lk}$ *and* $\beta_{ls}$ indicate variations in schools' growth rates due to the effect of these predictors. On the other hand, the differential effect of the covariates introduced in the model for each individual school is represented by the random parameters $\upsilon_{0kj}$, for the intercept, and by $\upsilon_{lkj}$, for the growth rates.

The value-added measure obtained from these growth models reflects the difference between the growth rate for the school $j$ and its expected growth, after controlling for the influence of factors external to the school on this growth (López-Martín, Kuosmanen, & Gaviria, 2014), and will equal:

$$\sum_{l=1}^{P} v_{lj} (t - t_0)^l \quad (6)$$

***Estimation of the technical efficiency***

In this second step, to estimate the efficiency of schools, value-added measures will be considered as an indicator of their productivity. On the other hand, all the variables that can be controlled by the schools or the authorities will be incorporated as educational inputs.

Out of the methodologies available to estimate efficiency, DEA was the approach chosen in this study. This non-parametric technique is the method of choice to analyze efficiency in the education sector (Worthington, 2001; Lopez-Martin, 2012), more than other parametric techniques such as stochastic frontier models. Among the reasons why this technique is so widely used to analyze efficiency in the public sector in general, and especially in education, are (Charnes, Cooper, Lewin, & Seiford, 1994; Cooper, Seiford, & Tone, 2007): the fact that it can be applied in contexts in which the production function is unknown; it does not require specifications a priori about weights and costs for inputs and outputs; there can be multiple inputs and multiple outputs which, in turn, can be measured on different scales; it can be adjusted for exogenous variables; and calculate specific estimates for preferred changes in inputs and outputs, so that the units under the productive frontier can be projected on it.

Since this approach was first introduced, several models of DEA have been proposed. These can be classified according to the orientation of the model (input-oriented or output-oriented), of the type of measure they produce (radial or non radial), or the classification of scale performances that characterize production technology (constant or variable performances). In the present work, taking into account that the inputs in the education system have usually already been established and education managers must center strategies on obtaining the best results with the resources that are available (output-oriented), it has been proposed to use the output-oriented CCR Model (Charnes, Cooper, & Rhodes, 1978), or the output-oriented BCC Model (Banker, Charnes, & Cooper, 1984), depending on whether the schools function under constant (CCR Model) or variable (BCC model) performances to scale.

| CCR Model– Output-oriented | BCC Model – Output-oriented |
|---|---|
| $\min_{u,v} \; w_0 = \sum_{i=1}^{m} v_i\, x_{i0}$ | $\min_{u,v} \; w_0 = \sum_{i=1}^{m} v_i\, x_{i0} - v_0$ |
| Subject to: | Subject to: |
| $\sum_{r=1}^{s} u_r\, y_{r0} = 1$ | $\sum_{r=1}^{s} u_r\, y_{r0} = 1$ |
| $\sum_{i=1}^{m} v_i\, x_{ij} - \sum_{r=1}^{s} u_r\, y_{rj} \geq 0;$ | $\sum_{i=1}^{m} v_i\, x_{ij} - \sum_{r=1}^{s} u_r\, y_{rj} - v_0 \geq 0;$ |
| $j = 1, \dots, n\,; \quad u_r, v_i \geq \varepsilon\,;\; r = 1, \dots, s\,;\; i = l, \dots, m.$ | $j = 1, \dots, n\,; \quad u_r, v_i \geq \varepsilon\,;\; r = 1, \dots, s\,; \quad i = l, \dots, m.$ |
| | $v_0 = free$ |

Figure 1. Output-oriented CCR and BCC models.

## Estimation of the technical efficiency of the schools in the Madrid Region (Spain)

In this section, the model proposed to assess the technical efficiency is applied to primary schools of the Madrid Region participating a longitudinal study[1], conducted over the academic years 2005-2006 and 2006-2007. Over this period, performance in reading comprehension and mathematics of a representative sample of students was assessed at the start and at the end of each academic year. The first assessment took place in November 2005 and the final assessment in 2007.

### *Description of the sample*

The sample of this study was estimated from the whole population of students matriculated in 5th year of Primary Education (E.P.) in schools of the Madrid Region

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[1] This study was a Research and Development project entitled `Value added in education and the educational production function: a longitudinal study´, sponsored by the Ministry of Science and Technology with reference SEC2003-09742.

(Spain) who, therefore, matriculated in 6th year of primary education in the academic year 2006-2007.

The sample was selected by following a multistage sampling strategy, and a total of 4,923 students were chosen from 109 primary schools. For financial reasons, during the second year of the study a random subsample of about one third of the original size was extracted. Because of this, and due to variations relating to experimental mortality (Campbell & Standley, 1963), the elimination of children with special education needs and the fact that only students with scores in all four tests of reading comprehension and mathematics were included, the final number of students and schools included in this work is recorded Table 1.

Table 1. Sample composition

| | Mathematics | Reading |
|---|---|---|
| Students | 2731 | 2739 |
| Schools | 92 | 92 |

### *Inputs and outputs considered*

To estimate value-added models, the students' performance in reading comprehension and mathematics in the four tests is included. The students' skills in these four variables were measured by tests constructed ad hoc, followed by a rigorous matching process to make the marks obtained in the four tests equivalent.

In order to control the influence of the students' individual and family characteristics and contextual factors, the following variables were introduced as Level 1 (student) predictors: gender, differential performance[2], first-generation immigrant status, socio-economic status and time devoted to reading. For Level 2 indicators (school) the average socio-economic status at the school and the percentage of students who were first generation immigrants. Operativization of these variables is presented in Table 2.

[2] The effect of the differential performance of each student in the first assessment (November, 2005) compared to the mean for the population on the schools' growth rate (linear and quadratic) has been controlled. The reason for entering this predictor in the model is that several studies have related the differences in the students' initial performances to the different rates of change (Choi & Seltzer, 2005, 2010; Klein & Muthén, 2006; Castro, Ruiz, & López, 2009).

Table 2. Variables included to estimate value added

| Variable | Values |
|---|---|
| Performance in Reading / Mathematics | Skills in reading /mathematics. The mean of these variables is 500 and the standard deviation is 100. |
| Gender | 0 = Male<br>1 = Female |
| Differential performance | Difference between the student's performance in the first test and the mean performance of the population. |
| First-generation immigrant status | 0 = No<br>1 = Yes |
| Socio-economic status | Index constructed from Rasch model (Rasch, 1960), taking into account the following variables: having more than 100 books at home, having an internet connection, educational level of the father and educational level of the mother. |
| Time devoted to Reading | 0 = Nothing<br>1 = 1 hour or less<br>2 = Between 1and 2 hours<br>3 = Between 2 and 3 hours<br>4 = More than 3 hours |
| Average socio-economic status at school | School's average student socioeconomic status |
| Students at school corresponding to first generation immigrants | Percentage of first generation immigrants at the school |

When calculating technical efficiency, the estimates of value added in mathematics and in reading comprehension calculated in the previous step are included as outputs of the education system. It is important to remember, as can be derived from equation 6, that value added is calculated from the distance between the real growth rate observed for a school and the predicted growth rate. Hence, it will have a negative value if the real growth is less than the predicted growth, equal to zero if the observed growth equals the predicted growth, and positive if the real growth is greater than the predicted growth. Therefore, estimates of value added have been transformed to a scale from 0 to 1, where a value of 0 is assigned to the school with the lowest added value and 1 to the school with the highest score. The formula applied to do this was:

$$X' = (X\text{-}Min) / (Max\text{-}Min) \qquad (7)$$

The following indicators were considered as inputs of the education system:

- *Teaching experience*. The number of years of experience of the teaching staff is related to the salary they receive and also to the effectiveness of their work. The following categories of teaching experience were, therefore, considered: a) the number of teachers with less than 5 years experience divided by the total number of students, b) the number of teachers with between 5 and 10 years experience divided by the total number of students c) the number of teachers with between 10 and 15 years experience divided by the total number of students and d) the number of teachers with more than 15 years teaching experience divided by the total number of students.
- *Students per class*. The teacher-student ratio significantly affects the resources used in education, since a smaller ratio implies the employment of more teachers. This is why the inverse of the number of students per class was

calculated (1/number of students) so that a higher value of this variable is associated with a greater productivity and vice versa.

- *Extracurricular activities carried out in schools.* The number of extracurricular activities carried out at the school divided by the total number of students.

***Results***

This next section presents the results obtained after applying the proposed model to calculate the technical efficiency of primary schools of the Madrid Region. After that, the efficiency scores obtained are compared with the results from using other models in which the level of efficiency of the schools is determined by the students' pass rates or academic performance in June 2007.

*Calculation of the value-added measures*

The growth models estimated are presented in Table 3. If the effects of personal, family and contextual characteristics are not taken into account, the results show a mean performance in fifth year of primary schooling of around 503 points in both academic subjects.

Regarding the growth undergone by schools over the study period, this is observed to follow a linear trajectory with a mean increase per test of around 11.244 points in mathematics and 13.505 points in reading comprehension. In the models estimated, the quadratic term associated with the mean growth rate of the schools ($\beta_{2.0}$), that informs about the mean acceleration or deceleration rate over time, was not significant. In any case, the random part of the models shows how the unexplained variance in both the linear ($\sigma_{v1}^2$) and the quadratic ($\sigma_{v2}^2$) growth rates of the schools are statistically significant. This implies that although the mean growth of schools is linear, at an individual level the growth of schools over time may follow non-linear trajectories.

Table 3. Basic and Contextualized Value-Added Models

| | | Basic VA models | | Contextualized VA models | |
|---|---|---|---|---|---|
| | | Mathematics | Reading | Mathematics | Reading |
| FIXED EFFECTS | | | | | |
| Intercept of performance average in the first evaluation | $\beta_{0.0}$ | 503.247 (3.736) | 503.444 (3.857) | 467.926 (8.833) | 466.414 (7.166) |
| Growth rates (linear term) | $\beta_{1.0}$ | 11.244 (2.483) | 13.504 (3.791) | 10.964 (2.386) | 9.798 (3.978) |
| Growth rates (quadratic term) | $\beta_{2.0}$ | 0.056 (0.735)* | 1.539 (1.052)* | 0.012 (0.746)* | 1.702 (1.087)* |
| Gender | $\beta_{0.1}$ | | | -12.614 (3.020) | 9.307 (2.849) |
| Immigrant status (First-generation) | $\beta_{0.2}$ | | | -36.031 (5.547) | -21.347 (4.793) |
| Socio-economic status (SES) | $\beta_{0.3}$ | | | 18.286 (1.853) | 14.188 (1.554) |
| Time devoted to reading | $\beta_{0.4}$ | | | 5.271 (1.677) | 7.290 (1.391) |
| Average SES in school | $\beta_{0.5}$ | | | 17.765 (7.145) | 14.964 (4.822) |
| Percentage of immigrant in school | $\beta_{0.6}$ | | | | -65.095 (19.055) |
| Differential performace x $(T_1 - T_0)$ | $\beta_{1.1}$ | | | -0.141 (0.004) | |
| Gender x $(T_1 - T_0)$ | $\beta_{1.2}$ | | | | 2.394 (0.957) |
| Immigrant status x $(T_1 - T_0)$ | $\beta_{1.3}$ | | | | |
| SES x $(T_1 - T_0)$ | $\beta_{1.4}$ | | | | |
| Time devoted to reading x $(T_1 - T_0)$ | $\beta_{1.5}$ | | | | |
| Average SES in school x $(T_1 - T_0)$ | $\beta_{1.6}$ | | | | |
| Percentage of immigrant in school x $(T_1 - T_0)$ | $\beta_{1.7}$ | | | | 15.947 (6.563) |
| Differential performace x $(T_1 - T_0)^2$ | $\beta_{2.1}$ | | | | |
| Gender x $(T_1 - T_0)^2$ | $\beta_{2.2}$ | | | | |
| Immigrant status x $(T_1 - T_0)^2$ | $\beta_{2.3}$ | | | | |
| SES x $(T_1 - T_0)^2$ | $\beta_{2.4}$ | | | | |
| Time devoted to reading x $(T_1 - T_0)^2$ | $\beta_{2.5}$ | | | | |
| Average SES in school x $(T_1 - T_0)^2$ | $\beta_{2.6}$ | | | | |
| Percentage of immigrant in school x $(T_1 - T_0)^2$ | $\beta_{2.7}$ | | | | |
| RANDOM EFFECTS | | | | | |
| Variance due to time (Level 1) | $\sigma^2_{\varepsilon}$ | 1568.559 (30.261) | 2630.566 (46.897) | 1327.414 (21.694) | 2621.382 (47.557) |
| Variance due to the initial performance level of the students (Level 2) | $\sigma^2_{\mu 0}$ | 5502.365 (182.894) | 3457.244 (118.163) | 5083.474 (153.702) | 3187.042 (112.917) |
| Variance for the growth rates of the students (Level 2) | $\sigma^2_{\mu 1}$ | 105.077 (13.017) | 44.842 (14.034) | | 41.279 (13.998) |
| Covariance for the initial performance level and the growth rates (Level 2) | $\sigma_{\mu 0 \mu 1}$ | -816.206 (41.906) | | | |
| Variance due to the initial performance level of the school (Level 3) | $\sigma^2_{v0}$ | 1002.508 (187.292) | 1125.757 (200.741) | 640.540 (130.860) | 602.662 (121.622) |
| Variance for the growth rates of the schools (linear) (Level 3) | $\sigma^2_{v1}$ | 411.702 (77.118) | 1061.352 (193.606) | 384.861 (76.013) | 1105.235 (201.512) |
| Covariance for the initial performance level and the growth rates (linear) of the schools (Level 3) | $\sigma_{v0v1}$ | -138.462 (33.024) | -767.638 (166.811) | | -650.045 (136.436) |
| Variance for the growth rates of the schools (quadratic) (Level 3) | $\sigma^2_{v2}$ | 34.470 (7.225) | 75.993 (14.875) | 37.041 (7.434) | 81.738 (15.869) |
| Covariance for the growth rates (linear) and the growth rates (quadratic) (Level 3) | $\sigma_{v1v2}$ | -113.926 (23.262) | -277.067 (52.846) | -117.202 (23.489) | -294.354 (55.784) |
| Covariance for the initial performance level and the growth rates (quadratic) of the schools (Level 3) | $\sigma_{v0v2}$ | | 181.783 (44.321) | | 159.999 (36.688) |
| Deviance | | 117775.59 | 123063.41 | 110179.79 | 117845.46 |
| Number of parameters | | 12 | 12 | 15 | 20 |
| Difference of deviances | | | | 7595.80 | 5217.95 |
| Difference of parameters | | | | 3 | 8 |
| P value | | | | 0.000 | 0.000 |

Note: standard errors in brackets

* No significant parameter

Regarding the latter point, it is important to note the covariances between the random parameters of Level 3. In the first place, the negative value of covariance between the initial performance level and linear growth rates ($\sigma^2_{v0v1}$) implies that schools with a greater initial performance present a smaller linear growth rate and, in the case of reading comprehension, the quadratic growth rate of these schools (greater initial performance) is even greater, in other words, the positive variance between the initial growth rate and the quadratic growth rate ($\sigma^2_{v0v2}$) reflects a greater acceleration by schools that present a greater initial performance.

After introducing in the models, the students' personal and family characteristics and the contextual factors considered, the mean performance of the schools becomes 467.926 points in mathematics and 466.414 points in reading comprehension. From the contextualized models estimated, it can be observed how the initial performance in both subjects varies according to the following predictors: gender (girls score approximately 13 points less than boys in mathematics, and 9 points more in reading comprehension), the condition of being first generation immigrant (the performance of these students is 36.03 points less in mathematics and 21.35 points less in reading comprehension), the socioeconomic level (for each point increase in the socioeconomic level of the parents the students' performance in mathematics increases by 18.286 points and in reading comprehension by 14.188 points and vice versa), time spent reading (the score in mathematics increases by 5.271 points and the score in reading comprehension by 7.290 points for each hour spent reading) and the mean socioeconomic level of students at the school (with a positive effect for this variable in both subjects). In turn, in the case of reading comprehension it can be observed how schools with a greater number of first generation immigrants obtain lower scores.

Taking into consideration students' linear growth, it can be observed how in mathematics growth over time of students with an initial level higher than the mean for the population is 0.141 points lower for each differential point. Similarly, for students with an initial performance lower than mean levels for the population, the growth rate is 0.141 points higher. On the other hand, the linear growth rate for students in reading comprehension is influenced by gender and by the percentage of first generation immigrants in the school. Regarding the first predictor, the results show how growth over time is greater for female than for male students. For the second predictor, schools with a higher percentage of immigrants presented higher growth rates in reading comprehension.

From the results of value-added contextualized models, the value added for each school will be estimated from the distance between the growth observed for a school $[\beta_{1.0}(t - t_0) + \beta_{2.0}(t - t_0)^2 + v_{1j}(t - t_0) + v_{2j}(t - t_0)^2]$ and the growth estimated for the group of schools $[\beta_{1.0}(t - t_0) + \beta_{2.0}(t - t_0)^2]$. In other words, the value added for each school will be equal to $v_{1j}(t - t_0) + v_{2j}(t - t_0)^2$ , such that schools with a growth rate higher than that estimated for the population will present a positive value added, while schools with a growth lower than this over the time period will have a negative value added. Figure 2 shows the value added in reading comprehension and mathematics for schools that form part of the sample in the last assessment (T=4).

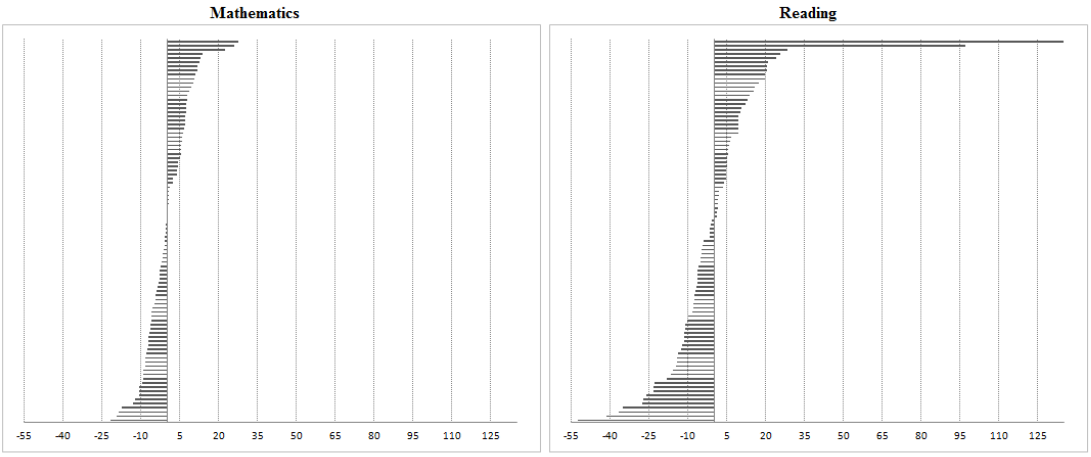


Figure 2. Value-added scores in Mathematics and Reading Comprehension.

*Calculation of the technical efficiency of schools*

To calculate efficiency Data Envelopment Analysis was used in the CCR[3] output-oriented version (Charnes et al., 1978). From this perspective, a school is considered to be more efficient than another if it produces greater value added when using the same amount of resources.

Table 4 shows the results obtained after estimating the technical efficiency of primary schools. The results show that of the 89 schools considered[4], only 29 present an efficient behavior. The mean level of efficiency was 25%. However, if only the scores of inefficient schools are considered, the mean level of inefficiency increases up to 38%. Given that the efficiency index estimated is a radial mean, the measures estimated refer to the equiproportional increase in performance (output) that the units must undergo to be located on the efficient frontier.

Table 4. Efficiency calculated after applying the proposed model

| Efficiency index | Model: VA |
|---|---|
| > 0.20 | 1 |
| 0.21 - 0.40 | 5 |
| 0.41 - 0.60 | 21 |
| 0.61 - 0.80 | 21 |
| 0.81 - 0.99 | 12 |
| Efficient units | 29 |
| Mean (All units) | 0.75 |
| Mean (Efficient units) | 0.62 |

When examining the distribution of efficiency scores, it is noteworthy that 6 of the 89 inefficient units present a level of inefficiency higher than 40%. A total of 42 of the schools have obtained values between 0.41 and 0.80, in other words, their level of

[3] If these scores are compared with the indices obtained when estimating the efficiency of schools under the principle of variable returns to scale (BCC Model), the correlation between both indices is very high ($\rho = 0.943$; $p = 0.000$). Therefore, in the case studied here, we can see that both constant and variable returns to scale provide very similar arrangements.

[4] Although in the previous step, value added in reading comprehension and mathematics was estimated in 92 schools, only 89 of these units had scores for both these academic subjects.

inefficiency was between 59% and 20%. Finally, 12 of the schools presented scores higher than 0.81, giving an index of inefficiency lower than 19%.

If the mean inefficiency of the schools that form part of the sample is studied in relation to ownership of the schools, it can be observed how, on average, the state schools are the most inefficient (0.68), followed by state-assisted schools (0.80) and, finally, by the private schools (0.91), with the latter groups presenting the lowest level of inefficiency. Results of the factorial analysis show that these differences are significant ($F = 6.106$; p. 0.003).

After concluding that the mean inefficiency of the schools differs in relation to ownership, the next step consisted in determining between which mean pairs these differences are observed. The post hoc Scheffe comparisons included in Table 5 show how the level of inefficiency of the state schools differs significantly from the index estimated for the private schools. However, the efficiency estimated for the state-assisted schools was not significantly different from the indices for state or for private schools.

Table 5. Comparison of the level of efficiency in relation to ownership

| | State | State-assisted | Private |
|---|---|---|---|
| State | | -0.127 (p = 0.054) | -0.234 (p = 0.013) |
| State-assisted | 0.127 (p = 0.054) | | -0.107 (p =0.426) |
| Private | 0.234 (p = 0.013) | 0.107 (p =0.426) | |

### *Comparison of the efficiency indices obtained when applying pass rates and overall performance as outputs of the education system*

This section compares the efficiency indices obtained with those resulting from applying other models that use indicators traditionally included in efficiency analyses within the non-university education system, in other words, students' pass rates[5] and overall performance[6]. Like the previous case, the technique used was Data Envelopment Analysis in the CCR output-oriented version.

Since estimation of the indices of efficiency did not take into account the effect of personal and family characteristics on education outcomes, a second step attempts to control the possible influence of these variables. The uncontrollable inputs considered corresponded to: the percentage of girls at the school; the percentage of first generation immigrant students, mean socioeconomic level of the students at the school, and the mean time spent reading.

Hence, first of all a regression model was calculated in which the radial index estimated is included as a dependent variable and individual and contextual factors are introduced as predictors. Next, after studying the uncontrollable *inputs* with a significant effect on the efficiency indices calculated (Appendix A), the indices initially obtained by

[5] This indicator represents the proportion of students above a given educational level in relation to the number initially matriculated.

[6] Overall performance refers to the performance obtained by students at a given moment in time, specifically in June 2007.

applying the following procedure were adjusted (Ray, 1991; Noulas & Ketkar, 1998; Cordero, 2006): a) estimation of the level of efficiency of each of the units analyzed ($y_i^{'} = \beta X_i$), b) calculation of the residuals ($\mu_i = y_i - y_i^{'}$); and c) sum of the greatest positive residual ($\mathrm{Max}(\mu_i)$) to the estimated efficiency score ($y_i^{''} = y_i^{'} + \mathrm{Max}(\mu_i)$).

In this way, $y_i^{''} - y_i$ represents the inefficiency that could be controllable by the education managers and $[(1 - y_i) - (y_i^{''} - y_i) = 1 - y_i^{''}]$ is the part of the inefficiency due to uncontrollable factors.

Table 6 shows the results obtained after estimating the different models. The number of schools on the productive frontier increases to 32 units in the model that takes into account the pass rates and to 37 units in the model that considers students' overall performance. Similarly, the mean technical efficiency of the schools is higher in these two models, especially when the output corresponds to the students' overall academic performance.

Table 6. Efficiency scores obtained in the different models

| | Model: VA | Pass rates model | Performance model |
|---|---|---|---|
| > 0.20 | 1 | 0 | 0 |
| 0.21 - 0.40 | 5 | 4 | 1 |
| 0.41 - 0.60 | 21 | 8 | 12 |
| 0.61 - 0.80 | 21 | 22 | 12 |
| 0.81 - 0.99 | 12 | 22 | 27 |
| Efficient units | 29 | 32 | 37 |
| Mean (All units) | 0.75 | 0.83 | 0.87 |
| Mean (Inefficient units) | 0.62 | 0.73 | 0.79 |

It is important to not only analyze the changes occurring in the indices, but also to observe if a school that is classified as `efficient´ when applying the model, can be located below the productive frontier if another output is considered, or vice versa. Table 7 shows, on the main diagonal, schools which have been classified as efficient or inefficient in both models (pass rate model and VA model). In the top right-hand corner, 9 schools can be found that the VA model considers to be inefficient, but that the pass rate model places on the productive frontier.

Similarly, the bottom left hand corner shows 5 schools that the VA model has classified as efficient but that the pass rate model indicates to be inefficient. The differences observed in the distribution of efficient and inefficient units, in relation to the model used, are statistically significant (Chi-squared = 37.192; degrees of freedom = 1; p = 0.000).

Table 7. Comparison of efficient units Model: VA vs. Model: Pass rate

| | | Model: Pass rate | | |
|---|---|---|---|---|
| | | Inefficient | Efficient | Total |
| Model: VA | Inefficient | 51 | 9 | 60 |
| | Efficient | 5 | 23 | 28 |
| | Total | 56 | 32 | 88 |

Note: The difference between the number of efficient units included in this table and those recorded in Table 4 is due to the presence of an efficient unit in the VA Model, for which the efficiency has not been estimated in the Pass model, as the pass rate was not available.

On the other hand, Table 8 compares the classification of schools after applying the VA Model and the Performance Model. Both models classify 48 units as inefficient and 25 units as efficient. However, four of the units located on the productive frontier in the VA model are considered to be inefficient in the Performance Model. By contrast, 12 schools classified as inefficient according to the VA Model, are considered to be efficient by the Performance Model. Moreover, the differences observed are statistically significant (Chi squared = 35.281; degrees of freedom = 1; p = 0.000).

Table 8. Comparison of efficient units according to the VA Model and the Performance Model

| | | Model: Performance | | |
|---|---|---|---|---|
| | | Inefficient | Efficient | Total |
| Model: VA | Inefficient | 48 | 12 | 60 |
| | Efficient | 4 | 25 | 29 |
| | Total | 52 | 37 | 89 |

## Conclusions and discussion

This work presents a model to assess the technical efficiency that incorporates the value added in schools as an output of the education system. The introduction of these value-added measures can be considered to be one of the best ways to control the effect of variables that influence the teaching-learning process, but that are outside the control of education managers.

Value-added models aim to isolate the schools' contribution to the students' academic development, by exclusively measuring the effect of factors related to the school, such as syllabuses or teachers, on academic growth. By considering the marks obtained by students at different moments in time, personal factors can be isolated, assuming that these variables affect both pre-test and post-test results. In this way, value-added measures give more accurate estimates than other types of outputs, such as overall performance or pass rates, which reflect the knowledge that students have accumulated over a lifetime. In efficiency models, if the inputs inform about the resources used at a specific moment in time, the outputs should do the same.

Moreover, calculating the value added in schools by applying multilevel models will increase still further the accuracy of the estimates. These models respect the nested structure of education data, permit linear and non-linear growth to be modeled, and help control the effect of uncontrollable inputs on initial performance and on growth rates.

These three aspects have been shown after applying the proposed model to assess technical efficiency in primary schools of the Madrid Region (Spain). Firstly, taking into account the nested structure of the data available (students grouped in schools) it has been possible to determine the achievement at each level. Secondly, it has been possible to observe in both academic subjects (mathematics and reading comprehension) how, although the mean growth of the schools is linear, the individual growth of each school can follow non-linear trajectories. Finally, the effect of the students' personal characteristics (Level 1) and the environmental variables (Level 2) on students' initial performance and on growth rates was controlled.

The resulting estimates of value added obtained are accurate measurements of the results achieved by schools, which can subsequently be introduced in models to assess efficiency. To not use this type of measure can produce biased estimates of efficiency, as any output would be conditioned, to a variable degree, by personal, family or contextual characteristics. This is extremely important in cases in which the schools have some control over the selection process of students in relation to some of these individual characteristics (for example, previous academic performance or socioeconomic level). Although uncontrollable inputs can be considered in a second step, this does not appear to be the most appropriate procedure since these characteristics are introduced as aggregated variables, attempting to control at the \`school level´ the effect of variables that really belong to the \`student level´.

A comparison of the results of the proposed model with efficiency indices obtained after applying other models, which use pass rates or overall performance as outputs of the education system, has revealed how the classification of schools as efficient or inefficient varies depending on which output is entered in the model. Hence, schools in which students show the greatest progress do not necessarily coincide with those with the greatest pass rates or the ones in which students achieve the highest marks in the performance tests. It is, therefore, important to define what exactly an efficient school is. Is it the one with the most pass rates? One in which the students show the highest levels of skills, regardless of whether these students already had these better results when they started at the school? Or, by contrast, the schools in which the students show the most progress? We consider the answer to be clear: an efficient school is the one in which students progress more than expected, adding extra value to the students' results in comparisons with other schools with similar initial student populations and which use the same amount of resources.

## BIBLIOGRAPHY:


Afonso, A., & St. Aubyn, M. (2006). Cross-country efficiency of secondary education provision: a semi-parametric analysis with non-discretionary inputs. *Economic Modelling, 23*(3), 476-491.

Agasisti, T. (2013). The efficiency of Italian secondary schools and the potential role of competition: a data envelopment analysis using OECD-PISA2006 data. *Education Economics, 21*(5), 520-544.

Agasisti, T., Bonomi, F., & Sibiano, P. (2014). Measuring the \`managerial´ efficiency of public schools: a case study in Italy. *International Journal of Educational Management, 28*(2), 120-140.

Aristovnik, A. (2013). ICT expenditures and education outputs/outcomes in selected developed countries: An assessment of relative efficiency. *Campus-Wide Information Systems, 30*(3), 222-230.

Banker, R. D., Charnes, A., & Cooper, W. W. (1984). Some models for estimating technical and scale inefficiencies in Data Envelopment Analysis. *Management Science, 30*(9), 1078-1092.

Bradley, S., Johnes, G., & Millington, J. (2001). The effect of competition on the efficiency of secondary schools in England. *European Journal of Operational Research, 135*(3), 545-568.

Campbell, D. T., & Stanley, J. C. (1963). *Experimental and quasi-experimental designs for research*. Boston, MA: Houghton-Mifflin.

Castro, M., Ruíz, C., & López, E. (2009). Forma básica del crecimiento en los modelos de valor añadido: Vías para la supresión de efecto de regresión. *Revista de Educación, 348*(1), 111-136.

Charnes, A., Cooper, W., Lewin, A. Y., & Seiford, L. M. (1994). *Data Envelopment Analysis. Theory, methodology and applications*. Boston: Kluwer Academic Publishers.

Charnes, A., Cooper, W. W., & Rhodes, E. (1979). Short communication: Measuring the efficiency of decision making units. *European Journal of Operational Research, 3*(4), 339.

Choi, K., & Seltzer, M. (2005). *Modeling heterogeneity in relationships between initial status and rates of change: Latent variable regression in a three-Level hierarchical mode*l. Los Angeles, CA: Center for the Study of Evaluation National Center for Research on Evaluation, Standards, and Student Testing. Retrieved January 10, 2009, from http://www.eric.ed.gov/ERICDocs/data/ericdocs2sql/content_storage_01/0000019b/80/1b/b4/ac.pdf

Choi, K., & Seltzer, M. (2010). Modeling heterogeneity in relationships between initial status and rates of change: Treating latent variable regression coefficients as random coefficients in a three-level hierarchical model. *Journal of Educational and Behavioral Statistics, 35*(1), 54-91.

Conroy, S. J., & Arguea, N. M. (2008). An estimation of technical efficiency for Florida public elementary schools. *Economics of Education Review, 27*(6), 655-663.

Cooper, W. W., Seiford, L. M., & Tone, K. (2007). *Data Envelopment Analysis. A comprehensive text with models, applications, references and DEA-Solver Software. (2nd Edition)*. New York, NY: Springer.

Cordero, J.M. (2006). *Evaluación de la eficiencia con factores exógenos mediante el Análisis Envolvente de Datos. Una aplicación a la Educación Secundaria en España*. Unpublished doctoral dissertation, Universidad de Extremadura, España. Retrieved June 20, 2010, from http://dialnet.unirioja.es/descarga/tesis/1488.pdf

Cordero, J. M., Muñiz, M. A., & Pedraja, F. M. (2006, 2-3 de febrero). *La confusión de la calidad en los servicios públicos: distintos conceptos, distintas medidas,* *distintas soluciones*. Paper presented at the XIII Encuentro de Economía Pública, Playadulce, Playadulce, Almería. Retrieved April 20, 2009, from dialnet.unirioja.es/servlet/fichero_articulo?codigo=3134014&orden=0

Cordero, J.M., Pedraja, F., & Salinas, J. (2005). Eficiencia en educación secundaria e *inputs* no controlables: sensibilidad de los resultados ante modelos alternativos. *Revista de Economía Púbica, 173*(2), 61-83.

Cordero-Ferrera, J.M., Pedraja-Chaparro, F., & Salinas-Jiménez, J. (2008). Measuring efficiency in education: an analysis of different approaches for incorporating non-discretionary *inputs*. *Applied Economics, 40*(10), 1323-1339.

De La Orden, A., Asensio, I., Carballo, R., Fernández, M. J., Fuentes, A, García, J. M., Guardia, S., & Navarro, M. (1997). Desarrollo y validación de un modelo de calidad universitaria como base para su evaluación. *Revista Electrónica de Investigación y Evaluación Educativas, 3*(1-2).

Farrell, M. J. (1957). The measurement of Productive Efficiency. En Journal of the Royal Statistical Society. *Series A (General), 120*(3), 253-290.

Giménez, V., Prior, D., & Thieme, C. (2007). Technical efficiency, managerial efficiency and objective-setting in the educational system: an international comparison. *Journal of the Operational Research Society, 58*(7), 996-1007.

Gómez, J., Buendía, F., Solana, J., & García, J. (2003). Estudio de la eficiencia de los centros de enseñanza secundaria de la ciudad de Murcia a través del Análisis Envolvente de Datos. *Revista de Investigación Educativa, 21*(1), 113-133.

Haelermans, C., & De Witte, K. (2012). The role of innovations in secondary school performance–Evidence from a conditional efficiency model. *European Journal of Operational Research, 223*(2), 541-549.

Hanushek, E. A. (1986). The economics of schooling: production and efficiency in public schools. *Journal of Economic Literature, 24*(3), 1141-1177.

Hernández Pascual, C., & Fuentes Pascual, R. (2003). Eficiencia y eficacia de los institutos públicos de bachiller de la provincia de Alicante. *Revista de Estudios Regionales, 65*, 15-42.

Johnes, J. (2015). Operational Research in education. *European Journal of Operational Research, 243*(3), 683-696.

Kirjavainen, T., & Loikkanen, H. A. (1998). Efficiency differences of Finnish senior secondary schools: An application of DEA and Tobit Analysis. *Economics of Education Review, 17*(4), 377-394.

Klein, A. G., & Muthén, B. O. (2006). Modeling heterogeneity of latent growth depending on initial status. *Journal of Educational and Behavioral Statistics, 31*(4), 357-375.

Leibenstein, H. (1966). Allocative efficiency vs. X-efficiency. *The American Economic Review, 56*(3), 392.415.

Leibenstein, H. (1978a). *General X-efficiency theory and economic development*. New York, NY: Oxford University Press.

Leibenstein, H. (1978b). On the basic proposition of X-Efficiency Theory. *The American Economic Review. Papers and Proceedings of the Ninetieth Annual Meeting of the American Economic Association (May, 1978), 68*(2), 328-332.

Levin, H.M. (1997). Raising school productivity: an X-efficiency approach. *Economics of Education Review, 16*(3), 303-311.

Lockheed, M. E., & Hanushek, E. A. (1994). Concepts of educational efficiency and effectiveness. Human resources development and operations policy working papers; no. HRO 24. Washington, DC: World Bank. Retrieved November 20, 2015, from http://documents.worldbank.org/curated/en/1994/03/698554/concepts-educational-efficiency-effectiveness

López-Martín, E. (2011). *Evaluación de la eficiencia técnica a partir del valor añadido en educación*. Unpublished doctoral dissertation, Universidad Complutense de Madrid, España. Retrieved April 28, 2012, from http://eprints.ucm.es/15220/

Lopez-Martin, E., Kuosmanen, T., & Gaviria, J. L. (2014). Linear and nonlinear growth models for value-added assessment: an application to Spanish primary and secondary schools' progress in reading comprehension. *Educational Assessment, Evaluation and Accountability, 26*(4), 361-391.

Maas, C. J. M., & Snijders, T. A. B. (2003). The Multilevel Approach to Repeated Measures for Complete and Incomplete Data. *Quality & Quantity, 37*(1), 71-89.

Mancebón, M. J., & Bandrés, E. (1999). Efficiency evaluation in secondary schools: the key role of model specification and of Ex Post Analysis of Results. *Education Economics, 7*(2), 131-152.

Mancebón, M. J., Calero, J., Choi, Á., & Ximénez-de-Embún, D. P. (2012). The efficiency of public and publicly subsidized high schools in Spain: Evidence from PISA-2006. *Journal of the Operational Research Society, 63*(11), 1516-1533.

Mancebón, M. J., & Muñiz, M. A. (2008). Private versus public high schools in Spain: disentangling managerial and programme efficiencies. *Journal of the Operational Research Society, 59*(7), 892-901.

Martínez, R. (2009). Usos, aplicaciones y problemas de los modelos de valor añadido en educación. *Revista de Educación, 348*(1), 217-250.

Martínez, R., Gaviria, J.L., & Castro, M. (2009). Concepto y evolución de los modelos de valor añadido en educación. *Revista de Educación, 348*(1), 15-45.

McCaffrey, D. F., Lockwood, J. R., Koretz, D. M., & Hamilton, L. S. (2003). *Evaluating value-added models for teacher accountability*. Santa Monica, CA: RAND Corporation. Retrieved June 3, 2009, from http://www.rand.org/pubs/monographs/2004/RAND_MG158.pdf

Muñiz, M. A. (2001). ¿Son realmente menos eficientes los centros LOGSE? (La evaluación DEA de los institutos de enseñanza secundaria). *Hacienda Pública Española, 157*(2), 169-196.

Noulas, A.G., & Ketkar, K. (1998). Efficient utilization of resources in public schools: a case study of New Jersey. *Applied Economics, 30*(10), 1299-1306.

OECD (2008). *Measuring Improvements in Learning Outcomes. Best Practices to Assess the Value-Added of Schools*. París: OECD Publishing.

Perelman, S., & Santín, D. (2011). Measuring educational efficiency at student level with parametric stochastic distance functions: An application to Spanish PISA results. *Education Economics, 19*(1), 29-49.

Podinovski, V. V., Ismail, I., Bouzdine-Chameeva, T., & Zhang, W. (2014). Combining the assumptions of variable and constant returns to scale in the efficiency evaluation of secondary schools. *European Journal of Operational Research, 239*(2), 504-513.

Raudenbush, S. (2004). What are value-added models estimating and what does this imply for statistical practice? *Journal of Educational and Behavioral Statistics, 29*(1), 121-129.

Raudenbush, S. W., & Bryk, A. S., (2002). *Hierarchical linear models: Applications and data analysis method (2nd Edition).* Newbury Park, CA: Sage.

Raudenbush, S. W., & Xiao-Feng, L. (2001). Effects of study duration, frequency of observation, and sample size on power in studies of group differences in polynomial change. *Psychological Methods, 6*(4), pp. 387-401

Ray, S. C. (1991). Resource-use efficiency in public schools: a study of Connecticut data. *Management Science, 37*(12), 1620-1628.

Ruggiero, J. (1996). Efficiency of educational production: An analysis of New York school districts. *The Review of Economics and Statistics, 78*(3), 499-509.

Santín, D. (2003). Eficiencia técnica y redes neuronales: un modelo para el cálculo del valor añadido en educación. Unpublished doctoral dissertation, Universidad Complutense de Madrid, España. Retrieved August 15, 2008, from http://biblioteca.ucm.es/tesis/cee/ucm-t26531.pdf

Seijas Díaz, A. (2004). *Evaluación de la eficiencia en la educación secundaria*. La Coruña: Netbiblo.

Waldo, S. (2007). On the use of student data in efficiency analysis - Technical efficiency in Swedish upper secondary school. *Economics of Education Review, 26*(2), 173-185

Worthington, A.C. (2001). An empirical survey of Frontier efficiency measurement techniques in education. *Education Economics, 9*(3), 245-268.

**Appendix A**

Table 9. The effect of uncontrollable inputs on efficiency indices.

| | Model: Pass rate | | | Model: Performance | | |
|---|---|---|---|---|---|---|
| | **B** | **Std. Error** | **P value** | **B** | **Std. Error** | **P value** |
| Constant | 0.691 | 0.153 | 0.000 | 0.744 | 0.119 | 0.000 |
| Percentage of girl in school | -0.046 | 0.150 | 0.759 | -0.024 | 0.116 | 0.839 |
| Percentage of immigrant in school | -0.009 | 0.139 | 0.947 | 0.149 | 0.108 | 0.170 |
| Average SES in school | 0.142 | 0.047 | 0.003 | 0.155 | 0.036 | 0.000 |
| Average time devoted to reading | 0.006 | 0.092 | 0.946 | -0.033 | 0.072 | 0.651 |