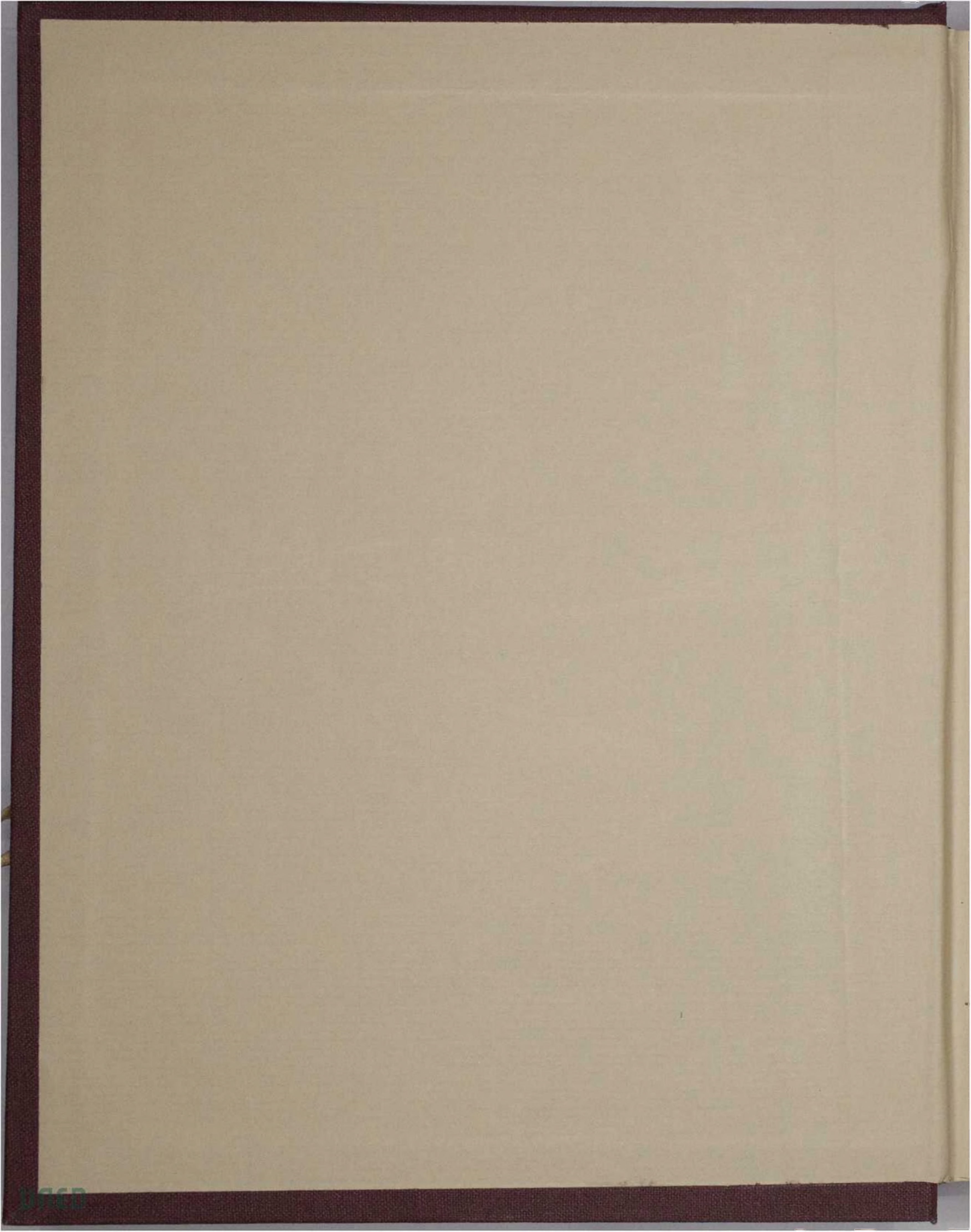


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PHILOSOPHICAL  
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A N D

COLLECTIONS,

To the End of the Year MDCC,

A B R I D G E D,

A N D

Dispos'd under GENERAL HEADS.

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V O L U M E I.

Containing all the

MATHEMATICAL PAPERS.

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By *JOHN LOWTHORP*, M.A. and F.R.S.

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The FIFTH EDITION, Corrected,  
In which the LATIN Papers are now first translated into ENGLISH.

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MDCCXLIX.



PHILOSOPHICAL  
TRANSACTIONS

COLLECTIONS

To the End of the Year 1743

A B R I E F

AND

Dispos'd under GENERAL HEADS

VOLUME I

Containing

MATHEMATICAL PAPERS

BY JOHN WALLIS, M.A. and F.R.S.

Printed by W. B. M. in the Strand

In which are contain'd the Philosophical Transactions for the Year 1743

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In which are contain'd the Philosophical Transactions for the Year 1743



To the HONOURABLE

Sir *ISAAC NEWTON*, Kt:

PRESIDENT;

And to the

COUNCIL and FELLOWS

OF THE

Royal Society of London

For the Advancement of

NATURAL KNOWLEDGE,

THESE

MATHEMATICAL PAPERS,

Abridged and Disposed under GENERAL HEADS,

Are most humbly Dedicated, by



*John Lowthorp.*



To the Honorable

SIR ISAAC NEWTON Kt.

PRESENT

and to the

COUNCIL and FELLOWS

of the

Royal Society of London

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NATURAL KNOWLEDGE

THE

MATHEMATICAL PAPERS

of Sir Isaac Newton

as they are contained in



John Sturges



T H E

P R E F A C E.

**T**HE Philosophical Transactions *having met with general Applause and Encouragement for many Years, it would be a needless Trouble to give any History of them: 'Tis enough to say, that many of the Discourses were composed, and all of them collected and published by particular Members of the Royal Society. I shall therefore employ these very few Pages, only to acquaint the Reader with my own Conduct in this Abridgment of them.*

*When I first resolved upon this Undertaking, I had two Sorts of Readers in view, whom I was desirous to serve; those who make use of Books for their private Instruction or Entertainment, and those who consult them in order to publish something of their own. To a Reader of the former Class, I thought it sufficient to give him the Substance of so many curious Papers, in such Order as would best suit with the Course of those Studies that might denominate him a general Scholar. But, for the Sake of the latter, I have, in the Margin, given the Title and Author of each Paper, and directed to the Number and Page of the Transactions or Collections, where he may meet with the Original itself. To the former, I designed this Abridgment to be as useful as the Volumes at large; and to serve the latter, instead of a not inconvenient Repertorium.: And in the Prosecution of this Design, I have generally confined myself to these Rules.*



I. *I have not only retained the Essential Parts of the Discourses, but I have kept in many Places to the very Words of their own Authors, (except where I was forced to vary them a little, to preserve the Connection:) For, I thought it very unwarrantable to obtrude any thing of mine under the Name of another Person.*

II. *But to shorten the whole Work, wherever I found any Personal Addresses, long and unnecessary Excursions, or pompous Citations of Books, I have taken the Liberty to suppress them; yet, I hope, without injuring the Force of the Author's Reasoning.*

III. *I have omitted all Accounts and Extracts of Books, which now, after so many Years Publication, seem almost useless: Yet, to put the Readers in mind of them, especially such as are about to furnish or enlarge their Libraries, I have added a Catalogue at the End of each Chapter to which they chiefly belong; and I have also directed them to such Additions, Emendations, or Refutations, as ought to be consulted, when those Books fall under their Examination.*

IV. *I have also omitted all Heads of Inquiries and Experiments simply proposed, without farther Prosecution; believing that the Answers already given to many of them, and other Discourses upon the same, or the like Subjects, will sufficiently direct the Notice of an Inquisitive Reader.*

V. *The previous Calculations of Eclipses, Lunar Appulses, and Satellite Eclipses and Occultations; also Tide-Tables, and many other curious Papers of that kind, have long ago outlived the Reason of their Publication.*

VI. *All Simple Catalogues of Natural Curiosities (as of Shells, Minerals, Plants, Animals, &c.) without particular Descriptions of them, are little instructive: and chiefly  
serve*



serve to enlarge the History of the Museum, where they are deposited: Which is no Part of the Design of these Volumes.

VII. I have commonly omitted such Papers as have been collected into just Volumes by their own Authors. For this Reason I have omitted some of those surprizing Microscopical Discoveries by the famous M. Leeuwenhoeck: But I farther confess, I was also less inclined to insert them here, because most of them treat of Subjects not at all convenient (in my Opinion) for common Readers.

VIII. But, to do all the Right I could to the ingenious Authors of those Papers, which the Limits of this Abridgment obliged me to omit, I have, at the End of each Chapter, annexed their Titles, and sometimes a short Account of them.

These are the Rules I have carefully observed thro' the whole Conduct of this tedious Work: Wherein I have faithfully aimed at the General Good of all sorts of Readers; if I have failed in the Performance, 'tis for want of Judgment to do it better: But I am bold to say, That if a kind Reception of this shall encourage a like Abridgment of the Foreign Philosophical Journals, in the same or a better Order, it will much facilitate the many Discoveries still ready to reward the Labours and Expences of all industrious Promoters of Natural Knowledge.



# Advertisement of the Booksellers

## FIFTH EDITION.

**T**HE *Philosophical Transactions*, since the Year 1700, having been abridged, and disposed under General Heads, (after the same Method with this Work) by Mr. *JONES*, and by him published in Two Volumes: We thought it would tend to make the whole complete; if the several *Indices* of the Five Volumes were made into One; by which means the Reader will have but one Trouble in seeking any Particular he has occasion for. To the same Purpose we have thrown the *Contents* of the several Volumes together, and prefixed them to the First.



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**V O L U M E** the First.

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T H E

# Mathematical Papers,

PUBLISHT'D and DISPERS'D

I N T H E

## Philosophical Transactions

A N D

# COLLECTIONS;

A B R I D G ' D,

A N D

## Dispos'd under GENERAL HEADS.

---

C H A P. I.

*Geometry, Algebra, Arithmetick, Logarithmotechny.*

I. 1.



S to what I formerly consider'd, about the Improvement of the Mathematical Sciences, the Result was chiefly this: While Men are destitute of Inclination, Genius, Assistances, and Leisure necessary for these Studies, no wonder if they make no greater Progress in them. Therefore it seems probable to me, that by the Help of the following Means, a tolerable good Remedy may be found for this Evil. That is, if,

*Dr. J. Pell's  
Idea of the Ma-  
thematically. An-  
1638. Phil. Col.  
N. 5. p. 127.*

§. 1. A Mathematical Monitor (as we may call it) be compos'd, which may give proper Answers to these three Questions. 1. What Advantages and of what



what Kind, may be expected from the Study of Mathematicks? 2. What Helps are now in being for attaining so advantageous a Knowledge? 3. What Order is to be observed in making use of those Assistances? Therefore this Monitor should contain,

1. An easy and perspicuous Discourse upon the Limits or Extent of the Mathematical Arts, and of the considerable Advantages that will accrue, not only to the Persons themselves that study them, but likewise to a Nation that abounds in skilful Mathematicians.

2. A Catalogue of Mathematicians, and of Works publish'd by them; which is to exhibit, 1. A Synopsis of all kinds of Mathematical Books, whether such as are already publish'd, or such as are yet unpublish'd, and, being in Manuscript, lie concealed in publick Libraries; proper Numbers or References being affix'd to every Kind. 2. A Chronological Catalogue of all the celebrated Mathematicians, disposed according to the Ages in which they flourished; always subjoining the Year of our Lord in which their Works were first printed. 3. A Catalogue of the same Works, according to the Series of Years, in which they were printed in any Language. In digesting of which I would proceed in such Manner, that marking the Year of our Lord, I would add (as in common Catalogues) the Names of all the Mathematical Books that were publish'd that Year, in any Country or any Language; 1. Shewing in each how much the Volume contain'd, by marking not only whether it was in Folio, Quarto, &c. but the whole Number of Pages, so that the Bulk of the Work might easily be known. 2. Before the Title mentioning the Year, to which any one might turn back, who should desire to know when the Book was wrote, and when it was last publish'd in any Language. 3. Marking in the Margin after the Title, 1. The Year in which any Work was last printed. 2. The Number referring the Reader to the Synopsis, which was given in the first Page of the Catalogue. Now by the Help of these Numbers any one might easily and readily run through all the Mathematical Books belonging to one Subject.

3. An Admonition to the Studious, which are the best Books in every Kind, in what Order and Method they are to be read, what is to be chosen and what omitted in reading some of the minor Mathematicians, how we are to proceed so as to retain every Thing in Memory.

4. An Exhortation and Encouragement to all those, who are sufficiently provided with Wealth, Opportunity, and Ingenuity for the Pursuit of these Studies; that, 1. Having Regard to the great Advantages that redound from hence not only to themselves but to all Mankind in general. 2. As likewise to that pure and sincere Pleasure which arises from the Search of hidden Truths, and from striving with difficult Problems and the Conquest of them; that they may seriously apply themselves to the Advancement of Science, and so much the rather, as, 3. More expeditious Methods are now found out than were known to our Ancestors, which save us much Labour, Time, and Expence. Then an Exhortation to all such as are eminent for setting a right Value on these Studies, and are likewise distinguished for Power and Wealth (which surely may be made instrumental to perpetual Fame, if prudently dispensed) that they  
may



may become Patrons to ingenious Men of this Kind, by proposing handsome Rewards to the most deserving of them, to encourage them to complete such Discoveries as their own Genius's may prompt them to. Lastly, To all Princes and Commonwealths, who cannot easily procure a greater Ornament to their Dominions, than by making it their Endeavour, 1. That they may abound with Persons skilled in these Arts. 2. That the Way leading to them may be made as little laborious and expensive as possible. 3. That Mathematical Genius's may be more publickly known, and meet with such Encouragement as they shall deserve.

For this end it will be very necessary that,

§. 2. A Publick Library may be founded, which may be furnish'd with all the Books abovementioned, and with one Instrument of every Sort that has been yet invented; and besides may have an Endowment sufficient, 1. To purchase Copies of all the Mathematical Books that shall be yearly publish'd any where abroad. 2. To maintain a Library-keeper, whose Business it should be,

1. To read over all the Books of this Kind, which are publish'd in his own Country. 1. Suppressing those which are not wrote according to the Rules of Art, that their Mistakes may not lead their Readers into Error. 2. To admonish Authors, lest they should only republish Things already known, and treated of by others.

2. On Peril of their Reputation that they should approve of notable Inventions, and heartily recommend the Inventors to proper Patrons.

3. To receive, to enter into their Catalogue, and dispose in their proper Repositories, one Copy of the Books so read over; when presented to the Library well bound up, at the Charge of the Author or Bookseller.

4. To give a civil and ready Answer to any studious Person, who shall consult him about any Problem, whether it is already solved or no; lest he should attempt any Thing that is well done already, or on the contrary suppress his Discoveries, out of Fear they may be already known, and perhaps discussed in some of the Books of the Library.

5. To receive, &c. all Manuscripts that may be presented to the Library, or bequeathed to it by Legacy.

6. To keep a constant Literary Correspondence with all Persons of this Kind, that reside in foreign Countries; lest he should be ignorant of what Books are publish'd there.

7. To take Notice among his Countrymen, who are fittest and most expert in instructing others in these Arts.

8. To have an Acquaintance with all kinds of Artificers, who excel in the constructing of Mathematical Instruments and Contrivances, whether they work in Wood, Loadstones, Metal, Glafs, &c.

9. After a fair Trial to give their Testimony, both of speculative Knowledge and practical Dexterity, to practical Men of all Kinds, whether Masters of Ships, Surveyors, Accomptants, &c. that such as have Occasion for this Kind of Men may not be imposed on by ignorant Pretenders, to their great Loss.



The Catalogue will easily inform, which in such a Multitude of Books that almost overwhelm the World, belong only to this Kind of Study. The Library will exhibit a Copy of every such Book, and inform where more Copies may be bought. It will also be a Kind of Storehouse both to Natives and Foreigners, whence they may easily learn, what Assistances that Country can supply to these Studies.

And this, in my Opinion, is the readiest Way of making use of the Helps we are already in Possession of. If more are wanting, it will be necessary, that by the Assistance of skilful Artists,

§. 3. The three following new Treatises may be composed and published.

1. Mathematical Pandects; containing, as perspicuously, methodically, compendiously, and ingeniously as can be done, whatever may be collected, or deduced by Way of Corollary, from the Mathematical Books or Discoveries made before our Time; quoting the most ancient Authors in which they are found, at the End of every Period or Proposition; and so marking in all the following Authors where they have been caught in a Theft, or where they have borrow'd without making any Acknowledgment, or (what is worst of all) have boldly claimed to themselves the Inventions of others. By this means that large Library would be contracted into a much narrower Compass, to a great saving of Labour, Time, and Expence for those that come after; and this much more than any one would imagine at present. But now since this Work would hardly make a portable Volume, there should be prepared also,

2. A Mathematical Companion, containing in a Manual (and therefore as concisely as may be) all the most useful Tables, with Precepts to shew their Application to solving of Problems, whether of pure Mathematicks, or applied to other Subjects.

Finally, that we may not always be confined to Books in this Kind of Learning, there should be contrived,

3. The Self-sufficient Mathematician, or an Instruction to shew how any Mathematician, who is no Enemy to Labour, may acquire so much Skill, that without the Assistance of Books or Instruments he may attain the Solution of any Mathematical Problem, and that as easily as another would solve it by turning over Books.

And this is that Idea of Mathematicks which, in my Manner, I have long ago figured to myself; being always firmly persuaded, that then only we can hope for Assistance in great Undertakings, when we have conceived an exact Idea of them in our Minds, and of the most apposite Means of putting them in Execution. And if we cannot express this Idea in fact, yet it is something to come as near it as may be. I imagine this is so far from being above human Power, that I think the Industry of one Man alone to be equal to it, who is not hindered by his own domestick Affairs, or immersed in a Multitude of busy Cares. For it is evident that the Library and Catalogue may easily be provided, if Money is not wanting. And as to the Pandects above described, if the Task of composing them were committed to me, I should impose upon myself much severer Conditions than I have mention'd there. For first I would delineate



delineate the infallible Process of human Reason in the Investigation of whatever it proposes to itself, by shewing how it proceeds from the first Principles or Rudiments, by an uninterrupted Chain, to the most sublime as well as the lowest Application of them. Which Art perhaps Men would not be long without, if hereafter they should carefully examine, by what Means such Thoughts have arose in the Minds of certain Men whom they admire, how such apt Means have been found out to attain such an End. How these Pandects may be abridg'd into a Manual, such as may be fit for common use, may not be difficult to understand. But so to fix them in their Minds, that they shall have no farther Need of Books (which is what is aimed at by our Self-sufficient Mathematician) will be thought by most to exceed the Power of the human Mind; since no one that I know of has yet ventured to conceive such a Thing in his Mind. Yet I believe that Men will dismiss something of their Incredulity, when they consider seriously with themselves what Arts have been found out for strengthening the Imagination, for assisting the Memory, and for directing the Reasoning Faculty, and what wonderful Effects may be produced by their Conjunction and constant Exercise.

2. It seems to me much more adviseable, if instead of all that Apparatus, Consider'd, Oct. 1639, by Merfennus, ibid. p. 135. which the Author of the Idea proposes, about collecting the various Writings of the Mathematicians, only the best and most deserving were selected. And first those ancient Authors should be chosen, whose Works are still extant; as *Euclid, Apollonius, Archimedes, Theodosius, Pappus, Ptolemy*, with their other Fragments and Manuscripts which have not yet seen the Light, some of which are in *Golius's* Custody at *Leyden*, and some are preserved at *Rome*. Then the more modern may be added to these, as *Vieta, Clavius*, and our *Herigon*. In like manner among the Opticians, should be chosen *Vitellio, Kepler, Aquilonius*, and *Dom. de Villes*. Among Arithmeticians after *Diaphantus*, the best are *Cardan, Tartaglea*, and your Countryman *Nepper*. For Spherical Triangles, and their Computation by Logarithms, you have *Briggs, Gordan, Pitiscus, Snellius*, and our *Morinus*. For Astronomical Matters, after *Ptolemy* and some *Arabians*, all those should be procured who have composed Tables, as *Alphonsus, John Regiomontanus, Kepler*, and our *Duretus*. And to be brief, for Fortification and Musick, eight or ten Authors might be selected, who have most excell'd in the Practice. In like manner for Mechanical Affairs, the Forces of Motion, Machines, and Water-works; so that ten or twelve Authors being perused, those that are curious in such Matters may be easily satisfied. And if twelve Men of good Understanding, having a friendly Correspondence with each other, would take this Affair upon them, so that each of them might compose a Treatise upon a Science, in one convenient and perspicuous Volume; such Things being supplied as are wanting in others, and unnecessary Things being pared away; without doubt we might have in twelve Volumes, in a short and nervous Manner, all that could be desired in this Matter. And in my Opinion, whatever belongs to Mathematicks, either pure or mixt, might be comprehended in those twelve Volumes. So one might deliver all the neater Part of Philosophy in three Books, and all the liberal Arts or mechanic Arts in three more;



so that Learning might be attained at a small Price. Now as to Mathematical Instruments, it would be of little Consequence to have an Apparatus of all that have hitherto been invented. It would be better to have four or five which are best in their Kind, and which shall be judged to be more convenient than any other.

*Answer'd by Dr.  
Pell, ibid. p.  
137.*

3. If I understand you right, learned Sir, you approve of all I have said, only you think I require a greater Apparatus than is necessary. You are of Opinion, that not all the Mathematical Books and Instruments should be collected, which I contend for, but only the best should be selected. I should not oppose this your Opinion, if it was agreed on by all, in such a Multitude of Authors which was the best, which should have the Preference before all others; and if I had this only in view, that something of Labour and Expence might be saved to the Studious in these Arts. But since I made it my Wish, to have the most perfect Foundation for the whole Ambit of the Mathematicks, I ought to delineate no other Scheme, than what might completely answer such a Design. The principal Part of such a Design, as I persuade myself, must consist in such an universal Library as I have described. I ought not to despise any Attempt, much less to condemn any one unheard, that shall throw in his Mite, and endeavour to promote this Undertaking. And if I may give my Judgment, the most trivial Writing or Mathematical Instrument should be preserved, one Copy at least even for its Errors, in some certain accessible Place of every Country. For we see many Things that have been ingeniously invented, in the ruder Instruments of former Ages, which are now not only worthy of observing, but even of being imitated: As some Writers of a lower Class may give very good Hints, and assist the Invention of those of a happier Genius; for we can often point at an Excellence which we cannot arrive at ourselves. We see many Lemmata that have been well demonstrated by this Kind of Writers; yet because of some one fundamental Fallacy, their whole Superstructure has come to the Ground. If you think many are to be rejected, as well for their Trifling and Verbosity, as for Error and false Conclusions; you should consider how different are the Notions and Taste of Mankind, nor should you estimate others by your own Sagacity. For there are some who can understand nothing, unless repeated to them an hundred Times, and that almost in the same Words; those Tautologies therefore are adapted to this Sort of People. And because we must always begin from Things more known, but the same Things are not more known to all; we must make very different Beginnings: So that you can hardly find a Learner, but who may be assisted by a rude Instrument or unpolished Author, and therefore he that undertakes the Office of a Mathematical Monitor should not be ignorant even of these. So that the complete Collection of Books before-mention'd seems to me to be quite necessary.

Now the more I am displeas'd with those minute Mathematicians, the more I should wish for a Library of this Kind, as being the only Method of curing that licentious Itch of Scribbling. For those prating Pretenders, ever trifling in a childish Manner, while they would seem to accommodate themselves to

the



the Capacity of Youth; may see that there are already more than enough, who have compiled Rudiments of this Kind. And they who fondly aim to advance the Mathematical Sciences by an Infinity of new Discoveries, when they see so many empty Paradoxes which have been condemned and laughed at by the Publick, may take Warning by the Miscarriages of others. But especially the Plagiaries, those Pests of all good Literature, will not have the Impudence to vend as their own any old Books, or any Part of them, which perhaps have not been printed more than once. On the other hand, Men of Candour and Ingenuity, able to deliver their Thoughts in a handsome Manner, when they see so many have gone before them, on almost every Subject, will be caution'd not to produce any Thing to the Publick but what is new and their own. Now what has been treated of already may be easily known, either by consulting such an ample Library, or if they will not be at this Trouble, they may be inform'd by the Library-keeper himself, to whose Custody I have committed it. And these are the Reasons in general, which I cannot retract, why I should prefer such an Universal Library as is above described.

4. I had no sooner read your Letter, learned Sir, but I became wholly yours, and was ready to subscribe to your Opinion, which I intirely approve: And likewise an unusual Ardour of Mind hurried me on; so that I would recommend this Undertaking of yours, great as it is, to the great Ones of the World, if I could have free Access to them. But where is the King, that will make a Beginning? For I cannot but call it a truly Royal Design.

*To the Satisfaction of Merfenus, Dec. 10, 1639. Ibid. p. 143.*

5. I inspected the Mathematical Idea only by the bye, and now only remember, that I found nothing in it from which I should dissent; and I much approved that, first, an Inventory of the whole Mathematical Furniture was there exhibited, and then the Self-sufficient Mathematician was described, as containing every Thing in himself. Almost in the same Sense I am used to distinguish two Things in the Mathematicks, the History and the Science. By the History, I mean all that is already found, and is committed to Books. And by the Science, the Skill of resolving all Questions, and therefore of finding by one's own Industry whatever can be found by human Ingenuity in that Science. He that possesses this Faculty does not much want Foreign Assistance, and therefore may very properly be call'd self-sufficient. Now it is much to be wish'd, that this Mathematical History, which lies dispersed in many Volumes, and is not yet intire and complete, were to be all collected into one Book. Nor for this Purpose would there be any Occasion to be at the Charge of seeking or purchasing Books. For since Authors transcribe many Things from one another, nothing is extant any where which may not be somewhere found in any Library that is but moderately furnish'd. Nor is Diligence in collecting all Things so necessary, as Judgment to reject what is superfluous, and Knowledge to supply such Things as are not yet found out. Now if such a Book were at hand, from thence any one might learn the whole Mathematical History, and a good Part of the Science also. But if any one should desire to have the whole that belongs to the Practice, as Instruments, Machines, Engines, &c. if

*The Judgment and Approbation of Des Cartes, Feb. 1640. Ibid. p. 144.*



he was a King, and had the Wealth of the whole World at Command, he could not supply the necessary Expences. Neither indeed is there any Occasion for them. It is enough if he can describe them all, and either knows how to make such as are wanting himself, or can set Artificers to work upon them.

Some of Euclid's Propositions, demonstrated independently from the rest, by Mr. Ash. N. 162. p. 672. 32. 1. E.

II. **T**HE Propositions which I shall endeavour to demonstrate independently from all others, shall be these; the 32d and 47th of the *First Book*; most of the *Second* and *Fifth Books*; the 1st and 16th of the *Sixth*; with their Corollaries. In order to demonstrate the 32d; I suppose it known what is meant by an Angle, Triangle, Circle, External Angle, Parallels, and that the Measure of an Angle is the Arch of a Circle intercepted between its Sides; that a Right Angle is measur'd by a Quadrant, and two Right Angles by a Semicircle. I say then, that in the Triangle  $A B C$ , the External Angle  $B C E$  is equal to the two opposite Internal ones  $A B C$ ,  $B A C$ ; for let a Circle be drawn,  $C$  being the Center, and  $B C$  the Radius; and let  $C D$  be drawn parallel to  $A B$ , those two Lines being always equidistant, will both have the same Inclination to any third Line falling upon them; that is, (by the Definition of Angle) they will make Equal Angles with it: For if any Part of  $C D$  (for Instance) did incline more to  $B C$  than to  $A B$ , upon that very Account they would not be parallel; it follows therefore that the Angles  $A B C$ ,  $B C D$ , are equal: Also  $B A C = D C E$ , because  $A E$  falls upon two Parallels; but the External Angle  $B C E = B C D + D C E$ , which was before prov'd to be equal to  $A B C$ ,  $B A C$ . *Q. E. D.* Hence may be inferr'd as a Corollary, That the three Angles of every Triangle are equal to two Right ones; for the Angles  $A C B + B C E$ , are measur'd by a Semicircle, and therefore equal to two Right Angles: Corollaries also from hence are the 20th, 22d, and 31st of the *Third Book*, which contain the Properties of Circles; whose Deduction from hence is most natural and obvious.

Fig. 1.

20, 22, 31. 3. E.

47. 1. E.

In order to demonstrate the 47th, I suppose the Meaning of the Terms made use of, to be known; and that two Angles or Superficies are equal, when one being put on the other, it neither exceeds, nor is exceeded. This being allow'd, I say, the Sides about the Right Angle are either equal or unequal; if equal, let all the Squares be describ'd; the whole Figure exceeds the Square of the Hypotenuse  $B C$ , by the two Triangles  $M$ ,  $V$ , and exceeds also the Squares of the other two Sides,  $A B$ ,  $A C$ , by the two Triangles,  $A B C$ , and  $S$ ; which Excesses are equal, for  $M$  is equal to  $A B C$ , the two Sides about the Right Angle, being two Sides of a Square upon  $A B$ , by Supposition equal to  $A C$ , and the third Side equal to  $B C$ ; therefore the whole Triangles are equal. After the same manner,  $S$  and  $V$  are proved to be equal; therefore the Square of  $C B$  is equal to the Squares of the two other Sides. *Q. E. D.*

Fig. 2.

Fig. 3.

But if the Sides be unequal, let the Squares be described, and the Parallelogram  $L Q$  compleated; the whole Figure exceeds the Square upon  $B C$ , by three Triangles,  $X$ ,  $R$ ,  $Z$ ; and exceeds also the Square  $L A$ ,  $A D$ , by the Triangle  $A B C$ , and the Parallelogram  $P Q$ : Which Excesses, I say, are equal;

equal;



equal; for Z is equal to A B C, the Side O C = B C, C D = A C, the Angle D = A, and O C D = B C A; which is manifest, by taking the common Angle A C O, out of the two Right Angles B C O, A C D; therefore by Superimposition the whole Triangles are equal. In like manner X is proved equal to A B C, also R; and the Parallelogram P Q to be double of the Triangle A B C: Thus the Excesses being proved equal, the Remainders also will be equal; viz. the Square of B C to the Square of A B, A C. Q. E. D. Manifest Corollaries from hence are the 35th and 36th of the *Third* 35, 36, 3. E. Book; also the 12th and 13th of the *Second*.

The first Ten Propositions of the *Second* Book are evidently demonstrated, 12, 13, 2. E. only by substituting Species or Letters instead of Lines, and multiplying 1, 2, &c. 2 E. them according to the Tenor of the Proposition: Thus, to instance in one or two, Call the whole Line A, and its Parts B and C, therefore  $A = B + C$ , and consequently  $A A = B B + C C + 2 B C$ , which is the very Sense of the *Fourth* of the *Second* Book. Thus also, Let a Line be cut into equal Parts F, F, and let another Line, S, be added thereto; 'tis manifest, that  $4 F F + 4 S F + 2 S S = 2 F F + 2 F F + 2 S S + 4 S F$ ; which is the *Tenth* Proposition of the same Book. 4. 2. E. Fig. 5. 10. 2. E.

Almost the whole Doctrine of Proportionals, viz. Permutation, Inversion, Conversion, Composition, Division of Ratio's, and Proportion *ex æquo*; and consequently the most useful Propositions of the *Fifth* Book, are clearly demonstrated by one Definition, and that is of Similar or Like Parts, which are said to be such as are after the same manner, or equally contain'd in their Wholes: Thus the Antecedents A and C, are either equal to their Consequents, or greater, or less; if equal, the thing is manifest; if less, then (by the Definition of Proportionals) A and C are Like Parts of B and E; therefore what Proportions the whole, B and E, have to one another, the same will A and C have; which is Permutation: Likewise  $E : C :: B : A$ , which is Inversion. So also if from Proportionals you take Like Parts, the Remainders will be proportional; whence Conversion and Division are demonstrated: And if to Proportionals you add Like Parts, the Wholes will still be proportional; which is Composition, &c. If the Antecedents be greater than the Consequents, the Consequents will be Like Parts of them, and the Demonstration exactly the same with the former. Fig. 6.

The first of the *Sixth* Book is prov'd, by considering the Generations of the Parallelograms, which are produced by drawing or multiplying the Perpendicular upon the Basis; that is, taking it so often as there be Parts and Divisions in the Base: Therefore the same Proportion that R X single, hath to N X single, the same hath R X multiplied by X Z, that is, repeated a certain number of times, to N X multiplied by X Z, that is, repeated the same number of times; which is as much as to say,  $R X : N X :: \text{Paral. } R Z : \text{Paral. } N Z$ . Now that this Proposition is also true in Oblique-angled Parallelograms, is proved, because they are equal to the Rectangled ones upon the same Basis, and between the same Parallels, as does thus independently appear; the Triangles R Q X, and M P Z, are equal; for  $R X = M Z$ ,  $Q X = P Z$ ,  $R M = Q P$ ; therefore adding to both  $M Q$ ;  $R Q = M P$ ; if therefore from these



equal Triangles you take what is Common, *viz.* M L Q, the Remainders will be equal, R X L M = Q L Z P; to both which add X L Z, and the whole Parallelograms will be equal, R Z = Q Z. *Q. E. D.* That Triangles also having a common Basis, are in the Proportion of their Altitudes, does hence follow; because they are the Halves of Parallelograms upon the same Basis. This also is true, and the Demonstration exactly the same, in Prisms, Pyramids, Cylinders, and Cones, having the same Basis.

16. 6. E.  
Fig. 6.

To prove the 16th of the *Sixth*, I suppose the 4 Lines, A, B, C, E, to be proportional, that is, granting A and C to be the lesser Terms, the same way that A is contained in B, so is C in E; and that D is the Denominator of the Ratio; 'twill follow then, that B is made up of A, multiplied by D, and E of C, multiplied by D; so that A D = B, and C D = E; draw therefore the Extremes upon one another, that is, A upon C D, and the Means, that is, C upon A D; the Factors being the same, I say, the Products A C D, and C A D, are the same, and consequently equal. *Q. E. D.*

The Problem proposed by M. Comier, (with Ostentation enough) as if it contain'd something New, though in reality it be nothing but the Old Business of doubling the Cube, a little disguised, is easily solved Algebraically, as follows,

Fig. 8.

8. 6. E.	1	$a : 2x :: x : \frac{2x^2}{a} = p$
47. 1. E.	2	$aa + 2ax - \frac{2x^3}{a} = \frac{4x^4}{aa}$
2 X by $aa$	3	$a^4 + 2a^3x = 4x^4 + 2x^3a$
$3 \div a + 2x$	4	$a^3 = 2x^3$ ; that is, The Cube upon $x$ is half the Cube upon $a$ .

The Squaring of  
the Hyperbola, by  
the Lord Viscount  
Brouncker.

N. 34. p. 645.  
Fig. 9.

III. Let A B be one Asymptote of the Hyperbola E d C; and let A E, and B C, be parallel to the other: Let also A E be to B C, as 2 to 1; and let the Parallelogram A B D E be equal to 1.

Supposing the Reader knows, that E A,  $\alpha\zeta$ , K H,  $\beta\eta$ ,  $d\theta$ ,  $\gamma\kappa$ ,  $\delta\lambda$ ,  $\epsilon\mu$ , C B, &c. are in an Harmonick Series, or a Series *Reciproca-Primanorum*, seu *Arithmetice Proportionalium*, (otherwise he is refer'd for Satisfaction to *Arithm. Infinitor. Wallisii*, Prop. 87, 88, 89, &c.) I say,

A B C d



( 11 )

$$ABCdEA = \frac{1}{1 \times 2} + \frac{1}{3 \times 4} + \frac{1}{5 \times 6} + \frac{1}{7 \times 8} + \frac{1}{9 \times 10}, \text{ \&c.}$$

$$EdDE = \frac{1}{2 \times 3} + \frac{1}{4 \times 5} + \frac{1}{6 \times 7} + \frac{1}{8 \times 9} + \frac{1}{10 \times 11}, \text{ \&c.}$$

$$EdCyB = \frac{1}{2 \times 3 \times 4} + \frac{1}{4 \times 5 \times 6} + \frac{1}{6 \times 7 \times 8} + \frac{1}{8 \times 9 \times 10}, \text{ \&c.}$$

For (in Fig. 10. and 11.) the Parallelogram,

And (in Fig. 12.) the Triangle,

Fig. 10, 11, 12.

	$CA = \frac{1}{1 \times 2}$	$EdC = \frac{1}{2 \times 3 \times 4} = \frac{\square dD - \square Fd}{2}$
$dD = \frac{1}{2 \times 3}$	$dF = \frac{1}{3 \times 4}$	$Ebd = \frac{1}{4 \times 5 \times 6} = \frac{\square br - \square bn}{2}$
$br = \frac{1}{4 \times 5}$	$bn = \frac{1}{5 \times 6}$	$dfC = \frac{1}{6 \times 7 \times 8} = \frac{\square fG - \square fk}{2}$
$fG = \frac{1}{6 \times 7}$	$fk = \frac{1}{7 \times 8}$	$Eab = \frac{1}{8 \times 9 \times 10} = \frac{\square aq - \square ap}{2}$
$aq = \frac{1}{8 \times 9}$	$ap = \frac{1}{9 \times 10}$	$bcd = \frac{1}{10 \times 11 \times 12} = \frac{\square cs - \square cm}{2}$
$cs = \frac{1}{10 \times 11}$	$cm = \frac{1}{11 \times 12}$	$def = \frac{1}{12 \times 13 \times 14} = \frac{\square et - \square el}{2}$
$et = \frac{1}{12 \times 13}$	$el = \frac{1}{13 \times 14}$	$fgC = \frac{1}{14 \times 15 \times 16} = \frac{\square gu - \square gb}{2}$
$gu = \frac{1}{14 \times 15}$	$gb = \frac{1}{15 \times 16}$	$\text{\&c.}$
$\text{\&c.}$	$\text{\&c.}$	

Note,



Note,  $\frac{1}{2}CA = dD + dF$   
 $\frac{1}{2}dD = br + bn$   
 $\frac{1}{2}dF = fG + fk$   
 $\frac{1}{2}br = aq + ap$   
 $\frac{1}{2}bn = cs + cm$   
 $\frac{1}{2}fG = et + el$   
 $\frac{1}{2}fk = gu + gb, \&c.$

And that therefore in the first Series half the first Term is greater than the Sum of the two next, and half this Sum of the Second and Third greater than the Sum of the four next; and half the Sum of those Four greater than the Sum of the next Eight, &c. *in infinitum*. For  $\frac{1}{2}dD = br + bn$ ; but  $bn > fG$ , therefore  $\frac{1}{2}dD > br + fG$ , &c. And in the Second Series, half the second Term is less than the Sum of the two next, and half this Sum less than the Sum of the four next, &c. *in infinitum*.

That the first Series are the even Terms, viz. the 2d, 4th, 6th, 8th, 10th, &c. and the second the odd, viz. the 1st, 3d, 5th, 7th, 9th, &c. of the following Series, &c.

$$\frac{1}{1 \times 2}, \frac{1}{2 \times 3}, \frac{1}{3 \times 4}, \frac{1}{4 \times 5}, \frac{1}{5 \times 6}, \frac{1}{6 \times 7}, \&c. \text{ in infinitum. } = 9.$$

Whereof  $a$ , being put for the Number of Terms taken at Pleasure,  $\frac{1}{a^2 + a}$

is the last,  $\frac{a}{a + 1}$  is the Sum of all those Terms from the Beginning, and

$\frac{1}{a + 1}$  the Sum of the rest to the End.

That  $\frac{1}{4}$  of the first Term in the third Series is less than the Sum of the two next, and  $\frac{1}{4}$  of this Sum less than the Sum of the four next, and  $\frac{1}{4}$  of this last Sum less than the next eight, I thus demonstrate:

Let  $a$  be equal to the third or last Number of any Term of the first Column, viz. of Divisors;

$$\frac{1}{a \times a - 1 \times a - 2} = \frac{1}{a^2 - 3a + 2} = \frac{16a^3 - 48a^2}{16a^6 - 96a^5 + 232a^4 + 56a - 24}$$

$$\frac{1}{288a^3 + 184a^2 - 48a} = A.$$

$$\frac{1}{2a \times 2a - 1 \times 2a - 2} = \frac{1}{8a^2 - 12a + 4}$$

$$\frac{1}{2a - 2 \times 2a - 3 \times 2a - 4} = \frac{1}{8a^3 - 36a^2 + 52a - 24}$$



$$\frac{16 a^3 - 48 a^2 + 56 a - 24}{64 a^5 - 384 a^4 + 880 a^3 - 960 a^2 + 496 a - 96} = B.$$

$$\frac{64 a^5 - 384 a^4 + 928 a^3 - 1152 a^2 + 736 a - 192}{64 a^5 - 384 a^4 + 880 a^3 - 960 a^2 + 496 a - 96} \times \frac{1}{4} A = B.$$

And  $48 a^4 - 192 a^3 + 240 a^2 - 96 a =$  Excess of the Numerator above the Denominator.

But the Affirmat.  $\succ$  the Negat.

That is,  $48 a^4 + 240 a^2 \succ 192 a^3 + 96 a$   
 Because  $a^4 + 5 a^2 \succ 4 a^3 + 2 a$   
 $a^4 + 5 a \succ 4 a^2 + 2$  } if  $a \succ 2$ .

Therefore  $B \succ \frac{1}{4} A.$

Therefore one fourth of any Number of A, or Terms, is less than their so many respective B; that is, than twice so many of the next Terms. Q. E. D.

By any one of which three Series it is not hard to calculate, as near as you please, these and the other like Hyperbolic Spaces, whatever be the rational Proportion of A E to B C; as for Example, When A E is to B C, as 5 is to 4. (Whereof the Calculation follows, after that where the Proportion is, as 2 to 1; and both by the *Third* Series.)

First then, when (in *Fig. 9.*)  $A E : B C :: 2 : 1.$

*Fig. 9.*

2x 3x 4) 1. (0. 0416666666]	0. 0416666666
4x 5x 6) 1. (0. 0083333333	} 0. 0113095237
6x 7x 8) 1. (0. 0029761904	
8x 9x10) 1. (0. 0013888888	} 0. 0029019589
10x11x12) 1. (0. 0007575757	
12x13x14) 1. (0. 0004578754	
14x15x16) 1. (0. 0002976190	
16x17x18) 1. (0. 0002042484	} 0. 0007306482
18x19x20) 1. (0. 0001461988	
20x21x22) 1. (0. 0001082251	
22x23x24) 1. (0. 0000823452	
24x25x26) 1. (0. 0000641026	
26x27x28) 1. (0. 0000508758	
28x29x30) 1. (0. 0000410509	
30x31x32) 1. (0. 0000336021)	



32x33x34) 1. (0. 0000278520	}	0. 0001829939
34x35x36) 1. (0. 0000233426		
36x37x38) 1. (0. 0000197566		
38x39x40) 1. (0. 0000168691		
40x41x42) 1. (0. 0000145180		
42x43x44) 1. (0. 0000125843		
44x45x46) 1. (0. 0000109793		
46x47x48) 1. (0. 0000096361		
48x49x50) 1. (0. 0000085034		
50x51x52) 1. (0. 0000075415		
52x53x54) 1. (0. 0000067193		
54x55x56) 1. (0. 0000060125		
56x57x58) 1. (0. 0000054014		
58x59x60) 1. (0. 0000048704		
60x61x62) 1. (0. 0000044068		
62x63x64) 1. (0. 0000040002		

0. 0416666666  
0. 0110095237  
0. 0029019589  
0. 0007306482  
3)0. 0001829939 (0. 0000609980

0. 05679179  
+ 0. 00006100  


---

0. 05685279 < E d C y

But 0. 0007306482 }  
0. 0001829939 } ∴  
0. 0000458315 }

Therefore 0. 05679179  
+ 0. 00004583  
+ 0. 00001528  


---

0. 05685290 > E d C y.

For it has been demonstrated, That  $\frac{1}{4}$  of any Term in the last Column is less than the Term next after it; and therefore that  $\frac{1}{3}$  of the last Term, at which you stop, is less than the remaining Terms; and that the Total of these is less than  $\frac{1}{4}$  of a third Proportional to the two last.

3

And



And therefore  $A B C y E$  being  $= 0.75$  —————  $0.57$   
and  $E d C y > 0.05685279$  ——— and  $< 0.05685290$

And  $A B C d E$  is  $< 0.69314720$  ——— and  $> 0.69314709$

But when  $A E : B C :: 5 : 4$ , or as  $E A$  to  $K H$ ; then will the Space *Fig. 9<sup>o</sup>*  
 $A B C E$ , or now, the Space  $A H K E$  ( $A H = \frac{1}{4} A B$ ) be found as follows.

$8 \times 9 \times 10$ ) I. (0.0013888888] 0.0013888888

$16 \times 17 \times 18$ ) I. (0.0002042484 }  
 $18 \times 19 \times 20$ ) I. (0.0001461988 } 0.0003504472

$32 \times 33 \times 34$ ) I. (0.0000278520 }  
 $34 \times 35 \times 36$ ) I. (0.0000233426 } 0.0000878204  
 $36 \times 37 \times 38$ ) I. (0.0000197566 }  
 $38 \times 39 \times 40$ ) I. (0.0000168691 }

0.0013888888  
0.0003504472  
3) 0.0000878204 (0.0000292735

0.0018271564  
+ 0.0000292735

0.0018564299  $< E a b$

But 0.0003504472 }  
0.0000878204 }  $\div$   
0.00002200737 }

Therefore 0.0018271564  
+ 0.0000220074  
+ 0.0000073358

0.0018564296  $> E a b$

Therefore  $E M b$  (*Fig. 12.*) being  $= 9.025$  —————  $0.025$   
 $E a b > 0.0018564292$  and  $< 0.0018564996$

$E M b a$  (*Fig. 12.*) or  $E K M$  (*Fig. 9.*)  $> 0.02685643$   $< 0.02685650$   
 $A H K E < 0.22314356$   $< 0.22314349$

Therefore



Therefore  $3 \text{ ABCdE} = 2.37944154$   
and  $\text{AHKE} = 0.2231435$

$\text{ABCdE}$  (when  $\text{AE} : \text{BC} :: 10 : 1.$ )  $= 2.3025850$

Therefore,  
The Logar. of 10,  
is to the Log. of 2,  
as 2.302585,  
to 0.693147.

*The Quadrature  
of a Circle, by  
M. Leibnitz,  
Phil. Coll. N. 7.  
p. 204.*

IV. The Quadrature of the Circle, or the turning it into an equal Share, or any other Right-lin'd Figure, (which depends upon the Ratio of the Circle to the Square of its Diameter, or of the Circumference to its Diameter) may be understood to be fourfold, to wit, either by Calculation, or by Linear Construction; and each of them again may be either perfectly exact, or else almost, or pretty near. The Quadrature by accurate Calculation, I call the Analytical; That which is done by accurate Construction, I call the Geometrical: That which is done by Calculation pretty near, I call the Approach; that which is by Construction pretty near, I call Mechanical.

The Approaches have been furthest carried on by *Ludolph van Ceulen*: *Vieta*, *Hugenius*, and others, have given several Mechanical. The accurate Geometrical Construction may be had, by which not only an entire Circle may be measured, but any Section or Arch of it also; which is by an exact and ordinate Motion, but such notwithstanding as suits with Transcendental Curves, which erroneously are accounted Mechanical, though in truth they are as Geometrical as those which are commonly so esteemed; though they are not Algebraical, nor can be reduced to Equations Algebraical, or of certain Degrees; they having Degrees proper to themselves, which though they be not Algebraical, are yet nevertheless Analytical.

The Analytical Quadrature, or that which is made by accurate Calculation, may be again subdivided into three Kinds; namely, into the Analytical Transcendent, the Algebraical, and the Arithmetical. The Analytical Transcendent is to be obtained, amongst others, by Equations of Degrees indefinite, hitherto considered by none: As if  $X^x + X$  be equal to 30, and  $X$  be sought, it will be found to be 3; because  $3^3 + 3$ , is  $27 + 3$ , or 30.

The Algebraical is done by Vulgar Numbers, though irrationally, or by the Roots of common Equations; which for the general Quadrature of the Circle, or its Sectors, is indeed impossible. Now there remains the Arithmetical Quadrature, which is performed by certain Series exhibiting the Quantity of the Circle exact by a Progression of Terms (first) Rational, such as I shall here propound.

I have found therefore, that if the Square of the Diameter be put 1, the

Area of the Circle will be  $\frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \frac{1}{13} - \frac{1}{15} + \frac{1}{17}$ , &c. to wit, the entire Square of the Diameter being

diminished (that it may not be too big) by a third Part; and again (because hereby too much is taken away) being augmented by one fifth; and again (because by this last too much is added) diminished by one seventh; and so onward, continually.

And



And the first Quantity will be too great, viz. 1. but the Error }  $\frac{1}{3}$   
will be less than ----- }  $\frac{1}{3}$

The next too little, viz.  $1 - \frac{1}{3}$ , but the Error will be less than  $\frac{1}{5}$   
 $\frac{1}{3}$   $\frac{1}{5}$

The 3d, too much, viz.  $1 - \frac{1}{3} + \frac{1}{5}$ , but, &c. -----  $\frac{1}{7}$   
 $\frac{1}{3}$   $\frac{1}{5}$   $\frac{1}{7}$

The 4th too little, viz.  $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7}$ , but, &c. -----  $\frac{1}{9}$   
&c.  $\frac{1}{3}$   $\frac{1}{5}$   $\frac{1}{7}$   $\frac{1}{9}$  &c.

The whole Series contains all the Approaches together, or the Values both greater than they ought to be, and less than they ought to be.

So that by continuing the Series, the Errors may be made less than any Fraction given, and consequently less than any assignable Quantity. Whence it follows, that the whole Series must give the true Value. And though the Sum of the whole Series cannot be expressed by one Number, and that the Series be infinitely continued; yet because it consisteth of one regular Method of Progression, the whole may sufficiently enough be conceived by the Mind. And if *Van Ceulen* could have given a Rule by which his Numbers 314159, &c. could have been continued *in infinitum*, he would have given us the Arithmetical Quadrature exact in whole Numbers, which we have here done in Fractions.

There are several things relating to this Quadrature which might be taken notice of, especially one, viz. That the Terms of this our Series,  $\frac{1}{1}, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}, \&c.$  are of Harmonical Progression, or in a continued Harmonical Progression; as will be evident to any one that shall examine them. And a Series

made by skipping, as  $\frac{1}{1}, \frac{1}{5}, \frac{1}{9}, \frac{1}{13}, \frac{1}{17}, \&c.$  is also of Harmonical Progression.

And  $\frac{1}{3}, \frac{1}{7}, \frac{1}{11}, \frac{1}{15}, \frac{1}{19}, \&c.$  is also a Series of Harmonical Proportionals. Wherefore since the Circle is  $\frac{1}{1} + \frac{1}{5} + \frac{1}{9} + \frac{1}{13} + \frac{1}{17}, \&c.$  —

$\frac{1}{3} - \frac{1}{7} + \frac{1}{11} - \frac{1}{15} + \frac{1}{19}, \&c.$  by subtracting the latter partial Series from the former partial Series, the Circle will be the Difference of two Series in Harmonical Progression. And because the Sum of any Number of Terms in Harmonical Progression, how many soever, may by some Compendium be obtained; hence Compendious Approaches (if after *Van Ceulen* there be any need of them) may be deduced; if one would in this our Series take out the Terms affected with the Sign —, by adding the two next into one, +





$\frac{1}{1} - \frac{1}{3}$ , and  $+\frac{1}{5} - \frac{1}{7}$ , and  $+\frac{1}{9} - \frac{1}{11}$ , and  $+\frac{1}{13} - \frac{1}{15}$ , and  $+\frac{1}{17} - \frac{1}{19}$ , and so onward, he will have a new Series for the Magnitude of the Circle, namely,  $\frac{2}{3}$  (that is,  $\frac{1}{1} - \frac{1}{3}$ )  $+$   $\frac{2}{35}$  (that is,  $\frac{1}{5} - \frac{1}{7}$ )  $+$   $\frac{2}{99}$  (that is,  $\frac{1}{9} - \frac{1}{11}$ )  $\&c.$  Wherefore

The Square inscrib'd being  $\frac{1}{4}$

The Area of the Circle shall be  $\frac{1}{3} + \frac{1}{35} + \frac{1}{99} + \frac{1}{195} + \frac{1}{323}$ ,  $\&c.$

But the Numbers 3, 35, 99, 195, 323,  $\&c.$  by Skipping are taken out of the Series of Square Numbers, 4, 9, 16, 25,  $\&c.$  diminished by an Unite, and so made the Series 3, 8, 15, 24,  $\&c.$  out of the Members of which Series, every fourth after the first, is a Number of this our Series. But I have

found (which is worth nothing) the Sum of this infinite Series,  $\frac{1}{3} + \frac{1}{8} + \frac{1}{15} + \&c.$  to be  $\frac{3}{4}$ . Nay, and by culling out by single Skipping, as

$\frac{1}{3} + \frac{1}{15} + \frac{1}{35}$ ,  $\&c.$  the Sum of this infinite Series maketh  $\frac{2}{4}$  or  $\frac{1}{2}$ . But if out of this again another Progression be culled by single Skipping, as

$\frac{1}{5} + \frac{1}{35} + \frac{1}{99}$ ,  $\&c.$  the Sum of that infinite Series shall be the Semicircle,

the Square of the Diameter being 1. Now because by the same Means the Arithmetical Quadrature of the *Hyperbola* is obtain'd, I thought it not amiss to represent to View the whole Harmony.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	$\&c.$
1	4	9	16	25	36	49	64	81	100	121	144	169	196	225	256	289	324	361	400	$\&c.$
0	3	8	15	24	35	48	63	80	99	120	143	168	195	224	255	288	323	360	399	$\&c.$
	$\frac{1}{3}$	$\frac{1}{8}$	$\frac{1}{15}$	$\frac{1}{24}$	$\frac{1}{35}$	$\frac{1}{48}$	$\frac{1}{63}$	$\frac{1}{80}$	$\frac{1}{99}$	$\frac{1}{120}$	$\frac{1}{143}$	$\frac{1}{168}$	$\frac{1}{195}$	$\frac{1}{224}$	$\frac{1}{255}$	$\frac{1}{288}$	$\frac{1}{323}$	$\frac{1}{360}$	$\frac{1}{399}$	$\&c. = \frac{3}{4}$
	$\frac{1}{3}$	$\frac{1}{15}$	$\frac{1}{35}$	$\frac{1}{63}$	$\frac{1}{99}$	$\frac{1}{143}$	$\frac{1}{195}$	$\frac{1}{255}$	$\frac{1}{323}$	$\frac{1}{399}$	$\frac{1}{486}$	$\frac{1}{578}$	$\frac{1}{685}$	$\frac{1}{807}$	$\frac{1}{945}$	$\frac{1}{1100}$	$\frac{1}{1275}$	$\frac{1}{1470}$	$\frac{1}{1685}$	$\&c. = \frac{2}{4}$
	$\frac{1}{8}$	$\frac{1}{24}$	$\frac{1}{48}$	$\frac{1}{80}$	$\frac{1}{120}$	$\frac{1}{168}$	$\frac{1}{224}$	$\frac{1}{288}$	$\frac{1}{360}$	$\frac{1}{440}$	$\frac{1}{528}$	$\frac{1}{624}$	$\frac{1}{735}$	$\frac{1}{860}$	$\frac{1}{1000}$	$\frac{1}{1155}$	$\frac{1}{1325}$	$\frac{1}{1510}$	$\frac{1}{1710}$	$\&c. = \frac{1}{4}$
	$\frac{1}{3}$	$\frac{1}{35}$	$\frac{1}{99}$	$\frac{1}{195}$	$\frac{1}{323}$	$\frac{1}{486}$	$\frac{1}{685}$	$\frac{1}{945}$	$\frac{1}{1275}$	$\frac{1}{1685}$	$\frac{1}{2245}$	$\frac{1}{2965}$	$\frac{1}{3855}$	$\frac{1}{4935}$	$\frac{1}{6225}$	$\frac{1}{7745}$	$\frac{1}{9505}$	$\frac{1}{11525}$	$\frac{1}{13815}$	$\&c. = 10$
	$\frac{1}{8}$	$\frac{1}{48}$	$\frac{1}{120}$	$\frac{1}{224}$	$\frac{1}{360}$	$\frac{1}{528}$	$\frac{1}{735}$	$\frac{1}{990}$	$\frac{1}{1296}$	$\frac{1}{1664}$	$\frac{1}{2112}$	$\frac{1}{2640}$	$\frac{1}{3240}$	$\frac{1}{3920}$	$\frac{1}{4680}$	$\frac{1}{5520}$	$\frac{1}{6440}$	$\frac{1}{7440}$	$\frac{1}{8520}$	$\&c. = 10$

the Circle A B C D, whose Inscribed Square is  $\frac{1}{4}$   
 the Hyperbola CBEHC, whose Power, A B C D, is  $\frac{1}{4}$

Fig. 13.

To the Asymptotes A F, A E, at Right Angles to each other, let there be described the Curve Line of an Hyperbola G C H, whose Vertex is C; and A B C D, the Power or Square, to which every Rectangle made of the Ordinate,



nate, as  $E H$ , and the intercepted Part,  $A E$ , is always equal. About this Square let a Circle be drawn, and let the Hyperbola be continued from  $C$  to  $H$ , so that  $A E$  be double to  $A B$ . Then putting  $A E$  to be  $1$ ,  $A B$  shall be  $\frac{1}{2}$ , and its Square,  $A B C D$ , shall be  $\frac{1}{4}$ , and the Circle, (whose Power  $A B C D$  is inscrib'd) shall be  $\frac{1}{3} + \frac{1}{35} + \frac{1}{99}$ , &c. but the Portion of the Hyperbola  $C B E H C$  (whose Power inscrib'd is the same Square  $\frac{1}{4}$ ) which represents the Logarithm of the Ratio of  $A E$  to  $A B$ , (or of  $2$  to  $1$ ) shall be  $\frac{1}{8} + \frac{1}{48} + \frac{1}{120}$ , &c.

V. 1. Let any Curve  $D Q$  be given, all the Points of which may be referred to any given Right Line  $E A B$ , by the Right Line  $D A$ ; whether  $E A B$  be the Diameter, or any other Line, or any other Lines are given at the same time, any of which, or their Powers, enter the Equation of the Curve, it matters little.

*Tangents to all Geometrical Curves; by Renatus Fran. Slusius. p. 90. Jan. An. 1673. Fig. 14.*

In the Analytical Equation, for the more easy Explanation, let  $D A$  be always called  $v$ , and  $B A$   $y$ . And let  $E B$ , and all the other given Quantities, be denoted by Consonants.

Then suppose the Line  $D C$  to be drawn, touching the Curve in  $D$ , and meeting  $E B$  (produced if necessary) in the Point  $C$ ; and let  $C A$  be always called  $a$ . Now to find  $A C$ , or  $a$ , this will be the general Rule.

1. Rejecting those Parts of the Equation in which  $y$  or  $v$  are not found, let all those be set on one Side of the Equation in which is  $y$ , and those on the other Side in which is  $v$ , with their Signs  $+$  or  $-$ . For Distinction Sake we will call this the Right Side, and that the Left.

2. On the Right Side, let there be prefixt the Exponent of the Power of  $v$  to all the Parts, or, which is the same thing, let the Parts be multiplied by it.

3. Let the same thing be done on the Left Side, that is, prefix to each Part the Exponent of the Power of  $y$ : And farther, in each of the Parts let one  $y$  be changed into  $a$ .

I say that the Equation so form'd will shew the Manner of drawing the Tangent at the given Point  $D$ . For when it is given, at the same time are given  $y$  and  $v$ , and the other Quantities which are expressed by Consonants; and then  $a$  will be known also.

If the Rule shall seem to be any thing obscure, it may be thus illustrated by some Examples. Let this Equation be given,  $b y - y y = v v$ ; in which let  $E B$  be  $b$ ;  $B A$ ,  $y$ ;  $D A$ ,  $v$ ; and let  $a$  or  $A C$  be sought, such as  $D C$  being joined may touch the Curve  $D Q$  in  $D$ . By the Rule nothing is here to be rejected, because either  $y$  or  $v$  is found in every Part of the Equation. Also it is so disposed, that all the Parts in which  $y$  is found are on one Side, and all in which there is  $v$  are on the other. Therefore we need only prefix



to each the Exponents of the Powers of  $y$  and  $v$ ; and on the Left Side one  $y$  is to be changed into  $a$ . Therefore it will be  $ba - 2ya = 2vv$ . I say now, that this Equation will shew the Manner of drawing the Tangent to the Point  $D$ ; that is,  $a = \frac{2vv}{b - 2y} = AC$ .

Thus if the Equation  $qq + by - yy = vv$  were given, the Equation for the Tangent would be the same as the former; for we must reject  $qq$  as the Rule prescribes.

Thus from the Equation  $2byy - y^3 = v^3$ , we have  $4bya - 3yya = 3v^3$ , or  $a = \frac{3v^3}{4by - 3yy}$ . And from the Equation  $bb y + zyy + y^3 = qv$ , we shall have  $bb a + 2zya + 3yya = 2qv$ , and thence  $a = \frac{2qv}{bb + 2zy + 3yy}$ . And from  $b^2 + by^3 - y^4 = qqvv + zv^3$ , we shall have  $3byya - 4y^3 a = 2qqvv + 3zv^3$ , and then  $a = \frac{2qqvv + 3zv^3}{3byy - 4y^3}$ .

And in all other Equations of this kind I think no Difficulty can arise. Perhaps there may be some in such Equations, some Parts of which consist of Products of  $y$  into  $v$ . As  $yv$ ,  $yyv$ ,  $y^3v^2$ , and such like. But this Difficulty will be but a slight one, as will appear from these Examples. For let there be given  $y^3 = bv^2 - yv^2$ . Now there is nothing to be rejected from hence, since  $y$  or  $v$  is found in all the Parts or Terms.

But that they may be disposed as the Rule prescribes, the Term  $yv^2$  is to be assumed twice, and placed as well on the Right Side of the Equation, in which are these Terms that have  $v$ , as on the Left Side, where are the Terms that have  $y$ ; since  $yv^2$  contains both  $y$  and  $v$ . Therefore we must make  $y^3 + v^2y = bv^2 - yv^2$ . Then changing, as before, this Equation into another  $3yya + vva = 2bv - 2yv$ , we shall have  $a = \frac{2bv - 2yv}{3yy + vv}$ .

For the Rule is to be so understood, that on the Left Side the Power of  $v$  is not considered, and therefore the Exponent of  $v^2$  is not to be prefixed to  $yv^2$ , but only that of  $y$ . And on the contrary on the other Side, the Power of  $y$  is not to be considered in  $yvv$ , but only that of  $v$ , and to this its Exponent is to be prefixed. Thus if the Equation were  $y^5 + by^4 = 2qqy^3 - y^2v^3$ , we must make  $y^5 + by^4 + v^3y^2 = 2qqv^3 - y^2v^3$ , and we should have the Equation for the Tangent  $5y^4a + 4by^3a + 2v^3ya = 6q^2v^3 - 3y^2v^3$ , and thence  $a = \frac{6q^2v^3 - 3y^2v^3}{5y^4 + 4by^3 + 2v^3y}$ .

And in these Examples I think I have comprehended all the Variety of Cases that can be proposed. But perhaps it may be of some use to apply what I have delivered in general, to some particular Curve. Therefore let the Curve  $BD$  be given, which has this Property, that taking any Point in it  $D$ , if  $BD$  be joined, and  $DE$  be raised perpendicular to it, meeting the



Right Line B E in E, the Right Line D E may always be equal to the given Right Line B F.

That we may have an Equation in Analytical Terms, make  $DA = v$ ,  $BA = y$ ,  $BF$  or  $DE = q$ . Therefore it will be  $EA = \frac{v v}{y}$ . And since the Square of D E is equal to the two Squares of D A and A E, we shall have the Equation  $q q = \frac{v^2}{y^2} + v^2$ ; or  $q^2 y^2 = v^2 + y^2 v^2$ ; which for the Tangent, as prescribed by the Rule, is to be reform'd to  $q q y^2 - v^2 y^2 = v^2 + y^2 v^2$ , and  $2 q q y a - 2 v^2 y a = 4 v^2 + 2 y^2 v^2$ , whence  $a = \frac{2 v^2 + y^2 v^2}{q^2 y - v^2 y}$ .

Now how such Equations are to be reduced to easier Terms, for the Sake of the Construction, will be no Mystery to skilful Mathematicians. As in this Example, because the Rectangle B A E is supposed equal to the Square of A D, if E A be called  $e$ , it will be  $v v = y e$ , and  $v^2 = y^2 e^2$ , and  $q q = y e + e e$ . Therefore instead of them substituting their Values in the Equation, it becomes  $a = \frac{2 y^2 e^2 + y^2 e}{e^2 y + e y^2 - e y^2}$ , or  $a = \frac{2 e y + y y}{e}$ , that is,  $a e = 2 e y + y y$ , and adding  $e e$  on both Sides,  $a e + e e = e e + 2 e y + y y = (e + y)^2$ . So that the three Quantities  $e$ ,  $e + y$ ,  $a + e$ , or E A, E B, E C, are in continual Proportion; whence the Construction becomes very easy.

But whereas we seem to have supposed hitherto, that the Tangent is always to be drawn towards B, when it may happen from the given Quantities, that it should be either parallel to A B, or should be drawn the contrary Way; we must now determine how this Diversity of Cases is to be distinguished in Equations. A Fraction therefore being made for the Value of  $a$ , as in the foregoing Examples, the Parts of the Numerator and the Denominator are to be considered, as also their Signs.

1. For in both if either all the Parts have the Sign +, or at least if the Affirmative prevail over the Negative, the Tangent must be drawn towards B.

2. If in the Numerator the Affirmative prevail over the Negative, but are equal in the Denominator, a Line drawn through D parallel to A B will touch the Curve in D: For in this case  $a$  will be of an infinite Length.

3. If as well in the Denominator as in the Numerator the affirmative Parts are less than the Negative; then all the Signs being changed, the Tangent must still be drawn towards B: For this case coincides with the first.

4. If in the Denominator they prevail, but in the Numerator are less; the Tangent must be drawn on the contrary Side, that is, A C must be taken towards E.

5. Lastly, if in the Numerator the affirmative Parts are equal to the Negative, however they may be in the Denominator, then  $a$  will become nothing. Therefore either A D will be the Tangent, or E A, or parallel to it, which is easily known from the given Quantities. Now all the Variety of these Cases may be shewn by the Equation to the Circle.

For



Fig. 16.

For let there be a Semicircle whose Diameter is  $EB$ , and let  $D$  be any given Point in it, from whence let fall the perpendicular  $DA = v$ . Make  $BA = y$ ,  $BE = b$ ; the Equation will be  $by - yy = vv$ . And drawing the Tangent  $DC$ , it will be  $AC$  or  $a = \frac{2vv}{b - 2y}$ . Now if  $b$  be greater than  $2y$ ; the Tangent must be drawn towards  $B$ ; if equal, it is parallel to  $EB$ ; if less, it must be drawn towards  $E$ . See 1, 2, 4.

Fig. 17.

Again, let there be given another Semicircle inverted, whose Points are to be conceived as referred to a Right Line parallel to the Diameter, and equal to the same, as in the Scheme. The Parts being denominated as before, and making  $NB = d$ , the Equation will be  $by - yy = dd + vv - 2dv$ .

Therefore  $AC$  or  $a = \frac{2vv - 2dv}{b - 2y}$ . Now since by the Example we have

supposed  $v$  always to be less than  $d$ ; if  $b$  is greater than  $2y$ , the Tangent must be drawn towards  $E$ ; if equal, it will be parallel; if less, then changing all the Signs, it must be drawn towards  $B$ ; as 4, 5, 3. But no Tangent would be drawn, or  $EB$  itself would be the Tangent, if we should suppose  $NB$  to be equal to the Semidiameter, or  $2d = b$  as at *n.* 5.

Fig. 18.

Lastly, let there be another Semicircle, whose Diameter  $NB$  is perpendicular to the Right Line  $BE$ , to which its Points are supposed to be referred. Let  $NB$  be called  $b$ , and let the other Parts be named as before. The Equation will be  $yy = bv - vv$ , and thence  $a = \frac{bv - 2vv}{2y}$ . Now if  $b$  is greater than  $2v$ , the Tangent must be drawn towards  $B$ ; if less, towards  $E$ ; if equal,  $DA$  itself will be the Tangent. As by 1, 4, 5.

And this, I think, is all the Variety of Cases, which can be observed from the Consideration of Equations.

The Lemmata,  
whereby the pre-  
ceding Method is  
demonstrated, by  
M. Slufius.

n. 95. p. 6059.

n. 97. p. 6126.

June, An. 1673.

(1) The Difference of two Dignities of the same Degree, applied to the Difference of the Sides, will give the several Parts of the inferior Degree of a

Binomial of the Sides. Thus  $\frac{y^3 - x^3}{y - x} = y^2 + yx + x^2$ ; as is easy to prove.

(2.) There are so many Parts arising from a Binomial of any Degree, as the Exponent of the Dignity next above has Units. That is, three in the Square, four in the Cube, &c. And this is generally known.

(3.) If the same Quantity is applied to two others, whose Ratio is given, the Quotients will be in the same Ratio reciprocally.

By these Lemmata my Method may easily be demonstrated, they being here disposed in such an Order, as will naturally lead to the Demonstration.

The Testudo  
Veliformis  
Quadrabilis  
Ænigmatically  
propos'd by V. V.

n. 195. p. 584.

Jan. An. 1693.

VI. 1. A Geometrical ÆNIGMA, concerning the wonderful Structure of a Hemispherical Cupola which is quadrable. By D. Pio Lisci, a minute Mathematician.

Among the venerable Monuments of ancient learned Greece, there is still in Being, and for ever will endure, a most August Temple dedicated to Divine Geometry, of a Circular Ichnography, which is cover'd by a Cupola perfectly



perfectly Hemispherical within. But in this there are the equal Areas of four Windows, (disposed about and upon the Base of the Hemisphere) of such a Configuration, Amplitude, and of such Ingenuity, that these being taken away, the remaining curved Superficies of the Cupola, adorned with curious Mosaick Work, is capable of a true Geometrical Quadrature.

Now it is inquired, which is that quadrable Part of the curved Hemispherical Superficies, which is like a distended Nautical Sail, and by what Art or Method the Geometrical Architect attained to it; and lastly, what quadrable Geometrical Plain is it equal to?

2. A Day or two ago, just as I was going to Bed, I received (learned Sir) your Letter, which being very busy I could not answer Yesterday. Within was inclosed a little printed Paper, which, you say, you received from Florence, to be transmitted to me.

*Solved by Dr. Wallis, ibid. p. 587.*

That Paper contains a Geometrical Ænigma, which when stript of its verbal Disguise seems to insinuate the following Problem. From the curved Superficies of a Hemisphere to cut off four equal Segments, so that what remains shall be capable of Quadrature.

And at the same Time it gives us a Hint, that something yet remains among the *Grecian* Monuments, by means of which this may be done.

I imagine this to be the Quadrature of the Lunula of *Hippocrates* of *Chius*.

For whereas *Archimedes* has demonstrated, that the curved Superficies of an Hemisphere is equal to two great Circles of the same Sphere, that is, to four Semicircles; and *Hippocrates* of *Chius* has taught us how to square a certain Lunula: If from every Quarter of this Hemispherical Cupola so much be taken away as that Lunula is deficient from a Semicircle, the Remainder will be equal to the Square that may be inscribed in the great Circle of the Sphere, on which the Hemispherical Cupola insists.

Now if besides the Ænigmatical Involution of the Problem, any Historical Fact about a Temple should be intended, I should imagine the Temple here meant is that of *Sancta Sophia* at *Constantinople*.

*SCHOLIUM.*] By the Quadrature of the Lunula of *Hippocrates* of *Chius*, (mentioned in the first Book of *Aristotle's* *Phyicks*, and in the Commentaries of *Simplicius* upon that Place) the Semicircle *A B D* being divided into two Quadrants, *A C D*, *B C D*, if the Subtense of the Quadrantal Arch *A D* be adapted, bisected in *H* by the Radius *C E*, and if Center *H* the Semicircle *A D F* is described: Because of the Square of the Right Line *A D* being subduple of the Square of the Right Line *A B*, the Semicircle *A D F* will be subduple of the Semicircle *A B D*, and therefore will be equal to the Quadrant *A C D*. Now taking away the common Segment *A D E* from each, the remaining Lunula *A E D F* will be equal to the remaining Triangle *A D C*. And four such *Lunulae* will be equal to four such Triangles, that is, to the whole Square *A D B G* inscribed in the Circle.

Fig. 197

Again, by what *Archimedes* has demonstrated, the Surface of the Sphere is equal to four of its great Circles; and therefore the curved Superficies of



an Hemisphere is equal to four such Semicircles, and a Quadrant of such an Hemispherical Surface is equal to one Semicircle.

Fig. 20.

Let the Circle  $A D B G$  be now the Base of a curved Hemispherical Surface, whose Pole is  $P$ , its Axis  $C P$  perpendicular to the Plain of the Base, and one of its Quadrants  $D P A$ , which is bisected by a Plain  $E P C$  passing through the Axis.

Also for the Convenience of Calculation, let the Radius of the Circle be call'd  $R$ , the Diameter  $D = 2 R$ , the Periphery  $P$ , and the Arch exhibited  $a$ .

And putting the Quadrantal Arch  $D E A = a = \frac{1}{4} P$ , the Semicircle  $A B D$  is  $a R = \frac{1}{4} R P$ : The Triangle  $A D C = \frac{1}{2} R^2 = \frac{1}{4} R D$ . And the Remainder of the Semicircle, when this Triangle is taken away, will be  $\frac{1}{4} R P - R D$ ; an equal to which is to be taken away from  $D P A$ , a Quadrant of the curved Hemispherical Surface, equal to the Semicircle  $A B D$ , that the Remainder may be equal to the exhibited Triangle  $A D C$ .

Now since this may be done various Ways, by what we have shew'd long ago, *An.* 1659. (at the End of the Treatise of the *Cycloid* then publish'd, p. 122. at § 68.) and again *An.* 1670, (in the Treatise concerning Motion, C. V. P. 24.) concerning a Plain Figure, which is equal to any Figure in a Spherical Surface, that is terminated by any Circles greater or lesser. It may be thus done in the simplest Manner.

Fig. 21.

Since the Segments of a Spherical Surface, cut off by parallel Plains, are proportional to the Segments of the Axis; (which obtains likewise in the exhibited Superficies  $D P A$  of the Quadrantal Cuneus) if in the Axis  $C P$  there be taken, as the Semicircle  $\frac{1}{4} R P$  to the Semicircle except the Triangle  $\frac{1}{4} R P - \frac{1}{4} R D$ , that is, as  $P$  to  $P - D$ , so  $C P$  to  $C Y$ : (or, which is the same thing, as  $P$  to  $D$ , so  $C P$  to  $P Y$ ;) A Plain through  $Y Z$ , parallel to the Base, will cut off a Portion of this curved Superficies, adjoining to the Pole, which will be equal to the Triangle  $A D C$ . And the same being done in the other Quadrants of the curved Surface, the whole so cut off, conterminous to the Pole, will be equal to the whole Square inscribed in the Base, and it will be distended as was required. *Q. E. F.*

Or thus more concisely. The curved Surface of the Hemisphere, (being equal to two great Circles) will be equal to  $R P$ . And the Square inscribed in a great Circle  $= 2 R R = R D$ : And that is to this as  $P$  to  $D$ . And therefore, because of the Segments of the Surface cut off by the parallel Plains, (being proportional to the Segments of the Axis) taking  $C P$  to  $P Y$  as  $P$  to  $D$ , not only the whole Superficies will be equal to  $R P$ , but also the Portion at the Pole, cut off by the Plain  $Z Y$ , will be equal to  $R D$ , or to the Square inscribed in the Base. *Q. E. F.*

If it be said that we proceed here upon the Presumption of the Quadrature of the Circle, or of the Ratio of the Perimeter of the Circle to its Diameter; this is true indeed, but not to be objected here. Because the proposed *Ænigma* does not require, that the Portions of the Hemispherical Surface which are cut off, or which are call'd the Windows, should be quadruple, but only the remaining Portion. And indeed if both were required, the absolute Geome-



Geometrical Quadrature of the Circle would be required at the same time; which it is plain is not yet found.

As to the manual Construction of the Cupola; upon a plain Base, situate out of the Base of the Hemisphere, but contiguous to it, whose two Sides meet in an Angle at A, within the protracted Right Lines D A, G A, (that there may be a free Prospect from the Windows on each Side) let a good strong Foundation be laid; so that as the Superstructure rises higher, its Edge may stretch out, supported by an Angle, making an Arch of a Circle as D Z, arising to the Height Y. And let the same be done at the other Angles D, B, G. Lastly upon these Structures, as it were so many Columns raised to the same Height, let the Cupola be placed, so excavated within, as the Surface of an Hemisphere requires. And thus the whole Work is finished as was required.

Fig. 19.

Otherwise. The same thing may be done, if any Square Q Q be given, for the Square inscribed in the Base, which is less than the curved Hemispherical Superficies. For it must be made, as R P the curved Hemispherical Superficies, to Q Q the given Square, so C P the Axis of the Hemisphere, to P Y the Portion of the Axis that is adjacent to the Pole. Then the Plain Z Y, parallel to the Base, will cut off a Portion of the Spherical Superficies which is capable of Quadrature, as being really equal to the given Square Q Q.

The same may be perform'd thus, but with more Trouble.

Since a Quadrant of the curved Hemispherical Surface D P A, as is shewn above, is equal to the Semicircle A B D, and its Segments cut off by Plains parallel to the Base are proportional to the Segments of the Axis; in the Quadrantal Arch D P let there be taken P Q an Arch of 60 Degrees; (which was suggested to me by Mr. Caswell.) Then a Circle Q T S described with the Pole P will bisect the Axis, because of the Versed Sine of 60 Degrees being equal to half the Radius: So that it will divide the Quadrant D P A of the curved Hemispherical Superficies into two equal Segments. One of these, which is the Quadrilinium D Q T S A, will be equal to the Circular Quadrant B C D; and the remaining Trilinium P Q T S will be equal to the Quadrant A C D: Whence if the Bilineum Q R S T be also taken away, equal to the Segment of the Circle A D E; the remaining Trilinium P Q R S will be equal to the Triangle A D C. And four such, in the four Quadrants of the Hemisphere, will be equal to the Square inscribed in the Base. But that Bilineum may be had by what we have formerly proved in the Places above cited.

Fig. 20.

The same may be thus done more universally.

Q being any where taken in the Arch D Z, so that D Q may not be greater than D Z; and how much the Quadrilinium D Q T S A is deficient from the whole to be taken away, so much let the Bilineum Q R S T supply. The Remainder will be equal to the Triangle A D C.

And therefore if Q be taken in D, so that the Quadrilinium may vanish; the Bilineum must be taken equal to the whole that is to be taken away.



But if  $Q$  is taken in  $Z$ , so that there is no Occasion for the Bilineum, the Quadrilinum will be equal to the whole that is to be taken away.

And all the same Things, concerning the Quadrilinum and Bilineum, which together compleat the whole to be taken away, are alike to be accommodated, *mutatis mutandis*, if instead of the Square inscribed in the Base, any other Square  $Q Q$  is substituted, which is not greater than the whole Surface of the Hemisphere.

After this was wrote, and in the Press, I was informed, that the learned Mr. *Leibnitz* had given an Answer to this Problem, and that it was inserted in the Journals of *Leipsick* for the Month of *June* 1692. Upon which I stopt the Press for some Weeks, that I might get a Sight of it, which with some Difficulty I at last obtained. There I saw that that great Man was of my Opinion, that the Problem is not of that Kind which is called Determinate, but may be solved in a great Variety of Manners, or rather infinite Ways.

*The Proposer's  
Solution demon-  
strated, by Dr.  
D. Gregory.  
N. 207. p. 25.  
Jan. An. 1694.*

The Author of this *Ænigma* has now given us a very ingenious and ready Construction of his Problem, in an *Italian* Treatise on the Formation and Mensuration of all Vaults and Cupolas, dedicated to the Grand Duke of *Tuscany*; wherein he has been pleased to give us his Name, by *V V.* the last Disciple of *Galileus*: Whereas before he concealed it by a Transposition of the Letters, as in an Anagram, under the feigned Name of *D. Pio Lischi, pusillo Geometra.*

But now the *Ænigma* is converted by the Author into the following Problem: *Upon the Surface of an Hemisphere to assign a Portion equal to a given Square.* Which he thus constructs.

Fig. 22.

Let the Sphere, whose Axis is equal to the Side of the given Square, be represented by the Circle  $A C B D$ , which is vertical in the proposed Sphere, whose Horizontal Diameter is  $A B$ , and Center  $E$ . Let the Sphere be perforated by two upright Cylinders, whose common Sections with the Plain  $A C B D$  are the Circles  $B L E G$ ,  $A H E I$ , described with the Diameters  $E B$ ,  $E A$ . I say the Thing is done; that is, from every Hemisphere, for Instance the upper  $A C B$ , four bilinear Figures are taken off by the perforating Cylinders, two on the anterior Side, and two on the posterior, which are similar and similarly posited, so that the remaining Hemispherical Superficies is equal to the Square of the Line  $A B$ . And because the Hemispherical Superficies, when the four said Bilinear Spaces are taken away, represents a Sail filled and extended by the Wind, and also an Hemispherical Cupola admitting Light by four Windows, which being constructed upon a Circular Base  $A E B$ , rests upon it at the Points  $A$ ,  $E$ ,  $E$ ,  $B$ . This he calls, according to a Right he has, *The Quadruple Florentine and Veliform Cupola.*

Then the Author in his Treatise delivers many Things which regard the Practice; how by the Assistance of the Lathe and Cylindrical Auger to make Models of this, as well as of five other Cupolas. And for this purpose, he constructs some other curious Problems; the Demonstrations of all which are omitted by the Author, but will easily follow from what is here delivered.

It appears plainly, that the four Windows in the Hemisphere, constructed as above, are Figures equal, similar, and similarly posited. It only remains, that



that we should prove the remaining Hemispherical Superficies is capable of a true Geometrical Quadrature.

In the Point E, equal to the Line E A, let a Line be supposed to be erected, perpendicular to the Plain C A D B; and upon the Periphery ACBD let there be an erect Cylindrical Superficies of the same Height. It is commonly known, that a Portion of the Spherical Surface, comprehended between two Plains parallel to the Circle A B C D, is equal to the Portion of the Cylindrical Superficies between the same Plains; and that like Portions of these Rings, cut off by Plains mutually intersecting one another in the Perpendicular erected at E, are equal also. Now by drawing innumerable Plains parallel to the Base A C B D, if in the Cylindrical Superficies Parts are conceived to be described in the aforesaid Manner, equal to the correspondent Spherical Parts; that which is represented by the Perforation of the Superficies, and taken away opposite to it, will be equal to it. So that it appears the remaining Superficies after the Perforation is equal to the remaining Cylindrical Superficies, excepting that which is determined by the said innumerable Plains, and is opposite to that which is taken away. Let any Diameter P M be drawn, cutting the Periphery A H E any how in H. Join H A, and through H let R T be drawn perpendicular to A B, and parallel to D C drawn through E, meeting the Periphery A C B D in R and T, and the Periphery A I E in I. Upon the Diameter R T let a Semicircle be drawn, whose Periphery is cut by H S and I Q perpendicular to R T, in the Points S and Q. Let the Plain of this Semicircle be conceived to be perpendicular to the Circle A B C D. Whence the Periphery R S Q T will be in the Hemispherical Superficies, and the Right Line H S, now perpendicular to the Plain A C B D, will be the Height of the perforating Cylindrical Superficies above the Point of the Base H. And the same Thing obtains in every Point of the perforating Cylindrical Superficies, *viz.* that its Height to the Superficies of the Sphere above any Point H of the Base, is the Right Line H S produced as before. But H S is equal to H A, the Right Sine of the Arch M A, because either of them is a mean Geometrical Proportional between P H and H M; one in the Circle M A P, the other in a great Circle of the Sphere, passing through the Points M, S, and P.

If in the Perpendicular to the Plain A C B D erected at E, from E be taken a Right Line equal to H S or H A, and from its Extremity parallel Lines are drawn to P M and U N; the Plain drawn through them will be parallel to the Plain A C B D, and these Lines will pass through the Points S and Q, and being produced as far as the Cylindrical Surface conscribed about the Hemisphere, from the Sides of the Cylinder will cut off Right Lines, which will likewise be equal to H S or H A, and will include Arches equal and correspondent to the Arches M N and V P. Now if another Plain Parallel to this at a very small Distance be conceived to be drawn in like Manner, by what is already shewn, these two will mark out in the Cylindrical Surface a Portion of a Ring, equal to the Portion between the same Plains, which is taken away from the Hemispherical Superficies by its Perforation.



foration. Now if the same Construction is supposed to be made at every Point in the Periphery  $AHE$ , all the Portions in the Cylindrical Superficies circumscribed about the Hemisphere, generated and marked out in the Manner aforesaid, will be equal to the Spherical Superficies taken away by the Perforation. Therefore the remaining Hemispherical Surface will be equal to the remaining Cylindrical Surface, composed of all the Right Lines  $HA$ , erected at the respective Points  $M, N, V$ , and  $P$ , or to the Figure of the Right Sines of the Semiperipheries  $ACB, ADB$ ; that is, by what has been long known to Geometricians, to four Times the Square of the Radius  $AE$ , or finally to the Square of the Diameter  $AB$ . And since the two intire Figures contained by the common Section of the aforesaid perforating Cylindrical Surface with the Surface of the Sphere, are equal to the four Halves of the same; it is plain that the remaining Hemispherical Superficies  $ACB$ , taking away the four Bilinear Spaces as in the Construction aforesaid, is equal to the Square of the Diameter  $AB$ . *Q. E. D.*

If the Semiperiphery  $AHE$  is so inflected, that it may coincide with the equal Quadrant of the Periphery  $ARC$ ; the Point  $H$  will fall upon the Point  $M$ , because of equal Arches  $AH, AM$ ; and  $HS$  the Altitude at  $H$  of the Cylindrical Surface insisting upon  $AHE$ , will coincide with its equal  $HA$ , the Altitude at  $M$  of the Figure of Right Sines erected upon  $AMC$ ; and the same Thing obtains in all other Points. Whence the Curve which is the common Interfection of the Spherical Surface with the Cylindrical Surface, inflected upon the Base  $AHE$ , though it does not lie in the same Plain, yet will coincide with it as said before, and therefore is equal to the Curve that terminates the Figure of Right Sines; that is, to the common Section of the Cylindric Surface erected upon the Quadrantal Arch  $ARC$ , with the Plain cutting the Plain of the Base in the Right Line  $BA$  at half Right Angles; or to a Quarter of the Elliptical Curve whose lesser Axis is  $AB$ , and its greater Axis is double in Power to the same. Therefore the Perimeter of the Quadrable *Florentine* Sail, consisting of four such Arches, is equal to the Perimeter of the aforesaid Ellipsis.

Moreover it will not be amiss to add, that the Superficies of two perforating Cylinders within the Sphere, are equal to the Spherical Surface remaining after the Perforation, or to a double *Florentine* Sail, that is, to the double Square of the Diameter. And this appears from hence, that the *Florentine* Sail is equal to four Figures of Right Sines of the Quadrant, and the perforating Superficies is also equal to the same, because it is congruous with them, if the Inflection is as above.

I shall only add one Word more, that the Consideration of the Figure of Right Sines, (the Parts also of which are easily changed into Squares) are sufficient for the Demonstration of all those Things, which are delivered concerning other Solids wrought by the Turning-Lathe, or perforated by a Cylinder, and their Superficies; by the most acute Geometrician *V. V. (Vincenzio Viviani)*, if I mistake not) the very worthy Disciple of *Galilæus*; when he instructs us in the Construction and Mensuration of Vaults or Cupolas. Particularly the Surface of the Roman Boat-like Cupola [*Volta a Schifo alla Romana*]



*mana*] consists of eight Figures of the Right Sines of the Quadrantal Arch, and therefore is equal to the *Florentine* Veliform Cupola. Whence it appears how two Cupolas may be constituted upon equal Squares, one of which is shut up on all Sides, the other perforated by Windows, each of which is double to the Square of the Base.

VII. 1. Drawing the streight Lines  $E A$ , and  $E B$  (cutting the Arc  $A B$  in  $G$ ,) and on  $A G$ , a Perpendicular  $E F$ , (which will therefore pass to the Center  $C$ , beause bisecting  $A G$  at Right Angles;) the Right-lined Triangle  $A F E$  is equal to  $A D E$ , the proposed Portion of the Lunula.

*The Quadrature of the Parts of the Lunula, by Mr. J. Perks, a little varied, by Dr. J. Wallis, N. 259. p. 411. Dec. An. 1699. Fig. 23.*

The Demonstration is to this purpose; *viz.*  $A D B$  being a Quadrantal Arc; the Angle  $A G B$  will be three Halves of a Right Angle; (and its conjunct Angle  $E G A$ , half a Right Angle) and that Angle (being external to the Triangle  $A G E$ ) is equal to the two opposite Intervals  $G E A + E A G$ . Whereof  $G E A$  (because an Angle in the Semicircle  $A E B$ ) is a Right Angle, and therefore  $E A G$  is half a Right Angle, (as are also  $F E G$ , and  $F E A$ ) and the three Triangles  $A F E$ ,  $G F E$  and  $G E A$ , each of them half a Square. And  $A G$  to  $A E$ , as  $\sqrt{2}$  to 1, (proportional to the respective Radii of the two Circles.) And the like Segments  $A D G$ ,  $A E$ , in their respective Circles (as the Squares of their respective Radii) as 2 to 1. And therefore the Semisegment  $A F D$ , equal to the Segment  $A E$ ; and consequently (one taking from the Triangle as much as the other adds to it) the Portion of the Lunula  $A D E$ , equal to the Triangle  $A F E$ . *Q. E. D.*

If the Point  $E$  chance to be in  $K$  (the middle of the Arc  $A E B$ ), there will be no Interfection at  $G$  (the Points  $G$ ,  $B$ , being then coincident, but without any Disturbance to the Demonstration): If it happen beyond it, toward  $B$ , then  $G$  will be on the other side; and what is here said of  $E G B$ , must be accommodated to  $E G A$ .

The Ground of the whole Procefs is plainly this: The Angle  $A C E$ , being an Angle at the Center of the greater Circle, but at the Circumference of the Lesser, the Line  $C D E$  (as it passeth from  $C A$  to  $C B$ ) doth, in the same Proportion, divide the Quadrantal Arc  $A D B$ , and the Semicircular  $A E B$ : Whence all the rest doth naturally follow.

2. If you compleat the two Circles, whose Arcs contain the *Lunula* of *Hippocrates*, the same is true as well of the Points in the other Semicircle  $A C B$ , as of those in the Semicircle  $A E B$ , and for the same Reasons; as appears in the Scheme annexed, wherein I have mark'd the Points in the Semicircle  $A C B$ , (correspondent to those of *Mr. Perks*, in  $A E B$ ) with the correspondent small Letters in the *Roman* and *Greek* Alphabets.

*Improved by Dr. Gregory, ibid. p. 414. Dec. An. 1699. Fig. 24.*

If *Mr. Perks* had made his Construction universal, by making both  $E A$  and  $E B$  meet with the greater Circle, (which he might have done by protracting these Lines and the greater Circle till they meet) he might have found that the Portions of the Spaces  $A C M$ ,  $B H C N$ , (supposing  $M C N$  parallel to  $A B$ ) are Quadrable as well as those of *Hippocrates's Lunula*, and that  $E A \gamma$  being



ing a streight Line, the Portion  $AED$  of *Hippocrates's Lunula*, is to  $A\epsilon\delta$  (the Correspondent of  $A\epsilon CM$ ) in duplicate Proportion of  $C\epsilon$  to  $A\epsilon$ ; for  $ER\epsilon$  (at  $R$  the Center of the lesser Circle) is, in this Case, a Right Angle.

Moreover, If you take any Point  $\epsilon$ , in the Semicircle  $ACB$ , and proceed according to Mr. *Perks's* Construction universaliz'd, as abovesaid, you will find, on the one side, the Trilineum  $A\epsilon\delta$  (contain'd by the Arcs  $A\epsilon$ ,  $A\delta$ , and the streight Line  $\epsilon\delta$ ) equal to the Rectilineal Triangle  $A\epsilon\phi$ ; and on the other side, the Trilineum contained by the Arc  $B\epsilon$  (the Complement of  $\epsilon A$  to the Semicircumference) and the Arc  $B\delta$  (the Complement of  $A\delta$  to the fourth Part of the Circumference) and the streight Line  $\epsilon\delta$ , (that is the Trilineum  $BHC\delta$  diminished by the Segment  $C\epsilon$ ) to be equal to the Rectilineal Triangle  $B\epsilon f$ ; and that those two Spaces  $A\epsilon\delta$ , and the Difference of  $BHC\delta$  from the Segment  $C\epsilon$  (Parts of the *Lunula*  $ACB g \gamma A$ ) taken together, are equal to the Triangle  $ACB$ , as well as to the two Spaces  $AED$  and  $BED$ , Parts of the *Lunula* of *Hippocrates*.

So that upon the whole it appears, that the two Circles (containing the *Lunula* of *Hippocrates*) being compleated, this *Lunula*,  $AEBGA$ , and the other,  $ACB g \gamma A$ , make up one System, and are conjugate Figures.

For, drawing a streight Line  $CDE$ , or  $C\epsilon\delta$ , or  $\epsilon Cd$ , at pleasure, thro'  $C$ , the Center of the greater Circle, and cutting those two Circles, the Space contained within two Arcs of these two Circles, and Part of the said streight Line (as  $AED$ , or  $A\epsilon\delta$ , or  $BH\epsilon d$ ) is equal to the rectilineal Triangle  $AEF$  or  $A\epsilon\phi$ , or  $B\epsilon f$ , respectively.

And so it happens, that if this Line going out from  $C$ , be on the same side of the Diameter  $MN$  with the *Lunula* of *Hippocrates*, the abovesaid Space (which receives a perfect Quadrature) is Solitary; such as are the Parts of *Hippocrates's Lunula*, and of the two Spaces,  $A\epsilon CM$ ,  $BHCN$  (which therefore are Parts of the *Lunula*, more nearly relating to one another.)

But if that Line going out from  $C$  be on the other side of  $MN$ , then the Space which is equal to the Rectilineal Triangle is the Difference of two Mixtilineal Figures (the one a Trilineum, the other a Segment of the lesser Circle), as is abovesaid; neither of which can be squared severally.

All these Particulars are plain from Mr. *Perks's* Demonstration; which, with a little Variation (such as is usual in the different Cases of the same Theorem), is applicable to them all; tho' perhaps he was not aware of it.

The like was done (without any Demonstration) by Mr. *Tschirnhaufe*, in the *Acta Lipsiæ* 1687, to this purpose: If from any Point  $E$ , in the Circumference of the lesser Circle, we let fall on  $AB$  a perpendicular Line cutting it in  $L$ , and draw the Line  $CL$ ; the Triangle  $CAL$  is equal to the Portion of the *Lunula*  $AED$  (and consequently the Triangle  $CBL$  equal to the Portion  $BED$ ); which I shall demonstrate so as the Demonstration may also reach the Portions of the conjugate Space  $ACB g \gamma A$ .

For the Triangles  $ACB$ ,  $AEL$ , are Like Triangles, each being the Half of a Square; and therefore, by 19 *El.* 6. the Triangle  $ACB$  is to the Triangle  $AEL$  in the duplicate Proportion of  $BA$  to  $AE$ , that is, by 8 *El.* 6. as  $BA$  is to  $AL$ : But, by 1 *El.* 6. the Triangle  $ACB$  is to the Triangle



$ACL$  as  $BA$  is to  $AL$ : Therefore, by 9 *El.* 5. the Triangles  $ACL$  and  $AEF$  are equal: But the Triangle  $AEF$  is (by Mr. *Perks*) proved equal to the Portion  $AED$ ; and therefore the Portion  $AED$  is also equal to the Triangle  $ACL$ .

3. On the Center  $B$ , Mr. *Caswell* draws by  $A$  a third Circle, which forms another *Lunula* than that of *Hippocrates*; and he doth (very dextrously) square the Portions of this *Lunula*; and doth hereby let us into a new System, which may be pursued in like manner as Dr. *Gregory* hath done that of *Hippocrates*. By Mr. Caswell, *ib.* p. 417.

4. M. *Tschirnhaufe*, letting fall from  $E$  (on  $AB$ ) a Perpendicular  $EL$ , determines the Angle  $ALC$  equal to the Portion  $ADE$ ; which being admitted, we may thus divide the *Lunula* in any given Proportion; if we divide  $AB$  at  $L$  in such given Proportion,  $CL$  will, in the same Proportion (because of the common Altitude) divide the Triangle  $ACB$  (which is equal to the whole *Lunula*), and  $LE$  (erected at Right Angles on  $ALB$ ) will determine the Point  $E$ ; from whence if we draw to  $C$  the straight Line  $EC$ , this will, at  $DE$ , divide the *Lunula* in the same Proportion. By Dr. Wallis, *ibid.* Fig. 25.

Mr. *Perks*, on  $EDC$  drawing the Perpendicular  $AF$ , determines the Semicquare  $AFE$  equal to the proposed Portion  $ADE$ ; which Semicquare is a Like Figure, and alike situate to  $AE$  as is  $ACB$  to  $AB$ . Fig. 26.

And therefore (because Like Figures are in duplicate Proportion of their respective Sides) if we so inscribe  $AE$ , as that the Square of  $AE$  be to the Square of  $AB$  in such given Proportion, the *Lunula* will, at  $DE$ , be so divided as is required.

And this will hold (if duly applied, according as the different Cases may require) though  $E$  be taken (in the Continuation of the Semicircle) beyond  $A$ ; for, still like Figures will be in duplicate Proportion of their respective sides, and  $CE = CD = DE$ ; and the same is yet improveable much further.

VIII. If upon  $BC$  you take any two Points  $D, E$ , and draw the Perpendiculars  $DH, EM$ , meeting  $BA$  in  $I$  and  $L$ , and cutting a Portion  $FGMH$  of the *Lunula*; the Solid generated by the Conversion of this Portion about the Axis  $BC$ , is equal to a Prism, whose Base is  $ILMH$ , and Height the Circumference of a Circle whose Diameter is  $BC$ ; and the Solid generated by the Semicircle  $BKA$ , is equal to a Prism or Semicylinder, whose Base is the Semicircle  $BKA$ , and Height the Circumference of a Circle whose Diameter is  $BC$ . The Dimension of Solids generated by the Conversion of Hippocrates's Lunula, by M. Ab. de Moivre, N. 265. p. 624. Jul. An. 1700. Fig. 27.

Having bisected  $BA$  in  $R$ , and  $BC$  in  $P$ , the Surface generated by the Conversion of the Arc  $HM$  about the Axis  $BC$ , is equal to  $\frac{c}{r} \times BP \times HM + BR \times DE$  (supposing the Ratio of the Radius to the Circumference to be as  $r$  to  $c$ ) and the Surface generated by the Semicircumference  $BKA$  is equal to a Rectangle whose Base is the Sum of that Semicircumference and Diameter  $BA$ , and Height, the Circumference of a Circle, whose



whose Diameter is  $BC$ . As for the Surface generated by the Arc  $GF$ , 'tis well known that it is equal to a Rectangle, whose Base is the Circumference of a Circle whose Radius is  $BC$ , and Height  $DE$ ; therefore the Surface generated by the Conversion of the Portion  $MHFG$  is known.

Fig. 28.

If upon  $BA$ , you take any two Points  $I, L$ , and draw  $IN, LV$ , perpendicular to it, cutting the Quadrant in  $O$  and  $T$ , and the Circumference in  $N$  and  $V$ ; the Solid generated by the Conversion of the Portion  $ONVT$  about the Axis  $BA$ , is equal to a Prism whose Base is  $IOTL$ , and Height the Circumference of a Circle whose Diameter is  $BA$ .

Having bisected  $BA$  in  $R$ , and drawn  $CR$  meeting the Quadrant in  $G$ , the Surface generated by the Conversion of the Arc  $OT$  about  $BA$ , is equal

$$\text{to } \frac{c}{r} \times CG \times IL - CR \times OT.$$

Fig. 27.

Bisect  $DE$  in  $I$ ; thro' the Center draw  $SQ$ , parallel to  $BC$ , meeting the Circumference  $BKA$  in  $S$ ,  $BK$  parallel to  $AC$  in  $V$ , and the Lines  $DH, EM$ , in  $N$  and  $O$ ; the Solid generated by the Conversion of the Portion  $FGMN$

about the Axis  $AC$ , is  $\frac{c}{r} \times \frac{1}{3} MO^3 - \frac{1}{3} NH^3 + PC \times NOMH +$

$CY \times DNOE - \frac{1}{3} EG^3 + \frac{1}{3} DF^3$ ; and the Solid generated by the Segment

$KBS$  is  $\frac{c}{r} \times \frac{2}{3} VK^3 + PC \times BVKS$ ; therefore the Solid generated by

the Semicircle  $BKA$  about  $AC$ , is  $\frac{c}{r} \times PC \times VQAK + PC \times BCQV$

$- \frac{1}{3} AC^3 + \frac{2}{3} VK^3 + PC \times BVKS$ , which by due Reduction, will be found equal to the Solid generated by the Conversion of the same Semicircle about the Axis  $BC$ .

Fig. 28.

The Solid generated by the Portion  $ONVT$ , about the Axis  $CP$ , is

$$\text{equal to } \frac{c}{r} \times \frac{1}{3} LV^3 - \frac{1}{3} IN^3 - \frac{1}{3} QT^3 + \frac{1}{3} PO^3 + CS \times PQIL.$$

Fig. 27.

From the Points  $M, H$ , drop the two Perpendiculars  $MZ, HW$ , upon  $CA$  prolonged, if need be; the Surface generated by the Conversion of the Arc  $HM$ , about the Axis  $CA$ , is equal to  $\frac{c}{r} \times PC \times HM$

$- RA \times WZ$ , when the Point  $Z$  is next to  $C$ ; or  $\frac{c}{r} \times PC \times HM + RA \times WZ$ ,

when the Point  $W$  is next to it.

Those that will think it worth their while to bestow some little Pains to find the Demonstration of this, may solve the following Problem.

Any



Any two Conic Sections being given, forming a Lunula by their Intersection, and a Right Line being given by Position, about which, as an Axis, this Lunula is imagined to turn, To find the Solid generated by the Conversion of any of its Parts, cut off by Lines perpendicular to that Axis, or parallel to it, or making any given Angle with it; as also the Surfaces made by that Conversion.

IX. Suppose D P V to be Half of an exterior Epicycloid, V B its Axis, V the Vertex, V L B Half the generant Circle, E its Center; D B the Base, C its Center: Bisect the Arc of the Semicircle V B in L, and on the Center C, thro' L, draw a Circle cutting the Epicycloid in P: Then, I say, the Curvilinear Triangle V L P will be = B E q in  $\frac{C E}{C B}$ ; that is, the Square of the Semidiameter of the generant Circle, will be to the Curvilinear Triangle V L P, as C B, the Semidiameter of the Base, to C E; which C E in the exterior Epicycloid is the Sum of the Semidiameters of the Base and Generant; but in the interior Epicycloid D p u, it is the Difference of the said Semidiameters.

*The Quadrature of a Portion of the Epicycloid, by Mr. Catwell. N. 217. p. 113. Oct. An. 1695. Fig. 29.*

COROLL. I.] In the interior Epicycloid, if C E is  $\frac{1}{2}$  C B, the Epicycloid then degenerating into a Right Line, the Quadrature of the Triangle l p u, will be in effect the same with the Quadrature of Hippocrates Chius.

COROLL. II.] If the Semidiameter of the Base is supposed infinite, the Epicycloid then being the common Cycloid, the Area of the said Triangle will be equal to the Square of the Radius of the Generant; and so it falls in with that Theorem which Lalovera found, and calls Mirabile.

The general Proposition from whence I deduced the abovesaid Quadrature, is this, viz. The Segments of the generant Circle are to the correspondent Segments of the Epicycloid, as C B to  $2 C E + C B$ . For Example; Suppose F m G, the Position of Part of the Generant, when the Point F of the exterior Epicycloid was designed, then the Segment F m G n is to the Segment D F n G, as C B to  $2 C E + C B$ ; and consequently the whole Epicycloid to the whole Generant in the same Proportion; which is the only Case demonstrated by M. de la Hire.

It follows also, that in the Vulgar Cycloid its Segments are triple of the correspondent Sectors of the Generant; which was first shewn by Dr. Wallis.

X. The Area of the Cycloid or Epicycloid, whether it be primitive, or contracted, or dilated, is to the Area of the generating Circle; and also the Areas of the generated Parts in the same Curves, to the Areas of the analogous Segments of the Circle; as the Sum of the double Velocity of the Center and the Velocity of the Circular Motion, to the Velocity of the Circular Motion.

*A general Proposition for measuring all Cycloids and Epicycloids, by Mr. Edm. Halley, n. 218. p. 125.*

Demonstration. Let any Epicycloid Y P S Q V B be described, by the Revolution of the Circle V L B, upon the Circular Base Y M N B. Let the

Fig. 30



Center of the generating Circle be  $c$ , and drawing  $c M K$ , let the Circle infist upon the Base in the Point  $M$ , and let  $S$  be the delineating Point. Now dividing the Motions, by the Circular Motion first let the Point  $S$  be transferred to  $R$ , that the Arch  $S M$  may be increased by the indivisible Particle  $R S$ ; then let the Center  $c$  go forward to  $C$ . By this Motion the Segment  $R S M$  being translated to the Situation  $Q T N$ , the Point  $Q$  will reach the Curve. It is plain that the Triangle  $R S M$  is the Moment or Fluxion of the Area of the Circular Segment: But the Trapezium  $Q S M N$  is the Fluxion of the Area of the Curvilinear Space generated at the same Time. Now since  $S M$ ,  $R M$ ,  $Q M$ , are supposed to differ from one another only by a Point, conceive the little Area  $Q S M N$  to consist of three Sectors  $R M S$ ,  $R M Q$ ,  $M Q N$ ; and therefore the little Area  $R M S$  is to the little Area  $Q S M N$ , as the Angle  $R M S$  to the Sum of the three Angles  $R M S + R M Q + M Q N$ . But the Angles  $R M Q + M Q N$  are equal to the Angles  $M C N + M K N$ , or to the Angle  $c M C$ ; because of the Lines  $R M$ ,  $Q N$  inclined to one another in an Angle equal to  $M K N$ , and because of the Angle  $M Q N$  being half  $M C N$ , by *Eucl.* 20. III. Therefore the Angle  $R M S$  is to the Angles  $R M S + c M C$ , that is, (by the same) the Arch  $\frac{1}{2} R S$  to the two Arches  $C c + \frac{1}{2} R S$ , or  $R S$  to  $2 C c + R S$ , as the little Area  $R S M$  to the little Area  $Q S M N$ ; or as the Moment of the Circular Segment  $Q T N$ , to the Moment  $Q S Y M N$  of the Epicycloid generated at the same Time. And as these Moments are always in the same Ratio, wherever the Point  $Q$  is taken, it is plain that the Areas themselves  $Q T N$ ,  $Q S Y M N$ , generated by these Moments, have the same constant Ratio, or that of the Velocity of the Circular Motion  $R S$ , to the double Velocity of the Center adding the Circular Motion, or  $2 C c + R S$ ; also as the Area  $V B Z$  to the Area  $Q V B N$ , and therefore the Semicircle  $V L B$  to the Curvilinear Space  $V Q Y N B$ . So that the Proposition is manifest. Now there is no other Difference in the Manner of demonstrating, if the generating Circle moves upon an Arch of a concave Base, except that the Angle  $c M C$  in this Case is the Difference of the Angles  $M C N$  and  $M K N$ . But if the Base be a Right Line, then  $M K N$  vanishing, and because of the Parallels  $R M$ ,  $Q N$ , the Proof becomes easier. Now in all these Curves there are quadrable Portions, analogous to those Portions in the primary Cycloid, which the learned Dr. *Wallis* has found to be capable of perfect Quadrature: Which easily follows from the Premises.

With Center  $K$  through the Point  $Q$  draw the Circular Arch  $Q Z$ , and draw  $Z B$  cutting off a Segment  $Z L B$ , equal to the Segment  $Q T N$ ; then bisect the Semicircle  $V B$  in  $L$ , and through the Point  $L$ , with the same Center  $K$  describe the Arch  $P L$ , cutting the Epicycloid in  $P$ , the generating Circle in  $T$ , and the Chords  $Q N$ ,  $Z B$ , in  $y$  and  $X$ . Now let the Arch  $V Z = a$ , and its Sine  $= s$ , the generating Radius  $= r$ , and the Radius of the Base  $= R$ . And make the Arch  $C E$ , or the Motion of the Center,  $= m$ . It is plain that the Sector  $C K E$  has the same Ratio to the Space  $X y N B$ , as the Square of  $K E$  has to the Difference of the Squares of  $K L$  and  $K B$ ; or as  $R R + 2 R r + r r$  to  $2 R r + 2 r r$ ; that is, as  $R + r$  to  $2 r$ ,



or  $KE$  to  $BV$ ; and therefore the Rectangle  $BE \times CE$ , or  $rm$ , is equal to the Space  $XyNB$ . But the Space  $VZB$  is equal to the Rectangle  $\frac{1}{2}ar + \frac{1}{2}sr$ ; and therefore according to our Proposition it will be, as  $a$  to  $2m$ , so is  $\frac{1}{2}ar + \frac{1}{2}sr$  to  $\frac{mar + msr}{a}$ , which is equal to the Curvilinear Space  $QVZLBNQ$ . From this subtract the Space  $XyNB = rm$ , and there will remain the Space  $QVZXY = \frac{mrs}{a}$ . And since the Spaces  $ZXL$ ,  $QYT$

are equal to each other, the Space  $QVLTQ$  will also be equal to  $\frac{mrs}{a}$ .

Therefore whenever  $a$  to  $m$ , or the Circular Motion to the progressive Motion of the Center, shall be in a given Ratio; the perfect Quadrature of the Curvilinear Spaces  $QVLTQ$  will be given also. Also the whole Space  $VPL$ , to the Square of the Radius  $BE$ , will be in the same Ratio of the Motions  $m$  to  $a$ , that is, in every primary Epicycloid in the Ratio of the Radii  $KE$  to  $KB$ ; which is the Proposition of Mr. Caswell. But the lesser Spaces  $QVLTQ$  will be to one another as the Sines of the Arches  $VZ$ ; and the Triangular Spaces  $QTP$ , by the same way of arguing, will be as the versed Sines of the Arches  $QT$  or  $ZL$ , and are therefore quadrable. In like manner it may be proved, that the Spaces  $PAR$ ,  $pLu$ ,  $p\lambda r$ , are always to the Square of the Radius  $BE$  (in all these Figures) in the aforesaid Ratio of  $m$  to  $a$ ; and their Portions  $pqt$  as the versed Sines of the intercepted Arches  $qt$ . But the remaining Segments, as  $qt\gamma\lambda$ ,  $qt\gamma\lambda$ , &c. will be as the Right Sines of the Complements of the same Arches  $qt$ .

Now the Ratio of the Velocities  $m$  to  $a$  is compounded of the Ratio of the Radii  $KE$ ,  $BE$ , and the Ratio of the Angles that are equably described at the same Time  $CKE$ ,  $VEZ$ ; and therefore that Ratio of the Angles being given, all the foresaid Epicycloidal Spaces will also be squared.

XI. 1. *A Curve is required with this Property, that the two Segments (of a Right Line drawn from a given Point through the Curve,) being raised to any given Power, and taken together, may every where make the same Sum. We leave it to Analysts to exhibit a general Solution.* A Problem proposed by M. J. Bernouilli. n. 224. p. 387. Jan. An. 1697.

2. The Problem (if I rightly understand it) may be thus proposed. A Curve  $KIL$  is required with this Condition, that if a Right Line  $PKL$  be any how drawn from some given Point or Pole  $P$ , meeting the same Curve in two Points  $K$  and  $L$ ; the Powers of its two Segments  $PK$  and  $PL$ , drawn from the given Point  $P$  to those Points of meeting, if they are raised to an equal Height, (that is, either Squares, or Cubes, or Biquadrates, &c.) in every Position of that Right Line they may make the same Sum,  $PK^q + PL^q$ , or  $PK^{cub.} + PL^{cub.}$  &c. A Solved by H. N. ibid. p. 389. Fig. 31.

*Solution.* Through any given Point  $A$  let an infinite Right-Line be drawn  $ADB$  given in Position, meeting the moveable Right Line  $PKL$  in the Point  $D$ ; and call  $AD$ ,  $x$ ; and  $PK$  or  $PL$ ,  $y$ ; and let  $Q$  and  $R$  be Quantities any how composed of any given Quantities and the Quantity  $x$ ; and let the Relation between  $x$  and  $y$  be denoted by this Equation,  $yy + Qy + R = 0$ .



$R = 0$ . And if  $R$  be a given Quantity, the Rectangle of the Segments  $PK$  and  $PL$  will be given. If  $Q$  be a given Quantity, the Sum of those Segments (conjoined by their proper Signs) will be given. If  $QQ - 2R$  is given, the Sum of the Squares ( $PKq + PLq$ ) will be given. If  $Q^3 - 3QR$  is a given Quantity, the Sum of the Cubes ( $PK^{\text{cub.}} + PL^{\text{cub.}}$ ) will also be given. If  $Q^4 - 4Q^2R + 2R^2$  be a given Quantity, then the Sum of the Biquadrates ( $PKqq + PLqq$ ) will also be given. And so on *ad infinitum*. Therefore let it be provided, that  $R, Q, Q^2 - 2R, Q^3 - 3QR, \&c.$  may be given Quantities, and the Problem will be resolved.  
Q. E. F.

In the same manner Curves may be found, which shall cut off three or more Segments having the like Properties. Let there be an Equation  $y^3 + Qy^2 + Ry + S = 0$ ; where  $Q, R, S$ , denote Quantities composed of any given Quantities whatever, and of the Quantity  $x$  any how involved; in which Case the Curve will cut off three Segments. Now if  $S$  be a given Quantity, the Solid contained by those three will be given. If  $Q$  be a given Quantity, the Sum of three such will be given. If  $QQ - 2R$  be a given Quantity, the Sum of the Squares of three such will be given.

The Use of Fluxions in the Solution of Geometrick Problems; by Mr. Abr. de Moivre. n. 216. p. 52. Mar. An. 1695.

XII. Here you have a Method for the Quadratures of Curvilinear Figures; for the Dimension of Solids generated by the Rotation of a Plain, and of their Superficies; for the Rectification of Curves; and for the Calculation of their Centers of Gravity. But before I go any farther, I would have you understand, that I make use of what the great *Newton* has demonstrated, in Pag. 251, 252, and 253 of his Philosophical Principles, about the momentary Increments or Decrements of Quantities, which either increase or decrease by perpetual Flux; and especially that the Moment of any Power  $A^{\frac{n}{m}}$ , is  $\frac{n}{m} a A^{\frac{n}{m}-1}$ .

Therefore the Fluxion  $\frac{n}{m} a A^{\frac{n}{m}-1}$  being given, on the contrary we may find the flowing Quantity  $A^{\frac{n}{m}}$ , first by removing  $a$  from the Fluxion. Secondly by increasing the Index of the Fluxion by Unity. And thirdly, by dividing the Fluxion by the Index so increased by Unity.

The Absciss of the Curve in what follows shall be denoted by  $x$ , its Fluxion by  $\dot{x}$ ; the Ordinate by  $y$ , and its Fluxion by  $\dot{y}$ .

This supposed, to proceed now to Quadratures, first let the Value of the Ordinate be obtained, by means of the Equation expressing the Nature of the Curve. Secondly, let this Value be multiplied by the Fluxion of the Absciss, and the Rectangle hence arising will be the Fluxion of the Area. Lastly, from this Fluxion of the Area let its Fluent be found, and we shall have the Area required.

Let



Let the Equation  $x^m = y^n$  be proposed, expressing the Nature of any Paraboloid, in which the Value of the Ordinate will be  $y = x^{\frac{m}{n}}$ ; which if multiplied by  $\dot{x}$ , the Rectangle  $x^{\frac{m}{n}} \dot{x}$  will be the Fluxion of the Area. Therefore the Area required will be  $\frac{n}{m+n} x^{\frac{m}{n}+1}$ , or putting  $y$  for  $x^{\frac{m}{n}}$ , it will be

$$\frac{n}{m+n} x y.$$

Again, let a Curve be proposed whose Equation is  $x^4 + a^2 x^2 = y^2$ , (which is the first among Mr. *Craig's* Examples) then assuming  $x \sqrt{x x + a a} = y$ , the Fluxion of the Area will be  $x \dot{x} \sqrt{x x + a a}$ . Now as it is involved in a Radical Sign, let us suppose  $\sqrt{x x + a a} = z$ ; whence  $x x + a a = z^2$ , and therefore  $x \dot{x} = z \dot{z}$ ; and putting  $z \dot{z}$  and  $z$  for  $x \dot{x}$  and  $\sqrt{x x + a a}$ , the Fluxion freed from Surds will be  $z^2 \dot{z}$ . This if we bring back to its Original  $\frac{1}{2} z^3$ , and restore  $\sqrt{x x + a a}$  for  $z$ , we shall have  $\frac{1}{2} x x + a a \sqrt{x x + a a}$  for the Area required.

But that it may farther appear with what Ease these Quadratures may be obtained, I will still add one Example more. Let the Equation of the Curve

be  $\frac{x^2}{x+a} = y^2$ ; therefore  $y = \frac{x}{\sqrt{x+a}}$ , and then  $\frac{x \dot{x}}{\sqrt{x+a}}$  will be the Fluxion of

the Area. Let us suppose  $\sqrt{x+a} = z$ , whence  $x = z z - a$ , and  $\dot{x} = 2 z \dot{z}$ .

Therefore  $\frac{x \dot{x}}{\sqrt{x+a}} = 2 z^2 \dot{z} - 2 a \dot{z}$ , and therefore  $\frac{2}{3} z^3 - 2 a z$ , or  $\frac{2}{3} x - \frac{4}{3} a$

$\sqrt{x+a}$  will be the Area required.

But it often happens that some Curves, such as the Circle, Ellipsis, or Hyperbola, are of such a Nature, that it would be in vain to endeavour to free their Fluxions from Surds; and then the Value of the Ordinate must be reduced to an infinite Series; then every Term of this Series being multiplied by the Fluxion of the Absciss, as above, the Fluent of every Term must be separately found, and the new Series thus arising will exhibit the Quadrature of the Curve proposed.

With the same Ease this Method may be accommodated to the Dimension of Solids form'd by the Rotation of a Plain, by assuming for their Fluxion the Product of the Circular Base into the Fluxion of the Absciss.

Let the Ratio of the Square to the inscribed Circle be called  $\frac{n}{1}$ , the Equa-

tion belonging to the Circle is  $y y = d x - x x$ , and therefore  $4 x \frac{d x \dot{x} - x^2 \dot{x}}{n}$



is the Fluxion of a Portion of the Sphere, and consequently  $4 \times \frac{\frac{1}{2} d x^2 = \frac{1}{3} x^3}{n}$   
 is the Portion itself. The Cylinder circumscribed about this is  $4 \times \frac{d x x - x^3}{n}$ ,  
 and therefore the Ratio of the Portion of the Sphere to the circumscribed Cy-  
 linder is as  $\frac{1}{2} d - \frac{1}{3} x$  to  $d - x$ .

The Rectification of Curves will be obtained, if the Hypothense of the Right-angled Triangle, whose Sides are the Fluxions of the Absciss and Ordinate, is considered as the Fluxion of the Curve. But Care must be taken in the Expression of that Hypothense, that one of the Fluxions only may remain, and only one of the Indeterminate Quantities, which must be that whose Fluxion is retained. This will be plain from the Examples.

Fig. 32.

From the given Right Sine CB, to find the Arch AC, making AB = x, CB = y, OA = r; let CE be the Fluxion of the Absciss, ED the Fluxion of the Ordinate, and CD the Fluxion of the Arch CA. The Property of the Circle is  $2 r x - x x = y y$ , whence  $2 r \dot{x} - 2 x \dot{x} = 2 y \dot{y}$ , and therefore

$$\dot{x} = \frac{y \dot{y}}{r - x}. \text{ But } CD q = \dot{y} \dot{y} + \dot{x} \dot{x} = \dot{y} \dot{y} + \frac{y^2 \dot{y}^2}{r r - 2 r x + x x} = \dot{y} \dot{y} + \frac{y y \dot{y} \dot{y}}{r r - y y} \\ = \frac{r r \dot{y} \dot{y}}{r r - y y}. \text{ Therefore } CD = \frac{r \dot{y}}{\sqrt{r r - y y}}. \text{ But } \frac{r \dot{y}}{\sqrt{r r - y y}}$$

of  $\frac{1}{\sqrt{r r - y y}}$  into  $r \dot{y}$ , or of  $(r r - y y)^{-\frac{1}{2}}$  into  $r \dot{y}$ ; so that if  $(r r - y y)^{-\frac{1}{2}}$  be reduced to an infinite Series, and all its Terms multiplied by  $r \dot{y}$ ; and if we find the Fluent of every Term, we shall have the Length of the Arch AC.

In a like manner the Arch may be found from the versed Sine being given. Let us resume the Equation found above  $2 r \dot{x} - 2 x \dot{x} = 2 y \dot{y}$ , which

$$\text{becomes } \dot{y} = \frac{r \dot{x} - x \dot{x}}{y}. \text{ But } CD q = x \dot{x} + \dot{y} \dot{y} = x \dot{x} + \frac{r r \dot{x} \dot{x} - 2 r x \dot{x} \dot{x} + x x \dot{x} \dot{x}}{y y} \\ = x \dot{x} + \frac{r r \dot{x} \dot{x} - 2 r x \dot{x} \dot{x} + x x \dot{x} \dot{x}}{2 r x - x x}, \text{ or reducing all to the same Denominator,}$$

and expunging those which destroy one another, 'tis  $CD q = \frac{r r \dot{x} \dot{x}}{2 r x - x x}$ , or

$$CD = \frac{r \dot{x}}{\sqrt{2 r x - x x}}. \text{ Therefore the Length of the Arch AC will easily be}$$

obtained, by what has been above delivered.

Sometimes the Fluxion of the Curve is found more easily by a Comparison between the similar Triangles CED and CBO. For we shall have this Proportion, CB . CO :: CE . CD. That is, for the Circle,  $\sqrt{2 r x - x x} . r :: x$

$$\frac{r \dot{x}}{\sqrt{2 r x - x x}},$$



The Curve-line of the Cycloid may be known in the same manner. Let  $ALK$  be a Semicycloid, whose generating Circle is  $ADL$ . Any Point  $B$  being assumed in the Diameter  $AL$ , let  $BI$  be drawn parallel to the Base  $LK$ , meeting the Periphery of the Circle in the Point  $D$ . Let the Rectangle  $AEIB$  be completed, and draw  $FH$  parallel to  $EI$ , and infinitely near it, cutting  $BI$  produced in  $G$ , and the Curve  $AK$  in  $H$ . Make  $AL = d$ ,  $AB = EI = x$ ,  $GH = x$ . It is known that the Right Line  $BG$  is every where the Aggregate of the Arch  $AD$ , and of the Right Sine  $BD$ ; hence it is manifest that the Fluxion  $IG$  is the Aggregate of the Fluxions of the Arch  $AD$ , and of the Right Sine  $BD$ . Now the Fluxion of the Arch  $AD$  is found to be  $= \frac{\frac{1}{2} dx}{\sqrt{dx - xx}}$ , and the Fluxion of the Right

$$\begin{aligned} \text{Sine } BD \text{ is } &= \frac{dx - 2xx}{2\sqrt{dx - xx}}. \text{ And therefore } IG = \frac{dx - xx}{\sqrt{dx - xx}}, \text{ and } IHq \\ &= IGq + GHq = \frac{ddxx - dx \dot{x}}{dx - xx}. \text{ Therefore } IH = \frac{x\sqrt{dd - dx}}{\sqrt{dx - xx}} = \\ &= \frac{\dot{x}\sqrt{d}}{\sqrt{x}} = d^{\frac{1}{2}} x^{-\frac{1}{2}} \dot{x}; \text{ and therefore } AI = 2d^{\frac{1}{2}} x^{\frac{1}{2}} = 2\sqrt{dx} = 2AD. \end{aligned}$$

This Conclusion may be deduced with very little Trouble from the known Property of the Tangent. For since its Particle  $IH$  is always parallel to the Chord  $AD$ , it causes the Triangles  $IGH$  and  $ABD$  to be similar. Whence  $AB \cdot AD :: GH \cdot IH$ . That is,  $x \cdot \sqrt{dx} :: x \cdot IH = \frac{x\sqrt{dx}}{x} = d^{\frac{1}{2}} x^{-\frac{1}{2}} \dot{x}$ .

Now by the Assistance of the Fluxion  $IH$  we may find the Area of the Cycloid. The Fluxion of the Area  $AEI$  is the Rectangle  $EIG = \frac{dx \dot{x} - x^2 \dot{x}}{\sqrt{dx - xx}} = x\sqrt{dx - xx}$ . But the Fluxion of the Portion  $ABD$  does not differ from this; therefore the Area  $AEI$  and the correspondent Portion of the Circle  $ABD$  are always equal.

Let  $AB$  be the Curve of a Parabola, whose Axis is  $AF$ , Parameter  $a$ . Make  $AE = x$ ,  $EB = y$ ,  $AB = z$ ,  $BD = x$ ,  $DC = y$ ,  $BC = z$ ; and assuming an Equation expressing the Nature of the Parabola, suppose  $ax = yy$ , it will be  $a\dot{x} = 2y\dot{y}$ , whence  $x = \frac{2y\dot{y}}{a}$ . But  $BCq = BDq + CDq$ , that is,  $z\dot{z} = x\dot{x} + y\dot{y} = \frac{4y^2\dot{y}\dot{y}}{aa} + y\dot{y} = \frac{4y^2\dot{y}\dot{y} + aa\dot{y}\dot{y}}{aa}$ , and therefore  $z = y \frac{\sqrt{4yy + aa}}{a}$ , or which is all one  $z = y \frac{\sqrt{y^2 + \frac{1}{4}a^2}}{\frac{1}{2}a}$ . If therefore

this



this Quantity is reduced to an infinite Series, the Curve A B may thence be known.

Fig. 35.

Now it easily appears, that if the Hyperbolical Space were given, this would be given also, and *vice versa*. For  $\frac{1}{2} a z = y \sqrt{yy + \frac{1}{4} a a}$ , and therefore  $\frac{1}{2} a z$  will be equal to the Space whose Fluxion is  $y \sqrt{yy + \frac{1}{4} a a}$ . But this Space is nothing else but the exterior equilateral Hyperbola, whose Semi-axis A B =  $\frac{1}{2} a$ , the Abscissa A E =  $y$ , and the Ordinate E G =  $x$ .

For the Mensuration of the Superficies produced by the Conversion of a Curve about its Axis, there must be assumed for its Fluxion a Cylindrical Superficies, whose Altitude is the Fluxion of the Curve itself, and whose Distance from the Axis is the Ordinate belonging to this Fluxion.

Fig. 32.

For Example, let A C be the Arch of a Circle, which by revolving about the Axis A B may generate a Spherical Superficies, which we undertake to measure.

The Fluxion of the Arch D C is already found to be  $\frac{r x}{\sqrt{2 r x - x x}}$ .

If we multiply this by the Circumference belonging to the Radius, B C, that is by  $\frac{c}{r} \sqrt{2 r x - x x}$ , (supposing the Ratio of the Circumference to the Ra-

dius to be =  $\frac{c}{r}$ ) we shall have the Fluxion of the Spherical Surface =  $c x$ , and therefore the Surface itself is  $c x$ .

As to what belongs to Centers of Gravity, having found the Fluxion of the Superficies or Solid, and multiplying this into its Distance from the Vertex, we must then return back to the Fluent. This divided by the Superficies or Solid itself, will give the Distance of the Center of Gravity from the Vertex.

Let the Center of Gravity of all the Paraboloids be to be found. Their Fluxion is thus expressed in a general Manner  $x^{\frac{m}{n}} x$ , which multiplied by  $x$  becomes  $x^{\frac{m}{n} + 1} x$ , whose flowing Quantity  $\frac{n}{m + 2n} x^{\frac{m}{n} + 2}$  being divided by the Paraboloids Area  $\frac{n}{m + n} x^{\frac{m}{n} + 1}$  will give  $\frac{m + n}{m + 2n} x$  for the Distance of the Center of Gravity from the Vertex.

The Center of Gravity in a Portion of a Sphere is found much in the same Manner. For its Fluxion  $4 \frac{d x x - x x x}{n}$  being drawn into  $x$  becomes  $4$

$\frac{d x^2 x - x^3 x}{n}$ , of which the Fluent is  $4 \frac{\frac{1}{2} d x^3 - \frac{1}{4} x^4}{n}$ , which divided by the

Solidity



Solidity of the Portion, that is, by  $4 \frac{\frac{1}{2} d x^2 - \frac{7}{3} x^3}{n}$ , produces  $\frac{\frac{1}{2} d - \frac{1}{4} x}{\frac{1}{2} d - \frac{1}{3} x} x$ , or  $\frac{4d - 3x}{6d - 4x} x$ , for the Distance of the Center of Gravity from the Vertex.

XIII. I. Prop. I. Prob.] *To find the Relation between the Fluxion of the Axis, and the Fluxion of the Ordinate, in the Curve called Catenaria.*

*The Catena; by Dr. Dav. Gregory. N. 231. p. 637. Aug. An. 1697. Fig. 36.*

Let F A D be a Chain hanging by its Ends F and D, whose lowest Point, or Vertex of the Curve, is A, its Axis A B perpendicular to the Horizon, and its Ordinate B D parallel to the same. We are to find the Relation between B b, or D d, and  $d\delta$ ; supposing the Point b to be infinitely near B, and b d to be parallel to B D, as also D d parallel to B A.

It appears from Mechanicks, that three Powers constituted in Æquilibrium have the same Ratio as three Right Lines that are parallel to their Directions, or which are inclined to them in a given Angle, being terminated by their mutual Concourse. So that if D d denotes the absolute Gravity of the Particle D d, (as would necessarily be in a Chain every where equally thick) then  $d\delta$  will represent that Part of the Gravity that acts perpendicularly upon D d, by which it is brought about, (because of the Flexibility of the Chain moving about d) that d D endeavours to reduce itself to a vertical Situation. Therefore if  $\delta d$ , or the Fluxion of the Ordinate B D, be supposed constant, the Action of Gravity exerted perpendicularly upon the correspondent Parts of the Chain d D, will also be constant, or every where the same. Let this be expounded by the Right Line a. Again, by the Mechanicks before cited, D d, the Fluxion of the Axis A B, will denote the Force, which is exerted according to the Direction of d D, which is equivalent to the aforesaid Endeavour of the heavy Line d D to reduce itself into a Vertical Situation, and which prevents its doing so. Now this Force arises from the heavy Line D A drawing according to the Direction d D, and therefore (*cæteris paribus*) is proportional to the Line D A. Therefore  $\delta d$ , the Fluxion of the Ordinate, is to  $\delta D$ , the Fluxion of the Absciss, as the constant Right Line a is to the Curve D A. Q. E. F.

COROL.] If the Right Line T D touches the Catenaria, and meets the Axis B A produced in T, it will be B D : B T :: ( $d\delta : \delta D ::$ ) a. Curve D A.

Prop. II. Theor.] *If to the Perpendicular A B as an Axis, with Vertex A, an Equilateral Hyperbola A H be described, whose Semiaxis A C is equal to a; and to the same Axis and Vertex, a Parabola A P be drawn, whose Parameter is equal to four Times the Axis of the Hyperbola; and if the Ordinate H B of the Hyperbola be continually produced, till H F is equal to the Curve A P; I say the Curve F A D, in which the Points F and D are found, (supposing B D = B F) will be the Catenaria.*



Make  $AB=x$ , then  $Bb=\dot{x}$ , and  $BH=\sqrt{2ax+xx}$ . Whence by the Method of Fluxions, the Fluxion of  $BH=\frac{a\dot{x}+x\dot{x}}{\sqrt{2ax+xx}}=mb$ . Again, because the Parameter of the Parabola  $AP$  is  $=8a$ , 'tis  $BP=\sqrt{8ax}$ . Whence  $np$  the Fluxion of  $BP$  will be  $\frac{2ax}{\sqrt{2ax}}$ . So that the Fluxion of the Curve  $AP=$

$Pp=\sqrt{np \times np + Pn \times Pn}=\sqrt{\frac{4a^2\dot{x}^2}{2ax} + \dot{x}^2}=\sqrt{\frac{2ax^2+x\dot{x}^2}{x}}$ , which by multiplying both Numerator and Denominator into  $\sqrt{2a+x}$  becomes  $\frac{2ax+xx}{\sqrt{2ax+xx}}$ . And since  $HE$  is every where equal to  $AP$ , the Fluxion of

the Right Line  $HF$ , that is  $mb+sf$ , will be equal to  $\frac{2ax+xx}{\sqrt{2ax+xx}}$ . But it

is already found that  $mb=\frac{a\dot{x}+x\dot{x}}{\sqrt{2ax+xx}}$ . Whence  $sf=\frac{a\dot{x}}{\sqrt{2ax+xx}}$ , which is the Fluxion of  $BF$  the Ordinate to the Axis of the Catenaria. Therefore the Fluxion of the Curve  $AF$ , or  $Ff=\sqrt{sf^2+Fs^2}=\frac{\sqrt{a^2\dot{x}^2}}{2ax+xx} + \dot{x}^2 = \frac{a\dot{x}+x\dot{x}}{\sqrt{2ax+xx}}$ , of which the Fluent is  $\sqrt{2ax+xx}$ , as

just now found. Therefore  $AF=\sqrt{2ax+xx}$ . And it appears that the Fluxion of the Ordinate  $BF$ , or  $\frac{a\dot{x}}{\sqrt{2ax+xx}}$ , is to  $\dot{x}$  the Fluxion of the

Absciss  $AB$ , as the given Line  $a$  to the Curve  $AF$ ; which is the Property of the Catenaria found above. Therefore the Points of the Catenaria are rightly determined by the foregoing Construction. *Q. E. D.*

*COROL. 1.]* From the Construction it appears, that  $BF$ , the Ordinate of the Catenaria, is equal to the Parabolical Curve  $AP$ , taking away  $BH$  the correspondent Ordinate of the conterminatè Hyperbola  $AH$ .

2. From the Demonstration it appears, that the Curve of the Catenaria  $AF$  is equal to  $BH$  the correspondent Ordinate of the conterminatè equilateral Hyperbola. For since the Fluxions of these Lines are equal, and the Lines themselves are nascent at the same Time, it is plain they must be always equal. Whence the Chain being given,  $AC$  or  $a$  will be given also, as being equal to the Semiaxis of the Equilateral Hyperbola whose Vertex is  $A$ , and Ordinate equal to the Absciss  $AB$  of the Chain  $AD$ .

3. All Catenaria are similar to one another, since they are generated by a like Construction of like Figures similarly posited. Whence two Right Lines alike inclined to the Horizon, drawn through the Vertices of the Chains, will



will cut off similar Figures, and Portions of the Chains which are proportional to the Right Lines so cutting them off.

4. If the Chain  $QAD$  is suspended at the Points  $Q$  and  $D$ , which are at unequal Heights, the Part of the Curve  $FAD$  continues the same as if it had been suspended at the Points  $F$  and  $D$ , which are equally high; because it is all one whether the Point  $F$  be fixt to the Horizontal Plain or not.

5. If the Force of the Chain, drawing according to the Direction  $dD$ , be denoted by  $Dd$ ; let it be divided (as is commonly known) into the Force  $d\delta$ , according to a Horizontal Direction, and a Force  $\delta D$ , according to a Vertical Direction. Therefore in the Extremity of the Chain, the Force of approaching directly to the Axis, is to the Force of perpendicular Descent in the same; or the Part of the sustaining Force acting according to the Direction  $BD$ , is to a Part of the same acting according to the Direction  $D\delta$ , as the Semiaxis of the conterminat Hyperbola  $AH$ , is to  $DA$  the Length of the Chain to the Vertex of the Curve. Whence when the Chain is given, this Ratio is given. And in the same Chain suspended more or less loosely, that Horizontal Force is as the Axis of the conterminat Hyperbola, since  $DA$  remains the same, when the Extremes of the Chain are equally high.

6. In a Vertical Plain, but in an inverted Situation, the Chain will preserve its Figure without falling, and therefore will constitute a very thin Arch or Fornix: That is, infinitely small, rigid, and polish'd Spheres, disposed in an inverted Curve of a Catenaria, will form an Arch, no Part of which will be thrust outwards or inwards by other Parts, but the lowest Parts remaining firm, it will support itself by means of its Figure. For since the Situation of the Points of the Catenaria is the same, and the Inclination of the Parts to the Horizon, whether in the Situation  $FAD$ , or in an inverted Situation, so that the Curve may be in a Plain which is perpendicular to the Horizon; it is plain that it must keep its Figure unchanged as well in one Situation as the other. And on the contrary, none but the Catenaria is the Figure of a true and legitimate Arch or Fornix. And when Arches of other Figures are supported, it is because in their Thickness some Catenaria is included. Neither would it be sustained, if it were very thin, and composed of slippery Parts. From *Corol. 5.* before, it may be collected, by what Force an Arch or Buttress presses a Wall outwardly to which it is applied. For this is the same with that Part of the Force sustaining the Chain, which draws according to a Horizontal Direction. For the Force which in the Chain draws inwards, in an Arch equal to the Chain drives outwards. All other Circumstances, concerning the Strength of Walls to which Arches are applied, may be geometrically determined from this Theory, which are the chief Things in the Construction of Edifices.

7. Instead of Gravity, if any other Power exerts its Force, acting in like manner upon a flexible Line, the same Curve will be produced. For Ex-  
ample,



ample, if the Wind be supposed equable, and should blow according to Right Lines parallel to a given Line; the Line thus inflated by the Wind would be the same as the Catenaria. For since all Things obtain in this other Force, as we have supposed in Gravity, it is evident the same Line must be produced.

Fig. 37.

Prop. 3. Theor.] *The Hyperbola aforesaid AH remaining, if through A a Right Line GAL be drawn perpendicular to the Axis AB, and a Curve KR be described of such a Nature, that BK may be a third Proportional to the Right Lines BA and AC, and to AC be applied a Rectangle AV equal to the interminate Space ABKRLA; the Concourse F of the Right Lines HB, VG, will be at a Catenaria.*

For by Construction 'tis  $BK = \frac{aa}{\sqrt{2ax+xx}}$ . Therefore the Fluxion of the Space ABKRLA is  $BK \dot{k}b = BK \times B\dot{b} = \frac{aa\dot{x}}{\sqrt{2ax+xx}}$ . And since  $BF = \frac{ABKRLA}{AC}$ , and AC is given; its Fluxion will be  $B\dot{F} = \frac{a\dot{x}}{\sqrt{2ax+xx}}$ . But in the Construction of the foregoing Proposition, the Fluxion of the Ordinate  $BF = \frac{a\dot{x}}{\sqrt{2ax+xx}}$ . Wherefore this Construction comes to the same as the Construction of the foregoing Proposition, and consequently the Point F is at a Catenaria. Q. E. D.

COROL.] As in the foregoing Proposition the Catenaria is described from the given Length of the Parabolical Curve; so in this its Description depends on the Quadrature of the Space, in which  $xxyy = a^4 - 2axy$ . For BK or  $y = \frac{aa}{\sqrt{2ax+xx}}$ .

Fig. 36.

Prop. 4. Theor.] *The Space AGF contained by the Catenaria AF, and the Right Lines FG, AG, parallel to AB, BF, is equal to the Rectangle under the Semiaxis AC, and DH the Distance of the Ordinates in the Hyperbola and Catenaria.*

For  $\dot{D}H = \dot{B}H - \dot{B}D =$  (by Prop. 2. of this)  $\frac{a\dot{x}+x\dot{x}}{\sqrt{2ax+xx}} - \frac{a\dot{x}}{\sqrt{2ax+xx}}$   
 $= \frac{x\dot{x}}{\sqrt{2ax+xx}}$ . Wherefore the Fluxion of the Rectangle under the given Line AC and HD is  $\frac{ax\dot{x}}{\sqrt{2ax+xx}} = x \times \frac{a\dot{x}}{\sqrt{2ax+xx}} = fs \times FG =$  the Fluxion of the Space AGF. And since these Figures are nascent together, it



it follows that the Rectangle under A C and D H is equal to the Space A G F.  
Q. E. D.

COROLL.] Hence it follows that the Space F A D, comprehended by the Chain F A D and the Horizontal Right Line F D, is equal to the Rectangle under F D and B A, subtracting the Rectangle under either Axis of the Hyperbola A H, and D H the Excess of the Right Line B H, or of the Curve A D, above the Ordinate B D.

Prop. 5. Theor.] *If the Rectangle L E, equal to the Hyperbolic Space A L H, be applied to the Right Line A L, the Point E will be the Center of Equilibrium of the Catenarian Curve A F D.*

Fig. 36

Let a heavy Curve F A be conceived to be poised upon the Axis G L. From the Doctrine of the Center of Gravity it follows, that the Moment of the Weight F A is expounded by the Surface of an upright Cylinder erected upon F A, and cut off by a Plain passing through G L, making half a Right Angle with the Plain of the Curve. And the Fluxion of this Surface, or  $\dot{F} A \times \dot{F} G$ , is equal to the Fluxion of the Space A L H, or  $\dot{B} H \times \dot{H} L$ ; because  $\dot{F} A$  and  $\dot{B} H$ , as also  $\dot{F} G$  and  $\dot{H} L$ , are equal. And therefore, since they are nascent at the same Time, the said Superficies of the erect Cylinder is equal to the Hyperbolic Space A L H. Therefore this applied to the heavy Line itself A F, or to the Right Line A L which is equal to it, it produces a Breadth A E equal to the Distance of the Center of Gravity from the Axis of Libration G L. Hence E will be the Center of Equilibrium of the Curve F A D, lying equally on each Side of the Axis A B. Q. E. D.

COROL. 1.] The Spaces A B H L, B A H, and A G F, are in Arithmetical Proportion. For the Fluxion of the Space A L H =  $\frac{a x + x x}{\sqrt{2 a x + x x}} \times x =$

$$\frac{a x + x x \times x}{\sqrt{2 a x + x x}} = \frac{2 a x + x x - a x \times x}{\sqrt{2 a x + x x}} = x \sqrt{2 a x + x x} - \frac{a x x}{\sqrt{2 a x + x x}} = \text{Flu-}$$

xion of the Space B A H, lessened by the Fluxion of the Space A G F, by Prop. 4. of this. And as these three Figures are nascent at the same Time, it will be  $B A H - A G F = (A L H =) B L - B A H$ . So that  $2 B A H = B L + A G F$ . Whence it follows that the Spaces B L, B A H, and A G F, are in Arithmetical Proportion.

2. The Center of Gravity of the Catenaria descends lower, than that of any other Line of the same Length, and having the same Extremities. For every heavy Body descends as low as it may. And since a Figure descends just so much as its Center of Gravity descends, a heavy flexible Line will so dispose itself, as that its Center of Gravity will be lower than if it assumes any other Figure. And from this Property of a heavy flexible Line, all its other Properties might be easily deduced.

3. If



3. If upon any Curves having the same Length, and the same Limits D and F as the Catenaria F A D, upright Cylinders were cut by a Plain passing through D F; of the Cylindrical Superficies so cut off, the greatest is that which insists upon the Catenaria. For these Superficies, if the Angle made by the Plains is half a Right Angle, applied to the Curves themselves, which in the present Case are of the same Length, produce Breadths equal to the Distances of the Centers of Gravity of the Curves from the Right Line D F. Now as in the Catenaria this Distance is the greatest, because of the greatest Descent of the Center of Gravity, the Cylindric Surface to be applied will also be the greatest. And because there is the same Ratio of Cylindrical Surfaces cut off by a Plain, containing any Angle with the Plain of the Base, as when the said Angle is half a Right Angle, the Proposition obtains universally.

Fig. 36.

LEMMA.] If upon any Ordinate F B perpendicular to the Axis A B of any Curve A F Q, that is described by the Evolution of another Curve K V, from the correspondent Point V in K V a perpendicular V R is let fall, meeting the Ordinate in R; if the Fluxion of the Axis A B remains the same, the Fluxion of the Fluxion of the Ordinate B F, the Fluxion of the Curve A F, and the Right Line F R, will be continual Proportionals.

Let the little Right Line F f be produced, till it meets the next Ordinate W φ in o. And because by the Hypothesis F s = f W, it will be o f = F f, and therefore o φ will be the Fluxion of f s, that is, the Fluxion of the Fluxion of the Ordinate. Moreover the Triangles o φ f and f F R are equiangular, because o φ f is equal to the Alternate φ f r, and f o φ = (F f r =) F f R, because their Difference R f r vanishes in Respect of either of them, since R r is nothing in Comparison of f r. Therefore it is o φ . φ f :: f F . F R. But φ f and f F are equal, since they only differ by the Fluxion of each. Therefore o φ . f F :: f F . F R. Q. E. D.

Fig. 36.

Prop. 6. Probl.] To find the Curve K V, by the Evolution of which the Catenaria A F Q is described.

Make A B = x, and B F = y, as before. Then by Prop. 2. of this, 'tis

$$y = \frac{a \dot{x}}{\sqrt{2ax + xx}}, \text{ or } 2ax\dot{y} + xx\ddot{y} = a a \dot{x} \dot{x}. \text{ Then by Newton's Method now in common use, 'tis } 2a\dot{x}\ddot{y} + 4ax\dot{y}\dot{y} + 2x\dot{x}\dot{y}^2 + 2x^2\ddot{y}\dot{y} = 2a^2\dot{x}\dot{x} = 0; \text{ (for } \dot{x} = 0, \text{ because } \dot{x} \text{ is a constant Quantity.) Therefore } \ddot{y} = \frac{-a\dot{x}\dot{y} - x\dot{x}\dot{y}}{2ax + xx} = \frac{a + x \times a \dot{x}^2}{2ax + xx \times \sqrt{2ax + xx}}, \text{ by substituting instead of } \dot{y} \text{ its}$$

Value  $\frac{a \dot{x}}{\sqrt{2ax + xx}}$ ; (for the Sign — prefixt to the Quantity  $\ddot{y}$  only shews, that the Place of the Point R in respect of F, is opposite to the Place of the Point

Point



Point F in respect of B, since the Curve A F Q is concave towards the Axis

A B.) And by the second *Prop.* of this,  $Ff = \frac{a+x \times x}{\sqrt{2ax+xx}}$ . Wherefore by

the foregoing *Lemma*,  $FR = \frac{Ffq}{j} = \frac{a+x \times x^2}{2ax+xx} \times \frac{2ax+xx \times \sqrt{2ax+xx}}{a+x \times a}$

$= \frac{a+x \times \sqrt{2ax+xx}}{a}$ . Again, because of the Right-angled Triangles

$Fsf$  and  $FRV$ , having equal Angles  $fFs$  and  $VFR$ , because  $VFs$  is the

Complement of each to a Right Angle, it is  $Fs : sf :: FR : VR$ , or  $x :$

$\frac{ax}{\sqrt{2ax+xx}} :: \frac{a+x \times \sqrt{2ax+xx}}{a} \cdot VR$ , which therefore is equal to  $a+x$ .

Therefore this is the Nature of the Curve  $KV$ , that if  $AB$  be called  $x$ , it

will be  $FR = \frac{a+x \times \sqrt{2ax+xx}}{a}$ , and  $VR = a+x$ . *Q. E. I.*

*COROL. 1.]*  $AC : CB :: BH : FR$ . For this is the Property of the Right Line  $FR$  found above.

2. The Right Line  $CB$  is equal to  $BI$  or  $VR$ ; for each of them is equal to  $a+x$ .

3. The evolving Right Line  $VF$  is a third Proportional to the Lines  $AC$  and  $CB$ . For because of equiangled Triangles  $fFs$  and  $VFR$ , it is  $sF :$

$Ff :: FR : VF$ . Or  $x : \frac{ax+xx}{\sqrt{2ax+xx}} :: \frac{a+x \times \sqrt{2ax+xx}}{a} \cdot VF$ , which there-

fore is equal to  $\frac{a+x \times \sqrt{2ax+xx}}{a}$ . Whence it is  $a : a+x :: a+x : VF$ , which also is

the Radius of a Circle which is equicurved to the Catenaria in the Point  $F$ .

4. When the Point  $F$  falls in  $A$ , or when the Vertex is described by Evo-

lution, that is, when  $x = 0$ , the Value of the evolving Right Line  $VF$ ,

which in this Case is  $KA$ , becomes  $\frac{a+x^2}{a} = a$ . That is, the Point  $K$ , where

the Curve  $VK$  meets the Axis, is as much above the Vertex  $A$  of the Chain,

as  $C$  is depressed below the same. Whence the Diameter of a Circle, equi-

curved to the Chain at its Vertex, is equal to the Axis of the conterminat

Hyperbola  $AH$ . Therefore the Chain  $AD$  and the Hyperbola  $AH$  have

the same Degree of Curvature at the Vertex  $A$ : For it is generally known

that the aforefaid Circle is equicurved to the Equilateral Hyperbola  $AH$  in

the Vertex  $A$ . Also this appears from the Nature of the Chain itself, by

what is demonstrated *Prop.* 2. of this. For the nascent Line  $FH = AP =$

the nascent  $BP = \sqrt{8ax}$  is double to the nascent Line  $BH$  or  $\sqrt{2ax+xx}$

$=$  (when



= (when  $x$  vanishes)  $\sqrt{2ax}$ . And therefore the same Point is both in the nascent Hyperbola and the nascent Catenaria, That is, the nascent Hyperbola  $AH$  coincides with the nascent Catenaria  $AD$ , and therefore these Lines are equicurved at the Vertex  $A$ .

5. The Curve  $KV$  is a third Proportional to the Right Line  $AC$ , and the Curve  $AF$ , or the Right Line  $AL$ . For from the Nature of Evolution

$$KV = VKA - KA = VF - KA = \frac{a + x^2}{a} - a = \frac{a^2 + 2ax + x^2}{a}$$

$$- a = \frac{2ax + xx}{a}. \text{ And therefore } a : \sqrt{2ax + xx} :: \sqrt{2ax + xx}. KV.$$

But  $\sqrt{2ax + xx} = AF$ , by *Cor. 2. Prop. 2.* Whence  $AC : AF :: AF : KV$ .

6. The Right Line  $KI$  is double to  $AB$ . For since  $BI = BC = CA + AB$ , it will be  $AI = CA + 2AB$ . But  $AK = CA$ , by *Corol. 4.* of this. Whence  $KI = 2AB$ .

7. The Rectangle of  $AC$  and  $BR$  is equal to twice the Hyperbolical Space  $BAH$ . For  $FR \times AC = \frac{a + x \times \sqrt{2ax + xx}}{a} \times a = a + x \times$

$\sqrt{2ax + xx} = x \times \sqrt{2ax + xx} + a \times \sqrt{2ax + xx} = AB \times BH + AC \times BH = AB \times BH + AC \times BD + AC \times DH$ . Therefore  $FR \times AC - BD \times AC = BR \times AC = AB \times BH + AC \times DH$ . But by *Prop. 4.* of this, 'tis  $AC \times DH = \text{Space } AGF$ . And therefore  $BR \times AC = ABHL + AGF = 2BAH$ , by *Cor. 1. Prop. 5.*

Fig. 38.

*Prop. 7. Theor.] If in the Logarithmic Curve  $LAG$ , whose given Subtangent  $HS$  is equal to the Right Line  $a$ , (determined *Cor. 2. Prop. 2.* of this) a Point  $A$  be taken, whose Distance  $AC$  from the Asymptote  $HP$  is equal to the Subtangent  $HS$ ; and from the Points  $H$  and  $P$ , any how taken in the Asymptote, equally distant from the Point  $C$ , if Ordinates  $HL, PG$  are erected to the Logarithmic Curve, to half the Sum of which  $HD$  or  $PF$  are made equal; the Points  $D$  and  $F$  will be in the Catenaria corresponding to the Right Line  $AC$ .*

Make  $AB = x$ , and therefore  $CB$  or  $DH$ , the half Sum of the Ordinates  $HL, PG$ , will be  $a + x$ ; let the half Difference of the same be called  $y$ . Then  $HL = a + x + y$ , and  $PG = a + x - y$ . And since from the Nature of the Logarithmic Curve  $CA$  is a mean Proportional between these, it will be  $aa + 2ax + xx - yy = aa$ , and therefore  $y = \sqrt{2ax + xx}$ . So that  $HL = a + x + \sqrt{2ax + xx}$ , and  $PG = a + x - \sqrt{2ax + xx}$ . Therefore the Fluxion of  $HL$ , or  $lm$ , is  $\frac{a^x + xx + x \sqrt{2ax + xx}}{\sqrt{2ax + xx}}$ .



And because of similar Triangles  $lmL$  and  $LHS$ , it is  $LH : HS :: lm :$

$mL$ . Whence  $mL$  or  $d\delta$ , the Fluxion of  $BD$ , is equal to  $\frac{ax}{\sqrt{2ax+xx}}$ .

That is, the Curve  $AD$  derived from the Logarithmic Curve in the foregoing Manner, is of such a Nature, that if its Axis is called  $x$  and its Fluxion

$\dot{x}$ , the Fluxion of the Ordinate  $BD$  will be  $\frac{ax}{\sqrt{2ax+xx}}$ . But this is the very

Property of the Catenaria to which  $a$  belongs, as demonstrated in *Prop. 1.* of this. Therefore the Curve  $FAD$  above described is no other but the Catenaria. *Q. E. D.*

*COROL. 1.]* As by the Help of the Logarithms the Catenaria may be described, so on the contrary by means of the Catenaria, which is constructed by Nature herself, the Logarithm of a given Number, or rather of a given Ratio, may be found. As supposing  $CA$  to be Unity, whose Logarithm is equal to  $o$ , let us find the Logarithm of the Number  $CQ$ , or of the Ratio between  $CA$  and  $CQ$ . To the Right Lines  $CQ$  and  $CA$  let the third Proportional be  $CV$ , and let half the Sum of  $CQ$  and  $CV$  be  $CB$ . The Ordinate to the Catenaria from  $B$ , that is  $BD$ , is the Logarithm required. The Reason is plain from the Proposition.

2. On the contrary, if from the Logarithm given  $CH$  or  $CP$  the correspondent Number  $HL$  or  $PG$  were required, or the Ratio  $HL$  to  $CA$ , or  $PG$  to  $CA$ ; from  $H$  or  $P$  let a Perpendicular be raised, meeting the Catenaria in  $D$  or  $F$ ; and let  $CR$  be made equal to  $HD$  or  $PF$ , that is to  $CB$ , and let it be terminated at the Horizontal Line  $AR$ . Then will  $AR$  be the Semidifference of the Lines required  $LH$ ,  $GP$ , as  $HD$  or  $CR$  is their Semisum, by what is demonstrated above about the Nature of the Catenaria. (For in three Quantities that are Geometrically proportional, such as  $HL$ ,  $CA$ ,  $PG$ , the Square of the half-sum of the Extreams lessened by the Square of the Mean, is equal to the Square of the half-Difference of the Extreams.) And therefore  $CR + AR$  and  $CR - AR$  are the Numbers  $HL$  or  $GP$ , belonging to the given Logarithm  $CH$  or  $CP$ .

3. From the Demonstration it is evident, that as  $HD$  the half Sum of the Ordinates  $HL$ ,  $PG$ , of the Logarithmick Curve, applied perpendicularly to  $CH$  in  $H$ , is the Ordinate of the Catenaria; so the half Difference of the same  $HL$ ,  $PG$ , applied perpendicularly to  $CA$  in  $B$ , is the Ordinate of the equilateral Hyperbola described with Center  $C$  and Vertex  $A$ ; and therefore, by *Cor. 2. Prop. 2.* of this, is equal to the Catenaria  $AD$ : For  $y = \sqrt{2ax+xx}$ . And since it is shewn in the foregoing Corollary, that  $AR$  also is the half Difference of the Right Lines  $HL$ ,  $PG$ , it is plain that  $AR$  is equal to the Portion of the Catenaria  $AD$ . Whence by the Way a Method is discovered, from the Chain  $AD$  being given to find  $C$  the Center of the conterminat Hyperbola, or that Point in the Asymptote of the Logarithmick



rithmick Curve GL. For if AR is taken equal to the Chain AD, and from the middle Point of the Right Line BR a Perpendicular to it is raised, this will meet BA the Axis of the Chain in the Point required C, as very plainly appears. For thus CR will be equal to CB.

4. Hence also it follows, that if the Angle BDT be made equal to ACR, the Right Line DT will touch the Catenaria in D. For thus in the similar Triangles DBT and CAR, it will be  $DB : BT :: CA : AR$ , or the Curve AD which is equal to it. And therefore by *Corol. Prop. 1.* of this, DT touches the Catenaria.

5. It also follows, that the Space ACHD is equal to the Rectangle of CA and AR. For because AYD, by *Prop. 4.* is equal to the Triangle under CA and AD—BD = (by *Corol. 3.* of this *Prop.*) AR—AY = YR, the Proposition is plain. And because CA is given, it is evident that the Space ACHD is as the Curve AD, or its Fluxion Hd is as the Fluxion of this Dd.

6. If through the Point K, where CR meets HD, a Line KZ is drawn parallel to PH, meeting the Right Line AC in Z, and CE be taken equal to half the Sum of BC, CZ; the Point E will be the Center of Equilibrium of the Curve FAD.

Upon FAD let it be conceived, that the upright Superficies of a Cylinder is erected, and cut by a Plain through PH, at half a Right Angle with the Plain of the Curve FAD. This Superficies will expound the Moment of the Curve FAD, when librated upon the Axis PH; and its Fluxion is

$$DH \times Dd + PF \times Ff = 2BC \times AD = 2 \times a + x \times \frac{ax + xx}{\sqrt{2ax + xx}} =$$

$$\frac{2aa\dot{x} + 4ax\dot{x} + 2xx\dot{x}}{\sqrt{2ax + xx}} = \frac{aa\dot{x}}{\sqrt{2ax + xx}} + \frac{aa\dot{x} + ax\dot{x}}{\sqrt{2ax + x}} + \frac{3ax\dot{x} + 2xx\dot{x}}{\sqrt{2ax + xx}},$$

of which the Fluent is  $a \times BD + a\sqrt{2ax + xx} + x\sqrt{2ax + xx} = CA \times BD + CB \times AD$ . Wherefore  $CA \times BD + CB \times AD$  is equal to the aforefaid Cylindrical Superficies, (for they are nascent together) which is equal to the Moment of the Curve FAD, when poised upon the Axis PH. Whence the Distance of the Center of Gravity of the Curve FAD

from the Point C is  $\frac{CA \times BD + CB \times AD}{2AD}$ , or  $\frac{1}{2} \frac{CA \times BD}{AD} + \frac{1}{2} CB$ .

Moreover because of ZK parallel to AR, it is  $AD : BD :: AR : ZK :: CA : CZ$ , whence  $CZ = \frac{CA \times BD}{AD}$ , and therefore CE, which by Con-

struction is equal to  $\frac{1}{2} BC + \frac{1}{2} CZ$ , will be equal to  $\frac{1}{2} \frac{CA \times BD}{AD} + \frac{1}{2} BC$ .

That is, the Center of Gravity of the Curve FAD, and the Point E determined



mined by the Construction, will be equally distant from C; but being in the same Right Line, and situate the same Way, they must necessarily coincide.

The Coincidence of the Point E, as above determined, with the Center of Equilibrium determined *Prop. 5.* of this, may be thus shewn synthetically. By *Corol. 1. Prop. 5.* 'tis  $2 B A X = A Y D + B A \times A R$ . Whence  $A H + 2 B A X = A C H D + B A \times A R =$  (by the foregoing *Corol.*)  $A R \times C A + B A \times A R$ . That is,  $B D \times A C + 2 B A X = A R \times C B$ , or  $B D \times A C = A R \times C B - 2 B A X$ . Whence  $B D \times A C + A D \times B C = A D \times B C + A R \times C B - 2 B A X = 2 A D \times B C - 2 B A X = 2 A D \times A C + 2 A D \times A B - 2 B A X$ . And by applying to  $2 A D$ , it will be  $\frac{B D \times A C}{A D} + \frac{1}{2} B C = A C + \frac{A B \times A D - B A X}{A D} = C A + \frac{A R X}{A R}$ .

But  $\frac{A R X}{A R}$  is the Distance of the Center of Equilibrium of the Chain from the Vertex A, by *Prop. 5.* and therefore by the same  $C A + \frac{A R X}{A R}$  is the

Distance of the Point E from C, and  $\frac{1}{2} \frac{B D \times A C}{A D} + \frac{1}{2} B C$  is the Distance of the same E from the same C, by this *Corol.* Whence it appears that these two Determinations of the Point E come to the same, because  $C A + \frac{A R X}{A R} = \frac{1}{2} \frac{B D \times A C}{A D} + \frac{1}{2} B C$ .

7. The Center of Gravity of the Space P F A D H is in I, the middle Point of the Right Line C E. For since the Center of Gravity of the Fluxion of A D, or D d and F f, is distant as far again from P H, as the Center of Gravity of the Fluxion of A C H D, or D H b d and F P p f, and  $\overline{D d + F f} \times A C$  is given, equal to  $D d b H + F f p P$ ; it is plain that the Center of Gravity E of the Fluent F A D is as far again distant from P H, as the Center I of the Fluent P F A D H. But I shall prove this otherwise, after the manner of the foregoing.

Let an erect Cylinder be supposed to be raised upon the Figure P F A D H, and cut off by a Plain passing through P H, making half a Right Angle with the Plain of the Base; that Solid will represent the Moment of the Figure P F A D H, when poised upon the Axis P H. The Fluxion of this Solid or of the aforesaid Moment, (that is, the Solids erected upon P F f p and H D d b) is produced, if the Moment of the Fluxion, or the Fluxion of the Moment of A D, is drawn into the given Line  $\frac{1}{2} A C$ . For by *Corol. 5.* of this Proposition,  $H D d b = D d \times A C$ . Wherefore the flowing Moment itself is produced by multiplying the Moment of the Curve F A D in respect of the Axis P H, determined by the foregoing *Corollary*, that is,  $C A \times B D + C B \times A D$  into  $\frac{1}{2} A C$ ; and therefore it will be  $\frac{1}{2} A C \times A C \times$   
H 2
B D +



$BD + \frac{1}{2} AC \times CB \times AD$ . Now if this be applied to the librated Figure  $PFADH$ , or  $2 CA \times AD$ , by *Cor. 5.* of this *Prop.* there will arise the Distance of the Center of Gravity of the Figure  $PFADH$  from the Axis  $PH$ , equal to  $\frac{1}{2} \frac{CA \times BD}{AD} + \frac{1}{2} CB$ , which is equal to half the Right Line  $CE$ , as above determined.

8. If through the Point  $N$ , where  $DT$  the Tangent to the Catenaria in  $D$  meets the Line  $AR$ , a Right Line be drawn parallel to  $BC$ , meeting a Right Line through  $E$  parallel to  $AR$  in the Point  $O$ ; this Point  $O$  will be the Center of Gravity of the Curve  $AD$ . For by *Corol. 6.* the Center of Gravity of the Curve  $AD$  is in the Right Line  $EO$ . But it will be also demonstrated, that it is in the Right Line  $NO$ , and therefore will be in the Point  $O$ . Let  $DA$  be conceived to be librated about the Axis  $HL$ ; the Moment of this is the Curve  $DA$ , drawn into the Distance of the Center of Gravity from  $HL$ , and therefore its Fluxion is  $DA \times Hb$ , ( $Hb$  is the Fluxion of the Distance of the Axis of Libration from the Center of Gra-

vity) which is equal to  $\sqrt{2ax+xx} \times \frac{ax}{\sqrt{2ax+xx}} = ax$ . And therefore the

Moment of the weighty Curve  $DA$ , librated about the Axis  $HL$ , is  $ax$ . Therefore the Distance of the Center of Gravity from the same Axis is  $ax$

applied to  $AD$ , or  $\frac{AC \times DY}{AR}$ . But because  $DT$  touches the Catenaria,

by *Cor. 4.* of this *Prop.* the Angle  $BDT$  or  $DN Y$  will be equal to  $ACR$ . And the Angles at  $A$  and  $Y$  are right; therefore in the equiangular Triangles  $RAC$  and  $DYN$ , 'tis  $RA : AC :: DY : YN$ . Whence  $YN =$

$\frac{AC \times DY}{RA}$ , that is,  $YN$  is the Distance of the Center of Gravity of the

Chain  $AD$  from the Axis  $HL$ ; or the said Center is in the Right Line  $NO$ .

9. If upon  $I$  a Right Line be drawn parallel to  $AR$ , meeting  $ON$  produced in  $W$ ; the Point  $W$  will be the Center of Gravity of the Space  $ACHD$ . For by *Corol. 7.* this Center is in the Right Line  $IW$ ; and it will be shewn presently, that it is in  $NW$ , and therefore is the very Point  $W$ . For in the same manner as in the foregoing, the Fluxion of the Moment of the Space  $ACHD$ , librated about  $HL$ , is shewn to be  $A CHD \times$

$Hb = AC \times AD \times Hb = ax \sqrt{2ax+xx} \times \frac{ax}{\sqrt{2ax+xx}} = aax$ . And

therefore the Moment of the Space  $A CHD$ , librated about  $HL$ , is equal to the Fluent of the Fluxion  $aax$ , that is to  $aax$ . This therefore applied to the Space itself  $A CHD$ , or  $a \sqrt{2ax+xx}$ , gives the Distance of the

Center of Gravity of the Space  $A CHD$  from  $HL$ , that is  $\frac{ax}{\sqrt{2ax+xx}} =$   
 $AC \times DY$



$\frac{AC \times DY}{RA}$ . But in the foregoing Corollary it is shewn, that  $YN =$

$\frac{AC \times DY}{RA}$ . Therefore the Center of Gravity of the Space  $ACHD$  is in

$NW$ . And by these two last *Corollaries* the Center of Gravity is found, of any Portion of a Catena that does not reach to the Vertex  $A$ , or of any Space of a Catenaria, comprehended by any Portion together with Right Lines.

10. Hence are measured the Superficies and Solids produced by the Rotation of a Catenaria, or of any Space comprehended by that and Right Lines, revolving about a given Axis. For the Figure generated by Rotation, as is commonly known, is equal to the revolving Figure drawn into the Periphery that is described by the Center of Gravity in the Rotation; which Periphery is given, since its Radius is given, or the Distance of the Center of Gravity from the given Axis. Thus if the Catenaria  $AD$  revolves about the Axis

$AB$ , the Periphery described by the Center of Gravity  $O$  will be  $\frac{\pi}{\rho} AN$ , if

$\frac{\pi}{\rho}$  denotes the Ratio of the Periphery of a Circle to its Semidiameter; and therefore the Superficies produced by the Rotation of the Catena  $AD$ , will

be  $\frac{\pi}{\rho} \times AN \times AD = \frac{\pi}{\rho} \times AN \times AR$ . That is, a Circle whose Radius is

equal in Power to the double of the Rectangle  $RAN$ , will be equal to the Superficies produced by the Rotation of the Chain  $AD$  about the Axis  $AB$ . In like manner it may be shewn, that a Solid generated by the Rotation of the Space  $ACHD$  about  $AC$ , is equal to a Cylinder whose Base is the aforesaid Circle, and its Height equal to  $AC$ . And so may the Superficies and Solids be measured, that are produced by the Rotation of these Figures about any other given Axis. For when the Centers of Gravity are known, the rest will easily follow.

2. What has been objected by an Anonymous Author, in his Animad-  
 versions upon our Demonstrations concerning the Catenaria, is this. *Act. Lips. M. Feb. An. 1699.* That I have undertaken to demonstrate, after my  
 Manner, a Matter found out and publish'd by others seven Years ago. This  
 is true, and I cannot find any thing in this that is Blame-worthy. Those  
 great Men *Huygens, Leibnitz, and Bernouilli*, have discovered and communi-  
 cated many Properties of the Catenaria, but without Demonstration. I have  
 contrived Demonstrations, which was the Thing I undertook to do.

But was this Matter (that is, the Nature and primary Properties of the Catenaria) found out and published by others? Surely that Property of the Catenaria, in *Cor. 6. Prop. 2.* was not at all mentioned by others before the Publication of these Demonstrations; although, if I am not mistaken, it may be reckoned among its primary Properties, is the most useful of all, and most easily reduced to the common Purposes of Life. From all Ages Architects  
 have

*The Animadver-  
 sions of . . . . .  
 by Dr. D. Gre-  
 gory. N. 259. Pa.  
 419. Dec. An.  
 1699.*



have made use of Arches in publick Buildings, as well for Strength as Beauty. Yet what was the true Geometrical Figure of an Arch, was not known before my Demonstrations came out.

The first Thing he finds Fault with is, that I affirm some Things are plain from Mechanicks, which he thinks should have been explained and applied more distinctly. As I undertook to demonstrate some Theorems to Geometricians, I did not think it necessary to pursue every Thing very minutely. But that I may oblige the Animadverter, I will now demonstrate that *Lemma, Prop. 1.* because I cannot express it more fully than I have already done, in the following Words:

“ Three Powers constituted in *Æquilibrium* have the same Ratio as three Right Lines, which are parallel to the Directions of the Powers, or are inclined in a given Angle, and terminated by their mutual Concourse.”

Fig. 39.

As suppose three Powers are in *Æquilibrium*, that either draw, press, or any how act according to three Right Lines  $PA, PB, PC$ ; and let the three Right Lines  $EF, FD, DE$ , be inclined to these Directions in any given Angle; that is, let the Angles  $EAP, FBP, DCP$ , be equal: I say the Powers  $A, B, C$ , are to one another as the Right Lines  $FE, FD, DE$ .

Let the Right Lines  $AP, BP, CP$ , be produced to  $G, H, K$ .

In the Quadrilaterum  $FABP$ , because by Hypothesis the external Angle  $EAP$  is equal to the internal and opposite Angle  $PBF$ , the two internal opposite Angles  $FAP$  and  $FBP$  will be equal to two Right Angles; and since all the four internal Angles are equal to four Right Angles, the other two Angles  $F$  and  $ABP$  opposite in the same Quadrilaterum, will also be equal to two Right Angles. But  $APB$  and  $BPG$  make two Right Ones; therefore the Angle  $F$  is equal to  $BPG$ . In like manner  $D$  and  $E$  may be shewn equal to  $BPK$  and  $APK$ .

Now because the three Powers are in *Æquilibrium*, they are immoveable, and therefore any one of them in respect of the two others that remain in *Æquilibrium*, may be considered as a Fulcrum. If  $B$  is the Fulcrum, by a most known Theorem in Mechanicks, the Power  $A$  is to the Power  $C$ , as the Sine of the Angle  $BPK$  to the Sine of the Angle  $BPG$ , that is, as the Sine of the Angle  $D$  to the Sine of the Angle  $F$ ; that is, as the Right Line  $FE$  to the Right Line  $DE$ . Again, supposing  $C$  the Fulcrum, the Power  $A$  is to the Power  $B$ , as the Sine of the Angle  $CPH$  to the Sine of the Angle  $CPG$ , or the Sine of the Angle  $BPK$  to the Sine of the Angle  $APK$ ; that is, the Sine of the Angle  $D$  to the Sine of the Angle  $E$ , or as the Right Line  $FE$  to  $FD$ . Therefore the three Powers  $A, B$ , and  $C$ , are as the Right Lines  $FE, FD$ , and  $DE$ . *Q. E. D.*

Fig. 40.

We must now say something about the Application of this Mechanical *Lemma*. If the absolute Gravity of the little Line  $dD$ , expounded by  $dD$ , as said above in *Prop. 1.* is conceived to be collected in its Center of Gravity  $M$ , and this heavy Line, by Virtue of its Gravity, endeavours to descend according to the Direction  $MF$  perpendicular to  $dD$ ; the Power drawing according to  $MD$ , which is in *Æquilibrium* with the said heavy Line, by the forego-



foregoing *Lemma* is to its Momentum or Power drawing according to  $M F$ , as  $d D$  is to  $d d$ . For the Angle  $d D d$ , in which  $D d$  is inclined to  $M D$ , is equal to the Angle  $d e F$ , in which  $d d$  is inclined to  $M F$ , for each is the Complement of the Angle  $d$  to a Right Angle. And this will obtain, as the Animadverter acknowledges, if the aforesaid Weight (as in the vulgar Mechanics) incumbering upon the Plain  $M F$ , is drawn by Help of a Pulley at  $M$ , by another Weight incumbering upon  $M D$ : Then this will be to that as  $D d$  to  $d d$ .

Other Things remaining as before, if the Manner of Application of these Powers is changed, so that to the middle Point  $M$  of the flexible Line  $d D$ , whose Extremity  $d$  is fixt, a Weight be applied exerting its Force according to  $M F$ ; (for in descending it would describe an Arch with Center  $d$  and Radius  $d M$ ) the Force of this Weight to bend the flexible Right Line at  $M$ , would be infinite in respect of the Force of its absolute Gravity. And the Force drawing according to  $M D$ , which is required in order to prevent the aforesaid bending, would also be infinite in respect of that which was before required, to support the Weight  $M$  in the Plain  $M F$ . So that the Powers which in the former Manner of Application were expounded by  $d d$  and  $d D$ , must now be expounded by infinitely greater Lines, which are still proportional to the former. For as before, the Weight  $M$  draws according to the Direction  $M F$ , and the Power sustaining it according to  $M D$ ; and that these two are in *Æquilibrium* appears from the Parts of the Chain being at rest. Therefore the Ratio of these remains the same as before. But the Cause which extends into a Right Line the flexible Line  $d D$ , (whose Extremity  $d$  is immoveable, and to whose middle Point  $M$  a Weight is applied, which is indeed infinitely little, but whose Force by this Manner of Application is made infinitely greater, and therefore in the Language of the Animadverter becomes assignable) is the Weight of the Chain  $D A$ , which is proportional to its Length. This therefore to the constant and assignable Quantity  $a$ , (proportional to the constant but not assignable Quantity  $d d$ ) is as  $D d$  to  $d d$ . And thus I hope it will appear to the Animadverter, that I have proved the Conclusion to be true, without making any erroneous Positions.

XIV. Let  $A C F$  be a Semicircle, whose Diameter is  $A F$ ;  $A D E$  is a *The Quadrature of Figures Geometrically irrational, by Mr. J. Craig. N. 232. p. 703. Sept. An. 1697. Fig. 41.* Curve Geometrically irrational, whose Ordinate  $B D$  cuts the Semicircle in  $C$ . Now let the Quantities be thus represented. The Diameter  $A F = 2 a$ , the Absciss  $A B = y$ , the Arch  $A C = v$ , and the Ordinate  $B D = z$ . Let  $z = r v y^n$  be a general Equation, expressing the Natures of the Geometrically irrational Curves  $A D E$ , in which  $r$  denotes any given and determinate Quantity, and  $n$  the indefinite Exponent of the indeterminate Quantity  $y$ . I say the Area of the Curve will be

$$A B D = \frac{r v y^{n+1}}{n+1} = q v + \sqrt{r y - y y} \text{ into } \frac{r a}{n+1} y^n + \frac{2 n r a^2 + r a^2}{n \times n + 1}$$



$$y^{n-1} + \frac{a A \times 2n-1}{n-1} y^{n-2} + \frac{a B \times 2n-3}{n-2} y^{n-3} + \frac{a C \times 2n-5}{n-3} y^{n-4} + \frac{a D \times 2n-7}{n-4} y^{n-5} + \frac{a E \times 2n-9}{n-5} y^{n-6}, \text{ \&c.}$$

Concerning this infinite Series the following Things are to be observed. (1.) That the Capitals A, B, C, D, E, &c. denote the Coefficients of the Terms immediately preceding. Thus  $A = \frac{r a^2 \times 2n+1}{n \times n+1}$ ,  $B = \frac{a A \times 2n-1}{n-1}$ ,  $C = \frac{a B \times 2n-3}{n-2}$ , and so on. (2.) That if the Exponent  $n$  represents any integer affirmative Number, or is  $= 0$ ; or if  $2n$  be an odd Number, then the Quadrature of the Space A B D is exhibited by a finite Quantity, because of the Series breaking off in these Cases. (3.) That  $q$  represents the last Term so breaking off. (4.) That all those Figures in which the Series breaks off have a Portion Geometrically quadrable, which is easily assigned by the Series itself. For if the Absciss  $y$  is taken equal to  $r^{n+1} \times n q + q^{n+1}$ , the Area belonging to this Absciss will be Geometrically quadrable. (5.) That only the irrational Term  $\sqrt{2 a y - y y}$  is to be multiplied into the Terms that follow it.

*Example 1.]* Let  $z = v$ . Now because in this Case 'tis  $r = 1$ ,  $n = 0$ , therefore  $\frac{r a}{n+1} y^n$  is the last Term breaking off. Therefore  $q = a$ , and  $A B D = v y - a v + a \sqrt{2 a y - y y}$ . Therefore by *Note 4*, if there be taken the Abscissa  $y = a$ , that is, if the Ordinate passes through the Center of the Circle, the Portion belonging to it will be Geometrically quadrable: For then Area  $= a a$ , or the Square of the Radius.

*Example 2.]* Let  $z = \frac{v y}{a}$ . Because in this Case  $r = \frac{1}{a}$ ,  $n = 1$ , therefore  $\frac{2 n r a a + r a a}{n \times n + 1} y^{n-1}$  is the last Term breaking off, so that  $q = \frac{3 a}{4}$ ,

whence  $A B D = \frac{v y^2}{2 a} - \frac{3 a v}{4} + \frac{y + 3 a}{4} \sqrt{2 a y - y y}$ , wherefore by *Note 4*, if we take  $y = \frac{\sqrt{3 a a}}{2}$ , the Geometrically quadrable Area belonging to

this Absciss will be  $\sqrt{\sqrt{6 a^4} - \frac{3 a^2}{2} \times \frac{\sqrt{3 a^2}}{3^2} \times \frac{3 a}{4}}$ .

*Example*



*Example 3.]* Let  $z = \frac{v y^2}{a a}$ , in this Case  $r = \frac{1}{a a}$ ,  $n = 2$ , and therefore

$\frac{a A \times 2 n - 1}{n - 1} y^{n-2}$  is the last Term breaking off. Therefore  $q = \frac{5 a}{6}$ ;

so that by the infinite Series it will be

$$A B D = \frac{6 v y^3 - 15 a^3 v + 2 a y^2 + 5 a^2 y + 15 a^3 \times \sqrt{2 a y - y y}}{18 a^2}. \quad \text{And}$$

therefore by *Note 4*, if we take  $y = \sqrt[3]{\frac{5 a^2}{2}}$ , the Area belonging to this Ab-

sciss will be Geometrically quadrable; that is, the Area  $\frac{2 a y^2 + 5 a^2 y + 15 a^3}{18 a}$

$\times \sqrt{2 a y - y y}$ .

Secondly, let  $A C F$  be a Parabola, whose Axis is  $A E$ , Vertex  $A$ , and Parameter  $B A$ . And let  $A D G$  be a Curve Geometrically irrational; whose Ordinate  $B D$  cuts the Parabola in  $C$ . Make the Absciss  $A B = y$ , the Ordinate  $B D = z$ , and the Arch of the Parabola  $A C = v$ . And let the general Equation, expressing the Natures of an Infinity of irrational Curves, be  $z = r v y^n$ , in which  $r$  denotes a given determinate Quantity, and  $n$  the indefinite Exponent of the indeterminate Quantity  $y$ . I say the Area

*Fig. 42.*

$$\begin{aligned} A B D &= \frac{r y^{n+1} \times v}{n+1} - q v + \sqrt{2 a y + y y} \text{ into } \frac{r}{n+2 \times n+1} y^{n+1} \\ &- \frac{r a}{(n+2 \times n+1)^2} y^n + \frac{r a a \times 2 n+1}{n \times n+2 \times n+1} y^{n-1} - \frac{a A \times 2 n-1}{n-1} y^{n-2} \\ &+ \frac{a B \times 2 n-3}{n-2} y^{n-3} - \frac{a C \times 2 n-5}{n-3} y^{n-4}, \text{ \&c.} \end{aligned}$$

Concerning this Series these Things are to be observed. (1.) That the great Letters  $A, B, C, \text{ \&c.}$  represent the Coefficients of the preceding Terms respectively. (2.) That if the Exponent  $n$  is an affirmative Integer, or equal to nothing, or likewise if  $2 n$  be an odd Number, then the Quadrature may be exhibited by a finite Number of Terms, the Series breaking off of itself. (3.) Then is  $q$  equal to the last Term breaking off. (4.) That of the Terms multiplying the Quantity  $\sqrt{2 a y + y y}$ , the last breaking off is to be doubled. (5.) That all those Figures in which  $n$  is an Integer positive and odd Number, or more generally, all the Figures in which the final Term has the Sign  $+$ , have a Portion Geometrically quadrable, which is easily assigned by the Series, by taking the Absciss as in *Not. 4.* of the foregoing Series.

*Example 1.]* Make  $z = v$ . Because in this Case  $r = 1$ ,  $n = 0$ , therefore the Term last breaking off is  $-\frac{r a}{n+2 \times n+1} y^n$ , whence  $q = -\frac{a}{2}$



by *Not.* 3. And because in this Case  $-\frac{a}{2}$  is the final Term, therefore  $-\frac{a}{2}$  is the last Term to be multiplied into  $\sqrt{2ay + yy}$ , by *Not.* 4. and therefore  $ABD = vy + \frac{av}{2} + \sqrt{2ay + yy} \times -\frac{1}{2}y - a$ .

*Example 2.]* Let  $z = \frac{vy}{a}$ . Because in this Case  $r = \frac{1}{a}$ ,  $n = 1$ , therefore the Term breaking off is  $\frac{raa + 2n + 1}{n \times n + 2 \times n + 1} y^{n-1} = \frac{a}{4}$ . Whence  $q = \frac{a}{4}$ , and  $\frac{a}{2}$  is the last Term to be multiplied into  $\sqrt{2ay + yy}$ . Therefore

$ABD = \frac{vyy}{2a} - \frac{av}{4} + \sqrt{2ay + yy} \times -\frac{yy}{6a} - \frac{y}{12} + \frac{a}{2}$ . And if we take  $y = \sqrt{\frac{aa}{2}}$ , the quadrable Area belonging to this Absciss will be  $\frac{1}{12}$

$$\sqrt{\sqrt{2a^2 + \frac{aa}{2}} \times 5a - \sqrt{\frac{aa}{2}}}$$

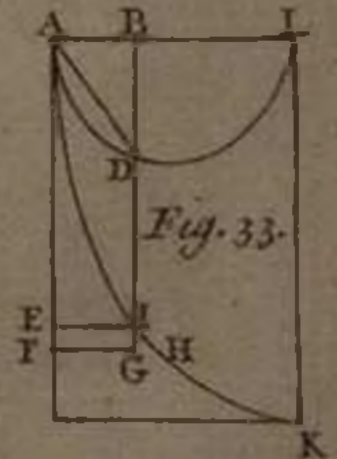
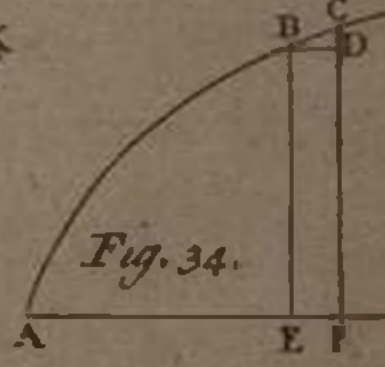
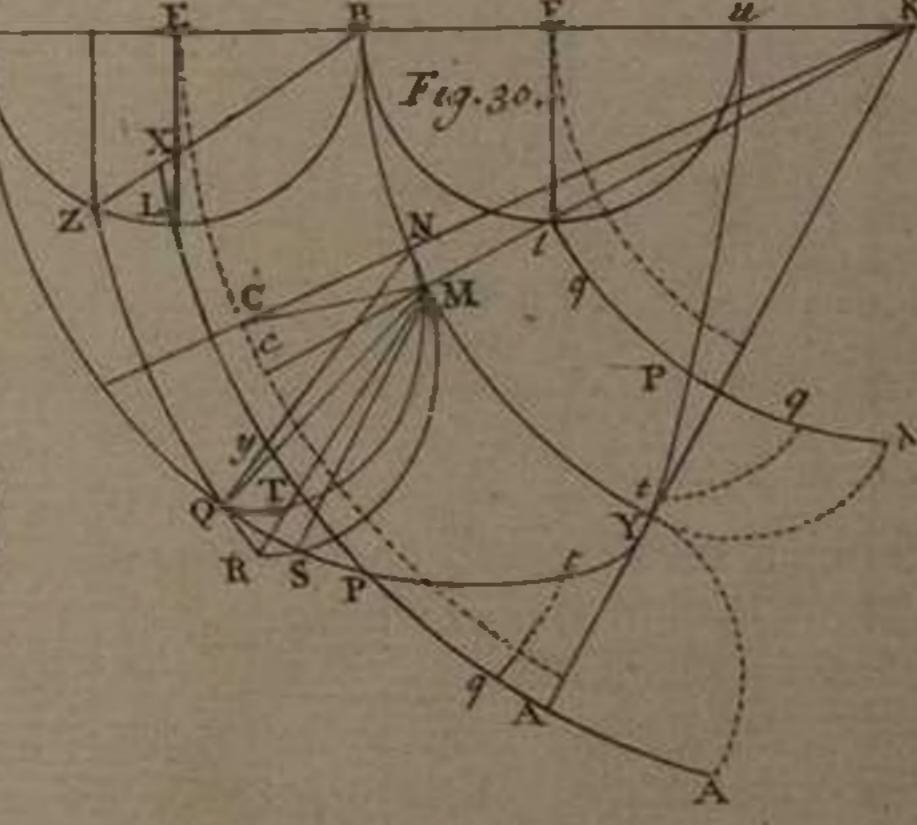
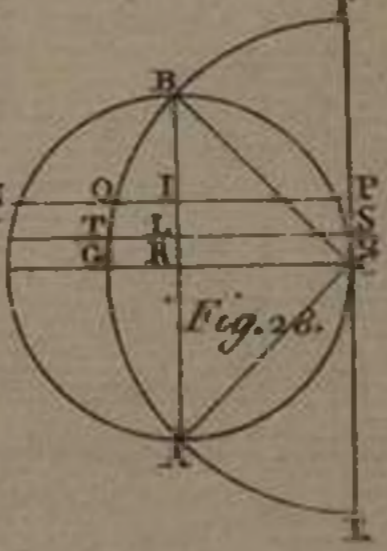
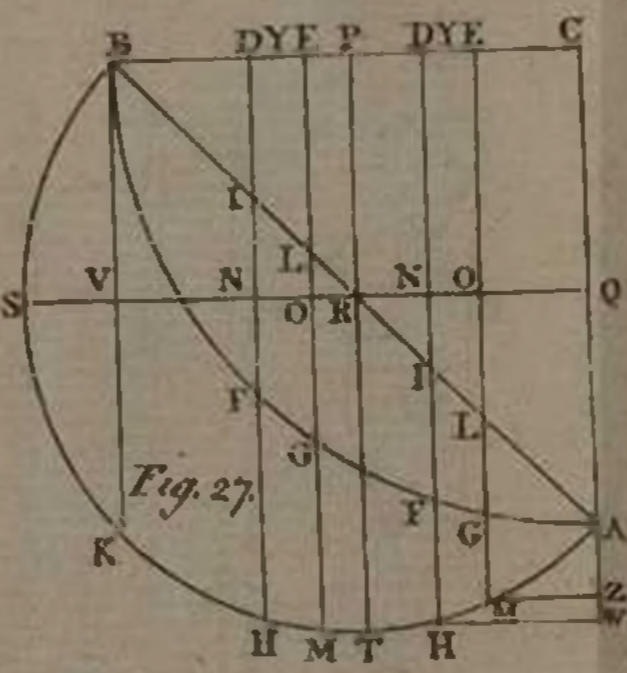
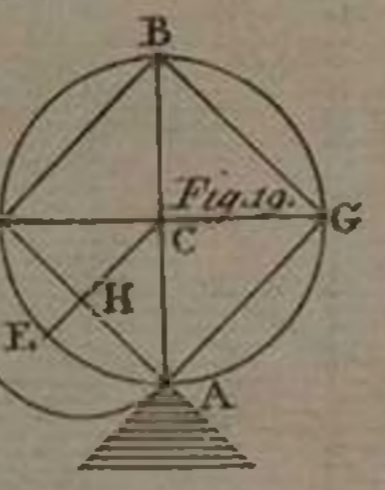
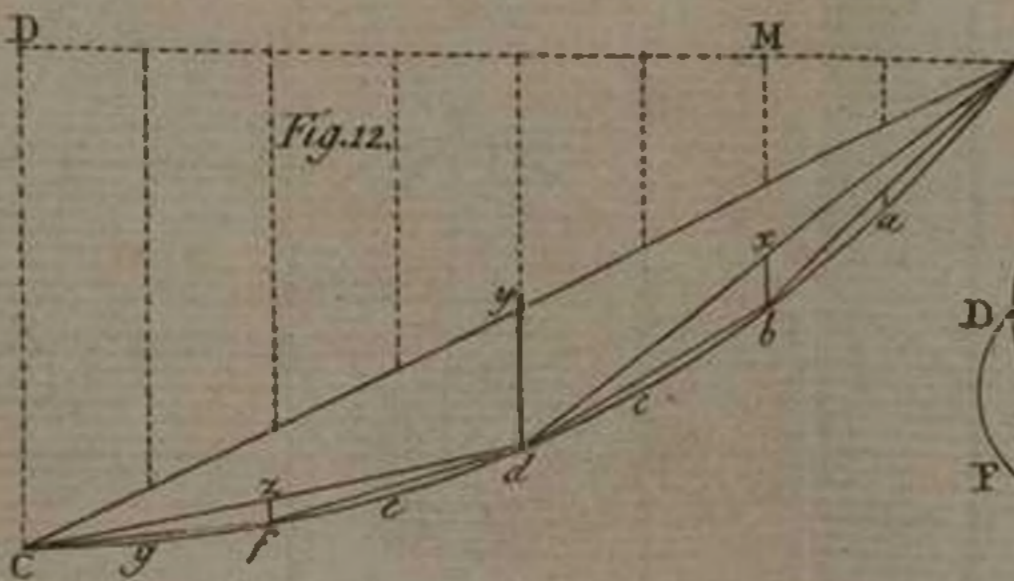
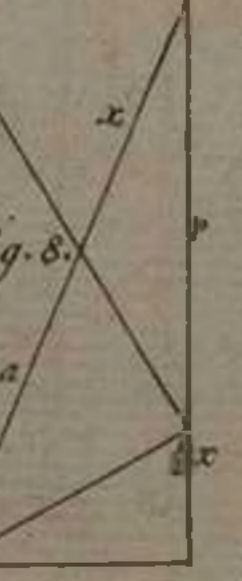
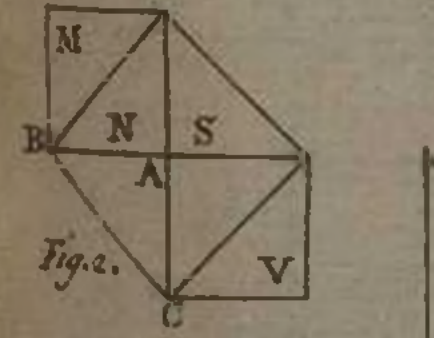
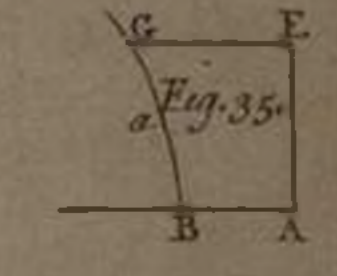
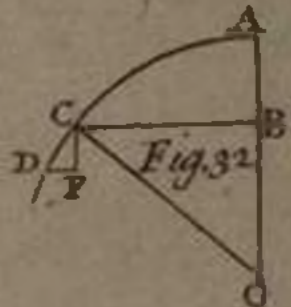
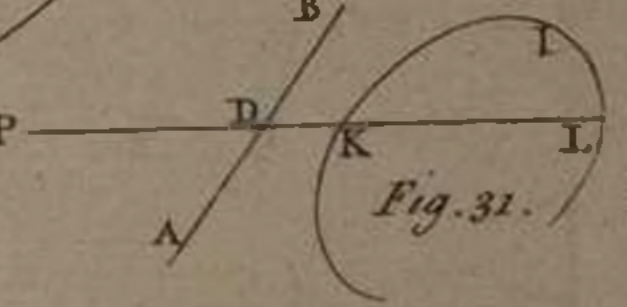
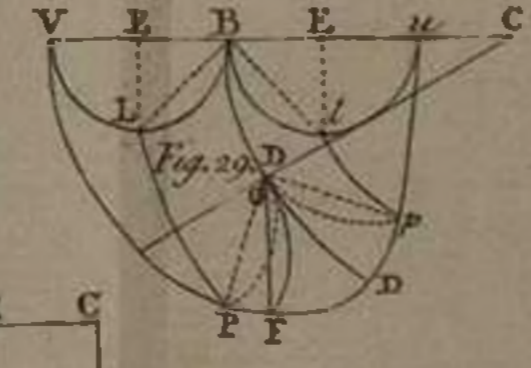
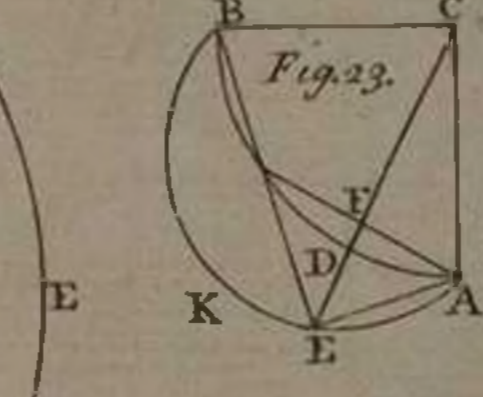
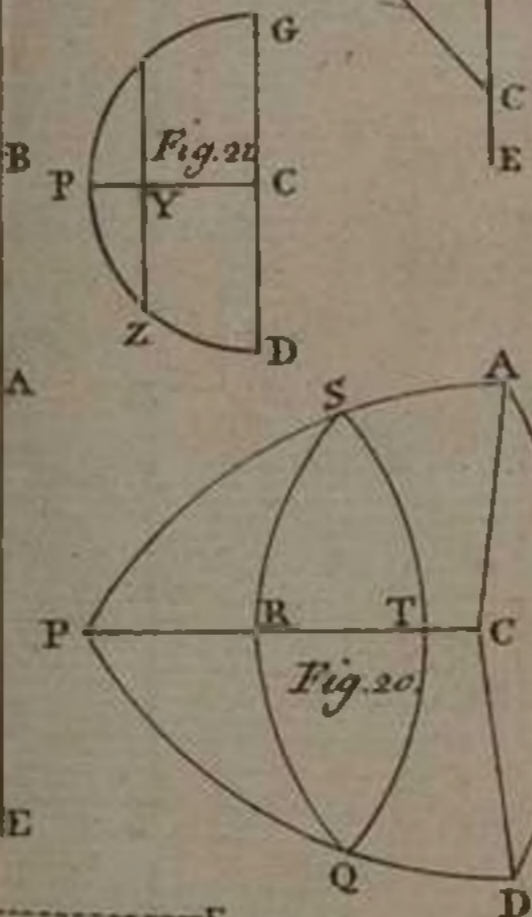
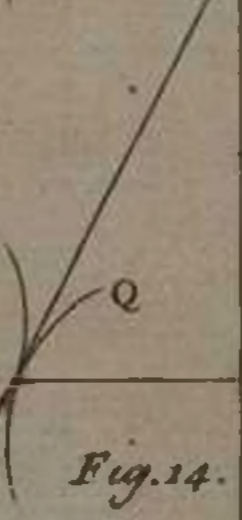
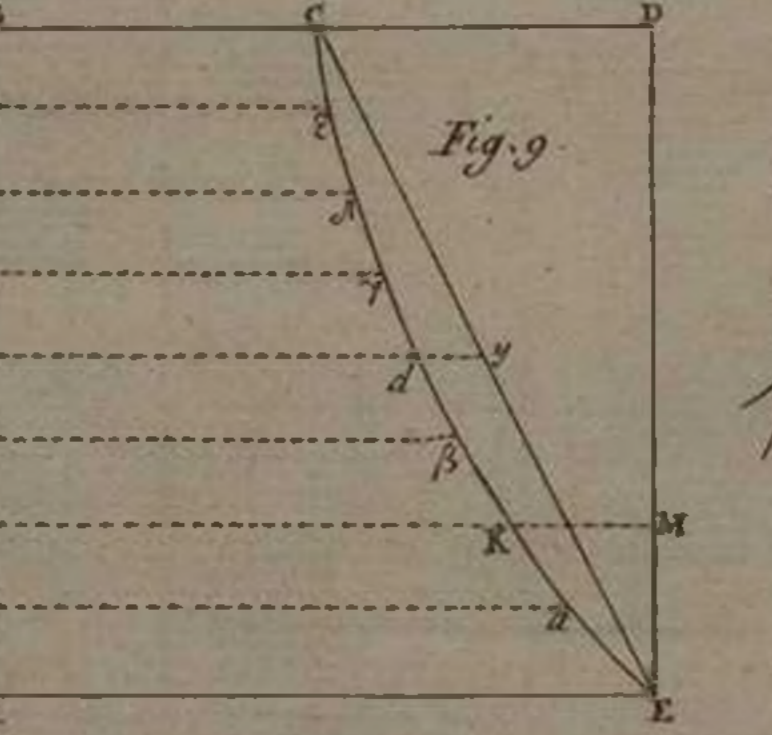
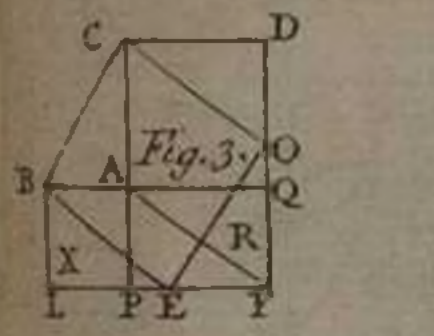
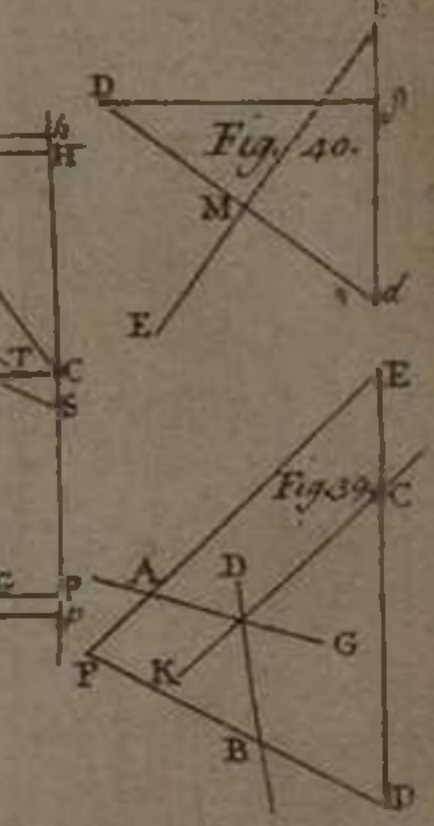
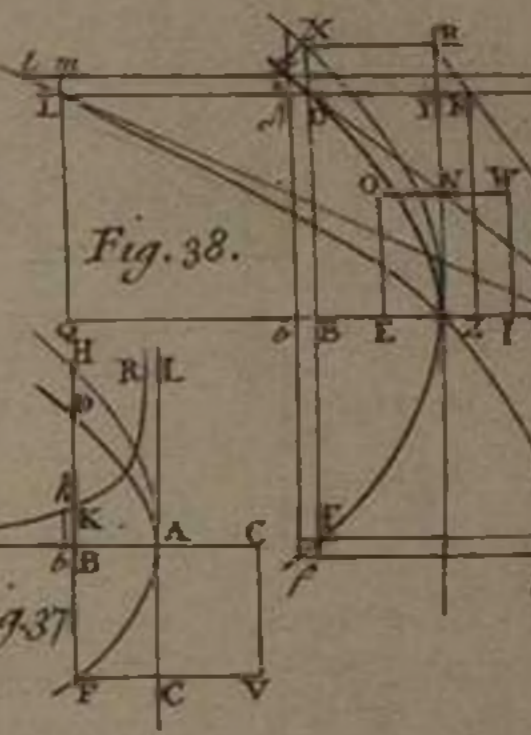
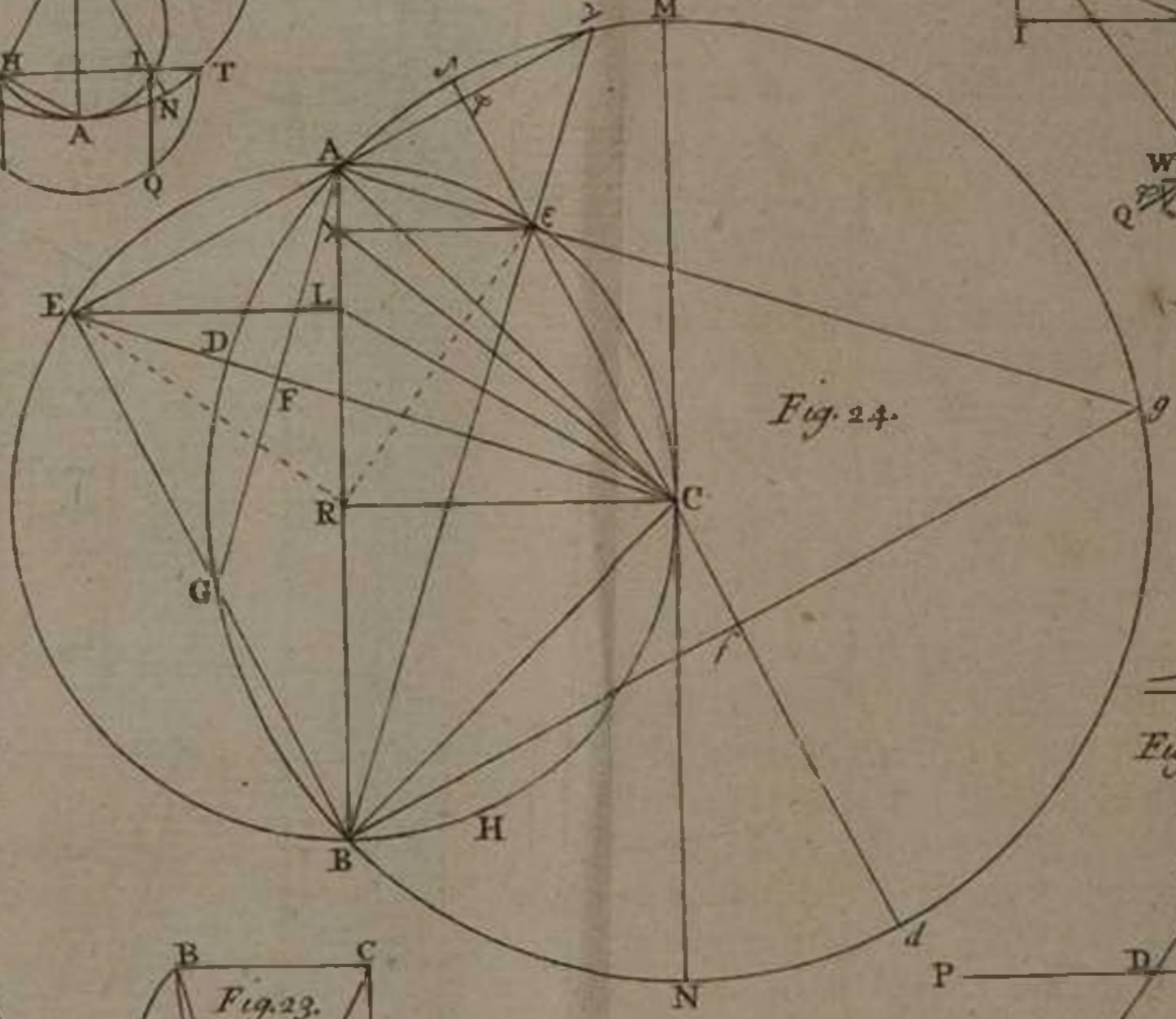
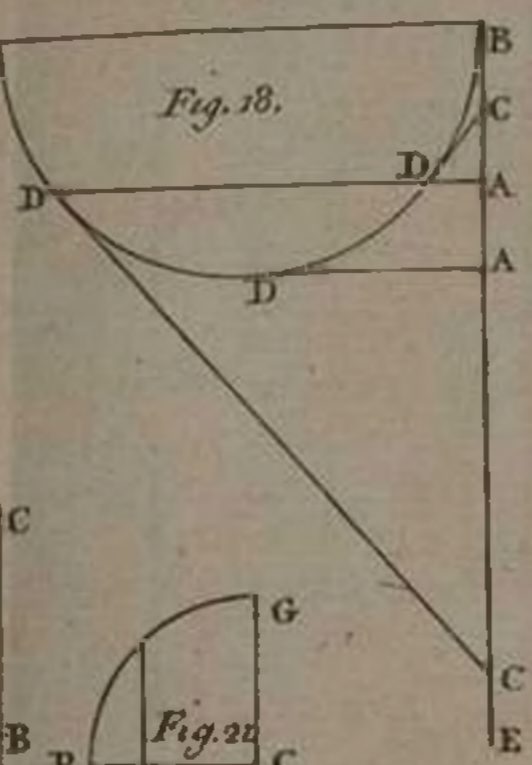
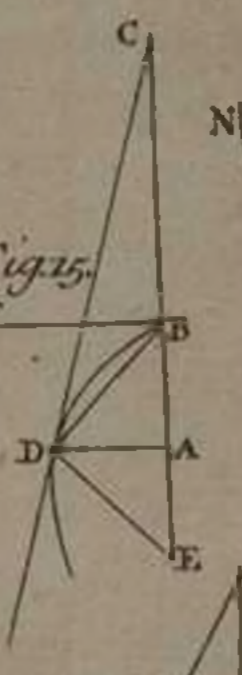
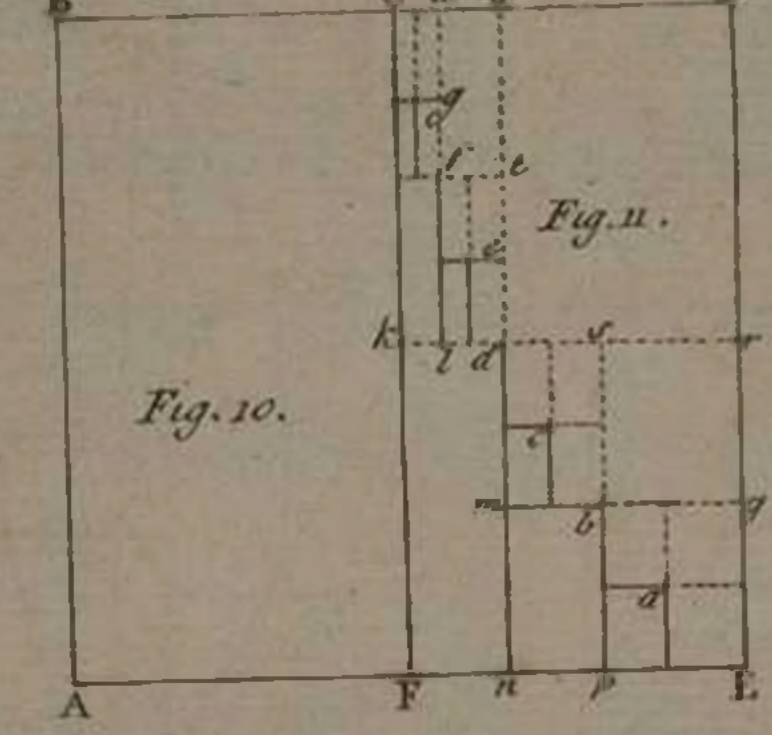
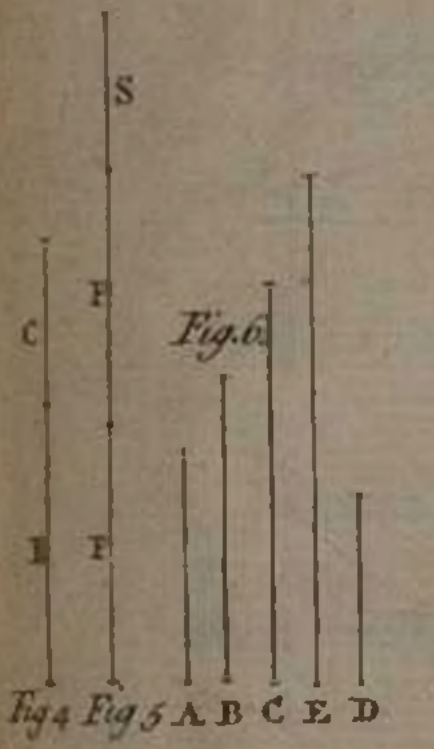
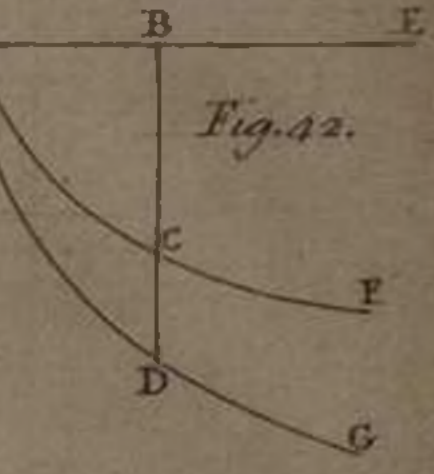
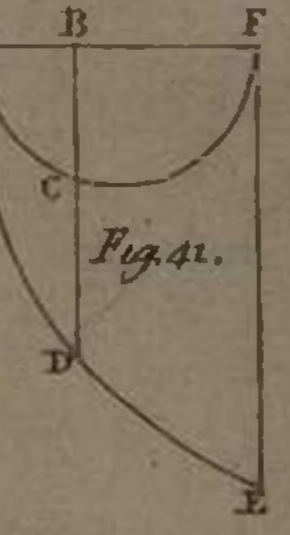
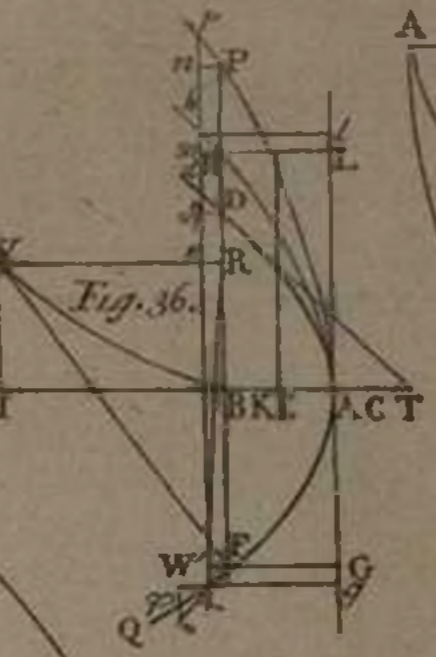
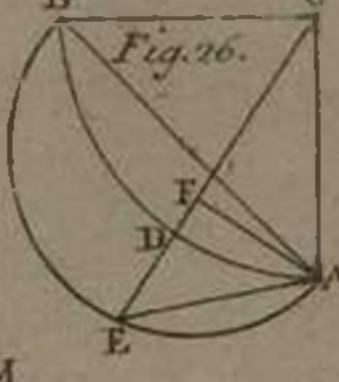
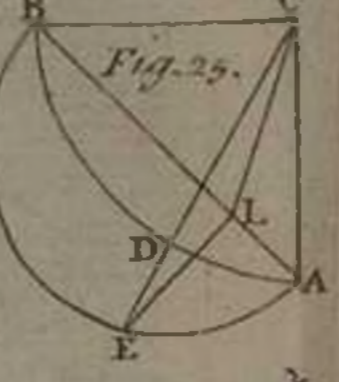
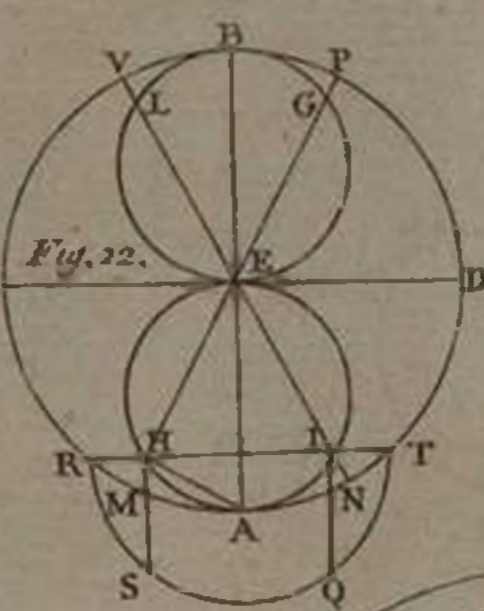
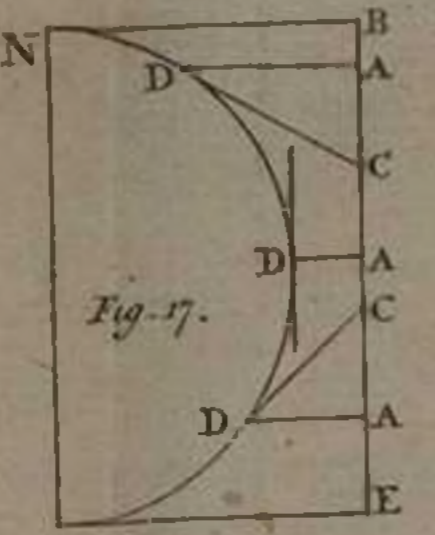
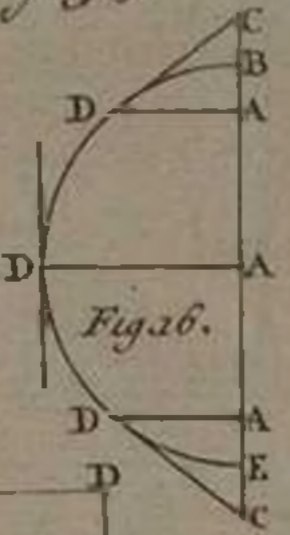
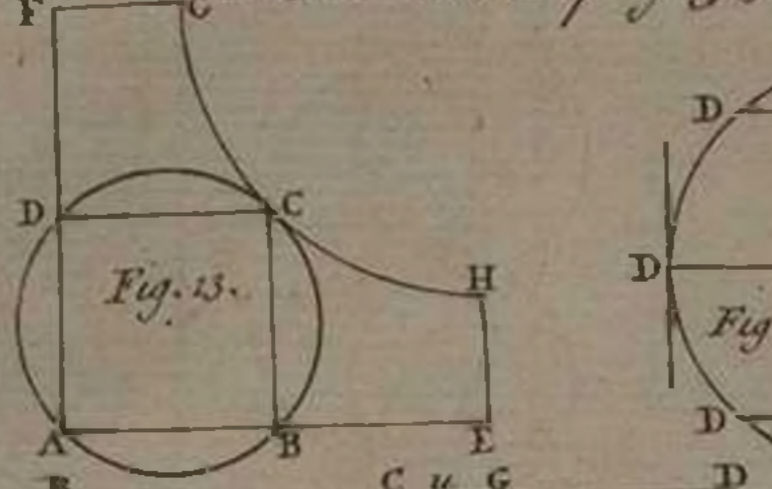
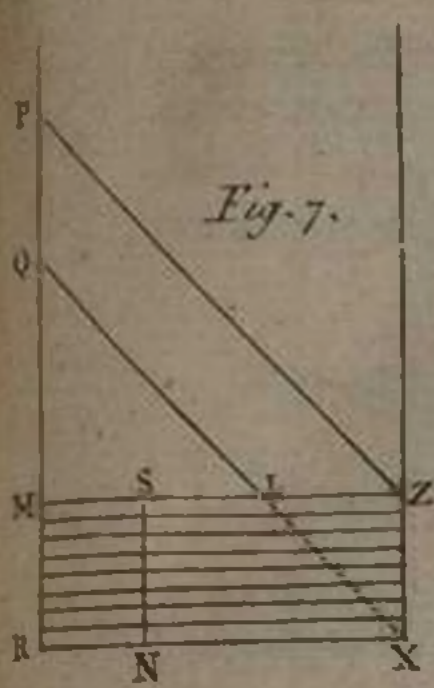
N. 235. p. 785.  
Fig. 41.

*Thirdly.]* Let  $ACF$  be a Semicircle,  $ADE$  a Curve Geometrically irrational, whose Ordinate  $BD$  cuts the Semicircle in  $C$ . Let the Quantities be represented as before, that is, the Diameter  $AF = 2a$ , the Absciss  $AB = y$ , the Arch  $AC = v$ , the Ordinate  $BD = z$ . And let the Equation expressing the Natures of the Curves  $ADE$  be  $z = r v^2 y^n$ , in which  $r$  denotes any given determinate Quantity, and  $n$  the indefinite Exponent of the indeterminate Quantity  $y$ . I say the Area is

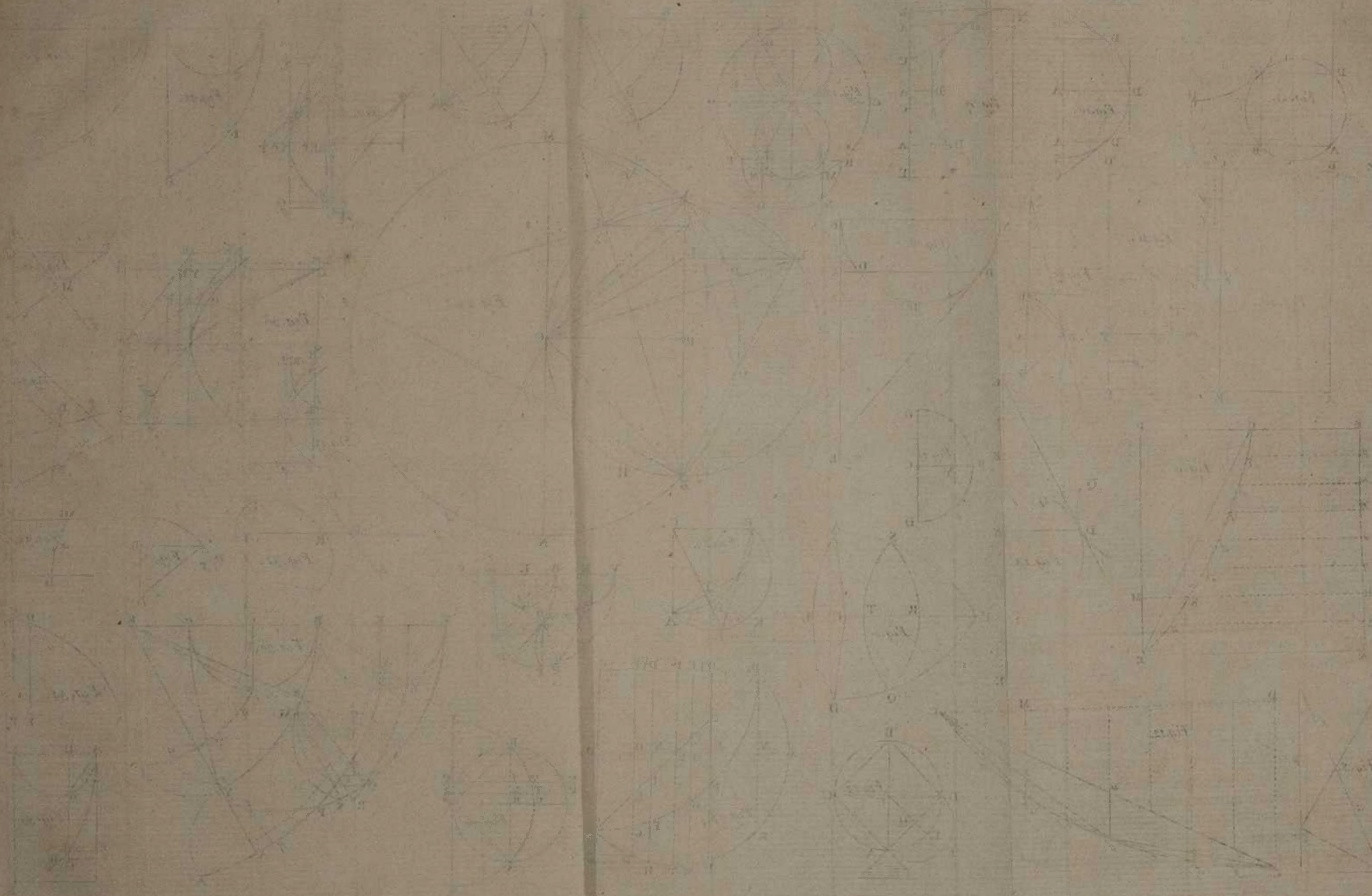
$$ABD = r v^2 y^n + \dots - q v^2 + v \sqrt{2ay - yy} \text{ into } \frac{2ra}{n+1} y^n + \frac{2ra^2 \times 2n+1}{n \times n + 1} y^{n-1} + \frac{aA \times 2n-1}{n-1} y^{n-2} + \frac{aB \times 2n-3}{n-2} y^{n-3} + \frac{aC \times 2n-5}{n-3} y^{n-4} + \frac{aD \times 2n-7}{n-4} y^{n-5}, \text{ \&c. } - \frac{2ra^2}{n+1} y^{n+1} - \frac{2ra^3 \times 2n+1}{n^2 \times n + 1} y^{n-1} - \frac{a^2 A \times 2n-1}{(n-1)^2} y^{n-2} - \frac{a^2 B \times 2n-3}{(n-1)^2} y^{n-3}, \text{ \&c.}$$

Of this Theorem these Things are to be observed; (1.) That it is compounded of two infinite Series, of which the first (connected by the Sign  $+$ ) is multiplied into  $v \sqrt{2ay - yy}$ ; but the Terms of the latter (affected with the Sign  $-$ ) are absolute. (2.) That in the former the great Letters  $A, B, C, D, \text{ \&c.}$  denote the Coefficients of the Terms that precede them: Also in the latter they have the same Values as in the former. (3.) That the











the Quadrature is exhibited by a finite Quantity, when  $n$  is an Integer positive Number, or equal to nothing, or when  $2n$  is an odd Number. For in these Cases both the Series break off. (4.) That  $2q$  is equal to the last Term breaking off of the former Series.

*Example 1.]* Let  $z = \frac{v^2}{a}$ . Because in this Case  $n = 0$ ,  $r = \frac{1}{a}$ , therefore it will be Area  $ABD = \frac{y v^2}{a} - v^2 + 2v \sqrt{2ay - yy} - 2ay$ .

*Corol.* The intire Figure  $A F E$  is equal to a double Square, the Side of which is  $A C F$ , taking away the Square of the Diameter.

*Example 2.]* Let  $z = \frac{y v^2}{a^2}$ . Because in this Case  $n = 1$ ,  $r = \frac{1}{a^2}$ , therefore the Area will be  $ABD = \frac{y^2 v^2}{2a^2} - \frac{3}{4} v^2 + v \sqrt{2ay - yy} \times \frac{v}{2a} + \frac{3}{2} + \frac{1}{4} yy - \frac{3ay}{2}$ .

*Example 3.]* Let  $z = \frac{y^2 v^2}{3a^3}$ . Because in this Case  $n = 2$ ,  $r = \frac{1}{a^3}$ , the Area will be  $ABD = \frac{y^3 v^2}{a^3} - \frac{5}{6} v^2 + v \sqrt{2ay - yy} \times \frac{2y^2}{9a^2} + \frac{5y}{9a} + \frac{5}{3} - \frac{2y^2}{27a} - \frac{5y^2}{18} - \frac{5ay}{3}$ .

XV. Let  $ONF$  be a Logarithmick Curve, whose Asymptote is  $AR$ , in which let such a Point  $A$  be taken, that its first Ordinate  $AO$  may be equal to the Subtangent, or to Unity. The Curvilinear Space  $AONM$  is required, comprehended by the two Ordinates  $AO$ , and  $MN$ , by the Absciss  $AM$ , and by the Logarithmic Curve  $ON$ .

*The Quadrature of the Logarithmick Curve, by Mr. J. Craig. N. 245. P. 373. Oct. An. 1693. Fig. 43.*

From  $O$  draw  $OE$  parallel to  $AM$ , cutting  $MN$  in  $E$ . I say that the Rectangle of the Segments  $ME$ ,  $EN$ , is equal to the Space required.

*Demonstration.]* Make the Ordinate  $MN = z$ , the Subtangent  $AO$  or  $ME = s$ , and to the Axis  $AR$  let another Curve  $H G Q$  be constructed, whose Equation is  $2sz = xx$ , where its Ordinate  $GM = x$ . I say it is the Quadratrix of the Logarithmic Curve, according to the Foundation of my Method: That is, its Subnormal is equal to the respective Ordinate of this, as according to the Calculation belonging to that Method may appear. Therefore according to what I have explained elsewhere, if to  $G$  be drawn  $GC$  perpendicular and equal to the Line  $GM$ , and also  $HD$  parallel to  $GC$ , meeting the Lines  $GM$ ,  $CM$ , in  $B$  and  $D$ ; it will be Trapezium  $GBDC$



$GBDC = AONM$ . But  $GBDC = GMC - BMD = \frac{1}{2}xx - \frac{1}{2}BMq = sz - \frac{1}{2}HAq$ . But  $HA = \sqrt{2}AOq$  by the Nature of the Curve  $HGQ$ . Therefore  $GBDC = sz - AOq = AO \times \overline{MN} - AO = ME \times \overline{MN} - ME = ME \times EN$ . Therefore also  $AONM = ME \times EN$ . *Q. E. D.*

*A Quadratrix to the Circle; being the Curve described by its Equable Evolution, by . . . . .*  
N. 260. p. 445.  
Jan. An. 1700.  
Fig. 44.

XVI. 1. By the Equable Evolution of a Circle, I mean such a gradual Approach of its Periphery to Rectitude, as that all its Parts do together, and equally, evolve or unbend; or so that the same Line becomes successively a less and less Arc of a reciprocally greater Circle.

2. Let  $AHK A$  be the Periphery of a Circle,  $AE$  a Tangent to the Point  $A$ . Let this circular Line be supposed cut or divided at  $A$ , and then to unbend (like a Spring,) its upper End remaining fixed to its Tangent  $AE$ , whilst the other Parts do equally evolve or extend themselves through all the Degrees of less Curvature (as in  $ABD$ ,  $AMC$ , &c.) till they become streight in Coincidence with the Tangent  $AE$ .

3. Let  $AMC$  be the evolving Curve in any middle Position between its first and last. Join the fix'd End  $A$ , and the moving End  $C$ , by the Chord-line  $AC$ , intersecting the first Circle at  $H$ ; I say, that  $AMC$  is a like Segment to  $A \cap H$ , cut off in the first Circle by the Chord  $AH$ : For, by the Supposition,  $AMC$  is the Arc of a Circle, having  $AE$  a Tangent common both to it and  $A \cap H$ ; and both Arcs are terminated in the same Right-Line  $AC$ .

4. Hence the Curve  $ADCE$  (described by the moving End of the Periphery in its Evolution) may be thus constructed: Let the Circle  $AHK A$  be by Bisections divided into any number of equal Parts; let  $H$  be one of the Points of such Division: Then say, As the Number of equal Parts in the Arc  $A \cap H$ , is to the Number of Parts in the whole Periphery  $AHK A$ ; so is the Chord  $AH$ , to a fourth Line, which let be  $AC$  in  $AH$  produc'd. So is  $C$  a Point in the Curve  $ADCE$ .

5. *Dem.* Upon  $AC$  describe  $AMC$ , an Arc like to the Arc  $A \cap H$ . Whence  $AH : AC :: A \cap H : AMC$ . But by Construction,  $AH : AC :: A \cap H : \text{Periph. } AHK A$ ; therefore is the Arc  $AMC$  equal to the whole Periphery  $AHK A$ , and like to the Arc  $A \cap H$ : Consequently  $AMC$  represents the evolving Periphery, in a Position like to the Arc  $A \cap H$ , and  $C$  is the describing Point.

6. After the same manner may be found other Points, thro' which the Curve may be drawn: But here (as in the old *Quadratrix* of *Dinostratus*) the Point  $E$  cannot be precisely determined; but the Curve may be brought so near it, that its Flexure or Tendency will so lead to the Point  $E$ , that  $AE$  shall be near enough to the Truth for common Uses.

7. Supposing the Point  $E$  found, a Tangent to any Point of the Curve may be drawn; and supposing a Tangent drawn, the Point  $E$  may be determined; the Property of the Tangent being this, that supposing  $RT$  a Tan-

gent



gent to the Point  $C$ , and  $CA$ ,  $CE$ , drawn from  $C$  to each End of the recti-  
fy'd Circle, the Angle  $ACT$  (the lesser Angle that  $AC$  makes with the  
Tangent) is equal to  $ACE$ , the Angle made by the two Lines drawn  
from  $C$ .

8. Let  $c$  be a Point in the Quadratrix indefinitely near to  $C$ ; and draw  $Ac$   
intersecting  $AHK A$  in  $b$ , and  $AMC$  in  $o$ . To  $Ac$  as a Chord, draw the  
Arc  $Amc$ , like unto the Arc  $Anb$ : To the Point  $C$  of the Arc  $AMC$   
draw the Tangent  $CL = AE$ , and join  $LA$ ; so is  $oC$  an indefinitely lit-  
tle Particle of the Arc coincident with its Tangent.

9. Because of the like Segments  $Anb A$ ,  $Amc A$ , as Chord  
 $Ac$  to Chord  $Ao$ , so is Arc  $Amc (= AMC)$  to Arc  $Amo$ : Or,  $Ac$ :  
 $Ao :: Amc (= AMC) : Amo$ ; and dividing,  $Ac - Ao (= co) : Ao ::$   
 $Amc - Amo (= Co) : Amo$ : That is,  $co : Ao :: Co : Amo$ ; and al-  
ternately,  $co : Co :: Ao : Amo$ . Put  $AC$  for  $Ao$ , and  $AMC$  for  $Amo$   
(as differing infinitely little) and then 'tis  $co : Co :: AC : AMC$ . But by  
*Construction*  $CL = AE = AMC$ , whence  $co : Co :: AC : CL$ ; and the  
Angle  $LCA = Coc$ , ( $oc$  being infinitely near to  $AC$ , is therefore parallel  
to it;) and therefore  $Coc$ ,  $ACL$ , are like Triangles.

10. Because of  $CL = AE$ , Angle  $EAC = LCA$ , ( $CL$  and  $EA$  be-  
ing Tangents to the two ends of the same Circular Arc  $AMC$ , make equal  
Angles with its Chord  $AC$ ) and  $AC$  common to both, the Triangles  $EAC$ ,  
and  $ACL$ , are like and equal: therefore are all three  $Coc$ ,  $ACL$ ,  $EAC$ ,  
like Triangles. Whence it follows, That the Angle  $ACE$  (in the Triangle  
 $EAC$ ) is equal to the Angle  $ocC$  (in the Triangle  $coC$ ) but  $ocC = ACT$ ,  
because  $oc$  and  $AC$  are parallel; therefore the Angle  $ACE = ACT$ .  
*Q. E. D.*

XVII. In a certain Epistle of mine, in Vol. 3. of my Mathematical Works, *The Dimensions of a Sphere and Cylinder compar'd; by D. Wallis. N. 263. p. 547. An. 1700.* among other Methods for Quadratures are to be found these two. One I call the Method of Convolution and Evolution, the other the Method of Com-  
plication and Explication. By Help of these I shew, which is the simplest Manner of measuring all Curve Figures, and particularly the Cycloid.

By a like Artifice may be shewn how to compare the Sphere and Cylin-  
der, which *Archimedes* thought fit to chuse for his Monument.

If to the Basis  $P$ , equal to the Circumference of a Circle, a Height  $R$  be  
assumed equal to Radius, there will be made a Rectangular Parallelogram =  
 $RP$ . This may be conceived as composed of an infinite Number of small  
Parallelograms, of the same Height, according to the received Method of  
Indivisibles.

Now if the Vertices of all these are conceived to be contracted into one  
Point, so that of those minute Parallelograms as many Triangles may be  
made, having the same Basis and an equal Height; each will be half of each  
of the others, and therefore all of all; and the Base being bent into the Cir-  
cumference of a Circle, a Circle will be made whose Radius is  $R$  and Center  
 $C$ , which therefore is half the Parallelogram, or  $\frac{1}{2} RP$ .

This

Fig. 45.

Fig. 46.



This is *Archimedes's* Dimension of a Circle, which is equal therefore to a Right-angled Triangle, one of whose Sides about the Right Angle is equal to the Periphery, and the other to the Radius of the proposed Circle. For  $\frac{1}{2} R$ , or half the Altitude of the Triangle, drawn into  $P$  the Base, exhibits the Magnitude of that Triangle  $\frac{1}{2} R P$ , which is equal to the Circle. And the same may be accommodated to the Circular Sector, taking the Arch  $A$  instead of the Periphery  $P$ .

Fig. 47.

Again, if to that Parallelogram  $= R P$ , as a Base, be taken in like manner an Altitude  $R$ , in order to a Hemisphere; there will be made a Parallelepiped  $= R R P$ . This in the same manner may be conceived as composed of an infinite Number of small Parallelepipeds of the same Height, insisting upon the minute Areas of that Plain; of all which the common Altitude is  $R$ , and the Aggregate of Bases  $= R P$ . Now if this Parallelogram, the Magnitude  $R P$  continuing, be supposed to be bent into a Cylindrical Surface (whose Base  $P$  is now bent into the Periphery of a Circle, and whose Altitude is  $R$ ) that those minute Parallelepipeds may be changed into so many Wedges, or Prisms with Triangular Bases, each of which are half their respective Parallelepipeds, and therefore all are half of all; having for their Vertices so many Points  $C$ , or minute Lines, in the Axis of the Cylinder, and thus filling it up; the Cylinder will become half the Parallelepiped, or  $\frac{1}{2} R R P$ .

Or in order to come at the intire Sphere, if on each Side is taken the Altitude  $R$ , so that the whole Altitude may be  $D = 2 R$ , and if a Convolution be made in like manner, a Cylinder will be produced as before, consisting of Wedges or Prisms infinite in Number, having their Points or Vertices in the Axis of the Cylinder, which will be equal to  $R R P = \frac{1}{2} R P \times 2 R$ , equal to the Product of  $\frac{1}{2} R P$ , or the Circular Base, into the Altitude  $2 R$ : Or which is the same Thing, it will be equal  $\frac{1}{2} R \times 2 R P$ , or equal to the Product of  $\frac{1}{2} R$ , the half of the common Altitude of the Wedges, into the Aggregate of the Bases  $2 R P$ .

Which Aggregate of the Bases is the curved Cylindric Superficies itself, which is equal to  $P \times 2 R$ , or to the Product of  $P$  the Periphery of the Circular Base drawn into the Altitude  $2 R$ , or equal to  $\frac{1}{2} R P \times 4$ , four of the great Circles of the Sphere. To which if we add the two opposite Circular Bases, there will be made the whole Superficies of the Cylinder circumscribed to the Sphere, equal to six great Circles,  $\frac{1}{2} R P \times 6 = 3 R P$ . And the Magnitude of the Cylinder,  $= R R P = \frac{1}{2} P P \times 2 R$ , equal to the Product of the Circular Base  $\frac{1}{2} R P$  drawn into the Altitude  $2 R$ , as before.

Now if the Vertices of all these Wedges that constitute the Axis of the Cylinder, are conceived to be contracted into one Point, so that these Wedges or Prisms may now become so many Pyramids, being on the same Bases and the same Height; each will be of each, and therefore all of all, in a Proportion subsesqui-tertia or as  $\frac{7}{4}$  to  $\frac{1}{2}$ ; and the Superficies which before was curved Cylindrical, will now become Spherical because of all its Points being equally remote from the Center, the Aggregate of the Bases remaining as before  $= 2 R P$ , or equal to four great Circles; we shall then have the

whole



whole Superficies of the Sphere  $= 2 R P = \frac{1}{2} R P \times 4 =$  four great Circles; and equal to the whole curved Cylindrick Surface, and the Parts respectively equal to the Parts that belong to the same Parts of the Axis; also the Magnitude of the Sphere  $\frac{2}{3} R R P = \frac{1}{3} R P \times 2 R$ , equal to the Product of  $\frac{1}{3} R$ , a third Part of the common Altitude of all the Prisms, drawn into  $2 R P$  the Aggregate of the Bases, which is now become the Spherical Surface.

Therefore both the Superficies and Magnitude of the Cylinder circumscribed to the Sphere, is sesqui-alter to the Superficies and Magnitude of the inscribed Sphere, or as 3 to 2: There because the Proportion is as six great Circles  $= 3 P P$  to four great Circles  $= 2 R P$ ; here because the Proportion is as  $R R P$  to  $\frac{2}{3} R R P$ : Which is the very Invention of *Archimedes* so much celebrated.

The same would be had a little shorter, if in the Parallelepiped upon the plain Base  $2 R P$  of the Altitude  $R$ , composed of minute Parallelepipeds, all their Vertices were immediately supposed to be contracted into one Point  $C$ : That the Aggregate of the Bases continuing as before  $= 2 R P$ , those Parallelepipeds may be reduced to so many Pyramids, having their Vertices meeting at the Center of the Sphere, whose Radius  $R$  is the common Altitude of all the Pyramids, and the Spherical Superficies is the Aggregate of all the Bases. For  $\frac{1}{3} R$ , a third Part of the common Altitude, drawn into  $2 R P$  the Aggregate of the Bases, exhibits as before the Magnitude of the Sphere  $\frac{2}{3} R R P$ , and the Surface of the Sphere  $= 2 R P$ .

In like manner this may be accommodated to the Spherical Sector, by drawing  $\frac{1}{3} R$ , a third Part of the common Altitude of all the Pyramids in it, into a Portion of the Spherical Surface cut off by a Plain: Which is to the whole Spherical Surface, as the Part cut off of the Diameter or Axis is to the whole Diameter; as was shewn above.

Now the Reason of this whole Process depends on these Principles. That a Figure composed of Triangles is half the Figure composed of Parallelograms, upon the same Bases and of equal Height. That I call a Convolute Figure, and this an Evolute. And that a Figure of Pyramids is a third Part of a Figure of Parallelepipeds, on the same Bases and equal Height. That I call a Complicate Figure, and this an Explicate. These Principles may be accommodated in a thousand Manners to Curvilinear Figures, whether Superficial or Solid, however perplexed and intricate.

XVIII. 1. It hath been observed by divers of this Nation, that in any Equation, howsoever affected, if you give a Root, and find the absolute Number or Resolvend, (which *Vieta* calls *Homogeneum Comparationis*;) and again give Roots and find more Resolvends; that if these Roots, or rather Rank of Roots, be assumed in Arithmetical Progression, the Resolvends, as to their first, second, or third Differences, &c. imitate the Laws of the pure Powers of an Arithmetical Progression, of the same Degree, that the highest Power, or first Term of the Equation is of. *E. g.* In this Equation  $a a a - 3 a a + 4 a = N$ .

*Improvements in England in the Resolution of Equations in Numbers, by Mr. J. Collins. N. 46. p. 929. Apr. An. 1669.*



If $n$ be =	}	10	Then N or the Absolutes or Re- solvends will be found to be,	}	740	218	1 diff.	2 diff.	3 diff.	
		9				522	170	48		
		8				352	128	42		6}
		7				224	92	36		6}
		6				132				

To wit, the 3d Differences of those Absolutes are equal, as in the Cubes of an Arithmetical Progression.

2. To find what Habitude those Differences have to the Coefficient of the Equation, 'tis best to begin from an Unit.

3. In any Arithmetical Progression, if you multiply Numbers by Pairs, you shall create a Rank of Numbers whose second Differences are equal; and if by Ternaries, then the third Differences of those Products shall be equal. And how to find the greatest Product of an Arithmetical Progression of any Number of Terms having any common Difference assigned, contained in any Number proposed, is shewed by *Pascal* in his Tract *Du Triangle Arithmetique*, where he applies it to the Extraction of the Roots of Simple Powers.

4. It appears, how this Rank may be carried easily by Addition, till you have a Resolvend either equal or greater or less than that proposed.

5. When you have a *Majus* and *Minus*, you may interpolate as many more Terms in the Arithmetical Progression as you will, that is to say, Subdivide the common Difference in the Arithmetical Progression, and render it less; and then renew, and find the Resolvends, which are easily obtained out of the Powers and their Coefficients, which are supposed known, and may be readily raised from a Table of Squares and Cubes, &c, with which kind the Reader may be furnished in *Guildini Centrobaryca*, and *Babington's Fireworks*. By this means you may obtain divers Figures of the Root; and then the General Method of *Vieta* and *Harriot* runs away more easily, and is so far improved, that after any Figure is placed in the Root, most certain Characters are given to know, by Aid of the subsequent Dividend and Divisor, whether the Figure before assumed be too great or too small: Or, lastly, it may be well concluded, that as in Logarithms, when you propose such an one as is not absolutely given in the Canon, you do, by proportional Work, using the Aid of their first Differences (when their absolute Numbers differ by Unite) find the absolute Numbers true to 5 or 6 Places farther than the Canon gives it (the Reason whereof is, that the first Differences do likewise agree to about the same Number of Places); that, I say, the like may be done in Equations, after divers of the first Figures of the Roots are found, provided there be the like Agreement in the first Differences of the Interpolated Resolvends.

Moreover, we ought here to take notice of a more subtile kind of Interpolation, common to all Gradual Ranks or Progressions of Numbers, wherein Differences happen to be equal: Of which kind the Reader may find Examples in *Briggii Arithmetica Logarithmica*, & *Trigonometria Britannica*, relating to Logarithms, Sines, and the Powers of an Arithmetical Progression:

but



But the Method there delivered may be rendered more easy and general, viz. by Aid of a Table of Figurate Numbers, by deriving Generating Differences sought from those given; a Doctrine that easily flows from *Mercator's Logarithmotechnia*, and of use in the Case in hand, should we suppose these Powers and their Coefficients unknown, or a Table of Squares and Cubes wanting, and give nothing more than a few Resolvends belonging to equal Moments or Spaces. And this may likewise be of good Use in *Gauging*, when having the Contents of a Solid, for every three Inches more or less given, without knowing the Dimensions of the Figure, and even in most Cases, when the Differences are progressive of one Kind, without knowing the Figure itself, having nothing given but its Contents at several equal parallel Distances, each such Distance may be sub-divided, and made as many as you please, and the respective Contents found by this general Method of Interpolation.

After one Root is obtained, the Methods of *Huddenius* and others will depress the Equation so as to obtain more, and consequently all of them.

6. It is easy by a Table of Figurate Numbers to give the Sum of any such Rank, or any Term in it relating to a known Part of the Series of Equals or Roots; but *è converso*, giving the Resolvend to find the Root, comes to an Equation as difficult as that proposed; as in *Dr. Wallis's Chapter of Figurate Numbers*.

7. Some affirm, they can give good Approaches for the obtaining a Root of any pure Power, affected Equation, or for the finding any of the mean Proportionals in any Rank between the two Extremes given.

8. Others pretend to have found out the Method (incited thereto by an Example in *Albert Gerard's Invention Nouvelle en Algebre, à Amsterdam 1629*) so much, by comparing of Equations, to increase or diminish the unknown Root of an Equation, as to render it a whole Number (or less differing therefrom than any Error assigned) and by *Albert Gerard's Method of Aliquot Parts* to find the same, and thereby the Root sought, altho' it be a mixt Number, Fraction, or Surd.

Probably this may sympathize with what is promised by the learned *Huddenius in Annexis Geometriæ Cartesianæ*, where he saith he intended not then to publish certain Rules he had ready; whereof one was to find out all the Irrational Roots both of Literal and Numeral Equations: This must be understood, when such Roots are possible; for 'tis certain there are Infinite Equations, whose Roots are no ways explicable, either in Whole or Mixed Numbers, Fractions or Surds, and can be no otherwise explained, but by a *quamproxime*.

9. The *Author of this Narrative* considering that the Conic Sections may *Ibid. p. 932.* be projected from lesser Circles placed on the Sphere, and thence easily (otherwise than hitherto hath been handled) described by Points, and that by their Interfections, some Spherick Problem is determined; accordingly, he found that this following Problem, according to the various Situation of the Eye, and of the projecting Plan, would take in all Cases.



*The Distances of an unknown Star are given from two Stars of known Declination and Right Ascension; the Declination and Right Ascension of the unknown Star is required.*

And saith, He hath observed, that, admitting the Mechanism of dividing the Periphery of a Circle into any Number of equal Parts, or (which is equivalent) the Use of a Line of Chords, that this Problem, wherever the Eye be placed, may be resolved by plain *Geometry*, and yet the Eye shall be so placed, as to determine it by the Intersection of the Conic Sections; consequently those Points of Intersection (the Species and Position of the Figures being given) may be found without describing any more Points than those sought; and the Lengths and Ordinates falling from thence on the Axes of either Figure calculated by Mixed *Trigonometry*, and hence likewise the Roots of all *Cubick* and *Biquadratic Equations* found by *Trigonometry*.

For giving, from the *Mesolabe* of *Slufius*, the Scheme that finds these Roots, it will then be required to fit those Sections into Cones, which have their Vertex either in the Center, or an assigned Point in the Surface of the Sphere, to which they relate as projected, and proceed to the Resolution of the Problem proposed; and how to fit in those Sections, see the *seven Books of Apollonius*, *Mydorgius*, the 3d Volume of *Des Cartes's Letters*, *Leotaudi Geometria Practica*, *Andersonii Exercitat. Geometricæ*.

As to the Problem itself, it is determined on the Sphere by the Intersections of the two lesser Circles of Distance, whose Poles are the known Stars. And this Problem hath divers Geometrick Ways of Resolution.

1. By plain Geometry (in the Sense before mentioned): Supposing a Plain to touch the Sphere or the North Pole; if the Eye be at the South Pole, projecting those Circles into the said Plane, they are still Circles (by reason of the sub-contrary Sections of the visual Cones) whose Centers fall in the Sides of the Right-lined Angle, made by the projected Meridians, that pass thro' the known Stars; and thus the Problem is easily solved in this Manner.

2. If it be required to be performed by Conic Geometry; In one Case it may be done, by placing the Eye at the Center of the Sphere, and projecting as before; to wit, when the longer Axes of the Figures being produced, concur above the Vertex: Here the Problem is determined by the Intersections of two Conic Sections (whereof a Circle cannot be one, unless its Center be in the Axis of the other Figure): And in this second Case, these Points of Intersection fall in the same Right Line or projected Meridian they did before, but at a more remote Distance from the Pole Point, to wit, in the former Supposition the Polar Distance was measured by a Right Line, that was the double Tangent of half the Arch; here it is the Tangent of the whole Arch. Hence it is evident how one Projection may beget another, yea, infinite others, altering the Scale, and how the lesser Circles in the *Stereographick* Projection help to describe the Conic Sections in the *Gnomonick* Projection: But (to reduce the Matter to the common Radius) if we suppose two Spheres equal, and so placed about the same Axis, that the Pole Point of the one shall pass thro' the Center of the other, and the Touch-Plain to pass thro' the said Center or Pole Point; and that a lesser Circle hath the  
same



same Position in the one as in the other: then, if the Eye be at the South Pole of the one, it is at the Center of the other; and any projected Meridian drawn from the projected Pole Point to pass thro' both the Projections of these lesser Circles, the Distances of the Points of Interfection are the Tangents of the half and the whole Arch of the Meridian so interfectcd. But as to the Points of Interfection, which determine the Problem proposcd, they may be found without the Aid of the former Way, from a *Gnomonick* and *Stereographick* Method of measuring and setting off the Sides and Angles of Spherical Triangles in those Projections, which is necessary in what follows.

3. If the Problem is to be performed by mixed Geometry, as by a Circle, and either a Parabola, Hyperbola, or Ellipsis, the Circle may be conceived to be the sub-contrary Section of a Cone projected by the Eye at the South Pole, and any of the rest of the Sections by the Eye at the Center of the Sphere.

4. If by any of the Conick Sections however posited, the projecting Plane may remain the same; but the Eye must be in some other Part of the Surface of the Sphere, and not in the Axis.

XIX. 1. The Construction delivered by *Des Cartes*, which easily finds the Roots of all Cubic and Biquadratic Equations, wherein the second Term is wanting, I may here suppose as known: Yet since the following depends upon it, I shall here subjoin the Rule taken from his Geometry, changing a few Things (as I hope) for the better.

*The Construction of Cubick and Biquadratic Equations, by a Parabola and Circle. By Mr. Edm. Halley. N. 183. p. 375. Jul. An. 1687. Fig. 43.*

When the second Term is wanting, all Cubic Equations may be reduced to this Form,  $z^3 + a p z + a a q = 0$ . And all Biquadratics to this,  $z^4 + a p z^2 + a a q z + a^3 r = 0$ . Where  $a$  denotes the *latus rectum* of any given Parabola, which may be used in the Construction; or making  $a = 1$ , the Equations will be  $z^3 + p z + q = 0$ , and  $z^4 + p z^2 + q z + r = 0$ .

Now the Parabola  $F A G$  being given, whose Axis is  $A C D K L$ , and its *latus rectum*  $a$  or Unity, let  $A C$  be its halt, and let it always be placed from the Vertex  $A$  within the Figure. Then take  $C D = \frac{1}{2} p$ , in the Line  $A C$  continued towards  $C$ , if in the Equation it is  $- p$ , or the other Way if it shall be  $+ p$ . Then from the Point  $D$ , or from  $C$  if  $p = 0$ , the Perpendicular to the Axis  $D E = \frac{1}{2} q$  is to be raised, to the Right Hand if it shall be  $- q$ , but on the other Side of the Axis if it shall be  $+ q$ . Then a Circle described with Center  $E$ , and Radius  $A E$ , if the Equation be only Cubic, will cut the Parabola in so many Points  $F$  and  $G$ , as it has true Roots, of which the Affirmative, as  $G K$ , will be to the Right Side of the Axis, and the Negative, as  $F L$ , will be on the Left Side.

But if the Equation is Biquadratic, the Radius of the Circle  $A E$  must be increased or diminished, by adding if it is  $- r$ , or subtracting if it is  $+ r$ , the Rectangle  $a r$  from its Square, or the Plain contained by the *latus rectum* and the given Quantity  $r$ ; which is easily done Geometrically. Now the Intersections of this Circle with the Parabola will exhibit all the true Roots of the Biquadratic Equation, by letting fall Perpendiculars to the Axis;



the Affirmatives to the Right Side of the Axis, and the Negatives to the Left Side. But I leave to *Des Cartes*, its Inventor, the Demonstration of the whole.

Here it is to be observed, that I endeavour always to have the Affirmative Roots on the Right Side of the Axis, to avoid that Confusion that will necessarily arise, from a Multitude of Cautions and Exceptions, the Reason of which would by no means be evident.

These Things being premised, that we may come to the Construction of such Equations also, where the second Term is present; the Rule for taking away the second Term must be considered, and for reducing the Equation to another, which may be constructed by the foregoing Method. Now all the Cubic Equations of this Class may be reduced to this Form,  $z^3 : b z^2 : a p z : a^2 q = 0$ , or to this,  $z^3 : b z^2 : * a^2 q = 0$ . And the Biquadratics to this  $z^4 : b z^3 : a p z^2 : a^2 q z : a^3 r = 0$ , or to this  $z^4 : b z^3 : * : a^2 q z : a^3 r = 0$ , or to this,  $z^4 : b z^3 : a p z^2 : * : a^3 r = 0$ , or lastly to this,  $z^4 : b z^3 : * : * : a^3 r = 0$ . Out of all these, as they may be differently connected by the Signs  $+$  and  $-$ , there arises a vast Variety. Whence a general Rule comprehending them all must needs be very difficult and obscure, unless it be treated in the following Manner, and so delivered from its Perplexities.

In Biquadratics the second Term is taken away, by making  $x = z + \frac{1}{4} b$ , if it be  $+ b$  in the Equation; or  $x = z - \frac{1}{4} b$ , if it be  $- b$ . Hence in the first Case  $x - \frac{1}{4} b$ , and in the second  $x + \frac{1}{4} b$ , is equal to  $z$ ; and in any Equation proposed instead of  $z$  substituting its equal Quantity, there arises a new Equation in which the second Term is wanting, all whose Roots  $x$  either exceed or are deficient from the Root sought  $z$ , by a given Difference  $\frac{1}{4} b$ .

*Example 1.*  $z^4 + b z^3 - a p z^2 - a^2 q z + a^3 r = 0$ . Make  $x - \frac{1}{4} b = z$ , then  $x^2 - \frac{1}{2} b x + \frac{1}{16} b^2 = z^2$ ,  $x^3 - \frac{3}{4} x^2 b + \frac{3}{16} x b^2 - \frac{1}{64} b^3 = z^3$ , and

$x^4 - b x^3 + \frac{3}{8} b^2 x^2 - \frac{1}{16} b^3 x + \frac{1}{256} b^4 = z^4$ . Hence by Substitution

$$x^4 - b x^3 + \frac{3}{8} b^2 x^2 - \frac{1}{16} b^3 x + \frac{1}{256} b^4 = + z^4$$

$$+ b x^3 - \frac{3}{4} b^2 x^2 + \frac{3}{16} b^3 x - \frac{1}{64} b^4 = + b z^3$$

$$- a p x^2 + \frac{1}{2} a b p x - \frac{1}{16} a p b^2 = - a p z^2$$

$$- a^2 q x + \frac{1}{4} a^2 b q = - a p q z$$

$$+ a^3 r = + a^3 r$$

The Sum of all these becomes a new Equation, in which the second Term is wanting, and which therefore may be constructed by *Cartes's* Rule; putting instead of  $\frac{1}{2} p$  half the Coefficient of the third Term divided by  $a$  or the la-



*tus rectum*, that is,  $-\frac{3bb}{16a} - \frac{1}{2}p$ ; and instead of  $\frac{1}{2}q$  half the Coefficient of the fourth divided by  $aa$ , or  $+\frac{b^3}{16a^2} + \frac{pb}{4a} - \frac{1}{2}q$ . The Parts affected with the Sign  $+$  are to be placed on the Left Side of the Axis, and those affected with  $-$  on the Right Side, that the Center of the Circle may be had, as is requisite for the Construction; so that the Intersections of this with the Parabola, by Perpendiculars let fall upon the Axis, may exhibit all the true Roots  $x$ , the Affirmatives on the Right Side of the Axis, and the Negatives on the Left Side. For since  $x - \frac{1}{4}b = z$ , by drawing a Line parallel to the Axis, on its Right Side and at the Distance  $\frac{1}{4}b$ , those Perpendiculars terminated at this Parallel will exhibit all the Roots  $z$ , the Affirmatives to the Right Hand and the Negatives to the Left. As for the Radius of the Circle, it will be had by adding the negative Parts, and subtracting the affirmative Parts of the fifth Term divided by  $aa$ , from the Square of the Line A E drawn from the Center E (when found) to the Vertex of the Parabola A. And this may generally be done by taking instead of the Line A E the Line A O, which is terminated at O the Intersection of the Parabola and the aforesaid Parallel. For its Square contains (as may easily be proved) all the Parts of the fifth Term accruing to the new Equation, by taking away the second Term: And it only remains that the Square of E O may be increased, if there is  $-r$  in the Equation, or diminished if there is  $+r$ , by the Addition or Subtraction of the Rectangle  $ar$ ; whence the Square of the Radius of the Circle required is composed.

This is Mr. Baker's Method of finding the Central Rule, but free from all his Cautions, and easy enough. The only Difference arises from hence, that I determine the Center of the same Circle by the Axis, and he by the Parallel to the Axis; and that I always find the affirmative Roots on the Right Side of the Axis, which he places sometimes on the Right and sometimes on the Left.

As for what belongs to Cubic Equations, they must be reduced to Biquadratics, before they can be constructed by the same general Rule. And this is done by multiplying the Equation proposed by its Root  $z$ , whence a Biquadratic Equation arises, in which the last Term, or  $r$ , is wanting. Wherefore taking away the second Term, and the Center E being found, the Line E O is the Radius of the Circle; since  $ar = 0$ , and in the new Equation the whole fifth Term arises, by only taking away the second Term.

*Example 2.]*  $z^3 - bz^2 + apz + aaq = 0$ . This multiplied by  $z$  becomes  $z^4 - bz^3 + apz^2 + aaqz = 0$ . To take away the second Term make  $x - \frac{1}{4}b = z$ .

$$\text{Then } x^4 + bx^3 + \frac{3}{8}b^2x^2 + \frac{1}{16}b^3x + \frac{1}{256}b^4 = z^4$$

$$-bx^3 - \frac{3}{4}b^2x^2 - \frac{3}{16}b^3x - \frac{1}{64}b^4 = -bz^3$$

$$+ apx^2$$



$$+ a p x^2 + \frac{1}{2} a b p x + \frac{1}{16} a p b^2 = + a p z^2$$

$$+ a^2 q x + \frac{1}{4} a^2 b q = + a^2 q z$$

In this new Equation, half the Coefficient of the third Term divided by  $a$ , that is  $-\frac{3 b b}{16 a} + \frac{1}{2} p$ , is to be used instead of  $\frac{1}{2} p$ ; and half the Coefficient of the fourth Term, divided by  $a a$  or the Square of the *latus rectum*, that is  $-\frac{b^3}{16 a^2} + \frac{p b}{4 a} + \frac{1}{2} q$ , is to supply the Office of  $\frac{1}{2} q$  in *Cartes's* Construction. Whence the Center  $E$  is determined. Then a Parallel being drawn to the Axis at the Distance of  $\frac{1}{4} b$ , on its Left Side, because of  $x + \frac{1}{4} b = z$ , and let the Interfection of this with the Parabola be  $O$ . A Circle described with Center  $E$ , and Radius  $E O$ , will cut or touch the Parabola in so many Points at the Equation has true Roots; which Roots or Values of  $z$  are exhibited by Perpendiculars let fall from those Points upon the Parallel to the Axis; the Affirmatives to the Right Hand, and the Negatives to the Left.

If in the Equation either the third or fourth Term, or both shall be wanting, there will be no Difference to be observed in investigating the Central Rule, but the Quantities  $p$  or  $q$  vanishing, those Parts of the Lines  $C D$  and  $D E$  arising from those Quantities will be absent; so that we must proceed with the remaining Coefficients of the third and fourth Terms in the new Equation, as is prescribed in the foregoing Examples.

Hitherto we have pursued *Baker's* general Method, than which we can expect none that will be easier or readier; whether a Parabola is assumed for the Construction, or any other Curve, and when the Equation ascends to the Biquadratic. And whilst I am writing this there occurs to me a Geometrical Effect of the Central Rule, which is expeditious beyond Hope, and will abundantly satisfy the Curious in these Matters.

Fig. 49.

The Parabola  $N A M$  being described, whose Vertex is  $A$ , its Axis  $A B C$ , and *latus rectum*  $a$ , let the Equation be reduced to this Form,  $z^3 : b z^2 : a p z : a^2 q = 0$ ; or to this,  $z^3 : b z^2 : a p z : a^2 q = 0$ , if it be a Cubic Equation. Then at the Distance of  $B D = \frac{1}{4} b$  let the Line  $D H$  be drawn parallel to the Axis, to the Left Hand if it be  $-b$ , or to the Right if  $+b$ , meeting the Parabola in the Point  $D$ , from whence let fall  $B D$  perpendicular to the Axis. In the Line  $A B$  continued towards  $B$  make  $B K = \frac{1}{2} a$ , and draw the Line  $D K$  both ways indefinitely. Then make  $K C = 2 A B$  in the Axis always continued beyond  $K$ ; and if the Quantity  $p$  is affected with the Sign  $-$ , take  $C E = \frac{1}{2} p$  also towards the same Side, but contrariwise if it is  $+p$ ; and from the Point  $E$  let there be raised  $E F$  perpendicular to the Axis, or from the Point  $C$  if the Quantity  $p$  is absent, meeting in  $F$  the Line  $D R$  produced if need be; which Point  $F$  is the Center of the Circle required if  $q$  be absent, but if it be present in  $F E$ , continued if need be, must be taken a Line  $F G = \frac{1}{2} q$ , to the Left if it is  $+q$ , but



to the Right if  $-q$ . And the Point G will be the Center of the Circle proper for the proposed Construction, and its Radius will be the Line G D, if  $r$  is wanting or if the Equation is only Cubical. In the Biquadratics the Square of this must be increased or lessened, according as it is  $-r$  or  $+r$ , by the Rectangle of  $r$  and the *latus rectum*. The Circle being thus described, a Perpendicular being let fall upon the Line D H from its Intersections with the Parabola, those that are on the Left Hand, as N O, will always represent the negative Roots of the Equation, and those on the Right Hand, as M L, the affirmative Roots.

Cubic Equations are constructed otherwise, and something more simply; by a Rule of *Van Schootens*, by which also the Roots are referred to the Axis. But because the Inventor himself does not deliver his Manner of finding it, nor the Demonstration of his Invention, it may not be amiss to shew here the Foundation of the same, and to make the Geometrical Effectation something more neat, and to free it from some Cautions in which it is involved.

This Rule is derived from hence, that every Cubic Equation may be reduced to a Biquadratic, in which the second Term is wanting. This is done by multiplying the Equation proposed into  $z - b = 0$ , if there is  $+b$  in the Equation; or into  $z + b = 0$ , if it be  $-b$ . Then the new Equation so produced will have the same Roots with the Cubic, and another besides equal to  $-b$ , if there is  $-b$  in the Equation; or on the contrary.

Suppose we were to construct  $z^3 - z^2 b + a p z + a a q = 0$ . Multiply this by  $z + b$ ; it becomes

$$\left. \begin{array}{l} z^4 - b z^3 + a p z^2 + a^2 q z \\ + b z^3 - b^2 z^2 + a b p z + a^2 b q \end{array} \right\} = 0.$$

Here the second Term is wanting, and the Coefficient of the third  $-bb + a p$  gives  $-\frac{bb}{2a} + \frac{1}{2}p$  to be substituted for  $\frac{1}{2}p$  or C D in *Cartes's* Construction; and of half the Coefficient of the fourth Term is made  $+\frac{1}{2}q + \frac{bp}{2a}$ , to be used instead of  $\frac{1}{2}q$  or D E, and so the Center of the Circle sought is determined. Then because one of the Roots of the new Equation is given, which is  $-$  or  $+b$ , a Point also in the Circumference will be given, and therefore the Radius. Lastly, the Circle being described, Perpendiculars let fall upon the Axis from its Intersections with the Parabola will exhibit the Roots of the Equation, the affirmative and negative as mentioned before.

Now the Center of the Circle is found by a very easy Construction, which is preferable to all others in Cubic Equations. Let A be the Vertex and A F the Axis of the described Parabola A M D. At a Distance equal to  $b$  let D K be drawn parallel to the Axis, at the Right or Left Hand according as it is  $+b$  or  $-b$ , which meets the Parabola in the Point D. With Centers D and A, and with equal Radius's, let occult Arches be described intersecting one another on both Sides, and through the Points of Intersection draw

Fig. 50.



draw the indefinite Right Line P C, which may be perpendicular to the supposed Line A D at its middle, and meet the Axis at the Point E. From E, either below or above according as there is either  $-p$  or  $+p$  in the Equation, make  $E F = \frac{1}{2} p$ ; and from F, or from E if  $p$  is wanting, draw the Perpendicular F G meeting the Line B C in the Point G; and in G F produced make  $G H = \frac{1}{2} q$ , to the Right if there is  $-q$  in the Equation, or otherwise to the Left. Then will H be the Center required, and H D the Radius of the Circle; which will shew all the Roots as before, by means of Perpendiculars let fall from its Intersections with the Parabola to its Axis.

The Number of  
Roots in such  
Equations, with  
their Limits and  
Signs; by Mr.  
Edm. Halley.  
N. 190. p. 389.  
Nov. An. 1687.

It appears from *Des Cartes* and from what has been already said, as well in Cubics as in Biquadratics, that the Roots may be represented by letting fall Perpendiculars upon the Axis, or a given Diameter of the given Parabola, from the Intersections of that Curve with a Circle. And since a Circle cutting a Parabola must cut it either in four or in two Points, it is plain that in Biquadratics there must necessarily be two or four true Roots, either affirmative or negative; as also if perhaps the Circle may touch it, in which we may infer an Equality of two Roots of the same Sign. But in Cubics, because one of the Intersections is required for the Construction, the one or the three remaining represent the one or the three Roots, as in the Case of Contact, whence it is plain that there are found two equal Roots, and that the Problem from whence the Equation results is truly a plane Problem.

Therefore all Cubic Equations however affected are explicable with one or with three Roots, as always possible; admitting negative Roots for true ones. So Biquadratics, of which the last Term  $r$  is affected with the Sign  $-$ , are explicable by two or four. But if there is  $+r$  in the Equation, and it is so great as that  $\sqrt{G D q - a r}$  is less than that a Circle can reach the Parabola in any Point, when described with that Radius and the Center G; the given Equation is totally impossible, nor is it explicable by any Root affirmative or negative. But of this more afterwards.

Now whereas there is so great a Difference between the Cases of Cubick and Biquadratick Equations, as that they cannot be comprehended together; I shall first treat of Cubicks, and then of the other. Now Cubicks may be constructed by an infinite Number of Circles in a given Parabola, but Biquadratics with one only, at least in this Method. And that because by putting  $z = e$ , or any Indeterminate, equal to nothing, the Cubick Equation is reduced to a Biquadratick, having the same Roots with the Cubick, and another besides which is equal to  $e$ . Whence it is that the Cubick may be constructed by so many different Circles as there are different Values of  $e$ , that is infinite. But among these Constructions that is by far the most easy, which I gave above in the last Paragraph. Yet there is another not much inferior to it, which seems more accommodated to the Discovery of the Number of the Roots and their Limits, and which derives its Original from taking away the second Term, by supposing (as is commonly done)  $x = z +$  or  $-$  the third Part of the Coefficient of the second Term. Here it follows:

The Parabola A B Y being given, its Vertex A, its Axis A E, and its *latus rectum*  $a$ ; let the Equation be reduced to the usual Form  $z^3 : b z^2 :$

$a p z :$

Fig. 4.

Fig. 51.



$apz : a^2 q = 0$ . Then at the Distance  $\frac{1}{2} b$  let  $BK$  be drawn parallel to the Axis, to the Right Hand if it be  $+ b$ , or else to the Left Hand, meeting the Parabola in  $B$ . And let an indefinite Perpendicular be erected  $DP$  on the middle of the supposed Line  $AB$ , meeting the Axis in the Point  $G$ . From  $B$  let fall  $BC$  perpendicular to the Axis, and let  $GE$  be always made equal to  $AC$  with its Direction downwards. From  $E$  make  $EH = \frac{1}{2} p$  with its Direction upwards if it be  $+ p$  in the Equation, but otherwise downwards, and from the Point  $H$  (or from  $E$  if  $p$  be wanting) let the Perpendicular  $HQ$  be raised meeting the indefinite Line  $DP$  in the Point  $O$ . Lastly in the indefinite Line  $HQ$  make  $OR = \frac{1}{2} q$ , to the Right from  $O$  if it be  $- q$ , to the Left if  $+ q$ . Then a Circle described with Center  $R$ , and Radius  $RA$ , will cut the Parabola in so many Points as the Equation proposed has true Roots, and these will be the Perpendiculars  $ZY$  let fall from the Points of Intersection  $Y$  upon  $BK$  parallel to the Axis; of which those on the Right Side of  $BK$  will be the affirmative Roots, and those on the Left Side the Negative.

The Convenience of this Construction consists in this, because it is performed by a Circle passing through the Vertex, just as if the second Term were wanting. Therefore for determining the Number of Roots, it is enough to know the Properties of the Locus or the Curve-Line which distinguishes the Spaces, where if the Center of the Circle be placed that passes through the Vertex of the Parabola, its Circumference will cut it either in one or in three Points: That is, to define the Nature of the Curve, upon which shall fall the Centers of all the Circles, that pass through the Vertex and then touch the Parabola.

But that Locus is the Paraboloid, which with the great *Wallis* we will call Semicubical, or in which the Cubes of the Ordinates are to one another as the Squares of the Portions of the Axis respectively. Its *latus rectum* is  $\frac{27}{8}$  of the *latus rectum* of the given Parabola, and its Vertex is the Point  $V$ ,  $AV$  being half the *latus rectum* of the same Parabola. That is, if Unity be assigned for the *latus rectum* of the given Parabola,  $\frac{8}{27}$  of the Cube of the Ordinate will be equal to the Square of the Part of the Diameter, or the Cube of  $\frac{2}{3} VH$  will be equal to the Square of  $HR$ , if  $R$  be the Center of the Circle that passes through the Vertex of the Parabola, and afterwards touches it. This is that Curve which first of all Mortals our Mr. *Neil* demonstrated to be equal to a Right Line, who on this Account has been celebrated among the chief of Geometricians. And its Properties have been inquired into by the learned Dr. *Wallis*, at the End of his Book concerning the Cissoïd, and *Huygens*, *Prop.* 8, 9. of the Evolution of Curve Lines, and other ingenious Men whose Writings the Reader may consult. This Curve being described on each Side of the Axis of the Parabola, that is,  $VNL$  and  $VPX$ , will comprehend a Space, in which if the Center of the Circle be put, which passes through the Vertex  $A$ , that Circle will cut the Parabola in three other Points. But the Spaces that are more remote from the Axis



will supply the Centers of Circles which will cut the Parabola but in one Point besides the Vertex.

These Things being rightly understood, we shall now proceed to determine the Number of the Roots. And first let the second Term be wanting, and let the *latus rectum* be Unity, or  $AV = \frac{1}{2}$ . In the Construction  $VH$  is  $\frac{1}{2} p$ , and  $HR = \frac{1}{2} q$ . And as  $\frac{1}{2} p$  is to be put upwards from  $V$  if it be  $+$   $p$ , the Center of the Circle is always to be placed without the Space  $L V X$ , and therefore the Equation is explicable by one Root only, affirmative if it be  $- q$ , negative if  $+$   $q$ , which Roots are investigated by *Cardan's* Rules. But if it is  $- p$ , then  $VH = \frac{1}{2} p$  must be taken below, and it may happen that  $HR$  may fall between the Axis and the Curve  $VX$  or  $VL$ , namely if the Cube of  $2 VH$  or  $\frac{1}{3} p$  is greater than the Square of  $\frac{1}{2} q$ , or  $\frac{1}{27} p^3$  greater than  $\frac{1}{4} q q$ , in which Case there are three Roots, two negative if it is  $- q$ , and one Affirmative equal to their Sum; or if it is  $+$   $q$ , then two affirmative and one negative. But if  $\frac{1}{27} p^3$  is less than  $\frac{1}{4} q q$ , then only one Root is found, affirmative if it is  $- q$ , but negative if  $+$   $q$ . And these Things are commonly taught by those who treat of this Part of Geometry.

Now let all the Terms be present, and first let this Equation be proposed  $z^3 - b z^2 + p z - q = 0$ , to which we have adapted Figure 51. In the Construction of this,  $BC = \frac{1}{3} b$ ,  $VG = \frac{1}{2} AC = \frac{1}{18} b b$ ,  $VE = \frac{1}{6} b b$ ,  $VH = \frac{1}{6} b b - \frac{1}{2} p$ ,  $GH = \frac{1}{9} b b - \frac{1}{2} p$ , or  $\frac{1}{2} p - \frac{1}{9} b b$ , hence  $HO = \frac{1}{27} b^3 - \frac{1}{6} b p$ , or  $\frac{1}{6} b p - \frac{1}{27} b^3$ , and  $HR$ , or the Distance of the Center of the Circle  $R$  from the Axis, is always the Difference between  $\frac{1}{6} b p$  and  $\frac{1}{27} b^3 + \frac{1}{2} q$ ; which if they are equal, the Center falls in the Axis; if  $\frac{1}{6} b p$  is greater than  $\frac{1}{27} b^3 + \frac{1}{2} q$ , it falls to the Left of the Axis, but if less, to the Right. If the Square-root of the Cube of  $\frac{2}{3} VH$ , (that is, of  $\frac{1}{9} b b - \frac{1}{3} p$ , which we may call  $d$ ,) that is, if  $\sqrt{d d d}$  is greater than  $HR$ , or the Difference between  $\frac{1}{27} b^3 + \frac{1}{2} q$  and  $\frac{1}{6} b p$ , the Center  $R$  is found within the Space  $HPV$ , which is circumscribed by the Paraboloids  $VPX$  and  $VNL$ , and the indefinite Right Line  $DNP$ . And therefore the Circle will cut the Parabola in three Points  $Y, Y, Y$ , situate to the Right Hand of the Line  $BK$ , and therefore the Equation has three affirmative Roots.



Roots. But the Center being without this Space, N U P, it can be explained but by one affirmative Root. Here we may take Notice, that the Right Line D P that touches the Paraboloid V P X in the Point P, E P being  $\frac{1}{27} b^3$ , will cut the other V N L in the Point N, so that letting fall the Perpendicular N F upon the Axis, V F will be a fourth Part of E V, or  $\frac{1}{24} b b$ , but N F will be  $\frac{1}{108} b^3$ . But V W, which from the Point V being drawn perpendicular to the Axis, and meets the Line D P in W, is equal to  $\frac{1}{54} b^3$ , or  $\frac{1}{2} E P$ .

Hence we may safely conclude, if in the Equation  $p$  is greater than  $\frac{1}{3} b b$ , or  $q$  greater than  $\frac{1}{27} b^3$ , there is only one Root to be had, and that affirmative. Therefore *Des Cartes's* Rule is not true, (*pag. 70. Edit. Amst. 1659*) wherein he asserts, that so many true Roots are to be had as in the Equation there are Changes of the Signs  $+$  and  $-$ ; nor is it to any Purpose that *Schooten* in his Commentaries excuses this Mistake. For infinitely more Equations may be contrived of the foregoing Form, having three Changes of the Signs, which have but one Root, than which have three Roots. Also the fifth Proposition of the fifth Section of our *Harriot's Ars Analytica*; as likewise the 18th Problem of *Vieta's Numerosa Potestatum Resolutio*, are not sufficiently sound, since from the Limitations they have there given, that should belong to the whole Parallelogram P I V W, which we have now proved to belong only to the Space N V P; that is, that it should afford a Center for a Circle, which cuts the Parabola in three other Points besides the Vertex.

Now the Quantity  $q$ , or the last Term, when  $b$  and  $p$  are given in such Conditions that  $p$  is less than  $\frac{1}{3} b b$ , is accurately limited by the foregoing Equation  $\sqrt{d d d} = \frac{1}{27} b^3 + \frac{1}{2} q \approx \frac{1}{6} b p$ , when the Circle touches the Parabola. Therefore  $\frac{1}{2} q$  ought to be less than  $\frac{1}{6} b p - \frac{1}{27} b^3 + \sqrt{d d d}$ . But if  $p$  is greater than  $\frac{1}{4} b b$ , it is necessary that  $\frac{1}{2} q$  should be greater than  $\frac{1}{6} b p - \frac{1}{27} b^3 - \sqrt{d^3}$ , lest the Center should fall within the little Space N V W. And with these Conditions the Equation is always explicable by three Roots, but otherwise by one only. But whether three or one, they are always affirmative, because of the Position of the Center R to the Right Hand of the Line D P.



And this is the most difficult Case, so that he that well understands what goes before will easily understand the rest. Now let be given the Equation  $z^3 - bz^2 + pz - q = 0$ . Here that three Roots may be had, it is necessary that the Center of the Circle may be somewhere within the Space  $P N \Delta$ , included by the Right Lines  $P N$ ,  $P \Delta$ , and by the Curve of the Paraboloid  $N \Delta$ . Wherefore since  $E F = \frac{1}{8} b b$ ,  $p$  must be less than  $\frac{1}{4} b b$ . Now for the Determination of the Quantity  $q$ , making  $d = \frac{1}{9} b b - \frac{1}{3} p$  as before, then  $\sqrt{d d d} + \frac{1}{27} b b b - \frac{1}{6} b p$  must always be greater than  $\frac{1}{2} q$ , that the Center of the Circle may be found in the aforesaid Space  $P N \Delta$ : Which when it happens, such an Equation has two affirmative Roots and one negative. But if  $p$  is greater than  $\frac{1}{3} b b$ , or  $\frac{1}{2} q$  greater than  $\sqrt{d^3} + \frac{1}{27} b^3 - \frac{1}{6} b p$ , the Equation is explicable but by one Root, and that will be negative.

Now let be proposed the Equation  $z^3 - bz^2 - pz - q = 0$ . That this Equation may have three Roots, the Center of the Circle must be found somewhere in the indefinite Space, between the Right Line  $D P D$  and the Curve of the Paraboloid  $P X$ . Hence the Quantity  $p$  will be subject to no Limitations, but  $\frac{1}{2} q$  must always be less than  $\sqrt{d^3} - \frac{1}{27} b^3 - \frac{1}{6} b p$  making  $d = \frac{1}{9} b b + \frac{1}{3} p$ . By this means there will be two negative Roots, and one affirmative; but otherwise if  $\frac{1}{2} q$  is greater than  $\sqrt{d^3} - \frac{1}{27} b^3 - \frac{1}{6} b p$ , one only affirmative Root will be exhibited.

In the fourth Place let the Equation be  $z^3 - bz^2 - pz + q = 0$ , which has two affirmative Roots and one negative, if the Center of the Circle is found in the indefinite Space between the Right Lines  $P \Delta$ ,  $P D$ , and the Curve of the Paraboloid  $\Delta L$ ; that is, supposing  $d = \frac{1}{9} b b + \frac{1}{3} p$ , if  $\frac{1}{2} q$  is less than  $\sqrt{d^3} + \frac{1}{27} b^3 + \frac{1}{6} b p$ ; but if  $\frac{1}{2} q$  is greater than this Quantity, there is only one negative Root.

Now the four remaining Equations, in which there is  $-b$ , as to the Limitation of the Number of Roots, do not differ from the foregoing, if the Sign of the last Term be changed, keeping the Sign of the third Term; but the Roots that in those were affirmative will here be negative, and *vice versa*. Thus in the Equation  $z^3 - bz^2 + pz - q = 0$ , either one or three are negative under the same Conditions, but none affirmative. So in



$z^3 + bz^2 + pz - q = 0$ , two are negative and one affirmative, if  $p$  is less than  $\frac{1}{3}bb$ , and  $\frac{1}{2}q$  less than  $\sqrt{d^3} + \frac{1}{27}b^3 - \frac{1}{6}bp$ , as in  $z^3 - bz^2 + pz + q = 0$ , two will be affirmative and one negative. But when  $p$  or  $q$  depart from the prescribed Limits, here is only one affirmative Root which there was negative. In like manner in  $z^3 + bz^2 - pz + q = 0$ , either there are two affirmative and one negative Root, or one negative only. Lastly for the same Reason in the Equation  $z^3 + bz^2 - pz - q = 0$ , there are two negative and one affirmative Root, or one affirmative only; as in the Equation  $z^3 - bz^2 - pz + q = 0$ , there are two affirmative and one negative, or only one Negative, according as  $\frac{1}{2}q$  is greater or less than  $\sqrt{d^3} + \frac{1}{27}b^3 + \frac{1}{6}bp$ .

If the third Term is wanting, or  $p = 0$ , the Center  $R$  always falls in the Line  $IP\Delta$ ; wherefore if it is  $z^3 - bz^2 - q = 0$ , or  $z^3 + bz^2 + q = 0$ , there can be only one Root, if 'tis  $-b$  the Root is affirmative, if  $+b$  negative. But if the Equation is  $z^3 - bz^2 + q = 0$ , or  $z^3 + bz^2 - q = 0$ , there may be two affirmative and one negative in the former, or one affirmative and two negative in the latter, the Center falling in the Line  $P\Delta$ , between  $P$  and  $\Delta$ , that is, if  $\frac{1}{4}q$  is less than  $\frac{1}{27}b^3$ : But if it be greater, then can be only one Negative in the former, or one Affirmative in the latter.

Hitherto we have fully explained the Number of Roots in Cubic Equations; now we must add something concerning the Quantity of those Roots. Here it is first to be observed, that every Equation having three Roots may be resolved expeditiously enough by means of the Table of Sines, that is,

by the Trisection of an Angle. For by making  $\sqrt{\frac{4}{9}bb - \frac{4}{3}p}$ , or  $\sqrt{4d}$

if it is  $+p$  in the Equation, or  $\sqrt{\frac{4}{9}bb + \frac{4}{3}p}$ , if it is  $-p$ , the Radius of the Circle; and making that the Angle to be trisected, whose Sine is  $\frac{1}{27}b^3 + \frac{1}{6}bp + \frac{1}{2}q$  in the Table of Sines; when this Angle is found

$\sqrt{d^3}$   
the Sine of a third Part of it, also the Sine of the third Part of its Complement to a Semicircle, and their Sum, will be known by the Table of Sines.

But these Sines are to be drawn into the Radius  $\sqrt{\frac{4}{9}bb + \frac{4}{3}p}$ , and we shall have the Quantities ( $y \&$ ,  $y \&$ ,  $y \&$ , in the Figure) the Sum or Difference of which, and of  $\frac{1}{3}b$ , as the Case requires, will exhibit the true Roots of the Equation. All these Things are derived from what has been found by



*Cartesius.* Now that I may comprehend all the Cases in as short a Compass as possible. I say that in the first Form of Equations, if the Center R falls in the Space V G P, the two Sections Y, Y, fall between A and B, and therefore each of the lesser Roots is less than  $\frac{1}{3} b$ , but the third and greater always exceeds  $\frac{1}{3} b$ , but is exceeded by  $b$ . Now if it falls in the Space G N V, two are greater than  $\frac{1}{3} b$ , but less than  $\frac{2}{3} b$ , and the third is  $b$  subtracting the two others, and therefore less than  $\frac{1}{3} b$ . But admitting a Limitation of the Quantity  $p$ , the Roots are included within narrower Bounds. For the greatest Root is less than  $\sqrt{\frac{4}{9} b b - \frac{4}{3} p} + \frac{1}{3} b$ , and greater than  $\sqrt{\frac{1}{4} b b - p} + \frac{1}{2} b$ . But when  $\frac{1}{4} b b$  is less than  $p$ , that Limit becomes  $\sqrt{\frac{1}{9} b b - \frac{1}{3} p} + \frac{1}{3} b$ . The middle Root is always less than  $\sqrt{\frac{1}{4} b b - p} + \frac{1}{2} b$ , but greater than  $\frac{1}{3} b - \sqrt{\frac{1}{9} b b - \frac{1}{3} p}$ . The greatest Root never exceeds this Limit, but vanishes with the Quantity  $q$ .

In the second Form by the Rules prescribed there are two affirmative and one negative Root, and the Center falling in the Space G P E, one of the Affirmatives is greater and the other less than  $\frac{1}{3} b$ . But the greater does not exceed  $b$ , and the Negative cannot be greater than  $\sqrt{\frac{1}{3} b b} - \frac{1}{3} b$ , and it is the Difference of  $b$  and of the Sum of the Affirmatives. But when the Center is placed in the Space E N G  $\Delta$ , either of the Affirmatives is greater than  $\frac{1}{3} b$ , but less than  $\sqrt{\frac{1}{3} b b} + \frac{1}{3} b$ . But the Negative is always less than  $\frac{1}{3} b$ . But from  $p$  being given, the Limits become nearer,  $\sqrt{\frac{1}{4} b b - p} + \frac{1}{2} b$  of the greatest affirmative Root, than which it is always less, and greater than  $\sqrt{\frac{1}{9} b b - \frac{1}{3} p} + \frac{1}{3} b$ . Yet the other affirmative Root, which diminishes with the Quantity  $q$ , is less than this Limit. But the negative Root is always less than  $\sqrt{\frac{4}{9} b b - \frac{4}{3} p} - \frac{1}{3} b$ , and vanishes when the Quantity  $q$  is absent.



In the third Form are two negative and one affirmative Root; in this, as also in the fourth, the Roots are not limited by the Quantity  $b$ . Now the

Affirmative is always less than  $\sqrt{\frac{4}{9} b b + \frac{4}{3} p + \frac{1}{3} b}$ , but greater than

$\sqrt{p + \frac{1}{4} b b + \frac{1}{2} b}$ . But the greatest of the Negatives is always greater

than  $\sqrt{\frac{1}{9} b b + \frac{1}{3} p - \frac{1}{3} p}$ , and less than  $\sqrt{p + \frac{1}{4} b b - \frac{1}{2} b}$ . But

the lesser of the Negatives always is diminished when the Quantity  $q$  is diminished.

In the fourth Form the Center falls within the Space  $L \Delta P D$ ; if two Roots are affirmative and one negative, the greatest of the Affirmatives cannot be greater than

$\sqrt{p + \frac{1}{4} b b + \frac{1}{2} b}$ , nor less than

$\sqrt{\frac{1}{9} b b + \frac{1}{3} p + \frac{1}{3} b}$ . But the lesser Root is diminished from this

Limit, as the Quantity  $q$  is diminished. The Negative is less than

$\sqrt{\frac{4}{9} b b + \frac{4}{3} p - \frac{1}{3} p}$ , but greater than  $\sqrt{p + \frac{1}{4} b b - \frac{1}{2} b}$ .

But here it must be noted, that the negative Roots are every where marked with the affirmative Sign, because these are the affirmative Roots of those four Equations, in which there is  $+ b$ , and  $q$  is marked with a contrary Sign, as I observed before. The Demonstration of all these follows from hence, that whenever the Center of the Circle  $R$  falls upon the Curve-Lines  $V P X$  or  $V \Delta L$ , its Circumference touches the Parabola in a

Point whose Distance from the Axis is  $\sqrt{\frac{2}{3}} V H$ . But when the Center

falls upon the Line  $D P D$ , one of the Roots becomes  $= 0$ , and therefore the Cubic is reduced to a Quadratic, or to  $z^2 - b z + p = 0$ , whose Roots mark out the Limits where the Quantity  $q$  vanishes. And the less  $q$  is, so much the nearer the Roots approach to these Limits. It is also a

Quadratic when the Center falls in the Axis; that is, when  $\frac{1}{2} q = \frac{1}{6} b p$

$- \frac{1}{27} b^3$ , in the first Form, or  $\frac{1}{2} q = \frac{1}{27} b^3 + \frac{1}{6} b p$  in the second.

In the third it is impossible, but in the fourth when  $\frac{1}{2} q = \frac{1}{27} b^3 + \frac{1}{6}$

$b p$ . In which Case the lesser of the affirmative Roots is  $\frac{1}{3} b$ , the greater

$\sqrt{\frac{1}{3} b^2 + p + \frac{1}{3} b}$ . And the negative Root is  $\sqrt{\frac{1}{3} b b + p - \frac{1}{3} b}$ .

In



In the first Formula the Roots are  $\frac{1}{3}b$  and  $\frac{1}{3}b \pm \sqrt{\frac{1}{3}bb - p}$ ; in the second  $\frac{1}{3}b$  and  $\sqrt{\frac{1}{3}bb - p} \mp \frac{1}{3}b$  are affirmative; but  $\sqrt{\frac{1}{2}bb - p} - \frac{1}{3}b$  is negative.

And this seems to be sufficient as to Cubics. But because of the excellent Use of that Method, which finds the Roots of these Equations by means of the Table of Sines; I have thought proper to add an Example or two, to shew the Compendiousness of this Praxis. Let there be proposed this Equation  $z^3 - 39z^2 + 479z - 1881 = 0$ , of which the Roots  $z$  are required.

We shall have  $\sqrt{\frac{1}{9}bb - \frac{1}{3}p} = \sqrt{9 \frac{1}{3}} = \sqrt{d}$ , whose double  $\sqrt{37 \frac{1}{3}}$  is the Radius of the Circle; and  $\frac{\frac{1}{27}b^3 + \frac{1}{3}q - \frac{1}{6}bp}{\sqrt{d^3}} = \frac{2197 + 940 \frac{1}{3} - 3113 \frac{1}{3}}{9 \frac{1}{3} \sqrt{9 \frac{1}{3}}} =$

$\frac{24}{9 \frac{1}{3} \sqrt{9 \frac{1}{3}}}$  is the Tabular Sine of the Angle, that is, making the Division by Help of the Logarithms, Log. 9. 9251560, to which answers the Angle  $57^\circ 19' 11'' \frac{1}{2}$ . A third Part of this is  $19^\circ 6' 24''$ , and of the Complement  $40^\circ 53' 36''$ . The Sines give Log. 9. 514933, and 9. 816011, which drawn into the Radius  $\sqrt{37 \frac{1}{3}}$ , produce Y & and Y &, Log. 0. 301030, = 2, and Log. 0. 601059, = 4. But the third Y & is equal to their Sum, or 6. Therefore the Roots are  $13 - 4 = 9$ ,  $13 - 2 = 11$ , and  $13 + 6 = 19$ ; of all of which the foresaid Equation is composed. Where it may be observed, that the two lesser Roots do not exceed  $\frac{1}{6}b$ , or 13, because in the Construction the Center R falls to the Right Hand of the Axis; that is,  $\frac{1}{6}bp$  is less than  $\frac{1}{27}b^3 + \frac{1}{2}q$ .

For another Example suppose  $x^3 - 15x^2 - 229x - 525 = 0$ , and let the Roots be sought. We have  $\sqrt{\frac{1}{9}bb + \frac{1}{3}p} = \sqrt{101 \frac{1}{3}} = \sqrt{d}$ , and the Radius of the Circle is  $\sqrt{405 \frac{1}{3}}$ . Then  $\frac{\frac{1}{27}b^3 + \frac{1}{2}bp + \frac{1}{2}q}{\sqrt{d^3}} = \frac{125 + 572 \frac{1}{3} + 262 \frac{1}{3}}{101 \frac{1}{3} \sqrt{101 \frac{1}{3}}} = \frac{960}{101 \frac{1}{3} \sqrt{101 \frac{1}{3}}} =$  Tabular Sine of an Arch, whose Log. 9. 9736426, and the Arch itself will be  $70^\circ 14' 22''$ , whose third Part is  $23^\circ 24' 47'' \frac{1}{2}$ , and of its Complement  $36^\circ 35' 12'' \frac{1}{2}$ ; whose Logarithmic Sines are 9. 599183, and 9. 775275; to which the Log. of  $\sqrt{405 \frac{1}{3}}$  being added, they become Log. 0. 903089. = 8, and Log. 1. 079181, = 12, and their Sum = 20. Hence it is concluded, that  $20 \mp \frac{1}{3}b$ , or 25,



is equal to the affirmative Root, and  $8 - \frac{1}{3}b$ , or 3, and  $12 - \frac{1}{3}b$ , or 7, are equal to the negative Roots. Now if the Equation had been  $x^3 + 15x^2 - 229x + 525 = 0$ , the Roots 3 and 7 would have been affirmative, and 25 negative. But other Cubic Equations, which are explicable by one Root only, are to be resolved by *Cardan's* Rules after the second Term is taken away; nor can I perceive how it may be done by less Calculation. But if this Root were desired, expressed by the Quantities  $b, p, q$ ;

I say in the first Form it would be  $\frac{1}{3}b$ , adding or subtracting the Sum or

Difference of the Cubic Roots of  $\sqrt{\frac{1}{4}qq - \frac{1}{108}p^2b^2 + \frac{1}{27}b^3q - \frac{1}{6}bpq + \frac{1}{27}p^3}$

$\pm \frac{1}{27}b^3 + \frac{1}{2}q - \frac{1}{6}bp$ ; that is, it will be  $+$  if  $\frac{1}{27}b^3 + \frac{1}{2}q$  is greater

than  $\frac{1}{6}bp$ , otherwise  $-$ . Also it will be the Sum as often as  $\frac{1}{3}bb$  is

greater than  $p$ , but if  $\frac{1}{3}bb$  is less, then the Difference. And in the other

Forms the Root is always made up of the same Elements, with Variations of the Signs  $+$  and  $-$ , as any one will easily perceive who has a Mind to try.

By Help of the Logarithmic Table of Versed Sines these Roots may be found readily enough; that is, if the Numeral Coefficients are Surds or Fractions, and the Roots are effable in Numbers, as it generally happens.

Now this is the Rule. In the first and second Formula, if  $\frac{1}{3}bb$  is less than

$p$ , make  $\frac{1}{3}p - \frac{1}{9}bb = d$ , and taking for Radius the Difference be-

tween  $\frac{1}{6}bp$  and  $\frac{1}{27}b^3 + \frac{1}{2}q$ , that is HR in the first, and between  $\frac{1}{6}$

$bp + \frac{1}{2}q$ , and  $\frac{1}{27}b^3$  in the second; let the Angle be found whose Tan-

gent is  $d\sqrt{d}$ . Then as the Cosine of this Angle to its Versed Sine, so is

the Difference taken for Radius to a fourth; whose Cubic Root by Trisection

will be had for a Logarithm. Then dividing  $\frac{1}{3}p - \frac{1}{9}bb$  by this Cubic

Root, from the Quotient subtract the Divisor, and the Remainder will be

the Quantity Y &. The Sum of  $\frac{1}{3}b$  and this Remainder, if the Center

fall on the Right Side the Axis, otherwise the Difference of the same, will

be the Root sought. Now if  $\frac{1}{3}bb$  is greater than  $p$ , taking HR for Ra-

dius, let  $d\sqrt{d}$  or the Distance of the Paraboloid from the Axis, be the Sine



of a certain Arch. Let the versed Sine of this be multiplied by the Radius, or  $\frac{1}{6} b p - \frac{1}{27} b^3 \pm \frac{1}{2} q$ , and the Logarithm of the Product being trisected, its Cubic Root will be had, by which let  $\frac{1}{9} b b - \frac{1}{3} p$  be divided. I say that the Sum of the Quotient and Divisor in the same Manner added to or taken from  $\frac{1}{3} b$ , will exhibit the Root required. And in like manner in the third and fourth Forms, unless that  $\frac{1}{27} b^3 + \frac{1}{6} b p \pm \frac{1}{2} q$  is to be assumed for the Radius, and  $\frac{1}{9} b b + \frac{1}{3} p$  into  $\sqrt{\frac{1}{9} b b + \frac{1}{3} b}$ , or  $d\sqrt{d}$  for the Sine. But these Precepts will be better understood by Examples.

Let the Cubic Equation be  $z^3 - 17z^2 + 54z - 350 = 0$ , and let the Root  $z$  be required. Here  $\frac{1}{3} b b$  is greater than  $p$ , but  $q$  is greater than the Cube of  $\frac{1}{3} b$ ; therefore the Equation is explicable by only one affirmative Root. Now  $\frac{289}{9} - \frac{54}{3}$  is  $d$ , and  $\frac{127}{9} \sqrt{\frac{127}{9}}$  is to be taken for the Sine to Radius  $\frac{4913}{27} + 175 - 153$ , that is  $\frac{5507}{27}$ . The Arch belonging to it is  $15^\circ : 3' : 49''$ . The Logarithm of the versed Sine of this is 8.5362376, which added to the Logarithm of the Radius 2.3095913, gives 0.8457889, whose third Part 0.2819276 is the Logarithm of the Cubic Root 1.91394. This dividing  $\frac{127}{9}$  or  $d$ , the Quotient is 7.37281. Then the Sum of the Quotient and Divisor adding  $\frac{1}{3} b$  is the Root sought, or 14, 9534, &c.

Having now done with Cubic Equations, we are to undertake Biquadratics. These always have either none, or two, or four true Roots, the Determination of which depends partly on the Coefficients, and partly on the Sign and Magnitude of the given absolute Number. In the Construction

of the Equation  $z^4 - b z^3 + p z^2 - q z + r = 0$ , make  $BD = \frac{1}{4} b$ ,  $AB = \frac{1}{16} b b$ ,  $BK = \frac{1}{2} p$ , or half the *latus rectum*,  $KC = 2 AB = \frac{1}{8} b b$ ,  $KE = -\frac{1}{8} b b - \frac{1}{2} p$ ,  $AE = \frac{1}{2} q = \frac{3}{16} b b - \frac{1}{2} p$ ,  $FE = \frac{1}{16} b^3 - \frac{1}{4} b p$ , and  $EG = \frac{1}{16} b^3 - \frac{1}{4} b p + \frac{1}{2} q$ . Which being constructed a

Circle



Circle with Center G and Radius  $\sqrt{GDq} - r$  will intersect the Parabola, either in none, two, or four Points, which exhibit all the Values of  $z$ , or all the Roots, by Perpendiculars upon the Line HD. Now that there may be four, it is plain the Center of the Circle must be placed somewhere within the Space, from any Point of which three Perpendiculars can be let fall upon the Curve of the Parabola; and at the same Time the Radius must be less than the greatest of those Perpendiculars, and greater than the middlemost. Now if the Center be placed without this Space, so that only one Perpendicular can be let fall upon the Parabola, than which the Radius is greater; or if it be less than the middlemost of the three Perpendiculars, but greater than the least of them, there can be only two Roots. But there will be none as often as the Radius  $\sqrt{GDq} - r$  is less than the least of the three, or than that one when there is but one. Now what Sort of Space this is, by what Limits it is circumscribed, and upon what Conditions the Radius of the Circle is less or greater than the said Perpendiculars, we must now inquire. And first it must be shewn, how a Perpendicular may be let fall upon the Parabola.

Let ABC be a Parabola, AE its Axis, AV its half *latus rectum*, and G the Point from whence the Perpendicular is to be drawn. Make GE perpendicular to the Axis, and let VE be bisected in F. And erecting the Perpendicular FH on the same Side of the Axis, make  $FH = \frac{1}{4} GE$ . I say, that a Circle described with Center H, and with Radius HA, will intersect the Parabola in three Points, or in one Z, to which the Right Lines GZ being drawn, will insift perpendicularly upon the Curve of the Parabola.

Fig. 53.

Now that there may be three such Intersections, it is required that the Center of the Circle H should be so placed, that it may be within the Space included by the Paraboloids. That is, that FH may be less than  $\sqrt{\frac{8}{27}}$

$VFc$ , or  $FHq$  less than the Cube of  $\frac{2}{3} VF$ , and therefore  $GE = 4FH$

will be less than  $\sqrt{\frac{8}{27}} VFc$  or  $4\sqrt{\frac{1}{27}} VEc$ , that is, the Square of GE

less than  $\frac{16}{27} VEc$ . Therefore these Limits coincide with the two Paraboloids of the same Kind with those which we made use of in Cubics, but whose

*latus rectum* is as little again; that is,  $\frac{27}{16}$  of the *latus rectum* of the Parabola,

or  $\frac{27}{8}$  of AV. Therefore it is the very Curve by the Evolution of which

the Parabola is generated, as Mr. Huygens has demonstrated, and which the Line DF always touches, which is perpendicular to the Parabola in the Point D. But the Point P, in which the Right Line DF touches the Pa-

Fig. 52.



paraboloid, is the Center of the Circle, which being described with Radius  $DP$  coincides with the Parabola in the Point  $D$ , or is of the same Degree of Curvity; which is manifest of itself.

Therefore such Paraboloids  $VXP$ ,  $VNA$ , being described on each Side the Axis, it is plain unless the Center of the Circle is fixed within these Limits, it cannot cut the Parabola in more than two Points. Whence we may determine under what Conditions the Coefficients of the intermediate Terms are restrained in Biquadratic Equations, in order to have four Roots. And

first it appears that  $p$  cannot be greater than  $\frac{3}{8} bb$ , in those Forms in

which it is  $\mp p$ , nor  $q$  greater than  $\frac{1}{16} b^3$ . But in general  $\frac{1}{16} b^3 \mp \frac{1}{4} pb$

$\mp \frac{1}{2} q$ , that is, the Distance of the Center from the Axis  $EG$ , must be

less than  $EH = 4 \sqrt{\frac{1}{27} V E c}$ ; that is, because of  $VE = \frac{2}{16} bb \mp \frac{1}{2} p$ ,

less than  $\frac{1}{4} bb \mp \frac{2}{3} p \sqrt{\frac{1}{16} bb \mp \frac{1}{6} p}$ ; the Signs  $\mp$  and  $-$  being left doubtful, that they may be varied according to the Exigence of any Equation; as we have shewn before in Cubics.

But the Limitation of the last Term  $r$  cannot so easily be found. And this because to let fall a Perpendicular upon the Curve of a Parabola is a solid Problem, and which cannot be resolved without the Solution of a Cubic Equation. Therefore first let the second Term of the Equation be wanting, or if it be present let it be taken away, that the Equation may have this Form  $z^4 : * : pz^2 : qz : r : = 0$ . Now if it be  $-r$ , the Equation may always be explained by two or four Roots: But that there should be four, the Center of the Circle must be had below the foresaid Paraboloids,

or that there may be  $-p$ , and  $qq$  less than  $\frac{8}{27} p^3$ , or than the Cube of  $\frac{2}{3}$

$p$ . Then let there be found the Roots of this Equation,  $y^3 : * : \frac{1}{2} py : \frac{1}{4}$

$q = 0$ , the same Signs being annexed to  $p$  and  $q$  as in the Biquadratic Equation. Now these Roots may easily be found by help of the Table of Sines.

Those three Values of  $y$  being found, (which are the Ordinates to the Axis of the Parabola, from the Points where the Perpendiculars fall upon the Curve, that is  $ZY$ ;) the Quantity  $py^2 - 3y^4$  composed of the lesser Root  $y$  will express the greatest Value of  $r$ , if it is  $-r$ ; than which if  $r$  is less, the Equation will have four Roots, otherwise but two. But if it is  $\mp r$ , it must be less than  $3y^4 - py^2$ , composed of the middlemost  $y$ ; for if it be greater, there can be but two Roots, at least if  $r$  be less than  $3y^4 - py^2$  composed of the greatest  $y$ . But if it be greater than this, the Equation is explicable by no true Root at all. Now these same Limits may be otherwise

marked



marked out by the Quantity  $q$ . That is,  $\frac{1}{2} qy - y^4$  in the first Case,  $y^4 - \frac{1}{2} qy$  in the second, and  $y^4 + \frac{1}{2} qy$  in the third.

But it may happen that the two lesser Quantities  $y$  are not far asunder, so that each of the Perpendiculars may be greater than the Right Line  $GA$ , that is, when  $qq$  is greater than  $\frac{4}{27} p^3$ , but less than  $\frac{8}{27} p^3$ , the Center falling within the Space intercepted by the Paraboloids, Fig. 51, 52. In this Case if it is  $+r$ , there can be but two Roots, the Quantity  $y^4 + \frac{1}{2} qy$  composed of the greatest  $y$  being greater than  $r$ , otherwise none at all. But if  $\frac{1}{2} qy - y^4$  composed of the least  $y$  is greater than  $r$  having the Sign  $-$ , and  $r$  is greater than  $\frac{1}{2} qy - y^4$  composed of the mean  $y$ , then there would be four Roots; and but two only if  $r$  should be either greater than the former or less than the latter.

But in the Equation if it should be  $+p$ , or if it is  $-p$  and  $qq$  greater than  $\frac{8}{27} p^3$ . the Equation  $y^3 : * : \frac{1}{2} py : \frac{1}{4} q = 0$ , is explicable only by one Root  $y$ , that is, one Perpendicular alone can be let fall from the Center of the Circle. Whence we may certainly conclude, that there can be had only two Roots from the given Equation, the Sum of which, if it is  $-r$ , increases with the Quantity  $r$ ; but if it is  $+r$ , the Quantity  $y$  being found, that Quantity  $r$  must be less than  $y^4 + \frac{1}{2} qr$ . For if it be greater, the proposed Equation will be absurd and impossible.

It would be a tedious and unnecessary Thing to run through all the Equations of this Sort, since it must be evident to any one that considers it, what Roots are negative and what affirmative; and that the Limits of these Roots may be had from the Quantities  $y$  being found. Yet for an Example, which may be imitated in others, let it be proposed to find the Limits or Conditions, under which in a Biquadratic Equation four affirmative Roots may be had. Now this will happen as often as the Center of the Circle  $G$  is placed in the Space  $VPK$ , and at the same Time there is  $+r$ , or the Radius of the Circle is less than  $GD$ . Whence it is plain that the Equation we are treating of must be of this Form  $z^4 - bz^3 + pz^2 - qz + r = 0$ . But  $p$  cannot be greater than  $\frac{3}{8} bb$ , nor in this Case  $\frac{1}{4} pb$  greater than  $\frac{1}{16} b^3 +$

Fig. 52

$\frac{1}{2} q$ . Also it is necessary that  $\frac{1}{4} bb - \frac{2}{3} p$  drawn into  $\sqrt{\frac{1}{16} bb - \frac{1}{6} p}$  must be greater than  $\frac{1}{6} b^3 + \frac{1}{2} q - \frac{1}{4} pb$ ; and from these Limitations

it



it will be certain, that the Center will be found within the Space V P K. Now that the Quantity  $r$  may be determined, we must first solve this Cubic

Equation  $y^3 : * : -\frac{3}{16} b^2 - \frac{1}{2} p y = \frac{1}{32} b^3 + \frac{1}{4} q - \frac{1}{8} p b$ , and the Points will be had, upon which the Perpendiculars from the Center will fall upon the Curve of the Parabola.

Now the three Values of this  $y$  being found,  $r$  must be less than  $\frac{3}{256} b^4 + \frac{1}{4} b q - \frac{1}{16} b^2 p + 3 y^4 - \frac{3}{8} b^2 y^2 + p y^2$ , composed of the middle Value of  $y$ , and greater than  $\frac{3}{256} b^4 + \frac{1}{4} b q - \frac{1}{16} b^2 p + 3 y^4 - \frac{3}{8} b^2 y^2 + p y^2$ , when composed of the least  $y$ . But if  $r$  exceeds these Limits, there can be only two Roots. Lastly if  $\frac{3}{256} b^4 + \frac{1}{4} b q - \frac{4}{16} b^2 p + 3 y^4 - \frac{3}{8} b^2 y^2 + p y^2$ , when composed of the greatest  $y$ , shall be less than  $r$ , the Equation is impossible.

It may also happen that there are four affirmative Roots, when the Center G is placed in the little Space V T S, drawing R T S perpendicular upon the middle of the Line A D. This happens when  $p$  is greater than

$\frac{5}{16} b b$ , and  $\frac{1}{4} b b - \frac{2}{3} p \sqrt{\frac{1}{16} b b} - \frac{1}{6} p$  is greater than  $\frac{1}{8} p b - \frac{5}{128} b^3 - \frac{1}{2} q$ . In which Case always two, and sometimes three, of the Roots are greater than  $\frac{1}{4} b$ .

Fig. 52.

But here it must be observed, that Limit produced from the least  $y$  is sometimes negative, or less than nothing; that is, whenever the greatest of the three Perpendiculars is greater than G D. This may be diminished to nothing, if it happens to be  $\frac{1}{4} r$  from the prescribed Limit composed of the middlemost  $y$ . And the Defect of the Limit from the least  $y$  shews, how great  $-r$  may be in the Equation, if there are three affirmative Roots in the Equation and one Negative: Which if it exceeds, there are only two Roots, one affirmative and one negative. All these Things are demonstrated from hence, that the aforesaid Limits of the Quantity  $r$  are the Differences of the Squares of the Line G D and of the Perpendiculars upon the Curve of the Parabola.

But because of the intricate Restrictions which the Diversity of Signs produces in these Equations, it is always best to take away the second Term, and then



then by the Precepts already delivered to find the Number of the Roots and their Signs; especially if those Quantities  $y$  are not far remote from one another.

Now of these four affirmative Roots, two are always less than  $\frac{1}{4}b$ ,

and two greater; that is, if  $DG$  is less than  $AG$ , or  $\frac{1}{4}pb$  than  $\frac{3}{64}b^3 +$

$q$ . But three of them are less than  $\frac{1}{4}b$ , as often as the middlemost Perpendicular, or that found from the middlemost  $y$ , is greater than  $AG$ , or

$\frac{3}{8}bb$  greater than  $3y^3 - py^2$  computed from the same middlemost  $y$ . But

the fourth and greatest Root is greater than the greatest  $y + \frac{1}{4}b$ , and is equal

to the Difference of  $b$  and the Sum of the other three Roots, and therefore is less than  $b$ . But I shall proceed no farther. Perhaps those who have a greater Insight into the Nature of the Parabola may be able to draw all these Conclusions more compendiously; but it may well be doubted, whether all these Quantities,  $b$ ,  $p$ ,  $q$ , and  $r$ , can be rightly determined, without the Solution of a Cubic Equation. For whatever can be done in this Matter by plane Equations, can only exhibit Approximations, and not the true Limits.

XX. Mr. de Lagny has lately given us two very compendious Rules for an Approximation to the Cubic Root; one of them rational, and the other irrational. That is, the Root of the Cube  $a^3 + b$  is between  $a + \frac{ab}{3a^2 + b}$  and  $\sqrt{\frac{1}{4}a^2 + \frac{b}{3a} + \frac{1}{2}a}$ . And the Root of the fifth Power  $a^5 + b$  he

thus expresses,  $\frac{1}{2}a + \sqrt{\sqrt{\frac{1}{4}a^2 + \frac{b}{5a} - \frac{1}{4}a^2}}$ . Now the aforefaid Rules

are demonstrated from the Genesis of the Cube and of the fifth Power. For supposing the Side of any Cube to be  $a + e$ , the Cube derived from thence

will be  $a^3 + 3a^2e + 3ae^2 + e^3$ . So that if  $a^3$  be supposed a Cubic Number which is the next less to a non-Cubic,  $e^3$  will be less than Unity, and the

Remainder or  $b$  will be equal to the other Members of the Cube  $3a^2e +$

$3ae^2 + e^3$ . And  $e^3$  being rejected because of its Smallness, it will be  $b =$

$3a^2e + 3ae^2$ . And since  $a^2e$  is much greater than  $ae^2$ ,  $\frac{b}{3a^2}$  will not much

exceed  $e$ ; and putting  $e = \frac{b}{3a^2}$ ,  $\frac{b}{3a^2 + 3ae}$  (to which the Quantity  $e$  is

nearly equal) will be found equal to  $\frac{b}{3a^2 + \frac{3ab}{3a^2}} = \frac{b}{3a^2 + \frac{b}{a}} = \frac{ab}{3a^3 + b}$

$= e$  ferè. So that the Side of the Cube  $a^3 + b$  is equal to  $a + e = a +$

$\frac{ab}{3a^3 + b}$

*The Extraction  
of all Roots with-  
out any previous  
Reduction; by  
Mr. Ed. Halley.  
N. 210. p. 136.  
May, An. 1694.*



$\frac{ab}{3a^2 + b}$ , nearly, which is the Rational Form of Mr. de Lagny. Now if  $a^3$  be a Cubic Number next greater than the given Number, the Side of the Cube  $a^3 - b$  by a like Way of reasoning will be found  $a - \frac{ab}{3a^2 - b}$ . And this Approximation to the Cubic Root, which is very easy and expeditious, makes but a very small Error in defect, since  $e$  the Remainder of the Root found by this Means, will be but little less than the true Remainder. Now the irrational Form may likewise be easily derived after the same Manner.

For because  $b = 3a^2e + 3ae^2$ , or  $\frac{b}{3a} = ae + e^2$ , and therefore

$$\sqrt{\frac{1}{4}a^2 - \frac{b}{3a}} = \frac{1}{2}a + e, \text{ and } \sqrt{\frac{1}{4}a^2 - \frac{b}{3a}} + \frac{1}{2}a = a + e \text{ which}$$

is the Root required. And after the same Manner will be had  $\frac{1}{2}a +$

$\sqrt{\frac{1}{4}a^2 - \frac{b}{3a}}$  for the Side of the Cube  $a^3 - b$ . Now this Form approaches something nearer to the true Root than the rational Form, erring in Excess as the other does in Defect, and seems to be more convenient for Practice, since the Restitution of the Calculation is nothing else but the continual Ad-

dition or Subtraction of the Quantity  $\frac{e^3}{3a}$ , as the little Supplement  $e$  becomes

known. So that it may rather be wrote  $\sqrt{\frac{1}{4}a^2 + \frac{b - e^3}{3a}} + \frac{1}{2}a$  in the

former Case, and  $\frac{1}{2}a + \sqrt{\frac{1}{4}a^2 + \frac{e^3 - b}{3a}}$  in the latter. Now by either Form the Figures already known in extracting the Root are at least tripled, which I believe will be a very acceptable Compendium to Arithmeticians, and therefore I congratulate the Inventor upon it. But that the Usefulness of these Rules may be the better perceived, I shall add an Example or two.

*Example I.* Let the Side of the double Cube be sought, or make  $a^3 + b = 2$ . Hence  $a = 1$ ,  $b = 1$ , and  $\frac{b}{3a} = \frac{1}{3}$ , and therefore  $\frac{1}{2} + \sqrt{\frac{7}{12}}$ , or 1, 26 will be found near the true Root. But the Cube of 1, 26 is

2, 000376, and therefore  $0, 63 + \sqrt{0, 3969 - \frac{0, 000376}{3, 78}}$  or  $0, 63 +$

$\sqrt{0, 3968005291005291} = 1, 259921049895 -$ ; which exhibits the Side of the double Cube as far as thirteen Figures with little Trouble, that is, by one Division and Extraction of the Square-root; whereas had it been found by the common Way of Operation, every Arithmetician knows what Labour it must have cost him. Now this Calculation may be continued at Pleasure, by



by increasing the Square by the Addition of  $\frac{e^3}{3a}$ : Which Correction in this Case brings only the Increase of an Unit to the fourteenth Figure.

*Example II.* Let the Side of a Cube be sought, which is equal in Measure to a Gallon, containing 231 solid Inches. The next lesser Cube is 216, whose Cubic Root is  $6 = a$ , and the Remainder  $15 = b$ . Therefore for

the first Approximation we have  $3 + \sqrt{9 + \frac{5}{6}} =$  to the Root. And because  $\sqrt{9, 8333 \dots} = 3, 1358 \dots$  it is plain that  $6, 1358 = a + e$ . Now make  $6, 1358 = a$ , and we shall have its Cube  $231, 000853894712$ , and

according to the Rule  $3, 0679 + \sqrt{9, 41201041 - \frac{0, 000853894712}{18, 4074}}$

$= 6, 13579243966195897$ , which is the Side of the given Cube very exactly, being true in the eighteenth Figure, and falling short in the nineteenth; which Calculation I performed within an Hour's Time. Now this Form is deservedly to be preferred before the rational one, which, because of its large Divisor, cannot be managed without a great deal of Trouble; whereas the Extraction of the Square-root proceeds much more easily, as manifold Experience has informed me.

Now the Rule for the Root of the pure Surfolid, or of the fifth Power, is of a little higher Inquiry, and yet performs the Matter much more perfectly. For it quintuples at least the given Figures of the Root, nor does it require much or very operose Calculation. But the Author has no where given his Method of Investigation or Demonstration of it, tho' it seemed most to be wanting: Especially as it is faultily printed in his Book, which may easily mislead the Unskilful. Now the fifth Power of the Side  $a + e$  is made up of these Members following,  $a^5 + 5a^4e + 10a^3e^2 + 10a^2e^3 + 5ae^4 + e^5 = a^5 + b$ ; whence  $b = 5a^4e + 10a^3e^2 + 10a^2e^3 + 5ae^4$ ;

(rejecting  $e^5$  because of its Smallness) wherefore  $\frac{b}{5a} = a^3e + 2a^2e^2 + 2ae^3 + e^4$ , and adding  $\frac{1}{4}a^4$  on each Side, and extracting the Square-root, it

will be  $\sqrt{\frac{1}{4}a^4 + \frac{b}{5a}} = (\sqrt{\frac{1}{4}a^4 + a^3e + 2a^2e^2 + 2ae^3 + e^4})^{\frac{1}{2}}$

$a^2 + ae + e^2$ . Then subtracting  $\frac{1}{4}a^2$  from each Side, and extracting the

Square-root again,  $\sqrt{\sqrt{\frac{1}{4}a^4 + \frac{b}{5a}} - \frac{1}{4}a^2} = (\sqrt{\frac{1}{4}a^2 + ae + e^2})$

$\frac{1}{2}a + e$ , to which if you add  $\frac{1}{2}a$ , 'tis  $a + e = \frac{1}{2}a + \sqrt{\sqrt{\frac{1}{4}a^4 + \frac{b}{5a}} - \frac{1}{4}a^2}$

$=$  Root of the Power  $a^5 + b$ . Now if it had been  $a^5 - b$ , (assuming  $a$



greater than it ought to be) the Rule would have been thus,  $\frac{1}{2} a +$

$$\sqrt{\sqrt{\frac{1}{4} a^4 - \frac{b}{5 a} - \frac{1}{4} a a}} = \text{Root of } a^5 - b.$$

And this Rule approximates wonderfully, so that there can scarce be any Need of Restitution. And upon farther considering this Matter, I have fallen upon a certain general Method of Forms for any Power, which is elegant enough, and which I cannot prevail upon myself to conceal; since in the higher Powers they triplicate at least the known Figures of the Root.

Now these Forms proceed after this Manner, as well the rational as the irrational.

$$\sqrt[2]{a^2 + b} = \frac{0}{1} a + \sqrt{\frac{1}{1} a a + \frac{b}{1 a^0}}, \text{ or } a + \frac{a b}{2 a^2 + \frac{1}{2} b}$$

$$\sqrt[3]{a^3 + b} = \frac{1}{2} a + \sqrt{\frac{1}{4} a a + \frac{b}{3 a}}, \text{ or } a + \frac{a b}{3 a^3 + \frac{2}{3} b}$$

$$\sqrt[4]{a^4 + b} = \frac{2}{3} a + \sqrt{\frac{1}{9} a a + \frac{b}{6 a^2}}, \text{ or } a + \frac{a b}{4 a^4 + \frac{3}{2} b}$$

$$\sqrt[5]{a^5 + b} = \frac{3}{4} a + \sqrt{\frac{1}{16} a a + \frac{b}{10 a^3}}, \text{ or } a + \frac{a b}{5 a^5 + \frac{4}{2} b}$$

$$\sqrt[6]{a^6 + b} = \frac{4}{5} a + \sqrt{\frac{1}{25} a a + \frac{b}{15 a^4}}, \text{ or } a + \frac{a b}{6 a^6 + \frac{5}{2} b} \text{ \&c.}$$

And so in other higher Powers. Now if  $a$  should be assumed greater than the Root sought, (which would be done with Advantage as often as the Power to be resolved is much nearer to the Power of the integer Number that is next greater, than it is to the next lesser) changing what is to be changed, the same Expressions of the Roots arise.

$$\sqrt[2]{a^2 - b} = \frac{0}{1} a + \sqrt{\frac{1}{1} a a - \frac{b}{1 a^0}}, \text{ or } - \frac{a b}{2 a^2 - \frac{1}{2} b}$$

$$\sqrt[3]{a^3 - b} = \frac{1}{2} a + \sqrt{\frac{1}{4} a a - \frac{b}{3 a}}, \text{ or } - \frac{a b}{3 a^3 - \frac{2}{3} b}$$

$$\sqrt[4]{a^4 - b} = \frac{2}{3} a + \sqrt{\frac{1}{9} a a - \frac{b}{6 a^2}}, \text{ or } - \frac{a b}{4 a^4 - \frac{3}{2} b}$$

$$\sqrt[5]{a^5 - b} = \frac{3}{4} a + \sqrt{\frac{1}{16} a a - \frac{b}{10 a^3}}, \text{ or } - \frac{a b}{5 a^5 - \frac{4}{2} b}$$

$$\sqrt[6]{a^6 - b} = \frac{4}{5} a + \sqrt{\frac{1}{25} a a - \frac{b}{15 a^4}}, \text{ or } - \frac{a b}{6 a^6 - \frac{5}{2} b} \text{ \&c.}$$

Now



Now between these two Limits the true Root always consists, being something nearer the irrational than the rational. But  $e$  found according to the irrational Formula always errs in Excess, as the Quote resulting from the rational always errs in Defect; therefore if it is  $+b$ , the irrational gives the Root greater than the Truth, and the rational less. And just the contrary if it be  $-b$ . And this may suffice concerning the finding of the Roots of pure Powers, which for ordinary Uses (and accurately enough) may be done more easily by Help of the Logarithms. But whenever the Root is to be extracted very accurately, and beyond the Reach of the Logarithms, we must necessarily have Recourse to such Methods as these. Besides, as from the Discovery and Consideration of these Forms, I have hit upon an universal Rule for affected Equations, which I believe will be made use of with good Advantage by all such as are studious of Geometry and Algebra; I was willing to lay open with all possible Perspicuity the Foundation of this Discovery.

About the Year 1687 I made publick a Method then newly invented by me, which contained a neat and very easy general Construction of affected Equations, not beyond Quadrato-quadratics; since which Time I have always had a very strong Desire of performing the same in Numbers. A little after that Mr. Raphson seemed in a good Measure to have accomplished my Wishes; till Mr. de Lagny shewed me, by his little Book on this Subject, that the Thing might be still performed more compendiously. Now my Method is this.

Let the Root of any Equation  $z$  be conceived to be composed of two Parts,  $a +$  or  $-e$ , of which let  $a$  be assumed (by Supposition) as near as may be to the Quantity  $z$ , (which is convenient indeed, but not absolutely necessary) and of the Quantity  $a + e$  let there be formed all the Powers, to which  $z$  arises in the given Equation, to which let their Numeral Coefficients be affixed respectively. Then let the Power to be resolved be subtracted from the Sum of the given Parts in the first Column, where  $e$  is not found, which is called the *Homogeneum Comparationis*, and let the Difference be  $+b$ . Then let there be found the Sum of all the Coefficients of the Side  $e$  in the second Column, which let be  $s$ . Lastly, in the third Column, let all the Coefficients be added of the Square  $ee$ , the Sum of which we may call  $t$ . Then the Root sought  $z$  will be had in the rational Form  $z = a +$

$\frac{sb}{ss + tb}$ , and in the irrational Form  $z = a + \frac{\frac{1}{2}s \pm \sqrt{\frac{1}{4}ss + bt}}{t}$ ; which

it may be worth while to illustrate by Examples. But as a convenient Help it may be proper to have at Hand a general Table, exhibiting the Genesis of all the Powers of  $a + e$ , which may be easily continued farther if necessary. I will carry it on to the seventh Power, because few Problems go beyond that. This Table may justly be called a general Analytical Speculum. Now the aforesaid Powers, arising from the continual Multiplication of  $z = a + e$ , are as follows with their Coefficients adjoined.



$$\begin{aligned}
cz &= ca + 1ca^0e \\
dz^2 &= da^2 + 2da^1e + 1da^0e^2 \\
fz^3 &= fa^3 + 3fa^2e + 3fa^1e^2 + 1fa^0e^3 \\
gz^4 &= ga^4 + 4ga^3e + 6ga^2e^2 + 4ga^1e^3 + 1ga^0e^4 \\
hz^5 &= ha^5 + 5ha^4e + 10ha^3e^2 + 10ha^2e^3 + 5ha^1e^4 + 1ha^0e^5 \\
kz^6 &= ka^6 + 6ka^5e + 15ka^4e^2 + 20ka^3e^3 + 15ka^2e^4 + 6ka^1e^5 + 1ka^0e^6 \\
lz^7 &= la^7 + 7la^6e + 21la^5e^2 + 35la^4e^3 + 35la^3e^4 + 21la^2e^5 + 7la^1e^6 + 1la^0e^7
\end{aligned}$$

$s$ 
 $t$ 
 $u$ 
 $w$ 
 $x$ 
 $y$

Now if it should be  $a - e = z$ , the Table would be composed of the same Parts, but only the odd Powers of  $e$  must be negative, as  $e, e^3, e^5, \&c.$  and the even Powers  $e^2, e^4, e^6, \&c.$  must be still affirmative. Let the Sum of the Coefficients of the Side  $e$  be denoted by  $s$ , of the Square  $e^2$  by  $t$ , of the Cube  $e^3$  by  $u$ , of the Biquadrate  $e^4$  by  $w$ , of the Surfolid  $e^5$  by  $x$ , of the Cubo-cube  $e^6$  be  $y$ ; and so on. And as  $e$  is supposed to be but a small Part of the Root required, all the Powers of  $e$  become much less than the like Powers of  $a$ , and therefore for the first Position the higher Powers may be rejected, as has been shewn in pure Powers; and a new Equation being formed, we shall have, as said before,  $\pm b = \pm se \pm te^2$ . For the better understanding of which take the following Examples:

*Example I.* Let the Equation proposed be  $z^4 - 3z^2 + 75z = 1000$ . For the first Supposition make  $a = 10$ , and the following Equation will arise.

$$\begin{aligned}
z^4 &= + a^4 \pm 4a^3e + 6a^2e^2 \pm 4ae^3 + e^4 \\
-dz^2 &= -da^2 + 2dae - de^2 \\
+cz &= +ca \pm ce \\
\hline
&= + 10000 \pm 400e + 600e^2 \pm 40e^3 + e^4 \\
&\quad - 300 \pm 60e - 3e^2 \\
&\quad + 750 \pm 75e \\
&\quad - 10000 \\
\hline
&+ 450 \pm 415e + 597e^2 \pm 40e^3 + e^4 = 0,
\end{aligned}$$

$s$ 
 $t$ 
 $u$

The Signs  $\pm$  are left in Doubt in respect of  $e$  and  $e^3$ , till it is known whether  $e$  is negative or affirmative, which may admit of some Difficulty, since in Equations that have many Roots, the *Homogeneum Comparationis* (as it is called) may be increased by diminishing the Quantity  $a$ , and contrary-wise may be diminished when it is increased. But the Sign of  $e$  is determined by the Sign of the Quantity  $b$ . For the Resolvend being taken away from the *Homogeneum* formed by  $a$ , the Sign of  $se$ , and therefore of the Parts prevailing in its Composition, will always be contrary to the Sign of the Difference  $b$ . Whence it will appear whether it should be  $-e$  or  $+e$ , or whether  $a$  is greater or less than the true assumed Root. But  $e$  is always equal



equal to  $\frac{\frac{1}{2}s - \sqrt{\frac{1}{4}ss - bt}}{t}$ , whenever  $b$  and  $t$  have the same Sign. But

when they are connected with different Signs, then the same  $e$  is  $\frac{\sqrt{\frac{1}{4}ss + bt} - \frac{1}{2}s}{t}$ .

Now after it is found that it must be  $-e$ , in the affirmative Parts of the Equation let  $e, e^3, e^5, \&c.$  be made negative, and in the negative let them be made affirmative, that is, let them all be wrote with a contrary Sign. But if it be  $+e$ , let their own Signs continue. Now in our Example we have 10450 instead of the Resolvend 10000, or  $b = +450$ ; whence it appears that  $a$  was taken above the true Root, and therefore it must be  $-e$ . Hence the Equation becomes  $10450 - 4015e + 597e^2 - 40e^3 + e^4 = 10000$ , that is,  $450 - 4015e + 597e^2 = 0$ . And therefore  $450 = 4015e - 597e^2$ ,

or  $b = se - te^2$ , whose Root  $e$  is  $\frac{\frac{1}{2}s - \sqrt{\frac{1}{4}ss - bt}}{t}$ , or if you had rather

$$\frac{s}{2t} - \sqrt{\frac{ss}{4tt} - \frac{b}{t}}; \text{ that is, in the present Case } e = \frac{2007\frac{1}{2} - \sqrt{3761406\frac{1}{4}}}{597}$$

whence arises the Root sought near the Truth  $z = 9, 886$ . Then I substitute this for a second Hypothesis, and there arises  $a + e = z = 9, 8862603936495 \dots$  very accurately, hardly exceeding the Truth by 2 in the last Figure; that is,

when  $\frac{\sqrt{\frac{1}{4}ss + bt} - \frac{1}{2}s}{t} = e$ . And even this, if there were Occasion, might

be verified much farther, by subtracting  $\frac{\frac{1}{2}ue^3 + \frac{1}{4}e^4}{\sqrt{\frac{1}{4}ss + tb}}$  if it is  $+e$ , or adding

$\frac{\frac{1}{2}ue^3 - \frac{1}{4}e^4}{\sqrt{\frac{1}{4}ss - tb}}$  if it be  $-e$ , from or to the Root before found: The Com-

pendium of which is so much the more to be valued, that sometimes from the first Supposition alone, but always from the second, you may continue the Calculation as far as you please, by keeping to the same Coefficients. But the foregoing Equation has also a negative Root, which is  $z = -10, 26 \dots$  which any one that pleases may pursue.

Exam.



*Example II.* Let  $z^3 - 17z^2 + 54z = 350$ , and make  $a = 10$ . Then according to the Rule.

$$\begin{aligned} z^3 &= a^3 + 3a^2e + 3ae^2 + e^3 \\ - dz^2 &= da^2 - 2dae - de^2 \\ + cz &= ca + ce \end{aligned}$$

$$\begin{array}{r} \text{That is,} \\ + 1000 + 300e + 30e^2 + e^3 \\ - 1700 - 340e - 17e^2 \\ + 540 + 54e \\ - 350 \end{array}$$

$$\text{Or,} \quad - 510 + 14e + 13e^2 + e^3 = 0.$$

Now since there is  $-510$ , it is plain that  $a$  was assumed less than the Truth, and therefore  $e$  is affirmative. And from  $510 = 14e + 13e^2$ , there

arises  $\frac{\sqrt{bt + \frac{1}{4}ss} - \frac{1}{2}s}{t} = e = \frac{\sqrt{6679} - 7}{13}$ ; whence 'tis  $z = 15,7\dots$

which is too much, because  $a$  was not taken near enough. Therefore, secondly, let us suppose  $a = 15$ , and by the same Way of arguing we shall have

$e = \frac{\frac{1}{2}s - \sqrt{\frac{1}{4}ss - tb}}{t} = \frac{109\frac{1}{2} - \sqrt{11710\frac{1}{4}}}{28}$ ; and therefore  $z = 14,954068$ .

Now if we would renew the Calculation a third Time, we should find the Root true to 25 Figures. But if we are contented with fewer, by writing

$tb \pm te^3$  instead of  $tb$ , or by adding or subtracting  $\frac{\frac{1}{2}e^3}{\sqrt{\frac{1}{4}ss + tb}}$  to or from

the Root before found, we shall soon arrive at our Purpose. Now the proposed Equation cannot be explained by any other Root, because the Power

to be resolved 350 is greater than the Cube of  $\frac{17}{3}$ , or  $\frac{1}{3}d$ .

*Example III.* Let it be that Equation which the learned *Wallis*, in Chap. 62. of his Algebra, makes use of in the Resolution of a most difficult Arithmetical Problem, in which he has attained the Root very accurately by the Method of *Vieta*. And the abovementioned *Mr. Raphson*, Pag. 25, 26.

brings the same Equation as an Example of his Method. The Equation is  $z^4 - 80z^3 + 1998z^2 - 14937z + 5000 = 0$ . Now this Equation is of

such a Form, as that it may have several affirmative Roots, and what increases the Difficulty, the Coefficients are very great in respect of the given

Resolvend. Now that it may be managed the better, let it be divided, and placed according to the known Rules of Punctuation,  $-z^4 + 8z^3 - 20z^2 + 15z = 0,5$ ; where  $z$  is but one tenth Part of  $z$  in the Equation

proposed. Then for the first Supposition let us make  $a = 1$ . Wherefore  $+2 - 5e - 2e^2 + 4e^3 - e^4 - 0,5 = 0$ . That is,  $1\frac{1}{2} = 5e +$

$2e^2,$



$2e^2$ , and hence  $\frac{\sqrt{\frac{1}{4}ss + bt} - \frac{1}{2}s}{t} = e = \frac{\sqrt{37-5}}{4}$ , and therefore  $z =$

1,27. Hence it appears, that 12,7 is the Root of the proposed Equation pretty near the Truth. Now, in the second Place, let it be supposed, that  $z = 12,7$ , and according to the Table of Powers,

$$\begin{array}{r}
- 26014,4641 - 8193,532e - 967,74e^2 - 50,8e^3 - e^4 \\
+ 163870,640 + 38709,60e + 3048, \dots e^2 + 80, \dots e^3 \\
- 322257,42 \dots - 50749,2 \dots e - 1998, \dots e^2 \\
+ 189699,9 \dots + 14937, \dots e \\
- 5000, \dots \\
\hline
+ \frac{298,6559}{b} - \frac{5296,132e}{s} + \frac{82,26e^2}{t} + \frac{29,2e^3}{u} - e^4 = 0
\end{array}$$

Therefore  $-298,6559 = -5296,132e + 82,26e^2$ , whose Root  $e$  according to Rule is  $\frac{\frac{1}{2}s - \sqrt{\frac{1}{4}ss - bt}}{t} = \frac{2648,066 - \sqrt{6987686,106022}}{82,26}$

$= 0,05644080331 \dots = e$  less than the Truth. Now that it may be corrected,  $\frac{\frac{1}{2}ue^3 - e^4}{\sqrt{\frac{1}{4}ss - bt}}$  or  $\frac{0,0026201 \dots}{2643,423 \dots}$  becomes  $0,00000099117$ , and

therefore  $e$  corrected will be  $0,05644179448$ . Now if more Figures of the Root be wanted, of  $e$  corrected let there be formed  $tue^3 - te^4 =$

$$\begin{array}{l}
0,43105602423 \dots, \text{ then } \frac{\frac{1}{2}s - \sqrt{\frac{1}{4}ss - bt - tue^3 + te^4}}{t}, \\
\text{or } \frac{2648,066 - \sqrt{6987685,67496597577 \dots}}{82,26} = 0,05644179448074402
\end{array}$$

$= e$ . Whence  $a + e = z$ , or the Root very accurately comes out 12,75644179448074402, such as was found by Dr. Wallis in the Place above quoted. Here it may be observed, that the renewing of the Calculus always triplicates the true Figures in the assumed  $a$ , which the first Cor-

rection, or  $\frac{\frac{1}{2}ue^3 - \frac{1}{2}e^4}{\sqrt{\frac{1}{4}ss - bt}}$ , makes quintuple, which Operation is easily per-

formed by the Logarithms. The other Correction after the first also adds a double Number of Figures, so that in the whole it makes the assumed Figures sevenfold. But the first generally is abundantly sufficient for all the Uses of Arithmetick. But what is here said about the Number of Figures rightly assumed in the Root, I would have so understood, that when  $a$  is distant from the true Root not above a tenth Part, the first Figure may be rightly assumed; if within an hundredth Part, the two first Figures; if within a thousandth Part, the three first Figures may be true. Then when managed according to our Rule, the true Figures will immediately become nine.





It remains that I should add a Word or two concerning our rational Form,  $e = \frac{s b}{s s \pm t b}$ , which will seem expeditious enough, and not much inferior to the former, since it is able to triplicate the given Figures. For an Equation being formed of  $a \mp e = z$ , as before, it will soon appear whether the assumed  $a$  be greater or less than the Truth, since  $s e$  must always have a Sign contrary to the Sign of the Difference of the Resolvend, and of its *Homogeneous* produced of  $a$ . Then supposing that  $\pm b \mp s e \pm t e = 0$ , the Divisor becomes  $s s - t b$  as often as  $b$  and  $t$  have the same Sign. But it becomes  $s s + t b$  if their Signs are different. But it seems more accommodated to Practice if the Theorem were written  $e = \frac{b}{s \pm \frac{t b}{s}}$ ; for then the

Thing would be performed by one Multiplication and two Divisions, which otherwise would require three Multiplications and one Division. Let us also take an Example of this Method from the Root of the foregoing Equation  $12, 7 \dots$  in which  $298, 6559, - 5296, 132 e + 82, 26 e^2 + 29, 2 e^3 - e^4$   
 $\begin{matrix} + b & - s & + t & + u \\ b & & & \end{matrix}$   
 $= 0$ , and therefore  $\frac{t b}{s - \frac{t b}{s}} = e$ . That is, let it be  $s$  to  $t$  so  $b$  to  $\frac{t b}{s} = 5296,$

$132) 298, 6559$  into  $82, 26$  ( $4, 63875$ . Wherefore the Divisor becomes  $s = \frac{t b}{s} = 5291, 49325 \dots$ )  $298, 6559$  ( $0, 056441 \dots = e$ , that is, five true Figures added to the assumed Root. But this Formula cannot be corrected as the foregoing irrational one; so that if more Figures of the Root are desired, it is better to repeat the Calculation by making a new Assumption; and the new Quotient, by triplicating the known Figures in the Root, will abundantly satisfy even the most scrupulous Computer.

*A Method of Raising an Infinite Multinomial to any given Power; by M. de Moivre. N. 230. p. 619. Jul. An. 1697.*

XXI. Theorem.]  $a z + b z^2 + c z^3 + d z^4 + e z^5 + f z^6$

$$\begin{aligned} & + g z^7 + h z^8 + i z^9 \&c.]^m = a^m z^m + \frac{m}{1} a^{m-1} b z^{m+1} \\ & + \frac{m}{1} \times \frac{m-1}{2} a^{m-2} b^2 z^{m+2} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} a^{m-2} b^3 z^{m+3} \\ & + \frac{m}{1} a^{m-1} e + \frac{m}{1} \times \frac{m-1}{1} a^{m-2} b c \\ & \qquad \qquad \qquad \times \frac{m}{1} a^{m-1} d \end{aligned}$$

$$+ \frac{m}{1}$$



$$+ \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} a^{m-4} b^4 z^{m+4}$$

$$+ \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{1} a^{m-3} b^2 c$$

$$+ \frac{m}{1} \times \frac{m-1}{1} a^{m-2} b d$$

$$+ \frac{m}{1} \times \frac{m-1}{2} a^{m-2} c^2$$

$$+ \frac{m}{1} a^{m-1} e$$

$$+ \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \times \frac{m-4}{5} a^{m-5} b^5 z^{m+5}$$

$$+ \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{1} a^{m-4} b^3 c$$

$$+ \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{1} a^{m-3} b^2 d$$

$$+ \frac{m}{1} \times \frac{m-1}{1} \times \frac{m-2}{2} a^{m-3} b c^2$$

$$+ \frac{m}{1} \times \frac{m-2}{1} a^{m-2} b c$$

$$+ \frac{m}{1} \times \frac{m-1}{1} a^{m-2} c d$$

$$+ \frac{m}{1} a^{m-1} f$$



$$\begin{aligned}
& + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \times \frac{m-4}{5} \times \frac{m-5}{6} a^{m-5} b^5 z^{m+5} \\
& + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \times \frac{m-4}{1} a^{m-5} b^2 c \\
& + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{1} a^{m-4} b^3 d \\
& + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{1} \times \frac{m-3}{2} a^{m-4} b^2 c^2 \\
& + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{1} a^{m-3} b^2 e \\
& + \frac{m}{1} \times \frac{m-1}{1} \times \frac{m-2}{1} a^{m-3} b c d \\
& + \frac{m-1}{1} \times \frac{m-1}{1} a^{m-2} b f \\
& + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} a^{m-3} c^2 \\
& + \frac{m}{1} \times \frac{m-1}{1} a^{m-2} c e \\
& + \frac{m}{1} \times \frac{m-1}{2} a^{m-2} d^2 \\
& + \frac{m}{1} a^{m-1} g, \text{ \&c.}
\end{aligned}$$

I suppose that the Infinite Number Multinomial is  $ax + bx^2 + cx^3 + dx^4 + ex^5, \text{ \&c.}$   $m$  is the Index of the Power, to which this Multinomial ought to be raised; or, if you will, 'tis the Index of the Root which is to be extracted: I say, That this Power or Root of the Multinomial is such a Series as I have expressed.

For the understanding of it, it is only necessary to consider all the Terms by which the same Power of  $z$  is multiplied; in order thereto, I distinguish two things in each of these Terms; 1<sup>o</sup>. The Product of certain Powers of the Quantities,  $a, b, c, d, \text{ \&c.}$  2<sup>o</sup>. The Unciæ (as *Oughtred* calls them) prefixed to these Products. To find all the Products belonging to the same Power of  $z$ , to that Product, for Instance, whose Index is  $m + r$  (where  $r$  may denote any Integer Number) I divide these Products into several Classes; those which immediately after some certain Power of  $a$  (by which all these Products begin) have  $b$ , I call Products of the first Class; for Example,  $a^{m-4} b^3 e$  is the Product of the first Class, because  $b$  immediately follows  $a^{m-4}$ ; those which immediately after some Power of  $a$  have  $c$ , I call Products of the second Class, so  $a^{m-3} c c d$ , is a Product of the second Class; those which immediately after some Power of  $a$  have  $d$ , I call Products of the third Class; and so of the rest.

This



This being done, I multiply all the Products belonging to  $z^{m+r-1}$  (which precedes immediately  $z^{m+r}$ ) by  $b$ , and divide them all by  $a$ ; 2<sup>o</sup>. I multiply by  $c$ , and divide by  $a$ , all the Products belonging to  $z^{m+r-2}$ , except those of the first Classis; 3<sup>o</sup>. I multiply by  $d$  and divide by  $a$ , all the Products belonging to  $z^{m+r-3}$ , except those of the first and second Classis: 4<sup>o</sup>. I multiply by  $e$  and divide by  $a$ , all the Terms belonging to  $z^{m+r-4}$ , except those of the first, second, and third Classis, and so on, till I meet twice with the same Term. Lastly, I add to all these Terms the Product of  $a^{m-1}$  into the Letter, whose Exponent is  $r + 1$ .

Here I must take notice, that by the Exponent of a Letter, I mean the Number which expresses what Place the Letter has in the Alphabet: So 3 is the Exponent of the Letter  $c$ , because the Letter  $c$  is the 3d in the Alphabet.

It is evident by this Rule, you may easily find all the Products belonging to the several Powers of  $z$ , if you have but the Product belonging to  $z^m$ , viz.  $a^m$ .

To find the Unciæ which ought to be prefixed to every Product, I consider the Sum of Units contained in the Indices of the Letters which compose it (the Index of  $a$  excepted) I write as many Terms of the Series  $m \times m - 1 \times m - 2 \times m - 3$ , &c. as there are Units in the Sum of these Indices; this Series is to be the Numerator of a Fraction whose Denominator is the Product of the several Series  $1 \times 2 \times 3 \times 4 \times 5$ , &c.  $1 \times 2 \times 3 \times 4 \times 5$ , &c.  $1 \times 2 \times 3 \times 4 \times 5 \times 6$ , &c. the first of which contains as many Terms as there are Units in the Index of  $b$ , the second as many as there are Units in the Index of  $c$ , the third as many as there are Units in the Index of  $d$ , the fourth as many as there are Units in the Index of  $e$ , &c.

*Demonstration.*] To raise the Series  $az + bz^2 + cz^3 + dz^4$ , &c. to any Power whatsoever, write so many Series equal to it, as there are Units in the Index of the Power demanded. Now it is evident that when these Series are also multiplied, there are several Products in which there is the same Power of  $z$ ; thus if the Series  $az + bz^2 + cz^3 + dz^4$ , &c. is raised to its Cube, you have the Products  $b^3 z^6$ ,  $abc z^6$ ,  $aad z^6$ , in which you find the same Power  $z^6$ . Therefore let us consider what is the Condition that can make some Products to contain the same Power of  $z$ ; the first thing that will appear in relation to it, is, that in any Product whatsoever, the Index of  $z$  is the Sum of the particular Indices of  $z$  in the multiplying Terms (this follows from the Nature of the Indices): thus  $b^3 z^6$  is the Product of  $b z^2$ ,  $b z^2$ ,  $b z^2$ , and the Sum of the Indices in the multiplying Terms, is  $2 + 2 + 2 = 6$ ;  $abc z^6$  is the Product of  $az$ ,  $b z^2$ ,  $c z^3$ , and the Sum of the Indices of  $z$  in the multiplying Terms is  $1 + 2 + 3 = 6$ ;  $aad z^6$  is the Product of  $az$ ,  $az$ ,  $d z^4$ , and the Sum of the Indices of  $z$  in the multiplying Terms is  $1 + 1 + 4 = 6$ . The next thing that appears, is, that the Index of  $z$  in the multiplying Terms is the same with the Exponent of the Letter to which  $z$  is joined: From which two Considerations it follows, that, to have all the Products belonging to a certain Power of  $z$ , you must find all



the Products where the Sum of the Exponents of the Letters which compose them, shall always be the same with the Index of that Power. Now this is the Method I use, to find easily all the Products belonging to the same Power of  $z$ , let  $m + r$  be the Index of that Power, I consider that the Sum of the Exponents of the Letters which compose the Products, must exceed by 1, those which belong to  $z^{m+r-1}$ : Now because the Excess of the Exponent of the Letter  $b$ , above the Exponent of the Letter  $a$ , is 1, it follows, that if each of the Products belonging to  $z^{m+r-1}$  is multiplied by  $b$ , and divided by  $a$ , you will have Products, the Sum of whose Exponents will be  $m + r$ ; likewise the Sum of the Exponent of the Letters which compose the Products belonging to  $z^{m+r}$  exceeds by 2 the Sum of the Exponents of the Letters which compose the Products belonging to  $z^{m+r-1}$ : Now because the Exponent of the Letter  $a$  is less by 2 than the Exponent of the Letter  $c$ , it follows, that if each Product belonging to  $z^{m+r-1}$  is multiplied by  $c$ , and divided by  $a$ , you will have other Products, the Sum of whose Exponents is still  $m + r$ : Now if all the Products belonging to  $z^{m+r-1}$  were multiplied by  $c$  and divided by  $a$ , you would have some Products that would be the same as some of them found before; therefore you must except out of them those that I have called Products of the first Classis. What I have said shews why all the Products belonging to  $z^{m+r-1}$ , except those of the first and second Classis, must be multiplied by  $d$ , and divided by  $a$ . Lastly, you see the Reason why to all these Products is added the Product of  $a^{m-1}$  by the Letter whose Exponent is  $r + 1$ ; 'Tis because the Sum of the Exponents is still  $m + r$ .

As for what relates to the Unciæ; observe, that when you multiply  $az + bzz + cz^3 + dz^4$ , &c. by  $az + bzz + cz^3 + dz^4$ , &c. each Letter,  $a, b, c, d$ , &c. of the second Series, is multiplied by each of the Letters  $a, b, c, d$ , &c. of the first Series. Thus the Letter  $a$  of the second Series is multiplied by the Letter  $b$  of the first Series, and the Letter  $b$  of the second Series is multiplied by the Letter  $a$  of the first; therefore you may have the two Planes,  $ab, ba$ , or  $2ab$ ; for the same Reason you have  $2ac, 2ad$ , &c. Therefore you must prefix to each Plane of those that compose the Square of the Infinite Series  $az + bzz + cz^3$ , &c. the Number which expresses how many Ways the Letters of each Plane may be changed; likewise if you multiply the Product of the two preceding Series by  $az + bzz + cz^3$ , each Letter,  $a, b, c, d$ , of the third Series, is multiplied by each of the Planes formed by the Product of the first and second Series: Thus the Letter  $a$  is multiplied by the Planes  $bc$  and  $cb$ ; the Letter  $b$  is multiplied by  $ac$  and  $ca$ ; the Letter  $c$  is multiplied by  $ab$  and  $ba$ ; therefore you have the six Solids,  $abc, acb, bac, bca, cab, cba$ , or six  $abc$ : Therefore you must prefix to each Solid whereof the Cube of the Infinite Series is composed, the Number which expresses how many Ways the Letters of each Solid may be changed; and, generally, you must prefix to any Product, whereof any Power of the Infinite Series  $az + bzz + cz^3$ , &c. is composed, the Number which expresses how many ways the Letters of each Product may be changed.

Now



Now to find how many Ways the Letters of any Product, for instance,  $a^{m-n} b^h c^p d^r$ , may be changed; this is the Rule which is commonly given: Write as many Terms of the Series  $1 \times 2 \times 3 \times 4 \times 5, \text{ \&c.}$  as there are Units in the Sum of the Indices, viz.  $m - n + h + p + r$ ; let this Series be the Numerator of a Fraction, whose Denominator shall be the Product of the Series,  $1 \times 2 \times 3 \times 4 \times 5, \text{ \&c.}$   $1 \times 2 \times 3 \times 4 \times 5, \text{ \&c.}$   $1 \times 2 \times 3 \times 4 \times 5 \times 6, \text{ \&c.}$   $1 \times 2 \times 3 \times 4 \times 5, \text{ \&c.}$  whereof the first is to contain as many Terms as there are Units in the first Index  $m - n$ ; the second as many as there are Units in the second Index  $h$ ; the third as many as there are Units in the third Index  $p$ ; the fourth as many as there are Units in the fourth Index  $r$ . But the Numerator and Denominator of this Fraction have a common Divisor, viz. the Series of  $1 \times 2 \times 3 \times 4 \times 5, \text{ \&c.}$  continued to so many Terms as there are Units in the first Index  $m - n$ ; therefore let both this Numerator and Denominator be divided by this common Divisor, then this new Numerator will begin with  $m - n + 1$ , whereas the other began with 1, and will contain so many Terms as there are Units in  $h + p + r$ , that is, so many as there are Units in the Sum of all the Indices excepting the first: As for the new Denominator, it will be the Product of three Series only; that is, of so many as there are Indices, excepting the first. But if it happens withal, that  $n$  be equal to  $h + p + r$ , as it always happens in our *Theorem*, then the Numerator beginning by  $m - n + 1$ , and being continued to so many Terms as there are Units in  $h + p + r$  or  $n$ , the last Term will be  $m$  necessarily: So if you invert the Series, and make that the first Term which was the last, the Numerator will be  $m \times m - 1 \times m - 2 \times m - 3, \text{ \&c.}$  continued to so many Terms as there are Units in the Sum of the Indices of each Product, excepting the first Index. And whatever is here said of Powers whose Index is an Integer, may be adapted to Roots or Powers whose Index is a Fraction.

XXII. *Theorem.*] If  $ax + bx^2 + cx^3 + dx^4 + ex^5 + fx^6, \text{ \&c.} = gy$  The Extraction of a Root of an Infinite Equation; by Mr. Abr. de Moivre. N. 240. p. 190. May, An. 1698.

$$+ byy + iy^3 + ky^4 + ly^5 + my^6, \text{ \&c.}$$

then will  $z$  be  $= \frac{g}{a}y + \frac{b - bAA}{a}y^2 + \frac{i - 2bAB - cA^2}{a}y^3 + \frac{k - bBB - 2bAC - 3cAAB - dA^3}{a}y^4 + \frac{l - 2bBC - 2bAD - 3cABB - 3cAAC - 4dA^2B - eA^3}{a}y^5 + \frac{m - 2bBD - bCC - 2bAE - cB^2 - 6cABC - 3cAAD - 6dAAB - 4dAC - 5eA^2B - fA^3}{a}y^6, \text{ \&c.}$

For the understanding of this Series, and in order to continue it as far as we please, it is to be observed, 1. That every Capital Letter is equal to the Coefficient of each preceding Term; thus the Letter B is equal to the Coefficient



cient  $\frac{b - b A A}{a}$ . 2. That the Denominator of each Coefficient is always  $a$ .  
 3. That the first Member of each Numerator is always a Coefficient of the Series  $g y + b y y + i y^3$ , &c. viz. the first Numerator begins with the first Coefficient  $g$ , the second Numerator with the second Coefficient  $b$ , and so on. 4. That in every Member after the first, the Sum of the Exponents of the Capital Letters is always equal to the Index of the Power to which this Member belongs: Thus, considering the Coefficient  $\frac{k - b B B - 2 b A C - 3 c A A B - d A^4}{a}$ , which belongs to the Power<sup>4</sup>,

we shall see that in every Member  $b B B$ ,  $2 b A C$ ,  $3 c A A B$ ,  $d A^4$ , the Sum of the Exponents of the Capital Letters is 4, (where I must take notice, that by the Exponent of a Letter, I mean the Number which expresses what place it has in the Alphabet; thus 4 is the Exponent of the Letter D) hence I derive this Rule for finding the Capital Letters of all the Members that belong to any Power; Combine the Capital Letters as often as you can make the Sum of their Exponents equal to the Index of the Power to which they belong. 5. That the Exponents of the small Letters, which are written before the Capitals, express how many Capitals there are in each Member. 6. That the Numerical Figures or Unciæ that occur in these Members, express the Number of Permutations which the Capital Letters of every Member are capable of.

For the *Demonstration* of this, Suppose  $z = A y + B y y + C y^3 + D y^4$ , &c. Substitute this Series in the room of  $z$ , and the Powers of this Series in the room of the Powers of  $z$ , there will arise a new Series: Then take the Coefficients which belong to the several Powers of  $y$  in this new Series, and make them equal to the corresponding Coefficients of the Series  $g y + b y y + i y^3$ , &c. and the Coefficients  $A, B, C, D$ , &c. will be found such as I have determined them.

But if any one desires to be satisfied, that the Law by which the Coefficients are formed will always hold, I'll desire them to have recourse to the *Theorem* I have given for *Raising an Infinite Series to any Power*, or extracting any Root of the same; for if they make use of it, for taking successively the Powers of  $A y + B y y + C y^3$ , &c. they will see that it must of necessity be so. I might have made the *Theorem* I give here much more general than it is; for I might have supposed,  $a z^m + b z^{m+1} + c z^{m+2} + d z^{m+3}$ , &c. =  $g y^m + b y^{m+1} + i y^{m+2}$ , &c. then all the Powers of the Series  $A y + B y y + C y^3$ , &c. designed by the universal Indices, must have been taken successively; but those who will please to try this, may easily do it, by means of the *Theorem* for raising an Infinite Series to any Power, &c.

This *Theorem* may be applied to what is called the Reversion of Series; such as finding the Number from its Logarithm given; the Sign from the Arch; the Ordinate of an Ellipse from an Area given to be cut from any Point in the Axis: But to make a particular Application of it, I will suppose we have this Problem to solve; viz. *The Chord of an Arc being given, to find*



the Chord of another Arc, that shall be to the first as  $n$  to 1. Let  $y$  be the Chord given,  $z$  the Chord required; now the Arc belonging to the Chord  $y$

is  $y + \frac{y^3}{6dd} + \frac{3y^5}{40d^4} + \frac{5y^7}{112d^6}$ , &c. And the Arc belonging to the Chord  $z$

is  $z + \frac{z^3}{6dd} + \frac{3z^5}{40d^4} + \frac{5z^7}{112d^6}$ , &c. The first of these Arcs is to the second

as 1 to  $n$ ; therefore multiplying the Extremes and Means together, we shall

have this Equation:  $z + \frac{z^3}{6dd} + \frac{3z^5}{40d^4} + \frac{5z^7}{112d^6}$ , &c.  $= ny + \frac{ny^3}{6dd} +$

$\frac{3ny^5}{40d^4} + \frac{5ny^7}{112d^6}$ , &c.

Compare these two Series with the two Series of the Theorem, and you

will find  $a = 1$ ,  $b = 0$ ,  $c = \frac{1}{6dd}$ ,  $d = 0$ ,  $e = \frac{3}{40d^4}$ ,  $f = 0$ , &c.  $g = n$ ,

$b = 0$ ,  $i = \frac{n}{6dd}$ ,  $k = 0$ ,  $l = \frac{3n}{40d^4}$ ,  $m = 0$ , &c. Hence  $z$  will be  $= ny +$

$\frac{n - n^3}{6dd} y^3$ , &c. or  $ny + \frac{1 - nn}{2 \times 3 dd} yy A$ , &c. supposing  $A$  to denote the

whole preceding Term; which will be the same Series as Mr. Newton has first found.

By the same Method this general Problem may be solved: *The Absciss corresponding to a certain Area in any Curve being given, to find the Absciss whose corresponding Area shall be to the first in a given Ratio.*

The Logarithmic Series might also be found without borrowing any other Idea, than that Logarithms are the Indices of Powers: Let the Number, whose Logarithm we enquire, be  $1 + z$ ; suppose its Log. to be  $az + bz^2 + cz^3$ , &c. Let there be another Number  $1 + y$ ; therefore its Logarithm

will be  $ay + byy + cy^3$ , &c. Now if  $1 + z = \sqrt[n]{1 + y}$ , it follows that  $az + bz^2 + cz^3$ , &c.  $: ay + byy + cy^3$ , &c.  $:: n : 1$ ; that is,  $az + bz^2 + cz^3$ , &c.  $= n ay + n byy + n cy^3$ , &c. therefore we may find a Value of  $z$

expressed by the Powers of  $y$ . Again, since  $1 + z = \sqrt[n]{1 + y}$ , therefore  $z = \sqrt[n]{1 + y} - 1$ ; that is,  $z = ny + \frac{n}{1} \times \frac{n-1}{2} yy + \frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3} y^3$ , &c. There-

fore  $z$  is doubly expressed by the Powers of  $y$ . Compare these two Values together, and the Coefficients  $a, b, c$ , &c. will be determined, except the first,  $a$ , which may be taken at pleasure, and gives accordingly all the different Species of Logarithms.

XXIII. You need be no longer in Concern, how the infinitesimal Differences of Infinitesimals are to be explained. For as you will easily grant, that

*Doctrine of Exhaustions; by Dr. Wallis.*

any N. 255. p. 282.  
Aug. An. 1699.



any Multiple of Nothing is still Nothing; I shall allow with the same Ease, that you may safely neglect infinitesimal Differences drawn into Infinitesimals. But though this may be done safely, yet it ought to be done cautiously. For in any kind of Quantities, those Things which differ by less than a given or assignable Difference, are to be considered as equal. Upon this the whole Doctrine of Exhaustions is founded, which is necessary both to the Antients and Moderns.

*The Approximation of the Antients in the Extracting of Roots, improv'd by Dr. Wallis. N. 215. p. 2. Jan. An. 1695.*

XXIV. It is agreed by all, that if a Number proposed be not a true Square, it is in vain to hope for a just Quadratic Root thereof, explicable by rational Numbers, Integers or Fracted. And therefore in such Cases we must content ourselves with Approximations (somewhat near the Truth) without pretending to Accuracy.

And so, for the Cubic Root of what is not a perfect Cube. And the like for superior Powers.

Now the Ancients (being aware of this) had their Methods of Approximation, which though scarce applied by them beyond the Quadratic, or perhaps the Cubic Root, yet are equally practicable (by due Adjustments) to the superior Powers also.

I shall begin with the Square Root: For which the ancient Method is to this Purpose.

From the proposed Non-quadrate (suppose  $N$ ) subtract (in the usual manner) the greatest Square in Integers therein contained (suppose  $Aq$ .) The Remainder (suppose  $B = 2AE + E^2$ ) is to be the Numerator of a Fraction, for designing the near Value of  $E$  (the remaining Part of the Root sought  $A + E = \sqrt{N}$ ) whose Denominator or Divisor is to be  $2A$  (the double Root of the subtracted Square) or  $2A + 1$  (that double Root increased by 1) the true Value falling between these two; sometime the one, sometime the other, being nearest to the true Value. But (for avoiding of Negative Numbers) the latter is commonly directed.

This Method Monsieur *De Lagny* affirms to be more than 200 Years old. And it is so; for I find it in *Lucas Pacciolus* (otherwise called *Lucas de Burgo*, or *de Burgo Sancti Sepulchri*) printed at *Venice* in the Year 1494, (if not even, sooner than so, for I find there have been several Editions of it.) And how much older than so, I cannot tell: For he doth not deliver it as a new Invention of his own, but as a received Practice, and derived from the *Moors* or *Arabs*, from whom they had their *Algorism*, or Practice of Arithmetick by the ten numeral Figures now in Use.

And it is continued down hitherto in Books of practical Arithmetick in all Languages, which teach the Extraction of the Square Root, and (therein) this Method of Approximation, in case of a Non-quadrate.

The true Ground of the Rule is this:  $Aq$  being (by Construction) the greatest Integer Square contained in  $N$ , 'tis evident that  $E$  must be less than 1; (otherwise not  $Aq$ , but the Square of  $A + 1$ , or some greater than so, would be the greatest Integer Square contained in  $N$ .) Now if the Remainder  $B = 2AE + E^2$  be divided by  $2A$ , the Result will be too great for  $E$ ,

(the



{the Divisor being too little; for it should be  $2A + E$ , to make the Quotient  $E$ .) But if (to rectify this) we diminish the Quotient by increasing the Divisor, adding 1 to it, it then becomes too little; because the Divisor is now too big. For ( $E$  being less than 1)  $2A + 1$  is more than  $2A + E$ ; and therefore too big.

As for Instance: If the Non-quadrate proposed be  $N = 5$ , the greatest Integer Square therein contained is  $Aq = 4$  (the Square of  $A = 2$  :) which being subtracted, leaves  $N - Aq = 5 - 4 = 1 = B = 2AE + E^2$ . Which divided by  $2A = 4$  gives  $\frac{1}{4}$ ; but divided by  $2A + 1 = 4 + 1 = 5$ , gives  $\frac{1}{5}$ . That too great, and this too little, for  $E$ . And therefore the true Root ( $A + E = \sqrt{N}$ ) is less than  $2\frac{1}{4} = 2.25$ , but greater than  $2\frac{1}{5} = 2.2$ : And this was anciently thought an Approach near enough.

If this Approach be not now thought near enough, the same Process may be again repeated; and that as oft as is thought necessary.

Take now for  $A$ ,  $2\frac{1}{5} = 2.2$ , whose Square is  $4.84 = Aq$ , (now considered as an Integer in the second Place of Decimal Parts.) This subtracted from  $5.00$ , (or, which is the same,  $0.84$ , the Excess of this Square above the former, from 1, which was then the Remainder) leaves a new Remainder

$B + 0.16$ : which divided by  $2A = 4.4$ , gives  $\frac{0.16}{4.40} = \frac{2}{55} = 0.03636 +$ ,

too much. But divided by  $2A + 1 = 4.5$ , it gives  $\frac{0.16}{4.50} = \frac{8}{225} = 0.03555 +$ , too little. The true Value (between these two) being  $2.236$  *proxime*, whose Square is  $4.99696$ .

If this be not thought near enough, subtract the Square from  $5.000000$ : the Remainder  $B = 0.000304$ , divided by  $2A = 4.472$ , or by  $2A + 1 = 4.473$ , gives (either way)  $0.000068 -$ ; which added to  $A = 2.236$ , makes  $2.236068 -$ , somewhat too big; but  $2.236067 +$  would be much more too little.

Which gives us the Square Root of 5, adjusted to the sixth Place of Decimal Parts, at three Steps. And by the same Method, if it be thought needful, we may proceed further.

For the Cubic Root the Rule is this:

From the Non-Cubic proposed, (suppose  $N$ ) subtract the greatest Cube in Integers therein contained, (suppose  $A^3$  :) the Remainder (suppose  $B = 3A^2E + 3AE^2 + E^3$ ;) is to be the Numerator of a Fraction for designing the Value of  $E$ , (the remaining Part of the Root sought  $A + E = \sqrt[3]{N}$ .) To this Numerator, if (for the Denominator or Divisor) we subjoin  $3A^2$ , the Result will certainly be too great for  $E$ , because the Divisor is too little: (For it should be  $3A^2 + 3AE + E^2$ , to give the true Value of  $E$ .) If for the Divisor we take  $3A^2 + 3A + 1$ , it will certainly be too little, because the Divisor is too great. (For  $E$  by Construction is less than 1.) It must therefore (between these Limits) be more than this latter. And therefore this latter Result being added to  $A$ , will give a Root whose Cube may be subtracted from the Non-Cubic proposed, in order to another Step.



But if, for the Divisor, we take  $3 A q + 3 A$ , (or even less than so) the Result may be too great; or (in case B be small) it may be too little, and oft is so. Which comes to pass from hence; because E (by Construction) is less than 1; and therefore  $3 A E$  less than  $3 A$ ; and perhaps so much as that the Addition of  $E q$  will not redress it. And when it so happens  $3 A q + 3 A$  is a better Divisor than  $3 A q + 3 A + 1$ , (or even somewhat less than either.) But because it doth not always so happen (though for the most part it doth) the Rule doth rather direct the other; as which doth certainly give a Root less than the true Value, whose Cube may always be subtracted from the Non-Cubic proposed: the Design being to have such a Cube as (being subtracted) may leave another B, to be ordered in like manner for a new Approach.

But, for the most part,  $3 A q$  may be safely taken for the Divisor. For tho' the Result will then be somewhat too big, yet the Excess may be so small as to be neglected; or, at least, we may thence easily judge what Number (somewhat less than it) may be safely taken. And if we chance to take it somewhat too big, the Inconvenience will be but this, that B for the next Step will be a Negative. Of which Case we shall speak anon.

Thus for Instance: If the Non-Cube proposed be  $9 = N$ : The greatest Integer Cube therein contained is  $8 = A c$ , (whose Cubic Root is  $A = 2$ .) Which Cube subtracted, leaves  $9 - 8 = 1 = B = 3 A q E + 3 A E q + E c$ . This divided by  $3 A q = 12$ , gives  $\frac{1}{12} = 0.08333 +$ , too big for E. But the same divided by  $3 A q + 3 A + 1 = 12 + 6 + 1 = 19$ , gives  $\frac{1}{19} = 0.05263 +$ , too little. Or if but by  $3 A q + 3 A = 12 + 6 = 18$ , it gives  $\frac{1}{18} = \frac{5}{90} = 0.05555 +$ , yet too little. For the Cube of  $A + 0.06 = 2.06$ , is but  $8.742 -$ , which is short of 9; and so much short of it, that we may safely take 2.07 as not too big: Or perhaps 2.08, which upon Trial will be found not too big; for the Cube of 2.08, is but  $8.998912$ .

If this first Step be not near enough, this Cube subtracted from  $9.000000$ , leaves a new  $B = 0.001088$ , which divided by  $3 A q = 12.9796$ , gives  $0.000084 -$ ; which will be somewhat too big, but not much. (For E is now so small, as that  $3 A E$  may be safely neglected, and  $E q$  much more.) So that if to 2.08, we add  $0.000084 -$ , the Result  $2.080084$  will be too big; but  $2.080083$  will be more too little, (as will appear if we take the Cube of each.) So that either of them, at the second Step, gives the true Root within an Unit in the sixth Place of decimal Parts. But when I say, taking the Cube of each, (which I do that the Thing may be more clearly apprehended) it is not necessary that we trouble ourselves with the whole Cube. For  $A c$  being already subtracted, for finding  $B = 3 A q E + 3 A E q + E c$ , we have no more to try, but whether  $3 A q E + 3 A E q + E c$ , be greater or less than B, according as we take  $0.000084$ , or  $0.000083$ , for E.

Which may conveniently be done in this manner: Take  $3 A + E$ , and multiply this by E, (or E by it) so have we  $3 A E + E q$ . To this add  $3 A q$ , and multiply the whole by E, (so have we  $3 A q E + 3 A E q + E c$ ) to see whether this be greater or less than B.

That is, in the present Case, if we take  $E = 0.000084$ , and add to this  $3 A = 6.24$ , then is  $6.240084 = 3 A + E$ . This multiplied by  $E = 0.000084$ ,



0.000084, is  $3 A E + E q = 0.000524 +$ . To which if we add  $3 A q = 12.9792$ , it is  $3 A q + 3 A E + E q = 12.979724$ . Which multiplied again by  $E = 0.000084$ , is  $0.0010902 + = 3 A q E + 3 A E q + E c$ , which is more than  $B = 0.001088$ .

But if we take  $E = 0.000083$ , and proceed as before, we shall have  $3 A q E + 3 A E q + E c = 0.001077 +$ , which is less than  $B = 0.001088$ . And therefore (if we subtract that from this) the Remainder, 0.000011, will be another B for the next Step, if we please to proceed farther.

Hitherto I have pursued the Method most affected by the Ancients, in seeking a Square or Cube (and the like of other Powers) always less than the just Value, that it might be subtracted from the Number proposed, leaving B a positive Remainder; thereby avoiding Negative Numbers.

But since the Arithmetic of Negatives is now so well understood, it may in this (and other Operations of like nature) be adviseable to take the nearest, whether it be greater or less than the just Value.

According to this Notion, for the Square Root of 5, I would say it is  $(2 +)$  somewhat more than 2; and enquire how much more. But for the Square Root of 8, I would say, it is  $(3 -)$  somewhat less than 3; and enquire how much less: taking (in both Cases) that which is nearest to the just Value.

Thus in the Cubic Root before us; I would take for E (in the last Enquiry)  $0.000084 -$  (where for the next Step we have  $B = -0.000002$ ) rather than  $0.000083 +$  (where for the next Step we should have  $B = +0.000011$ .) In the latter Case, we are to divide  $B = +0.000011$ , by  $3 A q = 12.980236 -$ , to find (by the Quotient) how much is to be added to  $0.000083$ . In the other Case, we are to divide  $B = +0.000002$ , by  $3 A q = 12.98028$ , to find (by the Quotient) what is to be abated of  $0.000084$ ; so have we  $\frac{0.000011}{12.980236} = 0.00000085 +$  to be added to  $6.240083$ :

Or  $\frac{0.000002}{12.980248} = 0.00000015 +$  to be abated of  $6.240084$ ; (Or it may suffice in either to divide by  $12.98 +$ , or even by  $13 -$ , without being incumbered with a long Divisor) either of which gives us, for the Root sought,  $2.08008385$  *proxime*; true (at the third Step) to the eighth Place of decimal Parts. And if this be not near enough, the Cube of this, compared with the Number proposed, will give us another B for the next Step: And so onwards as far as we please.

Now what is said of the Cube, is easily applicable to the higher Powers.

I shall omit that of the Biquadrate; because here perhaps it may be thought most adviseable to extract the Square Root of the Number proposed, and then the Square Root of that Root. But if we would do it at once, we are from N (the Number proposed, being not a Biquadrate) to subtract  $A q q$  (the greatest Biquadrate contained in it) to find the Remainder  $B = 4 A c E + 6 A q E q + 4 A E c + E q q$ . Which Remainder, if we divide by  $4 A c$ , the Quotient will certainly be too big for E, (though perhaps not much:) If by  $4 A c + 6 A q + 4 A + 1$ , it will certainly be too little (for Reasons be-



fore-mentioned.) And we are to use our Discretion in taking some intermediate Number. And if we chance not to hit on the nearest, the Inconvenience will be but this, that our Leap will not be so great as otherwise it might be. Which will be rectified by another B at the next Step.

For the Surfolid (of five Dimensions) we are, from N (the Number proposed, being not a perfect Surfolid) to subtract  $Aqc$  (the greatest Surfolid therein contained) to find the Remainder  $B = 5 AqqE + 10 AcEq + 10 AqEc + 5 AEqq + Eqc$ . Which (as before) if we divide by  $5 Aqq$ , the Result will be somewhat too big, (because the Divisor is too little :) If by  $5 Aqq + 10 Ac + 10 Aq + 5 A + 1$ , the Result will certainly be less than the true E. The just Value of E being somewhat between these two, where we are to use our Discretion what intermediate Number to take. Which according as it proves too great or too little, is to be rectified at the next Step.

But for the most part it will be safe enough (and least trouble) to divide by  $5 Aqq$ , which gives a Quotient somewhat too big; which we may either rectify at Discretion, by taking a Number somewhat less, or proceed to another B, (affirmative or negative, as the Case shall require) and so onward to what Exactness we please. Which is, for Substance, in a manner coincident with Mr. Raphson's Method, even for affected Equations.

Thus, in the present Case: If the Number proposed be  $N = 33$ , then is  $Aqc = 32$ , and  $B = 33 - 32 = 1 = 5 AqqE + 10 AcEq + 10 AqEc + 5 AEqq + Eqc$ . Which if we divide by  $5 Aqq = 5 \times 16 = 80$ , the Result  $\frac{1}{80} = 0.0125$ , is somewhat too big for E, but not much. And if we examine it, by taking the Surfolid of  $2.0125$ , or of  $2\frac{1}{80}$ , we shall find a Negative B (for the next Step), but not very considerable. Or if we think it considerable, we may proceed farther to another Step, or more than so.

The like Method may be applied (and with more Advantage) in the higher Powers, according as the Composition of each Power requires.

And the same Method may be of Use (with good Advantage) in long Numbers (if duly applied) even before we come to the Place of Units; for the same will equally hold there also. Which the Reader may easily apprehend, without a long Discourse upon it.

XXV. The very Idea of Magnitudes infinitely great, or such as exceed any assignable Quantity, does include a Negation of Limits: yet if we nearly examine this Notion, we shall find that such Magnitudes are not equal amongst themselves, but that there are really besides infinite Length, and infinite Area, three several Sorts of infinite Solidity: all of which are *Quantitates sui generis*; and that those of each Species are in given Proportions.

Infinite Length, or a Line infinitely long, is to be considered either as beginning at a Point, and so infinitely extended one Way, or else both Ways from the same Point; in which Case the one, which is a beginning Infinity, is the one half of the whole, which is the Sum of the beginning and ceasing Infinity; or, as I may say, of Infinity *à parte ante* and *à parte post*, which is analogous to Eternity in Time or Duration, in which there is always as

much

*The Proportion  
of infinite Quan-  
tities; by Mr. E.  
Halley. n. 195.  
p. 556.*



much to follow as is past, from any Point or Moment of Time: Nor doth the Addition or Subduction of finite Length, or Space of Time, alter the Case either in Infinity or Eternity, since both the one or the other cannot be any Part of the Whole.

As to Infinite Surface or Area, any Right Line infinitely extended both Ways on an infinite Plane, does divide that infinite Plane into equal Parts, the one to the right, and the other to the left of the said Line; but if from any Point in such a Plane, two right Lines be infinitely extended, so as to make an Angle, the infinite Area, intercepted between those infinite right Lines, is to the whole infinite Plane, as the Arch of a Circle on the Point of Concourse of those Lines as a Center, intercepted between the said Lines, is to the Circumference of the Circle; or as the Degrees of the Angle to the 360 Degrees of a Circle. For Example: Two right Lines meeting at a right Angle do include, on an infinite Plane, a quarter Part of the whole infinite Area of such a Plane.

But if so be two parallel infinite Lines be supposed drawn on such an infinite Plane, the Area intercepted between them will be likewise infinite; but at the same time will be infinitely less than that Space, which is intercepted between two infinite Lines that are inclined, tho' with never so small an Angle; for that in the one Case, the given finite Distance of the parallel Lines diminishes the Infinity in one Degree of Dimension; whereas in a Sector, there is Infinity in both Dimensions: and consequently the Quantities are the one infinitely greater than the other, and there is no Proportion between them.

From the same Consideration arise the three several Species of infinite Space or Solidity; for a Parallelopiped, or a Cylinder infinitely long, is greater than any finite Magnitude, how great soever; and all such Solids supposed to be formed on given Bases, are as those Bases in proportion to one another. But if two of these three Dimensions are wanting, as in the Space intercepted between two parallel Planes infinitely extended, and at a finite Distance; or with infinite Length and Breadth, with a finite Thickness, all such Solids shall be as the given finite Distances one to another; but these Quantities, tho' infinitely greater than the other, are yet infinitely less than any of those wherein all the three Dimensions are infinite. Such are the Spaces intercepted between two inclined Planes infinitely extended; the Space intercepted by the Surface of a Cone, or the Sides of a Pyramid, likewise infinitely continued, &c. of all which notwithstanding, the Proportions one to another, and to the  $\tau\acute{o}\ \pi\acute{\alpha}\nu$  or vast Abyss of infinite Space (wherein is the *Locus* of all Things that are or can be; or to the Solid of infinite Length, Breadth, and Thickness taken all manner of ways) are easily assignable. For the Space between two Planes is to the whole, as the Angle of those Planes to the 360 Degrees of the Circle. As for Cones and Pyramids, they are as the spherical Surface intercepted by them is to the Surface of the Sphere, and therefore Cones are as the versed Sines of half their Angles to the Diameter of the Circle: These three Sorts of infinite Quantity are analogous to a Line, Surface,

face,



face, and Solid, and after the same manner cannot be compared, 'or have' no Proportion the one to the other.

*In. finitely-infinite Fractions; by D. R. Wood. Pb. Col. n. 3. p. 45.*

XXVI. Infinitely-infinite Fractions, or all the Powers of the Fractions whose Numerator is 1, are all of them together equal to (1) an Unit.

		P.					
A.	R.	q.	c.	qq.			
$1 = \frac{1}{1}$	$= \frac{1}{2}$	$+$	$\frac{1}{4}$	$+$	$\frac{1}{8}$	$+$	$\frac{1}{16}$ &c.
$\frac{1}{2} = \frac{1}{3}$	$= \frac{1}{4}$	$+$	$\frac{1}{9}$	$+$	$\frac{1}{27}$	$+$	$\frac{1}{81}$ &c.
$\frac{1}{3} = \frac{1}{4}$	$= \frac{1}{5}$	$+$	$\frac{1}{16}$	$+$	$\frac{1}{64}$	$+$	$\frac{1}{256}$ &c.
$\frac{1}{4} = \frac{1}{5}$	$= \frac{1}{6}$	$+$	$\frac{1}{25}$	$+$	$\frac{1}{125}$	$+$	$\frac{1}{625}$ &c.
$\frac{1}{5} = \frac{1}{6}$	$= \frac{1}{7}$	$+$	$\frac{1}{36}$	$+$	$\frac{1}{216}$	$+$	$\frac{1}{1296}$ &c.
&c.	&c.						

			Or thus:				
$\frac{1}{1 \times 1} =$	$\frac{1}{2}$	$+$	$\frac{1}{1 \times 2} =$	$\frac{1}{2 \times 1} =$	$\frac{1}{3}$	$+$	$\frac{1}{2 \times 3} =$
$\frac{1}{2 \times 1} =$	$\frac{1}{3}$	$+$	$\frac{1}{3 \times 4} =$	$\frac{1}{3 \times 1} =$	$\frac{1}{4}$	$+$	$\frac{1}{3 \times 4} =$
$\frac{1}{3 \times 1} =$	$\frac{1}{4}$	$+$	$\frac{1}{4 \times 5} =$	$\frac{1}{4 \times 1} =$	$\frac{1}{5}$	$+$	$\frac{1}{4 \times 5} =$
$\frac{1}{4 \times 1} =$	$\frac{1}{5}$	$+$	$\frac{1}{5 \times 6} =$	$\frac{1}{5 \times 1} =$	$\frac{1}{6}$	$+$	$\frac{1}{5 \times 6} =$

2 anor	+	$\frac{1}{2^2} +$	$\frac{1}{1 \times 2^2} =$	$\frac{1}{2^3} +$	$\frac{1}{1 \times 2^3} =$		$\frac{1}{2^4} +$	&c.
3 anor	+	$\frac{1}{3^2} +$	$\frac{1}{2 \times 3^2} =$	$\frac{1}{3^3} +$	$\frac{1}{2 \times 3^3} =$		$\frac{1}{3^4} +$	&c.
4 anor	+	$\frac{1}{4^2} +$	$\frac{1}{3 \times 4^2} =$	$\frac{1}{4^3} +$	$\frac{1}{3 \times 4^3} =$		$\frac{1}{4^4} +$	&c.
5 anor	+	$\frac{1}{5^2} +$	$\frac{1}{4 \times 5^2} =$	$\frac{1}{5^3} +$	$\frac{1}{4 \times 5^3} =$		$\frac{1}{5^4} +$	&c.
6 anor	+	$\frac{1}{6^2} +$	$\frac{1}{5 \times 6^2} =$	$\frac{1}{6^3} +$	$\frac{1}{5 \times 6^3} =$		$\frac{1}{6^4} +$	&c.

A. Is a File or Row of absolute Numbers, or rather of all the Fractions, whose Numerator is 1; which Row is supposed to be continued in *infinitum* (downwards.)

R. Is



R. Is another File or Row of all the Roots (whose Numerator is 1) of all the Powers of such Fractions, supposed likewise to be continued *in infinitum* (downwards).

P. Are all the respective Powers of such Fractions, (*viz.* Squares, Cubes, &c.) or so many Ranks of Geometrical Proportionals, supposed to be continued *in infinitum*, both to the Right-hand, and also downwards.

Lemma.] Each of the said Ranks of Powers, together with their respective Roots, is equal to each of the several Numbers under A respectively.

Demonstration.] If from the Line  $ab$  you take (for instance)  $\frac{1}{4}$  part towards  $a$ , suppose  $ac$ ; and also from the other End of the same Line  $ab$ , you take two such Parts (or  $\frac{2}{4}$  Parts) towards  $b$ , suppose  $bd$ , (*viz.* a Number of Parts less by two than the whole Line  $ab$  was first supposed to be divided into) there will remain the Line  $cd = ac = \frac{1}{4}$  of  $ab$ . Then again, if from  $cd$  you take  $\frac{1}{4}$  Part thereof towards  $a$ , suppose  $ce$ , and from the other End  $\frac{1}{4}$  Part of the same  $cd$ , suppose  $df$ , there will remain only  $ef = ce = \frac{1}{4}$  of  $cd$ . And if you still go on without ceasing, to take on the Side towards  $a$ ,  $\frac{1}{4}$  Part of what was taken last before, and twice as much on the other Side towards  $b$ , there shall be found between the two Lines last taken always remaining  $\frac{2}{4}$  Part of the Line from which they were taken. From which  $\frac{1}{4}$  Part there may still after the same manner be supposed to be taken two other such Lines. But if this be supposed to be done infinite Times actually, then there will nothing more remain (between), and so the continued Division on either Side will come exactly to the Point  $g$ , supposing  $ag$  to be  $\frac{2}{3}$  of  $ab$ , and  $bg = 2 ag$ : For, because that which was taken away towards  $b$ , was always twice as much as that which was taken away towards  $a$ , the total Sum of all the Lines taken away towards  $b$ , (which all together do make up the Line  $bg$ ) must be twice as much as the Line  $ag$ , (which is the total Sum of all the Lines taken away towards  $a$ ) *viz.*  $bg = 2 ag$ ; and consequently  $bg + ag$  (or the whole Line  $ab$ ) is equal to  $3 ag$ ; and therefore  $ag = \frac{2}{3}$  of  $ab$ . Q. E. D.

Fig. 54.

The like *Construction* and *Demonstration* (*mutatis mutandis*) may be made use of in taking away any other Part of any Quantity, and the like Part again of the first mentioned Part, and so *in infinitum*. The total Sum of all the Parts so taken, or supposed to be taken, shall be equal to any certain Quantity, or Part, or Fraction, whose Denominator shall be less by an Unit than the Denominator of the first

mentioned Part; as  $\frac{1}{6} = \frac{1}{7} + \frac{1}{49} + \frac{1}{343} \&c.$   $\frac{1}{9} = \frac{1}{10} + \frac{1}{100} + \frac{1}{1000}$  &c. And so, that which the incomparable *Archimedes* (in his Squaring the *Parabola*) has only demonstrated in one particular Case, *viz.*  $\frac{1}{3} = \frac{1}{4} + \frac{1}{16} + \frac{1}{64}$

$+ \frac{1}{256} + \frac{1}{1024} \&c.$  and that too, not without an huge Apparatus of Preliminary Propositions, amounting to a whole Book, is here universally demonstrated



monstrated in all Cases (which are infinite) and by a very simple and easy Method (in *Des Cartes's* Way) and on one single Page.

Now if each of the said Ranks of Powers, together with their respective Roots, be equal to the several Numbers of Fractions under A; (as is demonstrated by the *Lemma*) then is A the Sum of them all, or equal to them all; that is to say,  $R + P = A = R + 1$ ; for R is the same with A wanting  $\frac{1}{I}$ , or 1, (as appears upon View) or but  $(\frac{1}{\infty})$  one infinite Part bigger. Wherefore  $P = 1$ , *Q. E. D. viz.* Infinitely-infinite Fractions are equal to Unity, that is, to the least Root that is an Integer.

*Corollaries.* Hence it is plain,

1. That a Progress *ad infinitum* must be allowed.
2. And not only to one Infinite, but to several; or rather to infinite Infinites, or infinitely Infinites.
3. And that this may be done, that is, this Calculation may be performed, by a very finite or bounded Capacity.
4. And that this whole Progress, or such infinite Progresses, may be cast up, or collected into one Sum.
5. And into a Sum that is not only not infinite, but so small that it is less than any Number.

It appears farther,

That of Infinites some are equal, and others unequal.

And that one Infinite may be equal to two, three, or several, either Finites or Infinites.

For 1. The infinite Powers of the first Rank are  $= \frac{1}{2} = \frac{1}{1 \times 2}$ , and also equal to all the infinitely-infinite Powers of all the other Ranks.

The infinite Powers of the second Rank are $= \frac{1}{6} = \frac{1}{2 \times 3}$	} <i>viz.</i> equal to the respective mean proportional Number between the Square Numbers respectively.  <i>e. g.</i> 4, 6, 9 9, 12, 16 16, 20, 25 25, 30, 36 &c. <i>in infinitum.</i>
Those of the third Rank are $= \frac{1}{12} = \frac{1}{3 \times 4}$	
Those of the fourth Rank are $= \frac{1}{20} = \frac{1}{4 \times 5}$	
Those of the fifth Rank are $= \frac{1}{30} = \frac{1}{5 \times 6}$	
&c. <i>in infinitum.</i>	





2. The infinite Powers of the two first Ranks are =  $\frac{2}{3}$

Those of the three first are =  $\frac{3}{4}$

Those of the four first are =  $\frac{4}{5}$

Those of the five first are =  $\frac{5}{6}$

&c. in infinitum.

3. All the Powers of all the infinite Ranks, except the first are =  $\frac{1}{2}$

All, except the two first, are =  $\frac{1}{3}$

All, except the three first, are =  $\frac{1}{4}$

All, except the four first, are =  $\frac{1}{5}$

&c. in infinitum.

The latter *Corollaries* may all appear by simple Addition and Subduction ; and so may many more.

XXVII. 1. That the Numeral Figures now in use, with the manner of The Antiquity of the Numeral Figures; by Dr. J. Wallis. N. 154. p. 399. Dec. An. 1683. Computation by them (and the Names of Algorithm, appropriated to that Way of Computation) came to us from the *Arabs* (but somewhat altered, as to the Shape of the Figures, in succeeding Ages) and to them from the *Indians*, is generally agreed. But it is not so generally agreed, of what Antiquity the Use of them, in our *European* Parts, hath been.

*Jo. Gerard Vossius* (*De Scientiis Mathematicis*) thinks they came not in Use here till about the Year of our Lord 1300 ; or at the farthest, later than the Year 1250. And *P. Mabillon* (*De Re Diplomatica*) tells us, that he hath not found them any where used sooner than the 14th Century. But I think their Use in these Parts was as old at least as the Times of *Hermannus Contractus*, who lived about the Year of our Lord 1050 (that is, about the middle of the 11th Century,) if not so frequently in ordinary Affairs ; yet at least in Mathematical Things, and especially in Astronomical Tables.

But I do not remember that I have any where seen any Monument of them more antient than the Mantle-tree of the Parlour Chimney at the Dwelling-House of Mr. *Will. Richards*, the Rector of *Helmdon* in *Northamptonshire*.

Fig. 55.

The Sides of the Chimney, by which the Mantle-tree is supported, are of Stone ; but the Mantle-tree itself is of Wood. It is all over as black as Ink, having by Age and Smoak contracted that Colour. It may yet continue many hundreds of Years ; for I did not discern in it any Thing either of Worm, or Rottenness, or any Tendency to it. The Length of it is five Foot nine Inches ; its Breadth or Depth at the Ends, (A B) is 11  $\frac{1}{2}$  Inches, but at



the Middle, as C D, somewhat less. It is also carved from End to End, and the lower Part of it is abated, as in the Mouldings of other Chimneys. On the Front of the upper Part there is (beginning at the Middle) on three Squares parted from each other by a deep Furrow or Channel, the Date (I suppose, when it was first made) described partly in Numeral Figures, A<sup>o</sup> Do<sup>i</sup> M<sup>o</sup> 133; on a fourth a Flower, and on a fifth the two Letters W. R. with an Escutcheon, representing (I suppose) the Name of him to whom it did then belong. Both the Letters and the Figures are of an Antique Form, agreeing well enough with that Age. They are not engraved or cut in, but prominent on their several Squares (by way of *Bas-relief*) the Wood being abated round about them. The o over the A is a round o, but that over the M is a square o.

Hence it appears, that the Use of such Figures here in *England*, even on ordinary Occasions, is at least as ancient as the Year 1133. And I judge it to have been yet somewhat ancients, because the Shape of the Figures, though not come just to the Shape which we now use, was even then considerably varied from the Shape of the *Arabick* Figures; which argues they had then been for some Time in use; such Change of Shape in Figures and Letters coming on gradually with Time.

It need not move any Scruple at all, that Part of the Numbers is expressed by the Numeral Letter M (or the Word *Millesimo*, of which M<sup>o</sup> is but a Contraction) while the rest is expressed in Numeral Figures: For the like doth oft occur in old *Manuscripts*; and sometimes even at this Day. And it doth rather favour the Simplicity of that Age, (not very nice in such Things, especially amongst Mechanics) than any Design of Imposture.

By Mr. Tho.  
Luffkin.  
N. 255. p. 287.  
N. 266. p. 677.  
Aug. An. 1699.

Fig. 56.

2. Over-against the *Market-place* in *Colchester* stands the House of Mr. *Furley*, a Linnen-draper; some of the backermost Part of which is an ancient *Roman* Building, but the Front is of lesser standing, and timbered. Upon the bottom Cell (which is almost in the Form of a triangular Prism) of one of the Windows of the Front, between two carved Lions, stands an Escutcheon, containing only these Figures 1090. The Periphery of the Cyphers, and Nine, are rather fracted than flected, prominent, large, and very fair; but to make them the more perspicuous, they are gilded by the Proprietor. The Window looks directly North, the Date being thereby preserved from the scorching Heat of the Sun; and by its Inclination (falling from the *Vertex* or *Perpendicular* by an Angle of about 60 Degrees) from Rain, Snow, &c. If it be objected, that the second and fourth Figures may represent that among the *Arabians*, which is with us as five; I answer, that the o is not used with all the *Arabs* for 5, but with some for a Cypher, and so it was used by the *Moors* in *Spain*, who first brought these Figures into our Parts; nor is the square o an *Arabick* Letter, but an *English* Letter of that Age. And the Form of these Figures soon degenerated from that of the *Arabs*, into such as we now use, if not at the first Reception from the *Arabs* [or *Moors*], certainly long before 1595, as this *Construction* would make it.

The Construction  
of Logarithms;  
by Mr. Edm.  
Halley. N. 216.  
p. 58.  
Mar. An. 1695.

XXVIII. The old Definition of *Logarithms*, that they are the equally-differing Associates of Proportional Numbers, is too scanty to define them fully: For

For



For they may much more properly be said to be *Numeri Rationum Exponentes*; wherein *Ratio* is considered as a *Quantitas sui generis*, beginning from the Ratio of Equality, or 1 to 1 = 0; being Affirmative when the Ratio is increasing, as of Unity to a greater Number, but Negative when decreasing; and these *Rationes* we suppose to be measured by the Number of the *Ratiunculæ* contained in each. Now these *Ratiunculæ* are to be so understood as in a continued Scale of Proportionals, infinite in Number between the two Terms of the Ratio; which infinite Number of mean Proportionals is to that infinite Number of the like and equal *Ratiunculæ* between any other two Terms, as the Logarithm of the one Ratio is to the Logarithm of the other. Thus if there be supposed between 1 and 10 an infinite Scale of mean Proportionals, whose Number is 100000, &c. *in infinitum*; between 1 and 2 there shall be 30103, &c. of such Proportionals, and between 1 and 3 there will be 47712, &c. of them; which Numbers therefore are the Logarithms of the *Rationes* of 1 to 10, 1 to 2, and 1 to 3; and not so properly to be called the Logarithms of 10, 2 and 3.

This being laid down, it is obvious that if between Unity and any Number proposed, there be taken any Infinity of mean Proportionals, the infinitely little Augment or Decrement of the first of those Means from Unity, will be a *Ratiuncula*, that is, the Momentum or Fluxion of the Ratio of Unity to the said Number: And seeing that in these continual Proportionals all the *Ratiunculæ* are equal, their Sum, or the whole Ratio will be as the said Momentum is directly; that is, the Logarithm of each Ratio will be as the Fluxion thereof. Wherefore if the Root of any infinite Power be extracted out of any Number, the *Differentiola* of the said Root from Unity, shall be as the Logarithm of that Number. So that Logarithms thus produced, may be of as many Forms as you please to assume infinite Indices of the Power whose Root you seek: as if the Index be supposed 100000, &c. infinitely, the Roots shall be the Logarithms invented by the Lord *Napier*; but if the said Index were 2302585, &c. *Mr. Briggs's* Logarithms would immediately be produced. And if you please to stop at any Number of Figures, and not to continue them on, it will suffice to assume an Index of a Figure or two more than your intended Logarithm is to have, as *Mr. Briggs* did, who to have his Logarithms true to 14 Places, by continual Extraction of the Square Root, at last came to have the Root of the 140737488355328th Power; but how operose that Extraction was, will be easily judged by who so shall undertake to examine his *Calculus*.

Now, tho' the Notion of an infinite Power may seem very strange, and to those that know the difficulty of the Extraction of the Roots of high Powers, perhaps impracticable; yet by the help of that admirable Invention of *Mr. Newton*, whereby he determines the *Unciæ* or Numbers prefix'd to the Members composing Powers, (on which chiefly depends the *Doctrinè of Series*) the Infinity of the Index contributes to render the Expression much more easy: For if the infinite Power to be resolved be put (after *Mr. Newton's* Method)

$$Q^2$$

$$\frac{p + pq}{p + pq^{\frac{1}{2}}}$$



$$\sqrt[m]{p + p q^{\frac{1}{m}}}, \text{ or } \sqrt[m]{1 + q^{\frac{1}{m}}}, \text{ instead of } 1 + \frac{1}{m} q + \frac{1 - m}{2 m m} q q + \frac{1 - 3 m + 2 m m}{6 m^2} q^3 + \frac{1 - 6 m + 11 m m - 6 m^3}{24 m^3} q^4, \text{ \&c.}$$

(which is the Root when  $m$  is finite), becomes  $1 + \frac{1}{m} q - \frac{1}{2 m} q q + \frac{1}{3 m} q^3 + \frac{1}{4 m} q^4 + \frac{1}{5 m} q^5, \text{ \&c.}$   $m$  being infinite; and conse-

quently whatever is divided thereby vanishing. Hence it follows that  $\frac{1}{m}$  multiplied into  $q - \frac{1}{2} q q + \frac{1}{3} q^3 - \frac{1}{4} q^4 + \frac{1}{5} q^5, \text{ \&c.}$  is the Augment of the first of our mean Proportionals between Unity and  $1 + q$ , and is therefore the Logarithm of the Ratio of 1 to  $1 + q$ ; and whereas the Infinite Index  $m$  may be taken at pleasure, the several Scales of Logarithms to such Indices will be as  $\frac{1}{m}$ , or reciprocally as the Indices. And if the Index be taken 10000, &c. as in the case of *Napier's* Logarithms, they will be simply  $q - \frac{1}{2} q q + \frac{1}{3} q^3 - \frac{1}{4} q^4 + \frac{1}{5} q^5 - \frac{1}{6} q^6, \text{ \&c.}$

Again; If the Logarithm of a decreasing Ratio be sought, the infinite Root of  $1 - q$ , or  $\sqrt[m]{1 - q^{\frac{1}{m}}}$  is  $1 - \frac{1}{m} q - \frac{1}{2 m} q^2 - \frac{1}{3 m} q^3 - \frac{1}{4 m} q^4 - \frac{1}{5 m} q^5 - \frac{1}{6 m} q^6, \text{ \&c.}$  whence the Decrement of the first of our Infinite Number of Proportionals will be  $\frac{1}{m}$  into  $q + \frac{1}{2} q q + \frac{1}{3} q^3 + \frac{1}{4} q^4 + \frac{1}{5} q^5 + \frac{1}{6} q^6, \text{ \&c.}$  which therefore will be as the Logarithm of the Ratio of Unity to  $1 - q$ : But if  $m$  be put 10000, &c. then the said Logarithm will be  $q + \frac{1}{2} q q + \frac{1}{3} q^3 + \frac{1}{4} q^4 + \frac{1}{5} q^5 + \frac{1}{6} q^6, \text{ \&c.}$

Hence the Terms of any Ratio being  $a$  and  $b$ ,  $q$  becomes  $\frac{b - a}{a}$  or the Difference divided by the lesser Term, when 'tis an increasing Ratio, or  $\frac{b - a}{b}$  when 'tis decreasing, or as  $b$  to  $a$ ; whence the Logarithm of the same Ratio may be doubly expressed; for, putting  $x$  for the Difference of the Terms  $a$  and  $b$ , it will be either



$$\frac{1}{m} \text{ into } \frac{x}{b} + \frac{x^2}{2 b b} + \frac{x^3}{3 b^3} + \frac{x^4}{4 b^4} + \frac{x^5}{5 b^5} + \frac{x^6}{6 b^6} \&c.$$

$$\text{or } \frac{1}{m} \text{ into } \frac{x}{a} - \frac{x^2}{2 a a} + \frac{x^3}{3 b^3} - \frac{x^4}{4 a^4} + \frac{x^5}{5 a^5} - \frac{x^6}{6 a^6} \&c.$$

But if the Ratio of  $a$  to  $b$  be supposed to be divided into two Parts, *viz.* into the Ratio of  $a$  to the *Arithmetical Mean* between the Terms, and the Ratio of the said *Arithmetical Mean* to the other Term  $b$ ; then will the Sum of the Logarithms of those two Rationes be the Logarithm of the Ratio of  $a$  to  $b$ ; and, substituting  $\frac{1}{2} z$  instead of  $\frac{1}{2} a + \frac{1}{2} b$ , the said *Arithmetical Mean*, the Logarithms of those Rationes will be, by the foregoing Rule,

$$\frac{1}{m} \text{ into } \frac{x}{z} + \frac{x x}{2 z z} + \frac{x^3}{3 z^3} + \frac{x^4}{4 z^4} + \frac{x^5}{5 z^5} + \frac{x^6}{6 z^6} \&c.$$

$$\text{and } \frac{1}{m} \text{ into } \frac{x}{z} - \frac{x x}{2 z z} + \frac{x^3}{3 z^3} - \frac{x^4}{4 z^4} + \frac{x^5}{5 z^5} - \frac{x^6}{6 z^6} \&c.$$

the Sum whereof  $\frac{1}{m} \text{ into } \frac{2 x}{z} * + \frac{2 x^3}{3 z^3} * + \frac{2 x^5}{5 z^5} * + \frac{2 x^7}{7 z^7} \&c.$  will be the

Logarithm of the Ratio of  $a$  to  $b$ , whose Difference is  $x$ , and Sum  $z$ . And this Series converges twice as swift as the former, and therefore is more proper for the Practice of making Logarithms; which it performs with that Expedition, that where  $x$  the Difference is but the hundredth Part of the Sum,

the first Step,  $\frac{2 x}{z}$ , suffices to seven Places of the Logarithm, and the second

Step to twelve; but if *Briggs's* first twenty *Chiliads* of Logarithms be supposed made, as he very carefully computed them, to fourteen Places, the first Step alone is capable to give the Logarithm of any intermediate Number, true to all the Places of those Tables.

After the same Manner may the Difference of the said two Logarithms be very aptly applied to find the Logarithm of prime Numbers, having the Logarithms of the two next Numbers above and below them: For the Difference of the Ratio of  $a$  to  $\frac{1}{2} z$ , and of  $\frac{1}{2} z$  to  $b$ , is the Ratio of  $a b$  to  $\frac{1}{4} z z$ , and the Half of that Ratio is that of  $\sqrt{a b}$  to  $\frac{1}{2} z$ , or of the Geometrical Mean to the Arithmetical. And consequently the Logarithm thereof will be the Half-difference of the Logarithms of those Rationes, *viz.*

$$\frac{1}{m} \text{ into } \frac{x x}{2 z z} + \frac{x^4}{4 z^4} + \frac{x^6}{6 z^6} + \frac{x^8}{8 z^8} \&c.$$

Which is a *Theorem* of good Dispatch to find the Logarithm of  $\frac{1}{2} z$ . But the same is yet much more advantageously performed by a Rule derived from the foregoing, and beyond which, in my Opinion, nothing better can be hoped. For the Ratio of  $a b$  to  $\frac{1}{4} z z$ , or  $\frac{1}{4} a a + \frac{1}{2} a b + \frac{1}{4} b b$ , has the Difference of its Terms,  $\frac{1}{4} a a - \frac{1}{2} a b + \frac{1}{4} b b$ , or the Square of  $\frac{1}{2} a - \frac{1}{2} b = \frac{1}{4} x x$ , which, in the present Case of finding the Logarithms of Prime Numbers,



is always Unity; and calling the Sum of the Terms  $\frac{1}{4} z z + a b = y y$ , the Logarithm of the Ratio of  $\sqrt{a b}$  to  $\frac{1}{2} a + \frac{1}{2} b$ , or  $\frac{1}{2} z$  will be found,

$$\frac{1}{m} \text{ into } \frac{1}{y y} + \frac{1}{3y^6} + \frac{1}{5y^{10}} + \frac{1}{7y^{14}} + \frac{1}{9y^{18}}, \text{ \&c.}$$

which converges very much faster than any *Theorem* hitherto published for this Purpose.

Here note, that  $\frac{1}{m}$  is all along applied to adapt these Rules to all Sorts of Logarithms. If  $m$  be 10000, &c. it may be neglected, and you will have *Napier's* Logarithms, as was hinted before; but if you desire *Briggs's* Logarithms, which are now generally received, you must divide your Series by 2, 302585-09299404568401799145468436420761010148862877976033328, or multiply it by the Reciprocal thereof, *viz.* 0, 43429448190325182765112891-8916605082294397005803666566114454.

But to save so operose a Multiplication (which is more than all the rest of the Work) it is expedient to divide this Multiplicator by the Powers of  $z$  or  $y$  continually, according to the Direction of the *Theorem*, especially where  $x$  is small and Integer, reserving the proper Quotes to be added together, when you have produced your Logarithm to as many Figures as you desire; of which Method I will give a Specimen, in the Logarithms of the first prime Numbers under 20 to sixty Places, computed by Mr. *Abraham Sharp*, as they were communicated to me by our common Friend, Mr. *Euclid Speidal*.

Num.	Logarithms,
2.	0, 301029995663981195213738894724493026768189881462108541310427
3.	0, 477121254719662437295027903255115309200128864190695864829866
7.	0, 845098040014256830712216258592636193483572396323965406503835
11.	1, 041392685158225040750199971243024241706702190466453094596539
13.	1, 113943352306837769206541895026246254561189005053673288598083
17.	1, 230448921378273028540169894328337030007567378425046397380368
19.	1, 278753600952828961536333475756929317951192337394497598906819

The next Prime Number is 23, which I will take for an Example of the foregoing Doctrine; and by the first Rules, the Logarithm of the Ratio of 22 to 23 will be found to be either

$$\frac{1}{22} - \frac{1}{968} + \frac{1}{31944} - \frac{1}{937024} + \frac{1}{25768160}, \text{ \&c.}$$

$$\text{or } \frac{1}{23} + \frac{1}{1058} + \frac{1}{36531} + \frac{1}{1119364} + \frac{1}{32181715}, \text{ \&c.}$$



As likewise that of the Ratio of 23 to 24 by a like Process.

$$\frac{1}{23} - \frac{1}{1058} + \frac{1}{36501} - \frac{1}{1119364} + \frac{1}{32181715}, \text{ \&c. or}$$

$$\frac{1}{24} + \frac{1}{1152} + \frac{1}{41471} + \frac{1}{1327104} + \frac{1}{39813120}, \text{ \&c.}$$

And this is the Result of the Doctrine of *Mercator*, as improved by the learned Dr. *Wallis*. But by the second *Theorem*, viz.  $\frac{2x}{z} + \frac{2x^3}{3z^3} + \frac{2x^5}{5z^5}, \text{ \&c.}$  the same Logarithms are obtained by fewer Steps; to wit,

$$\frac{2}{45} + \frac{2}{273375} + \frac{2}{922640625} + \frac{2}{2615686171875}, \text{ \&c. And}$$

$$\frac{2}{47} + \frac{2}{311469} + \frac{2}{1146725035} + \frac{2}{3546361843241}, \text{ \&c.}$$

which was invented and demonstrated in the Hyperbolic Spaces analogous to the Logarithms, by the excellent Mr. *James Gregory*, in his *Exercitationes Geometricæ*, and since further prosecuted by the aforesaid Mr. *Speidall*, in a late Treatise in *English* by him published on this Subject. But the Demonstration, as I conceive, was never till now perfected without the Consideration of the Hyperbola, which in a Matter purely Arithmetical, as this is, cannot so properly be applied. But what follows, I think, I may more justly claim as my own, viz. *That the Logarithm of the Ratio of the Geometrical Mean to the Arithmetical*, between 22 and 24, or of  $\sqrt{528}$  to 23, will be found to be either,

$$\frac{1}{1058} + \frac{1}{1119364} + \frac{1}{888215334} + \frac{1}{626487882248}, \text{ \&c. or}$$

$$\frac{1}{1057} + \frac{1}{3542796579} + \frac{1}{659676558485285}, \text{ \&c.}$$

All these Series being to be multiplied into 0,4342944819, &c. if you design to make the *Logarithm* of *Briggs*. But with great Advantage in respect of the Work, the said 0,4342944819, &c. is divided by 1057, and the Quotient thereof again divided by three times the Square of 1057, and that Quotient again by  $\frac{5}{3}$  of that Square, and that Quotient by  $\frac{7}{3}$  thereof, and so forth, till you have as many Figures of your Logarithm as you desire. As for Example, the Logarithm of the Geometrical Mean between 22 and 24 is found by the Logarithms of 2, 3, and 11, to be



			1,36131696126690612945009172669805
	1057)	43429 &c.	( - - - 41087462810146814347315886368
3	in 1117249)	41087 &c.	( - - - - - - - 12258521544181829460074
5	in 1117249)	12258 &c.	( - - - - - - - - - 6583235184376175
7	in 1117249)	65832 &c.	( - - - - - - - - - - - 4208829765
9	in 1117249)	42088 &c.	( - - - - - - - - - - - - - 2930
	Summa		1,36172783601757287886777711225119

Which is the Logarithm of 23 to thirty-two Places, and obtained by five Divisions with very small Divisors; all which is much less Work than simply multiplying the Series into the said Multiplier 0,43429, &c.

From the Logarithm given to find what Ratio it expresses, is a *Problem* that has not been so much considered as the former, but which is solved with the like Ease, and demonstrated by a like Process, from the same general *Theorem* of Mr. *Newton*; for as the Logarithm of the Ratio of 1 to  $1 + q$  was proved to be  $\overline{1 + q}^{\frac{1}{m}} - 1$ , and that of the Ratio of 1 to  $1 - q$  to be  $1 - \overline{1 - q}^{\frac{1}{m}}$ : so the Logarithm, which we will from henceforth call L, being given,  $1 + L$  will be equal to  $\overline{1 + q}^{\frac{1}{m}}$  in the one Case, and  $1 - L$  will be equal to  $\overline{1 - q}^{\frac{1}{m}}$  in the other: Consequently  $\overline{1 + L}^m$  will be equal to  $1 + q$ , and  $\overline{1 - L}^m$  to  $1 - q$ ; that is, according to Mr. *Newton's* said Rule,  $1 + mL + \frac{1}{2}m^2L^2 + \frac{1}{6}m^3L^3 + \frac{1}{24}m^4L^4 + \frac{1}{120}m^5L^5, \&c.$  will be = to  $1 + q$ ; and  $1 - mL + \frac{1}{2}m^2L^2 - \frac{1}{6}m^3L^3 + \frac{1}{24}m^4L^4 - \frac{1}{120}m^5L^5, \&c.$  will be equal to  $1 - q$ ; *m* being any infinite Index whatsoever: which is a full and general *Proposition* from the Logarithm given to find the Number, be the Species of Logarithms what it will. But if *Napier's* Logarithm be given, the Multiplication by *m* is saved, (which Multiplication is indeed no other than the reducing the other Species to his) and the Series will be more simple, *viz.*  $1 + L + \frac{1}{2}L^2 + \frac{1}{6}L^3 + \frac{1}{24}L^4 + \frac{1}{120}L^5, \&c.$  or  $1 - L + \frac{1}{2}L^2 - \frac{1}{6}L^3 + \frac{1}{24}L^4 - \frac{1}{120}L^5, \&c.$  This Series, especially in great Numbers, converges so slowly, that it were to be wished it could be contracted.

If one Term of the Ratio, whereof L is the Logarithm, be given, the other Term will be had easily by the same Rule: For if L were *Napier's* Logarithm



arithm of the Ratio of  $a$  the lesser to  $b$  the greater Term,  $b$  would be the Product of  $a$  into  $1 + L + \frac{1}{2} L^2 + \frac{1}{6} L^3$ , &c.  $= a + aL + \frac{1}{2} aL^2 + \frac{1}{6} aL^3$ , &c. But if  $b$  were given,  $a$  would be  $= b - bL + \frac{1}{2} bL^2 - \frac{1}{6} bL^3$ , &c. Whence, by the help of the *Chiliads*, the Number answering to any Logarithm will be exactly had to the utmost Extent of the Tables. If you seek the nearest next Logarithm, whether greater or lesser, and call its Number  $a$  if lesser, or  $b$  if greater; then the given  $L$ , and the Difference thereof from the said nearest Logarithm you call  $l$ ; it will follow that the Number answering to the Logarithm  $L$  will be either  $a$  into  $1 + l + \frac{1}{2} l^2 + \frac{1}{6} l^3 + \frac{1}{24} l^4 + \frac{1}{120} l^5$ , &c. or else  $b$  into  $1 - l + \frac{1}{2} l^2 - \frac{1}{6} l^3 + \frac{1}{24} l^4 - \frac{1}{120} l^5$ , &c. wherein as  $l$  is less, the Series will converge the swifter. And if the first 20000 Logarithms be given to 14 Places, there is rarely occasion for the three first Steps of this Series to find the Number to as many Places. But for *Ulacq's* great *Canon* of 100000 Logarithms, which is made but to ten Places, there is scarce ever need for more than the first Step  $a + al$ , or  $a + mal$  in one Case, or else  $b - bl$ , or  $b - mbl$  in the other, to have the Number true to as many Figures as those Logarithms consist of.

There is another Series which is not indeed so simple and uniform, yet the first Step thereof is most commodious for Practice, and exact enough for Tables not exceeding 14 Places: It is thus;  $a + \frac{al}{1 - \frac{1}{2}l}$  or  $b - \frac{bl}{1 + \frac{1}{2}l}$  will be the Number answering to the Logarithm given, differing from the Truth but by one half of the third Step from the former Series. But that which renders it yet more eligible is, that with equal Facility it serves for *Briggs's* or any other sort of Logarithms, with the only Variation of writing  $\frac{1}{m}$  instead of 1, that is

$$a + \frac{al}{\frac{1}{m} - \frac{1}{2}l}, \text{ and } b - \frac{bl}{\frac{1}{m} + \frac{1}{2}l} \text{ or } \frac{\frac{1}{m} a + \frac{1}{2} la}{\frac{1}{m} - \frac{1}{2}l} \text{ and } \frac{\frac{1}{m} b - \frac{1}{2} lb}{\frac{1}{m} + \frac{1}{2}l}, \text{ which}$$

are easily resolved into Analogies, *viz.*

As 42429 &c.  $- \frac{1}{2}l$ : to 43429  $+ \frac{1}{2}l$ : : so is  $a$ : } to the Number sought,  
Or, As 43429 &c.  $+ \frac{1}{2}l$ : to 43429  $- \frac{1}{2}l$ : : so is  $b$ : }



If more of this Series be desired, it will be found as follows,

$a + \frac{al}{2 - \frac{1}{2}l} - \frac{\frac{1}{2}al}{1 - l} + \frac{\frac{1}{4}al^2}{1 - 2l}$ , &c. as may easily be demonstrated by working out the Divisions in each Step, and collecting the Quotes, whose Sum will be found to agree with our former Series; which is no other than an easy *Corollary* to Mr. *Newton's* general *Theorem* for forming *Roots* and *Powers*.

XXIX. Papers of less General Use omitted.

*Tangents to  
Curves. N. 81.  
p. 4010.  
Mar. An. 1672.*

1. **A** Breviat of Dr. *Wallis's* two Methods of drawing *Tangents*; Extracted by him from his *Con. Sect.* and other Parts of his *Mathematical Works*.

*Rectification of  
Curves. N. 98.  
p. 6146, 6149.  
Nov. An. 1673.*

2. Mr. *Huygens* in his *Hor. Oscill.* having given M. *Huraet* the Honour of inventing a Curve equal to a Straight Line in the Year 1659; Dr. *Wallis* here asserts this Invention to Mr. *William Neile* (Son of Sir *Paul Neile*), who discovered and demonstrated the Equality of a *Paraboloid* to a Straight Line two Years before. The same was soon after otherwise demonstrated by my Lord *Brouncker*, and Sir *Christopher Wren*, in *June* and *July* 1657; and the Demonstrations inserted by Dr. *Wallis*, in his *Traët de Cycloide* 1659, with a fair Relation of the whole Matter. Besides, Sir *Christopher Wren* found a straight Line equal to that of a *Cycloid* in the Year 1658: Yet he freely confesses Mr. *Neile's* Invention of a Curve capable of *Rectification* the Year before.

*Ib. p. 6150.*

*Transformation  
of Curves. N. 214.  
p. 233.  
Nov. An. 1694.*

3. The *Abbot Galloys*, having, in the Year 1693, asserted that Mr. *James Gregory* and Dr. *Barrow* stole their General Propositions concerning the *Transformation of Curves* from Mr. *Robertvall*; Dr. *David Gregory* here fully refutes that Assertion. For Mr. *Gregory* published his Book at *Padua* 1668, and Dr. *Barrow* his *Lectiones Geometricæ* 1674, which Mr. *Robertvall* doubtless had a Sight of before he died (which was not till *October* 1675), yet he never complained of any such Injury done him.

*Cycloidal Spaces  
perfectly Quadrable.  
N. 217. p. 111.  
Oct. An. 1695.*

4. Besides that Segment of the *Semicycloidal Figure*, first observed by Sir *Christopher Wren*, and after him by Mr. *Huygens*, and a *Trilinear Part* of it, which are capable of being Geometrically Squared; Dr. *Wallis* here produces from his *Traëts de Cycloide*, and *de Motu*, some other Portions thereof equally capable of *Quadrature*.

*The Cycloid con-  
sider'd long ago.  
N. 229. p. 561.  
June An. 1697.*

5. Dr. *Wallis* finds among the *Mathematical Works* of *Bovillus*, published at several times between the Years 1501 and 1510, that the Curve (which is now called the *Cycloid*;) was then considered. But he also finds that *Bovillus* was not the first who considered it: For *Cardinal Cusanus*, as appears by an ancient Manuscript of his Works (transcribed by *J. Scoblant* in the Year 1451) had considered it some time before. The Figure indeed (thro' the Unskillfulness of the Transcriber) both in the MS. and the *Basil Edition*, A. 1565, is very ill drawn; but being corrected according to the true Meaning of that *Cardinal's* own Words, it evidently represents the modern *Cycloid*. From hence 'tis manifest, that this Curve was not first taken into Consideration either



either by *Mersennus* or *Galileo*, but some Ages before, tho' never well understood till this present Age.

6. Some Papers sent by Mr. *Jo. Collins* to Dr. *Wallis*, giving his Thoughts about some Defects in Algebra; which he did not live to finish.

*Defects in Algebra.* N. 159.  
p. 575.  
May An. 1684.

XXX. *Accounts of Books, with Editions, Emendations, &c. omitted.*

1. **E**Uclidis *Elementa Geometrica, novo ordine ac methodo demonstrata.* N. 15. p. 261.  
Lond. 1666.
2. *Archimedis Opera; Apollonii Perg: Conic. Libri iv; Theodosii Sphærica, Methodo nova illustrata, & succincte demonstrata: ab Isa. Barrow, R. S. S.* N. 114. p. 314.  
Lond. 1675. in 4to.
3. *Αρχιμήδους τῶν Συρακυσίων Ψαμμίτης, ἡ Κύκλου μέτρησις: Εὐτοκίῃ Ἀσκαλωνίτῃ εἰς αὐτὴν ὑπόμνημα, &c. Cum Versione & Notis Jo. Wallis, S. S. Th. D. Oxon.* N. 123. p. 567.  
1676.
4. *Theon Smyrnæus*, published at Paris by *Ismael Bulialdus* in Greek and Latin. N. 80. p. 3095.
5. *Diophanti Alexandrini Arithmeti corum Libri sex, & de Numeris Multangulis Liber unus; cum Commentariis C. G. Bacheti, & Observationibus D. P. de Fermat, Senatoris Tholosani: cui accessit Doctrinæ Analyticæ Inventum Novum.* Tolosæ 1670. in Folio. N. 72. p. 2185.
6. *The Works of Monsieur de Fermat.* N. 1. p. 15.
7. *Francisci du Laurens Specimina Mathematica, duobus Libris comprehensa. Horum prior Syntheticus agit de Genuinis Matheseos Principiis in genere; in specie autem de veris Geometriæ Elementis hucusque nondum traditis. Posterior Analyticus de Methodo Compositionis atque Resolutionis fuse differit, & multa nova complectitur, quæ subtilissimam Analyseos Artem mirum in modum promouent.* This Book is here censured, some Mistakes in it corrected, and the Censure vindicated, by Dr. *Wallis*.  
N. 34. p. 654.  
N. 38. p. 744.  
N. 39. p. 775.  
N. 41. p. 825.
8. *R. P. Andreae Taquet, è S. J. Opera Mathematica.* Antwerp. 1669. in Folio. N. 43. p. 869.
9. *A Mathematical Compendium, collected out of the Notes and Papers of Sir Jonas Moore, by Nicholas Stevenson.* Lond. 1674. in 12mo. N. 104. p. 83.
10. *R. P. Claudii Franc. Milliet de Chales, è S. J. Cursus seu Mundus Mathematicus, universam Mathesin tribus Tomis complectens.* Lugd. 1674. in Folio. N. 110. p. 229.
11. *The Mathematical Works of Dr. Jo. Wallis, Savilian Professor of Geometry in the University of Oxford, & F. R. S. in three Volumes in Folio.* Oxon. N. 216. p. 73.  
N. 154. p. 259.
12. *An Introduction to Algebra, translated out of High Dutch into English by Tho. Branker, M. A. much altered and augmented by Dr. J. Pell.* Also a Table of such odd Numbers as are less than One Hundred Thousand, shewing those that are *Incomposit*, and resolving the rest into their *Factors*, or *Coefficients.* Lond. in 4to. N. 35. p. 688.
13. *Labyrinthus Algebrae. Auth. Joh. Jac. Ferguson.* 1667. in 4to. N. 49. p. 996.



- N. 90. p. 5152.  
N. 95. p. 6073.  
N. 108. p. 192.  
N. 123. p. 21.  
N. 173. p. 1095.  
N. 233. p. 730.
14. The Elements of that Mathematical Science called *Algebra*; by *Jo. Kersey*. Lond. 1673. in *Folio*.
- N. 14. p. 253.  
N. 16. p. 289.
15. A Treatise of *Algebra*, both Historical and Practical; by *Jo. Wallis*, D. D. In the 109th Chapter there are some Numbers mistaken, which are here rectified by the Author.
16. *De Principiis & Ratiocinatione Geometrarum; contra Fastum Professorum Geometriæ*. Authore *Tho. Hobbes*. This Book is here animadverted on, and answered, by *Dr. Wallis*.
- N. 43. p. 971.
17. *Thomæ Hobbes Quadratura Circuli, Cubatio Sphære, Duplicatio Cubi, confutata*. Auth. *Jo. Wallis*. S. T. D. Oxon. 1669. in *Quarto*.
- N. 55. p. 1121.
18. *Thomæ Hobbes Quadratura Circuli, Cubatio Sphære, Duplicatio Cubi (secundo edita) denuo refutata*. Auth. *Jo. Wallis*. S. T. D. Oxon. 1669.
- N. 72. p. 2185.
19. *Rosetum Geometricum, cum Censura brevi Doctrinæ Wallisianæ de Motu*. Auth. *Tho. Hobbes Malmesburiensi*. Lond. 1671. in *Quarto*. This Book is here answered by *Dr. Wallis*.
- N. 73. p. 2202.
20. Four Papers of *Mr. Hobbs's*, published in the Months of *August* and *September* 1671. which are here answered by — —
- N. 75. p. 2241.
21. *Lux Mathematica, Collisionibus Johannis Wallisii, S. T. D. & Thomæ Hobbesii Malmesburiensis, excussa, multis & fulgentissimis aucta radiis*. Auth. *R. R. Adjuncta Censura Doctrinæ Wallisianæ de Libra, una cum Roseto Hobbesii*. Lond. 1672. in *Quarto*. This Book is here answered by *Dr. Wallis*.
- N. 86. p. 5047.
22. *Principia & Problemata aliquot Geometrica, ante desperata, nunc breviter explicata & demonstrata*. Auth. *T. H. Malmesburiensi*. Lond. 1673. in *Quarto*.
- N. 87. p. 5067.
23. *Le Grand & Fameux Probleme de la Quadrature du Cercle resolu Geometriquement par le Cercle & la Ligne droit, par M. Mallement de Messange*. à Paris. 1686. in *Twelves*. This Book is here refuted by *M. D. Cluverius*. R. S. S.
- N. 97. p. 6131.
24. *Nouveaux Elemens de Geometrie: Or a Mathematical Treatise, entituled New Elements of Geometry*. Paris 1667. in *Quarto*.
- N. 185. p. 245.
25. *Elemens de Geometrie; par le P. Ignace Gaston Pardies, de la Comp. de J. à Paris* 1671. in *Twelves*.
- N. 32. p. 625.
26. 1. *Vera Circuli & Hyperbolæ Quadratura, in propria sua Proportionis Specie inventa & Demonstrata*, à *Jac. Gregorio Scoto*. Patavii. in *Quarto*. This Subject is here further considered, and the Area of an Hyperbole explained; by *Mr. J. Collins*.
- N. 79. p. 3064.
2. *M. Huygens* having published Animadversions upon this Book, in the *Journal de Sçavans*, 1668. *Mr. Gregory* here answers them. To this *M. Huygens* replied in a following Journal of that Year; and *Mr. Gregory*, further to elucidate the Controversy, here returns a second Answer.
- N. 33. p. 640.
3. In the 48th Page of this Book, *Mr. Halley* has discovered and corrected a small Mistake in the Logarithm of 10.
- Ib. p. 641.
27. *Geometriæ pars Universalis, Quantitatum Curvarum Transmutationi & Mensuræ inserviens*. Auth. *Jac. Gregorio Scoto*. Patavii 1668. in *Quarto*.
- N. 37. p. 732.
28. *De Infinitis Spiralibus inversis, Infinitisque Hyperbolis, aliisque Geometricis*. Auth. *F. Stephano de Angelis Veneto*. Patavii. in *Quarto*.
- N. 44. p. 882.
29. *Michaelis*
- N. 216. p. 65.
- N. 35. p. 685.
- N. 37. p. 378.



29. *Michaelis Angeli Ricci Exercitatio Geometrica*. Romæ. in 4to. Reprinted N. 37. p. 738. at London, and annexed to *Mercator's Logarithmotechnia*.
30. *Renati Franc. Slusii Mesolabum. Cui accessit pars altera de Analyfi*, & N. 45. p. 903. *Miscellanea*. Leodii Eburonum 1668. in 4to.
31. *Elementa Geometriæ Planæ. Authore Ægidio Francisco de Gottignies* N. 67. p. 2054. *Bruxellensi. S. J.* Romæ. 1669. in 12mo.
32. *Synopsis Geometrica; cum tribus Opusculis, de Linea Sinuum & Cycloide; de Maximis & Minimis, Centuria; & Synopsis Geometriæ Planæ. Auth. Honor. Fabry. S. J.* Lugduni Galliarum 1669. in 12mo.
33. *Lectiones xiii. Geometricæ; in quibus (præsertim) Generalia Linearum Curvarum Symptomata declarantur, ab Isaaco Barrow.* Lond. 1669. in 4to. To these Lectures the Author here adds several *Corollaries* and *Theorems*.
34. *Erasmi Bartholini Selecta Geometrica.* Hauniæ. 1674. in 4to. N. 106. p. 137.
35. *Elemens des Mathematiques, ou Principes Généraux de toutes les Sciences qui ont les Grandeurs pour Object.* Par J. P. à Paris 1675. in 4to. N. 126. p. 638.
36. *Nouvelle Methode en Geometrie pour les Sections des Superficies Coniques & Cylindriques; qui ont pour Base des Circles, ou des Paraboles, des Ellipses, & des Hyperboliques; par Ph. de la Hire.* à Paris 1673. in 4to. N. 129. p. 745.
37. *De Cycloide & Sectionibus Conicis.* Ph. de la Hire. *Ibid.* p. 746.
38. *The Geometrical Key, or Construction of all Equations, Linear, Quadratic, Cubic, and Biquadratic, by a Circle and one only Parabola; by Mr. Tho. Baker.* N. 157. p. 549.
39. *Exercitatio Geometrica de Dimensione Figurarum. Auth. Davide Gregorio.* Edinb. 1684. in 4to. N. 163. p. 730.
40. *Methodus Figurarum Lineis Reëtis & Curvis comprehensarum Quadraturas determinandi, Auth. J. Craig.* Lond. 1685. in 4to. To this Tract the Author here makes an Addition; and takes notice of some Remarks made on it in the *Act. Lips.* by M. Leibnitz, and M. J. Bernoulli. *Ibid.* p. 186. N. 235. p. 786.
41. *Traëtatus Mathematicus de Figurarum Curvilinearum Quadraturis & Locis Geometricis. Auth. J. Craig.* Lond. 1693. in 4to. N. 209. p. 113.
42. *Traëtatus de Principiis Calculi Exponentialis. Auth. D. Bernoullio;* N. 245. p. 374. wherein a Mistake is here discovered and corrected, by Mr. Craig.
43. *Analysis Geometrica, sive nova & vera Methodus Resolvendi, tam Problemata Geometrica, quam Arithmeticas Quæstiones. Pars prima, de Planis. Auth. D. Antonio Hugone de Omerique Sanlucarense.* N. 257. p. 351.
44. *Stereometrical Propositions, variously applicable, but particularly intended for Gauging; by Rob. Anderson.* Lond. 1668. in 8vo. N. 39. p. 785.
45. *Gauging promoted; being an Appendix to Stereometrical Propositions; by Rob. Anderson.* Lond. 1669. in 8vo. N. 47. p. 960.
46. *Gauging Epitomized; by Mich. Dary.* Lond. 1699. upon one Folio Page. N. 52. p. 1054.
47. *Tabula Numerorum Quadratorum decies Millium, una cum ipsorum Latibus ab Unitate incipientibus, & Ordine Naturali usque ad 10000 progredientibus.* Lond. 1672. N. 82. p. 4050.
48. *The Description and Use of two Arithmetick Instruments, &c. by Sir Sam. Moreland.* Lond. 1673. N. 94. p. 6043.



N. 139. p. 980.

49. Johannis Wallisii S. T. D. *Exercitationes tres: 1. De Cometarum Distantiis investigandis. 2. De Rationum & Fractionum Reductione. 3. De Periodo Juliano.* Lond. 1678.

N. 38. p. 753.

*Ibid.* p. 756.

*Ibid.* p. 759.

50. *Logarithmotechnia* Nicholai Mercatoris. Lond. 1668. in Quarto. This Author's Method of squaring the Hyperbola, and of finding the Sum of the Logarithms is here improved by Dr. Wallis; and further explicated by the Author himself.

## CHAPTER II.

### Trigonometry, Surveying.

*A Chronographical Problem proposed by Mr. Rich. Townley, solved by Mr. John Collins. N. 69. p. 2093. Mar. An. 1674.*

I. Prob. **T**HE Distances of three Objects in the same Plane being given, as *A, B, C*; the Angles made at a fourth Place in the same Plane, as at *S*, are observed: the Distances from the Place of Observation to the respective Objects are required.

The Problem hath six Cases.

Fig. 57.

**C**ASE 1. If the Station be taken without the Triangle made by the Objects, but in one of the Sides thereof produced, as at *S*: find the Angle *ACB*; then in the Triangle *ACS*, all the Angles and the Side *AC* are known; whence either or both the Distances *SA*, or *SC*, may be found.

Fig. 58.

*Case 2.* If the Station be in one of the Sides of the Triangle, as at *S*: then having the three Sides, *AC, CB, BA*, given, find the Angle *CAB*; then again in the Triangle *SAB* all the Angles, and the Side *AB* are known; whence may be found either *AS*, or *SB*, Geometrically; if you make the Angle *CAD* equal to the observed Angle *CSB*, and draw *BS* parallel to *DA*, you determine the Point of Station *S*.

Fig. 59.

*Case 3.* If the three Objects lie in a right Line as *ACD* (suppose it done), and that a Circle passeth through the Station *S*, and the two Exterior Objects *A, B*: then is the Angle *ABD* equal to the observed Angle *ASC* (by 21. 3. *E.*) as inscribing on the same Arch *AD*; and the Angle *BAD* in like manner equal to the observed Angle *CSB*: By this means the Point *D* is determined. Join *DC*, and produce the same, then a Circle passing through the Points *A, B, D*, intersects *DC* produced, at *S*, the Place of Station.

*Calculation.*] In the Triangle *ABD*, all the Angles and the Side *AB*, are known, whence may be found the Side *AD*.

Then in the Triangle *CAD*, the two Sides *CA*, and *AD*, are known, and their contained Angle *CAD* is known; whence may be found the Angles *CDA*, and *ACD*, the Complement whereof to a Semicircle is the Angle *SCA*: in which Triangle the Angles are now all known, and the Side *AC*: whence may be found either of the Distances *SC*, or *SA*.

*Case*



*Case 4.* If the Station be without the Triangle made by the Objects, the Sum of the Angles observed is less than four Right Angles. The *Construction* is the same as in the last *Case*, and the *Calculation* likewise; saving that you must make one Operation more; having the three Sides  $AC$ ,  $CB$ ,  $BA$ , thereby find the Angle  $CAB$ , which add to the Angle  $EAD$ , then you have the two Sides, *viz.*  $AC$ , being one of the Distances, and  $AD$ , (found as in the former *Case*) with their contained Angle  $CAD$ , given, to find the Angles  $CDA$ , and  $ACD$ , the Complement whereof to a Semicircle is the Angle  $SCA$ : Now in the Triangle  $SCA$ , the Angle at  $C$  being found, and at  $S$  observed, and given by Supposition, the other at  $A$  is likewise known, as being the Complement of the two former to a Semicircle, and the Side  $AC$  given; hence the Distances  $CS$ , or  $AS$ , may be found.

Fig. 60.

*Case 5.* If the Place of Station be at some Point within the Plane of the Triangle, made by the three Objects, the *Construction* and *Calculation* are the same as in the last, saving only that instead of the observed Angle  $ASC$ , the Angle  $ABD$  is equal to the Complement thereof to a Semicircle, to wit, it is equal to the Angle  $ASD$ ; both of them insisting on the same Arch  $AD$ : And in like manner the Angle  $BAD$  is equal to the Angle  $DSB$ , which is the Complement of the observed  $CSB$ ; and in this *Case* the Sum of the three Angles observed, is equal to four Right Angles.

Fig. 61.

In these three latter *Cases* no use is made of the Angle observed between the two Objects, as  $A$  and  $B$ , that are made the Base-line of the *Construction*; yet the same is of ready use for finding the third Distance or last Side sought; as in the Triangle  $SAB$ , there is given the Distance  $AB$ , its opposite Angle equal to the Sum of the two observed Angles, and the Angle  $SAB$  attained, as in the fourth *Case*: Hence the third Side, or last Distance  $SB$ , may be found.

Fig. 60.

And here it may be noted, that the three Angles  $CAS$ ,  $ASB$ ,  $SBC$ , are together equal to the Angle  $ACB$ ; for the two Angles  $CSB$  and  $CBS$  are equal to  $ECB$ , as being the Complement of  $SCB$  to two Right Angles; and the like in the Triangle on the other Side. *Ergo, &c.*

*Case 6.* If the three Objects be  $A$ ,  $B$ ,  $C$ , and the Station at  $S$ , as before, it may happen, according to the former *Constructions*, that the Points  $C$  and  $D$  may fall close together, and so a right Line joining them, shall be produced with Uncertainty; in such *Case* the Circle may be conceived to pass through the Place of Station at  $S$ , and any two of the Objects, as through  $B$  and  $C$ ; wherein making the Angle  $DBC$  equal to the observed Angle  $ASC$ , and  $BCD$  equal to the Complement to  $180$  deg. of both the observed Angles in  $DSB$ ; thereby the Point  $D$  is determined, through which, and the Points  $C$ ,  $B$ , the Circle is to be described; and joining  $DA$ , (produced when Need requireth) where it intersects the Circle, as at  $S$ , is the Place of Station sought.

Fig. 62.

This *Problem* may be of good use for the due Situation of Sands and Rocks, that are within sight of three Places upon Land, whose Distances are well known; or for *Chorographical Uses*, &c. especially now there is a Method of observing Angles nicely accurate by the Aid of a Telescope.



Three Chorographic Problems  
 solved by a Member of the Philosophical Society at Oxford.  
 N. 177. p. 1231.  
 Dec. An. 1685.

II. The three following *Problems* may occur at Sea, in finding the Distance and Position of Rocks, Sands, &c. from the Shore; or in Surveying the Sea-Coast; when only two Objects, whose Distance from each other is known, can be seen at one Station: But especially they may be useful to one, that would make a Map of a Country by a Series of Triangles derived from one or more measured Bases; which is the most exact Way of finding the Bearing and Distance of Places from each other, and thence their true Longitude and Latitude; and may consequently occur to one that would in that Matter measure a Degree on the Earth.

Fig. 63.

Prob. 1.] *There are two Objects B and C, whose Distance BC is known; and there are two Stations at A and E, where the Objects B, C, being visible, and the Stations one from another, the Angles BAC, BAE, AEB, AEC, are known by Observation, (which may be made with an ordinary Surveying Semicircle, or Cross-Staff; or, if the Objects are beyond the View of the naked Eye, with a Telescopic Quadrant;) To find the Distances or Lines AB, AC, AE, EC.*

Fig. 64.

*Construction.*] In each of the Triangles, BAE, CAE, two Angles at A, E, being known, the third is also known; then take any Line  $\alpha\epsilon$  at pleasure, on which constitute the Triangles,  $\beta\alpha\epsilon$ ,  $\alpha\epsilon\gamma$ , respectively equiangular to the Triangles BAE, AEC; join  $\beta\gamma$ : Then upon BC constitute the Triangles BCA, BCE, equiangular to the correspondent Triangles  $\beta\gamma\alpha$ ,  $\beta\gamma\epsilon$ , join AE, and the Thing is manifestly done.

*The Calculation.*] Assuming  $\alpha\epsilon$ , of any Number of Parts, in the Triangles  $\alpha\beta\epsilon$ ,  $\alpha\gamma\epsilon$ , the Angles being given, the Sides  $\alpha\beta$ ,  $\alpha\gamma$ ,  $\epsilon\beta$ ,  $\epsilon\gamma$ , may be found by Trigonometry: Then in the Triangle  $\beta\alpha\gamma$ , having the Angle  $\beta\alpha\gamma$ , and the Legs  $\alpha\beta$ ,  $\alpha\gamma$ , we may find  $\beta\gamma$ . Then  $\beta\gamma:BC::\beta\alpha:BA::\beta\epsilon:BE::\gamma\alpha:CA::\gamma\epsilon:CE$ .

Fig. 65.

Prob. 2.] *Three Objects B, C, D, are given, or (which is the same) the Sides and consequent Angles of the Triangle BCD are given; also there are two Points or Stations A, E, such that at A may be seen the three Points B, C, E, but not D, and at the Station E, may be seen A, C, D, but not B; that is, the Angles BAC, BAE, AEC, AED, (and consequently EAC, AEC) are known by Observation: To find the Lines AB, AC, AE, EC, ED.*

Fig. 66.

*Construction.*] Take any Line  $\alpha\epsilon$  at pleasure, and at its Extremities make the Angles  $\epsilon\alpha\gamma$ ,  $\epsilon\alpha\beta$ ,  $\alpha\epsilon\gamma$ ,  $\alpha\epsilon\delta$ , equal to the correspondent observed Angles EAC, EAB, AEC, AED. Produce  $\beta\alpha$ ,  $\delta\epsilon$ , till they meet in  $\phi$ ; join  $\phi\gamma$ : then upon CB describe (according to 33. 3. E.) a Segment of a Circle, that may contain an Angle  $=\gamma\phi\beta$ ; and upon CD describe a Segment of a Circle capable of an Angle  $=\gamma\phi\delta$ : Suppose F the common Section of these 2 Circles; join FB, FC, FD; then from the Point C, draw for the Lines CA, CE, so that the Angle FCA may be  $=\phi\gamma\alpha$ , and FCE  $=\phi\gamma\epsilon$ : so A, E, the common Sections of CA, CE, with FB, FD, will be the Points required, from whence the rest is easily deduced.

*Calculation.*]



*Calculation.*] Assuming  $\alpha \varepsilon$  of any Number, in the Triangles  $\alpha \gamma$ ,  $\alpha \varepsilon$ ,  $\phi \varepsilon$ , all the Angles being given, with the Side  $\alpha \varepsilon$  assumed, the Sides  $\alpha \gamma$ ,  $\varepsilon \gamma$ ,  $\alpha \phi$ ,  $\varepsilon \phi$ , will be known; then in the Triangle  $\gamma \alpha \phi$ , the Angle  $\gamma \alpha \phi$ , with the Legs  $\alpha \gamma$ ,  $\alpha \phi$ , being known, the Angles  $\alpha \phi \gamma$ ,  $\alpha \gamma \phi$ , with the Side  $\phi \gamma$ , will be known: Then as for the rest of the Work, the Triangle BCD having all its Sides and Angles known, and the Angles BFC, BFD, being equal to the found  $\beta \phi \gamma$ ,  $\beta \phi \delta$ ; how to find FB, FC, FD, by *Calculation* (and also *Protraction*) has been already shewn above by Mr. Collins, as to all its Cases.

Sect. 1.

But it must here be noted, that if the Sum of the observed Angles BAE, AED, is 180 deg. then AB, and ED, cannot meet, because they are parallel, and consequently the given Solution cannot take Place; for which Reason I here subjoin another.

*Another Solution.*] Upon BC describe a Segment BAC, of a Circle, so that the Angle of the Segment may be equal to the observed Angle  $\beta \alpha \gamma$ , (which is shewn 33. 3. E.) and upon CD describe a Segment CED, of a Circle, capable of an Angle equal to the observed CED; from C draw the Diameters of these Circles CG, CH; then upon CG describe a Segment of a Circle GFC, capable of an Angle equal to the observed Angle ABC; likewise upon CH, describe a Circle's Segment CFA, capable of an Angle equal to the observed Angle CAE: suppose F the common Section of the two last Circles HFC, GFC; join FH, cutting the Circle HEC in E; join also FG, cutting the Circle GAC in A: I say, that A, E, are the Points required.

Fig. 67.

*Demonstration.*] For the Angle BAC is =  $\beta \alpha \gamma$ , by *Construction* of the Segment; also the Angles CEH, CAG, are right, because each exists in a Semicircle: Therefore a Circle being described upon CF, as a Diameter, will pass thro' E, A; therefore the Angle CAE = CFE = CEH = (by *Construction*) to the observed Angle  $\gamma \alpha \varepsilon$ . In like manner the Angle CEA = CFA = CFG = observ'd Angle  $\gamma \varepsilon \alpha$ .

If the Stations A, E, fall in a right Line with the Point C; the Lines GA, HE, being parallel, cannot meet; but in this Case the Problem is indeterminate, and capable of infinite Solutions. For, as before, upon CG, describe a Segment of a Circle capable of the observed Angle  $\gamma \varepsilon \alpha$ , and upon CH, describe a Segment capable of the observed Angle  $\gamma \alpha \varepsilon$ : then through C draw a Line any way, cutting the Circles in A, E, these Points will answer the Question.

*Problem 3.*] Four Points, B, C, D, F, or the four Sides of a Quadrilateral, with the Angles comprehended, are given; also there are two Stations A and E, such, that at A, only B, C, E, are visible, and at E, only A, D, F; that is, the Angles BAC, BAE, AED, DEF, are given: To find the Places of the two Points A, E; and consequently the Lengths of the Lines AB, AC, AE, ED, EF.

Fig. 68.

*Construction.*] Upon BC (by 33. 3. E.) describe a Segment of a Circle, that may contain an Angle equal to the observed Angle BAC; then from C draw the Chord CM, or a Line cutting the Circle in M, so that the Angle BCM, may be equal to the Supplement of the observed Angle BAE, i. e. its Residue



to 180 deg. In like manner on DF describe a Segment of a Circle, capable of an Angle equal to the observed DEF; and from D draw the Chord DN, so that the Angle FDN may be equal to the Supplement of the observed Angle AEF; join MN, cutting the two Circles in A, E: I say, A, E, are the two Points required.

*Demonstration.*] Join AB, AC, ED, EF; then is the Angle MAB = BCM (by 21. 3. E.) = Supplement of the observed Angle BAE, by *Construction*; therefore the constructed Angle BAE, is equal to that which was observ'd. Also the Angle BAC, of the Segment, is, by *Construction* of the Segment, equal to the observed Angle BAC. In like manner the constructed Angles AEF, DEF, are equal to the correspondent observed Angles AEF, and DEF; therefore A, E, are the Points required.

*Calculation.*] In the Triangle BCM, the Angle BCM, (= Supplement of BAE) and Angle BMC, (= BAC) are given, with the Side BC; thence MC may be found: in like manner DN, in the Triangle DNF, may be found. But the Angle MCD (= BCD - BCM) is known, with its Legs MC, CD; therefore its Base MD, and Angle MDC, may be known. Therefore the Angle MDN (= CDF - CDM - EDN) is known, with its Legs MD, DN; thence MN, with the Angles DMN, DNM, will be known. Then the Angle CMA (= DMC + DMN) is known, with the Angle MAC (= MAB + BAC) and MC, before found; therefore MA, and AC, will be known. In like manner in the Triangle EDN, the Angles E, N, with the Side DN, being known, the Sides EN, ED, will be known; therefore AE (= MN - MA - EN) is known. Also in the Triangle ABC, the Angle A, with its Sides BC, CA, being known, the Side AB will be known, with the Angle BCA; so in the Triangle EFD, the Angle E, with the Sides ED, DF, being known, EF will be found, with the Angle EDF. Lastly, in the Triangle ACD, the Angle ACD (= BCD - BCA) with its Legs AC, CD, being known, the Side AD will be known; and in like manner EC, in the Triangle EDC.

*Note,* That in this *Problem*, as also in the first and second, if the two Stations fall in a right Line with either of the given Objects, the *Locus* of A or E being a Circle, the particular Point of A or E cannot be determined from the Things given.

As to the other *Cases* of this third *Problem*, wherein A and E may shift Places, *i. e.* only D, F, E, may be visible at A, and only A, B, C, at E; or wherein B, D, E, may be visible at A, and only C, F, A, at E; or wherein A may be on one side of the Quadrilateral, and E on the other; or one of the Stations within the Quadrilateral, and the other without it; I presume that the *Surveyor* will easily direct himself, by what has been already said.

The *Solution* of this third *Problem* is general, and serves also for both the precedent. For suppose C, D, the same Point in the last Figure, and it gives the *Solution* of the second *Problem*; but if B, C, be supposed the same Points with D, F, by proceeding as in the last, you may directly solve the first *Problem*.



III. The Variation of the magnetic Needle is so commonly known, that I need not insist much on the Explication thereof; 'tis certain that the true solar Meridian, and the Meridian shewn by a Needle, agree but in very few Places of the World; and this too, but for a little Time (if a Moment) together; the Difference between the true Meridian and magnetic Meridian perpetually varying and changing in all Places, and at all Times; sometimes to the Eastward, and sometimes to the Westward.

*An Error of  
Common Survey-  
ors, in comparing  
Surveys taken at  
long Intervals of  
Time with the  
magnetic Nee-  
dle, demonst-  
rated; by Mr.  
William Moly-  
neux. N. 230.  
p. 625.  
July An. 1697.*

On which Account 'tis impossible to compare two *Surveys* of the same Place, taken at distant Times, by magnetic Instruments (such as the Circumferentor, by which the *Down Survey*, or Sir *William Petty's Survey of Ireland* was taken) without due Allowance be made for this Variation. To which purpose, we ought to know the Difference between the magnetic Meridian and true Meridian, at that time of the *Down Survey*, and the said Difference at the Time when we make a *New Survey* to compare with the *Down Survey*.

But here I would not be understood, as if I proposed hereby to shew, that a Map of the same Place, taken by magnetic Instruments at never so distant Times, should not at one Time give the same Figure and Contents as at another Time. This certainly it will do most exactly, the Variation of the Needle having nothing to do either in the Shape or Contents of the *Survey*. All that is affected thereby, is the Bearings of the Lines run by the Chain, and the Boundaries between Neighbours. And how this may cause a considerable Error (unless due Allowance be made for it) is what I shall prove most fully.

In order to which, let us suppose that about the Year 1657 (at which time the *Down Survey* was taken) the magnetic Meridian and true Meridian did agree at *Dublin*, or pretty nigh all over *Ireland*; that is to say, that there was no Variation. And indeed by Experiment it was at that time found, as I am well assured, that at *Dublin* it was hardly half a Degree.

Let us suppose, that in the Year 1695 the Variation was 7 Degrees from the North to the Westward: That it was really so, I believe, I am pretty well assured, from an Experiment made by *myself* with all Diligence. But this is not material; let us now only suppose it.

Let A, B, represent the Survey of two Town Lands, one in the Possession of A, and the other in Possession of B, taken by the *Down Survey*, Anno 1657, when there was no Variation.

Fig. 69.

Let the Line N S, running through the Point P, be the true Meridian, and consequently the magnetic Meridian also at that Time, because of the supposed no Variation; and let this Line N S, be also the Boundary between the two Town Lands A, and B.

In the Year 1695, when the Variation is 7 Degrees from the North to the Westward, B having a Map of the *Down Survey*, and being suspicious that his Neighbour A had encroached on him by a Ditch P Q, employs a Surveyor to enquire into the Matter: The Surveyor finds by his Map, that the Boundary between B and his Neighbour A, run from the Point P, through a Meadow, directly according to the magnetic Meridian S P N; but observing the Ditch P Q cast up much to the Eastward of the present magnetic Meridian, he concludes that A has encroached upon B, and that the Ditch ought to have

been



been cast up along the Line  $Pq$ , the Angle  $QPq$  being an Angle of 7 deg. that is, the present Variation of the Needle, and the Line  $Pq$ , the present magnetic Meridian; for which Variation not making any Allowance, he positively determines that B has all the Land in the Triangle  $QPq$ , more than he ought to have; and that his Ditch ought to run along the Line  $Pq$ .

'Tis true indeed, if the *Surveyor* go the whole Surround of the Land A, and B, he will find their Figure and Contents exactly agreeable to the Map here expressed. But then the Bearings of the Lines are all 7 deg. different from the Bearings in the Map, and they will run in and out upon the adjacent neighbouring Lands, and cause endless Differences between their Possessors; as is manifest from the Figure: Wherein the prick'd Lines represent the Disagreement in the Bearings of the Lines, protracted from the Point P; and we see A encroaching upon his Neighbours on the Westward, as he encroaches upon B, and B's Eastward Neighbours encroaching on him, and so forward and clear round. Whereas by a due Allowance for the Variation of the Needle, all this Confusion and Disagreement is avoided, and every thing hits right.

Thus, for Instance, in the Case before us, knowing that the magnetic Variation has caused the present magnetic Meridian to fall in the Line  $nqPs$ , 7 deg. from the North to the Westward; to reduce this to the magnetic Meridian at the Time of the *Down Survey*, I must make the Meridian of my Map to fall 7 deg. to the Eastward of my magnetic Meridian, as we see the Line  $PQ$  falls 7 deg. to the Eastward of the Line  $Pq$ .

What is here said on Supposition that the Magnet had no Variation at the Time of the first Survey taken, and that it had 7 deg. Variation Westward at the Time of the second Survey, may easily be accommodated to the Supposal of any other Variations at the first and second Surveys, *mutatis mutandis*; for knowing the Variations we know their Difference; and if we know their Difference, this gives us the Angle  $QPq$ , by which we reduce them to each other. The best Way therefore to make Maps invariable, constant, and everlasting, were for the Surveyors, who use magnetic Instruments, to make always Allowance for the magnetic Variation, and to protract and lay down Plats by the true Meridian.

Perhaps it may be objected, That Surveys may be taken without magnetic Instruments, and that therefore this Error arising from the magnetic Variation, and Change of the Bearing of Lines, may be avoided. To which I answer, First, That granting a Survey may be taken without magnetic Instruments, this is nothing against what we have laid down, relating to Surveys that are taken with magnetic Instruments, as the *Down Survey* actually was, and most *Surveys* at present actually are taken therewith. Secondly, Tho' a Survey may be taken truly without magnetic Instruments, so as to shew the exact Angles and Lines of the Plat, and consequently the true Contents; yet this will not give the true Bearings of the Lines, or shew my Position in relation to my Neighbours, or other Parts of the Country. This must be supplied by the Magnet, or something equivalent thereto, as finding a true meridian Line on your Land by celestial Observation. And I doubt not but the ancient *Ægyptians*, before the Discovery of the Magnet, were forced to some such Expedient in their Surveys



Surveys and Applotments of Lands, between Neighbour and Neighbour, after the *Inundations* of the *Nile*; which, we are told, gave the first Original to *Geometry* and *Surveying*: Absolute Necessity and Use having introduced these, as Delight and Diversion introduced *Astronomy* amongst the *Chaldeans*.

And this brings me to another Objection, which may be made against the Instance before laid down: It may be said, That certainly the Surveyor which B employed was very ignorant, who would chuse to judge of the Line P Q rather by its Bearing, than by determining the Point Q by measuring from H and G. To this I answer, What if both the Points H and G were vanished since the *Down Survey* was taken? What if the whole Face of the Country were changed, save only the Point P, and the Line P Q? How shall the Surveyor then judge of the Line P Q, but by its Bearing? That this is no extravagant Supposition, we have an Example in *Ægypt* above-mentioned, where the *Nile* lays all flat before it, and so uniformly covers all with Mud, that there is no Distinction. In such a Case your Bearing must certainly help you out; there is no other Way.

But I answer, secondly, To say that the Surveyor might have determined the Point Q by Admeasurement from G and H, or any other adjoining noted Points, as from F, K, I, &c. 'tis very true; but then 'tis against our *Supposition*. I am upon shewing an Error that arises from judging of the Line P Q by *magnetic Bearing*; and to tell me that this might be avoided by another Way, is to say nothing. I myself shew how it may be avoided, by allowing for the Variation; but still it is an Error till it be avoided.

But, thirdly, If B's Surveyor do not allow for the Variation of the Needle, he will never exactly determine even the Points G, F, H, K, &c. or any other Points in the Plat; but instead thereof, will fall on the Points *g, b, f, k*.

From what has been laid down, we may see the absolute Necessity of allowing for the Variation of the Magnet, in comparing old Surveys with new ones; for want of which, great Disputes may arise between neighbouring Proprietors of Lands: And it were to be wished, that our honourable and learned Judges would take this Matter into their Consideration, whenever any Business of this Kind comes before them.

IV. I have invented a *Level* with a Tube, with Glassess and a Thread, hanging between four Points, with a Weight in a Box so contrived, that as soon as the Instrument is set down, you have your *Point* of Horizon with a great deal of Exactness. I am making another, which playeth on one Steel Point, standing on a Diamond.

*A new Level;*  
by Mr. Butter-  
field. N. 141.  
p. 1026.  
Sept. An. 1678.

V. *An Account of a Book omitted; viz. The Art of Levelling; by M. N. 74. p. 2217.*  
*Mariotte.*



## C H A P. III.

## O P T I C S.

*A new Theory  
about Light and  
Colours; by Mr.  
Isaac Newton.  
N. 80. p. 3075.  
Feb. An. 1672.*

I. **I**N the Year 1666 (at which time I applied myself to the Grinding of Optic Glasses of other Figures than Spherical) I procured me a Triangular Glass-Prism, to try therewith the celebrated Phænomena of Colours. And in order thereto, having darkened my Chamber, and made a small Hole in my Window-shuts, to let in a convenient Quantity of the Sun's Light, I placed my Prism at its Entrance, that it might be thereby refracted to the opposite Wall. It was at first a very pleasing Divertisement, to view the vivid and intense Colours produced thereby; but after a while applying myself to consider them more circumspectly, I became surpris'd to see them in an oblong Form; which, according to the received Laws of Refractions, I expected should have been circular. They were terminated at the Sides with streight Lines, but at the Ends the Decay of Light was so gradual, that it was difficult to determine justly what was their Figure, yet they seem'd Semicircular.

Comparing the Length of this coloured *Spectrum* with its Breadth, I found it about five Times greater; a Disproportion so extravagant, that it excited me to a more than ordinary Curiosity of examining from whence it might proceed. I could scarce think, that the various Thickness of the Glass, or the Termination with Shadow or Darknes, could have any Influence on Light to produce such an Effect; yet I thought it not amiss, first to examine those Circumstances, and so tried what would happen by transmitting Light through Parts of the Glass of divers Thicknesses, or through Holes in the Window of divers Bignesses, or by setting the Prism without, so that the Light might pass through it, and be refracted, before it was terminated by the Hole: But I found none of those Circumstances material. The Fashion of the Colours was in all these Cases the same.

Then I suspected, whether by any Unevenness in the Glass, or other contingent Irregularity, these Colours might be thus dilated. And to try this, I took another Prism like the former, and so placed it, that the Light passing thro' them both, might be refracted contrary ways, and so by the latter returned into that Course from which the former had diverted it: For by this means I thought the regular Effects of the first Prism would be destroyed by the second Prism, but the irregular Ones more augmented, by the Multiplicity of Refractions. The Event was, that the Light, which by the first Prism was diffused into an oblong Form, was by the second reduced into an orbicular One, with as much Regularity as when it did not at all pass through them.

N. 83. p. 4061.

Fig. 70.

That this Experiment may be better apprehended; let E G design the Window; F, the Hole in it, thro' which the Light arrives at the Prisms; A B C, the first Prism, which refracts the Light towards P T, painting there the



the Colour in an Oblong, and  $\alpha\beta\gamma$ , the second Prism, which refracts back again the Rays to Q, where the long Image P T is contracted into a round one. I suppose the Plane  $\alpha\gamma$  parallel to B C, and  $\beta\gamma$  to A C, that the Rays may be equally refracted contrary ways in both Prisms. The Prisms also must be placed very near to one another; for if their Distance be so great, that the Colours begin to appear in the Light, before its Incidence on the second Prism, those Colours will not be destroyed by the contrary Refractions of that Prism. And if a *Lens* be placed in the Hole F, or immediately after the Prisms, so that its *Focus* be at the Image Q, or P T, the Perimeter of the Image Q, and the straight Sides of the Image P T, will become much better defined than otherwise. So that, whatever was the Cause of that Length, 'twas not any contingent Irregularity.

N. 80. p. 3076.  
Feb. An. 1672.

I then proceeded to examine more critically, what might be effected by the Difference of the Incidence of Rays coming from divers Parts of the Sun; and to that end, measured the several Lines and Angles belonging to the Image. Its Distance from the Hole or Prism was 22 Feet; its utmost Length  $13\frac{1}{4}$  Inches; its Breadth  $2\frac{5}{8}$ ; the Diameter of the Hole  $\frac{1}{4}$  of an Inch. The Angle which the Rays, tending towards the middle of the Image, made with those Lines, in which they would have proceeded without Refraction, was 44 deg. 56 min. and the Vertical Angle of the Prism, 63 deg. 12 min. Also the Refractions on both Sides the Prism, that is, of the Incident and Emergent Rays, were, as near as I could make them, equal, and consequently about 54 deg. 4 min. And the Rays fell perpendicularly upon the Wall. Now subducting the Diameter of the Hole from the Length and Breadth of the Image, there remains 13 Inches in the Length, and  $2\frac{3}{8}$  the Breadth, comprehended by those Rays, which passed through the Center of the said Hole; and consequently the Angle of the Hole, which that Breadth subtended, was about 31 min. answerable to the Sun's Diameter; but the Angle which its Length subtended, was more than five such Diameters, namely 2 deg. 49 min.

Having made these Observations, I first computed from them the refractive Power of that Glass, and found it measured by the Ratio of the Sines, 20 to 31; and then by that Ratio I computed the Refractions of two Rays flowing from opposite Parts of the Sun's *Discus*, so as to differ 31 min. in their Obliquity of Incidence, and found that the emergent Rays should have comprehended an Angle of about 31 min. as they did before they were incident.

But because this Computation was founded on the Hypothesis of the Proportionality of the Sines of Incidence and Refraction, which though by my own Experience I could not imagine to be so erroneous, as to make that Angle but 31 min. which in reality was 2 deg. 49 min. yet my Curiosity caused me again to take my Prism: And having placed it at my Window, as before, I observed, that by turning it a little about its Axis to and fro, so as to vary its Obliquity to the Light, more than an Angle of 4 or 5 Degrees, the Colours were not thereby sensibly translated from their Place on the Wall; and consequently by that Variation of Incidence, the Quantity of Refraction was not sensibly varied. By this Experiment therefore, as well as by the former

Com-



Computation, it was evident, that the Difference of the Incidence of Rays, flowing from divers Parts of the Sun, could not make them after Decussation diverge at a sensibly greater Angle, than that at which they before converged; which being, at most, but about 31 or 32 min. there still remained some other Cause to be found out, from whence it could be 2 deg. 49 min.

Then I began to suspect, whether the Rays, after their Trajection through the Prism, did not move in curve Lines, and according to their more or less Curvity tend to divers Parts of the Wall. And it increased my Suspicion, when I remember'd that I had often seen a Tennis-Ball struck with an oblique Racket, describe such a curve Line. For, a circular as well as a progressive Motion being communicated to it by that Stroke, its Parts on that Side, where the Motions conspire, must press and beat the contiguous Air more violently than on the other, and there excite a Reluctancy and Re-action of the Air proportionably greater. And for the same Reason, if the Rays of Light should possibly be globular Bodies, and by their oblique Passage out of one Medium into another acquire a circulating Motion, they ought to feel the greater Resistance from the ambient Æther on that Side where the Motions conspire, and thence be continually bowed to the other. But notwithstanding this plausible Ground of Suspicion, when I came to examine it, I could observe no such Curvity in them. And besides (which was enough for my purpose) I observed, that the Difference betwixt the Length of the Image, and the Diameter of the Hole through which the Light was transmitted, was proportionable to their Distance.

The gradual Removal of these Suspicions at length led me to the *Experimentum Crucis*, which was this; I took two Boards, and placed one of them close behind the Prism at the Window, so that the Light might pass through a small Hole, made in it for the purpose, and fall on the other Board, which I placed at about 12 Feet distance, having first made a small Hole in it also for some of that incident Light to pass through. Then I placed another Prism behind this second Board, so that the Light trajected through both the Boards might pass through that also, and be again refracted before it arrived at the Wall. This done, I took the first Prism in my Hand, and turned it to and fro slowly about its Axis, so much as to make the several Parts of the Image, cast on the second Board, successively pass through the Hole in it, that I might observe to what Places on the Wall the second Prism would refract them. And I saw by the Variation of those Places, that the Light, tending to that End of the Image towards which the Refraction of the first Prism was made, did in the second Prism suffer a Refraction considerably greater than the Light tending to the other End. And so the true Cause of the Length of that Image was detected to be no other, than that *Light* is not similar or homogeneous, but consists of *difform Rays, some of which are more refrangible than others*; so that without any Difference in their Incidence on the same Medium, some shall be more refracted than others; and therefore that, according to their *particular Degrees of Refrangibility*, they were transmitted through the Prism to divers Parts of the opposite Wall.

*Ibid.* p. 3081.



I shall now proceed to acquaint you with another more notable *Difformity* in its Rays, wherein the Origin of Colours is unfolded: Concerning which I shall lay down the Doctrine first; and then, for its Examination, give you an Instance or two of the Experiments, as a Specimen of the rest.

The Doctrine you will find comprehended and illustrated in the following Propositions.

1. As the Rays of Light differ in Degrees of Refrangibility, so they also differ in their Disposition to exhibit this or that particular Colour. Colours are not Qualifications of Light, derived from Refractions, or Reflections of natural Bodies (as 'tis generally believed) but original and connate Properties, which in divers Rays are divers. Some Rays are disposed to exhibit a Red Colour, and no other; some a Yellow, and no other; some a Green, and no other; and so of the rest. Nor are there only Rays proper and particular to the more eminent Colours, but even to all their intermediate Gradations.

2. To the same Degree of Refrangibility ever belongs the same Colour, and to the same Colour ever belongs the same Degree of Refrangibility. The least refrangible Rays are all disposed to exhibit a Red Colour; and contrarily, those Rays which are disposed to exhibit a Red Colour, are all the least refrangible: So the most refrangible Rays are all disposed to exhibit a deep Violet Colour; and contrarily, those which are apt to exhibit such a Violet Colour, are all the most refrangible: And so to all the intermediate Colours in a continued Series belong intermediate Degrees of Refrangibility. And this Analogy betwixt Colours and Refrangibility is very precise and strict; the Rays always either exactly agreeing in both, or proportionally disagreeing in both.

3. The Species of Colour, and Degree of Refrangibility proper to any particular Sort of Rays, is not mutable by Refraction, nor by Reflection from natural Bodies, nor by any other Cause that I could yet observe. When any one Sort of Rays hath been well parted from those of other Kinds, it hath afterwards obstinately retained its Colour, notwithstanding my utmost Endeavours to change it. I have refracted it with Prisms, and reflected it with Bodies, which in Day-light were of other Colours; I have intercepted it with the coloured Film of Air, interceding two compressed Plates of Glass, transmitted it through coloured Mediums, and through Mediums irradiated with other Sorts of Rays, and diversly terminated it; and yet could never produce any new Colour out of it. It would by contracting or dilating become more brisk, or faint, and, by the Loss of many Rays, in some Cases very obscure and dark; but I could never see it changed in Specie.

4. Yet seeming Transmutations of Colours may be made, where there is any Mixture of divers Sorts of Rays: For in such Mixtures, the component Colours appear not; but, by their mutual allaying each other, constitute a middling Colour. And therefore, if by Refraction, or any other of the aforesaid Causes, the difform Rays, latent in such a Mixture, be separated, there shall emerge Colours different from the Colour of the Composition. Which Colours are not new generated, but only made apparent by being parted; for



if they be again intirely mixed and blended together, they will again compose that Colour, which they did before Separation. And for the same Reason, Transmutations made by the convening of divers Colours are not real; for when the difform Rays are again severed, they will exhibit the very same Colours which they did before they entered the Composition; as you see Blue and Yellow Powders, when finely mixed, appear to the naked Eye, Green; and yet the Colours of the component Corpuscles are not thereby really transmuted, but only blended. For when viewed with a good Microscope, they still appear Blue and Yellow interspersedly.

5. There are therefore two sorts of Colours, the one Original and Simple, the other compounded of these. The original or primary Colours are, Red, Yellow, Green, Blue, and a Violet-Purple, together with Orange, Indico, and an indefinite Variety of intermediate Gradations.

6. The same Colours in Specie with these primary Ones, may be also produced by Composition. For a Mixture of Yellow and Blue makes Green; of Red and Yellow makes Orange; of Orange and Yellowish Green makes Yellow. And in general, if any two Colours be mixed, which in the Series of those generated by the Prism are not too far distant one from another, they by their mutual Alloy compound that Colour, which in the said Series appeareth in the Midway between them. But those which are situated at too great a Distance, do not so. Orange and Indico produce not the intermediate Green, nor Scarlet and Green the intermediate Yellow.

7. But the most surprizing and wonderful Composition was that of Whiteness. There is no one sort of Rays which alone can exhibit this. 'Tis ever compounded; and to its Composition are requisite all the aforesaid primary Colours, mixed in a due Proportion. I have often with Admiration beheld, that all the Colours of the Prism being made to converge, and thereby to be again mixed, as they were in the Light before it was incident upon the Prism, reproduced Light, entirely and perfectly White, and not at all sensibly differing from a direct Light of the Sun, unless when the Glasses I used were not sufficiently clear; for then they would a little incline it to their Colour.

8. Hence therefore it comes to pass, that Whiteness is the usual Colour of Light; for Light is a confused Aggregate of Rays indued with all sorts of Colours, as they were promiscuously darted from the various Parts of luminous Bodies. And of such a confused Aggregate, as I said, is generated Whiteness, if there be a due Proportion of the Ingredients; but if any one predominate, the Light must incline to that Colour; as it happens in the blue Flame of Brimstone; the yellow Flame of a Candle; and the various Colours of the Fixed Stars.

9. These Things considered, the Manner how Colours are produced by the Prism is evident. For, of the Rays, constituting the incident Light, since those which differ in Colour proportionally differ in Refrangibility, they by their unequal Refractions must be severed and dispersed into an oblong Form in an orderly Succession, from the least refracted Scarlet, to the most refracted Violet. And for the same Reason it is, that Objects when looked upon through a Prism, appear coloured. For the difform Rays, by their unequal Re-



Refractions, are made to diverge towards several Parts of the *Retina*, and there express the Images of things coloured, as in the former Case they did the Sun's Image upon a Wall. And by this Inequality of Refractions, they become not only coloured, but also very confused and indistinct.

10. Why the Colours of the Rainbow appear in falling Drops of Rain, is also from hence evident. For those Drops which refract the Rays, disposed to appear Purple, in greatest Quantity to the Spectator's Eye, refract the Rays of other sorts so much less, as to make them pass beside it; and such are the Drops on the Inside of the primary Bow, and on the Outside of the secondary or exterior One. So those Drops, which refract in greatest Plenty the Rays, apt to appear Red, toward the Spectator's Eye, refract those of other sorts so much more, as to make them pass beside it; and such are the Drops on the exterior Part of the primary, and interior Part of the secondary Bow.

11. The odd Phænomena of an Infusion of *Lignum Nephriticum*, Leaf-Gold, Fragments of coloured Glass, and some other transparently coloured Bodies, appearing in one Position of one Colour, and of another in another, are on these Grounds no longer Riddles. For those are Substances apt to reflect one sort of Light, and transmit another; as may be seen in a dark Room, by illuminating them with similar or uncompounded Light. For then they appear of that Colour only, with which they are illuminated; but yet in one Position more vivid and luminous than in another, accordingly as they are disposed more or less to reflect or transmit the incident Colour.

12. From hence also is manifest the Reason of an unexpected Experiment, which Mr. *Hook*, somewhere in his *Micrography*, relates to have made with two Wedge-like transparent Vessels, filled the one with a Red, the other with a Blue Liquor: namely, that though they were severally transparent enough, yet both together became opaque; for if one transmitted only Red, and the other only Blue, no Rays could pass through both.

13. I might add more Instances of this Nature, but I shall conclude with this general one: That the Colours of all natural Bodies have no other Origin than this, that they are variously qualified to reflect one sort of Light in greater Plenty than another. And this I have experimented in a dark Room, by illuminating those Bodies with uncompounded Light of divers Colours. For by that means any Body may be made to appear of any Colour. They have there no appropriate Colour, but ever appear of the Colour of the Light cast upon them; but yet with this Difference, that they are most brisk and vivid in the Light of their own Day-light Colour. *Minium* appeareth there of any Colour indifferently, with which it is illustrated, but yet most luminous in Red; and so *Bise* appeareth indifferently of any Colour, with which it is illustrated, but yet most luminous in Blue: and therefore *Minium* reflecteth Rays of any Colour, but most copiously those endowed with Red, and consequently when illustrated with Day-light, that is, with all sorts of Rays promiscuously blended, those qualified with Red shall abound most in the reflected Light, and by their Prevalence cause it to appear of that Colour. And for the same Reason *Bise*, reflecting Blue most copiously, shall appear Blue by the Excess of those Rays in its reflected Light; and the like of other Bodies. And that this is the en-



ture and adequate Cause of their Colours, is manifest, because they have no Power to change or alter the Colours of any sort of Rays incident apart, but put on all Colours indifferently, with which they are enlighten'd.

These things being so, it can be no longer disputed, whether there be Colours in the Dark, or whether they be the Qualities of the Objects we see; no, nor perhaps, whether Light be a Body. For, since Colours are the Qualities of Light, having its Rays for their intire and immediate Subject, how can we think those Rays Qualities also, unless one Quality may be the Subject of, and sustain another? which, in effect, is to call it Substance. We should not know Bodies for Substances, were it not for their sensible Qualities; and the Principal of those being now found due to something else, we have as good Reason to believe that to be a Substance also.

Besides, Who ever thought any Quality to be a heterogeneous Aggregate, such as Light is discovered to be? But to determine more absolutely what Light is, after what Manner refracted, and by what Modes or Actions it produceth in our Minds the Phantasms of Colours, is not so easy: And I shall not mingle Conjectures with Certainties.

Reviewing what I have written, I see the Discourse itself will lead to divers Experiments sufficient for its Examination: And therefore I shall not trouble you further than to describe one of those which I have already insinuated.

In a darkened Room make a Hole in the Shut of a Window, whose Diameter may conveniently be about a third Part of an Inch, to admit a convenient Quantity of the Sun's Light: And there place a clear and colourless Prism, to refract the entring Light towards the further part of the Room; which, as I said, will thereby be diffused into an oblong coloured Image. Then place a Lens of about 3 Foot Radius (suppose a broad Object-Glass of a three Foot Telescope) at the Distance of about 4 or 5 Feet from thence, through which all those Colours may at once be transmitted, and made by its Refraction to convene at a further Distance of about 10 or 12 Feet. If at that Distance you intercept this Light with a Sheet of white Paper, you will see the Colours converted into Whiteness again by being mingled. But it is requisite that the Prism and Lens be placed steddy, and that the Paper, on which the Colours are cast, be moved to and fro; for by such Motion you will not only find at what Distance the Whiteness is most perfect, but also see how the Colours gradually convene and vanish into Whiteness; and afterwards, having crossed one another in that Place where they compound Whiteness, are again dissipated and severed, and in an inverted Order retain the same Colours which they had before they entred the Composition. You may also see, that if any of the Colours at the Lens be intercepted, the Whiteness will be changed into the other Colours. And therefore, that the Composition of Whiteness be perfect, Care must be taken that none of the Colours fall beside the Lens. Thus in the Design of this Experiment, A B C expresseth the Prism set endwise to sight, close by the Hole F, of the Window E G. Its vertical Angle A C B may conveniently be about 63 deg. M N designeth the Lens. Its Breadth  $2\frac{1}{2}$ , or 3 Inches. S F, one of the straight Lines, in which *difform Rays* may be conceived to flow successively from the Sun. F P, and F R, two  
of



of those Rays unequally refracted, which the Lens makes to converge towards Q, and after Decussation to diverge again. And HI, the Paper, at divers Distances, on which the Colours are projected; which in Q constitute Whiteness, but are Red and Yellow in R,  $r$ , and  $e$ , and Blue and Purple in P,  $p$ , and  $\pi$ .

If you proceed further to try the Impossibility of changing any uncompounded Colour (which I have asserted in the third and thirteenth Propositions) 'tis requisite that the Room be made very dark, lest any scattering Light, mixing with the Colour, disturb and allay it, and render it compound, contrary to the Design of the Experiment. 'Tis also requisite, that there be a perfecter Separation of the Colours, than, after the Manner above described, can be made by the Refraction of one single Prism; and how to make such further Separations, will scarce be difficult to them that consider the discovered Laws of Refractions. But if Trial shall be made with Colours not thoroughly separated, there must be allowed Changes proportionable to the Mixture. Thus, if compound Yellow Light fall upon the Blue Bise, the Bise will not appear perfectly Yellow, but rather Green; because there are in the yellow Mixture many Rays endued with Green, and Green being less remote from the usual Blue Colour of Bise than Yellow, is the more copiously reflected by it.

In like manner, if any one of the prismatic Colours, suppose Red, be intercepted, on Design to try the asserted Impossibility of reproducing that Colour out of the others which are pretermitted; 'tis necessary, either that the Colours be very well parted before the Red be intercepted, or that together with the Red, the neighbouring Colours, into which any Red is secretly dispersed (that is, the Yellow, and perhaps Green too) be intercepted; or else, that Allowance be made for the emerging of so much Red out of the Yellow-Green, as may possibly have been diffused, and scatteringly blended in those Colours. And if these Things be observed, the new Production of Red, or any intercepted Colour, will be found impossible.

II. 1. To contract the Beams of the Sun without the Hole of the Window, and to place the Prism between the Focus of the Lens and the Hole.

2. To cover over both Ends of the Prism with Paper at several Distances from the Middle; or with moveable Rings, to see how that will vary or divide the Length of the Figure.

3. To move the Prism so, as the End may turn about, the Middle being steady.

4. To move the Prism by shoving it, till first the one Side, then the Middle, then the other Side pass over the Hole, observing the same Parallelism.

2. I suppose the Design of the Proposer of these Experiments is, to have their Events expressed, with such Observations as may occur concerning them.

Touching the first, I have observed, that the solar Image falling on a Paper placed at the Focus of the Lens, was by the interposed Prism drawn out in Length proportional to the Prism's Refraction or Distance from that Focus. And the chief Observable here, which I remember, was, that the streight Edges of the oblong Image were distincter than they would have been without the Lens.

*Some Experiments proposed in relation to this Theory.*  
N. 83. p. 4059.  
May, An. 1672.

*Observations on this Proposal by Mr. Newton.*  
N. 83. p. 4060.  
May, An. 1672.



Considering that the Rays coming from the Planet *Venus*, are much less inclined one to another, than those which come from the opposite Parts of the Sun's Disk; I once tried an Experiment or two with her Light. And to make it sufficiently strong, I found it necessary to collect it first by a broad Lens; and then interposing a Prism between the Lens and its Focus, at such Distance that all the Light might pass through the Prism, I found the Focus, which before appeared like a lucid Point, to be drawn out into a long splendid Line by the Prism's Refraction.

Concerning the *second* Experiment, I have occasionally observed, that by covering both Ends of the Prism with Paper at several Distances from the Middle, the Breadth of the solar Image will be increased or diminished as much, as is the Aperture of the Prism, without any Variation of the Length: Or, if the Aperture be augmented on all Sides, the Image on all Sides will be so much and no more augmented.

Of the *third* Experiment I have occasion to speak in my Answer to another Person; where you will find the Effects of two Prisms, in all cross Positions of one to another, described. But if one Prism alone be turned about, the coloured Image will only be translated from Place to Place, describing a Circle, or some other conic Section on the Wall, on which it is projected, without suffering any Alteration in its Shape, unless such as may arise from the Obliquity of the Wall, or casual Change of the Prism's Obliquity to the Sun's Rays.

The Effect of the *fourth* Experiment I have already insinuated, telling you that Light passing through Parts of the Prism of divers Thicknesses, did still exhibit the same Phænomena.

The Genuine  
Method of exam-  
ining this Theo-  
ry; by Mr.  
Newton. N. 85.  
p. 5004.  
July, An. 1672.

III. I cannot think it effectual for determining Truth, to examine the several Ways by which Phænomena may be explained, unless where there can be a perfect Enumeration of all those Ways. You know, the proper Method for enquiring after the Properties of Things, is to deduce them from Experiments. And I told, you that the *Theory* which I propounded, was evinced to me, not by inferring 'tis thus, because 'tis not otherwise; that is, not by deducing it only from a Confutation of contrary Suppositions, but by deriving it from Experiments concluding positively and directly. The Way therefore to examine it, is, by considering, Whether the Experiments which I propound do prove those Parts of the *Theory* to which they are applied, or by prosecuting other Experiments which the *Theory* may suggest for its Examination. And this I would have done in a due Method; the Laws of Refraction being thoroughly enquired into and determined, before the Nature of Colours be taken into Consideration. It may not be amiss to proceed according to the Series of these Queries; which I could wish were determined by the Event of proper Experiments, declared by those that may have the Curiosity to examine them.

1. Whether Rays, that are alike incident on the same Medium, have unequal Refractions? And how great are the Inequalities of their Refractions at any Incidence?

2. What



2. What is *the Law* according to which each Ray is *more or less refracted*; whether it be that the same Ray is ever refracted according to the *same Ratio* of the Sines of Incidence and Refraction; and divers Rays, according to divers Ratios; or that the Refraction of each Ray is greater or less without any certain Rule? That is, whether each Ray have a certain Degree of Refrangibility, according to which its Refraction is performed; or is refracted without that Regularity?

3. Whether *Rays*, which are endued with particular Degrees of *Refrangibility*, when they are by any Means separated, have particular *Colours constantly belonging to them*; viz. the least Refrangible, Scarlet; the most Refrangible, deep Violet; the middle, Sea-Green; and others, other Colours? And on the contrary?

4. Whether *the Colour* of any sort of *Rays apart* may be changed by *Refraction*?

5. Whether *Colours* by coalescing do really *change* one another to produce a *New Colour*, or produce it by *mixing* only?

6. Whether a *due Mixture of Rays*, indued with all variety of Colours, produces *Light perfectly like that of the Sun*, and which hath all the same Properties, and exhibits the same Phænomena?

7. Whether the *component Colours* of each Mixture be really *changed*; or be only *separated*, when from that Mixture various Colours are produced again by Refraction?

8. Whether there be any *other Colours* produced by *Refraction*, than such as ought to result from the *Colours* belonging to the *diversly Refrangible Rays*, by their being separated or mixed by *that Refraction*?

To determine by Experiments these and such like Queries, which involve the propounded Theory, seems the most proper and direct Way to a Conclusion. And therefore I could wish all Objections were suspended, taken from Hypotheses, or any other Heads than these two; of shewing the Insufficiency of Experiments to determine these Queries, or prove any other Parts of my Theory, by assigning the Flaws and Defects in my Conclusions drawn from them; or of producing other Experiments which directly contradict me, if any such may seem to occur. For if the Experiments, which I urge, be defective, it cannot be difficult to shew the Defects; but if valid; then by proving the Theory they must render all Objections invalid.

IV. 1. This so extraordinary an Hypothesis, which overturns the very Foundations of Dioptrics, and makes useless all the Practice that has hitherto obtained, is intirely built upon that Experiment of the Glass Prism, in which the Rays of Light entering at the Hole of a Window into a dark Room, and then striking against the Wall, or received upon a Paper, are not gathered into a round Spot, as Mr. *Newton* seemed to expect from the received Laws of Refractions, but appeared to be extended into an oblong Figure. Whence he concluded, that this oblong Figure proceeded from this Cause, that some of the Rays were refracted more and some less.

But

*Animadversion*  
upon this Theory.  
by R. P. Ign.  
Gaston Pardies.  
N. 84. p. 4087.  
June, An. 1672.



But it seems to me, that according to the common and received Laws of Dioptrics, that Figure ought not to be round but oblong. For since the Rays proceeding from the opposite Parts of the solar Disk have a different Inclination in passing through the Prism, they ought to be differently refracted; and since the Inclination of some is greater than the Inclination of others by at least 30 Minutes, the Refraction also must become greater.

Therefore the opposite Rays, emerging from the other Superficies of the Prism, diverge and divaricate more, than if all had proceeded without Refraction, or at least had been equally refracted. But that Refraction of the Rays is performed only towards those Parts which may be conceived to be in Plains perpendicular to the Axis of the Prism: For there is no Inequality of Refraction towards those Parts which are understood to be in Plains parallel to the Axis; as may be easily demonstrated. For the two Superficies of the Prism may be judged as parallel to one another, in respect to the Inclination of the Axis, since each is parallel to the Axis. But the Refraction through two parallel plain Superficies is reckoned as none, because as much as the Ray is deflected one Way by the first Superficies, so much it is turned the other Way by the other Superficies. Therefore since the Rays of the Sun passing from the Hole through the Prism are not refracted at the Sides, they proceed farther as if no Superficies of the Prism had intervened, (respect only being had, as I said before, to that lateral Divarication;) but since the same Rays are refracted towards the upper or lower Parts, some indeed more and some less, as being unequally inclined; it is necessary that they must divaricate more, and therefore must be extended into a longer Figure.

Now if a Calculation be rightly made; as the lateral Rays are found by the learned *Newton* in such a Breadth as subtends an Arch of 31 Minutes, which Arch answers to the Sun's Diameter; so I don't at all doubt, but that Height also of the Image being found, which subtends an Arch of  $2^{\circ}.49'$ . is that itself which in that Case answers to the same Diameter of the Sun after the unequal Refractions.

Fig. 72.

And in reality, supposing the Prism to be  $ABC$  whose Angle  $A$  is 60 Degrees, the Ray to be  $DE$ , which makes an Angle of 30 Degrees with the Perpendicular  $EH$ ; I find it at emerging through  $FG$  to make with the Perpendicular  $FI$  an Angle of  $76^{\circ}.22'$ . But supposing another Ray  $dE$ , which with the Perpendicular  $EH$  makes an Angle of  $29^{\circ}.30'$ ; I find this, as it emerges through  $fg$ , to make with the Perpendicular  $fi$  an Angle of  $78^{\circ}.45'$ . Whence those two Rays  $DE$ ,  $dE$ , which are supposed to proceed from opposite Parts of the Solar Disk, make an Angle with each other of 30 Minutes; but the same at emerging in the Lines  $FG$ ,  $fg$ , so diverge, as to constitute an Angle of  $2^{\circ}.23'$ . with each other. Now if two other Rays were assumed, that approach more to the Perpendicular  $EH$ , (which make, for instance, with the same Perpendicular, one an Angle of  $29^{\circ}.30'$ , the other an Angle of  $29^{\circ}.0'$ ;) then the same Rays at their emerging would diverge still more, and would make a greater Angle, even sometimes more than three Degrees. And moreover that Interval of the refracted Rays is farther increased from hence, that the two Rays  $DE$ ,  $dE$ , meeting in



in E, immediately begin to divaricate, and hit upon two separate Points of the other Superficies, that is, in F and f. Therefore it is not sufficient for duly performing the Calculation, from the Length of the Image projected upon the Paper to subtract the Magnitude of the Hole of the Window; since even supposing the Hole E. to be indivisible, there would be as it were another broad Hole in the other Superficies, which is Ff.

As to what he calls *Experimentum Crucis*, to me it seems to fall in with the commonly received Rules of Refraction. For, as I have now shewn, the Solar Rays, which approaching and converging make an Angle of 30 Degrees, after they have passed through the indivisible Foramen, diverge into an Angle of two or three Degrees. Therefore it is no wonder if the same Rays, singly impinging upon another Prism which is also opened with a small Hole, should be unequally refracted, since their Inclination is unequal. Nor is it to the purpose that those Rays are raised or depressed by the Rotation of the first Prism, the second Prism remaining without Motion, (which yet cannot be done in every Case;) or that, the first remaining immoveable, the second should be moved, that it may successively receive the coloured Rays of the whole Image, and transmit them through its own Hole. For in either Way it is necessary, that those extreme Rays, that is the Red and Violet, should fall upon the second Prism at an unequal Angle, and thereby that their Refraction should be unequal, so that That of the Violet Rays should be the greater.

Since therefore a manifest Cause appears, why the Figure of the Rays should be oblong, and that Cause arises from the very Nature of Refraction; there seems no Occasion to have Recourse to any other Hypothesis, or to admit any different Frangibility of the Rays.

What afterwards he has conceived concerning Colours, that indeed commodiously follows from the foregoing Hypothesis; yet even that is obnoxious to some Difficulties. For as he affirms, that no Colour at all, but rather Whiteness, will appear, when the Rays of all Colours are mixed promiscuously, that does not seem conformable to all the Phænomena. Surely the Varieties which are observed in the mingling of different Bodies, which are imbued with different Colours, the very same are perceived in the mingling of different Rays that are also imbued with different Colours. And he well observes, that as from a Yellow and Blue Body a Green Colour arises, so from a Yellow and Blue Ray a Green Colour will also arise. So that if all the Rays of all Colours were confounded together, it is necessary in that Hypothesis, that the same Colour should appear which really appears in a Mixture of all Paints. But now if those Colours, that is, Red and Yellow, together with Blue and Purple, and all others, if there be more, were beaten together and mingled; not White would appear, but a dark sad Colour. Therefore a like Colour would appear in the ordinary Rays of Light, if it consisted of a Mixture of all sorts of Colours.

Now at first Aspect nothing seems more ingenious or more proper, than what he says concerning the Experiment of the most acute Mr. *Hook*, in which two different Liquors, the one Red the other Blue, and both separately



rately transparent, but when mixed together become opaque. This, the learned *Newton* says, arises from hence, that one of the Liquors can only transmit the Red Rays, the other only the Blue; whence being mingled they can transmit none. This, I say, at first sight seems very opposite; nevertheless it should follow from hence, that a like Opacity would arise from the Mixture of any Liquors of different Colours, which yet is not true.

Answer'd by  
Mr. Newton.  
N. 84. p. 409 I.  
June, An. 1672.  
Fig. 73.

2. The Reverend Father *Pardies* makes the Refractions on the different Sides of the Prism as unequal as he can, whereas I make them equal, both in my Experiments, and in the Calculation founded upon those Experiments. Let  $ABC$  be a Section of the Prism perpendicular to its Axis,  $FL$ ,  $KG$ , two Rays crossing one another in  $x$  the middle of the Hole, and falling upon that Prism at  $G$  and  $L$ , and let their refracted Rays be  $GH$ ,  $Lm$ , and again  $HI$  and  $mn$ . And whereas I have supposed that their Refractions at the Side  $AC$  are nearly equal to those at the Side  $BC$ ; if  $AC$  and  $BC$  are put equal, the Rays  $GH$  and  $Lm$  will have a like Inclination to the Base of the Prism  $AB$ ; and therefore the Angle  $CLm = \text{Ang. } CHG$ , and  $\text{Ang. } Cml = \text{Ang. } CGH$ . Wherefore also the Refractions in  $G$  and  $m$  will be equal, as likewise in  $L$  and  $H$ , and so  $\text{Ang. } KGA = \text{Ang. } nmb$ , and  $\text{Ang. } FLA = \text{Ang. } BHI$ : And therefore the refracted Rays  $HI$  and  $mn$  will have the same Inclination to one another as the incident Rays  $FL$  and  $KG$ . Therefore if the Angle  $FxK$  be 30 Minutes, or equal to the Sun's Diameter, then also the Angle comprehended by  $HI$  and  $mn$  will be 30 Minutes, if the Rays  $FL$  and  $KG$  are supposed equally refrangible. But to me, who have tried it, that Angle comes out about  $2^\circ . 49'$ , which the Ray  $HI$  exhibiting the utmost Violet Colour, contains with the Ray  $mn$  of a Blue Colour; and therefore it must be necessarily granted, that those Rays are differently refrangible, or that the Refractions are performed according to an unequal Ratio of the Sines of Incidence and Refraction.

The Reverend Father adds besides, that for rightly performing the Calculation, it is not sufficient to subtract the Bigness of the Hole from the Length of the Image projected on the Paper; because even supposing the Hole to be indivisible, there would still be as it were another wide Hole in the posterior Superficies of the Prism. Yet this notwithstanding it seems to me, that the Refractions of the Rays, crossing at the anterior as well as posterior Surface of the Prism, may be rightly computed from the Principles laid down. But if the Matter were otherwise, the Breadth of the Chasm in the latter Superficies, which is as a Hole, could hardly make an Error of two Seconds; and in practical Matters I think it not worth while to take Notice of such minute Differences.

The Reverend Father does not contradict that which I called *Experimentum Crucis*, while he contends, that the unequal Refractions of the Rays, endued with different Colours, are produced by the unequal Incidences. For when the Rays pass through two very small, distant, and immoveable Holes, those Incidences (as I made the Experiment) were plainly equal, and yet these Refractions were very unequal. But if he has any Doubt about my Experiments, I beg that he would try himself, and measure the Refractions of the Rays endued



dued with different Colours, from equal Incidences, and he will find them to be unequal. If he does not like the Way I made use of to try this, (than which I think none can be more satisfactory) it is easy to contrive others; as I myself have experienced not a few, and with good Success.

It is objected to the Theory about Colours, that Powders of different Colours being mixed, they exhibit not a White, but a dark Brown Colour. Now to me White, Black, and all intermediate Brown Colours, which can be composed of White and Black mixed, seem to differ not so much in their Species of Colour as in the Quantity of Light. And in the Mixture of Paints, since the several Corpuscles reflect only their own Colour, and therefore the greatest Part of the incident Light is suppressed and stifled; the reflected Light must become obscure, and mixed as it were with Darkness, so that it cannot exhibit intense Whiteness, but such as would be caused by a Mixture of Black, that is, a dark Brown.

It is then objected, that from any Liquors of different Colours mixed in the same Vessel, as well as being contained in different Vessels, Opacity should be produced, which yet (he says) is not true. But I do not see the Consequence. For very many Liquors act upon one another, and secretly induce a new Configuration of Parts upon one another; whence they may become opaque, transparent, or endued with various Colours, such as could by no means arise from a Mixture of Colours. And for this I always thought, that Experiments of this kind were less fit, from whence Conclusions could be drawn. Yet here I will observe, that for this Experiment Liquors are required, which are endued with strong and intense Colours, which transmit but few Rays except those of their own Colour. Such are rarely to be met with, as may be seen by illuminating Liquors in a dark Room with the different Colours of the Prism. For few are found which appear very transparent in their own Colours, and opaque in others. It is also convenient, that the Colours made use of should be opposite to one another, such as I think Red and Blue to be, or Yellow and Violet, or also Green and that Purple which approaches to Scarlet. And perhaps some of these Liquors, whose tinging Particles will not come together, when mixed may become more opaque. But I am nothing solicitous about the Event, as well because the Experiment will be more perspicuous in Liquors when separate, as because I proposed that Experiment (as likewise that of the Rainbow, of the Tincture of *Lignum Nephriticum*, and the Phænomena of other natural Bodies) not so much to prove the Doctrine as to illustrate it.

Now as to the Reverend Father's calling my Theory *an Hypothesis*, I take it not amiss, because to him it may not yet appear. But I proposed it with another Intent, and it seems to contain nothing else but certain Properties of Light, which being now found I think it not difficult to prove; and which if I did not know to be true, I should rather choose to reject as vain and empty Speculation, than to own them as my Hypothesis.

3. In that Hypothesis which our *Grimaldus* explains at large, in which it is supposed that Light is a certain Substance moving with a most rapid Motion, some Diffusion of Light might be made after passing through the Hole,

*Some farther Objections, by R. P. Pardies. N. 85. p. 5012. Jul. An. 1672.*



and the Decussation of the Rays. Also in that Hypothesis, in which Light is supposed to proceed through certain Undulations of subtile Matter, as the very ingenious *Hook* explains; Colours might be accounted for by some kind of Diffusion and Expansion of Undulations, which may be made at the Sides of the Rays beyond the Hole, by Contact itself, and by the Continuation of the Matter. And indeed I have recourse to such an Hypothesis, in the Dissertation concerning the Motion of Undulation, which is the sixth Part of my *Mechanics*. Where I suppose, that those appearing Colours are produced by that Communication of Motion alone, which is propagated at the Sides by the Undulations going directly on. As if Rays entering at the Hole *a* go on towards *b*, the direct Undulations (having regard to strait and natural Motion) ought to be terminated at the Right Line *ab*; yet because of the Continuity of Matter, there will be some Communication of Motion towards the Sides *ec*, where a certain tremulous and subsultory Succussion will be excited. Now if Colours may be supposed to consist in that lateral subsultory Motion, I imagine all the Phænomena of Colours might be explained, as I have endeavoured to do more at large in the Dissertation now mentioned. From which being supposed it will also appear, why the Breadth of the Colours must be expanded farther than the Divarication of the Rays themselves would require.

As to the *Experimentum Crucis*, I do not at all doubt, but that in his Experiment he makes use of such a Situation, as that there may be an equal Inclination of the incident Rays; since he expressly affirms it was so done by him. But I was not able to guess that from what I had read before, where are supposed two small and very distant Holes, and one Prism near the first Hole which is in the Window; through which Prism the coloured Rays proceeding, fall upon the other distant Hole. It was also added, that in order that all the Rays might fall successively on that second Hole, the first Prism was turned about its Axis. But by this means it must necessarily be, that the Inclination of the Rays falling upon the second Hole must be changed. I also suggested, that the Thing would be the same, whether the first Prism remaining immoveable, the second Hole was raised or depressed, that it might receive successively all the Rays of the painted Solar Image; or, that second Hole remaining immoveable, the first Prism was turned about, so that the same Image might change its Situation, and all the Parts might successively impinge upon the second Hole. But without doubt the most sagacious *Newton* made use of other Cautions.

As to what I had objected about the Colours, I think it very well solved. And that I called his Theory an Hypothesis, I did it without any Design, but made use of the Word that first occurred. Therefore I desire him not to think, that I used that Word in any kind of Contempt.

4. The Reverend Father says, that it is possible to explain the Length of the Colours, without a different Refrangibility of the different Rays; suppose from the Hypothesis of *P. Grimaldus*, by the Diffusion of Light, which is supposed to be a certain Substance moved with the greatest Rapidity; or by the Hypothesis of our *Hook*, by the Diffusion or Expansion of Undulations,

tions,

*Answer'd by*  
*Mr. Newton.*  
*Ibid. p. 5014.*  
*July An. 1672.*



tions; which he pretends are excited in the *Æther* by lucid Bodies, and to be propagated every Way. To which I add, that by the Hypothesis of *Cartesius* may also be imagined a like Diffusion or Endeavour of the Pressure of the Globules, in like Manner as is supposed in the Explication of the Tail of a Comet. And the same Diffusion or Expansion may be feigned according to any other Hypothesis, in which Light is supposed to be a Force, Action, Quality, or Substance of any kind, emitted on all Sides from luminous Bodies.

That I may answer this, it is to be observed, that the Doctrine I have explained concerning Refractions and Colours, consists only in certain Properties of Light, neglecting all Hypotheses by which those Properties may be explained. Wherefore I thought proper here wholly to refrain from the Consideration of Hypotheses, as being no fit Place of Argumentation; and I will abstract the Force of the Objection, that it may receive a fuller and more general Answer.

Therefore by Light I understand any Being or Power of a Being, (whether it be a Substance, or any Power, Action, or Quality of it) which proceeding directly from a lucid Body is adapted to excite Vision. And by Rays of Light I understand the least Parts of it, or any indefinitely little Parts of it, which do not depend on one another; such as are all those Rays which luminous Bodies emit according to Right Lines, either at the same Time or successively. For those as well collateral as successive Parts of Light are independent, since some without others may be intercepted, and may be separately reflected or refracted any Way. And this being allowed, all the Force of the Objection consists in this, that the Colours may be stretched out in Length by some Diffusion of the Light beyond the Hole, which does not arise from the unequal Refrangibility of different Rays, or of the independent Parts of Light.

But I have shewn above, that they are not lengthened by any other Means; and that I might confirm the whole in the strictest Manner, I have added that Experiment which now is well known by the Name of *Experimentum Crucis*; of the Conditions of which since the Reverend Father has had some Doubt, I have thought fit to represent it by a Scheme. Let BC be the former little Board to which the Prism A is immediately prefixed, and let DE be the latter Board, at the Distance of about 12 Feet from the former, behind which the other Prism F is fix'd. But let the Boards be so perforated at *x* and *y*, that a little of the Light refracted by the former Prism may be transmitted through each of the Holes to the second Prism, and there again be refracted. Now let the former Prism be converted about its Axis by a reciprocal Motion, and the Colours falling upon the latter Board DE, will be raised and depressed by Turns, by which Means another and another Colour at Pleasure may be successively made to pass through its Hole *y* to the latter Prism, while the other Colours will fall upon the Board. Then you will see, that the Rays which are endued with different Colours will suffer a different Refraction in that latter Prism, because they will arrive to different Places of the opposite Wall, or of any Obstacle GH removed at the Distance

Fig. 75.

of



of some Feet farther. The Violet Rays will come suppose to H, the Red to G, and intermediate ones to intermediate Places. And yet because of the determinate Position of the Holes, the Incidence of the Rays of each Colour passing through each Prism must needs be alike. And thus it appears by measuring, that the Rays affected with different Colours will have different Laws of Refractions.

But I suspect what it was that led the Reverend Father into a State of Doubt; he seems to have placed his first Prism A behind the Board B C, and thus by converting it about its Axis, it is probable that the Inclination of the Rays, which come between the two Holes, may have suffered some Change because of the intermediate Refraction. But by the Description before explained, that Board ought to be placed after the Prism, that the Rays may fall directly between the Holes, as may appear from my Words; *I took two Boards, and placed one of them close behind the Prism at the Window.* And the Design of the Experiment requires the same Thing.

But farther I shall observe, that in this Experiment, because of the Refraction of the second Prism, the coloured Light is far less diffused, and divaricates less, than when it is quite White, so that the Image at G or H is almost Circular; especially if the Prisms are made parallel, and in a contrary Situation of their Angles, as is represented in the Scheme. And farther, if the Diameter of the Hole *y* is equal to the Breadth of the Colours, there will be no Extension of the same coloured Light in Length; but the Image which is formed by any Colour at G or H will be really Circular; supposing the Holes to be Circular, and the Refraction of the latter Prism not to be greater than of the former, the Rays being nearly perpendicular to the Obstacle. This shews the Diffusion mentioned before to proceed from a certain Law of the Refractions of every kind of Rays, and not from the Contact or Continuity of Matter undulating or moving very swiftly, or any the like Cause. But why that Image should be Circular in one Case, and in others something stretched out in Length, and how the Extension of the Light into Length may be lessened at Pleasure in any Case, I shall leave to be determined by Geometricians, and then to be compared with Experiment.

After the Properties of Light have been sufficiently examined by these and the like Experiments, by considering the Rays as Parts of it either collateral or successive, of which we have found by their Independence that they are distinct from one another; Hypotheses are thence to be judged of, and those are to be rejected which cannot be reconciled with the Phænomena. But it is an easy Matter to accommodate Hypotheses to this Doctrine. For if any one has a Mind to defend the *Cartesian* Hypothesis, he may say that the Globules are unequal, or that the Pressures of the Globules are some stronger than others, and thence they become differently refrangible, and proper to excite a Sensation of different Colours. And so according to Mr. *Hook's* Hypothesis, it may be said, that the Undulations of the *Æther* are some greater or denser than others. And so of the rest. For this seems to be the most necessary Law and Condition of Hypotheses, in which natural Bodies are supposed to consist of a Multitude of Corpuscles cohering together, that



that from the different Particles of lucid Bodies, or from the different Parts of the same Corpuscle, (according as they differ in Motion, Figure, Bulk, or any other Qualities) unequal Pressures, Motions, or moved Corpuscles, may be propagated every Way through the Æther, from which being confusedly mixed together, Light may be supposed to be constituted. And nothing can be more difficult in these Hypotheses than the contrary Supposition.

From the Aperture or Dilatation of the Light in the latter Face of the Prism, which the Reverend Father says is a Hole as it were, it is sufficient that no sensible Error can arise, if there is any at all. And if a Calculation is made exactly according to the Observations, the Error will be none. For the Diameter of the Hole being subtracted from the Length of the Image, a Length will remain which the Image would have, if the Hole before the Prism were an indivisible Point, and that notwithstanding the aforesaid Dilatation of the Light in the latter Face of the Prism; as may easily be shewn. Then from that given Length of the Image, and its Distance from the indivisible Hole, as also from the Position and Form of the Prism, and from the Inclination of the incident Rays to it, and from the Angle which the refracted Rays tending to the Middle of the Image make with the incident Rays from the Center of the Sun, all the other Things may be determined. And what determines the Refractions and Positions of the Rays are sufficient to make a true Calculation of those Refractions. But this Matter is not of such Consequence as to make any Difficulty.

As to the Reverend Father's calling our Doctrine an Hypothesis, I believe it proceeded from nothing else, but that he used the Word which first occurred to him, for a Custom has prevailed, that whatever is explained in Philosophy is called an Hypothesis. And I had no other Design in shewing my Dislike to that Word, than to obviate an Appellation, which may mislead those who desire to philosophize the right Way.

5. Mr. Newton's last Answer to my Objections has intirely satisfied me. The last Scruple that I had, about the *Experimentum Crucis*, is fully removed. And now I clearly understand by his Figure what I did not understand before. When the Experiment was performed after his Manner, every Thing succeeded, and I had nothing farther to desire.

*To the Satisfaction of P. Ig. Gast. Pardies. N. 85. p. 5018. July, An. 1672.*

V. The Consideration on my *Theories* consists in ascribing an Hypothesis to me, which is not mine; in asserting an Hypothesis, which, as to the principal Parts, is not against me; in granting the greatest Part of my Discourse, if explicated by that Hypothesis; and in denying some Things, the Truth of which would have appeared by an Experimental Examination.

*Some Considerations upon this Theory; by ---- Answer'd by Mr. Newton. N. 88. p. 5086. Nov. An. 1672.*

Of these Particulars I shall discourse in Order. And first of the Hypothesis, which is ascribed to me in these Words: *But grant this first Supposition, that Light is a Body, and that as many Colours or Degrees as there may be, so many Bodies there may be; all which compounded together would make White, &c.* This, it seems, is taken for my Hypothesis. 'Tis true, that from my *Theory* I argue the Corporeity of Light; but I do it without any absolute Positiveness, as the Word *perhaps* intimates; and make it at most but a very plausible





plausible Consequence of the Doctrine, and not a fundamental Supposition, nor so much as any Part of it; which was wholly comprehended in the precedent Propositions. And I somewhat wonder, how the Objector could imagine, that, when I had asserted the *Theory* with the greatest Rigour, I should be so forgetful as afterwards to assert the fundamental Supposition itself with no more than a *perhaps*. Had I intended any such Hypothesis, I should somewhere have explained it: But I knew, that the Properties, which I declared of Light, were in some measure capable of being explicated, not only by that, but by many other mechanical Hypotheses; and therefore I chose to decline them all, and to speak of Light in general Terms, considering it abstractly, as something or other propagated every Way in streight Lines from luminous Bodies, without determining what that Thing is; whether a confused Mixture of difform Powers, or Beings whatsoever. And for the same Reason I chose to speak of Colours according to the Information of our Senses, as if they were Qualities of Light without us. Whereas by that Hypothesis, I must have considered them rather as Modes of Sensation, excited in the Mind by various Motions, Figures, or Sizes of the Corpuscles of Light, making various mechanical Impressions on the Organ of Sense: as I expressed it in that Place, where I spake of the Corporeity of Light.

But supposing I had propounded that Hypothesis, I understand not why the Objector should so much endeavour to oppose it: For certainly it has a much greater Affinity with his own Hypothesis, than he seems to be aware of; the Vibrations of the *Æther* being as useful and necessary in this, as in his; for assuming the Rays of Light to be small Bodies, emitted every Way from shining Substances, those, when they impinge on any refracting or reflecting Superficies, must as necessarily excite Vibrations in the *Æther*, as Stones do in Water when thrown into it. And supposing these Vibrations to be of several Depths or Thicknesses, accordingly as they are excited by the said corpuscular Rays of various Sizes and Velocities; of what Use they will be for explicating the Manner of Reflection or Refraction, the Production of Heat by the Sun-Beams, the Emission of Light from Burning, Putrifying, or other Substances, whose Parts are vehemently agitated, the Phænomena of thin transparent Plates and Bubbles, and of all natural Bodies, the Manner of Vision, and the Difference of Colours, as also their Harmony and Discord; I shall leave to their Consideration, who may think it worth their Endeavour, to apply this Hypothesis to the Solution of Phænomena.

In the *second* Place, I told you, That the Objector's Hypothesis, as to the fundamental Part of it, is not against me. That fundamental Supposition is; *That the Parts of Bodies, when briskly agitated, do excite Vibrations in the Æther, which are propagated every Way from those Bodies in streight Lines, and cause a Sensation of Light by beating and dashing against the Bottom of the Eye, something after the Manner that Vibrations in the Air cause a Sensation of Sound by beating against the Organs of Hearing.* Now, the most free and natural Application of this Hypothesis to the Solution of Phænomena, I take to be this: That the agitated Parts of Bodies, according to their several Sizes, Figures, and Motions, do excite Vibrations in the *Æther* of various Depths or Bignesses,



Bignesses, which being promiscuously propagated through that Medium to our Eyes, effect in us a Sensation of Light of a white Colour; but if by any means those of unequal Bignesses be separated from one another, the largest beget a Sensation of a red Colour; the least or shortest, of a deep Violet; and the intermediate ones, of intermediate Colours; much after the manner that Bodies, according to their several Sizes, Shapes, and Motions, excite Vibrations in the Air of various Bignesses, which, according to those Bignesses, make several Tones in Sound: That the largest Vibrations are best able to overcome the Resistance of a refracting Superficies, and so break through it with least Refraction; whence the Vibrations of several Bignesses, that is, the Rays of several Colours, which are blended together in Light, must be parted from one another by Refraction, and so cause the Phænomena of Prisms, and other refracting Substances: And that it depends upon the Thickness of a thin transparent Plate or Bubble, whether a Vibration shall be reflected at its further Superficies, or transmitted; so that, according to the Number of Vibrations interceding the two Superficies, they may be reflected or transmitted for many successive Thicknesses. And since the Vibrations which make Blue and Violet, are supposed shorter than those which make Red and Yellow, they must be reflected at a less Thickness of the Plate: Which is sufficient to explicate all the ordinary Phænomena of those Plates or Bubbles, and also of all natural Bodies, whose Parts are like so many Fragments of such Plates.

These seem to be the most plain, genuine, and necessary Conditions of this Hypothesis. And they agree so justly with my Theory, that if the Animadversor think fit to apply them, he need not, on that Account, apprehend a Divorce from it. But yet how he will defend it from other Difficulties, I know not; for to me the fundamental Supposition itself seems impossible, namely, that the Waves or Vibrations of any Fluid, can, like the Rays of Light, be propagated in streight Lines, without a continual and very extravagant spreading and bending every Way into the quiescent Medium, where they are terminated by it. I mistake, if there be not both Experiment and Demonstration to the contrary. And as to the other two or three Hypotheses, which he mentions, I had rather believe them subject to the like Difficulties, than suspect the Animadversor should select the worst for his own.

What I have said of this, may be easily applied to all other mechanical Hypotheses, in which Light is supposed to be caused by any Pression or Motion whatsoever, excited in the Æther by the agitated Parts of luminous Bodies; for it seems impossible that any of those Motions or Pressions can be propagated in streight Lines, without the like spreading every Way into the shadowed Medium on which they border. But yet, if any Man can think it possible, he must at least allow, that those Motions, or Endeavours to Motion, caused in the Æther by the several Parts of any lucid Body, that differ in Size, Figure, and Agitation, must necessarily be unequal: Which is enough to denominate Light an Aggregate of *Difform Rays*, according to any of those Hypotheses. And if those original Inequalities may suffice to difference the Rays in Colour and Refrangibility, I see no reason, why they,