19 folding plts (14/5)

February 2. 1720-1.

At a Meeting of the *ROYAL SOCIETY*,

Sir ISAAC NEWTON, Prefident, *in the Chair*,

Mr. JONES prefented a Scheme and Specimen of an Abridgment of the PHILOSOPHICAL TRANSACTIONS, from the Year 1700 to the Year 1720. This Defign was approved of by the SOCIETY, and he was defired to proceed therein.

Edm. Halley, Secr. Reg. Soc.

Octob. 27. 1721.

IMPRIMATUR *Epitome* Tranfactionum Philofophicarum, *ab Anno* 1700 *ad Annum* 1720, *a D.* Hen. Jones *compofita.*

ISAAC NEWTON, *Reg. Soc. Præfes.*

THE

PHILOSOPHICAL TRANSACTIONS

(From the Year 1700, *to the Year* 1720.*)*

ABRIDG'D,

AND

Dispos'd under GENERAL HEADS.

In Two Volumes.

By *HENRY JONES*, M. A. and Fellow of *King*'s *College* in *CAMBRIDGE*.

VOL. IV. Containing

Part I. The MATHEMATICAL Papers.

Part II. The PHYSIOLOGICAL Papers.

The THIRD EDITION Corrected.

In which the *LATIN* PAPERS are now first translated into *ENGLISH*.

LONDON:

Printed for W. INNYS, R. WARE, J. and P. KNAPTON, D. BROWNE, T. LONGMAN, C. HITCH, J. HODGES, S. AUSTEN, A. MILLAR, J. and J. RIVINGTON, and J. WARD.

M. DCC. XLIX.

TO THE

Right Honourable

THOMAS

Earl of MACCLESFIELD,

Lord High CHANCELLOR

OF

GREAT BRITAIN, &c.

My LORD,

AS the conſiderable Improvements, which Learning has receiv'd, are chiefly owing to the favourable Encouragement and propitious Influence of the Great; So amongſt the many Patrons of the Age there is no one, who has ſhew'd more Encouragement to it than Your Lordſhip, or who is happy in a greater Share of it.

To

To Your Lordſhip therefore, as the worthieſt Patron, I moſt humbly offer theſe Diſcourſes of the greateſt Authors; which, if they have not ſuffer'd in paſſing through my Hands, are not only highly deſerving of your Acceptance, but ought more particularly in Honour to be inſcrib'd to Your Lordſhip's Name. For to whom could theſe Volumes be ſo properly Dedicated, as to one, who is a perfect Maſter of the Subjects here treated of? Who, like his great Predeceſſor the Lord Chancellor BACON, has taken in the wide Compaſs of Phyſical, as well as Civil Knowledge; and is throughly acquainted with the Laws of Nature, as well as thoſe of the Land? It was His Honour to lay, in ſome Meaſure, the Foundations of thoſe Improvements, which Philoſophy has ſince receiv'd; and it is the peculiar Honour and Advantage of that Philoſophy, to have fallen under Your Lordſhip's Protection in this its maturer State, in an Age abounding with uſeful Inventions and great Diſcoveries: Happy are thoſe Sciences in ſuch a Patron; they muſt certainly continue to flouriſh, when Your Lordſhip does not only encourage them by Your Liberality, but promote them by the Authority of Your own Example.

It is Matter of Surprize, my Lord, that one, whoſe whole Life has been employ'd in the active Part of the World, and in the Buſineſs of a Profeſſion very difficult and laborious, ſhould have any Inclination or Leiſure for thoſe other Parts of

Learning,

Learning, which your Lordſhip is by all allowed to poſſeſs in a very extraordinary Degree. Who, though you have receiv'd many public Addreſſes of this kind from eminent Authors, yet you have been able (ſuch are Your own natural Parts, and ſuch are Your Improvements of them) to equal thoſe Performances You have condeſcended to patronize. In the Study of Divinity, my Lord, You may well be ſaid, to be inferiour to no one; the late Dr. HICKES has long ſince told the World, that You are a Perſon, "Who to his great Underſtanding "in our Common and Statute Laws, and in the "*Engliſh* Conſtitution before and ſince the Con-"queſt, has added ſuch a Knowledge of all the "moſt uſeful Parts of Divinity, that it is not eaſy "to determine, whether he is better ſkill'd in Hu-"man or Divine Laws." And I may add, who to his great Acquiſitions in the more learned Studies has join'd no leſs Attainments in the politer Arts; whoſe own Speeches, on a very memorable Occaſion, will tranſmit his Character, as a conſummate Orator, to the lateſt Poſterity: Compoſitions applauded and admired by all; and, what is the trueſt Teſt of Merit, commended by thoſe, who diſlik'd the Subject of them.

Theſe Talents ſo various, theſe Qualifications ſo uncommon, have recommended Your Lordſhip to that High Office, which you now adorn: Others have ſtruggled with great Competitors, and contending Equals in the Paths of Ambition; but in Juſtice to your Lordſhip it is to be remember'd, that

You

You were ſollicited and importun'd to accept the greateſt Truſt in the Nation.

But I muſt not preſume to detain Your Lordſhip any longer, much leſs can I pretend to do Juſtice to Your Character: And as, in preſenting You with theſe Authors, I intend a Piece of great Reſpect to Your Lordſhip; ſo I muſt eſteem it a very great Honour done to me, that any Thing, in which I have had the ſmalleſt Share, can have the Favour of Your Lordſhip's Name, and obtain the Patronage of ſo good a Judge. I am,

My LORD,

Your Lordſhip's moſt Obedient

and moſt humble Servant,

Henry Jones.

THE

CONTENTS.

Vol. IV. Part I.

The MATHEMATICAL *Papers.*

CHAP. I.

GEOMETRY, ARITHMETIC, ALGEBRA, LOGARITHMOTECHNY.

The CONTENTS.

CHAP. II.

OPTICS.

CHAP. III.

ASTRONOMY.

XIX. *An*

The CONTENTS.

CHAP. IV.

MECHANICS, ACOUSTICS.

The CONTENTS.

CHAP. V.

HYDROSTATICS, HYDRAULICS.

CHAP. VI.

GEOGRAPHY, NAVIGATION.

CHAP. VII.

MUSIC.

THE

THE CONTENTS.

Vol. IV. Part II.

The PHYSIOLOGICAL *Papers.*

CHAP. I.

PHYSIOLOGY, METEOROLOGY, PNEUMATICS.

XIX. *A*

The CONTENTS.

CHAP. II.

HYDROLOGY.

CHAP.

The CONTENTS.

CHAP. III.

MINERALOGY.

CHAP.

The CONTENTS.

CHAP. IV.

MAGNETICKS.

CHAP. V.

AGRICULTURE, BOTANY.

THE

THE

Philosophical Transactions

ABRIDG'D.

PART I.

Containing the

Mathematical PAPERS.

CHAP. I.

Geometry, Arithmetic, Algebra, Logarithmotechny.

I. IT has heretofore passed for a current Maxim, That all Infinites are equal.

The Proportion of Mathematical Points to each other, by the Hon. Fr. Robartes, n. 334, p. 470. Vid. supra Vol. I. C. I. S. XXV.

The Position nevertheless is certainly erroneous, as Dr. *Halley* abundantly has shown in the *Philosophical Transactions* for *October* 1696. He there gives divers Instances of infinite Quantities which are in a determinate finite Proportion one to another, and some infinitely greater one than another. The like may be observed of infinitely small Quantities, *viz.* Mathematical Points, as the following Proposition will make appear.

PROP. I.] *The Points of Contact between Circles and their Tangents, are in subduplicate Proportion to the Diameters of the Circles.*

Fig. 1. Let two Circles *a d c h*, *a f b g*, touch one another from within at the Point *a*. Draw the Tangent *p a q*, and parallel to it the Line *m n*. From the Point *a* draw the Diameter *a c*.

Let *a c*, the Diameter of the great Circle, be equal to *R*, and *a b*, the Diameter of the lesser Circle, be equal to *S*.

Let *d h*, the Chord of the Arch *d a h*, be equal to *z*, and *f g*, the Chord of the Arch *f a g*, be equal to *y*, and let the Abscifs *a k* be equal to *x*.

If the Line *m n* be supposed to move till it becomes co-incident with the Tangent *p a q*, the Nature of a Circle will always give the following Equations.

$$zz = 4Rx - 4xx$$
$$yy = 4Sx - 4xx$$

When the Line is arrived at the Tangent, *z* and *y* will become the two Points of Contact, and then $zz = 4Rx$ and $yy = 4Sx$. ($4xx$ being laid aside as heterogeneous to the rest of the Æquation, by reason of *x* being become infinitely little.) Therefore

$$zz \,.\, yy :: 4Rx \,.\, 4Sx :: R \,.\, S.$$
$$\text{Therefore } z \,.\, y :: \surd R \,.\, \surd S. \quad Q.E.D.$$

PROP. II.] *The Point of Contact between a Sphere and a Plane, is infinitely greater than that between a Circle and a Tangent.*

Fig. 2. Let *a* be the Point of Contact between the Sphere *a d q f* and the Plane *b c*. About the Sphere describe the Cylinder *n p g m*.

Draw *k h* to represent a Circle parallel to the Plane. Let the Circle be supposed to move, till it becomes co-incident with the Plane. The Cylindrical Surface *k h g m* will always be equal (according to *Archimedes*) to the Spherical Surface *d a f*.

Now when these Surfaces become infinitely small, one terminates in the Point of Contact, and the other in the Periphery of the Base of the Cylinder. Therefore the Point of Contact is equal to the Periphery of the Base of the Cylinder (equal to a Periphery which has the same Diameter as the Sphere) and by Consequence is infinitely greater than any Point of Contact between a Circle and a Tangent. *Q. E. D.*

PROP. III.] *The Points of Contact by Spheres of different Magnitude are to one another as the Diameters of the Spheres.*

For by the second Proposition the Points of Contact are equal to the Peripheries of such Diameters, whose Proportion is the same as the Diameters. *Q. E. D.*

II. Let

II. Let $D E$ be the Transverse Axis of the Ellipsis, $A O$ the other Axis, and C the Center of the Section. Let P be any Point in its Circumference, $P Q$ a Tangent to the Curve at P, meeting the Transverse Axis at Q; the Points S, F; the Foci; $C P$, $C K$, Conjugate Semidiameters; $P H$ half the latus rectum to the Diameter $P C$; $P G$ a Perpendicular to the Tangent; let $H G$, perpendicular to $P C H$, meet this in the Point G, so that $P G$ may be the Radius of Curvature of the Ellipsis in the Point P. Also let $S T$, $C R$, and $F V$, be Perpendiculars let fall upon the Tangent $P Q$; let $S O$ be joined, and $P L$ a Perpendicular let fall upon the Axis. These Things supposed, I say that, *Some Properties of Conic Sections deduced from the Nature of Foci by Mr. Ab. de Moivre, n. 352. p. 622. Fig. 3.*

1. *The Rectangle of the Distances from each Focus of the Ellipsis, or* S P x P F, *is equal to the Square of the Semidiameter* C K.

Demonstration.

$P S q = P C q + C S q - 2 C S \times C L$, by 13. II. El.
$P F q = P C q + C S q + 2 C S \times C L$, by 12. II. El.
Whence $P S q + P F q = 2 P C q + 2 C S q$.
Now $P S + P F = D E = 2 C D$; and therefore
$P S q + P F q + 2 P S \times P F = 4 C D q$.
Therefore by Transposition, $2 P S \times P F = 4 C D q - 2 P C q - 2 C S q$.
And by halfing $P S \times P F = 2 C D q - P C q - C S q$.
But it is $C S q = C D q - C O q$, and therefore
$P S \times P F = C D q + C O q - P C q$.
But $C D q + C O q = P C q + C K q$; by 12. VII. Conicks of *Apollonius*.
Therefore $P S \times P F = C K q$. *Q. E. D.*

2. *The Distance from the Focus* S P *is to the Perpendicular let fall upon the Tangent, as the Semiconjugate* C K *is to the lesser Semiaxis* C O.

Demonstration.] Because of similar Triangles $S P T$ and $F P V$, it will be $P S . P F :: S T . F V$; and componendo, $P S + P F$ to $S T + F V$, or their Halves $C D$ to $C R$, as $P S$ to $S T$. Whence $C D \times C K$ to $C R \times C K$ as $P S$ to $S T$. But $C R \times C K$ is equal to the Rectangle of the Semiaxes $C D \times C O$, by 31. VII. *Conic.* Therefore $P S$ is to $S T$ as $C D \times C K$ to $C D \times C O$, or as $C K$ to $C O$. And in like Manner it may be demonstrated, that $P F$ is to $F U$ in the same Ratio. *Q. E. D.*

3. *Also the Transverse Semiaxis* C D, *to a Perpendicular* C R *let fall from the Center* C *to the Tangent, will be in the same Ratio.*

For since the Rectangle $C R \times C K$ is equal to the Rectangle $C D \times C O$, as said before; we shall have the Analogy $C D$ to $C R$ as $C K$ to $C O$. *Q. E. D.*

4. *Any Semidiameter* P C *is to the Distance of the Point* P *from the Focus* S, *or to* S P, *as the Distance from the other Focus* F P *to half the latus rectum belonging to the Vertex* P, *or to* P H.

This is manifest by *Prop.* I. since the Square of $C K$ is equal to the Rectangle $S P \times P F$.

 5. *The*

5. *The Rectangle of the Semiaxes* $CD \times CO$ *is to the Square of the Semiconjugate* CK, *as* CK *to the Radius of Curvature* PG *in the Point* P.

For the Triangles PCR, PGH are similar; whence CR is to PC as half the latus rectum PH to PG; that is, by Property 3 aforegoing, $\frac{CD \times CO}{CK} = CR$ is to PC, as $\frac{CKq}{PC} = PH$ is to $\frac{CKc}{CD \times CO} = PG$. Whence this Analogy $CD \times CO \,.\, CKq :: CK \,.\, PG$.

General Theorem 1.] *The Centripetal Force tending to the same Point* S, *in all Curves is always proportional to the Quantity* $\frac{SP}{PG \times STc}$

This Theorem was found by me many Years ago, and then communicated to my Friends. Since then it has been confirm'd by the Demonstrations of the learned Geometricians D. *J. Bernoulli* in the *Leipsic Acts*, D. *J. Keil* in n. 317 of these Transactions, and by D. *J. Herman* in his *Phoronomia*, p. 70. who may be consulted. Vid. *infra* C. IV. S. V.

Now if we write CKc for PG, by *Prop.* 5. and $\frac{SP}{CK}$, by *Prop.* 2. for ST, (because of CD, CO being given) the Centripetal Force tending to the Focus S of the Ellipsis, will always be as $\frac{SP \times CKc}{CKc \times SPc}$, that is as $\frac{SP}{SPc}$, or $\frac{1}{SPq}$; or reciprocally as the Square of SP. Whence it appears if the Section be an Ellipsis describ'd by the Motion of a Body, the Centripetal Force will be reciprocally as the Square of the Distance of the Center of Force. From these Properties follow some Corollaries that may deserve Observation.

Corol. 1.] *The Velocity of a Body revolving in an Ellipsis, at any Point* P, *to the Velocity in a Circle at the same Distance* SP *from the Center of Force, is in a subduplicate ratio of the Distance from the other Focus* PF, *to the transverse Semiaxis of the Section, or as a mean Proportional between* PF *and* CD *to* CD.

For the Velocity of a Body revolving in an Ellipsis at the Distance SP, to the Velocity of a Body revolving in a Circle or Ellipsis at the Distance of the Semiaxis CD or SO, is as CO to ST; that is, by *Prop.* 2. as $\sqrt{PF}$ to $\sqrt{SP}$. But the Velocity of a Body revolving in a Circle at the Distance CD, is to the Velocity of a Body revolving in a Circle at the Distance SP, as $\sqrt{SP}$ to $\sqrt{CD}$. Therefore *ex æquo* the Velocity of a Body revolving in an Ellipsis at the Distance SP, is to the Velocity of a Body revolving in a Circle at the same Distance, as $\sqrt{PF}$ to $\sqrt{CD}$.

Corol. 2.] *Having given the Velocity in an Ellipsis, the Position of the Tangent, and the Focus or Center of Force, it will be easy to determine the other Focus.*

For let the given Velocity be R; and let the Velocity by which a Circle would be describ'd at the given Distance SP from the Center be Q. Then

Then by the foregoing Corollary 'tis R to Q as $\sqrt{PF}$ to $\sqrt{CD}$, and therefore QQ is to RR as CD to PF, and $2QQ - RR$ will be to RR as SP to PF. But SP is given, and therefore PF is given in Magnitude. It is alſo given in Poſition, becauſe of the Angle VPF equal to SPT. Therefore the Point F the other Focus is given; which being known, the Section is eaſily deſcrib'd.

Now if $\frac{1}{2}RR$ is greater than QQ, the Quantity $2QQ - RR$ will be negative, and inſtead of an Ellipſis the Trajectory to be deſcribed will be changed into an Hyperbola. And it will be $RR - 2QQ$ to RR, as SP to PF, the Diſtance of the other Focus, to be transferr'd to the other Side of the Tangent, that the Focus F may be had. Now all the Properties which we have demonſtrated in the Ellipſis, changing what ought to be changed, will belong alſo to the Hyperbola.

Now if it ſhould happen that QQ is equal to half the Square of R, then $2QQ - RR = 0$, or the Quantity vaniſhes, or the fourth Proportional PF becomes infinite. Therefore the Trajectory to be deſcribed will be a Parabola, the other *Focus* paſſing to an infinite Diſtance. But the Axis of the Trajectory is given in Poſition, for it is parallel to PF, the Angle FPV being now equal to the given Angle SPT. Fig. 4.

Corol. 3.] *The Velocity of a Body revolving in a given Conic Section, at the Diſtance* SP, *is to the Velocity of the ſame revolving at any other Diſtance* SX, *as a mean Proportional between* FP *and* SX, *to a mean Proportional between* SP *and* FX.

For the Velocity in P is as $\sqrt{\frac{FP}{SP}}$, by *Prop* 2. And by the ſame, the Velocity in X is as $\sqrt{\frac{FX}{SX}}$. whence the Propoſition is manifeſt.

Corol. 4.] *Alſo the Ratio of the Velocities of two Bodies revolving in the ſame Syſtem, but in different given Conic Sections, the Diſtances of each being given from the common Focus of the Orbits, may be eaſily obtain'd by Corol.* 1.

For ſince the Velocity of the Body in P is to the Velocity in a Circle at the ſame Diſtance SP, as $\sqrt{PF}$ is to $\sqrt{CD}$; and in the other ſuppoſed Conic Section, whoſe Semiaxis is cd and Foci S, f; at the Diſtance Sp thoſe Velocities are as $\sqrt{pf}$ to $\sqrt{cd}$; but the Velocity of a Body revolving in a Circle at the Diſtance SP, is to the Velocity in a Circle at the Diſtance Sp, as $\sqrt{Sp}$ to $\sqrt{SP}$; theſe Ratio's being compounded, the Velocity in P will be to the Velocity in p, as $\sqrt{PF \times cd \times Sp}$ to $\sqrt{pf \times CD \times SP}$. Now if the other Section is a Parabola, cd, pf will be infinite, but in the Ratio of 1 to 2. Therefore the Ratio of the Velocities will be as $\sqrt{PF \times SP}$ to $\sqrt{2CD \times SP}$.

Corol. 5.] *If in the Hyperbola the Point* p *paſſes to Infinity, it is plain from the foregoing, that the laſt and leaſt Velocity with which a Body would aſcend for ever, is equal to that with which a Body at the Diſtance* CD, *equal to the tranſverſe Semiaxis, would deſcribe a Circle.*

Corol.

Corol. 6.] *From the given Distance from the Focus, the Position of the Tangent is also given, or the Angle* S P T *contain'd by the Distance* S P *and the Tangent* P T.

For by *Prop.* 2. 'tis PS to ST as CK to CO, or as $\sqrt{SP \times PF}$ to CO; and so is Radius to the Sine of Ang. SPT. But in Ellipses that approach near to Circles, it would be better to seek the Angle PST, the Complement of the same to a Quadrant. Now the Sine of this is to Radius as $\sqrt{SP \times PF - COq}$ is to $\sqrt{SP \times PF}$.

Corol. 7.] *And hence the Velocities follow with which the Distances* S P *increase or decrease.*

For from the preceding Corollary since it is as $\sqrt{SP \times PF}$ to $\sqrt{SP \times PF - COq}$, so Radius to the Sine of the Angle PSF; and in the same Ratio is the Velocity of the Body in P to the Velocity of the Moment of SP; but that Velocity in P is (by *Prop.* 2.) as $\sqrt{\frac{PF}{SP}}$; omitting the superfluous Quantities, $\frac{\sqrt{SP \times PF - COq}}{SP}$ will always be proportional to the Velocity wherewith the Distance SP increases or decreases.

General Theorem II.] *In every Curvilinear Trajectory the Angular Velocities about the Center of Forces are reciprocally proportional to the Squares of the Distances from the Center.*

For because of the equal Areæ of the nascent Sectors, the Arches subtended by these least Angles, or the Bases, are reciprocally as the Radii. Therefore the Angles of these least Sectors equal in Area, are to one another in a duplicate Ratio of the Radii reciprocally, or as the Squares of the Distances.

Corol 8.] *Hence the Angular Velocities of Bodies revolving in different given Ellipses may be compared to one another.*

For the Angular Velocities by which Circles would be described at Distances equal to the transverse Semiaxes, are reciprocally in the sesquialter Ratio of the Axes, or as $\frac{1}{CD\sqrt{CD}}$. But revolving Bodies have these mean Angular Velocities, when the Squares of the Distances are equal to the Rectangles of the Semiaxes of the Ellipses. Therefore by *Theor.* 2. it will be SPq to $CD \times CO$, so is $\frac{1}{CD\sqrt{CD}}$ to $\frac{CO}{SPq \times \sqrt{CD}}$; which Quantity is as the Velocity of the Angle at the Center S, described in a given least Time by the Motion of the right Line SP.

Corol. 9.] *The Angular Velocity by which the Tangent* P T *performs its Rotation, or the right Line* S T *perpendicular to the Tangent, is to the Angular Velocity of the right Line* S P, *as the transverse Semiaxis* C D *is to the Distance* P F *from the other Focus.*

Fig. 5. *Demonstration.*] Let the Points P, p, be very near each other, and drawing SP, Sp, let PT, pt be two Tangents, to which let be drawn the Perpen-

Perpendiculars ST, St. Parallel to theſe let the Radii of Curvature PG, pG, be drawn, meeting in G; and with Center S and Radius SP let the little Arch PE be drawn, meeting Sp in E. It is evident that the Angle PGp is equal to the Angle TSt, or to the Angular Velocity of the Perpendicular ST. But the Angle PSp is the Angular Velocity of the right Line SP. So that PGp is to the Angle PSp, as the Angular Velocity of ST is to the Angular Velocity of the right Line SP; that is, as $\frac{Pp}{PG}$ is to $\frac{PE}{PS}$. But $Pp \,.\, PE :: SP \,.\, ST :: CK \,.\, CO$, by *Prop.* 2. Therefore theſe Velocities are as $\frac{CK}{PG}$ to $\frac{CO}{PS}$. For PG write $\frac{CK\,cub.}{CD \times DO}$, by *Prop.* 5. and $\frac{CK}{PG}$ will become $\frac{CD \times CO}{CKq} = \frac{CD \times CO}{PS \times PF}$. Hence $\frac{CD \times CO}{PS \times PF}$ will be to $\frac{CO}{PS}$, or expunging what is ſuperfluous, CD to PF, as the Angle TSt is to the Angle PSp, or the Angular Velocity of the Tangent to the Angular Velocity of the Diſtance SP. Therefore the Velocity with which the Tangent revolves will always be proportional to the Quantity $\frac{CO \times \surd CD}{PF \times SPq}$.

In Sect 3. Lib. I. *Of the Principles of Natural Philoſophy, the Reader may find moſt of theſe Corollaries, derived from other Properties of the Conic Sections, or eaſily to be derived from them.*

III. I propoſe a Method of Tangents, (immediately derived from the Theory of the *Maxima* and *Minima*,) which is eaſy and ſufficiently general, nay the moſt general of all, as being with the ſame Labour applicable to all Curves. Nor ſhall I ſcruple to call it a new one, ſince no one of the celebrated Geometricians (as far as I have been able to learn) have ever publiſhed any Thing of this Kind. I ſhall here only produce a few Inſtances. *Tangents to Curves*, &c. *by Mr.* H. Ditton, n. 284. p. 1333.

Let A G H be a Curve, whoſe Vertex is A, its Axis A K, ordinate F D, and its Center (if it have any) the Point K. Taking L a Point in the Axis, make A L$=n$, A D$=x$, F D$=y$, F L$=z$. Of theſe Quantities the three laſt are flowing Quantities, and n is a conſtant Quantity; for this being always the ſame, anſwers to the others which are always variable. From the right-angled Triangle F D L, we have this Equation, $zz = yy + nn - 2nx + xx$; and determining z to be an Extream, there ariſes $2y\dot{y} - 2n\dot{x} + 2x\dot{x} = 0$; whence by interpreting $2\dot{y}y$ according to the particular Nature of the Curve, the Quantity n will be left, expreſs'd in Terms that will alſo be proper to the Curve. Fig. 6.

And now by this Means having z determined to its extreme Value, that is, having the Line F L either the greateſt or leaſt of all thoſe which can be drawn to the Curve from the Point L, and therefore perpendicular to the

the Curve in the Point F; it is evident that D L is the Subnormal, from whence the Subtangent is eaſily derived.

For an Example let us firſt take the *Apolonian* Parabola, which Curve we will ſuppoſe to be here delineated. Therefore we have $2 y \dot{y} = r \dot{x}$, ſuppoſing r to be the Parameter; whence $r \dot{x} - 2 n \dot{x} + 2 x \dot{x} = o$, and $n = \frac{r}{2} + x$. Therefore the Subnormal D L $= \frac{1}{2} r$. Now the Meaning of this Theorem is this. If beyond the Limit D of the Abſciſs A D there is taken D L equal to the Semiparameter, and from the Point L be drawn L F ſtrait to the Point F; the right Line ſo drawn will be perpendicular to the Parabola in the Point F, and the leaſt of all the Lines that can be drawn to the Curve from the Point L. I ſay it is the leaſt; for to any one that conſiders the Nature of the Curve, it is evident it cannot be the greateſt, (which I would have obſerved in what follows;) but it is neceſſarily either the greateſt or leaſt, and therefore the latter. And this is the firſt Part of Theor. 5. Lib. 7. of *de la Hire*'s Conicks.

Let the Ordinate E B be drawn, and join the Points E, L. make the intercepted Line B D $= f$, whence A B $= x - f$, and B L $= \frac{r}{2} + f$. Now L E q $= \frac{rr}{4} + r x + f f$, and F L q $= \frac{rr}{4} + r x$. Therefore L E q — F L q = B D q, which is the latter Part of the ſame Theorem.

The nearer the Point F approaches to the Point A, or to the Vertex, in which the Perpendicular cuts the Curve, the nearer alſo the Point L approaches to the ſame. Therefore when F coincides with A, and ſo the Ordinate F D vaniſhes, then the *Minimum* itſelf lies in the Axis A K, and will be equal to the Semiparameter. That is, in this Caſe $n = \frac{1}{2} r$ only; the Abſciſs x belonging to the vaniſhing Ordinate then alſo vaniſhing. If therefore A L $= n = \frac{1}{2} r$, taking the Point D between A and L, make A D $= \dot{x}$; then there ariſes F L q $= \frac{rr}{4} + \dot{x} \dot{x}$, and therefore F L q — A L q $= \dot{x} \dot{x}$, that is, F L q — A L q = A D q. As it is Theor. 2. L. 7. Conic. *de la Hire*.

Secondly, Let there be a certain Curve of a ſuperior Parabolic Order, whoſe Equation is $r^{p-q} x^q = y^p$

Then $yy = r^{\frac{2p-2q}{p}} x x^{\frac{2q}{p}}$, and therefore

$2y\dot{y} = \frac{2q}{p} r^{\frac{2p-2q}{p}} x^{\frac{2q-p}{p}} \dot{x}$. Now if we ſubſtitute this Value inſtead of $2y\dot{y}$ in the general Equation, which determines z to be an Extream, we ſhall have from thence

$n = \frac{q}{p} r^{\frac{2p-2q}{p}} x^{\frac{2q-p}{p}} + x$, and therefore the Subnormal is

$DL = \frac{q}{p} r^{\frac{2p-2q}{p}} x^{\frac{2q-p}{p}}$. Now this is eaſily apply'd to any of theſe Curves, if the Indices p and q are rightly expounded, according to the Nature and Genius of each Curve.

Thirdly, let it be ſuppoſed that the Curve is an Ellipſis, of which A K is half the greater Axis. Now it follows from its Equation that $2y\dot{y} = r\dot{x} - \frac{2rx\dot{x}}{q}$. Whence there ariſes $r\dot{x} - \frac{2rx\dot{x}}{q} - 2n\dot{x} + 2x\dot{x} = o$, and $n = \frac{r}{2} + x - \frac{rx}{q}$. Wherefore $\frac{r - rx}{2q} = DL$ the Subnormal. Now if inſtead of the Ellipſis a Circle were ſubſtituted, by proceeding with the Equation in the ſame Manner, we ſhould find $DL = r - x$, making r to be the Radius of the Circle.

But let us return back to the Ellipſis, another of whoſe Properties may be derived from hence, as was done in the Parabola.

Make $BD = f$, whence $AB = x - f$. Then we ſhall have $LEq =$ $(LBq + EBq =) \frac{rr}{4} - \frac{rrx}{q} + \frac{rrxx}{qq} + ff + rx - \frac{rxx}{q} - \frac{rff}{q}$. And $FLq = (FDq + LDq =) rx - \frac{rxx}{q} + \frac{rr}{4} - \frac{rrx}{q} + \frac{rrxx}{qq}$. Therefore $LEq - LFq = ff - \frac{rff}{q}$. Now this is Theor. 6. Lib. 7. Conic. *de la Hire.*

For that great Geometrician requires, that it may be $q \,.\, r :: \frac{q}{2} - x$. L D, whoſe Value therefore is $\frac{r - rx}{2q}$ as found above. Therefore it is a fourth Proportional to the three Quantities before exhibited. This being granted to him he evidently demonſtrates, that L F is the leaſt of all the Lines that can be drawn from the Point L to the Ellipſis. Moreover becauſe it is $q \,.\, q - r :: f \,.\, f - \frac{fr}{q}$. Therefore the Rectangle $ff - \frac{rff}{q} f \times f - \frac{rf}{q}$ is the ſame Rectangle which D. *de la Hire* calls his Specimen. But this Specimen, (according to his Definition) is a Rectangle like to the Rectangle that conſtitutes the Difference between the Square of the tranſverſe Axis and the Figure, (that is, the Rectangle $qq - qr$,) being beſides apply'd to the right Line B D or f. Now that the Rectangle $ff - \frac{rff}{q}$ has all theſe Conditions is very evident.

It may be obſerved, that it follows from the Value of n before found, that $n > \frac{r}{2}$. For $n = \frac{r}{2} + x - \frac{rx}{q}$. Therefore $qn + rx = \frac{qr}{2} + qx$. But becauſe $q > r$, 'tis $qx > rx$, and therefore $qn > \frac{qr}{2}$, and $n > \frac{r}{2}$.

When the Point *F* (as was juſt now obſerved in the Parabola) falls upon the Vertex *A*, the *Minimum* is determined in the Axis : And becauſe of x vaniſhing, we ſhall have $n = \frac{1}{2} r$. Then aſſuming any Point D between *A* and *L*, if A D is equal to any x, by Compariſon there ariſes $FLq - ALq = xx - \frac{rxx}{q}$. And this is Theor. 3. Lib. 7. of *de la Hire*'s Conicks. For becauſe it is $q \,.\, q - r :: x \,.\, x - \frac{rx}{q}$, it appears that the Rectangle $xx - \frac{rxx}{q}$ is the Exemplar, but apply'd to the Abſciſs x. And therefore this is the adequate Meaſure of the Defect of the Square of the leaſt Line, from the Square of any other right Line drawn from

from the ſame Point to the Curve. And this is what he demonſtrates in the Place above cited.

Now the Theorems belonging to the leſſer or conjugate Axis of the Ellipſis, (for hitherto we have inſiſted on the greater or tranſverſe Axis) are determined juſt in the ſame Manner. For now let A K, or half the leſſer Axis, be $\frac{c}{2}$, R the Parameter; and the Point L is now ſuppoſed to be placed beyond the Centre, on the other Side of G K. By working as before, we ſhall find A L or $n = \frac{R}{c} + x - \frac{Rx}{2}$, and the Subnormal $DL = \frac{R}{c} - \frac{Rx}{2}$. That is, $c . R :: \frac{c}{2} - x . \frac{R}{2} - \frac{Rx}{c}$; and therefore drawing F L, it will be the greateſt of all the Lines that can de drawn from the Point L to the Ellipſis; and $LFq - LEq = \frac{Rff}{c} - ff =$ to the Rectangle, which is the Exemplar, apply'd to B D or f. For it appears that this is the Exemplar, for it is $c . R - c :: f . \frac{Rf}{c} - f$, and therefore, according to the Definition, $\overline{\frac{Rf}{2} - f} \times f$ is equal to the Exemplar. Now this is Theor. 7. Lib. 7. of *de la Hire*'s Conicks.

Again, when the Point F coincides with A, becauſe of x vaniſhing with the Ordinate then vaniſhing, there is left $n = \frac{R}{2}$, and A L is the greateſt of all the Lines that can be drawn from the Point L to the Ellipſis, and $ALq - FLq = \frac{Rxx}{c} - xx =$ to the Exemplar apply'd to A D or x. And the ſame as to Theor. 4. of Lib. præd. Conicks.

But it ought to be obſerved at the foregoing Caſe, (which ſhould have been mentioned before) when we found $n = \frac{R}{2} + x - \frac{Rx}{c}$, that $n <$ $\frac{R}{2}$. For $cn + Rx = \frac{Rc}{2} + cx$; and becauſe $R > c$, therefore $Rx >$ cx, and there will be left $cn < \frac{Rc}{2}$, or $n < \frac{R}{2}$.

Now as the Matter is perform'd in the Ellipſis, ſo in the ſame Manner it might be perform'd in the Hyperbola, and the leaſt Lines may alſo be determined in this Curve. But there is ſuch a Connection between theſe two Curves, and the Tranſition from one to the other is ſo eaſy, that the Labour may ſeem unneceſſary even to Novices. Therefore nothing more remains to determine the Subnormal, than that the Sign — may be changed into +. For ſince in the Hyperbola it is $2y\dot{y} = r\dot{x} + \frac{2rx\dot{x}}{q}$, and $n = \frac{r}{2} + x + \frac{rx}{q}$; (the general Equation) there remains D L $= \frac{r}{2} + \frac{rx}{q}$.

Let it be conceived fourthly, that the Curve M S N (drawn on the other Side the Figure) is one of the Hyperboloids, whoſe Aſymptotes are A K, K H, and the right Line S R an Ordinate to the Aſymptote K H; make S R $= y$, S P $= z$, K R $= x$, K P $= n$, which here muſt needs be leſs than x, as will appear on Conſideration. The Equation proper to the Curve is $y^p x^q = r^q s^p$, inſtead of which, (becauſe of r and s being determinate Quantities) may be wrote $y^p = x^{-q}$, and therefore

$y^2 = x^{\frac{-2q}{p}}$ and $2y\dot{y} = -\frac{2q}{p}\dot{x}\, x^{\frac{-2q-p}{p}}$ Hence ſince it is

$zz = yy + xx - 2nx + nn$, for an Extream we have

$2y\dot{y} + 2x\dot{x} - 2n\dot{x} = 0$, that is, $-\frac{2q}{p}\dot{x}\, x^{\frac{-2q-p}{p}}$

$+ 2x\dot{x} = 2n\dot{x}$ and $n = x - \frac{q}{p}x^{\frac{-2q-p}{p}}$

Therefore the Subnormal P R $= (x - n =) \frac{q}{p}x^{\frac{-2q-p}{p}}$.

Laſtly, let us conceive the Curve A F G to be a primary Cycloid, and let the Radius be r, the Arch c, and the Ordinate of the generating Circle to be y, whoſe Diameter may be repreſented by A K, and the Center poſited between L and K. Then calling F D the Ordinate of the Cycloid a, and the reſt as before, the Equation of the Curve is $aa = yy + 2cy + cc$, and therefore $zz = (aa + nn - 2nx + xx =)\, yy + 2cy + cc + nn - 2nx + xx$, and z being determin'd for an Extream, $2y\dot{y} + 2c\dot{y} + 2y\dot{c} + 2c\dot{c} - 2n\dot{x} + 2x\dot{x} = 0$. But $\dot{y} = \frac{r\dot{x} - x\dot{x}}{y}$, and $\dot{c} = \frac{r\dot{x}}{y}$. Then ſubſtituting theſe Values, and duly reducing the Equation, we ſhall have $2r - x + \frac{2rc - 2xc}{y} + 2r + \frac{2cr}{y} = 2n - 2x$, and

therefore

therefore $2r - x + \frac{2rc - xc}{y} = n - x =$ D L the Subnormal.

The incomparable Dr. *Barrow* makes uſe of the Subtangent as already known, to determine the *Maximum* and *Minimum*. And Mr. *Newentiit*, in his Analyſis of Infinites, has done the ſame after him. But ſince the *Maxima* and *Minima* may be found by many other Methods, in which nothing need be preſuppoſed about the Tangents of Curves, it is plain that we may ſafely proceed from the *Maxima* and *Minima*, to inveſtigate the Method of Tangents.

Corol. 1.] In going over again the foregoing Examples, it will appear from each, that $2y\dot{y} - 2n\dot{x} + 2x\dot{x} = o$, by putting inſtead of n in this Equation its Value derived from the Nature of the Curve. For Example in the Hyperboloids

$$\frac{2q}{p}\dot{x} \quad x^{\frac{2q-p}{p}} - 2x\dot{x} - \frac{2q}{p}\dot{x}x^{\frac{2q-p}{p}} + 2x\dot{x} = o,$$

which appears by Inſpection. And the ſame will appear to be true in other Examples, without any Demonſtration.

Corol. 2.] From the Invention of the Subnormals we may eaſily determine the greateſt and leaſt Ordinates of Curves. In which Matter I ſhall add, if the Subnormal belonging to any Point of the Curve be put equal to nothing, we ſhall have the Ordinate of that Curve determin'd to be an Extream. And it will be the greateſt, if it is on the concave Side of the Curve, but the leaſt, if it is on the convex Side. For Example in the Circle, (making the Subnormal $= l$) it will be $l = r - x$. Let $r - x = o$, then $r = x$ and $y = r$; that is, the greateſt Ordinate is equal to the Radius. In like Manner in the Ellipſis $l = \frac{r}{2} - \frac{rx}{q}$; let $\frac{r}{2} - \frac{rx}{q} = o$, then $rq = 2rx$, or $x = \frac{q}{2}$. Therefore $yy = \frac{rq}{4}$, equal to a fourth Part of the Figure as they call it, or the Square of the conjugate Semiaxis, and therefore the greateſt y is equal to that Semiaxis. And the ſame Method may be uſed in other Curves. Let the Subnormal be found from the given Equation, and making that equal to nothing, we ſhall have the Ordinate of the Curve determin'd to a *Maximum* or *Minimum*; the firſt towards the concave Part of the Curve, and the other towards the convex Part.

POSTSCRIPT.

Firſt, it will be eaſy by this Method to determine the Tangent, by operating at the convex Side of the Curve, as before on the concave Side.

For

For let A C be the vertical Tangent, and C a Point in it taken at Pleaſure. Make $AC = n$, $CO = z$, (by which Symbol let all the Lines be denoted, which are drawn from the Point C to the convex Curve A E G.) Then drawing M O always perpendicular to A C, it will be $CM = n - y$. And ſince $OM = x$, it will be $zz = nn - 2ny + yy - xx$; and therefore (for an extream Value of z) $2y\dot{y} + 2x\dot{x} - 2n\dot{y} = o$. In which Equation, if $2x\dot{x}$ be expounded according to the Nature of the Curve, we ſhall have the Line C Z determined, which in this Place performs the Office of a Subnormal. This is too clear to want any Illuſtration by Examples.

Secondly, As in the foregoing Method we have found the Tangents of Curves, by determining to Extreams the Lines L E or C O, drawn from a given Point either in the Axis or in the vertical Tangent; thus by conſidering the Lines Q E, *&c.* drawn from a given Point in the Axis beyond the Vertex, the ſame may be perform'd, and that univerſally. For all the Lines Q E are of a flowing and variable Nature, but the Tangent Q F alone, (ſuppoſing Q F to touch the Curve) is conſtant and determin'd to one Value. Therefore in this Place we ſhall not inſiſt on the Hypotheſis of an Extream, but ſhall only conſider it as a permanent Quantity. Let two Points Q, L, be aſſumed, and thence to the ſame Point of the Curve E let two Lines L E, Q E, be always drawn. The Angle Q E L between the Point of Contact F and the Vertex, will always be obtuſe, but on the other Side of the Point F it will be acute; ſuppoſing, as ſaid before, that Q F touches the Curve, and F L is at right Angles to it. Make $QA = p$, $AL = n$, $AB = x$, $BE = y$, and $QE = z$. Alſo $VE = v$, which is intercepted between the Points E and V, where Q V falls perpendicularly from Q upon L E produced. Now becauſe of the obtuſe-angled Triangle Q E, we ſhall have this Equation,

$$zz = pp + 2pn - yy - xx \pm \sqrt{yy + nn - 2nx + xx} \times 2v;$$

or inſtead of $\frac{1}{2}\sqrt{yy + nn - 2nx + xx}$ writing f,

it will be $zz = pp + 2pn - yy - xx + 2nx - 2fv$, and thence $2z\dot{z} = y\dot{y} - 2x\dot{x} + 2n\dot{x} - 2f\dot{v} - 2v\dot{f}$. Now if z is a conſtant Quantity, in which Caſe Q E will coincide with the Tangent Q F, it will be then $-2y\dot{y} - 2x\dot{x} + 2n\dot{x} = o$, the Rectangle $2fv$, and therefore its Fluxion intirely vaniſhing. But this is the very general Equation, that was determined by the foregoing Method, which is deduced with the ſame Eaſe from the Suppoſition of a conſtant Quantity, as before from the Principle of an extream Quantity.

IV. Let

A Method of Squaring some Kinds of Curves, by Mr. Abr. de Moivre, n. 278. p. 1113.

IV. Let A be the Area of a Curve, whose Abscis is x, and Ordinate $x^m \sqrt{dx - xx}$. Let B be the Area of a Curve, whose Abscis is the same as the former, but its Ordinate is $x^{m-n} \sqrt{dx - xx}$. Let $\sqrt{dx - xx} = y$, the Area will be

$$d^n B \times \frac{2m+1}{2m+4} \times \frac{2m-1}{2m+2} \times \frac{2m-3}{2m} \times \frac{2m-5}{2m-2}, \&c. = P$$

$$-\frac{1}{m+2}\, x^{m-1}\, y^3 = -Q$$

$$-\frac{d}{m+1} \times \frac{2m+1}{2m+4}\, x^{m-2}\, y^3 = -R$$

$$-\frac{d^2}{m} \times \frac{2m+1}{2m+4} \times \frac{2m-1}{2m+2}\, x^{m-3}\, y^3 = -S$$

$$-\frac{d^3}{m-1} \times \frac{2m+1}{2m+4} \times \frac{2m-1}{2m+2} \times \frac{2m-3}{2m}\, x^{m-4}\, y^3 = -T, \&c.$$

Here it is to be observed, *first*, that n is supposed to be an integer and affirmative Number. *Secondly*, that the Quantity $d^n B$, in the Series denoted by P, must be multiply'd into so many Terms as there are Units in n. *Thirdly*, that so many of the following Series, denoted by $-Q$, $-R$, $-S$, &c. ought to be taken, as there as Units in n. Now that this may be made plain by an Example or two, I say, that if $n = 1$, then the Area will be

$$A = d^n B \times \frac{2m+1}{2m+4} - \frac{1}{m+2}\, x^{m-1}\, y^3.$$ And if $n = 2$, then

$$A = d^n B \times \frac{2m+1}{2m+4} \times \frac{2m-2}{2m+2} - \frac{1}{m+2}\, x^{m-1}\, y^3$$
$$-\frac{d}{m+1} \times \frac{2m+1}{2m+4}\, x^{m-2}\, y$$

Fourthly, if we make $y = \sqrt{dx - xx}$, then $A = Q - R + S - T$, &c. $\pm P$.

Corol.

Corol. 1.] If m be ſuppoſed equal to any Term of this

$$\text{Series, } -\frac{1}{2},\ \frac{1}{2},\ \frac{3}{2},\ \frac{5}{2},\ \frac{7}{2},\ \frac{9}{2},\ \text{\&c.}$$

the Quadrature of the Curve, whoſe Ordinate is $x^m\sqrt{dx-xx}$, or $x^m\sqrt{dx+xx}$ becomes finite, and will be exhibited by our Series. To make this plain by an Example, let the Area of the Curve be ſought, whoſe Ordinate is $x^{-\frac{1}{2}}\sqrt{dx-xx}$. Suppoſe this Curve to be compared with the Curve whoſe Ordinate is $x^{-\frac{3}{2}}\sqrt{dx-xx}$; becauſe in this Caſe $n=1$, therefore

$$A = d^n B + \frac{2m+1}{2m+4} - \frac{1}{m+2} x^{m-1} y^3$$

But $m = -\frac{1}{2}$, and therefore $2m+1=0$. So that

$$A = -\frac{1}{m+2} x^{m-1} y^3 = -\frac{2y^3}{3\sqrt{x^3}}$$

Here it muſt be obſerved, that the Area thus found will ſometimes be deficient from the true Area, and ſometimes exceed it by a given Quantity. Now that that Exceſs or Defect may become known, let the Area thus found be ſuppos'd to be increaſed or diminiſhed by a given Quantity q, and then putting $x=0$, let the Area thus increaſed or diminiſhed be made equal to 0. Thus in the preſent Caſe q will be found

$=\frac{2}{3}d\sqrt{d}$, and therefore

$$A = \frac{2}{3}d\sqrt{d} - \frac{2y^3}{3\sqrt{x^3}}.$$

Corol. 2.] If n is ſuppoſed equal to any Term of the following Series 3, 4, 5, 6, 7, &c. the Quadrature of the Curve, whoſe Ordinate is $x^{-n}\sqrt{dx-xx}$ or $x^{-n}\sqrt{dx+xx}$, becomes finite, and is exhibited by our Series. Let the Area of the Curve be required, whoſe Ordinate is $x^{-3}\sqrt{dx-xx}$. Suppoſe it to be compared with the Area of the Circle, which may be called A. Then $m=0$, $n=3$, and therefore $A=P-Q-R-S$. But ſince the Quantity $2m$ is infinitely little or nothing, and is found in the Denominator of the third Term by which $d^n B$ is multiply'd; the Quantity denoted by P becomes infinite, and for the ſame Reaſon the Quantity denoted by $-S$ becomes infinite; and therefore the Quantities A, $-Q$, $-R$ vaniſh. Therefore $P=S$.

Now

Now this Equation divided by $\frac{2m+1}{2m+4} \times \frac{2m-1}{2m+2}$ becomes

$d^n B \times \frac{2m-3}{2m} = \frac{dd}{m} x^{m-3} y^3$ or $d^n B \times \frac{2m-3}{2}$

$= d d x^{m-3} y^3$. And writing 0 and 3 for m and n, there arises

$$dB \times -\frac{3}{2} = \frac{y^3}{x^3}, \text{ or } B = -\frac{2y^3}{3dx^3}.$$

Corol. 3.] If m is suppos'd equal to any Term of the following Series, $-2, -1, 0, 1, 2, 3, 4,$ &c. The Quadrature of the Curve, whose Ordinate is $x^m \sqrt{dx - xx}$, depends on the Quadrature of the Circle. But the Area of the Curve, whose Ordinate is $x^m \sqrt{dx + xx}$, depends on the Quadrature of the Hyperbola; and the Relation of that Curve to the Circle or Hyperbola, is exhibited by our Series in finite Terms.

Corol. 4.] If m is expounded by any other Number different from any before assigned, the Curve, whose Ordinate is $x^m \sqrt{dx - xx}$ or $x^m \sqrt{dx + xx}$, is neither exactly squared, nor depends on the Circle or Hyperbola, but is reduced to a simpler Curve by our Series.

Theorem 2.] Let A be the Area of a Curve, whose Absciss is x and Ordinate $\frac{x^m}{\sqrt{dx-xx}}$. Let B be the Area of a Curve, whose Abscis is the same as the former, but its Ordinate is $\frac{x^{m-n}}{\sqrt{dx-xx}}$. Make $\sqrt{dx - xx}$ $= y$ Then

$$A = d^n B \times \frac{2m-1}{2m} \times \frac{2m-3}{2m-2} \times \frac{2m-5}{2m-4} \times \frac{2m-7}{2m-6} \text{ \&c.} = P.$$

$$-\frac{1}{m} \times x^{m-1} y = -Q$$

$$-\frac{d}{m-1} \times \frac{2m-1}{2m} \times x^{m-2} y = -R$$

$$-\frac{d^2}{m-2}\times\frac{2m-1}{2m}\times\frac{2m-3}{2m-2}\times x^{m-3}y=-S$$

$$-\frac{d^3}{m-3}\times\frac{2m-1}{2m}\times\frac{2m-3}{2m-2}\times\frac{2m-5}{2m-4}\times x^{m-4}y=-T, \text{ \&c.}$$

The Observations made upon the first Theorem obtain here also, and likewise in the following.

Corol. 1.] If m is supposed equal to any Term of the following Series, $\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$, $\frac{7}{2}$, $\frac{9}{2}$, &c. the Quadrature of the Curve, whose Ordinate is $\frac{x^m}{\sqrt{dx-xx}}$ or $\frac{x^m}{\sqrt{dx+xx}}$, becomes finite, and is exhibited by this Series.

Corol. 2.] If n is supposed equal to any Term of the following Series, 1, 2, 3, 4, 5, 6, 7, &c. every Curve, whose Ordinate is $\frac{x^{-n}}{\sqrt{dx+xx}}$ or $\frac{x^{-n}}{\sqrt{dx+xx}}$, is squared in finite Terms by this Series.

Corol. 3.] If m is expounded by any Term of the following Series, 0, 1, 2, 3, 4, 5, &c. the Curve whose Ordinate is $\frac{x^m}{\sqrt{dx-xx}}$ depends on the Quadrature of the Circle. But the Curve whose Ordinate is

Fig. 7. $\frac{x^m}{\sqrt{dx+xx}}$ depends on the Quadrature of the Hyperbola. For if Center C, and Diameter $AB=d$, a Circle AEB is describ'd, and $AD=x$ is taken; raise the Perpedicular DE, and join CE. The Sector AEC divided by $\frac{1}{8}dd$ is equal to the Area of the Curve whose Ordinate is

Fig. 8. $\frac{x^0}{\sqrt{dx-xx}}$. After the same Manner if Center C, and transverse Axis $AB=d$ an equilateral Hyperbola AE is described; let there be taken AD

$AD = x$, let DE be raiſed at right Angles, and CE be join'd. The Sector ACE divided by $\frac{1}{8} dd$ is equal to the Area of the Curve whoſe Ordinate is $\frac{x^0}{\sqrt{dx + xx}}$.

Corol. 4.] If m be ſuppoſed equal to any other Number not within the foregoing Limitations, the Curve whoſe Ordinate is $\frac{x^m}{\sqrt{dx - xx}}$ or $\frac{x^m}{\sqrt{dx + xx}}$, can neither be exactly ſquared, nor does it depend on the Circle or Hyperbola, but however is reduced to a ſimpler Curve.

Theor. 3.] Let A be the Area of a Curve whoſe Abſciſs is x, and Ordinate $x^m \sqrt{rr - xx}$. Let B be the Area of a Curve whoſe Abſciſs is the ſame x, and Ordinate $x^{m-2n} \sqrt{rr - xx}$: make $\sqrt{rr - xx} = y$. Then

$$A = r^{2n} B \times \frac{m-1}{m+2} \times \frac{m-3}{m} \times \frac{m-5}{m-2} \times \frac{m-7}{m-4}, \text{ \&c.} = P$$

$$- \frac{1}{m+2} x^{m-1} y^3 = -Q$$

$$- \frac{r^2}{m} \times \frac{m-1}{m+2} x^{m-3} y^3 = -R$$

$$- \frac{r^4}{m-2} \times \frac{m-2}{m+2} \times \frac{m-3}{m} x^{m-5} y^3 = -S, \text{ \&c.}$$

Corol. 1.] If m is expounded by any Term of the Series 1, 3, 5, 7, &c. the Qnadrature of the Curve whoſe Ordinate is $x^m \sqrt{rr - xx}$, or $x^m \sqrt{rr + xx}$, becomes finite, and is exhibited by this Theorem.

Corol. 2.] If n is interpreted by any Term of the following Series, 2, 3, 4, 5, 6, &c. the Curve whoſe Ordinate is $x^{-2n} \sqrt{rr - xx}$, or $x^{-2n} \sqrt{rr + xx}$, is ſquared exactly by this Theorem.

Corol. 3.] If m is interpreted by any Term of the following Series, -2, 0, 4, 6, 8, &c. the Quadrature of the Curve whoſe Ordinate is

$x^m\sqrt{rr-xx}$, depends on the Circle; and the Quadrature of the Curve whose Ordinate is $x^m\sqrt{rr+xx}$, depends on the Hyperbola.

Corol. 4.] If m is interpreted by any Number differing from those afore-mention'd, the Curve whose Ordinate is $x^m\sqrt{rr-xx}$, or $x^m\sqrt{rr+xx}$, is neither exactly squared, nor depends on the Circle or Hyperbola, but is reduced to a simpler Curve.

Theor. 4.] Let A be the *Area* of a Curve whose Abscis is x, its Ordinate $\frac{x^m}{\sqrt{rr-xx}}$, and let B be the Area of a Curve whose Abscis also is x, and its Ordinate $\frac{x^{m-2n}}{\sqrt{rr-xx}}$. Then

$$A = r^{2n}B \times \frac{m-1}{m} \times \frac{m-3}{m-2} \times \frac{m-5}{m-4} \times \frac{m-7}{m-6}, \&c. = P$$

$$-\frac{1}{m}x^{m-1}y = -Q$$

$$-\frac{r^2}{m-2} \times \frac{m-1}{m}x^{m-3}y = -R$$

$$-\frac{r^4}{m-4} \times \frac{m-1}{m} \times \frac{m-3}{m-2}x^{m-5}y = -S$$

$$-\frac{r^6}{m-6} \times \frac{m-1}{m} \times \frac{m-3}{m-2} \times \frac{m-5}{m-4}x^{m-7}y = -T, \&c.$$

Corol. 1.] If m be expounded by any Term of the following Series, 1, 3, 5, 7, 9, &c. the Quadrature of the Curve whose Ordinate is $\frac{x^m}{\sqrt{rr-xx}}$, or $\frac{x^m}{\sqrt{rr+xx}}$ is had by this Theorem in finite Terms.

Corol. 2.] If n is expounded by any Term in the following Series,

1,

1, 2, 3, 4, 5, &c. the Curve whose Ordinate is $\frac{x^{-2n}}{\sqrt{rr-xx}}$ or $\frac{x^{-2n}}{\sqrt{rr+xx}}$ is perfectly squared by this Theorem.

Corol. 3.] If m is expounded by any Term in the following Series, 0, 2, 4, 6, 8, &c. the Quadrature of the Curve whose Ordinate is $\frac{x}{\sqrt{rr-xx}}$ depends on the Quadrature of the Circle. For if Center C, Radius $CA = r$, a Circle AEG is described, make DE perpendicular to CD, and join CE; the Sector CAE divided by $\frac{1}{2}rr$ is equal to the Area of a Curve whose Ordinate is $\frac{x^0}{\sqrt{rr-xx}}$. In the same Manner if C be the Center, $CA = r$ the transverse Semiaxis of an equilateral Hyperbola EAM; draw $CF = x$ perpendicular to AC, draw FE parallel to the Axis till it meets the Hyperbola in E, and join CE. The Hyperbolical Sector ACE divided by $\frac{1}{2}rr$ is equal to the Area of a Curve whose Ordinate is $\frac{x^0}{\sqrt{rr+xx}}$.

Corol. 4.] If m is expounded by any Number different from the foregoing, the Curve whose Ordinate is $\frac{x^m}{\sqrt{rr-xx}}$, or $\frac{x^m}{\sqrt{rr+xx}}$, is neither squared exactly, nor does it depend on the Circle or Hyperbola, but is reduced to a simpler Curve.

Theor. 5.] Let A be the Area of a Curve whose Abscis is x, its Ordinate $\frac{x^m}{d-x}$; and let B be the Area of a Curve whose Abscis is also x, and its Ordinate $\frac{x^{m-n}}{d-x}$. Then the Area will be

$$d_n B - \frac{x^m}{m} - \frac{dx^{m-1}}{m-1} - \frac{d^2 x^{m-2}}{m-2}, \ \&c.$$

Let the Ordinate be $\frac{x^m}{d+x}$; the Area will be

$A=$

$$A = \frac{x^m}{m} - \frac{d\,x^{m-1}}{m-1} + \frac{d^2\,x^{m-2}}{m-2}, \text{ \&c. } \pm d\ B.$$

Corol.] If m is expounded by any Term of the following Series, 0, 1, 2, 3, 4, 5, &c. the Quadrature of the Curve whose Ordinate is $\frac{x^m}{d-x}$, or $\frac{x^m}{d+x}$, depends on the Quadrature of the Hyperbola.

Fig. 9. For drawing DE and EF at right Angles, take $EG = d$, draw GH perpendicular and equal to EG. Within the Asymptotes DE, EF, let an Hyperbola be describ'd, passing through H, then take $GK = x$ towards E in the first Case, and towards F in the second, and draw the Ordinate KL. The Area $HGKL$, divided by dd, is equal to the Area of the Curve whose Ordinate is $\frac{x^0}{d-x}$ or $\frac{x^0}{d+x}$. Hence the Solid generated by a Portion of the Cissoid, revolving about the Diameter of its generating Circle, is exhibited in finite Terms, the Quadrature of the Hyperbola being granted.

Theor. 6.] Let A be the Area of a Curve whose Abscis is x, its Ordinate $\frac{x^m}{rr+xx}$; let B be the Area of a Curve whose Abscis also is x, and its Ordinate $\frac{x^{m-2n}}{rr+xx}$. The Area will be

$$A = \frac{x^{m-1}}{m-1} - \frac{r^2\,x^{m-3}}{m-3} + \frac{r^4\,x^{m-5}}{m-5}, \text{ \&c. } \mp r^{2n}\ B.$$

Corol.] If m be expounded by any Term of the following Series, 0, 2, 4, 6, 8, &c. the Quadrature of the Curve whose Ordinate is $\frac{x^m}{rr+xx}$ depends on the Rectification of the circular Arch. For if Center C, Radius $CB = r$ a Circle AEG be describ'd; draw the Tangent $AK = x$, join CK meeting the Periphery in E. Then the Arch AE divided by rr is equal to the Area of the Curve whose Ordinate is $\frac{x^0}{rr+xx}$.

A

A General Corollary to these Six Theorems.

Any Mechanical Curve, whose Quadrature depends upon any of the infinite Number of Curves, the Ordinates of which can acquire any of the following Forms,

$$x^m\sqrt{dx\pm xx},\quad \frac{x^m}{\sqrt{dx\pm xx}},\quad x^m\sqrt{rr\pm xx},\quad \frac{x^m}{\sqrt{rr\pm xx}},$$

$\frac{x^m}{d\pm x}$, $\frac{x^m}{rr\pm xx}$, may be squared by these Series. We shall show this by one Example.

Supposing that the Cube of the circular Arch, corresponding to the versed Sine, be made the Ordinate of a Curve, whose Abscis is the same versed Sine; to find the Area of that Curve.

Let the Abscis be x, the circular Arch v; then the Fluxion of the Area will be $v^3\dot{x}$.

Let the Area be v^3x-q. Then $v^3\dot{x}+3v^2\dot{v}x-\dot{q}=v^3\dot{x}$, whence $\dot{q}=3v^2\dot{v}x$. But $\dot{v}=\frac{d\dot{x}}{2\sqrt{dx-xx}}$, and therefore $\dot{q}=\frac{3dv^2x\dot{x}}{2\sqrt{dx-xx}}$.

But by Theorem the 2d, $\frac{x\dot{x}}{\sqrt{dx-xx}}=\frac{d\dot{x}}{2\sqrt{dx-xx}}-\dot{y}=\dot{v}-\dot{y}$.

So that $\dot{q}=\frac{3}{2}dv^2\dot{v}-\frac{3}{2}dv^2\dot{y}$, therefore $q=\frac{1}{2}dv^3-$ Fl. $\frac{3}{2}dv^2\dot{y}$. Therefore we are come to this, that we must find the flowing Quantity of $\frac{3}{2}dv^2\dot{y}$.

Let this Quantity be $\frac{3}{2}dv^2y-r$.

Therefore $\frac{3}{2}dv^2\dot{y}+3dv\dot{v}y-\dot{r}=\frac{3}{2}dv^2\dot{y}$.

Therefore $\dot{r}=3dv\dot{v}y=\frac{3}{2}d^2v\dot{x}$. Make $r=\frac{3}{2}d^2vx-s$.

Therefore $\frac{3}{2}d^2v\dot{x}=\frac{3}{2}d^2v\dot{x}+\frac{3}{2}d^2x\dot{v}-\dot{s}$.

So that $\dot{s}=\frac{3}{2}d^2\dot{v}x=\frac{3d^3x\dot{x}}{4\sqrt{dx-xx}}=\frac{3}{4}d^3\dot{v}-\frac{3}{4}d^3\dot{y}$, by the second Theorem.

Therefore $s=\frac{3}{4}d^3v-\frac{3}{4}d^3y$. So that the Area required is $v^3x-\frac{1}{2}dv^3+\frac{3}{2}dv^2y-\frac{3}{2}d^2vx+\frac{3}{4}d^3v-\frac{3}{4}d^3y$.

Now

Now because Solids generated by the Rotation of Curves, Superficies produced by the same Rotation, the Rectification of Curves, and the Centers of Gravity of all these, depend upon the Quadrature of Curves; these will be easily computed if they depend upon any of these Curves.

After I had put these Theorems into Form, and had shew'd them to the great *Newton*, as the supreme Arbiter of these Matters; he was pleas'd to produce to me his own Manuscripts, by which it appear'd, that he had long been in Possession of a Method, by which, when any Trinomial Equation was given, expressing the Nature of a Curve, he could either square the same, or reduce it to some simpler Curve.

I question not but those learned Men, whose Writings in the Journals of *Leipsick* and elsewhere, have much contributed to the Improvement of the Mathematical Sciences; those Men (I say) have Methods akin to these, and therefore I assume nothing to myself, but that I have found these Theorems, not knowing whether they may already be extant; and that I have reduc'd them to so easy a Form, that all the Calculation requir'd is perform'd as it were by Intuition. Before I finish writing this, I think it may not be amiss, if I subjoin upon this Occasion a few Words in Answer to the Animadversions of the learned Mr. *Leibnitz*, upon a certain Series publish'd * by me, for finding the Root of an infinite Equation. That famous Man is of Opinion, that that Series is not sufficiently general, as not reaching the Cases in which the Quantities z and y are multiply'd into one another; and therefore he substitutes another Series for mine, which he asserts to be infinitely more general than mine. Now I imagine he was led into this small Mistake, because he took the Quantities a, b, c, d, *&c.* for given Quantities, whereas they may be made Use of as given or as indeterminate Quantities at Pleasure. Let me produce one Example, by which it will appear, that our Series includes all Cases whatever. Let the Equation be $nyz - z^3 = y^3$. In our Theorem make $a = ny$, $b = o$, $c = -1$, $g = 0$, $h = o$, $i = 1$. Or which is better, make $g = yy$, $h = o$, $i = o$. In either Case it will be $z = \frac{y^2}{n} + \frac{y^5}{n^4} + \frac{3y^8}{n^7} + \frac{12y^{11}}{n^{10}}$, *&c.*

* *Of finding the Root of an infinite Equation.* Vid. *sup.* Vol. I. C. 1. §. xxii.

The Quadrature of a Curve *of the third Order,* communicated *by Mr.* Ab. de Moivre, n. 345. p. 329.

V. I have looked a little farther into that Curve which fell lately under my Consideration. It is not the *Foliate* as I did at first imagine, but I believe it ought not to make a *Species* distinct from it. AEB is the Curve I thus describe. Let AB and BK be perpendicular to each other. From the Point A draw AR cutting BK in R, and make $RE = BR$, the Point E belongs to the Curve. Draw BC making an Angle of 45 *grad.* with AB, this Line BC touches the Curve in B; from the Point E draw ED perpendicular to BC, and calling BD, x; DE, y; AB, a; and making $\sqrt{8aa} = n$, the Equation belonging to that Curve,

Fig. 10.

Curve, is $x^3 + xxy + xyy + y^3 = nxy$ or $\frac{x^4 - y^4}{x - y} = nxy$. Taking $BG = AB$, and drawing GP perpendicular to BG, PG is an *Asymptote*. In the *Foliate* the Equation is $x^3 + y^3 = \frac{1}{2}nxy$, in which the two Terms $xxy + xyy$ of the former Equation are wanting; and its *Asymptote* is distant from B by $\frac{1}{3}BA$. Again, draw EF perpendicular to AB: let BF be called z and FE, v; the Equation belonging to the Curve AEB is $vv = \frac{azz - z^3}{a + z}$. In the Foliate the Equation is $vv = \frac{azz - z^3}{a + 3z}$. From these two last Equations, it seems that these Curves differ no more from one another than the *Circle* from the *Ellipsis*.

The Quadrature of the Curve here described has something of Simplicity, with which I was well pleased. With the Radius BA and Center B describe a Circle AKG, let the Square $HPST$ circumscribe it, so that HP be parallel to AG; prolong FE till it meet the Circumference of the Circle in M, and through M draw LMQ parallel to HP. The Area BFE is equal to the Area $KHLM$, comprehended by KH, HL, LM and the Arc KM. And the Area Bfe is equal to the Area $KmLH$ or $KMPQ$. Therefore if BF and Bf are equal, the two Areas BFE, Bfe taken together are equal to the Rectangle HQ, and therefore the whole Space comprehended by $BEAXBeYGZ$ (supposing Y and Z to be at an infinite Distance) is equal to the circumscrib'd Square HS.

N. B. *This Quadrature is easily demonstrated from the Equation: for by it* $a + z : a - z :: zz : vv$, *that is,* $AF : EF :: MF : FB$, *and so* φF *the* Fluxion *of* AF *to* Ll *the* Fluxion *of* MF. *Hence the* Areola $EF\varphi e$ *will be always equal to the* Areola $MLl\mu$, *and therefore the* Area AEF *always equal to the* Area MAL.

Hence it appears that this Curve requires the Quadrature of the Circle to square it; whereas the Foliate *is exactly quadrable, the whole* Leaf *thereof being but one Third of the Square of* AB, *which in this is above three Sevenths of the same. Again in our Curve, the greatest Breadth is when the Point* F *divides the Line* AB *in extreme and mean Proportion: Whereas in the* Foliate *it is when* AB *is triple in Power to* BF. *And the greatest* EF *or Ordinate in the* Foliate, *is to that of our Curve nearly as* 3 *to* 4, *or exactly as* $\sqrt{\frac{2}{3}\sqrt{\frac{1}{3}} - \frac{1}{3}}$ to $\sqrt{5\sqrt{\frac{5}{4}} - 5\frac{1}{2}}$.

But still these Differences are not enough to make them two distinct Species, *they being both defined by a like Equation, if the* Asymptote SGP *be taken for the Diameter. And they are both comprehended under the* fortieth

Kind *of the* Curves *of the* third Order, *as they ſtand enumerated by Sir* Iſaac Newton, *in his incomparable Treatiſe on that Subject.*

A general Method to determine the Quadratures *of Figures by Mr.* J. Craig. n. 284. p. 1346.

VI. I am not a little pleas'd, that the Method which I uſe for determining the Quadrature of Curvilinear Figures, is ſo well approved of by D. D. *Leibnitz* and *Cheyney*; ſo that the firſt acknowledges, it is not unlike the Method found by himſelf, and the other conjectures it has ſome Affinity with the Methods of Mr. *Newton.* He himſelf has purſued the ſame with ſuch Succeſs, that the inverſe Method of Fluxions has been vaſtly improved by him in a Book, which he has dedicated to D. *Archibald Pitcairn*, the Ornament of our Age and Country. But many neceſſary Things yet remain to be diſcover'd, for the Perfection of this inverſe Method. I ſhall now deliver ſome Reaſons in ſhort, which give me Occaſion to think, that what remains cannot be obtain'd by any Methods yet in Uſe.

And firſt, when from the given Relation between z and y, the Fluent of $z\dot{y}$ is required, all thoſe Methods demand, that z may be expreſs'd by y and given Quantities; which yet cannot be done, when the Equation involving that Relation aſcends beyond a Cubic or Biquadratic. For here the vulgar Algebra ſtops, to the great Reproach of that Science. Secondly, tho' a general Rule were known for finding the Roots of Equations of any Degree, yet it would be wholly uſeleſs in this inverſe Method. For the Root z would be involved in ſo many complicated Surds, that by no Art hitherto known we could return from the Fluxion to the Fluent. For theſe Reaſons I have attempted the Thing another Way, and with ſome Succeſs; a Specimen of which I ſhall now impart to the Publick.

Section 1.] Let the Equation expreſſing the Relation between the Ordinate z and the Abſciſs y, be $z^m + ay^n = bz^r y^e$, in which the Exponents m, n, e, r, denote any Numbers, integer or fracted, affirmative or negative. Make $r - n = c$. It will be

$$AREA = \frac{m}{m+n} zy + \frac{mc+ne}{m \times \overline{m+n} \times \overline{c+1} + n \times \overline{m+n} \times \overline{e+1}} \times \frac{b}{a} z^{e+1} y^{c+1}$$

$$+ \frac{\overline{m-e} \times \overline{c+1} + r \times \overline{e+1}}{m \times \overline{2c+1} + n \times \overline{2e+1}} \times \frac{Bb}{a} z^{2e+1} y^{2c+1}$$

B =

$$+\frac{\overline{m-e}\times\overline{2c+1}+r\times\overline{2e+1}}{m\times\overline{3c+1}+n\times\overline{3e+1}}\times\frac{bC}{a}z^{3e+1}y^{3c+1}$$

$$+\frac{\overline{m-e}\times\overline{3c+1}+r\times\overline{3e+1}}{m\times\overline{4c+1}+n\times\overline{4e+1}}\times\frac{bD}{a}z^{4e+1}y^{4c+1}$$

$$+\frac{\overline{m-1}\times\overline{4c+1}+r\times\overline{4e+1}}{m\times\overline{5c+1}+n\times\overline{5e+1}}\times\frac{bE}{a}z^{5e+1}y^{5c+1}, \text{\&c.}$$

Concerning this Series the following Things are to be obſerved. (1) The Capitals *B*, *C*, *D*, *&c.* denote the Coefficients of the Terms that immediately precede. (2) It exhibits the Quadratures of all quadrable Figures, whoſe Curves are defined by an Equation of three Terms. (3) Now they are always quadrable when $\frac{mr-r}{mn-rm-en}$ is an integer and affirmative Number, which we may call l. (4) Particularly $l+1$ gives the Number of Terms of the Series, to be taken from the Beginning, that conſtitute the required Area. (5) If we ſuppoſe $e=0$, this Series will be changed into the famous Theorem of *Newton* for the common Binomial, which Theorem is therefore a particular Caſe of this Series. (6) When Application is made of this Series to any particular Figure, theſe Rules are to be obſerved. *Firſt*, let the Equation defining the given Curve be reduced to the general Form, and by comparing the particular Equation with the general, let the Coefficients a and b be found, as alſo the Exponents m, n, e, r. *Secondly*, if the Exponents thus determined do not make l an integer affirmative Number, (according to the Condition enjoin'd in Not. 3.) then another Term of the particular Equation is to be freed from the Quantity z, and if the Exponents again determined do not give the Condition of Quadrability required, then the other Term is to be freed from the Quantity z. For every one of the three Terms, conſtituting the given Equation, cannot by any Means be freed from the Quantity z. *Thirdly*, if the aforeſaid Condition of Quadrability does not belong to the Equation, when managed according to the foregoing Rule, then by the Series the Complement of the Area, or Fluent of $y\dot{z}$, muſt be ſought; which being found, the Area required will become known. For it is well known, that zy — flu : $y\dot{z}$ = flu : $z\dot{y}$. And that the Complement may be obtain'd by the Series without any Confuſion, in the given Equation defining the particular Curve, for z we may write Y, and for y may be written Z. And this Change being made of the Ordinate into the Abſciſs, and of the Abſciſs into the Ordinate, the Equation may be managed according to the Precepts of the ſecond Rule, till the Condition of Quadrability is known, or till it appears that no ſuch Condition can be had.

Example 1.] Let $z^3 + y^3 = bzy$. Here becauſe $m = 3$, $n = 3$, $e = 1$, $r = 1$, $a = 1$, therefore $l = 1$, and $l + 1 = 2$. Then according to Not. 4. the two firſt Terms of the Series give the Area $= \frac{1}{2}zy - \frac{1}{6}bz^2y^{-1}$.

Example 2.] Let $z^7 + ay^3 = bzy^3$. Then $m = 7$, $n = 3$, $e = 1$, $r = 2$; which make $l = 2$. Therefore by Not 4. the three firſt Terms of the Series give the

$$\text{Area} = \frac{7}{10}zy - \frac{b}{15a}z^2 - \frac{2b^2}{15a^2}z^3y^{-1}.$$

Example 3.] Let $z^3 + ky^5 = bz^{-2}y^{11}$. Here $m = 1$, $n = 5$, $e = 2$, $r = 11$; but becauſe theſe do not make l an integer affirmative Number; therefore by the ſecond Rule I free the Term $bz^{-2}y^{11}$ from the Quantity z. Then the Equation becomes $z^5 - by^{11} = -kz^2y^2$, where $a = -b$, $b = -k$, $m = 5$, $n = 11$ $e = 2$, $r = e$; which make $l = 1$; whence the

$$\text{Area} = \frac{5}{16}zy - \frac{5}{16b}z^3y^{-5}.$$

Example 4.] Let $z^2 - by^2 = -kz^2y^2$. Here $m = 2$, $n = 2$, $e = 2$, $r = 2$; which do not make l an integer affirmative Number. Therefore I free the Term $-kz^2y^2$ from the Quantity z, and then $z^0 + ky^2 = bz^{-2}y^2$. Here $a = k$, $b = b$, $m = 0$, $n = 2$, $e = -2$, which make $l = 1$. Therefore the

$$\text{Area} = \frac{b}{k}z^{-1}y.$$

Example 5.] Let $z^2 - \frac{4g^2}{b}y^6 = -\frac{g}{b}z^2y^4$; where $m = 2$, $n = 6$,

$e = 2$, $r = 4$; which do not make l an integer affirmative Number. And the same Thing happens when each of the other Terms is freed from z. Therefore, according to the third Rule, I seek the Complement. Then, as before prescrib'd, making $z = Y$, and $y = Z$, the given Equation becomes

$$Y^2 - \frac{4g^2}{h} Z^6 = -\frac{g}{h} Z^4 Y^2;$$

which by Rule 1, reduced to the general Form will stand thus,

$Z^6 - \frac{h}{4g^2} Y^2 = -\frac{1}{4g} Z^4 Y^2$. Here $m = 6$, $n = 2$, $e = 4$, $r = 2$; which do not make l an integer affirmative Number. Therefore by Rule 2, I free the last Term from Z; then

$Z^2 + \frac{1}{4g} Y^2 = \frac{h}{4g^2} Z^{-4} Y^2$. Here $m = 2$, $n = 2$, $e = -4$, $r = 2$; whence $l = 1$, $a = -\frac{1}{4g}$, $b = \frac{h}{4g^2}$; whence the Complement of the Area required is

$\frac{1}{2} Z Y - \frac{h}{2g} Z^{-3} Y$, or $\frac{1}{2} zy - \frac{h}{2g} zy^{-3}$. And therefore the Area required is *Flu*: $z\dot{y} = \frac{1}{2} zy + \frac{h}{2g} zy^{-3}$.

Section 2.] Let $z^m + ay^n = bz^{2e} y^{2c+n} + fz^e y^{c+n}$ be an Equation, expressing the Relation between the Ordinate z and the Abscis y. The Area will be

$$Azy + Bz^{e+1} y^{c+1} + Cz^{2e+1} y^{2c+1} +$$

$$Dz^{3e+1} y^{3c+1} + Ez^{4e+1} y^{4c+1}, \text{ \&c.}$$

Making here $2c + n = r$, $c + n = s$, it will be $A = \frac{n}{m+n}$;

$$B = \frac{\overline{m-e+s} \times A + e - m}{m \times \overline{c+1} + n \times \overline{e+1}} \times \frac{f}{a}$$

$C =$

$$C=\frac{\overline{m-2e+r}\times bA+\overline{\overline{m-e}\times\overline{c+1}+r\times\overline{e+1}}\times fB+2eb-mb}{ma\times\overline{2c+1}+na\times\overline{2e+1}},$$

$$D=\frac{\overline{\overline{m-2e}\times\overline{c+1}+r\times\overline{e+1}}\times bB+\overline{\overline{m-e}\times\overline{2c+1}+s\times\overline{2e+1}}\times fC}{ma\times\overline{3c+1}+na\times\overline{3e+1}},$$

$$E=\frac{\overline{\overline{m-2e}\times\overline{2c+1}+r\times\overline{2e+1}}\times bC+\overline{\overline{m-e}\times\overline{3c+1}+s\times\overline{3e+1}}\times fD}{ma\times\overline{4c+1}+na\times\overline{4e+1}},$$

$$F=\frac{\overline{\overline{m-2e}\times\overline{3c+1}+r\times\overline{3e+1}}\times bD+\overline{\overline{m-e}\times\overline{4c+1}+s\times\overline{4e+1}}\times fF}{ma\times\overline{5c+1}+na\times\overline{5e+1}},$$

Concerning this Series, the Progreſſion of which may almoſt be perceived by Inſpection, the following Things are to be obſerved. (1) That thoſe Figures are quadrable, whoſe Curves are defined by the foregoing Equation, when the Exponents m, n, e, c, and the Coefficients a, b, f, have the Relations here aſſigned; that is, when $\frac{\overline{2c+m}\times\overline{n-2e}}{-cm-en}$ is an integer and affirmative Number, which we may call l: And l being greater than 2, when the Relation of the Coefficients is as follows.

$$\frac{\overline{m-2e}\times\overline{lc-c+1}+r\times\overline{le-e+1}}{\overline{e-m}\times\overline{lc+1}-s\times\overline{le+1}}\times\frac{bU}{f}=$$

m —

$$\frac{\overline{m-2e}\times\overline{lc-2c+1}+r\times\overline{le-2e+1}}{m\times\overline{lc+1}+n\times\overline{le+1}}\times\frac{bP}{a}+$$

$$\frac{\overline{m-e}\times\overline{lc-c+1}+r\times\overline{le-e+1}}{m\times\overline{lc+1}+n\times\overline{le+1}}\times\frac{fU}{a}.$$

Here U and P denote the Coefficients of two Terms, which immediately precede the laſt Term of the Area required. That is, U is the Coefficient of the Term next to the laſt, and P is the Coefficient of the Term remote from the laſt. As if $Fz^{5e+1}y^{5c+1}$ were the laſt Term of the Area required, then U would denote E, and P would denote D. (2) That laſt Term of the Area required is known from the Value of the Number l; for here alſo $l+1$ gives the Number of the Terms of the Series, which are to be taken from the Beginning, which conſtitute the Area required. (3) If $l=1$, then the Relation of the Coefficients muſt be this:

$$\frac{\overline{2e-m}\times\overline{1-A}+rA}{\overline{c-m}\times\overline{c+1}-s\times\overline{e+1}}\times\frac{b}{f}=\frac{\overline{e-m}\times\overline{1-A}+sA}{m\times\overline{c+1}+n\times\overline{e+1}}\times\frac{f}{a}.$$

If $l=2$, the Relation muſt be this:

$$\frac{\overline{m-2e}\times\overline{c+1}+r\times\overline{e+1}}{\overline{e-m}\times\overline{2c+1}+s\times\overline{2e+1}}\times\frac{bB}{f}=$$

$$\frac{\overline{2e-m}\times\overline{1-A}+rA}{m\times\overline{2c+1}+n\times\overline{2e+1}}\times\frac{b}{a}+$$

m —

$$\frac{\overline{m-e}\times\overline{c+1}+s\times\overline{e+1}}{m\times\overline{2c+1}+n\times\overline{2e+1}}\times\frac{fB}{a}.$$

Section 3.] Let $z^m = ay^n + bz^e y^{c+n} + fz^{2e} y^{2c+n} + gz^{3e} y^{3c+n}$, *&c.* be the Equation expreſſing the Relation between the Ordinate z and the Abſciſs y, and conſiſting of as many Terms as you pleaſe; the Area will be

$$Azy + Bz^{e+1}y^{c+1} + Cz^{2e+1}y^{2c+1} + Dz^{3e+1}y^{3c+1} + Ez^{4e+1}y^{4c+1}, \text{ \&c.}$$

which (if I miſtake not) is no contemptible Theorem. The Coefficients A, B, C, D, E, &c. are found by a very eaſy Calculation, as alſo the Conditions of Quadrability, and how many Terms of the Series the Area requires. The Number of theſe Conditions increaſes, with the Number of the Terms of which the Equation conſiſts, which defines the Relation of z and y. And particularly, if that Number of Terms is called N, then $N-2$ will be the Number of the Conditions of Quadrability; one of which ſhews the Relation of the Exponents m, n, e, c, when

$$\frac{Nc-2c+2e-Ne+m+n}{-cm-en}$$

is an integer and affirmative Number, which we call l. The other Conditions regard the Coefficients a, b, f, g, h, &c. And laſtly, $l+1$ gives the Number of the Terms of the Series, to be taken from the Beginning, which conſtitute the Area required.

Corol.] From this general Series, a Series may be deduced, which ſhall exhibit the Quadratures of Figures, whoſe Curves are defined by an Equation conſiſting of any Terms, which conſtitute the general Equation of the third Section. For to obtain this there is Need only to compute a Series for an Equation conſiſting of ſo many Terms of the general Equation taken from the Beginning, as the Equation defining the Curves includes Terms. Then from the Values of the Quantities A, B, C, &c. the Coefficients b, f, g, &c. may be eliminated, which do not belong

belong to the Equation propofed. The others will give the Area required. This will appear by an Example.

Section 4.] Let $z^m = ay^n + bz^e y^{c+n} + gz^{3e} y^{3c+n}$ be an Equation expreffing the Relation between z and y. Now becaufe

$$z^m = ay^n + bz^e y^{c+n} + fz^{2e} y^{2c+n} + gz^{3e} y^{3c+n}$$

is that Part of the Equation which includes the given Equation, taking the Terms in Order from the Beginning, which hereafter (for Brevity Sake) I will call the compleat Equation: Therefore the Areas of the Figures, whofe Curves are defined by the compleat Equation, will be

$$Azy + Bz^{e+1} y^{c+1} + Cz^{2e+1} y^{2c+1} +$$

$$Dz^{3e+1} y^{3c+1} + Ez^{4e+1} y^{4c+1} + Fz^{5e+1} y^{5c+1} \text{ \&c.}$$

Here the Coefficients a, b, f, g, enter the Values of the Quantities B, C, D, E, F, &c. If therefore in thefe Values we put every where $f = o$, (becaufe $fz^{2e} y^{2c+n}$ does not enter the given Equation) we fhall have the Values of the Quantities A, B, C, D, E, &c. which being fubftituted in the Series will give the Areas required. Now by the Calculation I have found that

$$A = \frac{m}{m+n}. \qquad B = \frac{c - \overline{m-c-n} \times A + m - e}{m \times \overline{c+1} + n \times \overline{e-1}} \times \frac{b}{a}$$

$$C = \frac{\overline{c+n} \times \overline{e+1} + \overline{m-e} \times \overline{c+1}}{m \times \overline{2c+1} + n \times \overline{2e+1}} \times \frac{bB}{a},$$

$$D = \frac{\overline{\overline{m-3e} \times \overline{1-A} + \overline{3c-n}} \times - Ag}{ma \times \overline{3c+1}}$$

$$\frac{+\overline{\overline{m-e}\times\overline{2c+1}+\overline{c+n}\times\overline{2e+1}}\times-bC}{+na\times\overline{3e+1}}$$

$$E=\frac{\overline{\overline{m-3e}\times\overline{c+1}+\overline{3c+n}\times\overline{e+1}}\times}{ma\times\overline{4c+1}}\qquad\frac{-gB+\overline{\overline{m-e}\times\overline{3c+1}+\overline{c+n}\times\overline{3e+1}}\times-bD}{+na\times\overline{4e+1}}$$

$$F=\frac{\overline{\overline{m-3e}\times\overline{2c+1}+\overline{3c+n}\times\overline{2e+1}}\times}{ma\times\overline{5c+1}}\qquad\frac{-gC+\overline{\overline{m-e}\times\overline{4c+1}+\overline{c+n}\times\overline{4e+1}}\times-bE}{+na\times\overline{5e+1}}$$

$$G=\frac{\overline{\overline{m-3e}\times\overline{3c+1}+\overline{3c+n}\times\overline{3e+1}}\times}{ma\times\overline{6c+1}}\qquad\frac{-gD+\overline{\overline{m-e}\times\overline{5c+1}+\overline{c+n}\times\overline{5e+1}}\times-bF}{+na\times\overline{6e+1}}$$

From hence appears the Progression of the rest *in infinitum*. And thus will be had a Series exhibiting the Quadratures of all the Figures, whose Curves are defined by this Equation of four Terms,

$$z^m=ay^n+bz^e y^{c+n}+gz^{3e}y^{3c+n}.$$

And

And it muſt be obſerved, that the Conditions of Quadrability, and the Number of the Terms of the Series that conſtitute any Area ſought, are the ſame with the Conditions of Quadrability, and Number of the Terms, which agree to the Figures whoſe Curves are defined by compleat Equations.

Corol.] Beſides theſe two Series in §. 2 and 4 for Figures of four Terms, in the ſame Manner infinite other Series may be computed for other Caſes of Figures of four Terms, which is alſo to be underſtood of all other Figures, whoſe Curves are defined by Equations conſiſting of any Number of Terms.

I have not Time at preſent to give a minute Deſcription of the Method, by which I arrive at theſe Series; yet to give ſome ſhort Account of it perhaps may not be amiſs. I aſſume a Series compoſed alike of z and y, ſuch as this following.

$$Azy + Bz^{p}y^{q} + Cz^{s}y^{h} + Dz^{l}y^{k}, \text{ \&c.} = \textit{Fluent} \text{ of } \dot{z}y;$$

of which all the Terms except the firſt have general Exponents. Then I form an Equation between two Values of the Quantity $\dot{z}$, one of which is derived from this Series, and the other is eaſily found by the direct Method of Fluxions, from the Equation exhibiting the Relation between z and y. From the Terms of this Equation duly reduced, firſt I determine the general Exponents p, q, g, h, l, k, &c. and then the Coefficients A, B, C, &c. And if there are more Compariſons than what are ſufficient for determining theſe Coefficients, from the reſt I deduce the Conditions of Quadrability. If you proceed the right Way, the Calculation will be very eaſy; and I have many Rules relating hereto, which perhaps I may give another Time: As alſo the Uſe of this Method in finding finite irrational Quadratures, when rational ones cannot be had. For the whole Affair is now in my Power.

VII. *The Solution of a Problem propoſed by Mr.* Jo. Bernoulli, *in a* French *Journal*, Feb. 1703.

Problem.] A Geometrical Curve being propoſed, to find others without Number which are equal to it in Length. *To find* other *Curves equal in Length to any given* Geometrical *Curve, by Mr.* J. Craig, n. 289. p. 1527.

Solution. Let the Co-ordinates of the given Curve be w, s, and thoſe of the Curve required be $x\,y$. Then from the Condition of the Problem it will be $\dot{w}\dot{w} + \dot{s}\dot{s} = \dot{x}\dot{x} + \dot{y}\dot{y}$. Let us ſuppoſe $\dot{x} = \dot{w} - m\dot{z}$, then it will be $\dot{y} = \sqrt{\dot{s}^2 + 2m\dot{w}\dot{z} - m^2\dot{z}^2}$. In this Equation inſtead of $\dot{s}$ let its Value be ſubſtituted and expreſſed by w, $\dot{w}$, and determinate Quantities; and for $\dot{z}$ let ſuch a Value be aſſumed, compoſed of w, $\dot{w}$, and determinate Quantities, as that the Fluents of $\dot{x}$ and $\dot{y}$ may be found. Thus x and y, the Co-ordinates of the Curve ſought, will be had. *Q. E. I.*

Example 1.] To find a Curve equal to the Parabolic Line. Let $2a$ be the latus rectum of the Parabola. Then $2as = w^2$, or $s = \frac{w^2}{2a}$ whence $\dot{s} = \frac{w\dot{w}}{a} = a^{-1}w\dot{w}$, and $\dot{s}^2 = a^{-2}w^2\dot{w}^2$, and therefore $\dot{y} = \sqrt{a^{-2}w^2\dot{w}^2 + 2m\dot{w}\dot{z} - m^2\dot{z}^2}$. That the Fluent of this may be found, assume $m\dot{z} = \frac{w^2\dot{w}}{a^2}$, whence $\dot{x} = \dot{w} - a^{-2}w^2\dot{w}$, and $\dot{y} = \sqrt{3a^{-2}w^2\dot{w}^2 - a^{-4}w^4\dot{w}^2} = \dot{w}\sqrt{3a^{-2}w^2 - a^{-4}w^4}$. Now the Fluents of these, by Methods already known, will be found to be $x = w - \frac{w^3}{3a^2}$, and $y = \frac{w^2 - 3a^2}{3a^2}\sqrt{3a^2 - w^2}$.

Example 2.] To find a Curve equal to the Circular Arch. Let a be the Radius of the Circle; then 'tis $s = \sqrt{a^2 - w^2}$; whence $\dot{s}^2 = \frac{w^2\dot{w}^2}{a^2 - w^2}$, and therefore $\dot{y} = \sqrt{\frac{w^2\dot{w}^2}{a^2 - w^2} + 2m\dot{w}\dot{z} - m^2\dot{z}^2}$. That the Fluent of this may be found, let us assume $m\dot{z} = \frac{4w^2\dot{w}}{a^2}$, and therefore $\dot{x} = \dot{w} - \frac{4w^2\dot{w}}{a^2}$, and $\dot{y} = \frac{-3a^2w + 4w^3}{a^2\sqrt{a^2 - w^2}} \times \dot{w}$. Now the Fluents of these are $x = w - \frac{4w^3}{3a^2}$, and $y = \frac{a^2 - 4w^2}{3a^2}\sqrt{a^2 - w^2}$.

Example 3.] To find a Curve equal to that of an Ellipsis. Let $2r$ be the latus rectum, a the transverse Axis. Then $s = \frac{r\sqrt{a^2 - w^2}}{a}$, whence $\dot{s}^2 = \frac{r^2w^2\dot{w}^2}{a^4 - a^2w^2}$, and therefore $\dot{y} = \sqrt{\frac{r^2w^2\dot{w}^2}{a^4 - a^2w^2} + 2m\dot{w}\dot{z} - m^2\dot{z}^2}$. That the Fluent may be had, make $m\dot{z} = \frac{2a + 2r}{a^3}w^2\dot{w}$; whence $\dot{x} = \dot{w} - \frac{2a + 2r}{a^3}w^2\dot{w}$, and $\dot{y} = \dot{w} \times$

$\sqrt{\frac{r^2 w^2}{a^4 - a^2 w^2} + \frac{4a + 4r}{a^3} w^2 + \frac{\overline{2a+2r}|^2}{-a^6} w^4}$; the Fluents of which, to be found by known Methods, are $x = w - \frac{2a + 2r}{3a^3} w^3$, and $y = \frac{2a^3 - ra^2 - 2aw^2 - 2rw^2}{3a^2} \sqrt{a^2 - w^2}$.

Example 4.] To find a Curve equal to the Cubical Parabola, whose Equation is $3a^2 s = w^3$. Thence $\dot{s}^2 = \frac{w^4 \dot{w}^2}{a^2}$, and therefore $\dot{y} = \sqrt{a^{-4} w^4 \dot{w}^2 + 2mw\dot{z} - m^2 \dot{z}^2}$. Now in order to find a Fluent, make $m\dot{z} = \frac{w^2 \dot{w}}{2a^2}$. Hence $\dot{x} = \dot{w} - \frac{w^2 \dot{w}}{2a^2}$, and $\dot{y} = \frac{w\dot{w}}{2a} \sqrt{3w^2 + 4a^2}$. The Fluents of these are $x = w - \frac{w^3}{6a^2}$, and $y = \frac{1}{18a} \times \overline{3w^2 + 4a^2}|^{\frac{3}{2}} = \overline{\frac{w^2}{6a} + \frac{2}{9}a} \times \sqrt{3w^2 + 4a^2}$.

From other infinite Values of the Quantity $m\dot{z}$ rightly assumed, may infinite Curves be derived, which are equal to the given Curve. And it may be observed, that this Problem has some Kind of Affinity with a certain Problem of *Diophantus*. His Problem is, to divide the Sum of two Squares into two other Squares, having their Sides rational. And *Bernoulli*'s Problem is, to divide the Sum of two Squares into two other Squares, the Fluents of whose Sides may be found. As the Solution of of *Diophantus*'s Problem depends only on the vulgar Algebra, so the Solution of *Bernoulli*'s Problem requires only the common inverse Method of Fluxions. The Artifice of each consists in a due Assumption of the Sides required; that of *Diophantus* that the Sides may be rational, that of *Bernoulli* that the Fluents of the Sides may be found.

VIII. The *Circle*, *Ellipsis* and *Hyperbola* being not geometrically quadrable (as infinite others) there have been two Ways made use of to find their *Area's*. By *Converging Series*, whereby Approaches are made nearer and nearer, according to the Exactness desired. 2. By *Quadratrices*, that is, mechanical Curves, which determine the Length of certain Lines, whose Squares or Rectangles give the *Area* of the Figure desired. Of this Sort is the old *Quadratrix* of *Dinostratus*, by which the Circle and Ellipse are squared; and another Sort (for the same Purpose)

A new Quadratrix *to the* Hyperbola, *by Mr.* Perks, n. 306. p. 2253.

I

Vid. supra, V. 1. C. 1. S. VII. I inserted in the *Transactions* about five Years ago. Since that, having found the Construction of a Curve, from whence (besides its own *Quadrature* and *Rectification*) the *Quadrature* of the *Hyperbola* is derived, I thought the following Account might not (to some) be unacceptable.

Fig. 11. Let AB, CD, be two strait Rulars joined at B, and there making a right Angle. (Their Length according to the Largeness of the Figure you will describe.) EF is another Rular somewhat longer than AB. Near the one End E, let a little *Truckle-Wheel* (represented edge-wise by $g\,h$, and made of a thin Plate of Brass or Iron) be fastened to the Rular by a Pin (i,) through its Center, so that the Wheel may turn about upon the Pin (i) tight to the Rular without joggling.

On the under Side of this Rular (the Side from the Eye in the Scheme) let there be pinn'd or glewed a little Piece of Wood (in the Form of a Quadrant, the Part which is seen being marked $k\,l$) whose Edge (or Limb) $k\,l$, is an Arch of a Circle of Center (i,) and *Radius* $i\,b$ (the same with the little Wheel.) The Design of this Piece of Wood is, that in the several Positions of the Rular EF, the circular Limb $k\,l$ always touching and sliding by the Edge of the Rular AB, the Center of the Wheel may be always in a Line ($i\,m$) parallel to the Rular AB.

In the Rular CD make $MB = i\,h$ or $i\,k$, and at M fasten a little Pin, and another to the Rular EF near the Wheel, as at P. To these two Ends let be fastened the two Ends of a String MR, so that its whole Length (from Pin to Pin) $+\,P\,i$, be equal to the intended Axis of the Curve TW.

The Instrument being thus prepared, let a strong Rular SO, be fastened (or held fast) upon the Paper or Plane that the Curve is to be drawn upon. Lay the Rular EF from M towards A, and parallel to AB, so that the String lie all strait along the Edge of the Rular EF from M to p, the Point $S\,k$ of the Quadrantal Piece of Wood resting upon the Edge of the Rular AB. Then with a small Pin at M keeping the String close to the Edge of the Rular EF, and with your other Hand upon the End E, keeping the Wheel tight to the Paper or Plane, move the Pin, String and Rular EF from M towards O, the Rular CD, sliding along by the fastened Rular SO in a right Line, the Wheel $g\,h$ will by its Motion describe the defined Curve TV.

Note, The Semidiameter of the little Wheel must be about the *Sum* of the Thicknesses of the two Rulars EF and AB, that it may touch the Paper. Also it will be convenient that its Edge be thin, and a little rough, that it may not slide flat-ways, and that it may leave a visible Impression.

From this Construction the following Properties are demonstrable:

1. It is evident from the Construction, that the *Sum of the Tangent and Subtangent* is every where equal to the same given Line ($= MR + Ri = TW$,) for the String (first strait at TW, afterwards making an Angle at R)

at R) being every where the fame; the Line $R\,i$ (or $R\,P + P\,i$) is always the Tangent, and the Remainder $R\,M$ the Subtangent; the Contact of the Wheel with the Plane, being the Point of the Curve to which they belong.

2. It hence follows, that any affignable Part of the Curve is *rectifiable*, or equal to any affignable ftrait Line. In *Fig.* 12. Let $F\,A\,E$ be a Part of the Curve, its Vertex F. $H\,D\,d$ is the Line defcrib'd by the Motion of the Pin R (in *Fig* 11.) and may be fhewn to be afymptote to the Curve. $F\,H$ a Perpendicular to $H\,D$. Let A be the given Point in the Curve, $A\,D$ the Tangent, and $B\,D$ the Subtangent to the fame Point A. Let a be another Point in the Curve infinitely near to A, to which let $a\,d$ be the Tangent, and $b\,d$ the Subtangent. Draw $A\,G\,a\,g$ perpendicular to $F\,H$ and $A\,B$, $a\,b$ perpendicular to $H\,D$. By the Conftruction $A\,D + D\,B = a\,d + d\,b$. Let $a\,\delta$ be made equal to $a\,D$, and draw $D\,\delta$. Then becaufe $a\,d + b\,d = A\,D + D\,B$. Subftract $b\,D$ and $a\,D$ (or $a\,\delta$) from both Sums (Equals from Equals) there remains $\delta\,d + d\,D = A\,a + B\,b$ (or $C\,a$). $A\,a\,C$, $D\,d\,\delta$ are like Triangles (or differing infinitely little from fuch) therefore $C\,a\;(B\,b) : A\,a :: \delta\,d : D\,d$, and compounding $B\,b + A\,a : A\,a :: \delta\,d + D\,d : D\,d$. Alternating $B\,b + A\,a : \delta\,d + D\,d :: A\,a : D\,d$. But $B\,b + A\,a = \delta\,d + D\,d$ (as is fhewn above) therefore $A\,a = D\,d$. $A\,a$ is the fluxional Particle of the Curve $F\,A$, and $D\,d$ is the fluxional Particle of the Line $H\,D$: Thefe Fluxions or Augments being equal, and their flowing Quantities beginning together, are themfelves therefore equal, *viz.* $F\,A = H\,D$.

Let $F\,G = x$. $G\,A\;(= H\,B) = y$. $A\,D = t$. $B\,D = S$. So is the Curve $F\,A = H\,D = y + S$: that is, *the Curve from the Vertex to any given Point therein, is equal to the Sum of its Ordinate, and Subtangent to the fame Point which is its fecond Property.*

3. The next Property (and whereupon I call it the *Hyperbolic Quadratrix*) is this: Let $F\,A\,E$ be a Part of the Curve, &c. (as before.) Fig. 12.
$F\,I\,K\,H$ is a Square upon the Line $F\,H$. $\lambda\,I\,L$ is an Equilater Hyperbola, whofe Vertex is I, its Affymptotes $H\,O$, $H\,R$; its Axis $H\,I\,\mu$. From a given Point L in the Hyperbola (below its Vertex I) draw $L\,A$ parallel to the Affymptote $R\,H$, interfecting the Diagonal $I\,H$ in M, $F\,H$ in G, and touching the Quadratrix in A. I fay, that the Hyperbolic Area $I\,L\,M$ is equal to a Rectangle, whofe Sides are the Ordinate $G\,A$, and twice $F\,H$, the Axis to the Quadratrix, that is, Trilin. $I\,L\,M = 2\,F\,H \times G\,A$.

Let $F\,H = a$, $F\,G = x$, $G\,A = y$. Becaufe of the Hyperbola $G\,L\,X$ $G\,H\,(L\,S) = F\,H\,q$, therefore $G\,L = \frac{F\,H\,q}{G\,H}$; and $L\,M = \frac{F\,H\,q}{G\,H} - G\,H\,(M\,G)$ that is, $L\,M = \frac{a\,a}{a\,x} - a + x = \frac{2\,a\,x - x\,x}{a - x}$, and confequently ly the Fluxion of the Area $ILM = \frac{2ax - xx}{a - x}\dot{x}$

In the Rectangle Triangle ADB, $AB = a - x$, $BD = S$, $AD = t = a - S$; then is $ADq = ABq + BDq$: or $aa - 2aS + SS = aa - 2ax + xx + SS$, which being thus reduced, gives

$$S = \frac{2ax - xx}{2a}$$

Let la be a right Line suppofed infinitely near and parallel to LA, and interfecting AB in C. Becaufe of like Triangles ACa, ABD;

$AB : BD :: AC : Ca$, that is $a - x : S \ (= \frac{2ax - xx}{2a}) :: \dot{x} : \dot{y}$.

therefore $\dot{y} = \frac{2ax - xx}{2aa - 2ax}\dot{x}$. Multiply each by $2a$, and 'tis $2a\dot{y} =$

$\frac{2ax - xx}{a - x}\dot{x}$. The flowing Quantity of $2a\dot{y}$ is $2ay$, and the flowing

Quantity of $\frac{2ax - xx}{a - x}\dot{x}$ is the Hyperbolic Area ILM (as is fhewn

before.) Thefe two Area's beginning together at F and I, and having every where equal *Fluxions*, or Augments, are therefore themfelves every where equal.

N. The Quadrature of the Trilinear Figure ILM being thus found, any other Area bounded with the Curve-line IL, and any other Right Lines is alfo given.

4. Suppofing the fame Things as in the precedent Propofition, I fay, that the Area of the Quadradrix $FabHF$ is equal to half the Square of Fg, wanting the Cube of Fg divided by $6FH$, or $FabHF$

$= \frac{xx}{2} - \frac{xxx}{6a}$. The Fluxion of this Area is the Rectangle $CabB$

$= \overline{a - x} \times \dot{y} = \overline{a - x} \times \frac{2ax - xx}{2aa - 2ax}\dot{x} = x\dot{x} - \frac{xx}{2a}\dot{x}$. The flowing

Quantity of $x\dot{x}$

Quantity of $\dot{x}x$ is $\frac{1}{2}\ xx$: And the flowing Quantity of $-\frac{xx}{2a}\dot{x}$ is $-\frac{xxx}{6a}$ [as is easily shewn by bringing back these flowing Quantities to their respective Fluxions.] And hence also it follows, that the whole Area continued on infinitely towards *E*, is *one third of the Square* $FIKH$; or $\frac{1}{3}\,aa$. For supposing $x=a$, the Area above becomes $\frac{aa}{2}-\frac{aa}{6}=\frac{aa}{3}$.

While I was considering the other Properties of this Curve, and had given some Account of them to my ingenious Friend Mr. *John Colson*, he returned me a Letter with the Addition of the Quadrature of the Curves Area, which I had not then enquired into.

5. Supposing still the same Things, I say that the Solid made by the Conversion of the Area $FabHF$ about the Line Hb as an Axis, is equal to a Cylinder whose Radius is $FH=a$, and Height equal to $\frac{xx}{2a}-\frac{x^3}{2aa}+\frac{x^4}{8a^3}$. And the whole Solid made by Conversion of the whole Figure infinitely continued, is equal to an eighth Part of a Cylinder, whose Radius and Height are each equal to FH or a.

Let $\frac{P}{D}$ express the Proportion of the Periphery and Diameter of a Circle. Then is $\frac{P}{D}\,ab$ quad. the Area of a Circle whose Radius is ab. And because $Ca=y=\frac{x-\frac{xx}{2a}}{a-x}\dot{x}$ the Fluxion of the Solid is $\frac{P}{D}\times a\ b.$ q. $\times\frac{x-\frac{xx}{2a}}{a-x}\dot{x}$, or $\frac{P}{D}\times\overline{a-x}^2\times\frac{x-\frac{xx}{2a}}{a-x}\dot{x}=\frac{P}{D}\times\overline{ax-\frac{3}{2}xx+\frac{x^3}{2a}}\times\dot{x}$;

 whose

whose flowing Quantity is $\frac{P}{D} \times \frac{a x^2 - x^3}{2} + \frac{x^4}{8 a}$. Which Solid being divided by $\frac{P}{D} a a$ (the Area of a Circle whose Radius is a) gives $\frac{x x}{2 a} - \frac{x x x}{2 a a} + \frac{x^4}{8 a^3}$ for the Height of a Cylinder on the said circular Base, and equal to the Solid made by Conversion of the Area $F a b H F$ about the Line $H b$ as an Axis. When $x = a$ (that is, when the whole Figure is turn'd about its Asymptote) the Height $\frac{x x}{2 a} - \frac{x^3}{2 a a} + \frac{x^4}{8 a^3}$ becomes $\frac{1}{8} a$.

6. The Curve Surface of the Solid generated by the Conversion of the Figure $F a b H F$ about $H B$, is equal to the Curve Surface of a Cylinder whose Radius is a, and Height equal to $\frac{x}{2} - \frac{x x}{4 a} + \frac{x x x}{12 a a}$. And the whole Curve Surface of the Solid infinitely continued, is equal to *one third Part of the Curve Surface of a Cylinder whose Radius and Height are equal to* F H or a. Which may be demonstrated after the Manner of the precedent Proposition.

7. The Radius of the Curvature of any Particle of the Quadratrix is

Fig. 13 $\frac{t t}{a - x}$ and this found Geometrically. $F A E$ is the *Quadratrix*, $H D$ the Asymptote, $A D$ the Tangent, $B D$ the Subtangent to a given Point A. Make $B V = A D$. Upon V raise the Perpendicular $V W$, from A draw $A W$ perpendicular to the Tangent $A D$, till it meet $A W$ in W. So is A W the Radius of the Curvature at A.

8. This Curve may be continued on infinitely above the Point F (but by a different and more operose Way of Construction) whose Properties will be these. 1. The *Difference* of its Tangent and Subtangent (taking the Subtangent in the Line $H S$) will be always equal to the same given Line $F H$ or a. That is, as $t + s = a$, below F, so $t - s = a$ above F.

2. As

2. As below F the Curve Line is equal to the *Sum* of its Ordinate and Subtangent, so above, it is equal to their *Difference*, or $s - y$. 3. As below F, $2ay = ILM$, so above, $2ay = I\lambda\mu$. All which (and its other Properties) may be demonstrated as the Precedent *mutatis mutandis*.

9. With a little Variation in the precedent Construction, may the *Logarithmick Curve* be constructed, which is also a *Quadratrix* to the Hyperbola. Omitting the String MRP, let the Distance MR be equal to the *Subtangent* of the intended Logarithmick Curve (which, as it is known, is invariable.) Stick a Pin at R in the Rular CD, to which apply the Rular EF, so that the Edge of the little Quadrant kl, resting upon the Rular AB, the Distance Mi be equal to MR. Then keeping the Rular EF tight to the Pin R and Rular AB, slide the Rular CD along in a strait Line (by the Rular or Line SO.) So will the Wheel gh describe a Part of the Logarithmick Curve TV, whose *Subtangent* is every where MR. Fig. 11.

10. Let FAE represent the *Logarithmick Curve*, whose Subtangent is equal to FH. $LI\lambda$ is an Equilater Hyperbola (&c. as before §. 3.) Let $FG = x$, $Ga = y$. $FH (= BD) = a$. $GH (= LS) = a - x$. $AC = \dot{x}$, $Ca = \dot{y}$. Then $AC : Ca :: AB : BD$, that is $\dot{x} : \dot{y} :: a - x : a :: a : \frac{aa}{a-x}$ therefore $a\dot{y} = \frac{aa}{a-x}\dot{x}$. The flowing Quantity of $a\dot{y}$ is ay; and the flowing Quantity of $\frac{aa}{a-x}\dot{x}$ is the Hyperbolick Area $FILG$ (for by the Nature of the Hyperbola $GL = \frac{aa}{a-x}$) therefore it the Hyperbolick Area $FILG$ equal to ay a Rectangle, whose Sides are the Subtangent ($BD = FH$) and Ordinate GA (as here accounted) of the Logarithmick Curve. Fig. 12.

IX. Lemma.] *To divide the Sum of two Squares into two other Squares.* *Of the Length of Curve Lines, by Mr.* J. Craig, n. 314, p. 64.

Let $\dot{z}^2$ and $\dot{s}^2$ be two given Squares whose Sum is $\dot{z}^2 + \dot{s}^2$. It is to be divided into two other Squares $\dot{x}^2$ and $\dot{y}^2$; and let m and n be any two Numbers taken at Pleasure. Now from the Condition of the Problem it is $\dot{x}^2 + \dot{y}^2 = \dot{z}^2 + \dot{s}^2$; whence (as may appear from *Diophantus*)

$$\dot{x} = \frac{\overline{mm - nn} \times \dot{z} + 2mn\dot{s}}{mm + nn}, \text{ and}$$

$$\dot{y} = \frac{\overline{nn - mm} \times \dot{s} + 2mn\dot{z}}{mm + nn}. \text{ Q. E. J.}$$

Problem.] *To find innumerable Curves, which are of the same Length with any proposed Curve, whether Algebraical or Mechanical.*

Let z and s represent the Co-ordinates of the Curve proposed ; x and y the Co-ordinates of the Curve required, which is to be of the same Length as the Curve proposed. Therefore it is evident from the Elements of Curves, that $\dot{x}^2 + \dot{y}^2 = \dot{z}^2 + \dot{s}^2$; and therefore by the foregoing Lemma

$$\dot{x} = \frac{\overline{mm - nn} \times \dot{z} + 2mn\dot{s}}{mm + nn},$$

$$\dot{y} = \frac{\overline{nn - mm}\,\dot{s} + 2mn\dot{z}}{mm + nn};$$

The Fluents of which are

$$x = \frac{\overline{mm - nn} \times z + 2mns}{mm + nn},$$

$$y = \frac{\overline{nn - mm}\,s + 2mnz}{mm + nn}.$$

And thus the Co-ordinates x and y of one of the Curves required will become known ; and in like Manner from this a second may be derived, and from the second a third, and so on, till as many as you please are found. *Q. E. J.*

I add no Examples now, becaufe there will be a fitter Occafion hereafter, in which this Method fhall be apply'd to feveral Problems of this Kind, and the Solution of this Problem fhall be illuftrated by a Variety of Examples. And I have fo plainly pointed out this Solution more than once, that it might eafily have been deduced, by any one verfed in thefe Matters, from what is fubjoin'd to the Solution of a particular Cafe of this Problem, in which the Curve propofed is Algebraical, and which I exhibited in the Philofophical Tranfactions for *Jan.* 1704. So that it may appear to Mr. *Jo Bernoulli*, the learned Propofer of the Problem, that its Solution may be obtain'd from the common Rules of the inverfe Method of Fluxions, fince he infinuated, in his private Letters to Dr. *Cheyney*, that the fame could not be exhibited by our Theorems publifh'd in the Philofophical Tranfactions for *March* 1703. And becaufe I perceive from the Acts of *Leipfick* of *Auguft* 1705, that our Solution did not pleafe that learned Man, though enough, and more than enough to the Purpofe; for that Reafon only I publifh the foregoing Solution, which can be liable to no Objection. Therefore the learned *Bernoulli* muft ingenuoufly acknowledge, that hardly any Problem can be propofed, the Solution of which is deduced with more Eafe from the inverfe Method of Fluxions, than this his Problem of the Transformation of Curves.

Vid. *fup.* S. VII.

Vid. *fup.* S. VI.

Now I fhall declare, in a few Words, what I cannot approve in Mr. *Bernoulli*'s Solution of his own Problem. *Firft*, That he has apply'd it only to Algebraical Curves. *Secondly*, That it is Mechanical, and depending wholly upon what he calls *Creeping Motion*. *Huygens* is certainly deferving of immortal Honour, for his Invention of the Motion of Evolution, becaufe from thence not only himfelf but others have derived admirable Theorems Geometrically. But neither *Leibnitz*'s Motion of Traction, nor *Bernoulli*'s Creeping Motion, will ever be comparable to *Huygens*'s Motion of Evolution, till thofe ingenious Men, as *Huygens* has done, fhall reduce the Curves generated by their Motions to the Laws of Geometry. Now, fince neither of them have yet perform'd this, the Solutions of Problems, depending upon Curves produced by their Motions, can only be reputed as Mechanical.

A General Solution of a Problem *concerning* Curves, *by* ---- n. 347. p. 399.

X. In the *Acta Eruditorum* for *October* 1698. pag. 471. Mr. *J. Bernoulli* writes thus. " At length I have obtain'd the general Method " I wifh'd for, for the orderly cutting of Curves given in Pofition, " whether Algebraical or Tranfcendental, in an Angle either right or " oblique, whether invariable or varying according to a given Law; " to which, according to the Opinion of Mr. *Leibnitz*, not a Jot can be " added for its farther Perfection, and for this Reafon, that it always " leads to an Equation. In which, if the indeterminate Quantities are " fometimes infeparable, the Method is not the lefs perfect for that; " for it belongs not to this, but to fome other Method to feparate them.

"I

" I intreat my Brother, that he will try his Strength in a Matter of this " Weight; nor will he repent of his Labour, if he happens to be suc- " cesſful. I know he will then forſake the Method he is now ſo fond " of, which can only be apply'd in a very few Occaſions.

Theſe three great Men had been uſed to exerciſe one another, for about the Space of four or five Years, in propoſing and ſolving ſuch Kind of Problems. It would be very difficult to give the very ſame Solution as that of Mr. *Bernoulli*, without one had the Spirit of Divination. It is ſufficient that the following Solution is general, and always brings us to an Equation.

Problem.] *A general Method is required for finding a Series of Curves, which ſhall cut at a given Angle, or at an Angle that ſhall vary in a given Law, Curves that are conſtituted in any other given Series.*

Solution.] The Nature of the Curves to be cut gives the Tangents of the ſame at any Points of Interſection; and the Angles of Interſection give the Perpendiculars of the cutting Curves; and two Perpendiculars coinciding, by their laſt Concourſe give the Center of Curvity of the cutting Curve at the Point of any Interſection. Let an Abſciſs be drawn in any convenient Situation, and let its Fluxion be Unity; and the Poſition of the Perpendicular will give the firſt Fluxion of the Ordinate belonging to the Curve required; and the Curvity of this Curve will give the ſecond Fluxion of the ſame Ordinate. And thus the Problem will always be reduced to Equation. *Q. E. F.*

Scholium.] It does not belong to this, but to another Method, to reduce the Equations, and to ſeparate the indeterminate Quantities, abſolutely if it may be done, if not, by infinite Series. As this Problem is hardly of any Uſe, for that Reaſon it has remain'd neglected and unſolved for many Years, in the *Acta Eruditorum*. And for the ſame Reaſon I ſhall not proſecute its Solution any farther.

Mr. Leibnitz's Problem *concerning* Curves, *ſolv'd by Dr.* B. Taylor, *n.* 354. p. 695.

XI. Since the deceaſed Mr. *G. G. Leibnitz*, in the Controverſy lately moved about the Inventer of the Method of Fluxions (which he has thought fit to call the Differential Method, and obſtinately to appropriate the Invention to himſelf) has given no Anſwer to thoſe Arguments which are alledged in Favour of Mr. *Newton*, as the Diſcoverer of that noble Method; yet by his Encouragement Mr. *Joh. Bernoulli* has propoſed a Problem, to be ſolved by the *Engliſh* Geometricians. But whether the Problem is ſolved by them or no, it can be no Prejudice to the Right of Mr. *Newton*. However, leaſt they ſhould make it an Occaſion of Triumph, if this Problem ſhould not be attempted by the *Engliſh*, I have ventured to give my Solution, ſuch as it is, tho' the Problem is no ways remarkable either for its Uſe or Difficulty.

The Problem at firſt propoſed by Mr. *Leibnitz* was ſo underſtood, as if nothing elſe had been required, than that Conic Hyperbola's, deſcrib'd

with

with the ſame Center and Vertices, ſhould be cut at right Angles. But when he was inform'd, that this Caſe had been immediately ſolved by ſome *Engliſh-men*, he wrote Word, that the Solution of a particular Caſe was not required, but a general Solution. For which Reaſon thoſe particular Solutions were not publiſh'd; tho' in the Philoſophical Tranſactions Numb. 347. [*See Sect.* X. *above*] a Solution appear'd which was univerſal. But Mr. *Leibnitz* and his Aſſociates were not content with this, but ſeem'd rather to deſpiſe it, as if the Author was not able to apply it to any particular Caſe. If they could not perceive how Equations were to be deduced from it, that is to be imputed to their Unſkilfulneſs. A little before the Death of Mr. *Leibnitz* the following Problem at laſt came out, which may be ſolved after different Manners, by purſuing the Steps of the general Solution before-mentioned; but at preſent we ſhall ſolve it as follows.

Problem.] *Upon the right Line* A G *as an Axis, from the Point* A *to draw an infinite Number of Curves, ſuch as* A B D, *which are to be of ſuch a Nature, that the Radii of Curvature* B O, *drawn every where in the ſeveral Points* B, *may be cut by the Axis* A G *in* C, *in a given Ratio, or ſo that it may be* B O . *B C* :: 1 . n. Fig. 14.

Then are to be conſtructed the Trajectories E B F, *which ſhall cut the former Curves* A B D *at right Angles.*

Firſt Part of the Solution.] To find the Curves *A B D*, which are to be cut. 1. Drawing the Ordinate *B H* perpendicular to the Axis *A G*, make the Abſciſs $AH = z$, the Ordinate $HB = x$, the Curve $AB = v$. Then by the direct Method of Fluxions it will be $BC = \frac{\dot{v} x}{\ddot{z}}$, and if v flows uniformly, $BO = \frac{\dot{v}\dot{x}}{\ddot{z}}$. Whence by the Condition of the Problem 'tis

$$BO\left(\frac{\dot{v}\dot{x}}{\ddot{z}}\right) . BC\left(\frac{\dot{v}x}{\dot{z}}\right) :: 1 . n, \text{ and therefore } \ddot{z}x - n\dot{z}\dot{x} = 0.$$

2. This Equation being compared with the ſecond formula of Fluxions, at the End of *Prop.* 6. of the Method of Increments, there is found $\dot{z}x^{-n} = \dot{v}a^{-n}$; a being a given Line, by the Value of which the Curve *A B D* may be accommodated to any Condition that is annex'd to the Problem.

3. Inſtead of $\dot{v}$ its Value $\sqrt{\dot{x}^2 + \dot{z}^2}$ being written, the Equation $\dot{z}x^{-n} = \dot{v}a^{-n}$ is changed into this $\dot{z} = \frac{\dot{x}x^n}{\sqrt{a^{2n} - x^{2n}}}$. Whence $\dot{z}$

is

is given when x is given, by the Quadrature of the Curve, whose Abscifs being x, its Ordinate is $\frac{x^n}{\sqrt{a^{2n}-x^{2n}}}$.

4. Let σ and τ be integer Numbers, either affirmative or negative, fuch as that the fimpleft of the Curves produced in this Manner may be that, whofe Abfcifs is y, and Ordinate is $y^{\frac{1-n+2\sigma n}{2n}} \times \overline{\;-y}|^{\tau-\frac{1}{2}}$; then it will be the fimpleft of all the Curves, by the Quadrature of which the Abfcifs z is given from the given Ordinate x.

5. The Curve ABD is a Geometrical Curve, as often as the Reciprocal of any odd Number is affumed for n.

6. Hitherto we have confidered the Curve ABD as concave towards the Axis AG, in which Cafe the greateft Ordinate x is equal to the given right Line a, which we may conveniently call the Parameter of the Curve. And in this Cafe the Curve will actually meet the Axis. Whence the Fluent of $\frac{\dot{x}\,x^n}{\sqrt{a^{2n}-x^{2n}}}$ being rightly taken, that is, fo that z and x may vanifh together, the Curve will pafs through the given Point A, as the Problem requires.

7. But if a Curve ABD be required, which is convex towards the Axis, in the fame Manner we fhall come to the Equation $\dot{z} = \frac{a^n\dot{x}}{\sqrt{x^{2n}-a^{2n}}}$; which alfo may be derived from the former Equation, by changing the Sign of n. And in this Cafe the Curve ABD is Geometrical, as often as the Reciprocal of any even Number is taken for n. But in this Cafe the leaft Ordinate x is equal to the Parameter a; and therefore the Curve no where meets with the Axis. Therefore the Problem is limited to the former Cafe.

8. From the foregoing it is eafily concluded, that all the Curves ABD are fimilar, and fimilarly pofited about the given Point A, their homologous Sides being proportional to the Parameters a.

The other Part of the Solution: Or the Invention of the cutting Curve.

9. From §. 2. tis $\dot{v} . \dot{z} :: a^n . x^n$. But it is $BC . BH :: \dot{v} . \dot{z}$. Therefore it is $BC . BH :: a^n . x^n$. But from the Condition of the Problem BC is a Tangent to the Curve fought EBF. Wherefore if we take now AH (z) and BH (x) for Co-ordinates of the Curve EBF, the Curve itfelf EB being call'd r; it will be by the direct Method of Fluxions $\dot{r} . -\dot{x} :: (BC . BH ::)\, a^n . x^n$. Whence it is $\frac{x^n}{a^n} = \frac{-\dot{x}}{\dot{r}}$.

10.

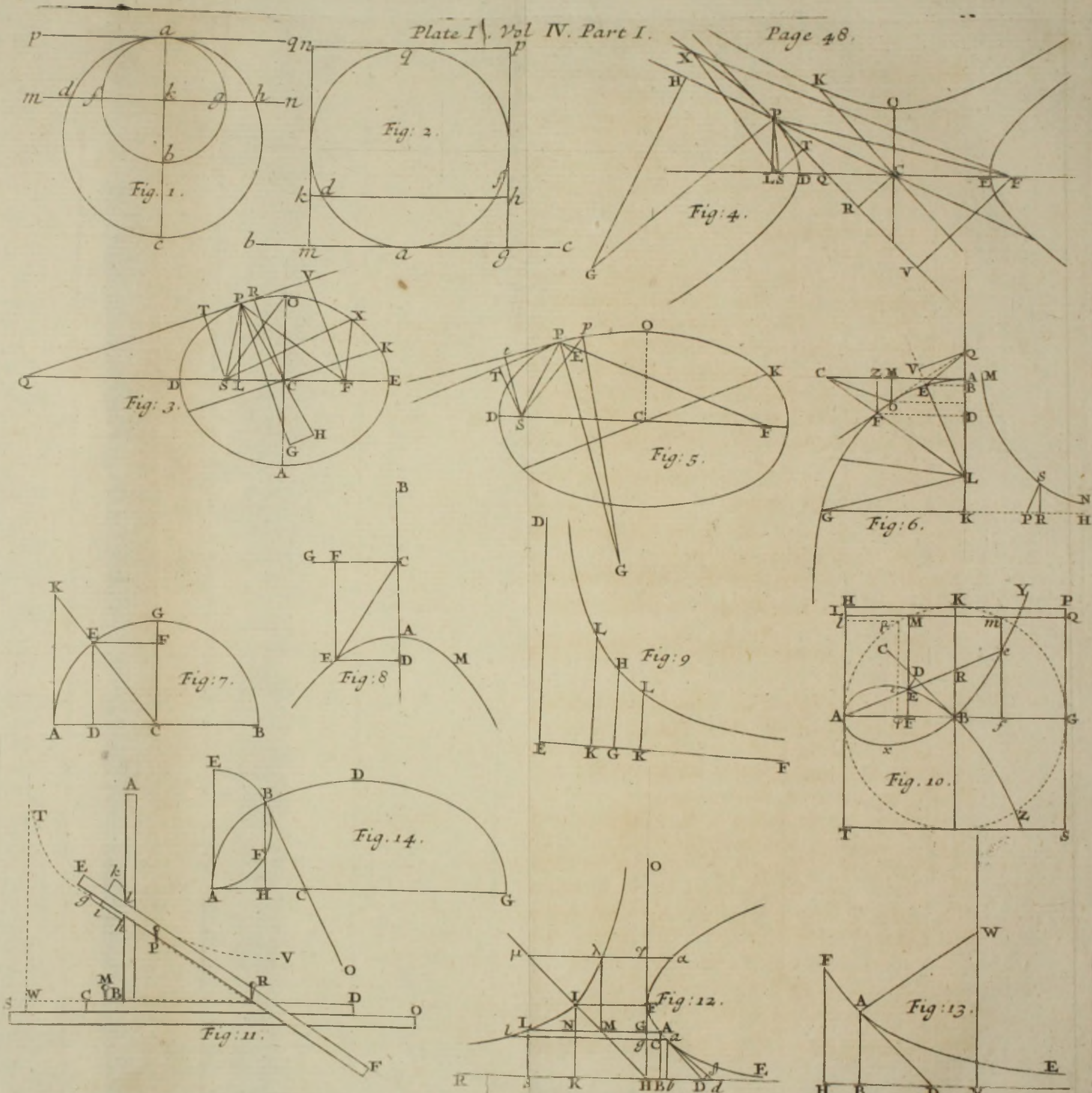
Fig: 1.
Fig: 2.
Fig: 3.
Fig: 4.
Fig: 5.
Fig: 6.
Fig: 7.
Fig: 8
Fig: 9
Fig. 10.
Fig: 11.
Fig: 12.
Fig: 13.
Fig. 14.

10. In the Curve ABD imagine the Equation $\dot{z} = \frac{\dot{x}x^n}{\sqrt{a^{2n} - x^{2n}}}$ to be transform'd into the Equation $\dot{z} = \frac{A\dot{x}x^n}{a^n} + \frac{B\dot{x}x^{3n}}{a^{3n}} + \frac{C\dot{x}x^{5n}}{a^{5n}}$, &c. which is not affected with Radical Signs; then by returning to the Fluents it will be $z = \frac{1}{n+1} \times \frac{Ax^{n+1}}{a^n} + \frac{1}{3n+1} \times \frac{Bx^{3n+1}}{a^{3n}} + \frac{1}{5n+1} \times \frac{Cx^{5n+1}}{a^{5n}}$, &c. where no new Coefficient is introduced, becauſe by the Condition of the Problem z and x are naſcent at the ſame Time. Here inſtead of $\frac{x^n}{a^n}$ ſubſtituting its Value $\frac{-\dot{x}}{\dot{r}}$, as found §. 9. we ſhall have $z = \frac{1}{n+1} Ax \times \frac{-\dot{x}}{\dot{r}} + \frac{1}{3n+1} Bx \times \frac{-\dot{x}^3}{\dot{r}^3}$, &c. which is a fluxional Equation of the firſt Degree belonging to the Curve required EBF. Now this is reduced to a more ſimple Form in finite Terms after the following Manner.

11. Let r flow equably, and a being a conſtant Quantity, make $\frac{-\dot{x}}{\dot{r}} = \frac{s^n}{a^n}$. This Value of $\frac{-\dot{x}}{\dot{r}}$ being ſubſtituted in the Equation laſt found, and the Equation being multiply'd by $\frac{s}{x}$, it will be transform'd into this $\frac{zs}{x} = \frac{1}{n+1} \times \frac{As^{n+1}}{a^n} + \frac{1}{3n+1} \times \frac{Bs^{3n+1}}{a^{3n}}$, &c. whence taking the Fluxions, it will be $\frac{\dot{s}zx + s\dot{z}x - sz\dot{x}}{x^2} = A\dot{s} \times \frac{s^n}{a^n} + B\dot{s} \times \frac{s^{3n}}{a^{3n}}$, &c. $= \frac{\dot{s}s^n}{\sqrt{a^{2n} - s^{2n}}}$. This laſt is manifeſt from the Analogy of the Series $A\dot{x} \times \frac{x^n}{a^2}$, &c. and $A\dot{s} \times \frac{s^n}{a^n}$, &c. Here for s and $\dot{s}$ ſubſtituting their

Values derived from the Equation $\frac{-\dot{x}}{\dot{r}} = \frac{s^n}{a^n}$, there will arise the Equation $n\dot{x}^2\dot{z}\cdot z - \ddot{x}x\dot{z}z - n\dot{x}x\dot{z}^2 - \ddot{x}\dot{x}x^2 = 0$; which is reduced to first Fluxions in the following Manner.

12. In the last Term $-\ddot{x}\dot{x}x^2$ instead of $\ddot{x}\dot{x}$ writing its Value $-\ddot{z}\dot{z}$, and then applying the Equation to $\dot{z}$, there arises $n\dot{x}^2 z - \ddot{x}xz - n\dot{x}x\ddot{z} + xx\ddot{z} = 0$. Which Equation multiply'd by x^{-n-1} is the Fluxion of the Equation $-\dot{x}x^{-n}z + x^{1-n}\dot{z} = a^{1-n}\dot{r}$, the Quantities a and $\dot{r}$ being constant. Therefore this Equation, or $\overline{\dot{z}x - z\dot{x}} \times a^{n-1} = \dot{r}x^n$ is a fluxional Equation of the first Degree, belonging to the Curve sought *E B F.*

13. Now in this Equation a is the Value of the Ordinate BH, when the Point H falls in the Point A.

14. It will not be very easy, while n continues to be general, to bring this Equation to an Equation involving only Fluents, or to the Quadrature of Curves. But the Points of the Curve EBF may conveniently be found by the Description of the Curve ABD, and of a certain Geometrical Curve. By a Geometrical Curve I understand one, into whose Equation no Fluxions enter, nor Fluents into the Indices of the Powers. For let the Curve ABD, whose Parameter is a, be cut in B by a Geometrical Curve whose Equation is $\alpha a^n x^n - z a^n x^n = x a^n \sqrt{a^{2n} - x^{2n}}$; then that Point of Intersection B will be in one of the Trajectories sought, which passes through the Point E; AE being equal to a, and perpendicular to AG.

15. Hence if ABD be a Geometrical Curve, EBF will also be a Geometrical Curve.

Scholium. The Equation $\overline{\dot{z}x - z\dot{x}} \times a^{n-1} = \dot{r}x^n$ may be found another Way. For by a certain Analysis, which at this Time I think fit to conceal, I have found the Equation $\frac{\dot{\alpha}}{\alpha} = \frac{\dot{r}\dot{r}}{\dot{z}z + \dot{x}x}$, which being compared with the Equation $\frac{x^n}{\alpha^n} = \frac{-\dot{x}}{\dot{r}}$ (§. 9.) by eliminating α and $\dot{\alpha}$, we at last arrive at the foregoing Equation $\overline{\dot{z}x - z\dot{x}} \times a^{n-1} = \dot{r}x^n$.

Example. A very simple Example may suffice to prove the Truth of this Solution. Make $n = 1$, in which Case ABD will be a Semicircle describ'd described with the Diameter AG; also EBF will be a Semicircle likewise, describ'd with the Diameter AE. But in this Case $\frac{\dot{x}x^n}{\sqrt{a^{2n}-x^{2n}}}$ $= \frac{\dot{x}x}{\sqrt{a^2-x^2}}$. Whence in §. 3. tis $\dot{z} = \frac{\dot{x}x}{\sqrt{a^2-x^2}}$. Therefore $z = a - \sqrt{a^2-x^2}$, which is an Equation to a Circle, described with the Diameter $AG = a$, as it ought to be. Also for n writing 1, the Equation $\dot{z}x - z\dot{x} \times a^{n-1} = \dot{r}x^n$ is changed into $\dot{z}x - z\dot{x} = \dot{r}x$. Whence exterminating $\dot{r}$ by Means of the Equation $\dot{r}\dot{r} = \dot{x}\dot{x} + \dot{z}\dot{z}$, there arises $\frac{2\dot{z}zx - \dot{x}z^2}{x^2} = -\dot{x}$, and therefore returning to the Fluents, it will be $\frac{zz}{x} = -x + a$, which is an Equation to a Circle described with Diameter $AE = a$, as it ought to be.

XII. Whereas in every Curve Line there is a certain Regularity of Curvature, tho' perhaps involved, according to which the Figure is determin'd; therefore Geometricians define the various Characters of Curves, by an Equation expressing the Relation of the Ordinates to the Abscisses of any Axis or Diameter. Now since the same Thing may be done from the Consideration of the Curves in respect of one given Center, nay the most simple Uniformity of Nature often requires this should be done in this Inquiry; therefore we shall have Recourse at present to this Method of considering Curves, and first we shall shew, how easy it will be (according to this Method of determining Curves, by the Assistance of the Arithmetick of Infinites) to derive the more complicate from the simple ones.

Construction and Measure of Curves, by Mr. C. Mac-laurin, n. 356. p. 803.

Sect. 1.] Let the Points L and l be as near as may be in the Curve BlL; let lo be an Arch described with Center S, perpendicular to SL, and Ll will be as the Moment of the Curve, and Lo the Moment of the Radius SL. And if the Ratio of Ll to Lo be given, or to lo in the Distance SL, the Equation of the Curve will be given at the Center S. Let LP, lp, be Tangents to the Curve in the Points L and l, upon which from S let be drawn the Perpendiculars SP, Sp, meeting them in the Points P and p. In like Manner upon all the Tangents of the Curve let Perpendiculars be drawn from the given Point S, and a Curve will be constructed passing through all the Intersections of the Tangents and Perpendiculars. The Elementary Triangle of this Pnp will be similar to the Triangle Lol, which therefore will be given from the given Curve BlL. For because of equal Angles SnP, PnL, and the

Fig. 15.

right

right Angles $S\,p\,n$, $S\,P\,L$, the Triangles $S\,p\,n$, $P\,n\,L$, will be equi-angular, and therefore $P\,n \,.\, p\,n :: L\,n \,.\, S\,n :: L\,o \,.\, l\,o$. Likewise because of equal Angles $P\,n\,p$, $S\,n\,L$, $L\,o\,l$, the Triangles $P\,n\,p$, $S\,n\,L$, $L\,o\,l$, will be similar. Since therefore there is the same Ratio of $L\,l$ to $l\,o$, as of $P\,p$ to $p\,n$, and $S\,L$ to $S\,P$; it is plain that the Ratio of $L\,l$ to $l\,o$ being given, and the right Line $S\,L$, the Ratio of $P\,p$ to $p\,n$ will be given, and the right Line $S\,P$, and therefore the Curve $D\,P\,p$. And by the same Method a third may be constructed from $D\,P$, and from this a fourth; and by proceeding thus an infinite Series of Curves may be derived, all which from one that is known will become known. Now if $L\,N$ and $l\,n$ are erected perpendicularly upon the Radii $S\,l$, $S\,l$, meeting one another in n; and if through all the Points of perpendicular Concourse that are alike defined, a Curve $E\,N$ is described: That will be the very Curve from whence $B\,L$ may be deduced, by the same Method as we constructed $D\,P$ from $B\,L$. In like Manner another Curve may be constructed from $E\,N$, and on this Side likewise an infinite Series of Curves may be constructed.

Fig. 10.

Sect. 2.] But of all the Curves produced in this Manner, the most simple will be, in which $L\,l$ is to $L\,o$ in the Ratio of some Power of the Radius; so that if a be a given Quantity, and r denotes the Radius of the Curve, and n any Number whatever; it may be as $L\,l$ to $l\,o$ so a^n to r^n, which will be their general Equation. But all these will have an Apsid, when $r = a$, because in that case $L\,l = l\,o$. To investigate the Equation of the Curve $D\,P$; since in $B\,L$ it is, as $L\,l$ to $l\,o$, so is a^n to r^n, so is r to $S\,P = \frac{r^{n+1}}{a^n}$, so is $a^{\frac{n}{n+1}} \times S\,P^{\frac{1}{n+1}}$ to $S\,P$, so is $a^{\frac{n}{n+1}}$ to $S\,P^{\frac{n}{n+1}}$, so is $P\,p$ to $p\,n$. Therefore if $\dot{s}$ represents the Moment of the Curve, $\dot{y}$ the circular Arch describ'd by the Radius from the Center S, and r the corresponding Radius; whatever the Curve be whose Equation is sought, the Equation of the Curve $B\,L$ will be $\dot{s} \,.\, \dot{y} :: a^n \,.\, r^n$. But the Equation of the Curve $D\,P$ is $\dot{s} \,.\, \dot{y} :: a^{\frac{n}{n+1}} \,.\, r^{\frac{n}{n+1}}$. And the Angle $P\,S\,p$ will be to the Angle $L\,S\,l$, as $\frac{p\,n}{S\,P}$ is to $\frac{l\,o}{S\,L}$, or as $\frac{P\,n}{S\,p}$ is to $\frac{L\,o}{S\,L}$, or as $\frac{\dot{x}}{x}$ to $\frac{\dot{r}}{r}$, if $S\,P = x$, and $S\,L = r$; that is (because $x = \frac{r^{n+1}}{a^n}$) as $\frac{n+1}{r}\,\dot{r}$ is to $\frac{\dot{r}}{r}$, or as $n+1$ to 1. Hence $B\,S\,P$ is to $B\,S\,L$

as

as $n+1$ to 1; whence the Curve $B P$ may be drawn more easily, without the Assistance of the Tangents. If the Angle $B S P$ to $B S L$ is taken in the Ratio of $n+1$ to 1, and a Perpendicular is let fall from L upon $S P$, the Meeting of the Perpendicular with $S P$ will be in the Curve $B P$, which was described before by the Help of the Tangents. Fig. 16.

Sect. 3.] We have shewn how from one, an infinite Series of Curves may be deduced; I shall now go on to demonstrate, how the Lengths of each may be known, from the Lengths of that and another being given. Since the Angle $S P p = S L l$, and $L S l$ is to $P S p$ as 1 to $n+1$; $L l$ will be to $P p$ as $S L$ to $\overline{n+1}\ S P$; or (because of $S L . S P :: L l . l o$,) as $L l$ to $\overline{n+1}\ l o$; and therefore $P p = \overline{n+1}\ l o$. But $l o = l n - o n = l n - L n + N n$. Therefore $P p = \overline{n+1} \times \overline{l n - L N + N n}$. But $l n - L n$ is the Moment of the right Line $L N$ perpendicular to $S L$; $P p$ is the Moment of the Curve $B p$; and $N n$ is the Moment of the Curve $B N$. And since $B P$, $B N$, $B L$, vanish together in B, they will be in the Ratio of their Moments, and therefore $BP = \overline{n+1} \times \overline{B N \mp LN}$. Whence the Curve $B P$ is to the Sum or Difference of the last Curve but one in the Series, and its Tangent intercepted by the intermediate Curve, as $n+1$ to n; or, putting m for the Index of the Equation of the Curve $B P$, (because $m = \frac{n}{n+1}$) as 1 to $1 - m$. Fig. 16.

Hence first, in the infinite Series of Curves above described, if the Lengths of two of the nearest are given, the Lengths of all will be given. For the Measure of every one depends always on the Measure of the last but one in the Series, and therefore one Pair will suffice for measuring all. If one Curve is commensurable to right Lines, or incommensurable, half the intire Series will be commensurable to right Lines, or incommensurable. Hence secondly, altho' the Curves $B P$ and $B N$ should be incommensurable to right Lines, yet the Difference of the Curve $B P$ from $n+1$ Part of the Curve $B N$, would be equal to an assignable right Line. Thirdly, if the Curve passes through S, the right Line $L N$ vanishing in S, it will be $B P S = \frac{B N S}{1-m}$.

Sect. 4.] Of all the Curves about which we have treated, whose Property is $\dot{s} . \dot{y} :: a^n . r^n$; the Circle is the most remarkable, S being in the Circumference whose Equation is $\dot{s} . \dot{y} :: a . r$, as is evident from the Similitude of the Triangles $L o l$, $B L S$. Therefore $n = 1$, and consequently $m = \frac{n}{n+1} = \frac{1}{2}$; and the Equation of the Curve $B P$ will be $\dot{s} . \dot{y} :: a^{\frac{1}{2}} . r^{\frac{1}{2}}$, which is the very Equation of the Epicycloid described by the Revolution of a Circle upon a Base equal to itself, to the Point where the describing Point touches the Base; which Mr. *Paschal* calls Mr. *Roberval's Snail*, and which Mr. *de la Hire* considers as a Conchoid with a circular Base, in the Memoirs Fig. 17.

Memoirs of the Academy of *Paris*, *An.* 1708. All the Perpendiculars LN, ln, concur in the Point B, and therefore $BN = 0$. Whence $BP = \frac{BN + NL}{1 - m} + 2\,BL$. Therefore the whole Curve $BPS = 2\,BS$, or the Length of the Epicycloid is always double to the Chord of the corresponding Arch in the Circle. *Secondly*, From the Epicycloid let the Curve $B\Pi S$ be described, in the same Manner as we described the Epicycloid from the Circle. In this Case $n = \frac{1}{2}$, and $m = \frac{n}{n+1} = \frac{\frac{1}{2}}{\frac{1}{2}+1} = \frac{1}{3}$, and therefore the Equation of the Curve $B\Pi S$ will be $\dot{s} . \dot{y} :: a^{\frac{1}{3}} . r^{\frac{1}{3}}$. The Length of the Curve will be $\frac{BL + LP}{1 - m} = \frac{3}{2} \times \overline{BL + LP} = \frac{3}{2} \times \overline{BL + LG}$, and therefore $B\Pi$ is sescuple the Sum of the circular Arch and its right Sine. Now if we take $CD = BD$, and with Radius SD and Center S describe a Circle meeting the right Line SP in H, and HK is made perpendicular to BS; because $DH = \frac{2}{3}\,BL$, it will be $B\Pi = DH + HK$. Hence the Arches $B\Pi$ are neither commensurable to right Lines nor to circular Arches, yet the Difference of the Arches $B\Pi$ and DH is the right Line HK. The Line LG vanishes in the Point S, and therefore $B\Pi S$ is $= \frac{3}{2}\,BLS$; whence the whole Curve is sescuple of the Semicircle. Yet no Part of this assignable Curve can be commensurable to the whole, nor is the intire Curve divisible in any given Ratio, so that the Portions may have an assignable Ratio to one another or to the whole. If this Curve could be divided Geometrically in any given Ratio, the Quadrature of the Circle would be compleated. For Instance, if it were $B\Pi$ to $B\Pi S$, as 1 to m, and BL to BLS as 1 to n, it would be $B\Pi = \frac{B\Pi S}{m} = \frac{3\,BLS}{2\,m} = \frac{3\,n\,BL}{2\,m} = \frac{3}{2} \times \overline{BL + LG}$. Whence it would be $BL = \frac{m\,LG}{n - m}$ and $BLS = \frac{n\,m}{n - m}\,LG$. *Thirdly*, by the Method already explain'd, from $B\Pi S$ let the Curve BR be constructed; and because $n = \frac{1}{3}$, it will be $m = \frac{n}{n+1} = \frac{1}{4}$, and the Equation of the Curve BR will be $\dot{s} . \dot{y} :: a^{\frac{1}{4}} . r^{\frac{1}{4}}$. Hence the Length of the Curve will be $\frac{4}{3} \times \overline{2\,BL + P\Pi}$, and the whole Length of the Curve BRS is $\frac{8}{3}$ of the Diameter SB. If the Constructions of these Curves are continued, there will arise such a Series of Equations as this following, which is easily continued at Pleasure.

The

The Equation of the Circle.	1.	$\dot{s} \,.\, \dot{y} :: a \,.\, r.$
Of the Epicycloid.	2.	$\dot{s} \,.\, \dot{y} :: a^{\frac{1}{2}} \,.\, r^{\frac{1}{2}}.$
Of the Second.	3.	$\dot{s} \,.\, \dot{y} :: a^{\frac{1}{3}} \,.\, r^{\frac{1}{3}}.$
Of the Third.	4.	$\dot{s} \,.\, \dot{y} :: a^{\frac{1}{4}} \,.\, r^{\frac{1}{4}}.$
Of any.	5.	$\dot{s} \,.\, \dot{y} :: a^{\frac{1}{n}} \,.\, r^{\frac{1}{n}}.$

Here it may be obſerved in general, that all thoſe are capable of perfect Rectification, the Denominators of whoſe Indices are even Numbers; and ſince every one to that before it is as 1 to $1 - m$, it will appear to any one that conſiders it, that the Length of any Curve will be $\frac{1}{1-m} \times \frac{1-2m}{1-3m} \times \frac{1-4m}{1-5m} \times \frac{1-6m}{1-7m}$, &c. $\times$ S B, continuing the Series till the Fraction is reduced to nothing. Now if the Denominator of the Index be an odd Number, the Curves will be incapable of perfect Rectification, and any of their Arches will be incommenſurable to each other, to the Wholes, to any right Lines, and circular Arches: Yet all may be expreſſed by circular Arches and right Lines. But the total Length of any Curve will be to the Semicircle, as $\frac{1}{1-m} \times \frac{1-2m}{1-3m} \times \frac{1-4m}{1-5m}$, &c. to Unity. *Laſtly*, if the little Area, deſcribed by a Body revolving in any one of theſe, be taken as conſtant, that is, if $r\dot{y} = 1$, the Subtenſe of the Angle of Contact, to which (becauſe of the Time being given if the Area is given) the Centripetal Force tending to S will always be proportional, will be reciprocally as the Power of the Diſtance whoſe Index is $2m + 3$. And this is no contemptible Privilege of theſe Curves, that in all of them the Centripetal Force tending to S, is as ſome Dignity of the Diſtance reciprocally; which is the moſt ſimple, and moſt uſeful Law of Centripetal Forces, in ſearching into Nature.

Sect. 5.] Of all the Curves in which $\dot{s} \,.\, \dot{y} :: a^{n} \,.\, r^{n}$, the right Line itſelf is next to be conſider'd, (which is indeed properly called a Curve) the Point S being without that right Line. In this Line becauſe of ſimilar Triangles P p n, P B S, if B S $= a$, and S P $= r$, it will be $\dot{s} \,.\, \dot{y} :: r \,.\, a$. By the direct Method nothing can be conſtructed from the right Line but the Point B; but by the inverſe Method, or from the Concourſe of the Perpendiculars P L, p l, a Curve may be conſtructed, Fig. 18.

whoſe

whose Index will be equal to $\frac{m}{1-m}$, if m be the Index of the Curve B P. For if the Index of the Curve B L is n, then it will be $m = \frac{n}{n+1}$, and therefore $n = \frac{m}{1-m}$. Whence in this Case, since $m = -1$, it will be $n = \frac{-1}{2}$, and the Equation of the Curve B L will be $\dot{s} . \dot{y} :: r^{\frac{1}{2}} . a^{\frac{1}{2}}$, which is an Equation of the Parabola in respect of its Focus. From this construct another, by making the Angle L S N = L S B, and raising L N perpendicular to S L, meeting S N in N. Now because $m = \frac{-1}{2}$ it will be $n = \frac{-1}{3}$ and the Equation of the Curve will be $\dot{s} . \dot{y} :: r^{\frac{1}{3}} . a^{\frac{1}{3}}$, and $BP = \frac{BN - LN}{1-m} = \frac{1}{2}\overline{BN - LN}$, and therefore $BN = 2BP + LN$; and therefore this Curve is rectifiable. If the Series is continued, the Equations will arise as before in this Order.

Equation of the right Line,	$\dot{s} . \dot{y} :: r . a.$
Of the Parabola,	$\dot{s} . \dot{y} :: r^{\frac{1}{2}} . a^{\frac{1}{2}}.$
Of the Second,	$\dot{s} . \dot{y} :: r^{\frac{1}{3}} . a^{\frac{1}{3}}.$
Of the Third,	$\dot{s} . \dot{y} :: r^{\frac{1}{4}} . a^{\frac{1}{4}}.$
Of any,	$\dot{s} . \dot{y} :: r^{\frac{1}{n}} . a^{\frac{1}{n}}.$

In this Series the first are the right Line and the Parabola, whence it appears that half this Series, as well as the former, are commensurable to right Lines; and the other half may be exhibited by right Lines and Arches of a Parabola. In all these the Centripetal Force at S is reciprocally as that Power of the Distance, the Index of which is $3 - 2m$; and therefore is always between the duplicate and triplicate Ratio of the Distance reciprocally.

Sect. 6.] The Equation of the Equilateral Hyperbola at the Center is $\dot{s} . \dot{y} :: r^{2} . a^{2}$. From which by the direct Method such a Series may be deduced,

1. $\dot{s} . \dot{y} :: r^{2} . a^{2}.$
2. $\dot{s} . \dot{y} :: a^{2} . r^{2}.$
3. $\dot{s} . \dot{y} :: a^{\frac{2}{3}} . r^{\frac{2}{3}}.$
4. $\dot{s} . \dot{y} :: a^{\frac{2}{5}} . r^{\frac{2}{5}}.$
5. $\dot{s} . \dot{y} :: a^{\frac{2}{2n-1}} . r^{\frac{2}{2n-1}}.$

Of

Of these Curves, those the Denominators of whose Indices are in this Progression, — 1, 3, 7, 11, *&c.* may be exhibited in right Lines and Arches of the Hyperbola; the rest in right Lines and Arches of the Curve, whose Equation to the Axis *AB*, (making x the Abscifs, and y the Ordinate) is $\overline{xx+yy}|^2 = a^2x^2 - a^2y^2$, and which is constructed (*See Fig.* 17.) by bisecting the Angle *BSL*, and taking *SN* a mean Proportional between *SB* and *SL*.

The Curves which may be constructed from the Hyperbola in the inverse Method, proceed as in this Series.

Of the Hyperbola, 1. $\dot{s} . \dot{y} :: r^2 . a^2$.

3. $\dot{s} . \dot{y} :: r^{\frac{2}{3}} . a^{\frac{2}{3}}$.

3. $\dot{s} . \dot{y} :: r^{\frac{2}{5}} . a^{\frac{2}{5}}$.

where the Curves, the Denominators of whose Indices are in the Progression 1, 5, 9, 13, *&c.* may be express'd in right Lines and Hyperbolic Arches; but the others in right Lines and the Arches of the Curve just now explain'd.

If other Curves were desired which should exhibit other Series, this may be done very easily by Means either of a Circle or of a right Line. For by one of them all the Curves may be constructed, in which $\dot{s} . \dot{y} :: a^n . r^n$; by taking (if the Problem is to be solved by Means of the Circle) *BRS* to *BSL* as 1 to n, and *SN* in $SR = a^{\frac{n-1}{n}} \times SL^{\frac{1}{m}}$. For the Equation of the Curve drawn through all the Points *N*, will be $\dot{s} . \dot{y} :: a^n . r^n$. In like Manner Curves may be constructed by Means of a right Line, whose Equation will be $\dot{s} . \dot{y} :: r^n . a^n$. Fig. 19.

We have exhibited two infinite Series of Curves, that are commensurable to right Lines; we have demonstrated another to be commensurable to circular Arches, another to Parabolical, another to Hyperbolical together with right Lines; but those seem reducible to the Measure of right Lines by infinite Art only, as they are expressed in right Lines only by an infinite Equation.

XIII. Whereas the great *Newton* has not extended his Method for describing Curves, to those of the third Order which are without a double Point, or to those of an higher Order destitute of a *punctum multiplex*; and pronounces their Description to be reckoned among the more difficult Problems of Geometry; I hope the following Method will not be unacceptable to Geometricians, by which Geometrical Curves of any Order are constructed, (by the Help only of given Angles and Right Lines,) though they may be without a *punctum duplex* or *multiplex*.

A New Method of describing all Kinds of Curves, *by Mr.* C. Maclaurin, n. 359. p. 939.

1. Lines of the first Order are only right Lines themselves, which can meet one another only in one Point. Lines of the second Order are

Conic Sections, which cannot be cut by a right Line in more than two Points. Now all these may be thus constructed, according to Lem. 21. Lib. 1. of *Newton*'s *Principia*. Let two given Angles M C R and L S N
Fig. 20. move about two given Points C and S, so that Q the Concourse of the Legs C M, S L, may always describe the indefinite right Line A E given in Position; then the Concourse of the other Legs C R and S N in the Point P, will describe a Line of the second Order, or a Conic Section.

Fig. 21. 2. Let the Angle M C R (as before) move about the given Point C; and the given Angle L N Q by its Angular Point N always run over the given right Line A E, so that the Leg N Q may always pass through the given Point S. First, if the Concourse of the Legs C R and S N, and also the Point Q, be drawn through the infinite Line A B, the Concourse of the Legs C M and N L will describe a Curve-line of the third Order, having a double Point in C. Secondly, the rest remaining as be-
Fig. 22. fore, if the Concourse of the Legs C M and N L is drawn through the indefinite right Line A B; the Concourse of the Legs C R and S N in P, will describe a Curve of the third Order, having a double Point in S.

Fig. 23. *Example of Case* 1. Let the Angles M C R and L N S be right Angles, and A E, D B, C S, be parallel; also let S A and S D be perpendicular to A E and D B respectively, and let S D = 2 S A. These things supposed, if S D be less than the Line C S, a Curve described according to the Rule of the first Case, will be a Parabola with a Node and an Oval, of the 68th Species of *Newton*'s Curves. Now if S D = C S, the Oval vanishes, and the Node becomes a Cuspis, and the Curve so described will be *Neil*'s or the semicubical Parabola. But if S D be greater than C S, the Curve will be a Parabola with a Point and Campaniform, of the 69th Species.

Fig. 24. 3. Let the given Angles R M T and K N L move in such Manner, that the Points M and N may run over the indefinite Lines B M and D N respectively, and let the Legs R M and K N always pass through the given Points C and S. First, if the Concourse Q of the Legs M T and N L is drawn through the indefinite right Line A Q; then the Concourse of the Legs M R and N R in P will describe a Line of the fourth Order, having two double Points, one in C and the other in S. But secondly,
Fig. 25. if the Concourse of the Legs M R and N K be drawn through the indefinite right Line A Q; then the Concourse of the Legs M T and N L will describe a Line of the fourth Order, which will have no double Point.

Fig. 24. 4. Now in the first Case of this Construction, if the right Lines C M R and S N K coincide together with C S; then the Points C and S become
Fig. 26. simple, and the Curve will be of the third Order without a double Point. For Example, let the right Lines B M, A Q, D N be parallel to one another, and all perpendicular to C S. Also let the Angles R M T and K N L be right; and if a Curve be described according to the Rule of the first Case, the Legs C M R and S N K will coincide with S C;

and

and by this Conſtruction may be deſcribed the Curves of *Newton*, 10, 11, 20, 21, 40, according to the various Poſitions of the Points C and S in reſpect of the three right Lines B M, A Q, D N; but all theſe Species will be without a double Point.

5. But Lines of the fourth Order, which have a treble Point, may be thus conſtructed. Fig. 27. Let there be three right Lines A Q, B N, D M, given in Poſition. Alſo let the Angles Q C T, S N M, N M L, be given and invariable. Let the Points N and M run through the right Lines B N and D M, ſo that the Leg N Q may always paſs through the given Point S. Let Q C T ſo revolve about C, that the Concourſe of the Legs C K and S N may run through a third right Line A Q. Then the Concourſe of the Legs C T and M L will deſcribe a Line of the fourth Order, having a triple Point in C.

6. I have ſhewn how Lines of the fourth Order may be deſcribed, which have a triple Point or two double Points. Others having only one double Point may be thus conveniently deſcribed. Let there be three Lines A Q, B N, D M, given in Poſition as before; alſo let the Angles S N K, S M L, R C T be given. Fig. 28. Let the Points N, M, S, be always in the ſame right Line. Let the Points N, M, move as before through the right Lines B N, D M; if the Concourſe of the Legs C R, N K, is drawn through the indefinite right Line A Q, then the Concourſe of the Legs C T, M L, will deſcribe a Line of the fourth Order, having one double Point in C. Now theſe two laſt Propoſitions ſupply us with new Methods for deſcribing Lines of the third Order, as well thoſe that have double Points, as thoſe that have none. But in this ſhort Specimen of our Method theſe muſt be omitted.

7. Let the Angles and right Lines remain, as in *Prop.* 3. and let the Fig. 29 Concourſe of the right Lines M T, N K, now be drawn through the indefinite right Line A Q; and the Concourſe of the Legs M R and N L will deſcribe a Line of the fifth Order, having a quadruple Point in S. I have alſo other Methods for deſcribing Curves of the fifth Order, which have a double or triple Point, or two double Points, or none but ſimple Points, but theſe may ſuffice to ſhew the Simplicity and Univerſality of the Method. But it muſt be obſerved, that in particular and ſimpler Circumſtances of the Angles and right Lines, ſometimes a Line will paſs into a Curve of an Order inferior to that which is explain'd in the Propoſition. Nay, all the Propoſitions ſupply particular Methods of deſcribing ſome Curves of every inferior Order.

8. *General Propoſition.* Let right Lines be taken at Pleaſure any Fig. 30. where poſited in the ſame Plain, of which let the Number be n; as B N, E R, F T, alſo let other right Lines be taken at Pleaſure, as D M, G L, H K, &c. of which let the Number be m. Let the Angles C N R, N R T, R T Q, &c. alſo the Angles S M L, M L K, L K Q, &c. be invariable, whilſt the Angular Points N, R, T, M, L, K, perambulate the indefinite right Lines B N, E R, F T, D M, G L, H K; let the Concourſe of the

Legs T Q and K Q be drawn thro' the indefinite right Line A Q; to find the Order of the Curve which ſhall be deſcribed by the Concourſe of the Leg S M with any one of the right Lines C N, N R, R T, T Q, *&c.* for Inſtance, with the Line R T.

In the Series of the right Lines C N, N R, R T, T Q, *&c.* let s denote the Number of the right Line R T, by the Concourſe of which with S M the Curve is to be deſcribed, from the Line C N incluſively; which in this Caſe is the third, or $n = 3$. Then will the Curve be of an Order which is expreſs'd by the Number $sm + s + n + 1$. Whence in the Caſe denoted by the Figure, ſince $s = m = n = 3$, the Curve will be of the ſixteenth Order.

In theſe Deſcriptions we have only poſtulated, that right Lines and Angles ſhould be given. But generally the more complicated Curves are more eaſily deſcribed by the Help of ſimpler Curves. And I have inveſtigated Propoſitions of this Kind, not leſs univerſal than theſe. But I omit theſe at preſent, with the Demonſtrations of theſe, becauſe they would be too prolix, tho' perhaps I may publiſh them hereafter.

The Doctrine of Combinations and Alternations, by Major Ed. Thornicroft, n. 299. p. 1961.

XIV. In order to underſtand what follows, it muſt be obſerved, *1ſt*, That as in the Notation of Powers, $a\,a\,a\,a\,b\,b\,b\,c\,c$ is deſigned by $a^4\,b^3\,c^2$, and univerſally p times the Poſition of a, q times the Poſition of b, r times the Poſition of c, by $a^p\,b^q\,c^r$, ſo in Things expoſed likewiſe; (unleſs where 'tis propoſed they ſhould be all different) which Indices, as they have here no Relation to Powers, but expreſs only the Occurrences of thoſe Things to which they reſpectively belong; I therefore call Indices of Occurrences.

2dly, That as often as I ſhall hereafter mention the Combination or Alternations of the $p^s\ q^s\ r^s$ or s^s, (which, conſidered by themſelves, are capable of no Variation) I mean of thoſe Things whoſe Indices they are.

3dly, That m is generally put for the whole Number of Things expoſed, whether all different or not, *i. e.* equal to the Sum of their Indices; and n, for ſuch a Number of them, as each Combination and Alternation muſt conſiſt of; (unleſs preſuppoſed equal) which explains what is hereafter meant by the Combinations and Alternations of m Things taken n and n; or of m Things taken m and m; and the like Expreſſion, by whatever Symbols the Number of Things, out of which the Combinations and Alternations are to be made, or of which they are to conſiſt, may be deſign'd.

Lemma 1.] If in a right Line, at any Diſtances, be placed any Number of Things $a\,b\,c\,d$, *&c.* the Number of the Intervals $a\,b$, $b\,c$, $c\,d$, *&c.* terminated each by two adjacent Things, is one leſs than the Number of Things.

For, whereas every Interval is terminated by two adjacent Things, if to any Number of Things, be added one Thing more, one Interval only is thereby added. *Q. E. D.*

Lemma 2.] The Number of the Alternations of m Things $a\,b\,c\,d$, *&c.* different from each other, taken m and m, is m times the Number of the Alternations of $m - 1$ Things $a\,b\,c$, taken $m - 1$ and $m - 1$.

For

For (by *Lem.* 1*st.*) the last Letter d, besides the Position it hath, may have $m-2$ Positions, *viz.* in the Intervals which are between $m-1$ Things $a\,b\,c$; but it may also have one more, for it may be put first of all, it may therefore have m Positions; and those in all the different Orders whereof $m-1$ Things are capable; which being all the possible Positions of d, in all the Varieties of $a\,b\,c$, is all the Variety whereof the whole Number of Things exposed $a\,b\,c\,d$, &c. is capable. *Q. E. D.*

Lemma 3.] The Number of the Alternations of m Things a, b, c, d, *&c.* different each from other, taken m and m, is equal to $m \times \overline{m-1} \times \overline{m-2} \times \overline{m-3} \times \overline{m-4}$, *&c.* continued to m Places.

For let $m\,O$ express the Number of the Alternations of m Things different from each other; $\overline{m-1}\,O$, of $m-1$ Things, and the like.

'Tis evident that if $m=1$, it will be $m\,O=m$; for there can be but one Order of one Thing.

And if m be greater than Unity, then it will be (by *Lem.* 2.) $m\,O=m \times \overline{m-1}\,O = m \times \overline{m-1} \times \overline{m-2}\,O = m \times \overline{m-1} \times \overline{m-2} \times \overline{m-3}\,O=$, *&c.* till we have an Equation consisting of m Places; *i. e.* $= m \times \overline{m-1} \times \overline{m-2} \times \overline{m-3} \times$ *&c.* continued to m Places. *Q. E. D.*

Lemma 4.] If $m\,\omega$ express the Number of the Alternations of m Things $a^p\,b^p\,c^p\,d^q\,e^q\,f^r$, *&c.* taken m and m, and α the Number of p's, β the Number of q's, γ the Number of r's, it will be

$$m\,\omega = \frac{m \times \overline{m-1} \times \overline{m-2} \times \overline{m-3} \times \overline{m-4} \times \overline{m-5} \times \text{\&c. continued to } m \text{ Places.}}{\overline{p \times \overline{p-1} \times \overline{p-2} \times \text{\&c.}}^{\,\alpha} \times \overline{q \times \overline{q-1} \times \text{\&c}}^{\,\beta} \times \overline{r \times \overline{r-1} \times \text{\&c.}}^{\,\gamma}}$$

each Series continued to p, q, r, *&c.* Places respectively.

For the Number of the Alternations of any Number of Things, however divided into Parts, is produc'd by a continual Multiplication of the Alternations of those Things among themselves respectively, which compose each Part, into the Number of their Alternations one amongst the other; *i. e.* in the present Case (the several Occurrences being supposed to compose the several Parts, and consequently the Number of the Alternations of the Things composing each Part equal to Unity) $m\,\omega =$ to the Number of the Alternations of the Things composing the Parts one amongst the other; but the Number of their Alternations one amongst the other, is the same in this Case, as if the Things exposed, being all different, were divided into the same Parts; for the Things which compose each Part in both Cases, are different from the rest of the Things exposed, *i. e.* by *Lem.* 3.

$$m\,\omega = \frac{m \times \overline{m-1} \times \overline{m-2} \times \overline{m-3} \times \overline{m-4} \times \overline{m-5} \times \text{\&c. continued to } m \text{ Places.}}{\overline{p \times \overline{p-1} \times \overline{p-2} \times \text{\&c.}}\,|^{\,\alpha} \times \overline{q \times \overline{q-1} \times \text{\&c.}}\,|^{\,\beta} \times \overline{r \times \overline{r-1} \times \text{\&c.}}\,|^{\,\gamma}}$$

each Series continued to p, q, r, Places respectively. *Q. E. D.*

Lemma 5.] The Number of the Combinations of m Things $a\,b\,c\,d$, *&c.* different from each other, taken n and n, is equal to

$$\frac{m \times \overline{m-1} \times \overline{m-2} \times \overline{m-3} \times \text{\&c.}}{n \times \overline{n-1} \times \overline{n-2} \times \overline{n-3} \times \text{\&c.}}$$

each Series continued to n Places.

For if the Things expos'd be divided in two Parts, *viz.* in the *Ratio* of n and $m-n$, 'tis evident that their different Combinations taken n and n, are produced by the Alternations of the Things composing the Parts one amongst the other: And therefore the Number of those = to the Number of these = to the Number of the Alternations of m Things taken m and m, the Indices of whose Occurrences are n and $m-n = \dfrac{m\times m-1\times n-2\times m-3\times \&c.\ \text{continued to } m \text{ Places.}}{n\times n-1\times \&c.\ \times\ m-n\times m-n-1\times \&c.}$ each Series continued to n and $m-n$ Places respectively (by *Lem.* 4.) *i. e.* because $n+m-n=m=\dfrac{m\times m-1\times m-2\times m-3\times \&c.}{n\times n-1\times n-2\times n-3\times \&c.}$ each Series continued to n Places. *Q. E. D.*

But the Number of the Alternations in every Combination is $=n\times n-1\times n-2\times n-3\times \&c.$ continued to n Places, (by *Lem.* 3.) therefore

Lemma 6.] The Number of the Alternations of m Things $a\ b\ c\ d$, &c. different each from other, taken n and n, is $=m\times m-1\times m-2\times m-3\times \&c.$ continued to n Places. *Q. E. D.*

Scholium.] Since in the Things exposed the same Things may occur more than once, and also n be less than m, the Indices of the Occurrences, which are in some of the Combinations of m Things taken n and n, may differ from those which are in others; but those Combinations, the Indices of whose Occurrences are the same, are said to be in the same Form: Therefore whereas n is equal to the Sum of the Indices which are in each Combination taken n and n, if n be expressed by all the different Combinations of such Indices only (being integer Numbers) whereof no one may exceed the highest Index of the Things exposed, and being more than one in a Combination, are each of them, which are in the same Combination, comprehended in a distinct Index thereof; these Expressions of n will necessarily be the several Forms of the Combinations taken n and n, whereof m Things are capable: Whence is derived a general Theorem for finding the Combinations and Alternations of m Things taken n and n universally: *i. e.* Whether m consist of Things all different or not, and whether n be equal to, or less than m.

Theorem.] If n be expressed, according to all the different Forms of Combination which the Things exposed are capable of,

$$\text{and}\left\{\begin{array}{l} p = \text{the highest Index} \\ q = \text{the next highest} \\ r = \text{the next highest} \\ s = \text{the next highest} \\ \&c. \end{array}\right. \left\{\begin{array}{l} \alpha = \text{the Number of } p' \\ \beta = \text{the Number of } q' \\ \gamma = \text{the Number of } r' \\ \delta = \text{the Number of } s' \end{array}\right\} \text{In every Form of Combination.}$$

$$\left\{\begin{array}{l} A = \text{the Number of all Indices not less than } p \\ B = \text{the Number of all Indices not less than } q \\ C = \text{the Number of all Indices not less than } r \\ D = \text{the Number of all Indices not less than } s \\ \&c. \end{array}\right\} \text{Which are in the Things exposed.}$$

and $b=a+\beta$, $c=b+\gamma$, $d=c+\delta$, &c. I say

I ſay the Number of the Combinations of m Things taken n, and n in any one Form of Combination, ſhall be $\frac{A\times A-1\times A-2}{\alpha\times\alpha-1\times\alpha-2}$ &c.

$$\times\frac{B-\alpha\times B-\alpha-1}{\beta\times\beta-1}\times\&c.\frac{C-b\times C-b-1}{\gamma\times\gamma-1}\times\&c.\times\frac{D-c\times D-c-1}{\delta\times\delta-1}\ \&c.$$

continued to ſo many Terms as there are different Indices in the Form of Combination, and each Term to α, β, γ, δ, &c. Places reſpectively, and this Number multiplied into

$$\frac{\times n-1\times n-2\times n-3\times n-4\times n-5\times n-6\ \&c.\ \text{continued to } n \text{ Places.}}{\overline{p\times p-1\times p-2\times\&c.}|^{\alpha}\times\overline{q\times q-1\times\&c.}|^{\beta}\times\overline{r\times r-1\times\&c.}|^{\gamma}}\times\&c.$$

each Series continued to p q r, &c. Places reſpectively, ſhall be the Number of their Alternations.

But the Sum of all the Combinations and Alternations which are in every Form of n, ſhall be the whole Number of Combinations and Alternations of m Things taken n and n.

Demonſtration.] Firſt then 'tis evident, That thoſe Combinations which are in different Forms, differ from each other.

Again, 'tis evident, that the Combinations of m Things, as a^p b^p c^p d^p e^q f^q g^q h^r i^r, &c. (the Indices ſimply conſidered) taken n and n, in a Form wherein are p' q' r', ſhall be equal to the Number of the Combinations of the p', which are in the Things expoſed, taken α and α, multiplied into the Number of the Combinations of the q' taken β and β, multiplied into the Number of the Combinations of the r' taken γ and γ.

But becauſe p and all leſſer Indices are comprehended in every Index, which is greater than themſelves; therefore is $A=$ to the Number of p', which are in the Things expoſed, and for the ſame Reaſon would $B=$ the Number of the q', and C the Number of r': But the Number of the p', which are in every Form of Combination, is $=\alpha$; therefore is $B-\alpha$ $=$ to the Number of q'; alſo becauſe the Number of p' and q' together, which are in every Form of Combination, wherein there are q', is $=\alpha+\beta=b$; therefore is $C-b=$ to the Number of r', and ſo on, how many ſoever were the different Indices in any Form of Combination.

But (by *Lemma* 5.) the Number of the Combinations of the p', which are in the Things expoſed, whoſe Number is A, taken α and α, is $=\frac{A\times A-1\times A-2}{\alpha\times\alpha-1\times\alpha-2}$, &c. continued to α Places, and the Number of the Combinations of the q', whoſe Number is $B-\alpha$, taken β and β, is $=\frac{B-\alpha\times B-\alpha-1}{\beta\times\beta-1}\frac{\times B-\alpha-2}{\times\beta-2}$, &c. continued to β Places, and the Number of the Combinations of the r', whoſe Number is $C-b$, taken γ and γ, is $=\frac{C-b\times C-b-1}{\gamma\times\gamma-1}$, &c. continued to γ Places. *Q. E. D.*

But

But every Combination in one and the ſame Form, affords the ſame Number of Alternations: Therefore the Number of Alternations, in any one Form, is ſo many Times the Number of Combinations, as is the Number of Alternations in any one of theſe Combinations.

But (by *Lem.* 4.) the Number of Alternations in any of thoſe Combinations ſhall be

$$\frac{n \times n-1 \times n-2 \times n-3 \times n-4 \times n-5 \times n-6 \times \text{\&c. continued to } n \text{ Places.}}{\overline{p \times p-1 \times p-2 \times \text{\&c.}}|^{\alpha} \times \overline{q \times q-2 \times \text{\&c.}}|^{\beta} \times \overline{r \times r-1 \times \text{\&c.}}|^{\gamma} \times}$$ each Series continued to $p\,q\,r$, &c. Places reſpectively. *Q. E. D.*

Now to make an Application of this general Rule, to thoſe particular Caſes which have already been conſidered by others, and which are contained in our 3d, 4th, 5th, and 6th *Lemma's*, and by us more generally demonſtrated; I ſay

If $n = m$, there can be but one Form of Combination, and but one Combination in that Form; and therefore the Number of Alternations

$$\frac{m \times m-1 \times m-2 \times m-3 \times m-4 \times \text{\&c. continued to } m \text{ Places.}}{\overline{p \times p-1 \times \text{\&c.}}|^{\alpha} \times \overline{q \times q-1 \times \text{\&c.}}|^{\beta} \times \overline{r \times \text{\&c.}}|^{\gamma} \times \text{\&c.}}$$ each Series to $p\,q\,r$, &c. Places reſpectively, *i. e.* (if $p = 1$) $= m \times m-1 \times m-2 \times m-3 \times m-4 \times$ &c. continued to m Places; which are the Caſes of the 4th and 3d *Lemma's*.

But if the Things expoſed are all different, and n be leſs than m, which is the Caſe of the 5th and 6th *Lemma's*, then alſo can there be but one Form of Combination, and it will be $A = m$ and $\alpha = n$, and the whole Number of Combinations $= \dfrac{A \times A-1 \times A-2 \times \text{\&c.}}{\alpha \times \alpha-1 \times \alpha-2 \times \text{\&c.}}$ *i. e.*

$= \dfrac{m \times m-1 \times m-2 \times \text{\&c.}}{n \times n-1 \times n-2 \times \text{\&c.}}$ each Series continued to n Places, and therefore the Number of Alternations $= m \times m-1 \times m-2 \times$ &c. continued to n Places.

But fully to illuſtrate this Theorem, which may ſeem ſomewhat too abſtracted, I ſhall ſubjoin one ſhort Example.

Example.] Let the Things expoſed be *a a a b b b c c*, or according to our Way of Notation $a^3\, b^3\, c^2$; 'Tis required to find the Number of their Combinations and Alternations taken 4 and 4.

Then (becauſe in the Things expoſed, there is no one Thing occurs more than thrice, nor more than three Things different from each other) will all the Forms of Combination, which the Things expoſed are capable of, be theſe,

$$viz. \left\{\begin{matrix} 3 \,.\, 1 \\ 2 \,.\, 2 \\ 2 \,.\, 1 \,.\, 1 \end{matrix}\right\} \text{Then}$$

In

In the 1ſt Form will $p=3$, $q=1$, $\alpha=1$, $\beta=1$, $A=2$, $B=3$,
In the 2d Form will $p=2$, ——, $\alpha=2$, ——, $A=3$, ——
In the 3d Form will $p=2$, $q=1$, $\alpha=1$, $\beta=2$, $A=3$, $B=3$.

The Number of Combinations in the 1ſt Form $=\frac{A}{\alpha}\times\frac{B-\alpha}{\beta}=\frac{2}{1}\times\frac{2}{1}=4$

The Number of Combinations in the 2d Form $=\frac{A\times\overline{A-1}}{\alpha\times\overline{\alpha-1}}=\frac{3\times2}{2\times1}=3$

The Number of Combinations in the 3d Form $=\frac{A}{\alpha}\times\frac{\overline{B-\alpha}\times\overline{B-\alpha-1}}{\beta\times\overline{\beta-1}}=\frac{2\times1}{2\times1}=3$

And the whole Number of Combinations $=10$

Alſo the Number of Alternations.

In the 1ſt Form $=4\times\frac{n\times\overline{n-1}\times\overline{n-2}\times\overline{n-3}}{\overline{p\times\overline{p-1}\times\overline{p-2}}|^{\alpha}\times q^{\beta}}=4\times\frac{4\times3\times2\times1}{\overline{3\times2\times1}|^{1}\times}=4\times4=16$

In the 2d Form $=3\times\frac{n\times\overline{n-1}\times\overline{n-2}\times\overline{n-3}}{\overline{p\times\overline{p-1}}|^{\alpha}}=4\times\frac{4\times3\times2\times1}{\overline{2\times1}|^{2}}=3\times6=18$

In the 3d Form $=3\times\frac{n\times\overline{n-1}\times\overline{n-2}\times\overline{n-3}}{\overline{p\times\overline{p-1}}|^{\alpha}\times q^{\beta}}=3\times\frac{4\times3\times2\times1}{\overline{2\times1}|^{1}\times12}=3\times12=36$

And the whole Number of Alternations $=70$

Many are the Properties of this *Theorem* in common with others, as, To find an *Unciæ* of a Multinomial raiſed to any integer Power. To raiſe an infinite Series to an integer Power, though of an interrupted Order, without introducing any Thing immaterial, or which muſt afterwards be expunged, and many others. But then ſo many Terms of the Series muſt be taken in at firſt as ſhall ſerve to the Purpoſes of the intended Approximation, otherwiſe, as often as it ſhall fall ſhort of that, the Operation muſt be begun *de novo.*

Many likewiſe are the Properties peculiar to this *Theorem*, and great Variety of Problems might be framed; and I ſcruple not to ſay, many may occur in Practice, which are ſolvable by this, and no other Method whatever.

Hence may be found the Number of Words whereof the 24 Letters are capable, from one Letter in each Word, to any Number of Letters given.

Hence

Hence may be found the Number of all Numbers, to any given Number of Places, which may be produced from any Number of Figures given.

Hence alſo the Compaſs of a Muſical Inſtrument being given, the Time and Number of the Bars, whereof each Tune ſhall conſiſt, the Number of Tunes may be found which that Inſtrument is capable of.

To give an Inſtance of the prodigious Variety that there is in Muſick, I have calculated the Number of Tunes in common Time, conſiſting of eight Bars each, which may be played on an Inſtrument of one Note Compaſs only, and it is this, *viz.* 27584. 270157. 013570. 368586. 999728. 299176. whereas the Changes on 24 Bells is but 620448. 401733. 239439. 360000. which is but

$\frac{1}{444588.\ 604583}$ of the Number of Tunes, and yet Dr. *Wallis* in his *Algebra* demonſtrates, could not be diſpatch'd in 31557. 600000. 000000 Years.

If then the Inſtrument were of as many Notes Compaſs as any Inſtrument now in Uſe, how prodigiouſly muſt the Number of Tunes be encreaſed! the Calculation of which (though much more intricate and operoſe) would be equally attainable by our Theorem.

The Univerſal Solution of Cubic and Biquadratic Equations, as well Analytically as Geometrically and Mechanically: By John Colſon, M. A. & F. R. S. *n.* 309. *p.* 2353.

XV. §. 1. Of the Univerſal Cubic Equation

$$\begin{array}{rrr} x^3 = 3px^2 & + 3q \times x & + 2r \\ & - 3p^2 & + p^3 \\ & & - 3pq \end{array}$$

The three Roots are

$$x = p + \sqrt[3]{r + \sqrt{r^2 - q^3}} + \sqrt[3]{r - \sqrt{r^2 - q^3}},$$

$$x = p - \frac{1 - \sqrt{-3}}{2} \times \sqrt[3]{r + \sqrt{r^2 - q^3}} - \frac{1 + \sqrt{-3}}{2} \times \sqrt[3]{r - \sqrt{r^2 - q^3}},$$

$$x = p - \frac{1 + \sqrt{-3}}{2} \times \sqrt[3]{r + \sqrt{r^2 - q^3}} - \frac{1 - \sqrt{-3}}{2} \times \sqrt[3]{r - \sqrt{r^2 - q^3}}.$$

Or to make the Arithmetical Calculation the more eaſy and ready, we may ſuppoſe $m + \sqrt{n}$ to repreſent the Cubic Root of the irrational Binomial $r + \sqrt{r^2 - q^3}$. Then the three Roots of the foregoing Equation will be $x = p + 2m$, and $x = p - m \pm \sqrt{-3n}$.

Therefore

Therefore when any Cubic Equation is given, we muſt make a Comparifon between its Terms, and the ſeveral Terms of the Univerſal Cubic Equation reſpectively, by which Means the Values of p, q, and r will be eaſily found: And when theſe are known, the Roots of the given Equation will thence be known. Of this Solution here follow ſome Examples in Numbers.

1. Let the Root x be propoſed to be found in this Equation, $x^3 = 2x^2 + 3x + 4$. Firſt, as order'd above it will be $3p = 2$, or $p = \frac{2}{3}$. Secondly, $3q - 3p^2 = 3$, that is, $3q - \frac{4}{3} = 3$, or $q = \frac{13}{9}$. Thirdly, $2r + \overline{p^2 - 3q} \times p = 4$, or $2r - \frac{70}{27} = 4$, that is, $r = \frac{89}{27}$; and $r^2 - q^3 = \frac{212}{27}$. Therefore $x = \frac{2}{3} + \sqrt[3]{\frac{89}{27} + \sqrt{\frac{212}{27}}} + \sqrt[3]{\frac{89}{27} - \sqrt{\frac{212}{27}}}$. The other two Roots are impoſſible.

2. In the Equation $x^3 = 12x^2 - 41x + 42$, it will be firſt, $3p = 12$, or $p = 4$. Secondly, $3q - 3p^2 = -41$, or $3q - 48 = -41$, that is, $q = \frac{7}{3}$. Laſtly, $2r + \overline{p^2 - 3q} \times p = 42$, that is, $2r + 36 = 42$, or $r = 3$. Thence $r^2 - q^3 = -\frac{100}{27}$. But the Cubic-root of $r + \sqrt{r^2 - q^3}$, that is, of the ſurd Binomial $3 + \sqrt{-\frac{100}{27}}$, being extracted by the Methods which the Arithmetick of Surds will ſupply, will be found to be $-1 + \sqrt{-\frac{4}{3}}$, which is repreſented by $m + \sqrt{n}$. And therefore the Root $x = (p + 2m =)\ 4 - 2 = 2$. Alſo $x = (p - m \pm \sqrt{-3n} =)\ 4 + 1 \pm \sqrt{4} = 7$ or 3. Or again, another Cubic-root (for it has three) of the ſame Binomial $3 + \sqrt{-\frac{100}{27}}$ is $\frac{3}{2}$

$+\sqrt{-\frac{1}{12}}$, which is represented by $m+\sqrt{n}$. Therefore the Root $x=(p+2m=)\ 4+3=7$. Also $x=(p-m\pm\sqrt{-3n}=)\ 4-\frac{3}{2}\pm\sqrt{\frac{1}{4}}=3$ or 2. Lastly, the third Cubic-root of the same Binomial $3+\sqrt{-\frac{100}{27}}$ is $-\frac{1}{2}-\sqrt{-\frac{25}{12}}(=m+\sqrt{n}.)$ And therefore the Root $x=(p+2m=)\ 4-1=3$; and also $x=(p-m\pm\sqrt{-3n}=)\ 4+\frac{1}{2}\pm\sqrt{\frac{25}{4}}=7$ or 2.

3. In the Equation $x^3=-15x^2-84x+100$, it will be $p=-5$, $q=-3$, $r=135$; and the Cubic-root of the Binomial $135+\sqrt{18252}$ is $3+\sqrt{12}$. Therefore the Root $x=-5+6=1$, and $x=-5-3\pm\sqrt{-36}=-8\pm\sqrt{-36}$, impossible.

4. In the Equation $x^3=34x^2-310x+1012$, it will be $p=\frac{34}{3}$, $q=\frac{226}{9}$, $r=\frac{5536}{27}$; and the Cubic-root of the Binomial $\frac{5536}{27}+\sqrt{\frac{707560}{27}}$ is $\frac{16}{3}+\sqrt{\frac{10}{3}}$. Therefore the Root $x=\frac{34}{3}+\frac{32}{3}=22$, and $x=\frac{34}{3}-\frac{16}{3}\pm\sqrt{-10}=6\pm\sqrt{-10}$, impossible.

5. In the Equation $x^3=28x^2+61x-4048$, it will be $p=\frac{28}{3}$, $q=\frac{967}{9}$, $r=-\frac{25010}{27}$; and the Cubic-root of the Binomial $-\frac{25010}{27}+\sqrt{-382347}$ is $\frac{41}{6}+\sqrt{-\frac{243}{4}}$. Therefore $x=\frac{28}{3}+\frac{41}{3}=23$, and $x=\frac{28}{3}-\frac{41}{6}\pm\frac{27}{2}=16$ or -11.

6. In the Equation $x^3=-x^2+166x-660$, it will be $p=-\frac{1}{3}$

$\frac{1}{3}$, $q = \frac{499}{9}$, $r = -\frac{9658}{27}$; and the Cubic-root of the Binomial $-\frac{9658}{27}$ $+\sqrt{-\frac{1147205}{27}}$ is $-\frac{22}{3} + \sqrt{-\frac{5}{3}}$. Therefore $x = -\frac{1}{3} - \frac{44}{3} = -15$, and $x = -\frac{1}{3} + \frac{22}{3} \pm \sqrt{5} = 7 \pm \sqrt{5}$, irrational.

7. In the Equation $x^3 = 63x^2 + 99673x + 9951705$, it will be $p = 21$, $q = \frac{100996}{3}$, $r = 6031680$; and the Cubic-root of the Binomial $6031680 + \sqrt{-\frac{4788717504 3136}{27}}$ is $183 + \sqrt{-\frac{529}{3}}$. Therefore $x = 21 + 366 = 387$, and $x = 21 - 183 \pm 23 = -139$ or -185.

And so we must proceed in other Examples. Now the Theorem may be investigated in the following Manner. I suppose the Root of some Equation to be $z = a + b$, and multiplying cubically there will arise $z^3 = (a^3 + 3a^2b + 3ab^2 + b^3 =)\ a^3 + 3ab \times \overline{a+b} + b^3$. Now instead of $a + b$ substituting its Value z, it will become $z^3 = 3abz + a^3 + b^3$, which is a Cubic Equation constructed from the Root $z = a + b$, in which Equation the second Term is wanting. Now that this may be reduced to a better Form, I assume the Equation $z^3 = 3qz + 2r$, which is now to represent the Equation $z^3 = 3abz + a^3 + b^3$. Therefore to transmute this into that, we shall have first $3q = 3ab$, and therefore $q^3 = a^3b^3$. Secondly, $2r = a^3 + b^3$, or $2ra^3 = (a^6 + a^3b^3 =)\ a^6 + q^3$. Then resolving this Quadratick Equation, we shall have $a^3 = r + \sqrt{r^2 - q^3}$, and thence $b^3 = 2r - a^3 = r - \sqrt{r^2 - q^3}$. So that at last it will be $a = \sqrt[3]{r + \sqrt{r^2 - q^3}}$, and $b = \sqrt[3]{r - \sqrt{r^2 - q^3}}$. Therefore in the Cubic Equation $z^3 = 3qz + 2r$, we shall have the Root $z = (a + b =)$ $\sqrt[3]{r + \sqrt{r^2 - q^3}} + \sqrt[3]{r - \sqrt{r^2 - q^3}}$.

But this Root is really three-fold, according to the three-fold Value which $\sqrt[3]{r + \sqrt{r^2 - q^3}}$ and $\sqrt[3]{r - \sqrt{r^2 - q^3}}$ can acquire. For the Cubic-root of any Quantity whatever is three-fold, forasmuch as the Cubic-root of Unity itself is either 1, or $-\frac{1}{2} + \frac{1}{2}\sqrt{-3}$, or $-\frac{1}{2} - \frac{1}{2}\sqrt{-3}$.

And

And this will appear by Cubing any one of theſe Quantities. Therefore if $r+\sqrt{r^2-q^3}$, or $1\times\overline{r+\sqrt{r^2-q^3}}$, be conſider'd as a Cube, its Cubic-root or Roots will be $\sqrt[3]{1}\times\sqrt[3]{r+\sqrt{r^2-q^3}}$; that is, firſt $1\times\sqrt[3]{r+\sqrt{r^2-q^3}}$, which we have above call'd $m+\sqrt{n}$, or $1\times\overline{m+\sqrt{n}}$. Secondly, $\frac{-1+\sqrt{-3}}{2}\times\sqrt[3]{r+\sqrt{r^2-q^3}}$, which therefore will be $\frac{-1+\sqrt{-3}}{2}\times\overline{m+\sqrt{n}}=\frac{-m-\sqrt{n}+m\sqrt{-3}+\sqrt{-3n}}{2}$. Thirdly, $\frac{-1-\sqrt{-3}}{2}\times\sqrt[3]{r+\sqrt{r^2-q^3}}$, which therefore will be $\frac{-1-\sqrt{-3}}{2}\times\overline{m+\sqrt{n}}=\frac{-m-\sqrt{n}-m\sqrt{-3}-\sqrt{-3n}}{2}$. In like Manner the Apotome $r-\sqrt{r^2-q^3}$ or $1\times\overline{r-\sqrt{r^2-q^3}}$, being conſider'd as a Cube, will have the Cubic-root or Roots $\sqrt[3]{1}\times\sqrt[3]{r-\sqrt{r^2-q^3}}$; that is, firſt, $1\times\sqrt[3]{r-\sqrt{r^2-q^3}}$, which we have call'd $m-\sqrt{n}$, or $1\times\overline{m-\sqrt{n}}$. Secondly, $\frac{-1+\sqrt{-3}}{2}\times\sqrt[3]{r-\sqrt{r^2-q^3}}$, which therefore will be $\frac{-1+\sqrt{-3}}{2}\times\overline{m-\sqrt{n}}=\frac{-m+\sqrt{n}+m\sqrt{-3}-\sqrt{-3n}}{2}$. Thirdly, $\frac{-1-\sqrt{-3}}{2}\times\sqrt[3]{r-\sqrt{r^2-q^3}}$, which therefore will be $\frac{-1-\sqrt{-3}}{2}\times\overline{m-\sqrt{n}}=\frac{-m+\sqrt{n}-m\sqrt{-3}+\sqrt{-3n}}{2}$. And by a due Connexion of theſe Roots, we ſhall have $z=\sqrt[3]{r+\sqrt{r^2-q^3}}+\sqrt[3]{r-\sqrt{r^2-q^3}}=m+\sqrt{n}+m-\sqrt{n}=2m$. And ſecondly, $z=\frac{-1+\sqrt{-3}}{2}\times\sqrt[3]{r+\sqrt{r^2-q^3}}+\frac{-1-\sqrt{-3}}{2}+\sqrt[3]{r-\sqrt{r^2-q^3}}=\frac{-m-\sqrt{n}+m\sqrt{-3}+\sqrt{-3n}}{2}+$

—

$\frac{-m+\sqrt{n}-m\sqrt{-3}+\sqrt{-3n}}{2} = -m+\sqrt{-3n}$. Laſtly, $z =$

$$\frac{-1-\sqrt{-3}}{2}\times\sqrt[3]{r+\sqrt{r^2-q^3}} + \frac{-1+\sqrt{-3}}{2}\times\sqrt[3]{r-\sqrt{r^2-q^3}}$$

$$= \frac{-m-\sqrt{n}-m\sqrt{-3}-\sqrt{-3n}}{2} + \frac{-m+\sqrt{n}+m\sqrt{-3}-\sqrt{-3n}}{2}$$

$= -m - \sqrt{-3}$. Theſe therefore will be the three Roots of the Cubic Equation $z^3 = 3qz + 2r$. Now that the Parts are duly connected in the foregoing Manner, may be farther proved by being continually multiply'd together in the uſual Method. Finally, make $z = x - p$, and, by Subſtitution, the Equation will become $x^3 - 3px^2 + 3p^2x - p^3 = 3qx - 3pq + 2r$; which, by Tranſpoſition, will be the ſame Equation as above, and its Roots are the ſame as are there exhibited.

Here it deſerves to be obſerved, that all the Roots of any Cubic Equation are then real and poſſible, when the irrational Member of the Binomial $\sqrt{r^2-q^3}$ includes an Impoſſibility; that is, when q is an affirmative Quantity, and at the ſame Time its Cube is greater than the Square of r. But if this Member $\sqrt{r^2-q^3}$ is poſſible, that is, if q is a negative Quantity, or being affirmative if its Cube be leſs than the Square of r; then the Equation has but one real and poſſible Root, and the other two will be impoſſible.

In this Theorem if it ſhould be $p = 0$, that is, if the ſecond Term of the Equation is wanting, we ſhall deſcend to the Caſe of thoſe Rules which are aſcribed to *Cardan*; the Solution of which is contain'd in what is here exhibited.

§. 2. In the Univerſal Biquadratic Equation

$$x^4 = 4px^3 + 2qx^2 + 8rx + 4s,$$
$$\qquad\qquad -4p^2 \quad -4pq \quad -q^2$$

The four Roots are $x = p - a \pm \sqrt{p^2+q-a^2-\frac{2r}{a}}$,

$$x = p + a \pm \sqrt{p^2+q-a^2+\frac{2r}{a}};$$

Here a^2 is the Root of this Cubic Equation following,

$$a^6 = p^2a^4 - 2pra^2 - r^2.$$
$$\qquad +q \qquad -s$$

Now when any Biquadratic Equation is given, a Compariſon muſt be made between its Terms, and the ſeveral Terms of this univerſal Equation, by which Means the Quantities p, q, r, s, will ſoon be found. And when theſe are known, the Value of a may be diſcover'd by the foregoing Theorem.

Theorem. And then all the Roots of the given Equation will become known.

An Example or two may suffice to illustrate this Solution.

1. Suppose we were to extract the Roots of this Biquadratic Equation, $x^4 = 8x^3 + 83x^2 - 162x - 936$. First, by what is prescrib'd we shall have $4p = 8$, or $p = 2$. Secondly, $2q - 4p^2 = 83$, or $q = \frac{99}{2}$. Thirdly, $8r - 4pq = -162$, or $r = \frac{117}{4}$. Lastly, $4s - q^2 = -936$, or $s = \frac{6057}{16}$. Hence $p^2 + q = \frac{107}{2}$, $2pr + s = \frac{7929}{16}$, $r^2 = \frac{13689}{16}$. Wherefore $a^6 = \frac{107}{2}a^4 - \frac{7929}{16}a^2 - \frac{13689}{16}$. Now that this Equation, which is really Cubic, may be resolved into its Roots, we must have Recourse to the foregoing Theorem: In which $p = \frac{107}{6}$, $q = \frac{22009}{144}$, $r = \frac{2903923}{1728}$, and $r^2 - q^3 = -\frac{11940075}{16}$. But the Cubic-root of the Binomial $\frac{2903923}{1728} + \sqrt{-\frac{11940075}{16}}$ is $-\frac{53}{12} + \sqrt{-\frac{400}{3}}$; and therefore $a^2 = \frac{107}{6} - \frac{53}{6} = 9$, and also $a^2 = \frac{107}{6} + \frac{53}{12} \pm \sqrt{400} = \frac{169}{4}$ or $\frac{9}{4}$. Or which comes to the same, the six Roots of the foregoing Equation, which in reality is Cubo-cubick, are $a = \pm 3$, $a = \pm \frac{13}{2}$, $a = \pm \frac{3}{2}$, any one of which may be taken indifferently for the true Root of the Equation, and will be subservient to our Purpose. Suppose in the present Case, that $a = 3$. Then by the Theorem it will be $x = p - a \pm \sqrt{p^2 + q - a^2 - \frac{2r}{a}} = 2 - 3 \pm \sqrt{4 + \frac{99}{2} - 9 - \frac{39}{2}} = -1 \pm 5 = 4$ or -6. Also $x = p + a \pm \sqrt{p^2 + q - a^2 + \frac{2r}{a}} = 2 + 3 \pm \sqrt{4 + \frac{99}{2} - 9 + \frac{39}{2}} = 5 \pm 8 = 13$ or -3; which are the four Roots of the given Equation.

2. In the Equation $x^4 = 20x^3 + 252x^2 - 6592x + 21312$, it will be $p = 5$, $q = 176$, $r = -384$, $s = 13072$. Hence $p^2 + q = 201$, $2pr + s = 9232$, and $r^2 = 147456$. And thence $a^6 = 201a^4 - 9232$

$a^2 +$

$a^2 + 147456$. Now in the Theorem for Cubics, it will be $p = 67$, $q = \frac{4235}{3}$, and $r = 65219$. And the Cubic Root of the Binomial $65219 + \sqrt{\frac{38889307072}{27}}$ will be $\frac{77}{2} + \sqrt{\frac{847}{12}}$. Therefore $a^2 = 67 + 77 = 144$, or $a = 12$. Therefore $x = 5 - 12 \pm \sqrt{25 + 176 - 144 + 64} = -7 \pm 11 = 4$ or -18. Alſo $x = 5 + 12 \pm \sqrt{25 + 176 - 144 - 64} = 17 \pm \sqrt{-7}$, which two Roots are impoſſible.

Now the Inveſtigation of this Theorem is in this Manner. By the Multiplication of the two Quadratic Equations $z^2 + 2az - b = 0$, and $z^2 - 2az - c = 0$ into each other, I conſtruct the Biquadratic Equation $z^4 = \overline{4a^2 + b + c} \times z^2 + \overline{2ac - 2ab} \times z - bc$, in which the ſecond Term is wanting, and which I make equivalent to this Equation $z^4 = ez^2 + fz + g$. Whence firſt, $4a^2 + b + c = e$, or $b = e - 4a^2 - c$. Secondly, $2ac - 2ab = f$, or $2ac - 2ae + 8a^3 + 2ac = f$, whence $c = \frac{f}{4a} + \frac{1}{2}e - 2a^2$, and thence $b = e - 4a^2 - \frac{f}{4a} - \frac{1}{2}e + 2a^2 = -\frac{f}{4a} + \frac{1}{2}e - 2a^2$. Thirdly, $-bc = g$, or $-\frac{ff}{16a^2} + \frac{1}{4}e^2 - 2ea^2 + 4a^4 = -g$, that is, $a^6 = \frac{1}{2}ea^4 - \frac{1}{4}ga^2 - \frac{1}{16}ea^2 + \frac{1}{64}ff$, which is as it were a Cubic Equation, compoſed of the Root a^2, and the known or aſſumed Quantities e, f, g. Now that Root may be exhibited by the foregoing Theorem, and by the ſame Calculation the Quantities b and c will be known. But the Roots of the Equations $z^2 + 2az - b = 0$, and $z^2 - 2az - c = 0$, are $z = -a \pm \sqrt{a^2 + b}$ and $z = a \pm \sqrt{a^2 + c}$, or $z = -a \pm \sqrt{\frac{1}{2}e - a^2 - \frac{f}{4a}}$, and $z = a \pm \sqrt{\frac{1}{2}e - a^2 + \frac{f}{4a}}$, which therefore will be the Roots of the Equation $z^4 = ez^2 + fz + g$, when a or a^2 is known from the Equation $a^6 = \frac{1}{2}ea^4 - \frac{1}{4}ga^2 - \frac{1}{16}ea^2 + \frac{ff}{64}$. Now that this Equation may become univerſal, and furniſhed with all its Terms, make $z = x - p$, then $x^4 - 4px^3 + 6p^2x^2 - 4p^3x + p^4 = ex^2 - 2pex + p^2e + fx - fp + g$, alſo $x = p - a$

 +

$\pm\sqrt{\frac{1}{2}e-a^2-\frac{f}{4a}}$, and $x=p+a\pm\sqrt{\frac{1}{2}e-a^2+\frac{f}{4a}}$. Laſtly, for Concinnity and Brevity Sake, make $e=2q+2p^2$, and $f=8r$; then $x^4-4px^3+4p^2x^2=2qx^2-4pqx+2p^2q+p^4+8rx-8pr+g$, $x=p-a\pm\sqrt{p^2+q-a^2-\frac{2r}{a}}$, alſo $x=p+a\pm\sqrt{p^2+q-a^2+\frac{2r}{a}}$, and $a^2=\overline{p^2+q}\times a^4-\overline{\tfrac{1}{4}g+\tfrac{1}{4}p^4+\tfrac{1}{2}p^2q+\tfrac{1}{4}q^2}\,a^2+r^2$. Finally, make $g=4s-q^2+8pr-p^4-2p^2q$, and the foregoing Equations become

$$x^4=\begin{array}{l}4p\\-4p^2\end{array}x^3+\begin{array}{l}2q\\-4pq\end{array}x^2+\begin{array}{l}8r\\-q^2\end{array}x+4s,\quad\text{and}\quad a^6=\begin{array}{l}p^2\\+q\end{array}a^4-2pra^2+\begin{array}{l}r^2;\\-s\end{array}$$

that is, all Things become as ſuppoſed above.

§. 3. Hitherto concerning the Analytical Solution of Cubic and Biquadratic Equations. Now becauſe their Geometrical Effection by the Parabola is commonly taught, and is much valued by ſome, I ſhall exhibit it here more univerſally, and yet more compendiouſly.

Any Cubic or Biquadratic Equation being given, a Compariſon muſt be made between its Terms, and the reſpective Terms of this Equation following.

$$x^4=\begin{array}{l}\frac{2p}{q}\\-4r\\ \\ \\\end{array}x^3+\begin{array}{l}\frac{4pr}{q}\\-4r^2\\+2s\\-1\end{array}x^2+\begin{array}{l}\frac{2p^2}{q}\\-\frac{2ps}{q}\\+4rs\\-2q\end{array}x+\begin{array}{l}p^2,\\-q^2\\-s^2\\+t^2\end{array}$$

by which Means the Values of p, q, r, s, t, will eaſily be found, any one of them being aſſumed at Pleaſure. Then in any given Parabola AVB, whoſe principal Vertex is V, its Axis VS, and VT perpendicular to the Axis; let there be taken $VS=p$, within the Parabola, and in the Angle SVT let there be inſcribed $ST=q$, which being produced let it cut the Parabola in two Points N and O. Let ON be biſected in M, and thro' M let MA be drawn, parallel to the Axis and meeting the Parabola in A. Draw AL parallel to ON, and let AL be the *Latus rectum* of the Parabola to the Diameter AM, and let the ſame be Unity. In AL, produced both Ways if neceſſary, take $AG=r$, and from G draw GR parallel to the Axis, ſo that it may cut the Parabola in B, from whence take $BR=s$. From the Point laſt found R draw RE parallel and equal to VT, and let it lie to the left Hand in reſpect of R, if q is an affirmative Quantity, otherwiſe to the right Hand if q be negative. And the ſame Thing is to be underſtood of AG and BR, which muſt be drawn on the contrary Side, if the Values of r and s happen to be negative.

Fig. 31.

Laſtly,

Laſtly, with Center E and Radius $EC = t$, let a Circle $CKkc$ be deſcribed, which will cut the Parabola in ſo many Points, as there are real Roots in the given Equation. For from thoſe Points C, K, let there be drawn CP, $k\Pi$, *&c.* parallel to ST, and terminated at the right Line GR produced if need be. Every one of theſe will be x, or the required Root of the given Equation. Thoſe lying to the right Hand will be affirmative Roots, and thoſe on the left Hand will be negative. A Point of Contact, if any ſuch ſhould be, is here taken for two Points of Interſection, that are infinitely near to each other.

This will be the only Difference between Cubic and Biquadratic Equations, conſtructed after this Manner, that in the former, becauſe of the laſt Term being abſent in the foregoing Equation, it will always be $pp - qq - ss + tt = 0$, or $t = \sqrt{ss + qq - pp}$. Therefore Center E and Radius $EC = \sqrt{BRq + (ERq)\, STq - USq}$ any Circle $CKkc$ being deſcribed, one of the Roots CP in the foregoing Conſtruction becomes nothing.

Now theſe Poſitions are demonſtrated in the following Manner. Suppoſing all Things as before conſtructed, and producing CP if needful till it meets AM in H, CH will be the Ordinate of the Parabola to the Diameter AH, and therefore $CHq = AL \times AH = AH$, becauſe of $AL = 1$. But $CH = CP + AG$, and $AH = GB + BP$, and therefore $CPq + 2AG \times CP + AGq = GB + BP$. But becauſe of the Nature of the Parabola it will be $AGq = BG$, whence $CPq + 2AG \times CP = BP$. Now from the Point C let there be drawn CD perpendicular to BP, which may alſo meet EI parallel to BP in the Point I. Now becauſe of ſimilar Triangles CDP and TVS, it will be

$DP = \frac{VS \times CP}{ST}$, and $CD = \frac{VT \times CP}{ST}$; and therefore $CPq + 2AG \times CP = BP = DP + DB = \frac{VS \times CP}{ST} + BR - IE$. Or $CPq + 2AG \times CP - \frac{VS}{ST} CP - BR = -IE$. But $IEq = CEq - CIq = CEq - CDq - VTq - 2CD \times VT = CEq - \frac{VTq \times CPq}{STq} - VTq - \frac{2VTq \times CP}{ST} =$ (becauſe of $VTq = STq - SVq$) $CEq - CPq + \frac{SVq}{STq} CPq - STq + SVq - 2ST \times CP + \frac{2SVq}{ST} CP$; this therefore will be equal to the Square whoſe Side is $CPq + 2AG \times CP - \frac{VS}{ST} CP - BR$. And when this Equation is reduced to the Terms p, q, r, s, t, it will become the very Equation propoſed.

 Hence

Hence it appears, that any the same Biquadratic Equation will admit of innumerable different Constructions by the Parabola, according to the different Values of that Quantity which we said might be assumed at Pleasure. But the most simple Case is, by making $VS = p = 0$, and the Construction passes into the common one (as to the Thing itself) in which the right Lines CP, *&c.* which are the Representatives of the Roots, are perpendicular to the Axis. Then the Equation becomes

$$x^4 = \begin{array}{l} -4rx^3 \\ \end{array} \begin{array}{l} -4r^2 x^2 \\ +2s \\ -1 \end{array} \begin{array}{l} +4rsx \\ -2q \\ \end{array} \begin{array}{l} -q^2 \\ -s^2 \\ +t^2 \end{array}$$

which is easily constructed as above.

§. 4. But least the Organical Description of the Parabola should seem too difficult, we may have Recourse to a certain Mechanical Artifice, to be perform'd by Means of a Plummet, or Thread with a Weight hanging at the End of it; by Help of which the last Equation may be constructed very easily and exactly, and therefore the Roots of any Cubic or Biquadratic Equations may be found. This Construction, which we may call a Mechanical one, is after this Manner.

Fig. 32. Against a smooth and upright Wall, or any other Plain perpendicular to the Horizon, at any Point F let there be hung a very fine flexible Thread FP, with any Weight P hung at its Extremity. In this Thread let any Point N be mark'd, which is at a sufficient Distance from the Point of Suspension F; or it may be tyed with a small Knot N. Then taking at Pleasure NO for Unity, at the middle Point M (and in the aforesaid Plain,) let the right Line AQ be drawn parallel to the Horizon, and produced both Ways as far as is necessary. These Things being prepared in general, for Application to any particular Case, make $AQ = r$; the Quantities q, r, s, t, being first determined in the last Equation, either Arithmetically or Geometrically, according to the Exigence of the given Equation. Then with a small Style or Bodkin, or with the slender Point of a Pair of Compasses, let the Thread be inflected and moved from its Place, till the Point of Inflection falls upon a certain Point B, and the Knot N falls at the same Time on the Point last found Q. In BQ from that Point B take $BR = S$, and at R raise the Perpendicular $RE = q$ to the Line BR. But those Lines AQ, BR, RE, must fall the contrary Way from their initial Points, if it should happen that the Values of r, s, q, should come out negative. *Lastly*, let one Leg of the Compasses be fixed in the Point found E, and let the other Leg, extended to the Distance $EZ = t$, be carried about with a circular Motion, taking with it the Thread FZP. By this Circulation of the Thread the Weight P will sometimes ascend, and sometimes descend with a reciprocal Motion, and the Knot N will sometimes be above and sometimes below the Horizontal Line AQ. But whenever the Knot N shall be found in the Line AQ, suppose in the Points $D\ d\ \Delta\ \delta$, successively; it will cut off the right Lines DQ, dQ, ΔQ, δQ, which will

will be all the real Roots of the given Equation: That is, those on the right Hand will be the affirmative Roots, and those on the Left the negative. The Demonstration will be manifest from what goes before, and attending to the Parabola that will pass through the Points $BCckK$. For making F the Focus of the Parabola, whose Distance from the Vertex is $\frac{1}{4}ON$, it is known that all the Lines as $FB + BQ$, $FC + CD$, *&c.* always make the same Sum.

And from the Principles here laid down it will not be difficult to construct an Instrument, which will be neat enough, and as accurate as you please, by the Help of which, and with very little Trouble, the Roots of all these Equations may be found, and exhibited to Ocular Inspection.

XVI. Let n be any Number whatever, y an unknown Quantity, and let a be any Quantity intirely known, or what they call the *Homogeneum Comparationis*; and let the Relation of these be expressed by the Equation.

Equations of the 3d, 5th, 7th, 9th, *&c. Powers solv'd Analytically by Mr.* Abr. de Moivre, n. 309. p. 2368.

$$ny + \frac{nn-1}{2\times3}ny^3 + \frac{nn-1}{2\times3}\times\frac{nn-9}{4\times5}ny^5 + \frac{nn-1}{2\times3}\times\frac{nn-9}{4\times5}\times\frac{nn-25}{6\times7}ny^7, \text{ \&c.} = a.$$

It is plain from the Nature of this Series, that if n is taken any odd Number, (that is, an Integer, but it is all one whether affirmative or negative) then the Series will stop of its own Accord, and the Equation becomes one of those described in the Title; the Root of which is

$$(1)\ y = \tfrac{1}{2}\sqrt[n]{\sqrt{1+aa}+a} - \frac{\frac{1}{2}}{\sqrt[n]{\sqrt{1+aa}+a}}$$

$$\text{or } (2)\ y = \tfrac{1}{2}\sqrt[n]{\sqrt{1+aa}+a} - \tfrac{1}{2}\sqrt[n]{\sqrt{1+aa}-a}$$

$$\text{or } (3)\ y = \frac{\frac{1}{2}}{\sqrt[n]{\sqrt{1+aa}-a}} - \tfrac{1}{2}\sqrt[n]{\sqrt{1+aa}-a}$$

$$\text{or } (4)\ y = \frac{\frac{1}{2}}{\sqrt[n]{\sqrt{1+aa}-a}} - \frac{\frac{1}{2}}{\sqrt[n]{\sqrt{1+aa}-a}}$$

For Examble, let it be this Equation of the fifth Power, $5y + 20y^3 + 16y^5 = 4$, whose Root is to be found. In this Case 'tis $n = 5$, and $a = 4$. Then the Root, according to the first Form, will be

$y =$

$y = \frac{1}{2}\sqrt[5]{\sqrt{17}+4} - \frac{\frac{1}{2}}{\sqrt[5]{\sqrt{17}+4}}$, which is reduced very expediously to common Numbers in this Manner. 'Tis $\sqrt{17}+4 = 8,1231$, whose Logarithm is 0, 9097164, the fifth Part of which is 0, 1819433, and the Number answering to this is 1, 5203 $= \sqrt[5]{\sqrt{17}+4}$. Now the Arithmetical Complement of 0, 1819433 is 9, 8180567, to which answers the Number 0, 6577 $= \frac{1}{\sqrt[5]{\sqrt{17}+4}}$. Therefore the half-Difference of these Numbers is 0, 4313 $= y$.

It may observed, that instead of the general Root it may be sufficient to take $y = \frac{1}{2}\sqrt[n]{2a} - \frac{\frac{1}{2}}{\sqrt[n]{2a}}$, whenever the Number n is very large in respect of Unity. As if the Equation were $5y + 20y^3 + 16y^5 = 682$; the Log. of $2a = 3, 1348143$, a fifth Part of which is 0, 6269628, and the Number answering to this is 4, 236. The Arithmetical Complement is 9, 3730372, whose Number is 0, 236, and the Half-difference of these Numbers is $2 = y$.

Again, if in the foregoing Equation the Signs are made alternately affirmative and negative, or which is the same Thing, if we had such a Series as this following,

$$ny + \frac{1-nn}{2\times3}ny^3 + \frac{1-nn}{2\times3}\times\frac{9-nn}{4\times5}ny^5 + \frac{1-nn}{2\times3}\times\frac{9-nn}{4\times5}\times\frac{25-nn}{6\times7}ny^7, \&c. = a,$$

the Root will be

$$(1)\ y = \tfrac{1}{2}\sqrt[n]{a+\sqrt{aa-1}} + \frac{\frac{1}{2}}{\sqrt[n]{a-\sqrt{aa-1}}}$$

$$\text{or } (2)\ y = \tfrac{1}{2}\sqrt[n]{a+\sqrt{aa-1}} + \tfrac{1}{2}\sqrt[n]{a-\sqrt{aa-1}}$$

$$\text{or } (3)\ y = \frac{\frac{1}{2}}{\sqrt[n]{a-\sqrt{aa-1}}} + \tfrac{1}{2}\sqrt[n]{a-\sqrt{aa-1}}$$

or

$$\text{or } (4)\ y = \frac{\frac{1}{2}}{\sqrt[n]{a - \sqrt{aa-1}}} + \frac{\frac{1}{2}}{\sqrt[n]{a + \sqrt{aa-1}}}$$

Here it is to be obſerved, that if $\frac{n-1}{2}$ ſhould be an odd Number, the Sign of the Root when found muſt be changed into its contrary.

Let this Equation be propoſed $5y - 20y^3 + 16y^5 = 6$; whence $n = 5$ and $a = 6$. Then the Root $y = \frac{1}{2}\sqrt[5]{6 + \sqrt{35}} + \frac{\frac{1}{2}}{\sqrt[5]{6+\sqrt{35}}}$.

Or becauſe $6 + \sqrt{35} = 11,916$, the Logarithm of this will be 1, 0761304, and its fifth Part 0, 2152561, and its Arithmetical Complement 9, 7847439. The Numbers belonging to theſe Logarithms reſpectively are 1, 6415 and 0, 6091, whoſe Semiſum 1, 1253 $= y$.

But if it ſhall happen that a is leſs than Unity, then the ſecond Form of the Root is rather to be made Choice of, as fitter for the Purpoſe. Thus if the Equation were $5y - 20y^3 + 16y^5 = \frac{61}{64}$, then $y = \frac{1}{2}\sqrt[5]{\frac{61}{64} + \sqrt{\frac{-375}{4096}}} + \frac{1}{2}\sqrt[5]{\frac{61}{64} - \sqrt{\frac{-375}{4096}}}$. And if by any Means the Root of the fifth Power of Binomials can be extracted, the Root would come out true and poſſible, notwithſtanding that the Expreſſion ſeems to include an Impoſſibility. Now the fifth Root of the Binomial $\frac{61}{64} + \sqrt{\frac{-375}{4096}}$ is $\frac{1}{4} + \frac{1}{4}\sqrt{-15}$; and of the Binomial $\frac{61}{64} - \sqrt{\frac{-375}{4096}}$ the fifth Root is alſo $\frac{1}{4} - \frac{1}{4}\sqrt{-15}$: And half the Sum of theſe Binomials is $\frac{1}{4} = y$. Now if this Extraction cannot be perform'd, or ſhould ſeem any Thing difficult; the Reſult may always be found by the Table of Natural Sines in the following Manner.

To

To Radius 1 let $a = \frac{61}{64} = 0,\ 95112$ be the Sine of a certain Arch, which therefore will be $72^{\circ}.\ 23'$, a fifth Part of which, (becaufe $n = 5$) is $14^{\circ}.\ 28'$. The Sine of this is 0, 24981, which is nearly $\frac{1}{4}$. Nor is it otherwife in Equations of higher Degrees.

The Method of Approximating, *in Extracting the* Roots *of* Equations *in Numbers*, improv'd by *Dr.* B. Taylor, n. 352. *p.* 610. * Vid. *fup.* V. I. C. I. S. XX.

XVII. Dr. *Halley* has publifh'd * a very compendious and ufeful Method of extracting the Roots of adfected Equations of the common Form in Numbers. This Method proceeds by affuming the Root defired nearly true to one or two Places in Decimals (which is done by a Geometrical Conftruction, or by fome other convenient Way) and correcting the Affumption by comparing the Difference between the true Root and the affumed, by Means of a new Equation whofe Root is that Difference, and which he fhews how to form from the Equation propofed, by Subftitution of the Value of the Root fought, partly in known, and partly in unknown Terms.

In doing this he makes Ufe of a Table of Products (which he calls *Speculum Analyticum*) by which he computes the Coefficients in the new Equation for finding the Difference mentioned. This Table, I obferved, was formed in the fame Manner from the Equation propofed, as the Fluxions are, taking the Root fought for the only flowing Quantity, its Fluxion for Unity, and after every Operation dividing the Product fucceffively by the Numbers 1, 2, 3, 4, *&c.* Hence I foon found that this Method might eafily and naturally be drawn from *Cor.* 2. *Prop.* 7. of my *Methodus Incrementorum*, and that it was capable of a further Degree of Generality, it being applicable, not only to Equations of the common Form, (*viz.* fuch as confift of Terms wherein the Powers of the Root fought are pofitive and integral, without any radical Sign) but alfo to all Expreffions in general, wherein any Thing is propofed as given which by any known Method might be computed; if *vice verfâ*, the Root were confidered as given: Such as are all radical Expreffions of Binomials, Trinomials, or of any other Nomial, which may be computed by the Root given, at leaft by Logarithms, whatever be the Index of the Power of that Nomial; as likewife Expreffions of Logarithms, of Arches by the Sines or Tangents, of Areas of Curves by the *Abfciffas*, or any other Fluents or Roots of fluxional Equations, *&c.*

For the Sake of this great Generality, it may not be improper to fhew how this Method is derived from the forefaid *Corollary*; therefore z and x being two flowing Quantities (whofe Relation to one another may be expreffed by any Equation whatfoever) by this *Corollary*, while

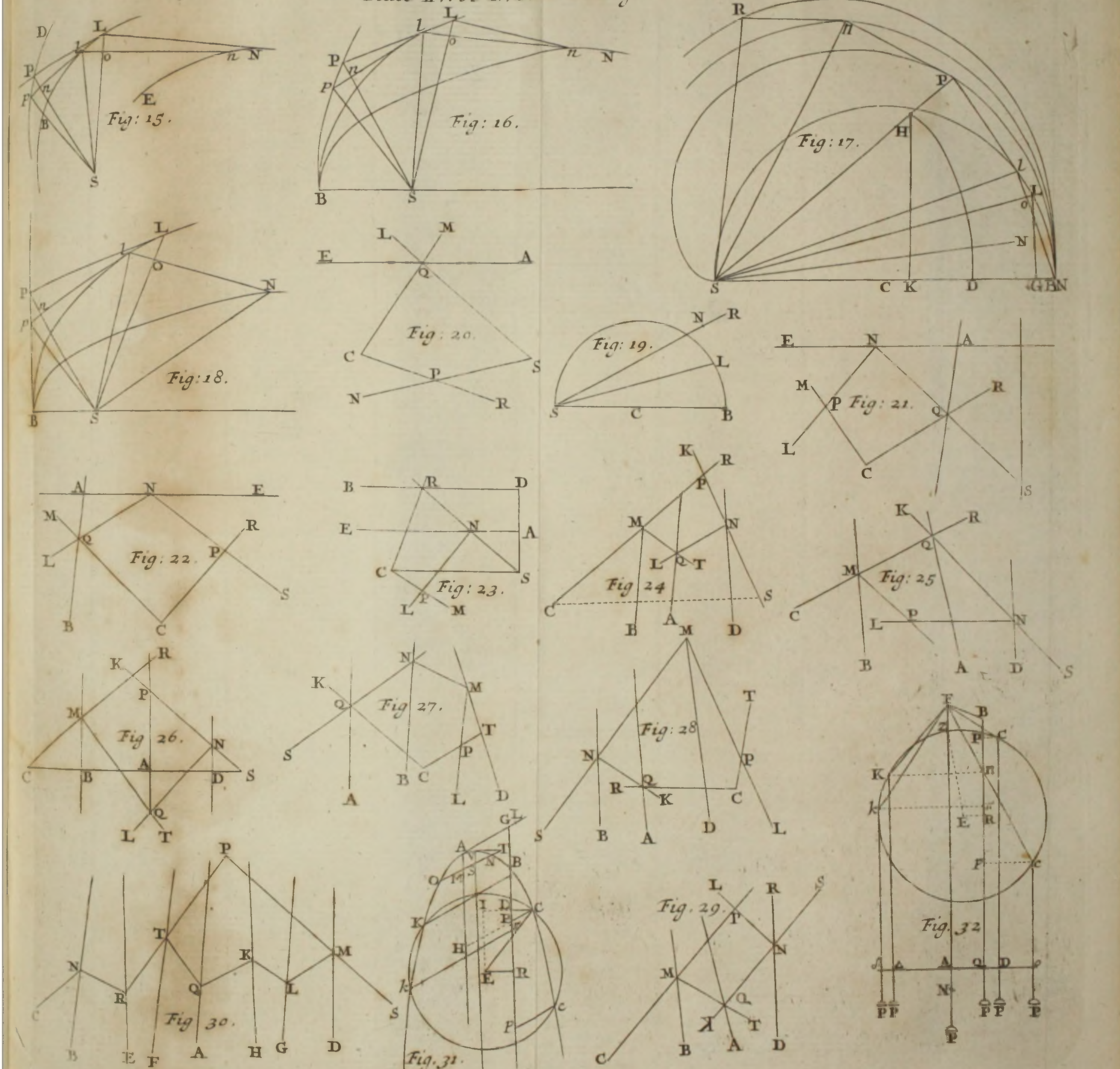
Fig: 15.
Fig: 16.
Fig: 17.
Fig: 18.
Fig: 20.
Fig: 19.
Fig: 21.
Fig: 22
Fig: 23.
Fig 24
Fig: 25
Fig 26.
Fig 27.
Fig: 28
Fig. 29.
Fig 30.
Fig. 31.
Fig. 32

z by flowing uniformly becomes $z+v$, x will become $x + \frac{\dot{x}}{1 \cdot \dot{z}} v$

$+ \frac{\ddot{x}}{1 \cdot 2 \dot{z}^2} v^2 + \frac{\dddot{x}}{1 \cdot 2 \cdot 3 \dot{z}^3} v^3 +$ *&c.* or $x \frac{\dot{x}v}{1} + \frac{\ddot{x}v^2}{1 \times 2} + \frac{\dddot{x}v^3}{1 \cdot 2 \cdot 3} +$ *&c.*

for $\dot{z}$ putting 1.

Hence if y be the Root of any Expreſſion formed of y and known Quantities, and ſuppoſed equal to nothing, and z be a Part of y, and x be formed of z and the known Quantities, in the ſame Manner as the Expreſſion made equal to nothing is formed of y; and let y be equal to $z+v$; the Difference v will be found by extracting the Root of

this Expreſſion $x + \frac{\dot{x}v}{1} + \frac{\ddot{x}v^2}{1 \cdot 2} + \frac{\dddot{x}v^3}{1 \cdot 2 \cdot 3} +$ *&c.* $= 0$. For in this

Caſe z being become $z+v=y$, x, which is now become $x + \dot{x}v + \frac{\ddot{x}v^2}{2}$

$+$ *&c.* muſt become equal to nothing.

The Root v in the Equation $x + \frac{\dot{x}v}{1} + \frac{\ddot{x}v^2}{1 \cdot 2} + \frac{\dddot{x}v^3}{1 \cdot 2 \cdot 3} +$ *&c.* $= 0$,

is to be found upon the Suppoſition of its being very ſmall with reſpect to z, (as it muſt be, if z be taken tolerably exact) by which Means

the Terms $\frac{\dddot{x}v^3}{1 \cdot 2 \cdot 3} + \frac{\ddddot{x}v^4}{1 \cdot 2 \cdot 3 \cdot 4} +$ *&c.* may be neglected, upon account of their Smallneſs with reſpect to the other Terms, ſo as to

leave the Equation $x + \frac{\dot{x}v}{1} + \frac{\ddot{x}v^2}{1 \cdot 2} = 0$, for finding the firſt Approximation of v.

By extracting the Root of this Equation, we have

$v = \sqrt{\frac{\dot{x}^2}{\ddot{x}^2} - \frac{2x}{\ddot{x}}} - \frac{\dot{x}}{\ddot{x}}$. That is, First, $\sqrt{\frac{\dot{x}^2}{\ddot{x}^2} - \frac{2x}{\ddot{x}}} - \frac{\dot{x}}{\ddot{x}}$, if $x + \dot{x}v + \frac{\ddot{x}v^2}{2} = 0$.

Second, $\sqrt{\frac{\dot{x}^2}{\ddot{x}^2} + \frac{2x}{\ddot{x}}} - \frac{\dot{x}}{\ddot{x}}$, if $-x + \dot{x}v + \frac{\ddot{x}v^2}{2} = 0$.

Third, $\frac{\dot{x}}{\ddot{x}} - \sqrt{\frac{\dot{x}^2}{\ddot{x}^2} - \frac{2x}{\ddot{x}}}$, if $x - \dot{x}v + \frac{\ddot{x}v^2}{2}$, &c. $= 0$.

Fourth, $\frac{\dot{x}}{\ddot{x}} - \sqrt{\frac{\dot{x}^2}{\ddot{x}^2} - \frac{2x}{\ddot{x}}}$, if $-x - \dot{x}v + \frac{\ddot{x}v^2}{2}$, &c. $= 0$.

This Approximation gives v exact to twice as many Places as there are true Figures in z, and therefore trebles the Number of true Figures in the Expression of y by $z + v$, which may be taken for a new Value of z, for computing a second v, seeking other Values of x, $\dot{x}$, $\ddot{x}$, &c. Though when z is tolerably exact (which it may be esteemed when it contains two or three or more Figures true in the Value of y, according to the Number of Figures the Root is proposed to be computed to) the Calculation may be restor'd without so much Trouble, only by

taking $\sqrt{\frac{\dot{x}^2}{\ddot{x}^2} \pm \frac{2x}{\ddot{x}} - \frac{2\dddot{x}}{2 \cdot 3\ddot{x}}v^3} - \frac{2\dddot{x}}{1 \cdot 2 \cdot 3 \cdot 4\ddot{x}}v^4$, &c. instead of

$\sqrt{\frac{\dot{x}^2}{\ddot{x}^2} \pm \frac{2x}{\ddot{x}}}$ taking every Time for v its Value last computed.

From the same Equation $x + \dot{x}v + \frac{\ddot{x}v^2}{2} + \frac{\dddot{x}v^3}{1 \cdot 2 \cdot 3} + \&c. = 0$,

may be gathered also a rational Form, *viz.* $v = \frac{-x}{\dot{x} - \frac{x\ddot{x}}{2\dot{x}}}$. For neglecting the Terms $\frac{\dddot{x}v^3}{1 \cdot 2 \cdot 3}$, &c. we have $v = \frac{-x}{\dot{x} + \frac{\ddot{x}}{2}v}$ which is nearly

=

$= \frac{-x}{\dot{x}}$. Therefore in the Divisor instead of v writing $\frac{-x}{\dot{x}}$ we have more exactly $v = \dfrac{-x}{\dot{x} - \dfrac{\ddot{x}x}{2\dot{x}}}$ that is,

1. $\dfrac{-x}{\dot{x} - \dfrac{\ddot{x}x}{2\dot{x}}}$, when $x + \dot{x}v + \dfrac{\ddot{x}v^2}{2}$ &c. $= 0$.

2. $\dfrac{x}{\dot{x} + \dfrac{\ddot{x}x}{2\dot{x}}}$, when $-x + \dot{x}v + \dfrac{\ddot{x}v^2}{2}$ &c. $= 0$.

3. $\dfrac{x}{\dot{x} - \dfrac{\ddot{x}x}{2\dot{x}}}$, when $x - \dot{x}v + \dfrac{\ddot{x}v^2}{2}$ &c. $= 0$.

4. $\dfrac{-x}{\dot{x} + \dfrac{\ddot{x}x}{2\dot{x}}}$, when $-x - \dot{x}v + \dfrac{\ddot{x}v^2}{2}$ &c. $= 0$.

This *Formula* will also triplicate the Number of true Figures in z. And the Calculation may be repeated, after every Operation, taking for a Divisor $\dot{x} \pm \dfrac{\ddot{x}}{2}v + \dfrac{\dddot{x}v^2}{1 \cdot 2 \cdot 3} + \dfrac{\ddddot{x}v^3}{1 \cdot 2 \cdot 3 \cdot 4} +$ &c. instead of $\dot{x} + \dfrac{\ddot{x}x}{2\dot{x}}$.

Dr *Halley* has fully explained the Manner of using both these *Formula's* in Equations of the common Form; wherefore I shall be the shorter in explaining two or three Examples of another Sort.

Ex. 1. Let it be propoſed to find the Root of this Equation $\overline{y^2+1}|^{\sqrt{2}} + y - 16 = 0$. In this Caſe, for y writing z, and for o writing x, we have $\overline{z^2+1}|^{\sqrt{2}} + z - 16 = x$. Whence by taking the Fluxions, we have $\dot{x} = 2\sqrt{2} \times z \times \overline{z^2+1}|^{\sqrt{2}-1} + 1$, and $\ddot{x} = 2\sqrt{2} \times \overline{8-4\sqrt{2}\,z^2} \times \overline{z^2+1}|^{\sqrt{2}-2}$. For finding the firſt Figures of the Root y, for $\sqrt{2}$ take $\frac{3}{2}$, and we have the Equation $\overline{y^2+1}|^{\frac{3}{2}} + y - 16 = 0$, which being expanded gives $y^6 + 3y^4 + 2y^2 + 32y - 255 = 0$.

By this Equation I find that for the firſt Suppoſition we may take $z = 2$. Therefore in order to find v, let us now make $\sqrt{2} = \frac{7}{5}$, (which is nearer than before) and we have $x = \overline{z^2+1}|^{\frac{7}{5}} + z - 16 = \overline{z^2+1}|^{\frac{7}{5}} - 14 = 5^{\frac{7}{5}} - 14 = -4,48$; $\dot{x} = 10,66$; $\ddot{x} = 4,72$. Whence by the ſecond rational Form

$$v = \cfrac{4,48}{10,66 + \cfrac{4,72 \times 4,48}{2 \times 10,66}} = 0,38;$$

which muſt be too big, becauſe $\frac{7}{5} < \sqrt{2}$, and therefore will require a larger Value of y to exhauſt the Equation, than where $\sqrt{2}$ is exact. For the ſecond Suppoſition therefore, let us take $z = 2,3$, and make $\sqrt{2} = 1,4142136$, and by help of the Logarithms we ſhall have $\overline{z^2+1}|^{\sqrt{2}} = 13,47294$, whence $x = -0,22706$; $\dot{x} = 14,93429$, and $\ddot{x} = 5,18419$. Hence by the 2*d* irrational *Formula*

$$v = \sqrt{\frac{14,93429^2}{5,18419^2} + \frac{0,45412}{5,18419}} - \frac{14,93429}{5,18419} = 0,0516,$$ which gives

$y = z + v = 2,31516$, which is true to ſix Places. If you deſire it more exact than to the Extent of the Tables of Logarithms, taking $z = 2,31516$ for the next Suppoſition, the Calculation muſt be repeated by computing $\overline{zz+1}|^{\sqrt{2}}$ to a ſufficient Number of Places; which muſt

muſt be done by the Binomial Series, or by making a Logarithm on Purpoſe, true to as many Places as are neceſſary.

Ex. 2. For another Example, let it be required to find the Number whoſe Logarithm is 0, 29, ſuppoſing we had no other Table of Logarithms, but Mr. *Sharp*'s of 200 Logarithms to a great many Places. This amounts to the reſolving this Equation $ly = 0,29$, or $ly - 0,29 = 0$. Hence therefore we have $x = lx - 0,29$, $\dot{x} = \frac{a}{z}$ (*a* being the *Modulus* belonging to the Table we uſe, *viz.* 0, 4342944819, &c.) $\ddot{x} = \frac{-a}{z^2}$, $\dddot{x} = \frac{2a}{z^3}$, $\ddddot{x} = \frac{-6a}{z^4}$, &c.

In this Caſe becauſe $\ddot{x}$ has a negative Sign, changing the Signs of all the Coefficients, the Canon for v will be found in the fourth Caſe, which in the irrational Form gives $v = \frac{\dot{x}}{\ddot{x}} \sqrt{\frac{\dot{x}^2}{\ddot{x}^2} + \frac{2x}{\ddot{x}} - \frac{2\dddot{x}}{2.3\ddot{x}} v^3 - \frac{2\ddddot{x}}{2.3.4\ddot{x}} v^4 \text{ \&c.}} = z - \sqrt{z^2 + \frac{2lz - 0,58}{a} \times z^2 + \frac{2v^3}{3z} - \frac{2v^4}{4z^2} + \frac{2v^5}{5z^3}, \text{ \&c.}}$ In this Caſe to avoid often dividing by z, it will be moſt convenient to compute $\frac{v}{z}$, which is got from this Equation $\frac{v}{z} = 1 - \sqrt{1 + \frac{2lz - 0,58}{a} + \frac{2v^3}{3z^3} - \frac{2v^4}{4z^4} + \frac{2v^5}{5z^5}, \text{ \&c.}}$ The neareſt Logarithm, in the Tables propoſed, to the propoſed Logarithm 0, 29 is 0, 2900346114, its Number being 1, 95. Therefore for the firſt Suppoſition taking $z = 1,95$, we have x ($= lz - 0,29 = 0,2900346114 - 0,29$) $= 0,0000346114$, and $\frac{2lz - 0,58}{a} = \frac{0,0000692228}{0,4342944819} = 0,00015939139$, and $1 + \frac{2lz - 0,58}{a} = 1,00015939139$. Whence

For

for the first Approximation we have $\frac{v}{z} = 1 - \sqrt{1,00015939139} = -0,00007969247$, and $v = -0,00015540032$, and $y = z + v = 1,94984459968$. Which is true to eleven Places, and may easily be corrected by the Terms $\frac{2v^3}{3z}$, &c. which I leave to the Reader's Curiosity.

Being upon the Subject of Approximations, it may not be amiss to set down here two Approximations I have formerly hit upon. The one is a Series of Terms for expressing the Root of any Quadratic Equation; and the other is a particular Method of approximating in the Invention of Logarithms, which has no Occasion for any of the Transcendental Methods, and is expeditious enough for making the Tables without much Trouble.

A general Series for expressing the Root of any Quadratic Equation.

2. Any Quadratic Equation being reduc'd to this Form $xx - mqx + my = 0$, the Root x will be express'd by this Series of Terms.

$$x = \frac{y}{q} + A \times \frac{1}{\frac{mq^2}{y} - 2} + B \times \frac{1}{a^2 - 2} + C \times \frac{1}{b^2 - 2}$$

$$+ D \times \frac{1}{c^2 - 2}$$ &c. which must be thus interpreted.

1. The Capital Letters A, B, C, &c. stand for the whole Terms with their Signs, preceding those whereen they are found, as

$$B = A \times \frac{1}{\frac{mq^2}{y} - 2}.$$

2. The little Letters a, b, c, &c. in the Divisors, are equal to the whole Divisors of the Fraction in the Terms immediately preceding; thus $b = a^2 - 2$.

For an Example of this, let it be required to find $\sqrt{2}$. Putting $\sqrt{2} = x + 1$, we have $x^2 + 2x - 1 = 0$, which being compared with

with the general *Formula*, gives $mq = -2$, and $my = -1$; therefore for m taking -1, we have $q = 2$, and $y = 1$, which Values substituted in the Series give $x = \frac{1}{2} - \frac{1}{2 \times 6} - \frac{1}{2 \times 6 \times 34} - \frac{1}{2 \times 6 \times 34 \times 1154} - \frac{1}{2 \times 6 \times 34 \times 1154 \times 1331714}$, &c. The Fractions here wrote down giving the Root true to twenty three Places.

3. This Method is founded upon these Considerations. *A New Method of computing* Logarithms.

1. That the Sum of the Logarithms of any two Numbers is the Logarithm of the Product of those two Numbers multiplied together.

2. That the Logarithm of Unit is nothing; and consequenly that the nearer any Number is to Unit, the nearer will its Logarithm be to 0. *3dly*, That the Product by Multiplication of two Numbers, whereof one is bigger, and the other less than Unit, is nearer to Unit than that of the two Numbers which is on the same Side of Unit with itself; for Example, the two Numbers being $\frac{2}{3}$ and $\frac{4}{3}$, the Product $\frac{8}{9}$ is less than Unit, but nearer to it than $\frac{2}{3}$, which is also less than Unit. Upon these Considerations, I found the present Approximation; which will be the best explain'd by an Example. Let it therefore be proposed to find the Relation of the Logarithms of 2 and of 10.

In order to this, I take two Fractions $\frac{128}{100}$ and $\frac{8}{10}$, *viz.* $\frac{\overline{2}|^7}{\overline{10}|^2}$ and $\frac{\overline{2}|^3}{\overline{10}|^1}$ whose Numerators are Powers of 2, and their Denominators Powers of 10; one of them being bigger, and the other less than 1. Having set these down in Decimal Fractions in the first Column of the Table annexed, against them in the second Column I set A and B for their Logarithms, expressing by an Equation the Manner how they are compounded of the Logarithms of 2 and 10, for which I write $l\,2$ and $l\,10$. Then multiplying the two Numbers in the first Column together, I have a third Number 1,024, against which I write C for its Logarithm, expressing likewise by an Equation in what Manner C is formed of the foregoing Logarithms A and B. And in the same Manner the Calculation is continued; only observing this *Compendium*, that before I multiply the two last Numbers already got in the Table, I consider what Power of one of them must be used to bring the Product the nearest to Unit that can be. This is found, after we have gone a little Way in the Table, only by dividing the Differences of the

the Numbers from Unit one by the other, and taking the Quotient with the nearest, for the Index of the Power wanted. Thus the two last Numbers in the Table being o, 8 and 1, 024, their Differences from Unit are o, 200 and o, 024; therefore $\frac{0,200}{0,024}$ gives 9 for the Index; wherefore multiplying the ninth Power of 1, 024 by o, 8, I have the next Number o, 990352031429, whose Logarithm is $D = 9C + B$. In seeking the Index in this Manner by Division of the Differences, the Quotient ought generally to be taken with the least: but in the present Case it happens to be the most, because instead of the Difference between o, 8 and 1, we ought strictly to have taken the Difference between the reciprocal 1, 25 and 1, which would have given the Index 10; and that would be too big, because the Product by that Means would have been bigger than 1, as 1,024 is. Whereas this Approximation requires that the Numbers in the first Column be alternately greater and less than 1, as may be seen in the Table.

When I have in this Manner continued the Calculation, till I have got the Numbers small enough, I suppose the last Logarithm to be equal to nothing. Which gives me an Equation, from which having got away the Letters by Means of the foregoing Equations, I have the Relation of the Logarithms proposed. In this Manner if I suppose $G = 0$, I have $2136\,l\,2 - 643\,l\,10 = 0$. Which gives the Logarithm of 2 true in seven Figures, and too big in the Eighth; which happens because the Number corresponding with G is bigger than Unit.

There is another Expedient which renders this Calculation still shorter. It is founded upon this Consideration, that when x is very small $\overline{1+x}|^n$ is very nearly $1 + nx$. Hence if $1 + x$, and $1 - z$ are the two last Numbers already got in the first Column of the Table, their Powers $\overline{1+x}|^m$ and $\overline{1-z}|^n$ are such as will make the Product $\overline{1+x}|^m \times \overline{1+z}|^n$ very near to Unit, m and n may be found thus: $\overline{1+x}|^m = \overline{1+mx}$ and $\overline{1-z}|^n = 1 - nz$, and consequently $\overline{1+x}|^m \times \overline{1-z}|^n = 1 + mx - nz - mnzx$, or (neglecting $mnzx$) 1 +

mx

$mx - nz$. Make this equal to 1, and we have

$$m : n :: z : x :: l\,\overline{1-z} : l\,\overline{1+x}.$$

Whence $x\,l\,\overline{1-z} + z\,l\,\overline{1+x} = 0$. To give an Example of the Application of this, let 1,024 and 0,990352 be the last Numbers in the Table, their Logarithms being *C* and *D*. Then we have $1{,}024 = 1 + x$, and $0{,}990352 = 1 - z$, and consequently $x = 0{,}024$, and $z = 0{,}009648$. Whence the Ratio $\frac{z}{x}$ in the least Numbers is $\frac{201}{500}$. So that for finding the Logarithms proposed we may have $500\,D + 201\,C = 48510\,l\,2 - 14603\,l\,10 = 0$; which gives $l\,2 = 0{,}3010307$, which is too big in the last Figure; but it is nearer the Truth than what is got from the Logarithm *F* supposed equal to nothing. So that by this Means we have saved four Multiplications, which were necessary to find the Number 9989595, &c. correspondent to *F*, and which must have been had if we would make the Logarithm true to the same Number of Places without this *Compendium*.

1,28000000000	$A = 7\,l\,2 - 2\,l\,10$	$l\,2 > 0{,}28$
0,80000000000	$B = 3\,l\,2 - l\,10$	$< 0{,}33$
1,02400000000	$C = 10\,l\,2 - 3\,l\,10$	$> 0{,}300$
0,990352031429	$D = 9\,C + B = 93\,l\,2 - 28\,l\,10$	$< 0{,}30107$
1,004336277664	$E = 2\,D + C = 169\,l\,2 - 59\,l\,10$	$> 0{,}301020$
0,998959536107	$F = 2\,E + D = 485\,l\,2 - 146\,l\,10$	$< 0{,}3010309$
1,000162894165	$G = 4\,F + E = 2136\,l\,2 - 643\,l\,10$	$> 0{,}30102996$
0,999936281874	$H = 6\,G + F = 13301\,l\,2 - 4004\,l\,10$	$< 0{,}301029997$
1,000035441215	$I = 2\,H + G = 28738\,l\,2 - 8651\,l\,10$	$> 0{,}3010299951$
0,999971720830	$K = I + H = 42039\,l\,2 - 12655\,l\,10$	$< 0{,}3010299959$
1,000007161046	$L = K + I = 70777\,l\,2 - 21306\,l\,10$	$> 0{,}30102999562$
0,999993203514	$M = 3\,L + K = 254370\,l\,2 - 76573\,l\,10$	$< 0{,}30102999567$
1,000000364511	$N = M + L = 325147\,l\,2 - 97879\,l\,10$	$> 0{,}3010299956635$
0,999999764687	$O = 18\,N + M = 6107016\,l\,2 - 1838335\,l\,10$	$< 0{,}3010299956640$
Com. Ar. 235313		
$0 = 364511\,O +$	$235313\,N = 2302585825187\,l\,2 - 693147400972\,l\,10$	$> 0{,}301029995663987$

I have computed this Table so far, that the Reader may see in what Manner this Method approximates; this whole Work, as it appears, requiring a little more than three Hours Time.

Of the summing of Infinite Series, Part I. by *P. R.* de Monmort. n. 353. p. 633.

XVIII. 1 *Prop.* 1. *Prob.* To find the Sum of any Number of Terms of this Series.

$$a \times \overline{a+n} \times \overline{a+2n} \times \overline{a+3n} \times \overline{a+4n}, \&c. \times \overline{a+\overline{p-1}\,n}$$
$$+ \overline{a+n} \times \overline{a+2n} \times \overline{a+3n} \times \overline{a+4n}, \&c. \times \overline{a+pn}$$
$$+ \overline{a+2n} \times \overline{a+3n} \times \overline{a+4n}, \&c. \times \overline{a+\overline{p+1}\,n}$$
$$+ \overline{a+3n} \times \overline{a+4n}. \&c. \times \overline{a+\overline{p+2}\,n}$$
$$+ \overline{a+4n}, \&c. \times \overline{a+\overline{p+3}\,n}$$
&c.

Here n is the given Difference both of the continued Factors a, $a+n$, $a+2n$, *&c.* of every the ſame Term, as of the homologous Factors of the ſucceſſive Terms of the Series continued; and p denotes the Number of ſuch Factors in every Term.

Solution. Let x denote the firſt of the Factors in the laſt of the Terms whoſe Sum is required; then that Sum will be

$$\frac{x \times \overline{x+n} \times \overline{x+2n}, \&c. \times \overline{x+pn} - \overline{a-n} \times a \times \overline{a+n} \times \&c. \times \overline{a+\overline{p-1}\,n}}{\overline{p+1}\,n}$$

Q. E. I.

Example 1. Let the Series of natural Numbers be propoſed, $1+2+3+4$, *&c.* and let the Sum be found of ſo many Terms as there are Units in the Number z, which in this Caſe is alſo the laſt of the Terms whoſe Sum is required. Then in this Caſe it will be $a=1$, $n=1$, $p=1$, and $x=z$. whence

$x \times \overline{x+n} \times \&c. \times \overline{x+pn} = z \times \overline{z+1}$, and $\overline{a-n} \times a \times \&c. \times \overline{a+\overline{p-1}\,n} = 0$, and $\overline{p+1}\,n = 2 \times 1$. Therefore the Sum required is $\frac{z \times \overline{z+1}}{2}$.

Example 2. In the Series $1+3+6+10$, *&c.* of Triangular Numbers, let the Sum of ſo many Terms be found as there are Units in the Number z. The Numbers in this Series may be wrote thus: $\frac{1\times2}{2}$, $\frac{2\times3}{2}$, $\frac{3\times4}{2}$, $\frac{4\times5}{2}$, *&c.* By this Means, if we ſet aſide the given Diviſor 2, the Series is reduced to the Form of the Propoſition, it being $a=1$, $n=1$, $p=2$, and $x=z$. Whence the double Sum of the Series is $\frac{x \times \overline{x+1} \times \overline{x+2} - 0}{3} = \frac{x \times \overline{x+1} \times \overline{x+2}}{3}$. And having Re-

gard

gard to the Divisor 2, the Sum of the Series will be $\frac{x \times \overline{x+1} \times \overline{x+2}}{2 \times 3}$,

or $\frac{z \times \overline{z+1} \times \overline{z+2}}{2 \times 3}$, in this Case x and z being the same. And after the same Manner the Sums of the other figurate Numbers may be found, whose Forms are now commonly known.

Example 3. Let $a = 1$, $n = 2$, $p = 3$, that the proposed Series may be $1 \times 3 \times 5 + 3 \times 5 \times 7 + 5 \times 7 \times 9$, &c. In this Case the Form of

the Sum is $\frac{x \times \overline{x+2} \times \overline{x+4} \times \overline{x+6} - \overline{1-2} \times 1 \times 3 \times 5}{4 \times 2} =$

$\frac{x \times \overline{x+2} \times \overline{x+4} \times \overline{x+6} + 15}{8}$. For Instance, if the Sum of ten

Terms is required, then $x = 19$, which is the tenth Term in the Series of Arithmetical Proportionals 1, 3, 5, 7, &c. and therefore the Sum is

$\frac{19 \times 21 \times 23 \times 25 + 15}{8} = 28630$. Now the Proposition is thus demonstrated.

Demonstration. Let there be a Series of Quantities, A, B, C, D, &c. whose Differences constitute the Series a, b, c, d, &c. so that it may be $a = B - A$, $b = C - B$, $c = D - C$, &c. Hence we immediately gather, that $a + b = C - A$, $a + b + c = d - A$, $a + b + c + d = E - A$; and in general, that the Aggregate of any Number of Terms of the Series a, b, c, d, &c. is equal to the next following Term of the Series A, B, C, D, &c. lessen'd by the first Term A. For A, B, C, &c. take the Terms.

$\frac{\overline{a-n} \times a \times \&c. \times \overline{a + \overline{p-1}\, n}}{\overline{p+1}\, n}$, $\frac{a \times \overline{a+n} \times \&c. \times \overline{a+pn}}{\overline{p+1}\, n}$

$\frac{\overline{a+n} \times \overline{a+2n} \times \&c. \times \overline{a + \overline{p+1}\, n}}{\overline{p+1}\, n}$, &c. that is, the successive Values of

$\frac{x \times \overline{x+n} \times \&c. \times \overline{x+pn}}{\overline{p+1}\, n}$; and their Differences being taken for a, b,

c, d, &c. then will $a \times \overline{a+n}, \times$ &c. $\times \overline{a + \overline{p-1}\, n}$, $\overline{a+n} \times \overline{a+2n}, \times$ &c.

$\times \overline{a + p\,n}$, &c. which are the very Terms of the Series propos'd. But by comparing these Series, if any Term of the latter Series is $x \times \overline{x+n}, \times$ &c. $\times \overline{x + \overline{p - 1}\,n}$, it is plain that the Term one Step farther in the former Series will be $\frac{x \times \overline{x+n}, \times \&c. \times \overline{x+p\,n}}{\overline{p+1}\,n}$. Therefore the Sum of the latter Series, as far as the Term $x \times \overline{x+n}, \times$ &c. $\times \overline{x + \overline{p - 1}\,n}$ inclusively, is

$$\frac{x \times \overline{x+n}, \times \&c. \times \overline{x+p\,n} - \overline{a-n} \times a, \times \&c. \times \overline{a + \overline{p-1}\,n}}{\overline{p+1}\,n} \quad Q.\ E.\ D.$$

Scholium 1. In this Proposition is contain'd some little Part of the *Methodus Incrementorum*, concerning which two Years ago my very good Friend Dr. *Brook Taylor*, Secretary to the Royal Society at *London*, publish'd a Book. He that would know more of this Method should consult that Work. It is sufficient for our Purpose to observe, how great an Affinity there is between this Method and the Method of Fluxions, or the Differential Method. For as in the Method of Fluxions, to find the Fluxion of x^m any Dignity of x, one Side is to be converted into the Fluxion $\dot{x}$, and the Quantity arising is to be multiply'd into m the Index of the Dignity, that the Fluxion sought $m\,\dot{x}\,x^{m-1}$ may be produc'd; so in the Method of Increments, to find the Increment of such a Product $x \times \overline{x+n} \times \overline{x+2\,n}$, (where the Factors x, $x+n$, $x+2\,n$, are in Arithmetical Progression, whose common Difference is n the given Increment of x,) the least of the Factors x is to be converted into the Increment, and the Quantity arising is to be multiply'd by the Number of the Factors, so that $3\,n \times \overline{x+n} \times \overline{x+2\,n}$ may be the Increment sought, the Number of Factors in the Case proposed being 3. Thus also the Increment of $x \times \overline{x+n}$ becomes $2\,n \times \overline{x+n}$.

2. Also by the same Rule are found the Increments of the Reciprocals of such Products. But here it must be observed, that as Division is contrary to Multiplication, instead of taking away the least of the Factors, now another Factor must be added, which is greater still by one Increment. Also that the Number of the Factors must be wrote with a nega-

negative Sign. By this means the Increment of $\frac{1}{x}$ will be $\frac{-1\times n}{x\times\overline{x+n}}$.

The Increment of $\frac{1}{x\times\overline{x+n}}$ will be $\frac{-2n}{x\times\overline{x+n}\times\overline{x+2n}}$. And so in all others of this Kind. This is easily proved by taking the Differences between two continued Values of the Integrals.

3. By treading in the Steps of the Direct Method, we may hence collect the Rules of the Inverse Method, by which the Integrals of any given Increments are to be found. For let the given Increment be apply'd to the known Increment of the Side; let a Factor be added which is still less by one Increment, and let the Quantity that arises be apply'd to the Number of the Factors so increased. Thus for Example, if the given Increment were $n\times x\times\overline{x+n}\times\overline{x+2n}$, it becomes first $x\times\overline{x+n}\times\overline{x+2n}$; then $\overline{x-n}\times x\times\overline{x+n}\times\overline{x+2n}$ adding the Factor $x-n$; lastly, $\frac{\overline{x-n}\times x\times\overline{x+n}\times\overline{x+2n}}{4}$, which is the Integral required. This obtains when the Factors are Multipliers; but when the Factors are in the Place of Divisors, then *mutatis mutandis* the Rule is thus. Let the given Increment be apply'd to the known Increment of the Side; let the greatest of the Factors be rejected, and let the Quantity arising be apply'd to the Number of the Factors remaining with a negative Sign. For Example, let be given the Increment $\frac{n}{x\times\overline{x+n}\times\overline{x+2n}}$: First it becomes $\frac{1}{x\times\overline{x+n}\times\overline{x+2n}}$, then $\frac{1}{x\times\overline{x+n}}$, lastly, $\frac{1}{-2\times x\times\overline{x+n}}$ or $\frac{-1}{2x\times\overline{x+n}}$, which is the Integral required.

In this last Case the Integral found, with a contrary Sign, is equal to the Sum of all the Increments in the Series being continued to Infinity. For Example it is $\frac{1}{2x\times\overline{x+n}}=\frac{n}{x\times\overline{x+n}\times\overline{x+2n}}$

+

$$+\frac{n}{\overline{x+n}\times\overline{x+2n}\times\overline{x+3n}}+\frac{n}{\overline{x+2n}\times\overline{x+3n}\times\overline{x+4n}}, \text{ \&c.}$$

For in this Case, x becoming at last infinite, $\frac{1}{2x\times\overline{x+n}}$ vanishes, that is, the last of the Terms A, B, C, *&c.* becomes nothing; and because of the Contrariety of the Signs of the Integral and Increment, instead of $-A$ the Aggregate is express'd by $+A$.

Lemma 1. In any Series of Numbers M, N, O, P, *&c.* let any Term be denoted by X, and let the Place of that Term in the Series be denoted by x; that is, let $x=1$, when X denotes the first Term or M; let $x=2$, when X denotes the second Term N, and so on. And of the Terms M, N, O, P, *&c.* let b be the first of the first Differences, c the first of the second Differences, d the first of the third, e the first of the fourth, and so on. Then it will be $X=M+b\times\frac{x-1}{1}+c\times\frac{x-1}{1}\times\frac{x-2}{2}+d\times\frac{x-1}{1}\times\frac{x-2}{2}\times\frac{x-3}{3}+e\times\frac{x-1}{1}\times\frac{x-2}{2}\times\frac{x-3}{3}\times\frac{x-4}{4}$, *&c.* This follows from the Table of Equations, *pag.* 60. of our Treatise call'd *Essay d'Analyse*, &c.

Lemma 2. The same Things supposed, let any Term in the Series of Arithmetical Proportionals a, $a+n$, $a+2n$, *&c.* be denoted by z, and now let it be $X=A+Bz+Cz\times\overline{z+n}+Dz\times\overline{z+n}\times\overline{z+2n}+Ez\times\overline{z+n}\times\overline{z+2n}\times\overline{z+3n}$, *&c.* Then the Values of A, B, C, D, E, *&c.* will be these following.

$$A=M+b\times\frac{-a}{n}+c\times\frac{-a}{n}\times\frac{-a-n}{2n}$$

$$+d\times\frac{-a}{n}\times\frac{-a-n}{2n}\times\frac{-a-2n}{3n}$$

+

$$+ e \times \frac{-a}{n} \times \frac{-a-n}{2n} \times \frac{-a-2n}{3n} \times \frac{-a-3n}{4n}, \&c.$$

$$B = \frac{1}{n} \times b + c \times \frac{-a-n}{n} + d \times \frac{-a-n}{n} \times \frac{-a-2n}{2n}$$

$$+ e \times \frac{-a-n}{n} \times \frac{-a-2n}{2n} \times \frac{-a-3n}{3n}, \&c.$$

$$C = \frac{1}{n} \times \frac{1}{2n} \times c + d \times \frac{-a-2n}{n} + e \times \frac{-a-2n}{n} \times \frac{-a-3n}{2n}, \&c.$$

$$D = \frac{1}{n} \times \frac{1}{2n} \times \frac{1}{3n} \times d + e \times \frac{-a-3n}{n}, \&c.$$

$$E = \frac{1}{n} \times \frac{1}{2n} \times \frac{1}{3n} \times \frac{1}{4n}\, e, \&c.$$

The Order of forming the Coefficients of b, c, d, e, *&c.* in these Values, is sufficiently manifest of itself.

Demonstration. Because by x and z the respective Terms of these Arithmetical Progressions are denoted, 1, 2, 3, 4, *&c.* and a, $a+n$, $a+2n$, $a+3n$, *&c.* therefore $x-1$ will denote the Number of the Differences n which is contain'd in z, so that it is $z = a + \overline{x-1}\, n$. Hence it is that $x - 1 = \frac{z-a}{n}$, $x - 2 = \frac{z-n-a}{n}$, $x - 3 = \frac{z-2n-a}{n}$, *&c.* Therefore by substituting these Values $x-1$, $x-2$, $x-3$, *&c.* in the Series of the foregoing Lemma, and reducing the Terms into Order, the Values of A, B, C, *&c.* come out as here exhibited.

Corol. When $a = n$, the Values of A, B, C, *&c.* become more simple, as

$$A = M - b + c - d + e, \&c.$$

$B =$

$$B = \frac{1}{n} \times \overline{b - 2c + 3d - 4e, \&c.}$$

$$C = \frac{1}{n} \times \frac{1}{2n} \times \overline{c - 3d + 6e, \&c.}$$

$$D = \frac{1}{n} \times \frac{1}{2n} \times \frac{1}{3n} \times \overline{d + 4e, \&c.}$$

Lemma 3. The Symbols X and x being interpreted in the ſame Manner as in the firſt Lemma, let $q, r, s, t, u,$ &c. be the Generators of the Arithmetical Triangle, whoſe tranſverſe Line is occupy'd by the Series $M, N, O, P, Q,$ &c. but in an inverted Order, ſo that $q\ (= M)$ may be the laſt Generator, r the laſt but one, s the laſt but two, and ſo on. Then it will be

$$X = q + r \times \frac{x-1}{1} + s \times \frac{x-1}{1} \times \frac{x}{2} + t \times \frac{x-1}{1} \times \frac{x}{2} \times \frac{x+1}{3}, \&c.$$

This is plain from the Conſideration of the Arithmetical Triangle itſelf, which we have exhibited *pag.* 63. of the Treatiſe *Eſſay d' Analyſe, &c.* where the ſame is more fully explain'd.

Lemma 4. The ſame Things being ſuppoſed, and the Symbol z being interpreted in the ſame Manner as in Lemma 2: If it is $X = A + Bz + Cz \times \overline{z+n}, \&c.$ as in *Lemma* 2. the Values of the Coefficients $A, B, C, D,$ &c. will be

$$A = q + r \times \frac{-a}{n} + s \times \frac{-a}{n} \times \frac{-a+n}{2n}$$

$$+ t \times \frac{-a}{n} \times \frac{-a+n}{2n} \times \frac{-a+2n}{3n}, \&c.$$

$$B = \frac{1}{n} \times r + s \times \frac{-a}{n} + t \times \frac{-a}{n} \times \frac{-a+n}{2n}, \&c.$$

$$C = \frac{1}{n} \times \frac{1}{2n} \times s + t \times \frac{-a}{n}, \&c.$$

D =

$$D = \frac{1}{n} \times \frac{1}{2n} \times \frac{1}{3n} \times t, \text{ \&c.}$$

The Order of the Coefficients is manifest in these Values, and the Lemma is demonstrated in the Manner of the second Lemma.

Cor. 1. When $a = n$, the Coefficients A, B, C, D, *&c.* come out in simpler Forms thus.

$$A = q - r, \qquad B = \frac{1}{n} \times \overline{r - s},$$

$$C = \frac{1}{n} \times \frac{1}{2n} \times \overline{s - t}, \qquad D = \frac{1}{n} \times \frac{1}{2n} \times \frac{1}{3n} \times \overline{t - u}, \text{ \&c.}$$

Cor. 2. Whence if some of the Generators q, r, s, t, u, *&c.* are equal to one onother, X will be exhibited by a simpler Form, some of the Coefficients A, B, C, *&c.* vanishing.

Thus for Example, a Series of Numbers being proposed 4, 69, 530, 2676, 10350, *&c.* which constitute the tenth transverse Line in the Arithmetical Triangle, whose three first Generators are 54, — 18, 5, and the seven last are equal to 4; it being $a = 1 = n$; the Term X is exhibited by a Form of four Terms only,

$$\frac{z}{1} \cdot \frac{z+1}{2} \cdot \frac{z+2}{3} \cdot \text{\&c.} \times \frac{z+6}{7} + 23 \cdot \frac{z}{1} \cdot \frac{z+1}{2}, \text{ \&c.}$$

$$\times \frac{z+6}{7} - 72 \cdot \frac{z}{1} \cdot \frac{z+1}{2} \cdot \text{\&c.} \times \frac{z+7}{8} + 54 \cdot \frac{z}{1} \cdot \frac{z+1}{2}$$

$\text{\&c.} \times \frac{z+8}{9}$; the first six Coefficients A, B, C, D, E, F, vanishing.

Prop. 11. *Prob.* To find the Sum of any Number of Terms of this Series.

$$\frac{M}{a \times \overline{a+n}, \text{ \&c.} \times \overline{a+p-1}^{\,n}} + \frac{N}{\overline{a+n}, \text{ \&c.} \times \overline{a+p}^{\,n}}$$

$+\frac{O}{\overline{a+2n},\ \&c.\times\overline{a+p+1n}}$, *&c.* Where the Numerators *M*, *N*, *O*, *&c.* constitute any Series of Terms whose Differences are given, whether first, second, third, *&c.* or which is the same, they constitute any transverse Line in any given Arithmetical Triangle. But the Denominators constitute the Series exhibited in *Prop.* 1.

Solution. Let the first of the Factors a, $a+n$, $a+2n$, *&c.* be represented by X in the Denominator of the same Term, so that X and z may be the same as in the foregoing Lemma's; and therefore let any Term of the Series be represented by

$\frac{X}{z\times\overline{z+n},\ \&c.\times\overline{z+p-n}}$. By Lemma 2 or 4, (as may seem convenient, either to admit the Differences, or the Generators of the Arithmetical Triangle,) let X be resolved into the Multinomium $A+B\times z+Cz\times\overline{z+n}+Dz\times\overline{z+n}\times\overline{z+2n}$, *&c.* By this Means (the Terms of the Multinomium being apply'd to the Denominator $z\times\overline{z+n},\ \&c.\times\overline{z+p-n}$,) every Term of the Series will be reduced to the Form $\frac{A}{z\times\overline{z+n}\ \&c.\ \times\overline{z+p-1n}}$

$$+\frac{B}{\overline{z+n},\ \&c.\times\overline{z+p-1n}}+\frac{C}{\overline{z+2n},\ \&c.\times\overline{z+p-1n}}\ \&c.$$

Whence (by *Schol.* 4. *Prop.* 1.) the Aggregate of the whole Series continued *in infinitum* from the Term $\frac{X}{z\times\overline{z+n},\ \&c.\times\overline{z+p-1n}}$

inclusively, is $\frac{A}{\overline{p-1}\times n\times z\times\overline{z+n}\ \&c.\times\overline{z+p-2n}}$

$$+\frac{B}{\overline{p-2}\times n\times\overline{z+n},\ \&c.\times\overline{z+p-2n}}$$

$$+\frac{C}{\overline{p-3}\times n\times\overline{z+2n}, \&c.\times\overline{z+p-2n},}\ \&c.$$

If this Aggregate be taken from the Value of the ſame Aggregate when $z=a$, the Remainder will be the Sum of all the Terms before the Term $\frac{X}{z+\&c.}$, that is, of ſo many Terms as there are Units in $\frac{z-a}{n}$.

Q. E. I.

Ex. 1. Let the firſt Example be the Series $\frac{5}{3\cdot5\cdot7\cdot9\cdot11\cdot13}$

$$+\frac{41}{5\cdot7\cdot9\cdot11\cdot13\cdot15}+\frac{131}{7\cdot9\cdot11\cdot13\cdot15\cdot17}$$

$$+\frac{275}{9\cdot11\cdot13\cdot15\cdot17\cdot19}+\frac{473}{11\cdot13\cdot15\cdot17\cdot19\cdot21}, \&c.$$

'Tis here $a=3$, $n=2$, $p=6$, $M=5$. And taking the Differences of the Numerators, it will be $b=36$, $c=54$, $d=0=e=\&c.$ Hence in the ſecond Lemma 'tis $A=5+36\times\frac{-3}{2}+54\times\frac{-3}{2}\times\frac{-5}{4}=\frac{209}{4}$,

$B=\frac{1}{2}\times36+54\times\frac{-5}{2}=\frac{-99}{2}$, $C=\frac{1}{2}\times\frac{1}{4}\times54=\frac{27}{4}$,

$D=0=E=\&c.$ Therefore the Sum of the whole Series is

$$\frac{209}{4\times5\times2\times3\cdot5\cdot7\cdot9\cdot11}+\frac{-99}{2\times4\times2\times5\cdot7\cdot9\cdot11}$$

$$+\frac{27}{4\times3\times2\times7\cdot9\cdot11}=\frac{283}{80\times3\cdot5\cdot7\cdot9\cdot11}.$$ And the Sum of the Terms in Number $\frac{z-3}{2}\left(=\frac{z-a}{n}\right)$ is $\frac{283}{80\times3\cdot5\cdot7\cdot9\cdot11}$

$$-\frac{209}{40\times z.\overline{z+2}.\overline{z+4}.\overline{z+6}.\overline{z+8}}+\frac{99}{16\times\overline{z+2}.\overline{z+4}.\overline{z+6}.\overline{z+8}}$$

$-\dfrac{27}{24\times\overline{z+4}.\overline{z+6}.\overline{z+8}}$. For Instance, let eight Terms be required; then it is $\frac{z-3}{2}=8$, or $z=19$; which Value being introduced into the Formula, the Sum is $\frac{155891}{2\cdot3\cdot3\cdot3\cdot3\cdot5\cdot5\cdot5\cdot7\cdot11\cdot19\cdot23}$.

The same Numerators possess the third transverse Line in the Arithmetical Triangle

$$\begin{array}{cccccc} 54 \cdot & 54 \cdot & 54 \cdot & 54 \cdot & 54 \cdot & 54 \cdot \&c. \\ & -18 \,. & 36 \,. & 90 \,. & 144 \,. & 198 \,.\ \&c. \\ & & 5 \,. & 41 \,. & 131 \,. & 275 \,.\ \&c. \end{array}$$

Whence in the Formula of *Lemma* 4. the Generators are $q=5$, $r=-18$, $s=54$, $t=0=$ &c. and the Coefficients come out $A=5-18\times\frac{-3}{2}+54\times\frac{-3}{2}\times\frac{-3+2}{4}=\frac{209}{4}$, $B=\frac{1}{2}\times\overline{-18+54\times\frac{-3}{2}}=\frac{-99}{2}$, $C=\frac{1}{2}\times\frac{1}{4}\times54=\frac{27}{4}$, $D=0=E=$ &c. the same as above.

Ex 2. Let the Series be $\frac{4}{1\cdot2\cdot3\cdot4\cdot5\cdot6\cdot7\cdot8\cdot9\cdot10\cdot11}+\frac{69}{2\cdot3\cdot\&c.\,12}+\frac{530}{3\cdot4\cdot\&c.\,13}+\frac{2676}{4\cdot5\cdot\&c.\,14}+\frac{10350}{3\cdot6\cdot\&c.\,14}$ &c. where it is $a=1$, $n=1$, $p=11$, and the Numerators constitute the Series exhibited *Cor.* 20. *L.* 4. Therefore applying the Value of X in that *Corollary* to the Denominator $z\times\overline{z+1}$, &c. $\times\overline{z+10}$, the Term

Term of the propoſed Series becomes

$$\frac{-1}{1\cdot2\cdot3\cdot4\cdot5\cdot6\times\overline{z+6}\cdot\overline{z+7}\cdot\overline{z+8}\cdot\overline{z+9}\cdot\overline{z+10}}$$

$$+\frac{23}{1\cdot2\cdot3\cdot4\cdot5\cdot6\cdot7\times\overline{z+7}\times\overline{z+8}\times\overline{z+9}\times\overline{z+10}}$$

$$-\frac{72}{1\cdot2\cdot3\cdot4\cdot5\cdot6\cdot7\cdot8\times\overline{z+8}\cdot\overline{z+9}\times\overline{z+10}}$$

$$+\frac{54}{1\cdot2\cdot3\cdot4\cdot5\cdot6\cdot7\cdot8\cdot9\times\overline{z+9}\times\overline{z+10}}.$$ Therefore by this

Propoſition the Sum of the Series continued from that Term *in infinitum*

is $$\frac{-1}{4\times1\cdot2\cdot3\cdot4\cdot5\cdot6\times\overline{z+6}\times\overline{z+7}\times\overline{z+8}\times\overline{z+9}}$$

$$+\frac{23}{3\times1\cdot2\cdot3\cdot4\cdot5\cdot6\cdot7\times\overline{z+7}\cdot\overline{z+8}\cdot\overline{z+9}}$$

$$-\frac{72}{2\times1\cdot2\cdot3\cdot4\cdot5\cdot6\cdot7\cdot8\times\overline{z+8}\times\overline{z+9}}$$

$$+\frac{54}{1\times1\cdot2\cdot3\cdot4\cdot5\cdot6\cdot7\cdot8\cdot9\times\overline{z+9}}.$$ Therefore for z aſſu-

ming 1, the Sum of the whole Series is

$$\frac{305}{12\times1\cdot2\cdot3\cdot4\cdot5\cdot6\cdot7\cdot8\cdot9\cdot10}.$$ And in general, the Sum of

the Terms in Number $\frac{z-1}{1}$ will be

$$\frac{305}{12\times1\cdot2\cdot3\cdot4\cdot5\cdot6\cdot7\cdot8\cdot9\cdot10}$$

$$+\frac{1}{4\times 1.2.3.4.5.6\times \overline{z+6}.\overline{z+7}.\overline{z+8}.\overline{z+9}}$$

$$-\frac{23}{3\times 1.2.3.4.5.6.7\times \overline{z+7}.\overline{z+8}.\overline{z+9}}$$

$$+\frac{72}{2\times 1.2.3.4.5.6.7.8\times \overline{z+8}\times \overline{z+9}}$$

$$-\frac{54}{1\times 1.2.3.4.5.6.7.8.9\times \overline{z+9}}.$$

Scholium 1. In computing the Sums of this Kind of Series, generally the Calculation is made easier by making Use of the Generators of the Arithmetical Triangle, than by making Use of the Differences. Therefore I have a Mind to take this Occasion, to shew how the Generators of the Arithmetical Triangle may be found, from the Differences being given.

Therefore let ω be the first Term of the Series, a the last Difference given, b the first of the last Order but one of the Differences, c the first of the next Order before, and so d, e, &c. and let t, u, x, y, &c. be the Generators of the Arithmetical Triangle required, whose transverse Line in Order p is occupied by the Series proposed. Then it is evident from the Consideration of the Arithmetical Triangle, that

$$a = t,$$

$$b = \frac{p-1}{1}t + u,$$

$$c = \frac{p-1}{1}\times\frac{p-2}{2}t + \frac{p-2}{1}u + x,$$

$$d = \frac{p-1}{1}\times\frac{p-2}{2}\times\frac{p-3}{3}t + \frac{p-2}{1}\times\frac{p-3}{2}u$$

$$+\frac{p-3}{1}x + y, \&c.$$

Whence

Whence the Values of the Generators are collected,

$$t = a,$$

$$u = b - \frac{p-1}{1}\,t,$$

$$x = c - \frac{p-1}{1} \times \frac{p-2}{2}\,t - \frac{p-2}{1}\,u,$$

$$y = d - \frac{p-1}{1} \times \frac{p-2}{2} \times \frac{p-3}{3}\,t - \frac{p-2}{1} \times \frac{p-3}{2}\,u$$

$$- \frac{p-3}{1}\,x, \text{ \&c.}$$

Now the last Generator is equal to ω the first Term of the Series.

2. After I had communicated these Things to D. *de Monsoury*, he found another Solution of this Problem, the Formula of which I shall here set down, because of its wonderful Simplicity. Therefore in the Series of Numerators let ω be the first Term, b the first of the first Order of Differences, c the first of the second, d the first of the third, and so on; and let the Denominator of the first Term be $z \times \overline{z+n}$, &c. $\times\, \overline{z + \overline{p-1}\,n}$. Then the Sum of the whole Series continued *in infinitum* will be exhibited by this Formula,

$$\frac{\omega}{n \times \overline{p-1} \times z \times \overline{z+n}, \text{ \&c. } \times \overline{z+\overline{p-2}\,n}}$$

$$+ \frac{b}{n^2 \times \overline{p-1} \times \overline{p-2} \times \overline{z+n}, \text{ \&c. } \times \overline{z+\overline{p-2}\,n}}$$

$$+ \frac{c}{n^3 \times \overline{p-1} \times \overline{p-2} \times \overline{p-3} \times \overline{z+2n}, \text{ \&c. } \times \overline{z+\overline{p-2}\,n}}, \text{ \&c.}$$

Let us take an Example in the Series $\dfrac{5}{3 \cdot 5 \cdot \text{\&c. } 13} + \dfrac{41}{5 \cdot 7 \cdot \text{\&c. } 15}$

+

$+\frac{131}{7\cdot 9\cdot \&c.\ 17}+\frac{275}{9\cdot 11\cdot \&c.\ 19}$, &c. the Sum of which we have already exhibited. In this Case it is $a=5$, $b=36$, $c=54$, $d=0=e=$ &c. Whence by the Formula the Sum of the whole Series is $\frac{5}{2\cdot 5\times 3\cdot 5\ldots 11}$

$$+\frac{36}{4\cdot 5\times 4\cdot 5\ldots\ldots 11}+\frac{54}{8\cdot 5\cdot 4\cdot 3\times 7\ldots 11}=\frac{283}{80\times 3\cdot 5\ldots 11},$$

as it was exhibited by our Formula. If it is required to find the Sum of the same Series beginning from the tenth Term $\frac{2273}{21\ldots\ldots 31}$, in this Case it is $a=2273$, $b=522$, $c=54$, and the Sum would be

$$\frac{2273}{2\cdot 5\times 21\ldots 29}+\frac{522}{4\cdot 5\cdot 4\times 23\ldots 29}+\frac{54}{8\cdot 5\cdot 4\cdot 3\times 25\ldots 29}.$$

This Formula is very convenient, and exhibits the Sum with very little Trouble, as often as the Sum of the whole Series is required; for the Differences are not too many. But when the Differences are many, and the whole Series is not required, but only some of the initial Terms, then our Forms will be the more convenient.

3. When the Terms of the Series are formed only by Multiplication, and are not affected by variable Divisors, the Sums may always be found by the Method delivered in *Prop.* 1. though the Formulæ are never so complicate: For they may always be reduced to such Terms as that *Proposition* requires. Thus if the Differences of z and x are m and n, and a Term in the Series is denoted by zx; this Term will be reduced to $\overline{a-n}\,z+\frac{n}{m}z\times\overline{z+m}$, whose Integral may be had by *Prop* 1. For because $dx=n$ and $dz=m$, 'tis $dx=dz\times\frac{n}{m}$. And returning to the Integrals it will be $x=\frac{n}{m}z+a$, (the invariable Quantity a being

added,

added, that an Account may be taken of the Relation between z and x in the firſt Term of the Series) which may be thus written, $\overline{a-n}+\frac{n}{m}\times\overline{z+m}$, that afterwards being drawn into z, it may acquire the neceſſary Form. And in the ſame Manner we may proceed in other Caſes of the ſame Kind. But when the Forms propoſed are affected with Diviſors, the ſame Difficulties occur as in the integral Calculus, as it is called, or in the inverſe Method of Fluxions, which are to be overcome with the ſame Induſtry. Nor can they always be overcome. For beſides as it hardly can be known for a Certainty what muſt be the Relation between the Numerator of the Fraction, and the Denominator, that the propoſed Formula may be reduced to an Integral; ſo it is often very difficult to find, whether ſuch a Relation is already in that Formula, or, if it is not, whether it can be introduced. What I have chiefly found of Uſe in this Matter, is contained in the three following *Propoſitions*,

Prop. III. *Prob.* The Quantities z, u, y, x, &c. increaſing by the given Differences n, m, l, o, &c. to find the Value of the integral Numerator N, ſo as that the Denominator being $z.\overline{z+n}$, &c. $\overline{z+pn}\times u.\overline{u+m}$, &c. $\overline{u+qm}\times y.\overline{y+l}$.&c.$\overline{y+rl}\times x.\overline{x+o}$.&c. $\overline{x+so}$, &c. the Fraction may be reduced to an Integral.

Solution. Make $N=\overline{z+pn}\times\overline{u+qm}\times\overline{y+rl}\times\overline{x+so}$, &c. $-zuyx$. &c. and the Integral will be a Fraction whoſe Denominator is $z.\overline{z+n}$.&c.$\overline{z+\overline{p-1}\,n}.u.\overline{u+m}$.&c. $\overline{u+\overline{q-1}\,m}.y.\overline{y+l}$ &c. $\overline{y+\overline{x-1}\,l}.\overline{x+o}$.&c. $\overline{x+\overline{s-1}\,o}$, &c. the Numerator being 1.

For the Difference of this Fraction is a Fraction whoſe Numerator is the exhibited Value of N, and the Denominator is the ſame rs the Denominator propoſed, as it ought to be.

Ex. 1. Let the Denominator propoſed be $z\times\overline{z+2}\times u\times\overline{u+3}$. In this Caſe Caſe 'tis $n=2$, $m=3$, $p=1$, $q=1$; Therefore $N=\overline{z+2}\times\overline{u+3}-zu=3z+2u+6$. And by $\frac{3z+2u+6}{z.\overline{z+2}\times u.\overline{u+3}}$ is re-

 preſented

presented a Term of a summable Series, the Sum of which, when continued *in infinitum*, is exhibited by $\frac{1}{zu}$. For Instance, let the first common Value of z and u be 1, and the Series to be sum'd will be $\frac{1}{1.3\times 1.4}$ $+\frac{23}{3.5\times 4.7}+\frac{35}{5.7\times 7.10}$, &c. for the Sum of the whole is 1. By p let be denoted the Order of any Term in this Series, then it will be $p=\frac{z-1+2}{2}=\frac{u-1+3}{3}$, and therefore $z=2p-1$, and $u=3p-2$; which Values being substituted for z and u, the Term will be denoted by this Form $\frac{12p-1}{\overline{2p-1}\times\overline{2p+1}\times\overline{3p-2}\times\overline{3p+1}}$. But the Sum of all the Terms before this, that is, of the initial Terms which are $\frac{z-1}{2}=p-1$ in Number, will be $1-\frac{1}{zu}=\frac{zu-1}{zu}$; that is, $\frac{6pp-7p+1}{\overline{2p-1}\times\overline{3p-2}}$. Wherefore writing $p+1$ for p, the Aggregate of so many initial Terms as there are Units in p, will be $\frac{p\times 6\overline{p+5}}{\overline{2p+1}\times\overline{3p+1}}$.

Ex. 2. The same z, u, n, m still remaining, let the Denominator be $z.\overline{z+2}.\overline{z+4}\times u.\overline{u+3}$. Then by the Formula the Numerator will be $\overline{z+4}\times\overline{u+3}-zu=3z+4u+12$, and the Sum of the Series will be exhibited by the Formula $\frac{1}{z.\overline{z+2}\times u}$. Of z and u let the first common Value be 1, and hence will be deduced the Series $\frac{19}{1.3.5\times 1.4}+\frac{37}{3.5.7\times 4.7}+\frac{55}{5.7.9\times 7.10}$, &c. $=\frac{1}{3}$.

Scholium.

Scholium. In the Series now exhibited there is every where the ſame Difference between the continual Factors of any the ſame Term, as between the homologous Factors of the continual Terms. In the following are ſome Examples of Series, whoſe Sums may be exhibited in a finite Number of Terms, although that Rule is not obſerved.

Prop. IV. *Prob.* The Quantity z increaſing by given Differences qn, to find the Integer Numerator N, ſo that the Fraction may be reduced to its Integral, whoſe Denominator conſiſts of a certain Number p of the Terms z, $z+n$, $z+2n$, &c. of Arithmetical Proportionals drawn into one another. But q muſt be an integer Number leſs than the Number of Factors p.

Solution. It will be $N = \overline{z+\overline{p-1}\,n} \times \overline{z+\overline{p-2}\,n}$. &c. $\times$ $\overline{z+\overline{p-q}\,n} - z \times \overline{z+n} \times$ &c. $\times\, \overline{z+\overline{q-1}\,n}$, the Integral being $\dfrac{1}{z \times \overline{z+n} \times \text{\&c.} \times \overline{z+\overline{p-q-1}\,n}}$. It is demonſtrated after the Manner of the foregoing Propoſition.

The Quantities n, p, q, being aſſumed at Pleaſure, and the firſt Value of z, hence will ariſe an infinite Number of ſummable Series, ſuch as the three following.

$$A = \frac{5}{1.2.3.4} + \frac{9}{3.4.5.6} + \frac{13}{5.6.7.8} + \frac{17}{7.8.9.10}, \text{ \&c.}$$

$$B = \frac{1}{1.2.3.4.5} + \frac{4}{4.5.6.7.8} + \frac{9}{7.8.9.10.11} + \frac{16}{10.11.12.13.14}, \text{ \&c.}$$

$$C = \frac{1}{1.2.3.4.5} + \frac{14}{5.6.7.8.9} + \frac{55}{9.10.11.12.13} +$$

$\frac{140}{13.14.15.16.17}$, &c.

I communicated these Series long ago to some principal Geometricians, to whom they did not seem contemptible. That very skilful Geometrician Mr. *Nicolas Bernoulli* thus writes to me in a Letter of *July* 25, 1716. " You will very much oblige me, Sir, if you will com-" municate to me the Solution of this Problem of yours. Having a " Series of Fractions given, the Numerators of which are any figurate " Number whatever, and of which the Denominators are formed of " the Product of an equal Numbers of Factors which are in Arith-" metical Progression; to find their Sum. And chiefly how you have " found these two Forms $\frac{p}{24 \times \overline{4p+1}}$ and $\frac{p \times \overline{p+1}}{12 \times \overline{3p+1} \times \overline{3p+2}}$

These Forms belong to the Series C and B, the Number of Terms whose Sum is required being denoted by p. And thus also Dr. *Taylor* writes to me, in his Letter of *Aug.* 22, 1716. " As also by what " Method you fell upon the Summation of the Series exhibited by you,

" I mean chiefly of the Series $\frac{1}{1.2.3.4.5} + \frac{4}{4.5.6.7.8} +$

" $\frac{9}{7.8.9.10.11}$, which seems to be of a deeper Inquiry.

But now let us return to our Examples.

In the Series A 'tis $p = 4$, $q = 2$, $n = 1$, the first Value of z being 1. Therefore 'tis $\overline{z+3} \times \overline{z+2} - z \times \overline{z+1} = 2 \times \overline{2z+3}$; whence (rejecting the given Number 2) are derived these Numerators 5, 9, 13, 17, &c. Also the Formula of the Sum is $\frac{1}{z \times \overline{z+1}}$. Then taking an Account of the Number 2, which we rejected out of the Numerators, the Sum of the whole Series, continued *in infinitum* from the Term in which is z, will be exhibited by the Formula $\frac{1}{2z \times \overline{z+1}}$; and therefore the Sum of the intire Series is $\frac{1}{2 \times 1 \times 2} = \frac{1}{4}$.

In

In the Series B 'tis $n = 1$, $p = 5$, $q = 3$, the first Value of z being 1. Therefore $N = \overline{z+4} \times \overline{z+3} \times \overline{z+2} - z \times \overline{z+1} \times \overline{z+2}$ $= 6 \times \overline{z+2}^2$. But the continued Values of $z + 2$ are 3, 6, 9, &c. which because they are all divisible by 3, making $z + 2 = 3x$, it will be $N = 6 \times \overline{3x}^2 = 6 \times 9x^2 = 54x^2$, the continued Values of x being 1, 2, 3, &c. Therefore the given Number 54 being rejected, hence proceed the Numerators $1, 2^2, 3^2$, &c. that is, 1, 4, 9, &c. Also the integral Formula is $\frac{1}{z \times \overline{z+1}}$, wherefore taking Account of the Number 54 rejected out of the Numerators, the Sum of the Series continued *in infinitum* from the Term in which is z, will be $\frac{1}{54z \times \overline{z+1}}$. Whence the Sum of the intire Series is $\frac{1}{108}$.

Lastly, in the Series C 'tis $n = 1$, $p = 5$, $q = 4$, and the first Value of z is 1. Whence $N = \overline{z+4} \times \overline{z+3} \times \overline{z+2} \times \overline{z+1} - z \times \overline{z+1}$ $\times \overline{z+2} \times \overline{z+3} = 4 \times \overline{z+1} \times \overline{z+2} \times \overline{z+3}$. But the Values of N arising by this Formula can always be divided by $4 \times 2 \times 3 \times 4 = 96$. Therefore this Divisor being rejected, there come out the Numerators 1, 14, 55, 140, &c. And the Formula of the Sum, admitting the Number 96, is $\frac{1}{96z}$. And therefore the Sum of the intire Series is $\frac{1}{96}$.

Scholium. By these two last Propositions we may easily find as many summable Series as we please. And on the contrary, having a Series given of this Kind, if it can be sum'd its Sum may generally be reduced to one of these two Propositions. Yet there is Need of good Sagacity in the Tryal. But it proceeds best if the Terms of the given Series are reduced to the Form of *Prop.* III. Thus for Instance, having this Series proposed $\frac{7}{3 \cdot 5 \cdot 7 \cdot 9 \cdot 11} + \frac{11}{7 \cdot 9 \cdot 11 \cdot 13 \cdot 15} + \frac{15}{11 \cdot 13 \cdot 15 \cdot 17 \cdot 19}$, &c. the Denominators may thus be written, $3 \cdot 7 \cdot 11 \times 5 \cdot 9$, $7 \cdot 11 \cdot 15 \times 9 \cdot 13$, $11 \cdot 15 \cdot 19 \times 13 \cdot 17$ &c. whence according to *Prop.* 3. 'tis $n = 4$, $m = 4$, $p = 2$, $q = 1$, the first Value of z is 3, the first Value of u is 5. Hence the Form of the Numerator is found to be $4 \times \overline{z + 2u + 8}$. But this is always divisible by 3; wherefore

wherefore rejecting the given Divisors 4 and 3, by this Formula the Numerators come forth, 7, 11, 15, &c. the same as the Numerators in the proposed Series, which therefore may be sum'd by that Proposition.

2. After I had communicated those Series *A*, *B*, *C*, to Dr. *Taylor*, he wrote me Word, that he had found their Sums, of the first *A* and the third *C* by reducing them to simple Cases of his Method of Increments; that he had reduced the third *C* to this Form

$$\frac{1}{24} \times \overline{\frac{1}{1\cdot5} + \frac{1}{5\cdot9} + \frac{1}{9\cdot13} + \frac{1}{13\cdot17}},\ \&c.$$ that the Sum might be had by the Precepts delivered in the *Scholium* of *Prop.* 1. But in the second Series *B*, when this did not succeed so well, he used the following Analysis, which, because of its great Elegance, having first obtained his Leave, I shall here insert. "The Term of that Series (to use his own Notation) is exhibited by this Formula

$$\frac{\overline{z+2}\times\underset{.}{z}}{27z\times\overline{z+1}\times\underset{,}{z}\times\overline{\underset{,}{z}+1}}$$; writing in the Denominator $\underset{,}{z}$ for $z+3$, because it is $\underset{.}{z}=3$. Suppose $\frac{B}{27C}$ to be equal to the Integral required, that is $\frac{B}{C}$ is the Integral of $\frac{\overline{z+2}\times\underset{.}{z}}{z\times\overline{z+1}\times\underset{,}{z}\times\overline{\underset{,}{z}+1}}$, setting aside the given Divisor 27. But the Increment of $\frac{B}{C}$ is $\frac{\underset{.}{B}C-B\underset{.}{C}}{C\underset{,}{C}}$. Therefore $\frac{\underset{.}{B}C-B\underset{.}{C}}{C\underset{,}{C}}$ and $\frac{\overline{z+2}\times\underset{.}{z}}{z\times\overline{z+1}\times\underset{,}{z}\times\overline{\underset{,}{z}+1}}$ ought to be the same. Then comparing the Denominators, it is found that $C=z\times\overline{z+1}$. And taking the Increments it becomes $\underset{.}{C}=2z\underset{.}{z}+\underset{.}{z}^2+\underset{.}{z}$ $(=2z\underset{.}{z}+4\underset{.}{z})$ because it is $\underset{.}{z}=3$.) These Values being substituted in the Place of C and $\underset{.}{C}$, there arises $\underset{.}{B}C-B\underset{.}{C}=\overline{zz+2}\,\underset{.}{B}-2\underset{.}{z}\times\overline{z+2}\,B$, which ought to be the same as $\overline{z+2}\times\underset{.}{z}$. Let $B=a+v$, a being the invariable Part of B, and v the variable Part. Then taking the Increments 'tis $\underset{.}{B}=\underset{.}{v}$. Whence to find a and v we have the Equation

$\overline{zz+z\underset{.}{v}} - 2\underset{.}{z}\times\overline{z+2}\times\overline{a+v} = \overline{z+2}\times z$, which may thus be written $\overline{zz+z\underset{.}{v}} - 2\underset{.}{z}\times\overline{z+2}\,v = \underset{.}{z}\times\overline{z+2}\times\overline{1+2a}$, or also $C\underset{.}{v}$ $-\underset{.}{C}v = \underset{.}{z}\times\overline{z+2}\times\overline{1+2a}$. Make $1+2a=0$, (whence $a = \frac{-1}{2}$,) and it becomes $C\underset{.}{v} - \underset{.}{C}v = 0$, in which it may be $v=0$, (because each Term of the Equation is affected either by v or $\underset{.}{v}$,) hence it is $B = a = \frac{-1}{2}$, and therefore $\frac{B}{C} = \frac{-1}{2z\times\overline{z+1}}$. Then restoring the Divisor 27, the Integral required will be $\frac{-1}{54\times z\times\overline{z+1}}$. Now by comparing the Equation $C\underset{.}{v} - \underset{.}{C}v = 0$ with the general Formula $\frac{\underset{.}{B}C - B\underset{.}{C}}{CC_{,}} = 0$, we may thence conclude, that $\frac{v}{C}$ is equal to a given Quantity, because its Increment is 0. So that assuming n for any given Number, it will be $v = nC$, and $B = -\frac{1}{2} + nC$. By which the Integral required becomes $\frac{B}{C} = \frac{-\frac{1}{2}+nC}{C} = \frac{-1}{2C} + n$; which differs from the Integral found before only by the given Quantity n. This proceeds from hence, that as in the Quadrature of Curves the Area when found may be increased or diminished by a given Area, so in the Method of Increments, the Integral when found may be increased or diminished by a given Quantity. But by the first Integral, where n is absent, the Sum of the Series is exhibited when continued *ad infinitum*.

Prop. 5. When z increases by Units, and a, b, c, &c. are given Quantities, none of which are equal to one another, to find the Integral of $\frac{1}{z\times\overline{z+a}\times\overline{z+b}\times\overline{z+c}\times \&c.}$

Solution. Multiplying both the Numerator and Denominator of the Fraction into the Terms $z+1$, $z+2$, &c. $z+a+1$, $z+a+2$, &c.

z

$z+b+1$, $z+b+2$, &c. $z+c+1$, $z+c+2$, &c. deficient in the Denominator, let the Denominator be reduced to the Form $z\times\overline{z+1}\times\overline{z+2}$, &c. of the Denominator in *Prop.* 1. *Schol. n.* 3. Then let the Numerator be reduced to the Form $A+Bz+Cz\times\overline{z+1}+Dz\times\overline{z+1}\times\overline{z+2}$, &c. Then applying the Terms to the new Denominator $z\times\overline{z+1}\times\overline{z+2}$, &c. let the Fraction be reduced to this Form

$$\frac{A}{z\times\overline{z+1},\ \&c.}+\frac{B}{\overline{z+1}\times\overline{z+2},\ \&c.}+\frac{C}{\overline{z+2}\times\overline{z+3},\ \&c.}$$

$$+\frac{D}{\overline{z+3}\times\overline{z+4},\ \&c.},$$ &c. whence lastly let the Integral be sought by *Schol. Prop.* 1. *n.* 3.

The Reason of the Solution is manifest of itself.

Scholium 1. The whole Difficulty of this Solution lies in the Reduction of the Numerator to the Form required, which yet how it may be done will appear from one Example. Therefore let the Product $\overline{z+2}\times\overline{z+3}\times\overline{z+7}$ be proposed to be reduced to the required Form. Therefore I evolve the Terms by Degrees as follows. The first Factor $z+2$ I thus write $2+z$, whose first Term 2 I multiply into $3+x$, whence it becomes $6+2x$. The second Term z I multiply by $2+\overline{z+1}$ $(=z+3)$ whence it is $2z+z\times\overline{z+1}$. Then collecting the Products it is $\overline{z+2}\times\overline{z+3}=\begin{matrix}6+2z\\+2z\end{matrix}+z\times\overline{z+1}=6+4z+z\times\overline{z+1}$. It remains to multiply this into $z+7$. Therefore I multiply the first Term 6 into $7+z\ (=z+7)$ whence it is $42+6z$. I multiply the second Term $4z$ into $6+\overline{z+1}\ (=z+7)$ whence it is $24z+4z\times\overline{z+1}$; the third Term $z\times\overline{z+1}$ I multiply into $5+\overline{z+2}\ (=z+7)$ whence it is $5z\times\overline{z+1}+z\times\overline{z+1}\times\overline{z+2}$. Therefore the Products being collected together as before 'tis $\overline{z+2}\times\overline{z+3}\times\overline{z+4}=42+30z+9z\times\overline{z+1}+z\times\overline{z+1}\times\overline{z+2}$. And so we may proceed in other Cases.

2. Let us take an Example of the Proposition in the Fraction

$\frac{1}{z\times\overline{z+2}\times\overline{z+5}}$. By restoring the Factors $\overline{z+1}\times\overline{z+3}\times\overline{z+4}$,

which

which are deficient in the Denominator, the Fraction becomes

$$\frac{\overline{z+1}\times\overline{z+3}\times\overline{z+4}}{z\times\overline{z+1}\times\overline{z+2}\times\overline{z+3}\times\overline{z+4}\times\overline{z+5}}.$$

Therefore the Numerator is to be reduced to the Form required. Now by the Method already delivered, 'tis first $\overline{z+1}\times\overline{z+3}=1\times\overline{3+z}+z\times\overline{2+\overline{z+1}}$ $=3+\overline{z+2}\,z+z\times\overline{z+1}=3+3z+z\times\overline{z+1}$. Then $\overline{z+1}$ $\times\overline{z+3}\times\overline{z+4}=3\times\overline{4+z}+3z\times\overline{3+\overline{z+1}}+z\times\overline{z+1}$ $\times\overline{2+\overline{z+2}}=12+\overline{3z+9z+3z\times\overline{z+1}+2z\times\overline{z+1}+z\times}$ $\overline{z+1}\times\overline{z+2}=12+12z+5z\times\overline{z+1}+z\times\overline{z+1}\times\overline{z+2}$. By applyiug this Product to the Denominator $z\times\overline{z+1}\times$ &c. $z+5$, the Fraction will be reduced at last to this Form

$$\frac{12}{z\times\overline{z+1}\times\overline{z+2}\times\overline{z+3}\times\overline{z+4}\times\overline{z+5}}+$$

$$\frac{12}{\overline{z+1}\times\overline{z+2}\times\overline{z+3}\times\overline{z+4}\times\overline{z+5}}+$$

$$\frac{5}{\overline{z+2}\times\overline{z+3}\times\overline{z+4}\times\overline{z+5}}+\frac{1}{\overline{z+3}\times\overline{z+4}\times\overline{z+5}},$$

the Integral of which is $\dfrac{-12}{5z\times\overline{z+1}\times\overline{z+2}\times\overline{z+3}\times\overline{z+4}}+$

$$\frac{-12}{4\times\overline{z+1}\times\overline{z+2}\times\overline{z+3}\times\overline{z+4}}+\frac{-5}{3\times\overline{z+2}\times\overline{z+3}\times\overline{z+4}}+$$

$$\frac{-1}{2\times\overline{z+3}\times\overline{z+4}}.$$

3. When there are two Factors only z and $z+a$, the Integral will also be exhibited by the Formula $-\frac{1}{2}-\frac{1-a}{2z\times\overline{z+1}}-\frac{\overline{1-a}\times\overline{2-a}}{3z\times\overline{z+1}\times\overline{z+2}}-\frac{\overline{1-a}\times\overline{2-a}\times\overline{3-a}}{4z\times\overline{z+1}\times\overline{z+2}\times\overline{z+3}}$, &c. where the Series is to be continued till it breaks off by the vanishing of the Terms. If there are two Factors z and $z-a$, the Integral will be exhibited by the Formula $\frac{-1}{z-1}-\frac{-1+a}{2\times\overline{z-1}\times\overline{z-2}}-\frac{\overline{-1+a}\times\overline{-2+a}}{3\times\overline{z-1}\times\overline{z-2}\times\overline{z-3}}$, &c. The same Integral may be express'd either Way, according as the Factor of the given Fraction is taken either less or greater than z.

4. If the first Value of z is $a+1$, the latter Formula will be changed into this $\frac{-1}{a}\times\frac{1}{1}\times\frac{1}{2}\times\frac{1}{3}$, &c. to $\frac{1}{a}$ inclusive, by which with a contrary Sign is exhibited the Sum of this Series continued *in infinitum*, $\frac{1}{1\times\overline{1+a}}+\frac{1}{2\times\overline{2+a}}+\frac{1}{3\times\overline{3+a}}$, &c. For Instance, let $a=1$, the Series will be $\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}$, &c. $\times\frac{1}{1}\times\frac{1}{1}=1$. If $a=2$, the Series will be $\frac{1}{1\times3}+\frac{1}{2\times4}+\frac{1}{3\times5}$ &c. $=\frac{1}{2}\times\overline{\frac{1}{1}+\frac{1}{2}}=\frac{3}{4}$. If $a=3$, the Series will be $\frac{1}{1\times4}+\frac{1}{2\times5}+\frac{1}{3\times6}+\frac{1}{4\times7}$, &c. $=\frac{1}{3}\times\frac{1}{1}+\frac{1}{2}+\frac{1}{3}=\frac{11}{18}$.

5. From

5. From the ſame Series $\frac{1}{1 \times \overline{1+a}} + \frac{1}{2 \times \overline{2+a}} + \frac{1}{3 \times \overline{3+a}}$, &c. according to the different Values of *a*, ſeveral Series will ariſe, which in Form will be elegant enough. To ſet ſome of theſe before the Eye of the Reader, perhaps will not be unacceptable.

If the even Numbers 2, 4, 6, 8, &c. are made ſucceſſively equal to *a*, the Series will be as follows.

If $a = 2)\ \frac{1}{1 \times \overline{1+2}} + \frac{1}{2 \times \overline{2+2}} + \frac{1}{3 \times \overline{3+2}} + \frac{1}{4 \times \overline{4+2}}$, &c.

$4)\ \frac{1}{1 \times \overline{1+4}} + \frac{1}{2 \times \overline{2+4}} + \frac{1}{3 \times \overline{3+4}} + \frac{1}{4 \times \overline{4+4}}$, &c.

$6)\ \frac{1}{1 \times \overline{1+6}} + \frac{1}{2 \times \overline{2+6}} + \frac{1}{3 \times \overline{3+6}} + \frac{1}{4 \times \overline{4+6}}$, &c.

$8)\ \frac{1}{1 \times \overline{1+8}} + \frac{1}{2 \times \overline{2+8}} + \frac{1}{3 \times \overline{3+8}} + \frac{1}{4 \times \overline{4+8}}$, &c.

Or $\frac{1}{4-1} + \frac{1}{9-1} + \frac{1}{16-1} + \frac{1}{25-1}$, &c.

$\frac{1}{9-4} + \frac{1}{16-4} + \frac{1}{25-4} + \frac{1}{36-4}$, &c.

$\frac{1}{16-9} + \frac{1}{25-9} + \frac{1}{36-9} + \frac{1}{49-9}$, &c.

$\frac{1}{25-16} + \frac{1}{36-16} + \frac{1}{49-16} + \frac{1}{64-16}$, &c.

Or $\frac{1}{4-1} + \frac{1}{9-1} + \frac{1}{16-1} + \frac{1}{25-1}$, &c.

$$\frac{1}{4+1}+\frac{1}{9+3}+\frac{1}{16+5}+\frac{1}{25+7},\ \&c.$$

$$\frac{1}{4+3}+\frac{1}{9+7}+\frac{1}{16+11}+\frac{1}{25+15},\ \&c.$$

$$\frac{1}{4+5}+\frac{1}{9+11}+\frac{1}{16+17}+\frac{1}{25+23},\ \&c.$$

If for a are taken the odd Numbers successively, 1, 3, 5, 7, &c. the Series will be

$$a=1)\ \frac{1}{1\times\overline{1+1}}+\frac{1}{2\times\overline{2+1}}+\frac{1}{3\times\overline{3+1}}+\frac{1}{4\times\overline{4+1}},\ \&c.$$

$$3)\ \frac{1}{1\times\overline{1+3}}+\frac{1}{2\times\overline{2+3}}+\frac{1}{3\times\overline{3+3}}+\frac{1}{4\times\overline{4+3}},\ \&c.$$

$$5)\ \frac{1}{1\times\overline{1+5}}+\frac{1}{2\times\overline{2+5}}+\frac{1}{3\times\overline{3+5}}+\frac{1}{4\times\overline{4+5}},\ \&c.$$

$$7)\ \frac{1}{1\times\overline{1+7}}+\frac{1}{2\times\overline{2+7}}+\frac{1}{3\times\overline{3+7}}+\frac{1}{4\times\overline{4+7}},\ \&c.$$

Or $\frac{1}{2}\times\frac{1}{1}+\frac{1}{3}+\frac{1}{6}+\frac{1}{10},\ \&c.$

$$\frac{1}{2}\times\frac{1}{3-1}+\frac{1}{6-1}+\frac{1}{10-1}+\frac{1}{15-1},\ \&c.$$

$$\frac{1}{2}\times\frac{1}{6-3}+\frac{1}{10-3}+\frac{1}{15-3}+\frac{1}{21-3},\ \&c.$$

$$\frac{1}{2}\times\frac{1}{10-6}+\frac{1}{15-6}+\frac{1}{21-6}+\frac{1}{28-6},\ \&c.$$

Or $\frac{1}{2}\times\frac{1}{1+0}+\frac{1}{3+0}+\frac{1}{6+0}+\frac{1}{10+0},\ \&c.$

$$\frac{1}{2} \times \frac{1}{1+1} + \frac{1}{3+2} + \frac{1}{6+3} + \frac{1}{10+4}, \text{ \&c.}$$

$$\frac{1}{2} \times \frac{1}{1+2} + \frac{1}{3+4} + \frac{1}{6+6} + \frac{1}{10+8}, \text{ \&c.}$$

$$\frac{1}{2} \times \frac{1}{1+3} + \frac{1}{3+6} + \frac{1}{6+9} + \frac{1}{10+12}, \text{ \&c.}$$

6. Some Years ago that great Geometrician Mr. *James Bernoulli* found the Sum of any Series, whoſe Numerators conſtitute a Series of Equals, and the Denominators were a Series of Squares diminiſh'd by any given Square *Q*, or a Series of Triangles diminiſh'd by any given Triangle *T*. This he found by obſerving, that ſuch Series ariſe, by taking away a truncated Series of harmonically Proportionals from the ſame Series when intire; that is, ſo that the Number of deficient Terms in the truncated Series may be either double of the Side of the given Square *Q*, or the double increaſed by Unity of the Side of the given Triangle *T*. He obſerved alſo, that the Sum of a reciprocal Series of Squares would be ſought after in vain. And the ſame is true alſo of the Reciprocals of Cubes, or of any other Powers of Numbers in Arithmetical Progreſſion. The Reaſon is, becauſe no Difference intercedes between the Factors of the Denominators, which is always required for ſuch Summations, as appears from the Method of taking the Differences explain'd in the *Scholium* of *Prop.* 1. For if the Sum required could be exhibited by any Formula, the Difference of that Formula would exhibit the Terms of the propoſed Series; but in ſuch a Difference the Denominator is always affected by the Factors which are different from one another; which becauſe it does not take Place in the aforeſaid Series, the Sums of ſuch Series cannot be had in finite Terms. Almoſt in the ſame Manner, by an Argument derived from *Prop.* 3, 4. it may be demonſtrated, that the Sums of Series cannot be exhibited in a finite Number of Terms, whoſe Numerators conſtitute a Series of Equals, but the Denominators conſiſt of a certain Number of Terms in Arithmetical Progreſſion, the greateſt Factor of every Term being leſs than the leaſt Factor in the Term next following, ſuch as is this Series $\frac{1}{1 \cdot 2} + \frac{1}{3 \cdot 4} + \frac{1}{5 \cdot 6} + \frac{1}{7 \cdot 8}$, *&c.*

7. Now I might give ſome Rules which I have contrived for certain ſingular Caſes; but this would lead us too far. It may ſuffice therefore to have explain'd the more general, and to take Notice at the ſame Time, that nothing would more conduce to the Improvement of this new Doctrine of Infinite Series, than if ſome very general Forms of Sums were digeſted in Order, from the Differences of which being computed by the

the Rule above, Canons might afterwards be form'd of summable Quantities; just as is already done in the Integral Calculus, or what Sir *I. Newton* calls the Inverse Method of Fluxions.

8. By restoring the Factors which are deficient in the Denominator, the present Problem might be reduced to *Prop.* 2. Also it might be proposed in more general Terms, or for the Numerator might any Formula be taken, of which any Difference is given. Yet with this Condition, that the Dimensions of the Denominator should exceed the Dimensions of the Numerator at least by two, for otherwise the Sum of the Series could not be had in finite Terms. Let there be an Example of this in the Series $\frac{1}{1\cdot3\cdot5\cdot7}+\frac{4}{2\cdot4\cdot6\cdot8}+\frac{9}{3\cdot5\cdot7\cdot9}+\frac{16}{4\cdot6\cdot8\cdot10}$, &c. where the Numerators are the Squares of the natural Numbers. Then applying both the Numerators and Denominators to the natural Numbers, the Series will be reduced to a more simple Form $\frac{1}{3\cdot5\cdot7}+\frac{2}{4\cdot6\cdot8}+\frac{3}{5\cdot7\cdot9}+\frac{4}{6\cdot8\cdot10}$, &c. The natural Numbers 1, 2, 3, 4, 5, &c. being denoted by p, a Term of the Series will be represented by the Form $\frac{p}{\overline{p+2}\times\overline{p+4}\times\overline{p+6}}$, or by the Formula $\frac{z-2}{z\times\overline{z+2}\times\overline{z+4}}$ writing z for $p+2$. Now in proceeding from Term to Term, because z is increased by Units, the deficient Factors $z+1$, $z+3$, are to be restored in the Denominator, and by this Means the Term of the Series will be reduced to this Form $\frac{\overline{z-2}\times\overline{z+1}\times\overline{z+3}}{z\times\overline{z+1}\times\overline{z+2}\times\overline{z+3}\times\overline{z+4}}$.
By the Method already explain'd in this Proposition, the Numerator is reduced to $-6-6z-z\times\overline{z+1}+z\times\overline{z+1}\times\overline{z+2}$. Whence having Respect to the Denominator, the Term is reduced to this Form

$$\frac{-6}{z\times\overline{z+1}.\,\&c.\times\overline{z+4}}+\frac{-6}{\overline{z+1}\times\overline{z+2}\times\overline{z+3}\times\overline{z+4}}+$$

$$\frac{-1}{\overline{z+2}\times\overline{z+3}\times\overline{z+4}}+\frac{1}{\overline{z+3}\times\overline{z+4}}.$$ Therefore by taking the Integral we shall find it

$$\frac{6}{4z\times\overline{z+1}\times\overline{z+2}\times\overline{z+3}}+\frac{6}{3\times\overline{z+1}\times\overline{z+2}\times\overline{z+3}}+$$

2 x

$\frac{1}{2\times\overline{z+2}\times\overline{z+3}} + \frac{-1}{z+3}$; by which with a contrary Sign will be exhibited the Sum of the Series continued *in infinitum*, beginning from the Term $\frac{z-2}{z\times\overline{z+2}\times\overline{z+4}}$. Therefore the Sum of the whole Series beginning from the Term $\frac{1}{3\cdot5\cdot7}$ will be $\frac{31}{240}$.

If we had a Mind to proceed by *Prop.* 2. from the Formula $\overline{z-2}\times\overline{z+1}\times\overline{z+3}$ the firſt Numerators being collected, 24, 70, 144, 252, taking their Differences we ſhould have $46=b$, $28=c$, $6=d$, $e=0$ $=$ &c. M being 24; whence by *Lem.* 2. the Formula $-6-6z-z\times\overline{z+1}+z\times\overline{z+1}\times\overline{z+2}$ would ariſe, by which the Term is denoted as above. And proceeding by *Prop.* 2. the Sum is had.

Prop. 6. *Prob.* To find the Sum of any Number of Terms of a Series of Fractions, whoſe Numerators and Denominators make any two tranſverſe Lines in *Paſchal*'s Arithmetical Triangle; that is, whoſe Generators are Units.

Solution. Let the Order of the Series of Numerators in the Arithmetical Triangle be denoted by n, and let p be the Difference between the Order of the Numerators and Denominators, and let the Number of the Terms whoſe Sum is required be denoted by q. Then if the Denominators are of more Dimenſions than the Numerators, the Sum will be exhibited by the firſt Formula following. But if the Dimenſions of the Numerators are more than thoſe of the Denominators, the Sum will be exhibited by the ſecond Formula.

Formula I.

$$\frac{n+p-1}{p-1} - \frac{n\,.\,\overline{n+1}\,.\,\overline{n+2}\,.\,\&c.\ \overline{n+p-1}}{\overline{p-1}\times\overline{n+q}\,.\,\overline{n+q+1}\,.\,\&c.\ \overline{n+q+p-2}}$$

Formula II.

$$-\frac{n-p-1}{p+1} + \frac{\overline{q+n-1}\,.\,\overline{q+n-2}\,.\,\&c.\ \overline{q+n-p-1}}{\overline{p+1}\times\overline{n-1}\,.\,\overline{n-2}\,.\,\&c.\ \overline{n-p}}$$

Ex. 1. Let it be propoſed to find the Aggregate of the ſix firſt Terms of the Series $\frac{1}{1}+\frac{4}{7}+\frac{10}{28}+\frac{20}{84}+\frac{35}{210}+\frac{56}{462}$, &c. where the

the Numerators constitute the fourth Line, and the Denominators the seventh, in the Arithmetical Triangle. Therefore it is $n=4$, $p=3$, $q=6$, and because the Dimensions of the Denominators exceed the Dimensions of the Numerators, the Sum will be given by the first Formula; that is, $\frac{4+3-1}{3-1} - \frac{4\cdot5\cdot6}{\overline{3-1}\times\overline{4+6}\times\overline{4+7}} = 3 - \frac{6}{11} = 2\frac{5}{11}$.

Ex. 2. Let the Sum be required of the six first Terms of this Series, $\frac{1}{1}+\frac{7}{4}+\frac{28}{10}+\frac{84}{20}+\frac{210}{35}+\frac{462}{56}$, &c. the Terms of which are the Reciprocals of the Terms of the foregoing Series. 'Tis therefore $n=7$, $p=3$, $q=6$, and therefore by the second Formula the Sum is $-\frac{3}{4}+\frac{12\cdot11\cdot10\cdot9}{4\times6\cdot5\cdot4}=24$.

Scholium 1. Two Years ago I communicated the Forms in this Proposition, to those learned Geometricians M. de *Moivre* and the *Bernoulli's*. They may easily be derived from the Precepts delivered in *Prop.* 1. We will take for Example the foregoing Series $\frac{1}{1}+\frac{4}{7}+\frac{10}{28}$, &c. The Place of the Term in this Series being denoted by p, the Term will be exhibited by the Formula $\frac{4\cdot5\cdot6}{\overline{p+3}\cdot\overline{p+4}\cdot\overline{p+5}}$; whence returning to the Integral, the Sum of the Series beginning from that Term will be exhibited by the Formula $\frac{4\cdot5\cdot6.}{2\times\overline{p+3}\times\overline{p+4}}$.

Therefore taking 1 for p, the whole Series is $\frac{4\cdot5\cdot6}{2\cdot4\cdot5}$, $=3$, and the Sum of the six first Terms will be $3-\frac{4\cdot5\cdot6}{2\cdot10\cdot11}$, just as it was now exhibited by the Formula.

2. In the first Formula the Sum of the Series continued *in infinitum* is $\frac{n+p-1}{p-1}$, the other Part of the Formula now vanishing. But in the

Case

Case of the second Formula, the Sum is an infinite Quantity, whose Species, in respect of the infinite Number q, is exhibited by the other Part of the Formula, which in this Case becomes $\frac{q^{p+1}}{\overline{p+1}\times\overline{n-1}.\overline{n-2}.\&c.\overline{n-p}}$.

3. Concerning Series of this Kind, that great Geometrician Mr. *Leibnitz*, in a Letter of *May* 1716, wrote to me in the following Manner: The Death of which great Man, lately taken from us, we now lament.

"It seems to me that heretofore I sum'd certain Series of this Kind, "such as $\frac{1}{1}+\frac{2}{4}+\frac{3}{10}+\frac{4}{20}+\frac{5}{35}+\frac{6}{56}$, &c. The Term of this "Series exprefs'd Analytically is

"$$\frac{x}{x.\overline{x+1}.\overline{x+2}\times\frac{1}{1}.\frac{1}{2}.\frac{1}{3}}=\frac{1.2.3}{\overline{x+1}.\overline{x+2}}=\frac{6}{xx+3x+2}.$$

"'Tis required to find the Sum of a given Series, one of the Terms of "which is $\frac{ll}{xx+3lx+2ll}$, where x signifies the natural Numbers "1, 2, 3, &c. and l signifies Unity, or the Difference of the several x. "Let us suppose that the Term required of the Summing Series is "$\frac{fx}{mx+nl}=\frac{\odot}{☽}$. But the Difference of $\frac{\odot}{☽}=\frac{-\odot}{☽}+\frac{\odot+d\odot}{☽+d☽}=$ "$\frac{☽d\odot-\odot d☽}{☽☽+☽d☽}$. But $d\odot=fdx$, and $d☽=mdx=ml$. Therefore "the Difference of $\frac{\odot}{☽}=\frac{nfll}{mmxx+2mnlx+nnll+mmlx+mnll}$. Now we must make

$$\frac{nfll}{mmxx+2mnlx+nnll+mmlx+mnll}=\frac{mfll}{mmxx+3mmlx+2mmll},$$ that is

"these two Forms must be identified, wherein the given Quantity is multi-"ply'd by $\frac{nf}{mm}$. Then making the respective Terms equal, since the "x's are the same, we shall have by simple x, $2n+m=3m$, that is to "say, $m=n$. Then by the absolute Terms $nn+mn=2mm$, which "is again $m=n$. Therefore the Identification succeeds, and we may "make $n=m=l=1$, and $f=1$, (for f may be taken at Pleasure) and

" the Term of the Summing Series will be $\frac{x}{x+1}$. For the Difference " of $\frac{x}{x+1}$ gives $-\frac{x}{x+1}+\frac{x+1}{x+2}=\frac{1}{xx+3x+2}$, and confe- " quently $\frac{6x}{x+1}$ gives the Sum of the $\frac{2}{x.\overline{x+1}.\overline{x+2}\times\frac{1}{1}.\frac{1}{2}.\frac{1}{3}}$, 3, " 4, $\frac{9}{2}$, $\frac{24}{5}$, 5, $\frac{36}{7}$, &c. the Summing Series whose Term is $\frac{6x}{x+1}$. " Also $\frac{1}{1}+\frac{2}{4}+\frac{3}{10}+\frac{4}{20}+\frac{5}{25}$, &c. is the Series to be sum'd, " whose Term is $\frac{x}{x.\overline{x+1}.\overline{x+2}.\times\frac{1}{1}\times\frac{1}{2}\times\frac{1}{3}}$. And to make Use " of them for Summations, the five Terms, for Example, of the given " Series shall be $\frac{36}{7}-3=\frac{15}{7}$. And in general, the Sum of " the Terms as far as any Term $\frac{x}{x.\overline{x+1}.\overline{x+2}\times\frac{1}{1}\times\frac{1}{2}\times\frac{1}{3}}$ exclu- " sively, shall be $\frac{6x}{x+1}-3$. And for the Sum of the intire Series " *ad infinitum*, x then becomes infinite, and therefore $\frac{6x}{x+1}=6$: " Therefore the Sum of the whole Series is $6-3=3$, as you have " found it.

" This Method is the Calculation of Differences apply'd to Numbers; " and I must own to you, that before I apply'd it to Figures, and even " before I commenced Geometrician, I practised it in some Measure " upon Numbers: Having found, when I was yet but young, that " Series whose Numerators are Units, and the Denominators any figu- " rate Numbers, as Triangular, Pyramidal, &c. were the first, second, " third, &c. Differences multiply'd by the constant Quantities of this " Series $\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}$, &c. and consequently were summable. " But after I became something of a Geometrician and an Analyst, I " saw it was possible to arrive at these Summations by a general Me- " thod, when it could be done; and that the Calculus of Differences " was still more convenient in Geometry than in Numbers, because there

" there are more frequent Coincidences, and that the Differences have " Place in the Tangents, as the Sums have in Quadratures. This ge- " neral Method of finding the Summing Series of the Series given, " when it is poſſible, always ſucceeds, when the Term of the Series " given expreſs'd Analytically has no variable Quantity involved in a " Radical Sign, nor entring in the Exponent ; and then one may al- " ways determine the Summing Series, or prove it impoſſible to be " found. And the Thing very often ſucceeds, even when the variable " Quantity enters the Exponent. But as there are ſometimes particular " Quadratures of ſome Portions of a Figure, when one cannot give the " general Quadrature, or the Quadratrix ; ſo ſometimes one may find " the Sum of the whole Series, or of a certain Part, though one cannot " find the Sum of every Part. Then we muſt have Recourſe to parti- " cular Methods, which we have not always in our Power, our Analy- " ſis being not yet carried to its due Perfection.

Prop. 7. *Prob.* To find the Sum of a Series, whoſe Numerators conſtitute any erect Line in *Paſchal*'s Arithmetical Triangle, and the Denominators conſtitute any tranſverſe Line.

Solution. Let p denote the Order of the erect Line, and q the Order of the Tranſverſe Line ; and let m be the Aggregate of ſo many of the firſt Terms in the erect Line of the Order $p + q - 1$ as there are Units in $q - 1$; and the Sum required will be $\overline{2^{p+q-2} - m} \times$

$$\frac{1 \cdot 2 \cdot 3 \cdot \&c. \overline{q - 1}}{p \cdot \overline{p + 1} \cdot \&c. \overline{p + q - 2}}.$$

Ex 1. Let this Series be propoſed $\frac{1}{1} + \frac{5}{4} + \frac{10}{10} + \frac{10}{20} + \frac{5}{35} + \frac{1}{56}$, where the Numerators conſtitute the ſixth erect Line, and the Denominators poſſeſs the fourth tranſverſe Line. Therefore in this Caſe 'tis $p = 6$, $q = 4$, $p + q - 1 = 9$, $q - 1 = 3$, and therefore $m = 1 + 8 + 28 = 37$, that is, equal to the three firſt Terms of the ninth erect Line. Whence the Sum required will be $\overline{2^{8} - 37} \times \frac{1 \cdot 2 \cdot 3}{6 \cdot 7 \cdot 8} = \frac{219}{56}$.

Ex. 2. Let the Numerators conſtitute the hundredth erect Line, and let the Denominators be the Trigonal Numbers, which poſſeſs the third tranſverſe Line. Then it will be $p = 100$, $q = 3$, $m = 102$, and therefore the Sum required is $\overline{2^{101} - 102} \times \frac{1 \cdot 2}{100 \cdot 101}$.

Cor. If $q = 2$, the Formula becomes $\frac{2^{p} - 1}{p}$, by which the Aggregate will be exhibited, of the whole firſt Term, with half the ſecond, a third Part of the third, a fourth Part of the fourth, and ſo on, of any erect Line of the Order p of *Paſchal*'s Arithmetical Triangle. Thus for Inſtance, $\frac{1}{1}+\frac{5}{2}+\frac{10}{3}+\frac{10}{4}+\frac{5}{5}+\frac{1}{6}=\frac{2^6-1}{6}=10\frac{1}{2}$.

Prop. 8. *Prob.* To find the Sum of the ſame Series, when the Signs of the Terms are alternately + and —.

Solution. The Sum required will be exhibited by this very ſimple Formula, $\frac{q-1}{p+q-2}$.

Ex. Let it be propoſed to find the Sum of this Series, $\frac{1}{1}-\frac{6}{9}+\frac{15}{45}-\frac{20}{165}+\frac{15}{495}-\frac{6}{1287}+\frac{1}{3003}$, where the Numerators conſtitute the ſeventh erect Line, and the Denominators the ninth tranſverſe Line. Therefore in the Formula for p and q writing 7 and 9, and the Sum will be $\frac{8}{14}=\frac{4}{7}$.

The ſame Series of Numerators remaining, (that is, the ſeventh erect Line) if for the Series of Denominators be taken the ſecond, third, fourth, &c. tranſverſe Lines ſucceſſively, the Sums will be $\frac{1}{7}$, $\frac{2}{8}$, $\frac{3}{9}$, $\frac{4}{10}$, $\frac{5}{11}$, &c. which may be wrote thus, $\frac{1}{7}$, $\frac{7}{28}$, $\frac{28}{84}$, $\frac{84}{210}$, $\frac{210}{462}$, &c. where both the Numerators and Denominators are taken out of the tranſverſe Line of the ſeventh Order. The ſame would obtain if inſtead of the ſeventh, the Numerators ſhould conſtitute any other erect Line of the Order p. For the Sums would ariſe from the Application of the Terms of the tranſverſe Line of the ſame Order p to the Terms next following in the ſame Line.

Theſe two laſt Propoſitions are rather neat than uſeful; therefore we will leave the Demonſtration of our Formulæ, to be found by the Ingenuity of the Reader, and betake ourſelves now to the laſt Propoſition, which contains a third Species of Series, remarkable enough for its great Uſe.

Lemma 5. Let there be any Series $\frac{M}{b}$, $\frac{N}{b^2}$, $\frac{O}{b^3}$, $\frac{P}{b^4}$, &c. the Denominators of whoſe Terms conſtitute any Geometrical Progreſſion, b, b^2,

b^2, b^3, b^4, &c. Alſo let the firſt of the Numerators $A\,(=M)$ be the firſt of the firſt Differences B, the firſt of the ſecond C, the firſt of the third D, of the fourth E, and ſo on. And let $\frac{\alpha}{b}$, $\frac{\beta}{b^2}$, $\frac{\gamma}{b^3}$, $\frac{\delta}{b^4}$, &c. reſpectively be the Aggregates of one, two, three, four, or more Terms of the Series $\frac{M}{b}$, $\frac{N}{b^2}$, $\frac{O}{b^3}$, &c. and let the firſt of the Numerators be $a\,(=\alpha)$ the firſt of the firſt Differences be b, the firſt of the ſecond c, the firſt of the third d, and ſo on. And let $b-1=q$. Then the Values of a, b, c, d, &c. will be

$$\begin{aligned}
a &= A=\alpha=M\\
b &= bA+B\\
c &= qbA+bB+C\\
d &= q^2bA+qbB+qbC+D, \text{ and so on.}
\end{aligned}$$

Demonſtration. It is plain that $a=\alpha=A=M$.

The Terms $\frac{M}{b}$, $\frac{N}{b^2}$. $\frac{O}{b^3}$, $\frac{P}{b^4}$, &c. (the Numerators M, N, O, P, &c. being expreſs'd by A, B, C, D, &c.) are transform'd into the Terms $\frac{A}{b}$, $\frac{A+B}{b^2}$, $\frac{A+2B+C}{b^3}$, $\frac{A+3B+3C+D}{b^4}$, &c.

Whence by collecting the Sums of the Terms are found the Numerators α, β, γ, δ, &c. that is,

$$\begin{aligned}
\alpha &= A\\
\beta &= \overline{b+1}\,A+B\\
\gamma &= \overline{b^2+b+1}\,A+\overline{b+2}\,B+C\\
\delta &= \overline{b^3+b^2+b+1}\,A+\overline{b^2+2b+3}\,B+\overline{b+3}\,C+D, \text{ \&c.}
\end{aligned}$$

Whence taking the Differences, they become

$$\begin{aligned}
b &= bA+B\\
c &= qbA+bB+C\\
d &= qqbA+qbB+bC+D, \text{ and so on, as in the Propoſition.}
\end{aligned}$$

Cor. 1. Of the Numerators M, N, O, P, &c. if the firſt, ſecond, third, or any other Difference is given; in the Series A, B, C, D, &c. all the Terms after ſome of the firſt vaniſhing, the Differences b, c, d, &c. will at laſt become a Geometrical Progreſſion in the Ratio of 1 to q. For Example, of the Numerators M, N, O, P, &c. if the firſt Difference B is given, then c, d, &c. will be in the continued Geometrical Ratio of 1 to q; as appears by their Values $qbA+bB$, $qqbA+qbB$, &c. it being $C=0=D$, &c.

Cor. 2.

Cor. 2. But the Order of the first of the Differences B, C, D, &c. which vanish in this Manner, is the same as the Order of the Differences b or c, &c. whence that Geometrical Progression begins. Thus if $B = 0 = C$, &c. then b, c, d, &c. will be in a Geometrical Progression. If $C = 0 = D$, &c. then c, d, &c. will be in a Geometrical Progression. And so on.

Lemma 6. The same Things being supposed, let r be that Term by which the Geometrical Progression begins in the Series of Differences b, c, d, &c. and by $p + 1$ let the Order in the Term be denoted in the Series $\frac{\alpha}{h}$, $\frac{\beta}{h^2}$, $\frac{\gamma}{h^3}$, $\frac{\delta}{h^4}$, &c. Then that Term will be denoted by a Fraction, the Denominator of whieh being h^{p+1}, the Numerator is

$$\frac{a + bp + cp \times \frac{p-1}{2} + dp \times \frac{p-1}{2} \times \frac{p-2}{3}, \text{ \&c.} + \frac{r}{q^n}}{\times h^p - 1 - qp - q^2 p \times \frac{p-1}{2} - q^3 p \times \frac{p-1}{2} \times \frac{p-2}{3}, \text{ \&c.}}$$

the Order of the vanishing Difference of the Series B, C, D, &c. being denoted by n, as also the Number of the Terms $a + bp$, &c. and likewise of the Terms $- 1 - qp$, &c.

Demonstration. By *Lemma* 1, the Numerator of that Term is exhibited by the Formula $a + bp + cp \cdot \frac{p-1}{2} + dp \times \frac{p-1}{2} \times \frac{p-2}{3}$, &c. (if $p + 1$ supplies the Place of x in that *Lemma.*)

Therefore if it is for Example $n = 2$, by *Lem.* 5. *Cor.* 2. it will be c, d, &c. in the continued Ratio of 1 to q. Therefore the Numerator in this Case is $a + bp + cp \times \frac{p-1}{2} + cqp \times \frac{p-1}{2} \times \frac{p-2}{3} + cq^2 p \times \frac{p-1}{2} \times \frac{p-2}{3} \times \frac{p-3}{4}$, &c. But if the Terms $cp \times \frac{p-1}{2} + cqp \times \frac{p-1}{2} \times \frac{p-2}{3}$, &c. are multiplied by $\frac{q^2}{c}$, and to the Product be added the Terms $1 + qp$, a Series will arise, by which the Dignity $\overline{1+q}|^p = h^p$ of the Binomial $1 + q$ is express'd. Therefore that Product is equal to $h^p - 1 - qp$; and therefore the Terms $cp \times \frac{p-1}{2} + cqp \times \frac{p-1}{2} \times \frac{p-2}{3}$, &c. $= \frac{c}{q^2} \times \overline{h^p - 1 - qp}$, by which the Numerator becomes

comes $a + bp + \frac{c}{qq} \times \overline{h^p - 1 - qp}$; the two Terms being $a + bp$, as alſo the two $- 1 - qp$, according to the Senſe of the Propoſition, becauſe $n = 2$. And there is the ſame Demonſtration in other Caſes. As to the Denominator, the Thing is manifeſt of itſelf.

Prop. 9. *Prob.* To find the Sum of any Number of Terms of any Series $\frac{M}{h}$, $\frac{N}{h^2}$, $\frac{O}{h^3}$, $\frac{P}{h^4}$, *&c.* of the Terms of which the Denominators conſtitute any Geometrical Progreſſion h, h^2, h^3, *&c.* and the Numerators are Quantities having any conſtant Difference.

Solution. Of the Numerators $M, N, O, P,$ *&c.* let the firſt be A, the firſt of the firſt Differences B, the firſt of the ſecond C, the firſt of the third D, and ſo on. And of $A, B, C, D,$ *&c.* let the Number be n, and $h - 1 = q$. Then let $a = A (= M)$ $b = hA + B$, $c = qhA + hB + C$, $d = q^2 hA + qhB + hC + D$, *&c.* that there may be ſo many Terms, $a, b, c,$ *&c.* as there are Units in $n + 1$. Let the laſt of thoſe Terms be called r, and by $p + 1$ let the Number of Terms $\frac{M}{h}$, $\frac{N}{h^2}$, $\frac{O}{h^3}$, $\frac{P}{h^4}$, be denoted, whoſe Sum is required. I ſay that Sum will be exhibited by a Fraction, whoſe Denominator being h^{p+1}, the Numerator will be

$$\overline{a + bp + cp \times \frac{p-1}{2} + dp \times \frac{p-1}{2} \times \frac{p-2}{3}, \&c. + \frac{r}{q^n}}$$
$$\overline{\times h^p - 1 - qp - q^2 p \times \frac{p-1}{2} - q^3 p \times \frac{p-1}{2} \times \frac{p-2}{3}, \&c.}$$
$$- q^{n-1} p \times \frac{p-1}{2}, \&c.$$

Demonſtration. For (by *Lem.* 6.) by this Formula is repreſented the $p + 1$ Term in order of the Series $\frac{\alpha}{h}$, $\frac{\beta}{h^2}$, $\frac{\gamma}{h^3}$, $\frac{\delta}{h^4}$, *&c.* which Term (by the Conſtruction of *Lem.* 5.) is equal to the Aggregate of Terms in Number $r + 1$ of the propoſed Series $\frac{M}{h}$, $\frac{N}{h^2}$, $\frac{O}{h^3}$, $\frac{P}{h^4}$, *&c.* *Q. E. D.*

Ex. 1. Let there be found the Sum of nine Terms of the Series $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{8}$, $\frac{4}{16}$, *&c.* In this Caſe 'tis $h = 2$, $q = h - 1 = 1$, $p + 1 = 9$, $p = 8$, $A = 1$, $B = 1$, $C = 0 = D$, *&c.* and therefore $n = 2$, (becauſe there are two, A, B,) Hence it is $a = A = 1$, $b = hA + B = 2 \times$

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$1+1=3$, $c=qbA+bB+C=2\times1+2\times1+0=4=r$; and therefore by the Formula the Sum required is

$$\frac{1+3\times8+\frac{4}{1^2}\times\overline{2^8-1-1\times8}}{2^9}=\frac{1013}{512}.$$

Ex. 2. Let the Sum of fix Terms be required of this Series $1\times3+3\times3^2+6\times3^3+10\times3^4+15\times3^5+21\times3^6$, *&c.* In this Cafe 'tis

$b=\frac{1}{3}$, $q=\frac{-2}{3}$, $p+1=6$, $p=5$, $A=1$, $B=2$, $C=1$, $D=0$ $=E$, *&c.* and therefore $n=3$, and $a=1$, $b=\frac{1}{3}+2=\frac{7}{3}$, $c=\frac{-2}{9}+\frac{2}{3}+1=\frac{13}{9}$, $d=\frac{4}{27}-\frac{4}{9}+\frac{1}{3}=\frac{1}{27}=r$. Whence the Sum required is 19956, or

$$\frac{1+\frac{7}{3}\times5+\frac{13}{9}\times5\times\frac{4}{2}+\frac{-1}{8}\times\overline{\frac{1}{3^5}-1+\frac{2}{3}\times5-\frac{4}{9}\times5\times\frac{4}{2}}}{\overline{\frac{1}{3}}\big)^6}.$$

Corol. 1. The Sum of the fame Series, continued from the firft Term $\frac{M}{b}$ *in infinitum*, is exhibited by a very fimple Formula $\frac{A}{b-1}+\frac{B}{\overline{b-1}\big)^2}+\frac{C}{\overline{b-1}\big)^3}+\frac{D}{\overline{b-1}\big)^4}$, *&c.*

Corol. 2. If $b=2$, the Sum of the whole Series continued infinitely is had by the Addition only of the Terms A, B, C, D, *&c.* And this Sum is the fame as the Sum of the erect Line anfwering to the firft Term A in the Arithmetical Triangle, whofe tranfverfe Line is occupied by the Numerators M, N, O, P, *&c.* which eafily appears from the Confideration of the Triangle. If therefore M, N, O, *&c.* are figurate Numbers of any Order n, the Sum of the Series $\frac{M}{2}+\frac{N}{4}+\frac{O}{8}+\frac{P}{16}$, *&c.* will be equal to this Dignity of the Number 2, that is $\overline{2}\big)^{n-1}$. Thus the

the Series $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}$, &c. $2^{1-1}=1$, as is commonly known. The Series $\frac{1}{2}+\frac{2}{4}+\frac{3}{8}+\frac{4}{16}$, &c. $=2^{2-1}=2$; and the Series $\frac{1}{2}+\frac{3}{4}+\frac{6}{8}+\frac{10}{16}$ &c. $=2^{3-1}=2^2=4$. And so on.

Scholium. The celebrated Mr. *James Bernoulli*, in his Treatise about infinite Series, has solved this Problem. " To find the Sum of an in-" finite Series of Fractions, whose Denominators increase in any Geo-" metrical Progression, but the Numerators proceed either according " to the natural Numbers 1, 2, 3, 4, &c. or Trigonals 1, 3, 6, 10, " &c. or Pyramidals 1, 4, 10, 20, &c. or according to Squares 1, 4, " 9, 16, &c. or Cubes 1, 8, 27, 64, &c. or the Multiples of these. The Reader may consult his Solution. Mr. *Nic. Bernoulli* his Nephew found another Solution much more general, and was pleas'd to communicate it to me in a Letter of *September* 18, 1715, after I had sent him this, but without a Demonstration. His Letter was full of admirable Discoveries, such as that learned Gentleman often imparts to me. Concerning this Problem he writes thus. As to the Sum of any determinate Number n of Terms of the Series of your seventh Theorem, (the first Corollary of this Proposition) I have found this Formula $\frac{1}{m^n}\times$

$$\frac{n-1}{m-1}a+\frac{A-n}{m-1}b+\frac{B-n\,.\,\frac{n-1}{2}}{m-1}c+\frac{C-n\,.\,\frac{n-1}{2}\,.\,\frac{n-2}{3}}{m-1},\ d\ \&c.$$

where the Letters A, B, C, &c. denote the Coefficients of the Terms immediately preceding. And in this Formula putting $p+1$ for n, b^m for m, and multiplying the whole by e^{m-1}, we shall have the Solution of your *Prob.* 9. And this able Mathematician acquaints me, that this his general Formula will be changed into our particular one, (*Corol.* 1. of this *Proposition*) when $n=\infty$. For then $1, n, n, \frac{n-1}{2}, n\,.\,\frac{n-1}{2}\,.\,\frac{n-2}{3}$, &c. will vanish in respect of the Terms m^n, A, B, C, &c. so that the Series in that Case will be $\frac{1}{m-1}a+\frac{A}{m-1}b+\frac{B}{m-1}c$, &c. which intirely coincides with ours $\frac{a}{m-1}+\frac{b}{\overline{m-1}|^2}+\frac{c}{\overline{m-1}|^3}$, &c.

Dr. *Taylor* has found another Solution of this Problem, very different from theſe by the Help of his Method of Increments. At the Deſire of that very learned Man I had ſent him my ſecond Formula for the Solution of *Prob.* 2. as alſo the other Forms belonging to the third, fourth, and fifth Propoſitions, but without their Demonſtrations. For I did not doubt but ſo acute a Man, and the Inventer of the Method of Increme ts, would be able to find out theſe, or others like to them. He wrote back that he had found the Solutions, and at the ſame Time communicated ſeveral other Things, tending much to the Improvement of this Method. Theſe, at my Requeſt, he has thought fit to ſubjoin to this Diſcourſe of mine.

An Appendix, *treating of the ſame Matter in a different Manner; by Dr.* B. Taylor, *ib. p.* 676.

At the Requeſt of the ingenious Author we exhibit the following Propoſitions, which we propos'd to reſerve for another Occaſion, but that we thought proper to comply with the Deſire of a Friend, who propoſed ſome of the foregoing Propoſitions to our Inquiry, and gave us the Occaſion of finding them.

Definitions. 1. I denote the preſent Value of any variable Quantity by the Letter ſimply wrote down, as x; the foregoing Values I diſtinguiſh by little Lines put at the Top of the Letter, and the following Values I diſtinguiſh by little Lines put under the Letter. So that by the Force of this Definition $\overset{\prime\prime}{x}, \overset{\prime}{x}, x, \underset{\prime}{x}, \underset{\prime\prime}{x}$, are five continual Values of the ſame variable Quantity, x being the preſent Value, $\overset{\prime}{x}$ the Value laſt paſt, $\overset{\prime\prime}{x}$ the ſecond paſt; $\underset{\prime}{x}$ is the Value next to come, and $\underset{\prime\prime}{x}$ the ſecond to come. And ſo of others. After the ſame Manner are to be underſtood the little Lines which are put to Increments. Thus $\underset{\cdot\cdot}{\overset{\prime\prime}{x}}, \underset{\cdot\cdot}{\overset{\prime}{x}}, \underset{\cdot\cdot}{x}, \underset{\prime\cdot\cdot}{x}, \underset{\prime\prime\cdot\cdot}{x}$, are five ſucceſſive Values of $\underset{\cdot\cdot}{x}$; and ſo $\underset{\cdot\cdot}{\overset{\prime\prime}{x}}$ is the ſecond Increment of $\overset{\prime\prime}{x}$, and $\underset{\cdot\cdot}{\overset{\prime}{x}}$ is the ſecond Increment of $\overset{\prime}{x}$; and the like of others.

Cor. By vertue of this Definition $\overset{\prime}{x} + \underset{\cdot}{\overset{\prime}{x}} = x$, $x + \underset{\cdot}{x} = \underset{\prime}{x}$, $\underset{\cdot}{x} + \underset{\cdot\cdot}{x} = \underset{\prime\cdot}{x}$. And ſo of all others of this Kind.

When Occaſion requires that a variable Quantity, ſuppoſe x, is to be look'd upon as an Increment, I denote its Integral by the Letter included between two Hooks [ ]. Alſo the Integral of the Integral [x] or the ſecond Integral of x. I denote by putting the Number 2 over the firſt of the Hooks, as $\overset{2}{[}\, x\,]$. Alſo the Integral of this Integral, or the third Integral of x, I denote in the ſame Manner by putting the Number

3

3 over the Hooks, as $\overset{3}{[x]}$. And so on. Therefore by virtue of this Definition $\overset{3}{[x]}, \overset{2}{[x]}, [x], x$, constitute a Series of Terms, every one of which is the first Increment of the Quantity immediately before it; so that it is $\overset{2}{[x]} = \overset{3}{[\underset{\cdot}{x}]}$, $[x] = \overset{2}{[\underset{\cdot}{x}]}$, $x = [\underset{\cdot}{x}]$.

Lemma. The Increment of the Product xv, made by the Multiplication of the two variable Quantities x and v, is $\underset{\cdot}{x}v + \underset{/}{x}\underset{\cdot}{v}$.

For the Variables being increased by their proper Increments, becomes the new Product $\overline{x+\underset{\cdot}{x}} \times \overline{v+\underset{\cdot}{v}}$, or $xv + \underset{\cdot}{x}v + \overline{x+\underset{\cdot}{x}} \times \underset{\cdot}{v}$, that is, $xv + \underset{\cdot}{x}v + \underset{/}{x}\underset{\cdot}{v}$, writing $\underset{\cdot}{x}$ for $\underset{/}{x} + x$, by *Def.* 1. From whence taking the former Product xv, there remains the Increment $\underset{\cdot}{x}v + \underset{/}{x}\underset{\cdot}{v}$.

Prop. 1. *Theor.* The Increment of the same Product xv, either first, second, third, or any other, the Order of which is denoted by the Symbol n, will be exhibited by this general Formula.

$$\underset{n}{x}v + n \times \underset{n-1}{\underset{\prime}{x}} \times \underset{\cdot}{v} + n \times \frac{n-1}{2} \times \underset{n-2}{\underset{\prime\prime}{x}} \times \underset{\cdot\cdot}{v} + n \times \frac{n-1}{2} \times \frac{n-2}{3} \times \underset{n-3}{\underset{\prime\prime\prime}{x}} \times \underset{\therefore}{v}, \&c.$$

In this Formula these Things are to be observed. First, the Coefficients of the Terms $1, n, n \times \frac{n-1}{2}, n \times \frac{n-1}{2} \times \frac{n-2}{3}$, &c. are the same as of the Binomial raised to the Power n. Secondly, the Numbers n, $n-1$, $n-2$, &c. written below every x, denote the Number of the Points by which the Increments are determin'd. Thirdly, the little Lines $_{\prime}$, $_{\prime\prime}$, $_{\prime\prime\prime}$, &c. written under x, are to be interpreted according to *Def.* 1. Fourthly, in every Term the Number of the Points together written under x and v, always make n. For Instance, let $n = 4$, then by the Formula the fourth Increment of xv, will be $\underset{::}{x}v + 4\underset{\therefore}{\underset{/}{x}}\underset{\cdot}{v} + 6\underset{\cdot\cdot}{\underset{//}{x}}\underset{\cdot\cdot}{v} + 4\underset{///}{x}\underset{\therefore}{v} + \underset{////}{x}\underset{::}{v}$.

Prop. 2. *Theor.* The first Integral of xv, or $[xv]$, is exhibited by the Series $[x]v - \overset{2}{[\underset{/}{x}]}\underset{\cdot}{v} + \overset{3}{[\underset{//}{x}]}\underset{\cdot\cdot}{v} - \overset{4}{[\underset{///}{x}]}\underset{\therefore}{v}$, &c.

But the Series is thus terminated, that it may be $[xv] = [x]v - \left[[\underset{\prime}{x}]\underset{\cdot}{v}\right] = [x]v - \overset{2}{[\underset{\prime}{x}]}\underset{\cdot}{v}\ \left[\overset{2}{[\underset{\prime\prime}{x}]}\underset{\cdot\cdot}{v}\right] = \&c.$

For by taking the Increments, the propofed Quantity xv will be reftored.

Cor. 1. Two of thofe $[x]$, $[xv]$, $\left[[\underset{\prime}{x}]\underset{\cdot}{v}\right]$, being given, the third will be given. Alfo any three of thefe being given $[x]$, $\overset{2}{[\underset{\prime}{x}]}$, $[xv]$, $\left[\overset{2}{[\underset{\prime\prime}{x}]}\underset{\cdot}{v}\right]$, the fourth is given; and fo on.

Cor. 2. If $\underset{\cdot}{v} = 0$, then $[xv]$ is given when $[x]$ is given. If $\underset{\cdot\cdot}{v} = 0$, then $[xv]$ is given from the two $[x]$ and $\overset{2}{[x]}$ being given. If $\underset{\cdot\cdot\cdot}{v} = 0$, then $[xv]$ is given from the three $[x]$, $\overset{2}{[x]}$, $\overset{3}{[x]}$ being given. And fo on.

Ex. 1. For an Example of this Formula, let us find the Integral of $\frac{v}{z\underset{\prime}{z}\underset{\prime\prime}{z}\underset{\prime\prime\prime}{z}}$ from $\underset{\cdot}{z}$ being given, and $\underset{\cdot\cdot\cdot}{v} = 0$; which is a particular Cafe of of the fecond Propofition of the foregoing Treatife of Mr. *Monmort*. Making therefore $x = \frac{1}{z\underset{\prime}{z}\underset{\prime\prime}{z}\underset{\prime\prime\prime}{z}}$, 'tis $[x] = \frac{-1}{3\underset{\cdot}{z}z\underset{\prime}{z}\underset{\prime\prime}{z}}$, $\overset{2}{[\underset{\prime}{x}]} = \frac{1}{2\underset{\cdot}{z}\times 3\underset{\cdot}{z}\underset{\prime}{z}\underset{\prime\prime}{z}}$ and $\overset{3}{[\underset{\prime\prime}{x}]} = \frac{-1}{1\underset{\cdot}{z}\times 2\underset{\cdot}{z}\times 3\underset{\cdot}{z}\underset{\prime\prime}{z}}$. Whence by the Formula it is $[xv]$, that is

$$\left[\frac{v}{z\underset{\prime}{z}\underset{\prime\prime}{z}\underset{\prime\prime\prime}{z}}\right] = -\frac{v}{3\underset{\cdot}{z}z\underset{\prime}{z}\underset{\prime\prime}{z}} - \frac{\underset{\cdot}{v}}{2\underset{\cdot}{z}\times 3\underset{\cdot}{z}\underset{\prime}{z}\underset{\prime\prime}{z}} - \frac{\underset{\cdot\cdot}{v}}{1\underset{\cdot}{z}\times 2\underset{\cdot}{z}\times \underset{\cdot}{z}\underset{\prime\prime}{z}}$$

Ex. 2. Let another Example be the Invention of the Integral of $n a^z$, where $z = 1$, and a is given. Then for x affuming a^z, and for v taking n, 'tis $x = a^{\underset{\prime}{z}}$, that is $\underset{\prime}{x} = ax$, or $x + \underset{\cdot}{x} = ax$, and therefore $\underset{\cdot}{x} = \overline{a-1}$

x,

x, and $x = \frac{\dot{x}}{a-1}$. Therefore returning to the Integrals, it is $[x] = \frac{x}{a-1}$; also $\overset{2}{[x]} = \frac{[x]}{a-1} = \frac{x}{\overline{a-1}|^{2}}$. Also $\overset{3}{[x]} = \frac{x}{\overline{a-1}|^{3}}$; and so on. Therefore because $x_{\prime} = a\,x$, 'tis $[x] = \frac{x}{a-1}$, $[x_{\prime}] = \frac{a\,x}{\overline{a-1}|^{2}}$, $[x_{\prime\prime}] = \frac{a^2\,x}{\overline{a-1}|^{3}}$, &c. Whence by the Formula there comes out $[n\,a^z]$

$$= \frac{a^z n}{a-1} - \frac{a^{z+1}\dot{n}}{\overline{a-1}|^{2}} + \frac{a^{z+2}\ddot{n}}{\overline{a-1}|^{3}}, \text{ \&c.}$$

In this Example is contained the Solution of the Problem, treated of by Mr. *Monmort* in the ninth Proposition. And the Formula coincides with that which he gives us in his first Corollary of the same Proposition.

Scholium. Other Values also of the Integral required may be derived from this Form, according to the different Manner in which the Factors of the proposed Increment are interpreted. Thus in the second Example the Integral of $n\,a^z$ may be exhibited by the Formula $a^z\,[n] - \overline{a-1}\,a^z\,\overset{2}{[n_{\prime}]} + \overline{a-1}|^{2}\,a^z\,\overset{3}{[n_{\prime\prime}]}$, &c. that is, taking n for x, and a^z for v. But we may treat more of this perhaps on another Occasion.

Prop. 3. *Theor.* The Integral of the same $x\,v$, either first, second, third, or any other whose Order is denoted by the Symbol n, is exhibited by a Series proceeding in this general Form, $\overset{n}{[x\,v]} = \overset{n}{[x]}\,v - n\,\overset{n+1}{[x_{\prime}]}\,\dot{v} + n \times \frac{n+1}{2}\,\overset{n+2}{[x_{\prime\prime}]}\,\ddot{v} - n \times \frac{n+1}{2} \times \frac{n+2}{3}\,\overset{n+3}{[x_{\prime\prime\prime}]}\,\dddot{v}$, &c.

The Form of the Series being derived from the foregoing Proposition, the Coefficients $1, -n, n \times \frac{n+1}{2} - n \times \frac{n+1}{2} \times \frac{n+2}{3}$, &c. are thus found by the Method of Increments. Suppose $\overset{n}{[x\,v]} = A\,\overset{n}{[x]}\,v + B\,\overset{n+1}{[x_{\prime}]}\,\dot{v} + C\,\overset{n+2}{[x_{\prime\prime}]}\,\ddot{v} + D\,\overset{n+3}{[x_{\prime\prime\prime}]}\,\dddot{v}$, &c. Then n being increased by its Augment $\dot{n} = 1$, and the Quantities A, B, C, D, &c. by their synchro-

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nal Increments $\underset{\cdot}{A}, \underset{\cdot}{B}, \underset{\cdot}{C}, \underset{\cdot}{D}$, &c. that they may now become $\underset{,}{n}, \underset{,}{A}, \underset{,}{B}, \underset{,}{C}, \underset{,}{D}$, &c. a new Integral (which is the Integral of $[x v]^{n}$) will be had, that is

$$[x v]^{n+1} = \underset{,}{A}[x]^{n+1} v + \underset{,}{B}[\underset{,}{x}]^{n+2} \underset{\cdot}{v} + \underset{,}{C}[\underset{,,}{x}]^{n+3} \underset{\cdot\cdot}{v} + \underset{,}{D}[\underset{,,,}{x}]^{n+4} \underset{\therefore}{v}, \text{ \&c.}$$

Therefore the first Increment of this ought to coincide with the Integral supposed above. Therefore taking the Increments, it will be $[x v]^{n} =$

$$\begin{array}{llll} \underset{,}{A}[x]^{n} v & + \underset{,}{A}[\underset{,}{x}]^{n+1} \underset{\cdot}{v} & + \underset{,}{B}[\underset{,,}{x}]^{n+2} \underset{\cdot\cdot}{v} & + \underset{,}{C}[\underset{,,,}{x}]^{n+3} \underset{\therefore}{v}, \text{ \&c.} \\ & + \underset{,}{B} & + \underset{,}{C} & + \underset{,}{D} \end{array}$$

the same as the Integral above supposed. Therefore comparing the homologous Terms with one another, it is first $\underset{,}{A} = A$. Whence A is a given Quantity. But when $n = 0$, then $A = 1$; therefore $A = 1$. Secondly, $B = \underset{,}{B} + \underset{,}{A}$, that is, $B = B + \underset{\cdot}{B} + 1$, or $\underset{\cdot}{B} = -1 = -\underset{\cdot}{n}$. Therefore returning to the Integrals it is $B = -n + a$. But when $n = 0$, it is $B = 0$. Therefore $a = 0$, and $B = -n$. Thirdly, $C = \underset{,}{C} + \underset{,}{B}$, that is, $\underset{\cdot}{C} = \underset{,}{n}$. And returning to the Integrals 'tis $C = \frac{n\underset{,}{n}}{2} + b$. But when $n = 0$, 'tis $C = 0$; therefore $b = 0$, and $C = \frac{n\underset{,}{n}}{2}$, that is, $n \times \frac{n+1}{2}$. Fourthly, in the same Manner it will be found, that $D = -n \times \frac{n+1}{2} \times \frac{n+2}{3}$. And proceeding thus the other Coefficients will be found.

Scholium 1. In this Proposition compared with the first Proposition, it may be perceived that there is a certain singular Relation between Increments and their Integrals. For as in vulgar Arithmetick Multiplication and Division are so contrary, that if Multiplication is denoted by an affirmative Index, Division will be denoted by an Index with a contrary Sign; so in the Method of Increments, if an Increment be denoted by an affirmative Index, a negative Index will affect the Integral. Thus in the first Proposition, if for n be taken the Number 2, by the Formula the second Increment of $x v$ will be exhibited, that is $\underset{\cdot\cdot}{x} v + 2 \underset{,\cdot}{x} \underset{\cdot}{v} + \underset{,,}{x} \underset{\cdot\cdot}{v}$.

But

But if for n the negative Number — 2 is taken, that now may be sought the negatively second Increment (if we may say so) of $x v$, which is the same Thing as the second Integral, the Coefficients come out the same as if n were taken affirmatively in the present Proposition; and as the Quantities $x, \underset{\prime}{x}, \underset{\prime\prime}{x}$, &c. moreover being interpreted by $[\overset{2}{x}], [\underset{\prime}{\overset{3}{x}}], [\underset{\prime\prime}{\overset{4}{x}}]$, &c. the Series becomes intirely the same as by the former Proposition, where the second Integral is required.

2. And from these Formulæ, as it were on their own Accord, proceed the Formulæ of the eleventh and twelfth Propositions of the Book concerning the Method of Increments. For if for the Increments are wrote the Fluxions, and the Increments vanishing all the $x, \underset{\prime}{x}, \underset{\prime\prime}{x}, \underset{\prime\prime\prime}{x}$, &c. become equal; then this second Proposition will immediately be changed into that eleventh, and the present third into that twelfth, which is a remarkable Example of the *Newtonian* Method, by which he collects the Ratio's of the Fluxions from the last Ratio's of the vanishing Increments, or the first Ratio's of the nascent Increments.

Supplement. Being wholly engaged in the printing of the foregoing Treatise, and taking care to correct the Errors of the Press, and upon that Occasion thinking often on these Matters, that Artifice came into my Mind, which Mr. *James Bernoulli* formerly made Use of, in the Invention of certain Series, by the Help of an harmonical Progression, which Mr. *Monmort* mentions in *Schol.* 6. *Prop.* 5. aforegoing: That it may be conveniently apply'd to the finding Mr. *Monmort*'s Propositions 2, 3, 4, 5, and others of that Kind perhaps something more general. To shew this in the few following Words, I thought would not be unacceptable to the Reader. *A Postscript by the same.* ibid. p. 683.

Theor. Let there be an Arithmetical Progression $p, p+n, p+2n, p+3n$, &c. whose several Terms may successively be denoted by x. And let b, c, d, &c. be any Multiples of the given Difference n of the Terms of that Progression. Let A, B, C, D, &c. be any given Numbers, and let them constitute any Fractions $\frac{A}{x}, \frac{B}{x+b}, \frac{C}{x+c}, \frac{D}{x+d}$, &c. For x writing successively its Values $p, p+n, p+2n$, &c. and from any of these Fractions will arise a Series of harmonically Proportionals. Thus for Instance, from the first Fraction $\frac{A}{p}$ arises the Series $\frac{A}{p}, \frac{A}{p+n}, \frac{A}{p+2n}$, &c. I say that the Aggregate of any Number of such Series continued *in infinitum* may be exhibited in a finite Number of Terms,

if

if only the Aggregate of the Numerators A, B, C, D, &c. be equal to nothing. This will be evident from the two following Examples.

Ex. 1. Let there be only two Fractions $\frac{A}{x}$ and $\frac{-A}{x+3n}$, it being $b = 3n$. Let harmonical Series be wrote arising from these Forms, in such an Order, that the Terms in which are the equal Denominators may answer one another, and the Sums of the homologous Terms being collected, the Aggregate of the several Series will arise in a finite Number of Terms, as may be seen in the Calculation following.

$$\frac{A}{p}+\frac{A}{p+n}+\frac{A}{p+2n}+\frac{A}{p+3n}+\frac{A}{p+4n}, \text{ \&c.} = \text{to the Series from } \frac{A}{x}.$$

$$+\frac{-A}{p+3n}+\frac{-A}{p+4n}, \text{ \&c.} = \text{to the Series from } \frac{-A}{x+3n}.$$

$$\frac{A}{p}+\frac{A}{p+n}+\frac{A}{p+2n}+0+0 \text{ \&c.} = \text{Aggregate of the two Series.}$$

Ex. 2. Let there be three Fractions $\frac{A}{x}$, $\frac{B}{x+2n}$, $\frac{C}{x+3n}$, and let $b = 2n$, $c = 3n$, and $A+B+C=0$. In this Case the Calculation is thus.

$$\frac{A}{p}+\frac{A}{p+n}+\frac{A}{p+2n}+\frac{A}{p+3n}, \text{ \&c.} = \text{to the Series arising from } \frac{A}{x}.$$

$$+\frac{B}{p+2n}+\frac{B}{p+3n}, \text{ \&c.} = \text{to the Series from } \frac{B}{x+2n}.$$

$$+\frac{C}{p+3n}, \text{ \&c.} = \text{to the Series from } \frac{C}{x+3n}.$$

$$\frac{A}{p}+\frac{A}{p+n}+\frac{A+B}{p+2n}+\left(\frac{A+B+C}{p+3n}=0\right) = \text{to the Aggregate of the}$$

three Series.

Here the Aggregate of the Series comes forth in a finite Number of Terms, that is $\frac{A}{p}+\frac{A}{p+n}+\frac{A+B}{p+2n}$, because the Aggregate of the Numerators A, B, C, is equal to nothing. And in the same Manner may the Theorem be demonstrated in all Cases whatever.

Cor. 1. From these Principles may be derived an Infinity of infinite Series, which yet are summable in finite Terms.

Cas. 1. Let $\frac{1}{x}$ and $\frac{-A}{x+b}$ be the Formulæ of two harmonical Series,

ries, the Aggregate of which comes out in a finite Number of Terms by what is demonſtrated above. Then thoſe Formulæ being collected, $\frac{Ab}{x \times \overline{x+b}}$ becomes the Formula of the ſummable Series. Make for Inſtance $A = \frac{1}{6}$, $p = 1$, $n = 2$, and $b = 3n = 6$. Then the Formulæ of the harmonical Series will be $\frac{1}{6x}$ and $\frac{1}{6 \times \overline{x+6}}$. The Formula of the compound ſummable Series will be $\frac{1}{x \times \overline{x+6}}$, that Series being $\frac{1}{1 \times 7} + \frac{1}{3 \times 9} + \frac{1}{5 \times 11} + \frac{1}{7 \times 13}$, *&c.* and the Sum of the Series, by the Calculus demonſtrated before will be $\frac{1}{6 \times 1} + \frac{1}{6 \times 3} + \frac{1}{6 \times 5}$. Let there be three Formulæ of harmonical Series, $\frac{A}{x}$, $\frac{B}{x+b}$, $\frac{C}{x+c}$, it being $A + B + C = 0$, that the Aggregate of the three Series may be finite, by what goes before. Then the Formulæ being collected into one, will be

$$\frac{A \times \overline{x+b} \times \overline{x+c} + Bx \times \overline{x+c} + Cx \times \overline{x+b}}{x \times \overline{x+b} \times \overline{x+c}},$$

or (the Terms being reduced to the Form of the Factors x, $x \times \overline{x+b}$, $x \times \overline{x+b} \times \overline{x+c}$)

$$\frac{Acb + \overline{Ac + \overline{c-b}\,B} \times x + \overline{A+B+C} \times x \times \overline{x+b}}{x \times \overline{x+b} \times \overline{x+c}},$$

that is, (becauſe of $A + B + C = 0$) $\frac{Acb + \overline{Ac + B \times \overline{c-b}} \times x}{x \times \overline{x+b} \times \overline{x+c}}$, the Formula of the ſummable Series,

If there are four Fractions $\frac{A}{x}$, $\frac{B}{x+b}$, $\frac{C}{x+c}$, $\frac{D}{x+d}$, if $A + B + C + D = 0$, the formula of the ſummable Series will be found in the ſame Manner to be

$$\frac{\overline{Abcd + Acd + B \times \overline{c-b} \times \overline{d-b}} \times x + \overline{Ad + B \times \overline{d-b} + C \times \overline{d-c}} \times x \times \overline{x+b}}{x \times \overline{x+b} \times \overline{x+c} \times \overline{x+d}}.$$

And ſo we might go on to ſtill more compounded Formulæ.

Case 2, And if there are several Formulæ of such summable Series, the Factors of whose Denominators are taken out of different Arithmetical Progressions, by the Addition of any Number of those Formulæ into one, a new Formula will be composed of a summable Series. For Example Sake let there be two Formulæ of summable Series $\frac{1}{x\times\overline{x+3}}$ and $\frac{1}{z\times\overline{z+2}}$, x being taking out of the Arithmetical Progression 1, 2, 3, 4, *&c.* and z out of the Progression 1, 3, 5, 7, *&c.* then of these Formulæ collected into one Sum will be made a new Formula $\frac{z\times\overline{z+2}+x\times\overline{x+3}}{x\times\overline{x+3}\times z\times\overline{z+2}}$, or $\frac{\overline{2x-1}\times\overline{2x+1}+x\times\overline{x+3}}{x\times\overline{x+3}\times\overline{2x-1}\times\overline{2x+1}}$, when z is expounded by x and given Numbers.

Cor 2. Hence every infinite Series is summable, whose Terms are denoted by a Fraction, the Factors of whose Denominator are taken out of any Arithmetical Progression; and the Numerator is a Multinomium whose Dimensions are at least fewer by two than the Dimensions of the Denominator. For every Fraction of this Kind may be resolved into so many simple Fractions, as are the Dimensions (that is, the Number of Factors) of the Denominator, of which Numerators the Aggregate is nothing. For Example, let the proposed Formula be $\frac{\alpha+\beta x+\gamma x\times\overline{x+b}}{x\times\overline{x+b}\times\overline{x+c}\times\overline{x+d}}$. Suppose this Formula to be equal to the Aggregate of the Fractions $\frac{A}{x}+\frac{B}{x+b}+\frac{C}{x+c}+\frac{D}{x+d}$. Then those Fractions being collected into one Sum, it will be $Abcd+\overline{Acd+B\times\overline{c-b}\times\overline{d-b}}\times x+\overline{Ad+B\times\overline{d-b}+C\times\overline{d-c}}\times x\times\overline{x+b}+\overline{A+B+C+D}\times x\times\overline{x+b}\times\overline{x+c}$, apply'd to $x\times\overline{x+b}\times\overline{x+c}\times\overline{x+d}$, is equal to the Fraction $\frac{\alpha+\beta x+\gamma x\times\overline{x+b}}{x\times\overline{x+b}\times\overline{x+c}\times\overline{x+d}}$.

Then by comparing the homologous Terms, it is $Abcd=\alpha$, $Acd+B\times\overline{c-b}\times\overline{d-b}=\beta$, $Ad+B\times\overline{d-b}+C\times\overline{d-c}=\gamma$, $A+B+C+D=0$, and therefore $A=\frac{\alpha}{bcd}$, $B=\frac{\beta-Acd}{\overline{c-b}\times\overline{d-b}}$,

$C=$

$C = \frac{\gamma - Ad - B \times \overline{d-b}}{d-c}$, $D = -A - B - C$. By which Means the propofed Formula is refolved into the fimple Fractions $\frac{\alpha}{bcdx} + \frac{\beta - Acd}{\overline{c-b} \times \overline{d-b} \times \overline{x+b}} + \frac{\gamma - Ad - B \times \overline{d-b}}{\overline{d-c} \times \overline{x+c}} + \frac{-A-B-C}{x+d}$. by which the Aggregate of the arifing Series, that is, the Sum of the Series arifing from the propofed Formula $\frac{\alpha + \beta x + \gamma x \times \overline{x+b}}{x \times \overline{x+b} \times \overline{x+c} \times \overline{x+d}}$, comes forth in finite Terms, by what has been faid. Now that the Dimenfions of the Numerator in the propofed Formula muft be fewer by two at leaft than the Dimenfions of the Denominator, will appear from hence, that in the Reduction of the Fractions $\frac{A}{x}$, $\frac{B}{x+b}$, $\frac{C}{x+c}$, $\frac{D}{x+d}$, every Numerator A, B, C, D, is multiply'd into all the Denominators except one, which is its own. Whence the Dimenfions of the Numerator come forth one lefs than the Dimenfions of the Denominator. But by the Equation $A + B + C + D = 0$ the higheft Dimenfion in the Numerator is loft; whence there remain the Dimenfions of the Numerator fewer at leaft by two than the Dimenfions of the Denominator. Now to this Corollary may be reduced Mr. de *Monmort*'s Propofitions 2d and 5th.

Cor. 3. Alfo a Formula being propofed according to *Caf.* 2. *Cor.* 1. ftill more compounded, it may be perceived from the fame Principles whether the Series be fummable. Let there be two Arithmetical Progreffions 1, 3, 5, &c. 2, 4, 6, &c. the homologous Terms of which may be denoted by x and z, and let the propofed Formula of the Series be $\frac{\alpha + \beta x + \gamma x^2}{x \times \overline{x+2} \times z \times \overline{z+2}}$, or (for z writing $x+1$, and the Factors of the Denominator being reduced into Order,) $\frac{\alpha + \beta x + \gamma x^2}{x \times \overline{x+1} \times \overline{x+2} \times \overline{x+3}}$. Suppofe this Formula to be equal to the Aggregate of the Formulæ $\frac{P}{x \times \overline{x+2}}$, $\frac{Q}{\overline{x+1} \times \overline{x+2}}$, of Series that are fummable by what is ſaid above, that (when theſe laſt Formulæ are collected into one Sum) it may be

$$\frac{P \times \overline{x+1} \times \overline{x+3} + Q \times x \times \overline{x+2}}{x \times \overline{x+1} \times \overline{x+2} \times \overline{x+3}}, \text{ or}$$

$$\frac{3P + \overline{4P+2Q}\, x x + \overline{P+Q}\, x x^2}{x \times \overline{x+1} \times \overline{x+2} \times \overline{x+3}} = \frac{\alpha + \beta x + \gamma x^2}{x \times \overline{x+1} \times \overline{x+2} \times \overline{x+3}}.$$

Hence comparing the homologous Terms, theſe Equations ariſe, $3P = \alpha$, $4P + 2Q = \beta$, $P + Q = \gamma$. Whence P and Q being eliminated by juſt Analytical Reductions, there will ariſe the Equation $2\alpha - 3\beta + \gamma = 0$, by which the Relation will be determined, which muſt obtain among the Coefficients α, β, γ, in order that the Series may be ſummable, which ariſes from the propoſed Formula

$$\frac{\alpha + \beta x + \gamma x^2}{x \times \overline{x+1} \times \overline{x+2} \times \overline{x+3}}.$$

In the ſame Manner if the Factors of the propoſed Formula of the Denominator are taken out of three Arithmetical Progreſſions, two Equations will be found, by which the Relations of the Coefficients of the Numerator will be determined, in order that the Series may be ſummable. If there are four Arithmetical Progreſſions, the Relation of the Coefficients will be determin'd by three Equations. And ſo on. And in ſuch Formulæ, in order that the Series may be ſummable, it is farther to be obſerved; firſt, that the Dimenſions of the Numerators may at leaſt be leſs by two than the Dimenſions of the Denominators; likewiſe that out of each of the Arithmetical Progreſſions be taken at leaſt two Factors of the Denominator. Laſtly, if two or more Factors of the Denominator are equal to each other, it muſt be ſuppoſed that ſo many Arithmetical Progreſſions, out of which they are taken, are alſo equal. The Premiſes being duly conſider'd, theſe Things will be evident. Now to this Corollary are eaſily reduced the third and fourth Propoſitions of Mr. de *Monmort.*

XIX. *Propo-*

XIX. *Proposition.* To find a Parabolical Line, which shall pass through the Extremities of any Number of equidistant Ordinates. *The* Newtonian *Differential Method explain'd, by Mr.* J. Stirling, n. 362. p. 1050. Fig. 33.

$$\begin{array}{ccccccccccccccccc}
A & & A2 & & A3 & & A4 & & A5 & & A6 & & A7 & & A8 & & A9 \\
& B & & B2 & & B3 & & B4 & & B5 & & B6 & & B7 & & B8 & \\
& & C & & C2 & & C3 & & C4 & & C5 & & C6 & & C7 & & \\
& & & D & & D2 & & D3 & & D4 & & D5 & & D6 & & & \\
& & & & E & & E2 & & E3 & & E4 & & E5 & & & & \\
& & & & & F & & F2 & & F3 & & F4 & & & & & \\
& & & & & & G & & G2 & & G3 & & & & & & \\
& & & & & & & H & & H2 & & & & & & & \\
& & & & & & & & I & & & & & & & &
\end{array}$$

Cas. 1. Let A, $A2$, $A3$, $A4$, &c. denote equidistant Ordinates insisting upon an Abscis in a given Angle. Collect their Differences B, $B2$, $B3$, $B4$, &c and the Differences of these C, $C2$, $C3$, &c. and the Differences of these D, $D2$, $D3$, &c. and of these E, $E2$, $E3$, &c. and of these F, $F2$, $F3$, &c. And so on. Now the Differences must be collected by always taking the former from the latter. That is, making $B = A2 - A$, $B2 = A3 - A2$, $B3 = A4 - A3$, $B4 = A5 - A4$, &c. Then $C = B2 - B$, $C2 = B3 - B2$, $C3 = B4 - B3$, &c. Then $D = C2 - C$, $D2 = C3 - C2$, $D3 = C4 - C3$, &c. And all the following Differences must be collected in the same Manner. Or let α, β, γ, δ, ϵ, ζ, η, &c. be equal to A, $A2$, $A3$, $A4$, $A5$, $A6$, $A7$, &c. respectively; then $A = \alpha$, $B = \beta - \alpha$, $C = \gamma - 2\beta + \alpha$, $D = \delta - 3\gamma + 3\beta - \alpha$, $E = \epsilon - 4\delta + 6\gamma - 4\beta + \alpha$, $F = \zeta - 5\epsilon + 10\delta - 10\gamma + 5\beta - \alpha$, $G = \eta - 6\zeta + 15\epsilon - 20\delta + 15\gamma - 6\beta + \alpha$, &c. In these Values the numeral Coefficients of α, β, γ, δ, ϵ, &c. are generated as in the integer Powers of the Binomial $\overline{1-z}|^0$, $\overline{1-z}|^1$, $\overline{1-z}|^2$, $\overline{1-z}|^3$, $\overline{1-z}|^4$, &c. by writing the Numbers 1, 2, 3, 4, 5, &c. in the Series $1 \times \frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3} \times \frac{n-3}{4}$, &c. instead of n successively. Now let PQ be any Ordinate intermediate to the rest, and let AP its Distance from the first Ordinate A be called z, then it will be

$$PQ = A +$$

$$B \times \frac{z}{1} +$$

$$C \times \frac{z}{1} \times \frac{z-1}{2} +$$

$$D \times \frac{z}{1} \times \frac{z-1}{2} \times \frac{z-2}{3} +$$

$$E \times \frac{z}{1} \times \frac{z-1}{2} \times \frac{z-2}{3} \times \frac{z-3}{4} +$$

$$F \times \frac{z}{1} \times \frac{z-1}{2} \times \frac{z-2}{3} \times \frac{z-3}{4} \times \frac{z-4}{5} +$$

$$G \times \frac{z}{1} \times \frac{z-1}{2} \times \frac{z-2}{3} \times \frac{z-3}{4} \times \frac{z-4}{5} \times \frac{z-5}{6} + \text{\&c.}$$

Here the Sign of z is to be changed, whenever PQ falls on the other Side of the first Ordinate, as pq.

Cas. 2. Now let $A5$ be an Ordinate in the Middle of all; make $A = B4 + B5$, $B = D3 + D4$, $C = F2 + F3$, $D = H + H2$, &c. and $a = C4$, $b = E3$, $c = G2$, $d = I$, &c. that is, if it be $A6 = \alpha$, $A7 = \beta$, $A8 = \gamma$, $A9 = \delta$, &c. $A4 = \kappa$, $A3 = \lambda$, $A2 = \mu$, $A = \nu$, &c. Make $A = \alpha - \kappa$, $B = \beta - 2\alpha + 2\kappa - \lambda$, $C = \gamma - 4\beta + 5\alpha - 5\kappa + 4\lambda - \mu$, $D = \delta - 6\gamma + 14\beta - 14\alpha + 14\kappa - 14\lambda + 6\mu - \nu$, &c. $a = \alpha - 2A5 + \kappa$, $b = \beta - 4\alpha + 6A5 - 4\kappa + \lambda$, $c = \gamma - 6\beta + 15\alpha - 20A5 + 15\kappa - 6\lambda + \mu$, $d = \delta - 8\gamma + 28\beta + 56\alpha + 70A5 - 56\kappa + 28\lambda - 8\mu + \nu$, &c. And let $A5P$ be called z. Then it will be

$$PQ = A5 + \frac{Az + azz}{1 \cdot 2} +$$

$$\frac{2Bz + bzz}{1 \cdot 2} \times \frac{zz-1}{3 \cdot 4} +$$

$$\frac{3Cz + czz}{1 \cdot 2} \times \frac{zz-1}{3 \cdot 4} \times \frac{zz-4}{5 \cdot 6} +$$

$$\frac{4Dz + dzz}{1 \cdot 2} \times \frac{zz-1}{3 \cdot 4} \times \frac{zz-4}{5 \cdot 6} \times \frac{zz-9}{7 \cdot 8} +$$

$$\frac{5Ez + ezz}{1 \cdot 2} \times \frac{zz-1}{3 \cdot 4} \times \frac{zz-4}{5 \cdot 6} \times \frac{zz-9}{7 \cdot 8} \times \frac{zz-16}{9 \cdot 10} + \text{\&c.}$$

Cas. 3.

Caſ. 3. Now let $A\,4$, $A\,5$, be two Ordinates in the Middle of all. Make $A = \frac{A\,4 + A\,5}{2}$, $B = \frac{C\,3 + C\,4}{2}$, $C = \frac{E\,2 + E\,3}{2}$, $D = \frac{G + G\,2}{2}$, &c. $a = B\,4$, $b = D\,3$, $c = F\,2$, $d = H$, &c. Or let $A\,5 = \alpha$, $A\,6 = \beta$, $A\,7 = \gamma$, $A\,8 = \delta$, $A\,4 = \varkappa$, $A\,3 = \lambda$, $A\,2 = \mu$, $A = \nu$, &c. Then it will be $2\,A = \alpha + \varkappa$, $2\,B = \beta - \alpha - \varkappa + \lambda$, $2\,C = \gamma - 3\,\beta + 2\,\alpha + 2\,\varkappa - 3\,\lambda + \mu$, $2\,D = \delta - 5\,\gamma + 9\,\beta - 5\,\alpha - 4\,\varkappa + 9\,\lambda - 5\,\mu + \nu$, &c. And $a = \alpha - \varkappa$, $b = \beta - 3\,\alpha + 3\,\varkappa - \lambda$, $c = \gamma - 5\,\beta + 10\,\alpha - 10\,\varkappa + 5\,\lambda - \mu$, $d = \delta - 7\,\gamma + 21\,\beta - 35\,\alpha + 35\,\varkappa - 21\,\lambda + 7\,\mu - \nu$, &c. And let O be the middle Point between $A\,4$, $A\,5$, and let $O\,P$ be called z. Then the Ordinate will be

$$P\,Q = \frac{A + az}{4^0} +$$
$$\frac{3\,B + bz}{4^1} \times \frac{4zz - 1}{2 \cdot 3} +$$
$$\frac{5\,C + cz}{4^2} \times \frac{4zz - 1}{2 \cdot 3} \times \frac{4zz - 9}{4 \cdot 5} +$$
$$\frac{7\,D + dz}{4^3} \times \frac{4zz - 1}{2 \cdot 3} \times \frac{4zz - 9}{4 \cdot 5} \times \frac{4zz - 25}{6 \cdot 7} +$$
$$\frac{9\,E + ez}{4^4} \times \frac{4zz - 1}{2 \cdot 3} \times \frac{4zz - 9}{4 \cdot 5} \times \frac{4zz - 25}{6 \cdot 7} \times \frac{4zz - 49}{8 \cdot 9} + \text{\&c.}$$

Alſo in theſe two Caſes z is negative, when the Ordinate $P\,Q$ falls on the other Side of the Beginning of the Abſciſs. And in all the three Caſes the Common Diſtance of the Ordinates is ſuppoſed Unity.

All the three Caſes are very eaſily demonſtrated by Calculation. In the firſt Caſe for $P\,Q$ I write ſucceſſively α, β, γ, δ, &c. and for z at the ſame Time 0, 1, 2, 3, 4, &c. which are the Lengths of the Abſciſs following in Order. Thence ariſe theſe Equations,

$\alpha = A$, $\beta = A + B$, $\gamma = A + 2\,B + C$, $\delta = A + 3\,B + 3\,C + D$, $\varepsilon = A + 4\,B + 6\,C + 4\,D + E$, &c.

$\beta - \alpha = B$, $\gamma - \beta = B + C$, $\delta - \gamma = A + 2\,C + D$, $\varepsilon - \delta = B + 3\,C + 3\,D + E$, &c.

$\gamma - 2\,\beta + \alpha = C$, $\delta - 2\,\gamma + \beta = C + D$, $\varepsilon - 2\,\delta + \gamma = C + 2\,D + E$, &c.

$\delta -$

$\delta - 3\gamma + 3\beta - \alpha = D$, $\varepsilon - 3\delta + 3\gamma - \beta = D + E$, *&c.*
$\varepsilon - 4\delta + 6\gamma - 4\beta + \alpha = E$, *&c.*

These Equations are easily resolved, by taking their Differences, as may here be seen. And they give us the same Values of A, B, C, D, *&c.* as are before supposed in the Solution. The other two Cases are demonstrated after the same Manner.

Every one of these three Series will converge to the Value of the Ordinate PQ, when the Differences of the given Ordinates are of a proper Magnitude. But when they do not converge, other Expedients are to be try'd. At present we shall add a few Things about the Use of this Proposition.

Let α, β, γ, δ, ε, ζ, η, θ, *&c.* represent any equidistant Terms, whose Differences are very small; and let the Relations which they have to one another be nearly represented by the following Equations, which arise by taking the Differences, and the Differences of the Differences continually, and making them equal to nothing.

$$\alpha - \beta = 0$$
$$\alpha - 2\beta + \gamma = 0$$
$$\alpha - 3\beta + 3\gamma - \delta = 0$$
$$\alpha - 4\beta + 6\gamma + 4\delta + \varepsilon = 0$$
$$\alpha - 5\beta + 10\gamma - 10\delta + 5\varepsilon - \zeta = 0$$
$$\alpha - 6\beta + 15\gamma - 20\delta + 15\varepsilon - 6\zeta + \eta = 0$$
$$\alpha - 7\beta + 21\gamma - 35\delta + 35\varepsilon - 21\zeta + 7\eta - \theta = 0$$
$$\alpha - 8\beta + 28\gamma - 56\delta + 70\varepsilon - 56\zeta + 28\eta - 8\theta + \kappa = 0$$
$$\alpha - 9\beta + 36\gamma - 84\delta + 126\varepsilon - 126\zeta + 84\eta - 36\theta + 9\kappa - \lambda = 0$$
&c.

This Table must be kept for Use, to be consulted when Occasion requires. Now that these Equations either obtain accurately, or approximate to the Truth when the Differences of the Terms are small, may appear from the Demonstration of the Proposition of the first Case.

Let any Series be assumed, $\frac{1}{101}$, $\frac{1}{102}$, $\frac{1}{103}$, $\frac{1}{104}$, $\frac{1}{105}$, $\frac{1}{106}$, *&c.* and let the Term be sought which stands the next before $\frac{1}{101}$. It is plain this is $\frac{1}{100}$; let us see then what this Method will make it. Let α denote the Term required, then it will be

$\frac{1}{101} = \beta = 0099,0099,0099,0,$	Equation. 1	gives α	0099,0099,0099,0
$\frac{1}{102} = \gamma = 0098,0392,1568,7,$	2		0099,9805,8629,3
$\frac{1}{103} = \delta = 0097,0873,7864,1,$	3		0099,9994,3455,0
$\frac{1}{104} = \varepsilon = 0096,1538,4615,4,$	4		0099,9999,7824,8
$\frac{1}{105} = \zeta = 0095,2380,9523,8,$	5		0099,9999,9895,8
$\frac{1}{106} = \eta = 0094,3396,2264,2,$	6		0099,9999,9993,1

Therefore it is evident, that this Method continually approximates. If the Differences of the Terms had been leſs, the Values would have approached faſter to the Truth, and on the other Hand ſlower, if thoſe Differences ſhould be greater. Hence if a Term ſhould be wanting in Numeral Tables, it may be interpolated by this Method.

Alſo by this Method come forth the very ſame Algebraical Series themſelves, that are uſed to ariſe by other Methods. Let $\overline{1+zz}|^{-1}$ be propoſed as the Ordinate of a Curve which is to be ſquared. It is the firſt in the regular Series $\overline{1+zz}|^{-1}$, $\overline{1+zz}|^{0}$, $\overline{1+zz}|^{1}$, $\overline{1+zz}|^{2}$, $\overline{1+zz}|^{3}$, *&c.* of Ordinates, all which except the firſt give their Areas z, $z+\frac{1}{3}z^3$, $z+\frac{2}{3}z^3+\frac{1}{5}z^5$, $z+\frac{3}{3}z^3+\frac{3}{5}z^5+\frac{1}{7}z^7$, *&c.* conſtituting a new Series, of which the firſt Term will be the Area required. This therefore will be found by putting α to repreſent it, and β, γ, δ, ε, *&c.* for the reſt in their Order. The firſt Equation gives $\alpha = z$: The ſecond $\alpha = z - \frac{1}{3}z^3$: The third $\alpha = z - \frac{1}{3}z^3 + \frac{1}{5}z^5$: The fourth $\alpha = z - \frac{1}{3}z^3 + \frac{1}{5}z^5 - \frac{1}{7}z^7$: And ſo for the reſt. Therefore the Area required is univerſally $z - \frac{1}{3}z^3 + \frac{1}{5}z^5 - \frac{1}{7}z^7 + \frac{1}{9}z^9 - \frac{1}{11}z^{11}$, *&c.* Which Series is the Arch whoſe Tangent is z, in a Circle the Radius of which is Unity. This was found by our Mr. *James Gregory*, and communicated to Mr. *Collins* the Beginning of the Year 1671, from whom, by Means of Mr. *Oldenburg*, it came to the Hands of Mr. *Leibnitz.*

Now let *&c.* e, d, c, b, a, P, α, β, γ, δ, ε, *&c.* be a Series proceeding both Ways *in infinitum*, where all the Terms are given except P in the Middle of all. Make $A = \alpha + a$, $B = \beta + b$, $C = \gamma + c$, $D = \delta + d$, $E = \varepsilon + e$, *&c.* and it will be

$$P = \frac{A}{2} + \frac{A-B}{6} + \frac{5A - 8B + 3C}{60} +$$

$$\frac{7A - 14B + 9C - 2D}{140} +$$

$$\frac{42A - 96B + 81C - 32D + 5E}{1260} +$$

$$\frac{66A - 165B + 165C - 88D + 25E - 3F}{2772} +$$

$$\frac{429A - 1144B + 1287C - 832D + 325E - 72F + 7G}{24024} + \&c.$$

This Series is found by the Equations, by taking thoſe alternately in which the Number of the Terms is odd. For their Differences will leave the Terms in this Series, which therefore may be produced at Pleaſure.

Let $\overline{1+z}|^{-1}$ be the Ordinate of an Hyperbola, and let its Area be ſought, which lies above the Abſciſs z, when that becomes Unity. This Ordinate is the Middlemoſt in the Series of Ordinates, &c. $\overline{1+z}|^{-5}$, $\overline{1+z}|^{-4}$, $\overline{1+z}|^{-3}$, $\overline{1+z}|^{-2}$, $\overline{1+z}|^{-1}$, $\overline{1+z}|^{0}$, $\overline{1+z}|^{1}$, $\overline{1+z}|^{2}$, $\overline{1+z}|^{3}$, &c. which are equidiſtant, and proceed both Ways *in infinitum.* Therefore the Areas generated by theſe Ordinates will conſtitute a like Series, whoſe middle Term will be the Area required; which therefore may be obtained by the foregoing Series. When z is Unity, as in the preſent Caſe, the Areas of the Curves become &c. $\frac{15}{64}$, $\frac{7}{24}$, $\frac{3}{8}$, $\frac{1}{2}$, and 1, $\frac{3}{2}$, $\frac{7}{3}$, $\frac{15}{4}$, &c. Hence $A = 1 + \frac{1}{2}$, $B = \frac{3}{2} + \frac{3}{8} = \frac{15}{8}$, $C = \frac{7}{3} + \frac{7}{24} = \frac{21}{8}$, $D = \frac{15}{4} + \frac{15}{64} = \frac{255}{64}$, &c. Theſe being ſubſtituted in the Series, P or the Area of the Hyperbola comes out $\frac{3}{4} - \frac{3}{48} + \frac{3}{480} - \frac{4}{4480}$, &c.

that is, $\frac{3}{4} - \frac{A}{4 \cdot 3} - \frac{2B}{4 \cdot 5} - \frac{3C}{4 \cdot 7} - \frac{4D}{4 \cdot 9} - \frac{5E}{4 \cdot 11}$, &c. where now A, B, C, D, &c. (after *Newton's* Manner) denote the Terms in Order from the Beginning. I add the Calculation.

TERMS.

TERMS.

Affirmative.	*Negative.*
7500,0000,0000,0000,0	0625,0000,0000,0000,0
62,5000,0000,0000,0	6,6964,2857,1428,5
7440,4761,9047,6	845,5086,5800,8
97,5586,9130,8	11,3818,4731,9
1,3390,4086,1	1585,7062,8
188,7745,5	22,5708,7
2,7085,0	3260,2
393,4	47,5
5,7	7
+ 7563,2539,3900,7494,1	— 0631,7821,3370,8041,1

Subtracting the negative Sum from the affirmative, I have for the Area, that is, for the Hyperbolic Logarithm of 2, the Number 6931,4718,0559,9453.

The Series that follows is very convenient for the Construction of any Numeral Tables. Let *&c.* e, d, c, b, a, α, β, γ, δ, ϵ, *&c.* denote the alternate Terms in a Series proceeding both Ways *in infinitum*; put $A = \alpha + a$, $B = \beta + b$, $C = \gamma + c$, $D = \delta + d$, $E = \epsilon + e$, *&c.* and the Term between α and a will be

$$\frac{A}{2} +$$

$$\frac{1}{1} \times \frac{A - B}{2^4} +$$

$$\frac{1 \cdot 3}{1 \cdot 2} \times \frac{2A - 3B + C}{2^7} +$$

$$\frac{1 \cdot 3 \cdot 5}{1 \cdot 2 \cdot 3} \times \frac{5A - 9B + 5C - D}{2^{10}} +$$

$$\frac{1 \cdot 3 \cdot 5 \cdot 7}{1 \cdot 2 \cdot 3 \cdot 4} \times \frac{14A - 28B + 20C - 7D + E}{2^{13}} +$$

$$\frac{1 \cdot 3 \cdot 5 \cdot 7 \cdot 9}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} \times \frac{42A - 90B + 75C - 35D + 9E - F}{2^{16}} +$$

$$\frac{1 \cdot 3 \cdot 5 \cdot 7 \cdot 9 \cdot 11}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} \times \frac{132A - 297B + 275C - 154D + 54E - 11F + G}{2^{19}} +$$

&c.

This Series follows from the third Case of the Proposition, by making $z = 0$. The numeral Coefficients of the Letters are thus produced; for Example, in the fourth Term the Coefficient of the last Letter but one C is 5; make $5 + 1 = n$, and the Numbers which arise from the Multiplication of the Terms $1 \times \frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3} \times \frac{n-3}{4} \times \frac{n-4}{5}$, *&c.* will be 1, 6, 15, 20, *&c.* The Differences of these 5, 9, 5, *&c.* are the Numbers required; and therefore the Series may be produced at Pleasure.

Having given the Logarithms of the Numbers 46, 48, 50, 52, 54, 56, 58, 60, to find the Logarithm of the Number 53, which is in the Middle of all. Make $l.\ 52 + l.\ 54 = A = 3,4483,7910,34$: $l.\ 50 + l.\ 56 = B = 3,4471,5803,13$: $l.\ 48 + l.\ 58 = C = 3,4446,6923,08$: $l.\ 46 + l.\ 60 = D = 3,4409,0908,19$. These Values being wrote in the Series, the first four Terms will give 1,7242,2586,96 for the Logarithm of the Numb r 53. And in the same Manner any other intermediate Number may be found.

Therefore in the Construction of Tables it is sufficient, first to find some Terms at due Distances, for the rest may be inserted in this Manner. For the Terms at first found are to be continually intercalated, 'till we arrive at last at those that are desired. By this Means the whole Table will be had, from a few given Terms at first as a Foundation for the Work. But it is not convenient that the Terms we first seek are all equidistant thro' the whole Table; for if we omit them by Turns, where their Difference is the greatest, we may elsewhere *per saltum* omit two, three, twenty, or perhaps more Terms. But the Number of Terms that are omitted, consisting between two given ones, ought always to be one of the following, 1, 3, 7, 15, 31, 63, *&c.* if we would insert them by this Series; for this by no Means would be any Hindrance to the Work.

But for Practice the Terms may be collected into one Sum, as you see done in this Table. The first Expression is the first Term; the second is the Sum of the first and second; the third is the Sum of the first, second, and third; and so on.

2 | $\frac{A}{2}$

4 | $\frac{9A - B}{16}$

6 | $\frac{150A - 25B + 3C}{256}$

8 | $\frac{1225A - 245B + 49C - 5D}{2048}$

10 | $\frac{39690A - 8820B + 2268C - 305D + 35E}{65536}$.

Thus ſome of the alternate Terms being given, the intermediate ones will be preſently found by theſe Expreſſions, without taking any Notice of the particular Nature of the Table. For theſe Rules are the ſame in all. The Areas of Curves are nearly equal to the Areas of the Parabolical Figure which paſſes through the Extremities of its Ordinates. But becauſe it would be too laborious always to have Recourſe to the Parabola, I have computed the following Table, by which the Areas are exhibited directly from the Ordinates being given.

1 | $\frac{A}{1}R$

3 | $\frac{A + 4B}{6}R$

5 | $\frac{7A + 32B + 12C}{90}R$

7 | $\frac{41A + 216B + 27C + 272D}{840}R$

9 | $\frac{989A + 5888B - 928C + 10496D - 4540E}{28350}R$

11 | $\frac{16067A + 106300B - 48525C + 272400D - 260550E + 427368F}{598752}R$

Here

Here the Number of the Ordinates is odd, A is the Sum of the firſt and laſt, B of the ſecond and laſt but one, C of the third and laſt but two; and ſo on, till you come to that which is in the Middle of all, which is repreſented by the laſt Letter in every Expreſſion. R is the Baſe, or that Part of the Abſciſs which is intercepted between the firſt and the laſt Ordinate. The Expreſſions are the Areas contained between the Curve, the Baſe, and the extream Ordinates on both Sides. I have not added the Table for Ordinates that are even in Number, becauſe when every Thing elſe is alike, the Area is defined more accurately from an odd Number of Ordinates.

Let the Area be required which is generated by the Ordinate $\overline{1+zz}|^{-1}$, and lies upon the Abſciſs z when it becomes Unity. In $\overline{1+zz}|^{-1}$ for z write $\frac{0}{10}, \frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}, \frac{5}{10}, \frac{6}{10}, \frac{7}{10}, \frac{8}{10}, \frac{9}{10}, \frac{10}{10}$; and eleven Ordinates will ariſe $1, \frac{100}{101}, \frac{25}{26}, \frac{100}{109}, \frac{25}{29}, \frac{4}{5}, \frac{25}{34}, \frac{100}{149}, \frac{25}{41}, \frac{100}{181}, \frac{1}{2}$. Hence $A = 1 + \frac{1}{2} = \frac{3}{2}$, $B = \frac{100}{101} + \frac{100}{181} = \frac{28200}{18281}$, $C = \frac{25}{26} + \frac{25}{41} = \frac{1675}{1066}$, $D = \frac{100}{109} + \frac{100}{149} = \frac{25800}{16241}$, $E = \frac{25}{29} + \frac{25}{34} = \frac{1575}{986}$, $F = \frac{4}{5}$. Theſe Values being ſubſtituted in the laſt Expreſſion, and Unity for R, you will find the Area to be 785398187; which Number is true in the ſeventh Figure, but in the eighth it is too much by 2.

If eleven Ordinates do not give the Area to ſufficient Exactneſs, you muſt erect more; and conceive the Area to be divided into more Parts; then ſeeking every one ſeparately, you may have the Whole to what Degree of Truth you pleaſe.

The Value of $\overline{1+Q}|^{n}$ may be expreſs'd by any one of the three following Series,

$$\overline{1+Q}|^{n} = 1 +$$

$$Q \times \frac{n}{1} +$$

$$Q^2 \times \frac{n}{1} \times \frac{n-1}{2} +$$

$$Q^3 \times \frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3} +$$

$$Q^4 \times \frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3} \times \frac{n-3}{4} +$$

$$Q^5 \times \frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3} \times \frac{n-3}{4} \times \frac{n-4}{5} + \&c.$$

Or

Or $\overline{1+Q}^{n} = 1 +$

$$R \times \frac{n}{1} +$$

$$R^2 \times \frac{n}{1} \times \frac{n+1}{2} +$$

$$R^3 \times \frac{n}{1} \times \frac{n+1}{2} \times \frac{n+2}{3} +$$

$$R^4 \times \frac{n}{1} \times \frac{n+1}{2} \times \frac{n+2}{3} \times \frac{n+3}{4} +$$

$$R^5 \times \frac{n}{1} \times \frac{n+1}{2} \times \frac{n+2}{3} \times \frac{n+3}{4} \times \frac{n+4}{5} + \&c.$$

Here you muſt make $\dfrac{1+Q}{Q} = R.$

Or $\overline{1+Q}^{n} = 1 +$

$$\frac{2+\overline{n+1}\times Q}{\overline{1+Q}^{1}} \times Q \times \frac{n}{1.2} +$$

$$\frac{4+\overline{n+2}\times Q}{\overline{1+Q}^{2}} \times Q^3 \times \frac{n}{1.2} \times \frac{nn-1}{3.4} +$$

$$\frac{6+\overline{n+3}\times Q}{\overline{1+Q}^{3}} \times Q^5 \times \frac{n}{1.2} \times \frac{nn-1}{3.4} \times \frac{nn-4}{5.6} +$$

$$\frac{8+\overline{n+4}\times Q}{\overline{1+Q}^{4}} \times Q^7 \times \frac{n}{1.2} \times \frac{nn-1}{3.4} \times \frac{nn-4}{5.6} \times \frac{nn-9}{7.8} +$$

$$\frac{10+\overline{n+5}\times Q}{\overline{1+Q}^{5}} \times Q^9 \times \frac{n}{1.2} \times \frac{nn-1}{3.4} \times \frac{nn-4}{5.6} \times \frac{nn-9}{7.8} \times \frac{nn-16}{9.10}$$

$+ \&c.$

The

The two firſt Series are demonſtrated by *Caſ.* 1. of the Propoſition.

For if $\overline{1+Q}|^0$, $\overline{1+Q}|^1$, $\overline{1+Q}|^2$, $\overline{1+Q}|^3$, $\overline{1+Q}|^4$, &c. denote ſo many equidiſtant Ordinates in the Parabolical Figure; then will $\overline{1+Q}|^n$ be an Ordinate of the ſame, whoſe Diſtance from $\overline{1+Q}|^0$ will be n. And ſo comes forth the firſt Series. But if in another Parabola the equidiſtant Ordinates are $\overline{1+Q}|^0$, $\overline{1+Q}|^{-1}$, $\overline{1+Q}|^{-2}$, $\overline{1+Q}|^{-3}$, $\overline{1+Q}|^{-4}$, &c. then will $\overline{1+Q}|^1$ be an Ordinate in the ſame, the Diſtance of which from $\overline{1+Q}|^0$ will be $-n$. So will the ſecond Series come forth. Now in a third Parabola let &c. $\overline{1+Q}|^{-4}$, $\overline{1+Q}|^{-3}$, $\overline{1+Q}|^{-2}$, $\overline{1+Q}|^{-1}$, $\overline{1+Q}|^0$, $\overline{1+Q}|^1$, $\overline{1+Q}|^2$, $\overline{1+Q}|^3$, $\overline{1+Q}|^4$, &c. be a Series of equidiſtant Ordinates proceeding both Ways *in infinitum*, and in the ſame the Ordinate $\overline{1+Q}|^n$ will be removed from the middle Term $\overline{1+Q}|^0$ at the Diſtance n. And thus the third Series will come forth by the Second Caſe of the Propoſition. The firſt breaks off when n is an integer affirmative Number; the ſecond when n is integer and negative; and the third breaks off in either Caſe. The third converges much faſter than either of the other; its ſecond Term may be uſed as a Correction, when an Extraction is to be perform'd by the Repetition of the Calculus. By any of theſe the Roots of Numbers may conveniently be reduced to Series.

Dr. *Halley* in his Method of conſtructing the Logarithms, from the firſt of theſe Series demonſtrates *Mercator*'s Series for the Quadrature of the Hyperbola. Let its Ordinate be $\overline{1+z}|^{-1}$ or $\overline{1+z}|^{n-1}$, n being here an infinitely ſmall Number. Whence by the Method for Quadratures, the Area which lies above the Abſciſs z, that is, the Logarithm of the Number $1+z$, will be $\dfrac{\overline{1+z}|^{n}-1}{n}$. But by the firſt Series 'tis $\overline{1+z}|^n$ $= 1 + \dfrac{n}{1}z + \dfrac{n}{1}\times\dfrac{n-1}{2}z^2 + \dfrac{n}{1}\times\dfrac{n-1}{2}\times\dfrac{n-2}{3}z^3$, &c. And therefore in the preſent Caſe, in which n is infinitely ſmall, it will be $\overline{1+z}|^n = 1 + \dfrac{n}{1}z - \dfrac{n}{2}z^2 + \dfrac{n}{3}z^3 - \dfrac{n}{4}z^4 + \dfrac{n}{5}z^5$, &c. which being ſubſtituted in the Value of the Area, it becomes $z - \frac{1}{2}z^2 + \frac{1}{3}z^3 - \frac{1}{4}z^4 + \frac{1}{5}z^5$, &c. which is *Mercator*'s Series.

In like Manner by the ſecond Series this Rule comes forth. Let the given Number $1+z$; make $R = \dfrac{z}{1+z}$, and its Logarithm will be $R + \frac{1}{2}R^2 + \frac{1}{3}R^3 + \frac{1}{4}R^4 + \frac{1}{5}R^5$, &c.

By

By the third Series comes out this Rule. Let R be any Number; make $z = \frac{\overline{R-1}^2}{2R}$, and its Logarithm will be $\frac{RR-1}{2R} - \frac{1}{3}Az - \frac{2}{5}Bz - \frac{3}{7}Cz - \frac{4}{9}Dz - \frac{5}{11}Ez$, &c. where A, B, C, D, &c. after *Newton*'s Method, denote the Terms of the Series in Order. This Series, as well as that from whence it is deduced, approximates much faster than the other two, and is express'd much more generally than that which we gave before, from a Foundation not unlike this, for finding the Logarithm of the Number 2.

A Method for finding the Values of Arithmetical Series, that converge never so slowly.

In some Series the Sum of the Terms cannot be had, except to a very few Places of Figures; till some other Artifice is made Use of, more than their meer Addition. Now let any Series be proposed, all whose Terms are affected with the same Signs, and whose nearest Terms continually tend to Equality. Such are the following $\frac{1}{1 \cdot 2} + \frac{1}{3 \cdot 4} + \frac{1}{5 \cdot 6} + \frac{1}{7 \cdot 8}$, &c. $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25}$, &c. Collect the Sum of some of the Terms from the Beginning, and let those that are next to be added be $\alpha, \beta, \gamma, \delta, \varepsilon$, &c. In Numbers near the Truth, let $r = \frac{\alpha\gamma - \beta\beta}{\alpha\beta - 2\alpha\gamma + \beta\gamma}$, and of the Quantities $\alpha \times \frac{\alpha + r\beta}{\alpha - \beta}$, $\overline{\alpha + \beta} \times \frac{\beta + r\gamma}{\beta - \gamma}$, $\overline{\alpha + \beta + \gamma} \times \frac{\gamma + r\delta}{\gamma - \delta}$, $\overline{\alpha + \beta + \gamma + \delta} \times \frac{\delta - r\varepsilon}{\delta - \varepsilon}$, $\overline{\alpha + \beta + \gamma + \delta + \varepsilon} \times \frac{\varepsilon + r\zeta}{\varepsilon - \zeta}$, &c. let the Differences be a, b, c, d, e, &c. Then in the nearest Numbers let $s = \frac{ac - bb}{ab - 2ac + bc}$, and of the Quantities $a \times \frac{a + sb}{a - b}$, $\overline{a + b} \times \frac{b + sc}{b - c}$, $\overline{a + b + c} \times \frac{c + sd}{c - d}$, $\overline{a + b + c + d} \times \frac{d + se}{d - e}$, &c. let the Differenees be A, B, C, D, &c. and let $t = \frac{AC - BB}{AB - 2AC + BC}$; and so proceed as far as you please. Then will $\alpha + \beta + \gamma + \delta + \varepsilon +$

 ζ, &c.

ζ, &c. $= \alpha \times \frac{\alpha + r\beta}{\alpha - \beta} + a \times \frac{a + sb}{a - b} + A \times \frac{A + tB}{A - B}$, &c. And there will seldom be Occasion to proceed beyond the two first Terms of this new Series.

As if the Value of this Series were desired, $\frac{1}{1.2} + \frac{1}{3.4} + \frac{1}{5.6} + \frac{1}{7.8}$, &c. collect the first 21 Terms, the Sum of which I find to be ,6813, 8410, 1885. The Terms next to be added are $\alpha =$, 0005, 2854, 1226, $\beta =$, 0004, 8309, 1787, $\gamma =$,0004, 4326, 2411, $\delta =$,0004, 0816, 3265, &c. Hence it is that $r = 1$, nearly, and $\alpha \times \frac{\alpha + r\beta}{\alpha - \beta} =$, 0117, 6449, 6282, $a = -$,0000, 0017, 5096, $b = -$, 0000, 0014, 7410, $c = -$, 0000, 0012, 4986, &c. Whence $s = \frac{1}{3}$ nearly, and $a \times \frac{a + sb}{a - b} = -$, 0000, 0141, 8111, which because of the negative Sign I subtract from $\alpha \times \frac{\alpha + r\beta}{\alpha - \beta}$, and there remains , 0117, 6307, 8171; this being added to the Sum before found, ,6813, 8410, 1885, gives the Number , 6931, 4718, 0056 for the Sum of the whole Series, which is true in the ninth Decimal. But before these two Corrections, the Sum was true in the first Figure only. If you have a Mind to come nearer the Mark, you may proceed to the following Approximations. If the Terms of the Series have different Signs, they are to be so join'd, that all may have the same. Thus in the Series $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9}$, &c. by collecting the Terms by two and two, it becomes $\frac{2}{1.3} + \frac{2}{5.7} + \frac{2}{9.11} + \frac{2}{13.15}$, &c. But here it is to be noted, that the Differences a, b, c, d, e, &c. as also A, B, C, D, &c. must be collected by subtracting the antecedent Quantities from the subsequent. And in all this Kind of Series, if p, q, r, represent three Terms following one another in Order, p the first, q the second, and r the third; and the Rectangle $\frac{p + r}{2} \times q$ is not greater than pr, the Value of the Series will be infinitely great: But it will always be finite when the contrary happens. This Rule may some-

ſometime fail, when the Terms p, q, and r are but little diſtant from the Beginning of the Series; but if they are ſuch Terms as are pretty remote from the Beginning, the Rule then becomes very ſafe and ſure.

To other Kinds of Series other Rules muſt be apply'd. Let there be a Series of Regular Polygons inſcribed in a Circle, the Radius being Unity; as

$H =$	2,0000,0000,0000,000	4
$G =$	2,8284,2712,4746,190	8
$F =$	3,0614,6745,8920,718	16
$E =$	3,1214,4515,2258,051	32
$D =$	3,1365,4849,0545,938	64
$C =$	3,1403,3115,6954,752	128
$B =$	3,1412,7725,0932,772	256
$A =$	3,1415,1380,1144,299	512

Now let the laſt Polygon be called A, the laſt but one B, the laſt but two C, and the reſt in their Order, but backwards, D, E, F, &c. and the Area of the Circle required will be $A + \frac{A-B}{3} + \frac{4A-5B+C}{3 \cdot 15} +$

$$\frac{64A - 84B + 21C - D}{3 \cdot 15 \cdot 63} + \frac{4096A - 5440B + 1428C - 85D + E}{3 \cdot 15 \cdot 63 \cdot 255},$$

&c. where if for A, B, C, D, E, &c. are wrote their proper Values, the firſt four Terms will give the Area of the Circle 3, 1415, 9265, 3589, 790. Now this Series is general, and does not at all depend on the Nature of the Circle. It is applicable whenever the former Differences of the approximating Numbers are as it were quadruple of the latter. The Factors in the Denominators are the integral Powers of the Number 4 diminiſh'd by an Unit. Which being had, the Coefficients of the Letters in the different Terms are form'd by the continual Multiplication of the Numbers $1, \frac{n}{3}, \frac{n-3}{15}, \frac{n-15}{63}, \frac{n-63}{255}$, &c. where the laſt of the Factors of the Denominators muſt be ſubſtituted for n.

The laſt of the Quantities $x - 1$, $2\sqrt[2]{x} - 2$, $4\sqrt[4]{x} - 4$, $8\sqrt[8]{x} - 8$, $16\sqrt[16]{x} - 16$, &c. is equal to the Logarithm of the Number x. For x write 2, and by a repeated Extraction of the Square Root, the following Numbers will ariſe.

 $M =$

$M=$ 1,0000,0000,0000,0000,
$L=$ 8284,2712,4746,1901,
$I=$ 7568,2864,0010,8843,
$H=$ 7240,6186,1322,0613,
$G=$ 7083,8051,8838,6214,
$F=$ 7007,0875,6931,7337,
$E=$ 6969,1430,7308,8294,
$D=$ 6950,2734,2438,7611,
$C=$ 6940,8641,2851,8363,
$B=$ 6936,1658,4759,4014,
$A=$ 6933,8182,9699,9493,

Let the laſt of the Numbers be called A, the laſt but one B, and ſo on backwards; and the Logarithm required will be $A+\frac{A-B}{1}+\frac{2A-3B+C}{1\cdot 3}$ $+\frac{8A-14B+7C-D}{1\cdot 3\cdot 7}+\frac{64A-120B+70C-15D+E}{1\cdot 3\cdot 7\cdot 15}$, &c.

The firſt five Terms will give 6931, 4718, 0559, 9457 for the Hyperpolic Logarithm of the Number 2. And how this Series proceeds *in infinitum* is eaſily infer'd from what we have ſaid of the former. It is alſo univerſal, having no Regard to the particular Properties of the Hyperbola.

Alſo this Differential Method may be extended to the Reſolution of Equations, and to many other Speculations which we forbear to mention here. And it contains the moſt general Foundations of Series, as perhaps I may ſhew in a ſhort Time, by applying it to the Reduction of Irrational Equations, as alſo Fluxional Equations.

A General Method of making Logarithms, *by Mr.* J. Craig, n. 328. p. 191.

XX. For the Perfection of this moſt uſeful Part of Arithmetick, this only ſeems to be wanting, the Diſcovery of ſome General Method for finding all the Logarithmical Series. Now ſuch is this that follows, being eaſy and genuine, as being derived from the very Nature of the Logarithms.

By the Letter l prefix'd to any Number is denoted (as is commonly known) the Logarithm of that Number. Now becauſe the Logarithm of any Number propoſed may be found in two Manners, therefore we ſhall conſtitute two Parts of this Logarithmotechny. In the firſt we deduce immediately the Logarithm from the Number. In the latter we derive the Logarithm of the propoſed Number from the known Logarithms of ſome antecedent Numbers.

Part 1. Let $a+1$ be any Number propoſed, and x its Logarithm to be found. Now by the Hypotheſis $x=l\,.\,\overline{a+1}$, which Equation may be call'd a general Canon. (1.) Let there be an Equation among Terms any how compoſed of a and y, with any other Numbers, combined

bined in any Manner by Addition, Subtraction, Multiplication, Division, or Extraction of Roots. (2.) By the Help of the Equation so assumed at Pleasure, let a be exterminated out of the general Canon, and an Equation will be had expressing the Relation between the indeterminate Quantities x and y. (3.) By *Bernoulli*'s Rule let the Differential of this Equation be found, [or its Fluxion,] and by known Methods let the Integral [or Fluent] of this be found, expres'd by an infinite Series. This will give the Value of the Logarithm x required.

Example 1. Let it be assumed $a = y$; then by the general Canon $x = l.\overline{1+y}$, whose Fluxion is $\dot{x} = \frac{\dot{y}}{1+y}$. And the Fluent of this expres'd by an infinite Series is $x = y - \frac{1}{2}y^2 + \frac{1}{3}y^3 - \frac{1}{4}y^4 + \frac{1}{5}y^5 - \frac{1}{6}y^6$, &c.

Example 2. Assume $y = \frac{a}{a+2}$, whence $a + 1 = \frac{1+y}{1-y}$, and therefore by the general Canon $x = l.\frac{1+y}{1-y}$, the Fluxion of which is $\dot{x} = \frac{2\dot{y}}{1-yy}$, and the Fluent of this expres'd by a Series is $x = 2$ into $y + \frac{1}{3}y^3 + \frac{1}{5}y^5 + \frac{1}{7}y^7$, &c. Where the Number 2 prefix'd must be multiply'd into all the Terms of the Series. Nor is there Occasion to add more Examples here, since from hence it appears, how innumerable Logarithmical Series may be found, which, without any Respect to the Logarithms of other Numbers, exhibit the Logarithm of the Number proposed. *Q. E. I.*

Lem. 1. Let z be the Logarithm of any Fraction $\frac{b}{a+1}$, x the Logarithm of the Denominator $a+1$; then will $l.\overline{b - z} = x$. Or if z is the Logarithm of the Fraction $\frac{a+1}{b}$, it will be $l.\overline{b + z} = x$.

Lem. 2. *Let* e be the Exponent of any Power of the Number b; then will $l.b^e = e \times l.b$. Therefore the Logarithm of the Number b^e, and the Exponent e being given, the Logarithm of b will also be given. Both these *Lemmata* are plain from the Nature of Logarithms.

Part 2. Let $a + 1$ be the Number, as before, whose Logarithm is to be found, and let b^e be a Number produced by the Multiplication of Numbers, the greatest of which is less than $a+1$. And let z be the Logarithm of the Fraction $\frac{b}{a+1}$, that is, $z = l.\frac{b}{a+1}$. And let this Equation be called the general Canon. Then (1.) for b let there be taken a Quantity any how compoſed of a, and any determinate Numbers, and let this Value of the Number b, ſo taken at Pleaſure, be ſubſtituted in the Fraction $\frac{b}{a+1}$, whence it will be expreſs'd by a and given Numbers. (2.) Let there be any Equation between y and a, with Numbers taken at Pleaſure; and by the Help of this let a be exterminated out of the general Canon; whence an Equation will be had, expreſſing the Relation between the Indeterminates x and y. (3.) By *Bernoulli*'s Rule let the Fluxion of this Equation be found, and then by known Methods find the Fluent of this expreſs'd by an infinite Series, which will give the Logarithm z of the Fraction $\frac{b}{a+1}$. And when z is found, the Logarithm $x = l.\overline{b-z}$ of the propoſed Number $a+1$ will be had by *Lem.* 1. For by Hypotheſis b^e is produced by the Multiplication of Numbers, the greateſt of which is leſs than $a+1$; and by Hypotheſis the Logarithms of all Numbers are known, which are leſs than the propos'd Number $a+1$. Therefore alſo the Logarithm of the Number which is the Product of all, or b^e; and therefore (by *Lem.* 2.) the Logarithm of b will be given.

Example 1. Aſſume if you pleaſe $b = a$, whence $z = l.\frac{a}{a+1}$. Then (by Art. 2.) take *ad libitum* $y = 2a+1$, by which let a be exterminated, and it will be $z = l.\frac{y-1}{y+1}$, whoſe Fluxion is is $\dot{z} = \frac{2\dot{y}}{yy-1}$. The Fluent of this, expreſs'd by a Series, is $z = -2$ into $\frac{1}{y} + \frac{1}{3y^3} + \frac{1}{5y^5} + \frac{1}{7y^7}$, &c. whence by *Lem.* 1.

$$x = l.b + 2 \times \overline{\frac{1}{y} + \frac{1}{3y^3} + \frac{1}{5y^5} + \frac{1}{7y^7} + \frac{1}{9y^9}}, \text{ \&c.}$$

Example 2. Make $b = \sqrt{aa+2a}$, whence $z = l.\frac{\sqrt{aa+2a}}{a+1}$. Take alſo at Pleaſure $y = aa+2a$, whence $z = l.\frac{1}{y}\sqrt{yy-4}$, of which the Fluxion is $\dot{z} = 4\dot{y} \times \overline{y^3 - 4y}|^{-1}$, and the Fluent of this is $z = -2$ into $\frac{1}{y^2} + \frac{2^2}{2y^4} + \frac{2^4}{3y^6} + \frac{2^6}{4y^8}$, &c. whence by *Lem.* 1.

$$x = l.b + 2 \times \overline{\frac{1}{y^2} + \frac{2^2}{2y^4} + \frac{2^4}{3y^6} + \frac{2^6}{4y^8} + \frac{2^8}{5y^{10}}}, \text{ \&c.}$$

Example 3. Make $b = \sqrt{aa + 2a}$, as in the foregoing, but now assume $yy = 2aa + 4a + 1$; if by these two Equations are extermi-nated b and a out of the general Canon, it will be $z = l . \frac{\sqrt{yy - 1}}{\sqrt{yy + 1}}$, of which the Fluxion is $\dot{z} = 2y\dot{y} \times \overline{y^4 - 1})^{-1}$, of which the Fluent expreſs'd by a Series is $z = -\frac{1}{y^2} - \frac{1}{3y^6} - \frac{1}{5y^{10}} - \frac{1}{7y^{14}}$, &c.

Therefore by *Lem.* 1. $x = l . b + \frac{1}{y^2} + \frac{1}{3y^6} + \frac{1}{5y^{10}} + \frac{1}{7y^{14}} + \frac{1}{9y^{18}}$, &c.

But it muſt be obſerved, that the Number 2 prefix'd in the Series of the firſt and ſecond Examples, is ſuppoſed to be multiply'd into all the Terms of the following Series. And that like Series may be derived from $z = l . \frac{a + 1}{b}$ in the ſame Manner; but then $x = l . \overline{b + z}$, as appears from the ſecond Part of the firſt *Lemma*. Therefore from hence it appears very plain, that the Logarithmotechny now explain'd is very eaſy and genuine, and ſo general, that by theſe two Ways innumerable Series may be found, exhibiting the Logarithm of any Number propoſed. For we may aſſume innumerable Equations at Pleaſure, expreſſing the Relation between y and a, every one of which will give us a new Logarithmic Series. Yet Care ſhould be taken that ſuch Equations may be aſſumed, that ſhall cauſe the Terms to converge as faſt as may be, ſo that the Logarithm may be found with the leaſt Trouble poſſible. To perform this, the Series exhibited in the laſt Examble will be very proper, which is the ſame as that given by the learned Dr. *Halley*, the firſt Inventer of it, in his very elegant Method of conſtructing the Logarithms.

Here by the Way I deſire the Reader to take Notice, that the Curve, which is derived from our Analyſis of the Problem, concerning the Length of Curve-Lines, publiſhed in the Philoſophical Tranſactions for the Year 1708, is the ſame with that propoſed. As I was wholly intent about the Analyſis, I took no Notice of the Coincidence of the Curve propoſed with that which was found, till the learned D. *Jo. Bernoulli* inform'd me of it, in his Letter to Mr. *Will. Burnett*, F. R. S. By which alſo that learned Man was pleaſed fully to ſatisfy all my Objections againſt his *Creeping Motion*; as I now readily own, out of that pure Love which I bear to Truth.

XXI. *Log.*

A New Method for making Logarithms, communicated by Mr. J. Long, n. 339. p. 52.

XXI.

Log.	*Nat. Num.*	*Log.*	*Nat. Num.*
0,9	7,943282347	0,00009	1,000207254
0,8	6,309573445	0,00008	1,000184224
0,7	5,011872336	0,00007	1,000161194
0,6	3,981071706	0,00006	1,000138165
0,5	3,162277660	0,00005	1,000115136
0,4	2,511886432	0,00004	1,000092106
0,3	1,995262315	0,00003	1,000069080
0,2	1,584893193	0,00002	1,000046053
0,1	1,258925412	0,00001	1,000023026
0,09	1,230268771	0,000009	1,000020724
0,08	1,202264435	0,000008	1,000018421
0,07	1,174897555	0,000007	1,000016118
0,06	1,148153621	0,000006	1,000013816
0,05	1,122018454	0,000005	1,000011513
0,04	1,096478196	0,000004	1,000009210
0,03	1,071519305	0,000003	1,000006908
0,02	1,047128548	0,000002	1,000004605
0,01	1,023292992	0,000001	1,000002302
0,009	1,020939484	0,0000009	1,000002072
0,008	1,018591388	0,0000008	1,000001842
0,007	1,016248694	0,0000007	1,000001611
0,006	1,013911386	0,0000006	1,000001381
0,005	1,011579454	0,0000005	1,000001151
0,004	1,009252886	0,0000004	1,000000921
0,003	1,006931669	0,0000003	1,000000690
0,002	1,004615794	0,0000002	1,000000460
0,001	1,002305238	0,0000001	1,000000230
0,0009	1,002074475	0,00000009	1,000000207
0,0008	1,001843766	0,00000008	1,000000184
0,0007	1,001613109	0,00000007	1,000000161
0,0006	1,001382506	0,00000006	1,000000138
0,0005	1,001151956	0,00000005	1,000000115
0,0004	1,000921459	0,00000004	1,000000092
0,0003	1,000691015	0,00000003	1,000000069
0,0002	1,000460623	0,00000002	1,000000046
0,0001	1,000230285	0,00000001	1,000000023

This

This Table is what I ſometimes make Uſe of for finding the Logarithm of any Number propoſed, and *vice verſâ*, for finding the Number correſponding to a Logarithm given. For Inſtance: Suppoſe I had Occaſion to find the Logarithm of 2000, I look in the firſt Claſs of my Table, (the whole Table conſiſts of 8 Claſſes) for the next leſs to 2, which is 1.295262315, and againſt it is 3, which conſequently is the firſt Figure of the Logarithm ſought. Again, dividing the Number propoſed 2, by 1.995262315, the Number found in the Table, the Quotient is 1.002374467; which being look'd for in the ſecond Claſs of the Table, and finding neither its equal, nor a leſſer, I add 0 to the Part of the Logarithm before found, and look for the ſaid Quotient 1.002374467 in the third Claſs, where the next leſs is 1.002305238, and againſt it is 1, to be added to the Part of the Logarithm already found; and dividing the Quotient 1.002374467, by 1.002305238, laſt found in the Table, the Quotient is 1.000069070; which being ſought in the fourth Claſs gives 0, but being ſought in the fifth Claſs gives 2, to be added to the Part of the Logarithm already found; and dividing the laſt Quotient by the Number laſt found in the Table, *viz.* 1.000046053, the Quotient is 1.000023015, which being ſought in the ſixth Claſs, gives 9 to the Part of the Logarithm already found: and dividing the laſt Quotient by the new Diviſor, *viz.* 1.000002072, the Quotient is 1.000000219, which being greater than 1.000000115, ſhews that the Logarithm already found, *viz.* 3.3010 99 is leſs than the Truth by more than half an Unit; wherefore adding 1, you have *Briggs*'s Logarithm of 2000, *viz.* 3.3010300.

If any Logarithm be given, ſuppoſe 3.3010300, throw away the Characteriſtic, then over-againſt theſe Figures 3...0..1...0..3..6..0, you have in their reſpective Claſſes 1.995262315.....0.....1.002305238......0.....1.000069080....0...0 which multiplied continually into one another, the Product is 2.000000019966, which by reaſon the Characteriſtic is 3, becomes 2000.000019966, that is, 2000, the natural Number deſired. I ſhall not mention the Method by which this Table is framed, becauſe you will eaſily ſee that from the Uſe of it.

It is obvious to the intelligent Reader, that theſe Claſſes of Numbers are no other than ſo many Scales of mean Proportionals. In the firſt Claſs, between 1 and 10; ſo that the laſt Number thereof, *viz.* 1.258925412 is the tenth Root of 10, and the reſt in order aſcending are the Powers thereof. So in the ſecond Claſs, the laſt Number 1.023292992 is the hundredth Root of 10, and the reſt in the ſame Manner are Powers thereof. So 1.002305238 in the third Claſs, is the tenth Root of the laſt of the ſecond, and the reſt its Powers, &c. Or, which is all one, each Number in the preceding Claſs, is the tenth Power of the correſponding Number in the next following Claſs: Whence 'tis plain, that to conſtruct theſe Tables requires no more than one

one Extraction of the fifth or ſurſolid Root for each Claſs, the reſt of the Work being done by the common Rules of Arithmetick; and for extracting the fifth Root, you will find more than one very compendious Rule in *Num.* 210. of theſe *Tranſactions*, if any one ſhall deſire to examine the *computus* of theſe Tables.

The Proceſs is exactly the Reverſe of Mr. *Briggs*'s Doctrine, in *Cap.* XII. of his *Arithmetica Logarithmica Vlacq*'s Edition; and had *Briggs* been apprized hereof, it would have greatly eaſed the Labour of deducing the Logarithms of the firſt prime Numbers, which appear to have coſt him ſo much Pains.

A Letter of Mr. l'Abbe Conti *to Mr.* Leibnitz, *concerning the* Invention *of the Method of* Fluxions, n. 359. p. 923.

XXII. 1. I have defer'd till now to anſwer your Letter, becauſe I had a Mind to accompany my Anſwer with that which Mr. *Newton* [a] has lately made, to the Poſtſcript which you have added to it. I ſhall not enter into the Particulars of the Diſpute between you and Dr. *Keil*, or rather Mr. *Newton*. I can only relate Matters of Fact, what I have ſeen, and what I have read, and what I ſhall ſtill ſee and read, in order to make a true Judgment of the Affair.

I have read with great Attention, and without the leaſt Prepoſſeſſion, the *Commercium Epiſtolicum*, and the little Book [b], which contains an Extract from it. I have ſeen at the *Royal Society* the original Papers of the Letters of the *Commercium*; a ſmall Letter [c] wrote in your Hand to Mr. *Newton*; and an old Manuſcript [d] that Mr. *Newton* ſent to Dr. *Barrow*, and which Mr. *Jones* has lately publiſh'd.

From all which I collect, that if we leave out of the Diſpute all foreign Digreſſions, all we have to do is to examine, whether Mr. *Newton* had the *Calculus* of Fluxions, or of Infiniteſimals, before you, or whether you had it before him. You publiſh'd it firſt, that is true; but you have likewiſe own'd, that Mr. *Newton* had given great Hints of it, in the Letters that he had wrote to Mr. *Oldenburg*, and to others. This is proved at full Length in the *Commercium*, and in the Extract from it; what is your Anſwer to this? This is what the Publick wants, in order to make a ſure Judgment in this Matter.

Your Friends expect your Anſwer with much Impatience, and it is their Opinion, that you cannot avoid returning ſome Anſwer; if not to Dr. *Keil*, yet at leaſt to Mr. *Newton* himſelf, who gives you a Challenge in expreſs Terms, as you will ſee in his Letter.

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[a] In his Letter dated *Feb.* 26. 1715-16. *ſt. vet.* and printed at the End of *Raphſon*'s Hiſtory of Fluxions.

[b] Printed in the *Philoſophical Tranſactions*, N. 342. and in Tome VII. *du Journal Literaire.*

[c] Dated 17 *March* 1693. and printed at the End of *Raphſon*'s Hiſtory of Fluxions.

[d] Entituled, *Analyſis per Series numero terminorum infinitas.*

I

I ſhould be glad to ſee you on good Terms with one another. The Publick receives but ſmall Advantage from ſuch Diſputes, but rather loſes for many Ages all the Improvements, which ſuch Diſputes deprive it of.

His Majeſty has been pleaſed to lay his Commands upon me, to acquaint him with all that has paſſed between Mr. *Newton* and you. I did it to the beſt of my Power, and I wiſh it might be with Succeſs to you both.

Your Problem has been reſolved very eaſily, and in a little Time. Several Geometricians, both at *London* and *Oxford*, have given a Solution of it. It is general, for it extends to all Sorts of Curves, whether Geometrical or Mechanical. The Problem is propoſed a little equivocally; but I think M. *de Moivre* is not miſtaken, when he ſays, that our Ideas of it ſhould be reſtrained to a Series of Curves. For Example, we may ſuppoſe it to have the ſame Subtangent to the ſame Abſciſs; which will not only agree to the Conic Sections, but to infinite other Curves, as well Geometrical as Mechanical. Other Suppoſitions might be made, to fix the Idea of it.

I ſhall ſpeak to you another Time concerning Mr. *Newton*'s Philoſophy. We muſt firſt agree upon the Method of Philoſophizing, and very carefully diſtinguiſh between the Philoſophy of Mr. *Newton*, and the Conſequences that many are apt to draw from it, though very raſhly. Many Things are aſcribed to this great Man, which he does not own, as he has proved to thoſe *French* Gentlemen, who came to *London*, on Account of the great Eclipſe.

I am, with all poſſible Reſpect,

London, *Sir*, your, *&c.*
March 1716.

N. B. *Mr.* l'Abbé Conti *ſpent ſome Hours alſo in looking over the old Letters and Letter Books kept in the Archives of the* Royal Society, *to ſee if he could find any Thing which made either for Mr.* Leibnitz, *or againſt Mr.* Newton, *and had been omitted in the* Commercium Epiſtolicum Collinii & aliorum; *but could find nothing of that Kind.*

S I R, *Hanover*, *Apr.* 14, 1716.

2. Not to make you wait, I ſhall tell you before Hand, that I have anſwer'd already to the Letter which I had the Honour of receiving from you, and at the ſame Time to that which Mr. *Newton* has wrote to you. I have ſent the Whole to Mr. *Remond* at *Paris*, who will not fail of tranſmitting them to you. I made uſe of this Way, that I might have impartial and intelligent Witneſſes of our Diſpute: And Mr. *Remond* will alſo communicate them to others. I have ſent him at the ſame Time, a Copy of your Letter, and of that of Mr. *Newton*. After this you will be able to judge, whether the Petulance of ſome of your new Friends gives me much Diſturbance. *Mr.* Leibnitz's *Anſwer.*

As to the Problem, of which ſome among them have thought fit to reſolve ſome particular Caſes, to fix their Ideas, as they call it; it is probable they have pitched upon ſome eaſy Caſes. For there are ſome ſuch among the tranſcendent Curves, as well as among thoſe that are common. But the Buſineſs is to find a general Solution. This Problem is no new one. Mr. *John Bernoulli* has already propoſed it for the Month of *May* in the *Leipſic* Journal, 1697. p. 211. And as Mr. *Facio* deſpiſed what we had done, the Propoſal was repeated for him, and for others like him, in the Journal of *May* 1700. p. 204. It may ſtill ſerve to this Day, to ſhew ſome People, how far they are gone in Methods, and whether they have gone as far as we. And, in the mean Time, till they find out the Means of arriving at a general Solution, they may try what they can do in fixing their Ideas upon a particular Caſe, which we here propoſe to them in the Paper hereto annexed. Its Solution proceeds ſtill from the ſame Mr. *Bernoulli*. So I hope you will have the Goodneſs, not to give yourſelf up too much to the Inſinuations of thoſe who are oppoſite to us; as when they would make you believe, that our Problem was eaſy to them. *I am*, *SIR*, *with much Zeal*, *yours*, &c.

A Problem containing a particular Caſe of the general Problem, for finding a Series of Curves, every one of which is perpendicular to another Series of Curves.

Fig. 34. *Upon the right Line* AG, *as an Axis*, *from the Point* A *any Number of Curves being conſtructed*, *ſuch as* ABD, *of ſuch a Nature*, *that the Radius of Curvature* BO, *drawn from all the Points* B *of the ſeveral Curves*, *may be cut by the Axis* AG *in* C, *alway in the ſame given ratio*: *That is*, *that it may be as* BO *to* BC, *ſo is* m *to* n. *Now let Trajectories be conſtructed ſuch as* ENF, *that may cut the former Curves* ABD *at right Angles.*

Fig. 35. *Thus far this Letter.*] Mr. *Leibnitz* firſt propoſed the general Problem to M. *l'Abbé Conti* in theſe Words; *To find a Line* BCD, *which may cut at right Angles all the Curves of a determinate Series of the ſame Kind*; *for Example*, *all the Hyperbola's*, AB, AC, AD, *which have the ſame Vertex and the ſame Center*; *and this by a general Method.* And in the *Acta Euriditorum* for *October*, 1698. p. 470, 471. he calls the Curves in this determinate Series, *Curves given as to their Ordinates*, *and given in Poſition*, *and given in Poſition as to their Ordinates.* And by all this, the Series of Curves to be cut is given, and nothing more is to be found, than the other Series which is to cut it at right Angles. But Mr. *Leibnitz* being told, that his Problem was ſolved, he changed it into a new one, of finding both the Series to be cut, and the other Series which is to cut it. And the particular Problem, propoſed in this Letter, is a ſpecial Caſe, not of the general Problem firſt propoſed, as it ought to have been, but of this new double Problem. And the firſt Part of this double Problem, (*viz.* by any given Property of a Series of Curves to

find the Curves) is a Problem harder than the former, and of which a general Solution is not yet given. Mr. *Leibnitz*, in a Letter to Mr. *John Bernoulli*, dated 16 *December*, 1694. and publifhed in the *Acta Eruditorum* for *October*, 1698. p. 471. fet down his Solution of the Problem, when the given Series of Curves is defined by a finite Equation, expreffing the Relation between the Abfcifs and Ordinate. The fame Solution holds, when the Equation is a converging Series, or when the Property of the Curve to be cut, can be reduced to fuch an Equation, by the *Analyfis* by Series that are infinite in the Number of their Terms. But Mr. *Leibnitz* was for folving the Problem without converging Series.

XXIII. In an Epiftle for an eminent Mathematician, *Act. Lipf.* 1716. I am accufed of Plagiarifm, as if I arrogated to myfelf the Inventions of *Bernoulli* and others. Let them produce their Examples, and then they fhall have an Anfwer. 'Tis true I have treated of many Things in common with others, but I have by no Means ufed other Mens Inventions as my own. I have every where ufed by own Analyfis, (if you will except the Problem of Ifoperimeters, of which Mention fhall be made hereafter) that it cannot be faid in any wife I have cheated others. They fhould have named their Authors, from whom I have taken my Methods. I have fo great a Veneration for the illuftrious Names of *Huygens*, *de l'Hofpital*, *Varignon*, *Leibnitz*, and others, that I cannot tell but that I have err'd on the contrary Side, when I may feem to have been wanting to myfelf, who always thought it an Honour to myfelf to quote fuch Men as thefe. Perhaps there might be a little Lazinefs in the Matter, that being wholly intent upon Things, I neglected little Pieces of Hiftory. Yet I hoped I could not fall under the Sufpicion of fuch a Fraud, fince the celebrated Works of fuch great Men would eafily difcover it. What Problems I have treated of in common with *Bernoulli* are, of the Funicularia, of the Center of Ofcillation, and of Ifoperimeter's. In the two firft I have ufed my own Analyfis entirely. In the Ifoperimeters I ufed the Analyfis of the Author *James Bernoulli*, a Man very deferving in Mathematicks, to whom I now pay the Honour which is due to him. My Solution of the Problem concerning the Center of Ofcillation was communicated to my Friends ever fince the Beginning of the Year 1712, as I can appeal to the manufcript Letters of Dr. *Keil* for Witneffes. As alfo my Book was in the Cuftody of the Royal Society, and communicated to almoft all our Mathematicians, from the Month of *April* of the Year 1714; which I thought neceffary to mention here, left *Bernoulli* fhould claim alfo that Solution to himfelf. His two Solutions are extant [a], both publifhed in the fame Year; the latter of which fo perfectly agrees with mine, as to its Principles, that you would fwear they were both invented by the fame Perfon. The Matter of the Ifoperimeter's was

Dr. Taylor's *Apology against* J. Bernoulli, n. 360. p. 955.

[a] One in the *Leipfic* Journal of *June*, the other in the Memoirs of the *French* Academy for *Auguft*.

first

first invented by *James Bernoulli*, as hinted above. His Solution with the Analysis is extant in the *Leipsic* Journal for the Year 1701. His Brother's Analysis is extant in the Memoirs of the Royal Academy of Sciences for the Year 1706. A Solution is also extant in my Book. *Bernoulli* has lately published a Commentary about the same Subject in the *Leipsic* Acts for the Year 1718. There, least he should seem to do the same Thing over again [b], he spitefully endeavours to detract, not only from mine, but from his Brother's Solutions also, objecting Prolixity to his Brother [c], and Obscurity to me [d]. He promises every Thing that is great of those his new Undertakings [e], and by the Help of a certain Principle, fetch'd from the Law of Uniformity, which nobody has hitherto observed, he will compleat the whole Matter almost without Calculation, and with very little Trouble. But I know not by what Fatality, in this Matter about the Isoperimeters, *Bernoulli* never finds the Gods propitious. For *first*, that former Analysis of his from Beginning to End makes only one continued Blunder. *Secondly*, that so much boasted Principle of his, fetch'd from the Law of Uniformity, which nobody has hitherto observed, (for so he boldly affirms) has already been observed by me. *Lastly*, the Analysis which he here exhibits as a new one, is merely that of his Brother. For it is the Precepts which make the Analysis, according to which the Calculation is afterwards performed; which is not itself the Analysis, but only the Instrument of the Analysis. The Precepts being once laid down, every one easily performs the Calculation, each in his own Way, one more copiously, another closer or neater, according as his Genius directs.

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[b] P. 16, *&c.* Therefore for these and other Reasons, I shall not seem to do the same Thing over again, *&c.*

[c] Here the Reader will find no Rocks, which the operose Analysis of my Brother throws in his Way, and the Intricacies and Thorns of third Differences, with which the Way is every where beset ; he will find no such Things in my Method. —— Neither has he to fear the Prolixity of my Brother's Calculation, nor the Obscurity of *Taylor*'s, which is equally offensive and troublesome, p. 18. —— which my Brother has deduced by his most operose Analysis —— not only those Things which were formerly proposed by my Brother with great Pomp, and solved with no less Labour and Difficulty, I have solved by the Law of Uniformity alone, without any Analytical Calculus, *&c.*

[d] See the foregoing Note — also what is now taken from p. 18.

[e] I trust with Joy that the Publick will give him Thanks, that I have had Occasion of publishing such Things now, which perhaps, with many other Things, might have lain buried for ever in my Papers, altho' they will not a little enlarge the Boundaries of the sublimer Geometry, p. 17. What was omitted there by Incogitancy, I shall here make Amends for, by a new Method of Solution, which dispatches Problems with singular Facility, not only all those which my Brother proposed concerning Isoperimeters, but innumerable others of a like Kind, *ib.* —— by Help of a certain Principle derived from the Law of Uniformity, which no one has hitherto observed, by the Inspection of the Figure alone, and almost without any Calculation, I shall deduce Equations for Curves required, that offer themselves as it were of their own Accord, *&c.* as in Note [c]. I shall not seem to do the same Thing over again, if in this Argument, which is difficult of itself, I shew a Way or Method that is short, plain, clear, and easy, by which any one, endued but with a moderate Capacity, may arrive at those abstruse Truths, not upon the Credit of others, but be convinced with his own Eyes ; so that, *&c.* as in Note [c].

It

It must not be denied, that *Bernoulli* has made the Calculation more neat and elegant, but he has done it in his Brother's Analysis and not his own. Nor is it to be doubted but that his Brother, if he had lived till now, would have illustrated this Matter as well. We said before, that the Analysis consists wholly in the Precepts; but all the Precepts are his Brother's. For that he considers a little Arch of the Curve required, as composed of three little Elementary right Lines, is wholly owing to his Brother, as he himself has confess'd[f]. That from the given Length of that little Arch he seeks the ratio of the Differences of the Ordinates in his Lemma's, is from his Brother. That he seeks the same ratio over again, by supposing the little nascent Area, composed of the Functions as he calls them, to be either the greatest or least, is from his Brother. *Lastly*, that from that double Expression of that same ratio he obtains the Equation, by which the Nature of the Curve sought is determined, is from his Brother. But these are the Things which constitute the Solution: Therefore the Solution is entirely his Brother's. I said that I heretofore made Use of that Principle, which *Bernoulli* arrogates to himself with so much Ostentation. Here are two Examples of it in the same Page. In Page 113. of my Book are found these,

$\frac{\underset{,}{m}}{\underset{,}{R}} = \frac{m}{R}$. But $\frac{\underset{,}{m}}{\underset{,}{R}}$ is a new Value of $\frac{m}{R}$. Whence $\frac{m}{R}$ will be a given Quantity. Here it is as clear as the Light, that in this Place, from observing the Uniformity of the Expressions $\frac{\underset{,}{m}}{\underset{,}{R}}$ and $\frac{m}{R}$, I concluded that $\frac{m}{R}$ is a given Quantity. I did the same in what follows: Suppose $\frac{\underset{''}{m n}}{n\underset{,}{R}} = \frac{\overset{,}{m}}{R}$, that is $\frac{\underset{'',}{m n n}}{\underset{,}{R}} = \frac{\underset{,}{\overset{\cdot}{m} n n}}{R}$, *&c.* where that the Uniformity might appear between the *Formulæ*, $\frac{\underset{'',}{m n n}}{\underset{,}{R}}$ and $\frac{\underset{,}{\overset{\cdot}{m} n n}}{R}$, I transform'd the Equation. I fancy you will now perceive how happily I have penetrated into *Bernoulli*'s profound Mysteries. Will he say that this is obscure too?

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[f] For this I shall use, (as he has done in his Analysis) the Notion of a very small Arch, *&c.* p. 18.

Now

Now I come to the firſt Part of my Undertaking, which is to ſhew, that *Bernoulli*'s former Analyſis was extremely corrupt. *Firſt* by a Subſtitution, which is ridiculous enough, fetch'd I ſuppoſe from his profound Speculations, he transforms the Equation $FO \times \Delta RO = \varphi\omega \times \Delta\rho\omega$ into this $FO \times \Delta PF = \varphi\omega \times \Delta\pi\varphi$; which in a particular Caſe, (that is, when the Functions are as the Squares of the Ordinates) comes to this, that at the ſame Time $FO \times RO = \varphi\omega \times \rho\omega$, and $FO \times PF = \varphi\omega \times \pi\varphi$. Whence it follows that $PF \,.\, RO :: \pi\varphi \,.\, \rho\omega$. But this is impoſſible, becauſe it is $PF < RO < \rho\omega < \pi\varphi$, or elſe $PF > RO > \rho\omega > \pi\varphi$; neither of which can be reconciled with the propoſed Analogy. For if $PF < RO < \rho\omega < \pi\varphi$ by the Analogy it will be alſo $\pi\varphi < \rho\omega$ (becauſe of $PF < RO$) contrary to the Hypotheſis. Or if $PF > RO > \rho\omega > \pi\varphi$, by the Analogy it will be alſo $\pi\varphi > \rho\omega$, contrary to the Hypotheſis. *Secondly*, he very unſkilfully ſuppoſes the Curvature in F to be to the Curvature in φ, as φO to FO; ſince there is nothing in the whole Analyſis that can reſtrain this Property to the Point O, rather than to any other Point ω in the little Arch $FO\omega\varphi$ taken any where: Nor indeed can Curvitude be eſtimated in ſo ridiculous a Manner. *Thirdly*, with but little Skill he makes $mn = \ddot{x}$, $nl = \ddot{y}$, and $ml = \frac{\dot{t}\ddot{y}}{\dot{x}}$; when they ought to be $mn = \frac{1}{2}\ddot{x}$, $nl = \frac{1}{2}\ddot{y}$, and $ml = \frac{\dot{t}\ddot{y}}{2\dot{x}}$. *Laſtly*, what is worſt of all, to theſe very erroneous Principles he has affix'd a very perfect Concluſion. I ſay this in the firſt Problem; for in the ſecond the Off-ſpring is more worthy of ſuch Parents. You imagine that I am only expoſing ſome of *Bernoulli*'s old obſolete Blunders: But it is not ſo, for thus he goes on. "All theſe "Things I have laid-by a good While, and now diſcuſſing them over "again very accurately, I have weigh'd them in the Scale of a ſevere "Examination [g]. And it is to be noted, that the Solution of the firſt "Problem, in my Paper inſerted in the Memoirs of the Academy, "p. 235. is perfectly right [h]." Therefore he has again adopted his old Miſtakes. Now perhaps any one would enquire, by what Right he pretends to the firſt Rank in the ſublimer Analyticks, with ſuch a ſtubborn Ambition? So that nobody can make any Advances in it, but he muſt be immediately accuſed of having penetrated into *Bernoulli*'s profounder Science [i]. Whence does it appear to be true, what has been lately affirm'd by ſomebody, that the Rules now extant in the Treatiſe *Analyſe des Infiniment Petits*, were firſt derived from *Bernoulli* [k]? That

[g] P. 16. [h] P. 17. [i] P. 18. See alſo Ep. for an eminent Mathematician, and *Bernoulli*'s own Writings almoſt every where.

[k] He allows the Marquis *de l'Hoſpital* underſtood that Method, and knows that illuſtrious Perſon learn'd it from the great *Bernoulli*; and is well aſſured, that the Rules in the ſaid Book (the Analyſis of infinitely ſmall Quantities) owe their Original to the famous *Bernoulli*. *Act. Leipſ. Ann.* 1718. p. 464.

the Praiſe uſually given to the moſt excellent Marquis *de l'Hoſpital*, muſt be now transfer'd to his Preceptor? Is ſuch a Man fit to teach others the Rules of Differencing Differences [a]? With many other Things which there is no Occaſion now particularly to enumerate.

XXIV. Twelve Years ago I undertook the Defence of that learned Man Mr. *James Gregory* my Uncle, againſt the Calumnies of Abbot *Galloyſe*; who alſo impeached before the learned World the great Dr. *Barrow* [b], as if he had ſtolen from *Robervall* his Propoſitions concerning the Transformation of Curves. Now ſince *Galloyſe* has thought fit to revive the ſame Controverſy again [c], give me Leave again to vindicate my Uncle's Reputation. *A Vindication of Mr.* J. Gregory, *by Dr.* D.Gregory, n. 308. p. 2236.

Robervall lived ſeven Years after *Gregory*'s Book was publiſhed. He that was catching at every little Advantage, was challenging every Thing to himſelf, and would leave no one in quiet Poſſeſſion of his own; would he ſuffer himſelf to be rifled of his Propoſitions, while he was alive and had the Uſe of his Eyes? But *Galloyſe* ſays, he did not ſee it, he read no new Book all that Time, he patiently ſuffer'd himſelf to be robb'd of all his Diſcoveries, he gave up his Fame together with his Mathematicks. I wonder with what Face he can throw out ſuch Fictions as theſe, which can ſo eaſily be refuted. There is ſo little Truth in his Aſſertion, that from the Year 1668, *Robervall* lived in Retirement, remote from the Converſation of learned Men, and had renounced his Mathematical Studies; that from the Year 1670, he was a Profeſſor of Mathematicks in the Academy of *Paris*, and communicated to the Royal Academy of Sciences his Invention of a *new Balance*, as their Acts teſtify, which were publiſhed for that Year [d]. Therefore *Robervall* was preſent at the Aſſemblies of the Academicians; and if he then read nothing himſelf, yet can it be thought he heard nothing in Converſation about Mr. *Gregory*'s Inventions, which were then ſo celebrated in *France?* Did he hear nothing about them from Mr. *Huygens*, who at that Time diſputed very eagerly againſt *Gregory* among the Academicians [e]? But if there was no Familiarity between him and *Huygens*, as *Galloyſe* affirms, (perhaps becauſe he was diſpleaſed that *Huygens* has found out the chief and moſt uſeful Property of his *Trochoid*) could he hear nothing for the whole Space of ſeven Years, from all the reſt of the Academicians? Or if he did hear, did he make no Complaint to his Brethren and Friends? Who can believe he had ſuch a Contempt for Fame, that has but once heard of his Squabbles with the *Italians*, with his own People, and with every body? If of a ſudden he was be-

[a] In the mean Time it may be concluded, that he, together with Mr. *Newton* at the Beginning, remain'd in that Error, till at laſt they were deliver'd from it by the Uſe of the *Calculus Differentialis*, and were taught the *Rules of Differencing Differences by the famous* Bernoulli, *ib.* p. 465.

[b] *Memoirs of the Academ.* for 1693.

[c] *Memoirs of the Acad.* for 1703.

[d] *Journal des Sçavans, An.* 1670.

[e] *Journal des Sçavans, An.* 1668.

come ſo indolent, and ſo indifferent to Reputation, that he could eaſily ſuffer to ſee all his Diſcoveries aſcribed to others; and that what he had happily invented rather to lie dormant in his *Ecritoire*, than to bring them to Light; How could it be that *Gregory* ſhould ſteal theſe Things from him? Let us ſee by what Strength of Argument *Galloyſe* proceeds to fix this Accuſation upon him. *Firſt* (ſays he) *it appears* [g], *that this Method for the Transformation of Curves, which was invented by* Roberyall, *was known in* Italy *before the Year* 1668; *for* Torricellius, *who died An.* 1647, *teſtifies in his Letters, that it was communicated to him by* Robervall. Secondly, *the Adverſary, however unwilling, is obliged to confeſs, that this Method is the ſame with that of* Gregory. Thirdly, *it muſt therefore appear very probable, that* Gregory, *when upon his Travels in* Italy, *might learn this Method from the* Italians, *which had been ſo long known in* Italy.

That this Method, which came out in the Year 1692, under the Name of *Robervall*, is the ſame as that which *Gregory* had publiſhed 24 Years before, Prop. XI. *Math. Univerſ.* as it is plain to any one that views them both, ſo I had granted it without any Heſitation. Indeed I ſaid, that in the Writings of the *French*, wherein it is aſcribed to *Robervall*, it was dreſs'd out with a miſerable and ſhameful Demonſtration. But that it was the ſame with that of *Gregory*, I never once queſtion'd, nor made any Diſpute about it; tho' *Galloyſe* made this the chief Point of the Controverſy, and triumphs as tho' I had yielded him the Victory. But I by no Means grant him, either that it was known before to the *Italians*, or that it was communicated by them to *Gregory*. For how does it appear that it was known to them? Becauſe *Robervall* had communicated it to *Torricellius*. How does this appear? From the Letter of *Torricellius* himſelf. But where is this Letter? *Galloyſe* has it. When was it wrote? About 60 Years ago. Where has it been hid ſo long? Where all wonderful Things are hid, in *Robervall*'s own Treaſury. Whether this Epiſtle is genuine or no, or whether there be any ſuch Thing or no, we muſt not preſume to doubt, ſince there are ſo many credible Witneſſes. But by what Literary Monuments does it appear, that *Torricellius* communicated theſe Inventions to the *Italians*? About this there is ſtill a profound Silence. Or if he had imparted theſe to any, they might by this Time have been quite extinct and unknown, ſince *Torricellius* himſelf had been dead 20 Years before *Gregory* went into *Italy*. Or if they had not been yet out of Memory, *Galloyſe* ſhould tell us, who among the *Italian* Mathematicians imparted theſe Secrets to *Gregory*, which had been intruſted to him by *Torricellius*. Perhaps he will ſay, (for he can take the Liberty of ſaying any Thing) that they were known to many in *Italy*. But would the *Italians* truſt theſe Geometrical Secrets to *Gregory*, a meer Foreigner, which they had concealed from every body for 20 Years? Would he dare, in the Midſt of *Italy*, (for his Book was printed at *Padua*) publiſh Things as his own, which he had but juſt learn'd of the *Italians*? Or if he had been ſo de-

[g] *Oeuvres des Mathem. per Meſſ. de l'Acad. Roy.*

void

void of Shame, could he have done it without being censured by the *Italians*, whereas he was rather applauded by them? This I confess is beyond my Faith to believe.

XXV. *A Paper omitted.*

Logometria, auctore *Rogero Cotes*, Trin. Coll. Cantab. Soc. Astron. N. 338. p. 5.
& Ph. Exp. professore *Plumiano*, & R. S. S.

XXVI. *Accounts of Books*, &c. *omitted.*

1. *Lexicon Technicum*, or an Universal *English* Dictionary of Arts and N. 292.
Sciences; explaining not only the Terms of Art, but the Arts them- p. 1699.
selves, by *J. Harris*, M. A. and F. R. S. *Folio*, 1704.

2. *Euclidis* que supersunt omnia Gr. Lat. ex Recensione *Davidis Gre-* N. 289.
gorii M. D. Astronomiæ professore *Saviliano*, & R. S.S. *Oxon.* 1703. *Folio.* p. 1558.

3. *Apollonii Pergæi* Conicorum Libri octo, & *Sereni Antissensis* de Sec- N. 354.
tione Cylindri & Coni Libri duo. *Fol.* Reg. e Theatro *Oxon.* 1710. p. 732.

The 5th, 6th, *and* 7th *Books of* Apollonius *are here translated out of* Arabic *from a MS. by Dr.* Hally, *in which Language they are only to be found*; *who has also endeavour'd to restore the* 8th *Book, which was wholly lost. The* Greek *Text of* Serenus Antissensis *was never publish'd before.*

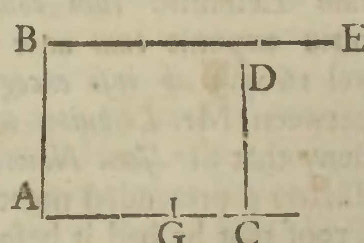
4. De *Locis Solidis* secunda Divinatio Geometrica, in quinque Libros in- N. 291.
juria Temporum amissos *Aristæi* senioris Geometræ; Auctore *Vincentio* p. 1607.
Viviani, Magni Ducis *Etruriæ* Mathematico Primario, & Regalis Societatis *Londini* Sodali. Opus *Conicum* in lucem prolatum, An. 1701. *Folio.*

5. *Methodus Incrementorum*, Auctore *Brook Taylor*, LL. D. & R. S. S. N. 345.
An Error *in the* 25th *Proposition of this Book is here corrected by Dr.* Taylor. p. 339.

This Error does not affect the Reasoning by which I find the Distance of the Center of Percussion from the Axis of Rotation; but it is this, that I supposed the Center of Percussion to be in the Plane passing thro' the Center of Gravity, and perpendicular to the Axis of Rotation; which is a Mistake, and is corrected by the following Proposition.

PROP. PROB.] *To find the Distance of the Center of Percussion from the Plane passing through the Center of Gravity and perpendicular to the Axis of Rotation.*

SOLUTION.] Let this *Figure* be supposed in the Plane passing through the Axis of Rotation, and in which the Center of Percussion is sought.



Let AB be the Axis of Rotation, AGC be the Intersection of this Figure with the Plane passing through the Center of Gravity, and perpendicular to the Axis of Rotation; G be the Point whereon a Line, raised perpendicular to this Figure, will pass thro' the Center of Gravity; BE be a Line parallel to AG, wherein is the Center of *Percussion*. Then to find the Distance AB, let p stand for an Element of the Body proposed, standing per-

pendicularly

pendicularly on any Point *D*. Draw *DC* perpendicular to *AGC*, and *AB* will be equal to the Sum of all the Quantities $p \times GC \times CD$ taken with their proper Signs, divided by the Body itſelf multiplied into the Diſtance *AG*.

Having thus found the Diſtance *AB*, ſuppoſe the Plane of the Figure in *Prop.* 25. to cut the preſent Figure at Right-Angles in the Line *BE*, and the Center of Percuſſion will be rightly determined by that Propoſition.

N. 342. p. 173. *Vid.* ſupra p. 162.

6. *Commercium Epiſtolicum Collinii & aliorum de Analyſi promota.* Publiſhed by the *Order* of the *Royal Society*, in relation to the Diſpute between Mr. *Leibnitz* and Dr. *John Keill*, about the Right to the Invention of the Method of *Fluxions*, by ſome call'd the *Differential Method.*

This Book conſiſts of Letters, and other Papers, which paſs'd many Years ago between Mr. *Collins*, Sir *Iſaac Newton*, Mr. *Leibnitz*, Dr. *Barrow*, Dr. *Wallis*, Mr. *Oldenburg*, Mr. *J. Gregory*; the Occaſion of their being publiſh'd was this:

The Editors of the *Acta Lipſienſia* for *January* 1705, (in giving an Account of Sir *Iſaac Newton*'s *Treatiſe of* Quadratures) *began* to repreſent, that Mr. *Leibnitz* was the *firſt* Inventor of the *Differential* Method, and that Sir *Iſaac Newton* had ſubſtituted *Fluxions* for *Differences*; Dr. *John Keill* upon this, in a Paper publiſh'd in the *Philoſophical Tranſactions*, (for *September* and *October* 1708.*) aſſerted the Invention to Sir *Iſaac Newton*, appealing to the Letters which Dr. *Wallis* had printed in the Collection of his Works, publiſh'd many Years ſince. Mr. *Leibnitz* upon this complain'd to the *Royal Society* of Dr. *Keill* in 1711; whereupon the *Society* appointed a numerous *Committee* of Gentlemen of *ſeveral Nations* to ſearch old Letters and Papers, and to report their Opinion, which was,

* *Vid.* infra C. IV. S. V.

That from theſe Papers it appears, that Sir *Iſaac Newton* had found the Method of *Fluxions* in or before the Year 1669.

That it does not appear from theſe Papers, that Mr. *Leibnitz* had the Method of *Fluxions* or *Differential* Method before the Year 1677.

Dr. *Wallis* (in the ſecond Volume of his Works publiſh'd) in 1695, aſſerted the Invention to Sir *Iſaac Newton* in theſe Words.—— *Newtoni* Methodus de Fluxionibus — *quam ego deſcripſi ex binis* Newtoni *Literis, aut earum alteris*, Junii 13, & Octob. 24, 1676, *ad* Oldenburgum *datis, cum* Leibnitio *tum communicandis* —— *ubi* Methodum hanc Leibnitio exponit *tum* ante decem Annos *nedum plures* [i. e. ann. 1666 vel 1665.] *ab ipſo excogitatam* —— Several Letters follow'd hereupon, between Mr. *Leibnitz* and Dr. *Wallis*, in which Mr. *Leibnitz* did not deny that Sir *Iſaac Newton* had the Method *Ten Years* before thoſe *Two Letters*; pretended not that he had the Method ſo early; brought no Proof that he had it before the Year 1677; no other Proof beſides the Conceſſion of Sir *Iſaac* that he had it ſo early; affirm'd not that he had it earlier; and commended Sir *Iſaac Newton* for his Candour in this Matter.

When Mr. *Fatio* in 1699 ſuggeſted that Mr. *Leibnitz*, the *Second* Inventor of this Calculus, might borrow ſomething from Sir *Iſaac Newton*,

ton, the *oldest* Inventor by many Years, Mr. *Leibnitz*, in his Answer in the *Act. Erud.* for *May* 1700, allow'd that Sir *Isaac* had found the Method apart, and did not deny that Sir *Isaac* was the oldest Inventor by many Years; and asserted then no more to himself, than that he also had found the Method apart, or without the Assistance of Sir *Isaac*; nor did he put in his Claim to be the *first* Inventor, till after the Death of Dr. *Wallis*, the last of the old Men who were acquainted with what had pass'd between the *English* and Mr. *Leibnitz* forty Years ago. The Doctor died in *October* 1703, and Mr. *Leibnitz* began not to put his *new* Claim before *January* 1705.

CHAP. II.

OPTICS.

I. THE *Manner of separating the primitive Colours of Light to such a Degree, that if any one of the separated Lights be taken apart, its Colour shall be found unchangeable, was not publish'd before Sir* Isaac Newton'*s Optics came abroad. For want of knowing how this was to be done, some Gentlemen of the* English *College at* Liege, *and Monsieur* Mariotte *in* France, *and some others, took those for primitive Colours which were made by immitting a Beam of the Sun's Light into a dark Room through a small round Hole, and refracting the Beam by a triangular Prism of Glass placed at the Hole. And by trying the Experiment in this Manner, they found that the Colours thus made were capable of Change, and thereupon reported that the Experiment did not succeed. And lately the Editor of the* Acta Eruditorum *for* October 1713, pag. 447, *desired that Sir* If. Newton *would remove this Difficulty.* The Objections (says he) which have been made by learned Men, both in *France* and *England*, against that Theory of Colours, have been very succesfully answer'd by the most perspicatious *Newton*; as is abundantly manifest from the *English Transactions*, N. 84, 85, 88, 96, 97, 121, 123, 128. Whence it is desired by many, that he would be pleased to explain himself upon that Difficulty, which was started by the most ingenious Mr. *Mariotte*, in his Treatise of *Colours*, p. 207, *&c.* who while he lived was a very indefatigable as well as succesful Enquirer into the Nature of Things. His Difficulty was this. At the Distance of about 25 or 30 Feet he received upon a Paper an entire Ray, let in through a small Hole into a darkned Chamber, which was transmitted through a triangular glass Prism; and the violet Colour, which possess'd a Space of above three Lines, he let pass through a Slit of two Lines, which he received upon another Prism placed very obliquely. When this was done, he observed some Part of this Light to be changed into Red and Yellow. In like Manner he found,

Some of Sir Isaac Newton's *Experiments of Light and Colours repeated, by Mr.* J.T.Desaguliers, N. 348. p. 433.

Vid. supra, V. I. C. III.

found, that Part of this red Light would be changed into Blew and Violet. Now if this Transmutation is admitted, the whole *Newtonian* Theory must fall, as is plain from the *Transactions* for 1706, p. 60. Now *Mariotte* took the Distance of 30 Feet, lest in a smaller Distance any one should pretend, that there was not a compleat Separation of the heterogeneous Rays. To us the Experiment of *Mariotte* would appear decisive, if the whole Blue Light had been changed into some other. *Thus far the Editor of the* Acta. *In Answer to which it is to be observed, that the Red and Yellow which came out of the Violet, and the Blue and Violet which came out of the Red, might proceed from the very bright Light of the Sky next encompassing the Sun, and that several Sorts of Rays which come from several Parts of the Sun's Body are intermixt in all Parts of the colour'd* Spectrum, *which falls upon a Paper at any Distance from the Prism. In this Manner of Trial, for making the Experiment succeed, the Light of the bright Clouds, immediately surrounding the Sun, should be intercepted by an opake Skreen placed in the open Air without, at the Distance of ten or twenty Foot from the Hole through which the Sun shines into the dark Room. And in the Skreen there should be a small Hole for the Sun to shine through. The Hole may be either round or oblong, and not above one eighth or one tenth Part of an Inch broad; so that the Skreen may intercept not only the bright Light of the Clouds next encompassing the Sun's Body, but also the greatest Part of the Sun's Light: For thereby the Colours will become less mixed. The Beam of Light which passes through this Hole, must afterwards pass thro' the other Hole into the dark Room, and the Prism must be placed parallel to the oblong Hole in the Skreen, and the refracting Angle thereof be sixty Degrees or above. In this Manner the Experiment may be tried with Success, but the Trial will be less troublesome if it be made in such a Manner as is described in the fourth Proposition of the first Book of Sir* Isaac Newton'*s Optics.*

Sir Isaac Newton *therefore, upon reading what has been cited out of the* Acta Eruditorum, *desired Mr.* Desaguliers *to try the Experiment in the Manner described in the said Proposition; and he tried it accordingly with Success before several Gentlemen of the* Royal Society, *and afterwards before Monsieur* Monmort *and others of the* Royal Academy *of* Sciences: *How this and other concomitant Experiments were tried and succeeded, is described as follows.*

Experiment 1.] Having sew'd together end-wise two Pieces of Ribbon four Inches long each, the one Blue and the other Red, whose common Breadth was $\frac{3}{4}$ of an Inch; I caused it to be held in such Manner, that the Light which fell from the Clouds thro' the Window was so reflected, that the Angle made by the Rays of Light, which came in at the Midde of the Window, with the Plane of the Ribbon produced, was equal to the Angle made by a Line drawn from the Ribbon to my Eye, and the said Plane of the Ribbon. My Eye was placed as far behind the Ribbon as the Window was before it, the Distance from which to me was about 12 Feet. Then looking thro' a Prism at the Ribbon, it appear'd broken asunder in the Place where the Blue and Red Half join'd.

join'd. If the Prifm was held with the refracting Angle downwards (or laid with one of its Planes flat upon the Nofe) the blue Half of the Ribbon appear'd to be carried down lower than the Red, as at *B*, *R*; but if the refracting Angle of the Prifm was turn'd upwards, (as when the Prifm has one of its Planes laid flat to the Forehead) then the blue Half of the Ribbon was lifted up, as at *ϐ ρ*. Fig. 36.

The Prifm was of white Glafs, having every Angle of 60 Degrees; but when inftead of it, one of a greenifh Sort of Glafs, fuch as Object Glaffes of Telefcopes are made of, was ufed, having the refracting Angle which I look'd through of about 48 Degrees; the fame Pnænomenon was more diftinct, this Glafs having no Veins, but the Red and Blue were nearer to a ftreight Line; in fuch Manner, that if *A* reprefent the Ribbon feen through the firft Prifm, *B* will reprefent the Ribbon feen through the fecond Prifm, *Fig.* 37. If the refracting Angle of the Fig. 37. laft Prifm had been as great as that of the firft, the Light being tranfmitted through too great a Body of greenifh Glafs, the Phænomenon would not have fucceeded fo well.

The blue Ribbon being fomewhat too pale, and the Red a little dull, I repeated the Experiment with a Skain of Blue, and one of red Worfted join'd together in the Middle as the Ribbons were before; and the Colours of both being very intenfe, the Experiment fucceeded better with both Prifms. All that were prefent trying the Experiment found it to fucceed, and that every Circumftance anfwer'd to the Account given in *Prop.* 1. *Theor.* 1. *Book* 1. of Sir *Ifaac Newton*'s *Optics*, as far as the Directions there given were follow'd. So that it appear'd that the Blue being carried lower than the Red in the firft Cafe, and lifted higher in the fecond, was owing to the greater Refraction of the blue Ray: For though each Part of the Ribbon or Worfted reflected all Manner of Rays, yet the Phænomenon was very apparent; as alfo that the blue Ribbon or Worfted reflected the blue Rays more copioufly than the red Rays, and that the red Ribbon or Worfted reflected the red Rays more than the blue ones, becaufe the Red of the blue Half feen through the Prifm was lefs intenfe than that of the red Half, and the Blue or Purple of the red Half feen thro' the Prifm was lefs intenfe than that of the blue Half.

N. B. If the Ribbon or Worfted is laid upon any enlightened Body, the Phænomenon will not fucceed fo well; the Colours of the Body feen through the Prifm mixing with thofe of the Ribbon or Worfted. Even a black Body will not do, if Light fall upon it: But there muft be a black Cloth behind, in fuch Manner, that no Light falling upon it can be reflected fo as to difturb the Phænomenon. And if a fhortfighted Perfon looks through the Prifm, a concave Lens between his Eye and the Prifm, will render the Phænomenon more diftinct than it would otherwife be.

Exp. 2.] Some Days after the Sun fhining, I made two Holes *H*, *h*, Fig. 38. in the Window Shut *S*, *s*, of a darkned Room; through which letting the Sun's Beams pafs, by Means of two Prifms *A B*, (one near each Hole)

Hole) I opened the Rays coming from the Sun into the two coloured *Spectra* α, β, where the following Colours were very distinct, *viz.* red, Orange, Yellow, Green, Blue, Purple and Violet. Now the Reason of their being more distinct than ordinary was, that the Prisms which I made Use of, were made of the greenish Glass mentioned before; which is very free from those Veins by which the Colours are too much thrown into one another, by the best white Prisms of the common Sort.

The forementioned colour'd *Spectra* being thrown into the Room, to the Distance of about 20 Feet from the Window where the Sun's Light came in, I caused a Piece of white Paper ϖ, ¼ Inch broad, and 5 Inches long, to be held within the refracted Rays (at a Distance of 10 Feet from the Window) which produced these Colours in such Manner, that by turning the Prisms round their Axis, I could make the red Ray of the *Spectrum* made by the one Prism, fall upon one Half of the Paper, and the purple Ray of the *Spectrum* made by the other Prism fall upon the other Half; for the *Spectra* were both vertical, the Lines which terminated the long Sides of them towards each other just touching, as ap-

Fig. 38. pears in *Fig.* 38. Then at the Distance of 9 Foot, looking through the Prism *C* at the Paper thus colour'd, the red half appear'd very much separated from the Purple, the one seeming lifted up from the other; the Red or Purple appearing the highest, according as the refracting Angle of the Prism was either held upwards or downwards. The Phænomenon is much much distinct this Way than any other; for the Paper not only seems divided into two, when it is colour'd by a red and

Fig. 39. a purple Ray, but also by a red and blue, *Fig.* 39. by a red and a green

Fig. 40. Ray, *Fig.* 40, or indeed by any two Colours that are different, how near soever their Places in the *Spectra* be to each other. The Halves of the Paper appear, when view'd through the Prism, to be farther from each other, when the Paper is tinged with such Colours, as are farther from each other in the Series of Colours in the *Spectrum*; and nearest, though still divided, when neighbouring Colours fall upon the Paper, as Yellow and Green, or a light and a deep Green. But the Paper appears no

Fig. 41. Way divided, when colour'd with the Red of the two *Spectra*, *Fig.* 41. if those Reds are equally intense; and so of the other Colours.

Exp. 3.] I held a Lens of about three Foot Radius, at the Distance of six Feet from the oblong Paper, (on which a red and a purple Ray falling, made it look half red and half purple) and I projected the Image of the said coloured Paper at the Distance of about six Foot on the other Side of the Lens, on a white Sheet of Paper; where it was observable, that when the red Half was distinctly painted on the white Paper (which was known by the Edges of the Image being regularly terminated) then the blue Half of the Image was confused: But if the white Paper was brought about two Inches nearer to the Lens, the Image of the blue Half became distinct, and that of the red Half confused.

I tried the Experiment with a Paper coloured half Red and half Blue, the Red with Carmine, and the Blue with Smalt, making the Candle to enlighten

enlighten the Paper, (the Room being otherwiſe dark) and the Experiment ſucceeded in the ſame Manner. The Experiment thus made, is the ſame that Sir *Iſaac Newton* gives an Account of, *Book* 1. *Part* 1. *Theor.* 1. *of his Optics.* Only it is to be obſerved, that when the oblong Paper is coloured with Red and Blue from the Priſms, the focal Place, where the red Part of the Image is diſtinct, is more diſtant from the Place where the blue Part of the Image is diſtinct, than when the Paper is coloured with the Painter's Powders, and much more vivid.

The 42[d] Figure ſhews the Projection of the Paper tinged with the Rays; and *Fig.* 43. the Projection of it when painted; where a black Thread is wrapp'd round the red and the blue Part, that the Diſtinctneſs of the Image of the Thread may ſhew when the red or when the blue Part of the Image of the Paper is moſt diſtinct. Fig. 42. Fig. 43.

N. B. When the Candle enlightens the painted Paper, ſet an opaque Body as *B* between the Candle and the Lens; leſt the Image of the Candle being alſo projected, ſhould diſturb the Experiment.

Exp. 4.] Having made an Hole of $\frac{1}{4}$ Inch Diameter in the Window-Shut of the darkened Room, I ſuffer'd a Sun-Beam to come into the Room, which I intercepted with a Priſm at the Diſtance of five Inches from the Hole; and after its Refraction in paſſing through the Priſm, I received it upon a Sheet of white Paper, where it was coloured, making an oblong Image of the Sun or *Spectrum* of about nine Inches in Length, and two in Breadth; which Breadth was nearly equal to the Diameter of the round Image of the Sun received upon a Paper at the ſame Diſtance from the Hole, which here was 18 Foot. Or if the Sun be too high, a Looking-Glaſs being put in the Room of the Priſm, will throw a white round *Spectrum* upon the Paper, which held at the ſaid Diſtance of 18 Foot, will have its Diameter equal to the Breadth of the colour'd *Spectrum.*

The Colours of the *Spectrum* were theſe; red, orange, yellow, green, blue, purple, and violet, though the Violet was ſo faint in this as to be ſcarce perceivable. See *Fig.* 44. Fig. 44.

N. B. The Axis of the Priſm in this, and all the other Experiments hereafter mentioned, muſt be perpendicular to the Ray that falls on it; and the Plane, into which the Ray enters, muſt be held in ſuch a Poſition, that the Angle, which ſuch a Ray makes with that Plane when it enters, may be equal to the Angle made by the middle Line of thoſe Rays which emerge after Refraction, on the other Side of the refracting Angle of the Priſm, with the Plane out of which they emerge. That is $\angle BDG = \angle AEH$.

If the Plane *AC*, on which the Sun-Beam falls, be turned nearer to a Perpendicular to the Sun-Beam than before, the *Spectrum* will be much longer: If it be more inclined to the ſaid Beam, the *Spectrum* will be ſhorter, and in both Caſes leſs diſtinct. See the *Spectrum DE* and the *Spectrum de* where *Hh* repreſents the Hole in the Window-ſhut in each Caſe; *AC*, *ac* the Plane of the Priſm on which the Rays enter; *BC*, *bc* that out of which they emerge; *P*, *p* the perpendicular, and *C*, *c* the refracting Angle. Fig. 45, 46.

Fig. 47. If the Plane *AC* be ſtill more oblique to *HF*, all the Light will be reflected, and there will be no colour'd Image or *Spectrum* made by Refraction at all.

But if it be held ſo as to be more nearly perpendicular to the Sun-Beam than in *Fig.* 47. the whole Beam will indeed enter the Priſm; but meeting with *BC* the lower Surface of the Priſm, or rather the Surface of the Air contiguous to it, ſome of the Light will by the Plane *BC* be reflected to *de*, paſſing almoſt perpendicularly through *AB*; and the reſt will emerge through *BC*, and by Refraction make the imperfect *Spectrum DE*. See *Fig.* 45. Fig. 45.

If the Sun-Beam enter *AC* perpendicularly and in the Middle of it the Light will be all reflected as in *Fig.* 48. ſome of it by the Plane *BC* to *R*, and the reſt by the Plane *AB* to *ρ*. But if the Beam fall nearer to *A*, (ſtill perpendicularly) it will be all reflected by the Plane *AB*; if nearer to *B*, it will be all reflected by the Plane *BC*. Fig. 48.

In order therefore to have the coloured *Spectrum* as it ought to be, Care muſt be taken that the emerging coloured Light may make the ſame Angle with the Plane *BC*, as the immerging Light does with the Plane *AC*; that is, the Angle *AEH* muſt be equal to *BDG*, as was ſaid before, *Fig.* 44. which may alſo be ſeen on the enlightened Duſt in the Air. But the beſt Way is to turn the Priſm on its Axis, and at the ſame Time look at the coloured *Spectrum*, which will riſe and fall and become longer and ſhorter as you turn your Priſm; and between the Aſcent and Deſcent of the Image, will appear ſtationary; there ſtop the Priſm, and the Reflection will be ſuch as is required for all the Experiments hereafter mention'd. Fig. 44.

In order to have the Priſm move freely on its Axis, and ſtop any where, I fix'd each End of it into a triangular Collar of Tin, from the End of which came a Wire, which was the Axis of the Priſm produced; and ſo I laid it on two wooden Pillars, with a Notch on the Top to receive the Wires, and fixed it to a ſmall Board juſt broad enough to ſtand faſt. See *Fig.* 49. Fig. 49.

Exp. 5.] I took the Priſm *CD*, and thro' it looked at the coloured *Spectrum RP*, which appeared then round and white as at *S*, juſt as if it had been the Sun's Light received on a Paper from the Hole *H*, and ſeen with the naked Eye. In this Caſe the Priſm *CD* muſt be held *in directum* with *AB*, and the refracting Angles in the two Priſms muſt be equal. This *Spectrum* appearing white but juſt in one Point, is not ſo readily found; but the beſt Way is to look through the ſame Priſm *AB* which makes the *Spectrum*, which may eaſily be done if it be pretty long, and then *RP* will be ſeen white and round, and as at *S*, as if coming directly from *H*. See *Fig.* 50. Fig. 50.

Exp. 6.] I held a broad Lens *Ll*, ground to a Radius of 2 ½ Feet, in ſuch Manner that the whole coloured *Spectrum* fell upon it; and after Refraction all the Colours appeared to converge, if received on a Paper at *pp*; but when the Paper was held in the Focus at *F* in the Poſi- Fig. 51.

tion

tion $\pi F \pi$, the *Spectrum* was round and perfctly white by the Union of all the coloured Rays. If the Paper was held at $\pi\ \pi$, the Colours appeared to diverge from each other, but then the red was uppermoſt, which before uſed to be the loweſt, and ſo on in an inverted Order.

I tried the ſame Experiment with a Lens of one Foot Radius, with one of 9 Inches, and with another of 7, and the Succeſs was the ſame. See *Fig.* 51. where the *RO*, *Y*, *G*, *B*, *B*, *P*, *V*, expreſs the Colours. Fig. 51.

N. B. Care muſt be taken that the very End of the Red, and the Extremity of the Violet, be taken in by the Lens; otherwiſe the *Spectrum* will not be perfectly white with the Glaſs's Focus.

There is no fixed Diſtance of the Priſm from the Lens, but it ought to be brought ſo near the Priſm, that the two Ends of the *Spectrum* may fall nearer the Axis of the Lens than the Edges of the Lens; becauſe there the Refraction is not ſo regular.

Behind the Lens *L*, which made the Colours converge into white at the diſtinct Baſe or Focus *F*, I placed the Lens *l*, which made the white be at *f* the diſtinct Baſe of the two Glaſſes combined; and the Experiment ſucceeded as before. *Fig.* 52. Fig. 52.

When the Paper was held in the Focus of the Lens, ſo as to receive the white Image of the coloured *Spectrum* projected by the Lens; if with a Card I intercepted the red Ray, the white appeared tinged with purple, and if I intercepted the violet or purple Ray, or both, the white appeared tinged with red; and if the red was intercepted at the ſame Time, the *Spectrum* appeared to be a Mixture of yellow, green and blue. If any ſingle Colour was ſuffered to fall upon the Lens, the reſt being intercepted, that Colour would continue the ſame; only it would be more intenſe in the Focus of the Lens.

Exp. 7.] I took a Board (*Fig.* 53.) *q b s* which ſtood reclining on a Prop *t*, having an Hole of a Quarter of an Inch Diameter at *b*, and behind it a Priſm *B* ſupported on two Props as abovementioned, ſo as to turn eaſily about its Axis; and having ſet this Board on the Ground with the Priſm behind it at *B*; by turning the Priſm *AC* about its Axis, I firſt made the red Ray of the coloured *Spectrum* paſs through the Hole *b*, and fall obliquely upon the ſecond Priſm *B*. This Ray after its Refraction in paſſing through the ſecond Priſm, was carried up to the Cieling of the Room at the Place marked *R*; then I made the purple Ray fall upon the Board, and paſs through the Hole *b*, as the red had done before; and after Refraction through the Priſm *B* it was carried up the Ceiling at *P*. And the green Ray being afterwards made to paſs the ſecond Priſm in the ſame Manner, went up to *G*: And ſo of all the intermediate Rays, which were by this ſecond Refraction thrown to the intermediate Places on the Ceiling between *R* and *P*. Fig. 53.

Care is to be taken that the ſecond Priſm be placed oblique to the Rays which come through the Hole *b*, leſt they be reflected, as they would be, if the Board being in the Poſition *QS*, and the ſecond Priſm in the Poſition *LNM*, the Ray from the firſt Priſm be *ṣb*; for

 then

Fig. 54. then it will be reflected upwards to σ inſtead of being refracted, *Fig.* 54. Neither muſt the Plane of Immerſion be too oblique, leſt the Incident Ray be reflected downwards by it, as the Ray *R b* is by the Priſm *B*
Fig. 55. thrown to *E*, in *Fig.* 55. Several have confeſſed to me that they at firſt uſed to fail in this Experiment, for want of ſetting the ſecond Priſm in a due Inclination.

Though the Colours by the ſecond Refraction on the Ceiling appeared unchanged, when ſeen by the naked Eye, yet if viewed through a Priſm, they afforded new Colours, (except ſome Part of the Red, and ſome Part of the Violet) which was owing to their not being fully ſeparated; for which Reaſon I made the following Experiment, to prove, that if the Colours be well ſeparated, they are truly homogeneal and unchangeable.

N. B. When the Priſms are good, and no Clouds are near the Sun, the Extremity of the Red or Violet will afford unmixed Colours in this Experiment; otherwiſe not.

Exp. 8.] Having made a Hole in the Window-Shut 2 Inches wide,
Fig. 56. I applied to it a Tin-plate, which ſliding up and down hid all this Hole in the Wood, and only tranſmitted a ſmall Beam through its own Hole *H*, whoſe Diameter was $= \frac{1}{16}$ Inch. This Beam, by Means of the Looking-Glaſs *L*, placed on the Board of the Window *XW*, I reflected horizontally to the other End of the Room. But to correct the Irregularity of the Reflection of the Looking-Glaſs, I made Uſe of the Frame of Paſte-board *P p*, which had an Hole in it *b* of $\frac{1}{16}$ Inch likewiſe; and placing it at *P p*, I ſuffered ſome of the reflected Beams to paſs through it, ſo as to fall upon the Lens *FE* (convex on both Sides, and ground to a *Radius* of $4\frac{1}{2}$ Feet) at the Diſtance of 9 Feet, ſo that the Image of the Hole *b* was projected to *f* on the other Side of the Glaſs, at the Diſtance of 9 Feet more. Juſt behind the Lens, which by a Screw in the Stand *S* might be raiſed or ſet down, ſo as always to receive the Beam along its Axis, I placed a Priſm *A* (upright on one of its Ends and eaſily moveable about its Axis, by reaſon of its Wire turning freely in an Hole in the ſolid Piece of Wood *T*, which ſtood on another Stand behind the Lens) as near as I could to the Lens *EF*, ſo that the Image of *b* inſtead of being round, white, and projected to *f*, was caſt ſidewiſe on a white Paper ſtretched on a Frame, and appear'd coloured, and 30 or 40 Times its Breadth, as at *MN*. The Colours in this Caſe were very vivid and well ſeparated, only the Violet had ſome pale Light darting from its End, upon Account of ſome Veins in the Priſm *A*, and the Light not coming directly from the Sun, reflected; which ought not to have been, if the Sun had been low enough to have thrown the Rays a good Way into the Room without the Help of a Looking-Glaſs.

To ſhew that the Colours in this *Spectrum* were ſimple and homogeneal Lights, I made the following Experiments.

Exp. 9.] Having made an Hole *b* in the Paper which received the coloured *Spectrum*, I ſuffer'd the red Light to paſs; which being refracted

fracted by a ſecond Priſm, fell upon another Paper at *T*, where it appeared ſtill red, whether ſeen with the naked Eye or Priſms of different refracting Angles. To the Eye which ſaw it through the Priſm *V*, it appeared indeed lower as at *t*, but red, round and unchanged. I made the Experiment upon all the Colours, which by this Means appeared to be ſimple and homogeneal. See *Fig.* 57. where the ſame Letters denote the Lens, Priſm and firſt Paper. Fig. 57.

Through the ſame Lens and Priſm the *Spectrum* was made to fall on a Book; then through the Priſm *F* it appeared unchanged; and the Letters in the Book which croſs'd the *Spectrum* were as diſtinct as when ſeen with the naked Eye. See *Fig.* 58. Fig. 58.

N. B. The Axis of the Priſm *F* ought to be perpendicular to the long Axis of the *Spectrum s m* thrown on the Book, which will appear as at $\sigma\ \mu$; and the Priſm in the Poſition repreſented at *F*, with its flat Side towards the Noſe: For that is the moſt convenient Poſition for looking at the *Spectrum* in theſe Experiments.

I ſuffer'd the purple Ray only to paſs through the Hole *h*, and fall upon a Book at *P*, the Letters of which appear'd at π, and were as diſtinct through the Priſm *Q* as when ſeen with the naked Eye; and I had the ſame Succeſs with all the other Rays. See *Fig.* 59. Fig. 59.

But if a Sun-beam as *r* comes through the Hole *H* directly upon the Book at *W*, an Eye looking at it through a Priſm at *X* will ſee this Beam at Υ oblong and coloured, and the Letters on which it falls, confuſed. See *Fig.* 59.

N. B. The Lens ought to be very good, without Veins or Blebs, and ground to no leſs a Radius than I mention'd in the Experiment; though a Radius of a Foot or two longer is not amiſs. The Priſm ought to be of the ſame Glaſs as the Object-Glaſſes of Teleſcopes, the white Glaſs, of which Priſms are uſually made, being commonly full of Veins. And the Room in theſe laſt Experiments ought to be very dark.

A few Days after, having got very good Priſms made for the Purpoſe of the abovementioned Glaſs, I made all the Experiments over again, before ſeveral Members of the *Royal-Society*, with better Succeſs; and had the *Spectrum* very regularly terminated, without any pale Light darting from the Ends of it.

For a further Account of Experiments to this Purpoſe, ſee Sir Iſaac Newton's Optics, *Book* 1. *Part* 1. to which I might have referr'd the Reader altogether; but that I was willing to be particular in mentioning ſuch Things as ought to be avoided in making the Experiments abovementioned; ſome Gentlemen abroad having complained that they had not found the Experiments anſwer, for want of ſufficient Directions in Sir *Iſaac Newton*'s *Optics*; though I had no other Directions than what I found there.

An Experiment *to confirm the Doctrine of* Refrangibility, *by the ſame*, n. 348. p. 448.

II. After the *Experimentum Crucis* made by two Priſms, I ſhould not give the following Experiment, but that it is ſo eaſy to be made, that by by it thoſe who want the *Apparatus* (or are unwilling to be at the Pains) to make the *Experimentum Crucis*, may at any Time ſatisfy themſelves of the Truth of the forementioned Doctrine.

Let the Candle *A* be ſet before the Bar of a Chimney Looking-Glaſs, Fig. 60. ſuch as is repreſented by *HH*, Fig. 60. which is a Piece of Looking-Glaſs Plate conſiſting of four Planes, ſeen in the Section of it $\alpha f d \beta$, *viz* $d\beta$, which is quick-ſilver'd behind, $f\alpha$ a Plane parallel to it, fd one of the Side-planes bezzell'd towards $d\beta$, or inclined to it in an Angle of about 40 Degrees, (though from 30 to 40 will do, but the greater the Angle the better, if it does not exceed 45°.) $\alpha\beta$ the other Side-plane inclined in the ſame Angle to βd.

The Rays of the Candle which come from *A* to γ fall obliquely on the Plane $\alpha\beta$, ſo that inſtead of going on to *a*, they are by Refraction made to incline more towards the Perpendicular *pp*, namely, to go on in the Line γc, and then are reflected from the Point *c* on the quick-ſilver'd Surface, in the Direction $c\kappa$, ſo as to make the Angle $\kappa c d = \gamma c \beta$. Now as the Rays which would go to κ, if not refracted, emerge obliquely from the Plane $\alpha\beta$ they leave the Direction $c\kappa$, and decline from the Perpendicular $\pi\pi$, and, being differently refracted, open into four differently colour'd Rays, *viz.* *bR* a red Ray, *bYO* a Ray made up of orange and yellow; *bGB* a Ray made up of green and blue, or a Sea-green, and *bP* a purple Ray.

If from the Place *Ee* you look full upon the Point *b*, the *Spectrum* or Image of the Candle at *b* will appear double; but not mixed; that is, there will appear a Sea-green Spot, and a red Spot, as it were, one upon another, but not ſo as to produce a mixed or intermediate Colour. Then if the right Eye or Eye at *E* be ſhut, there will appear only a green Spot to the Eye at *e*; if the Eye at *e* be ſhut, the Eye at *E* will ſee only a red Spot.

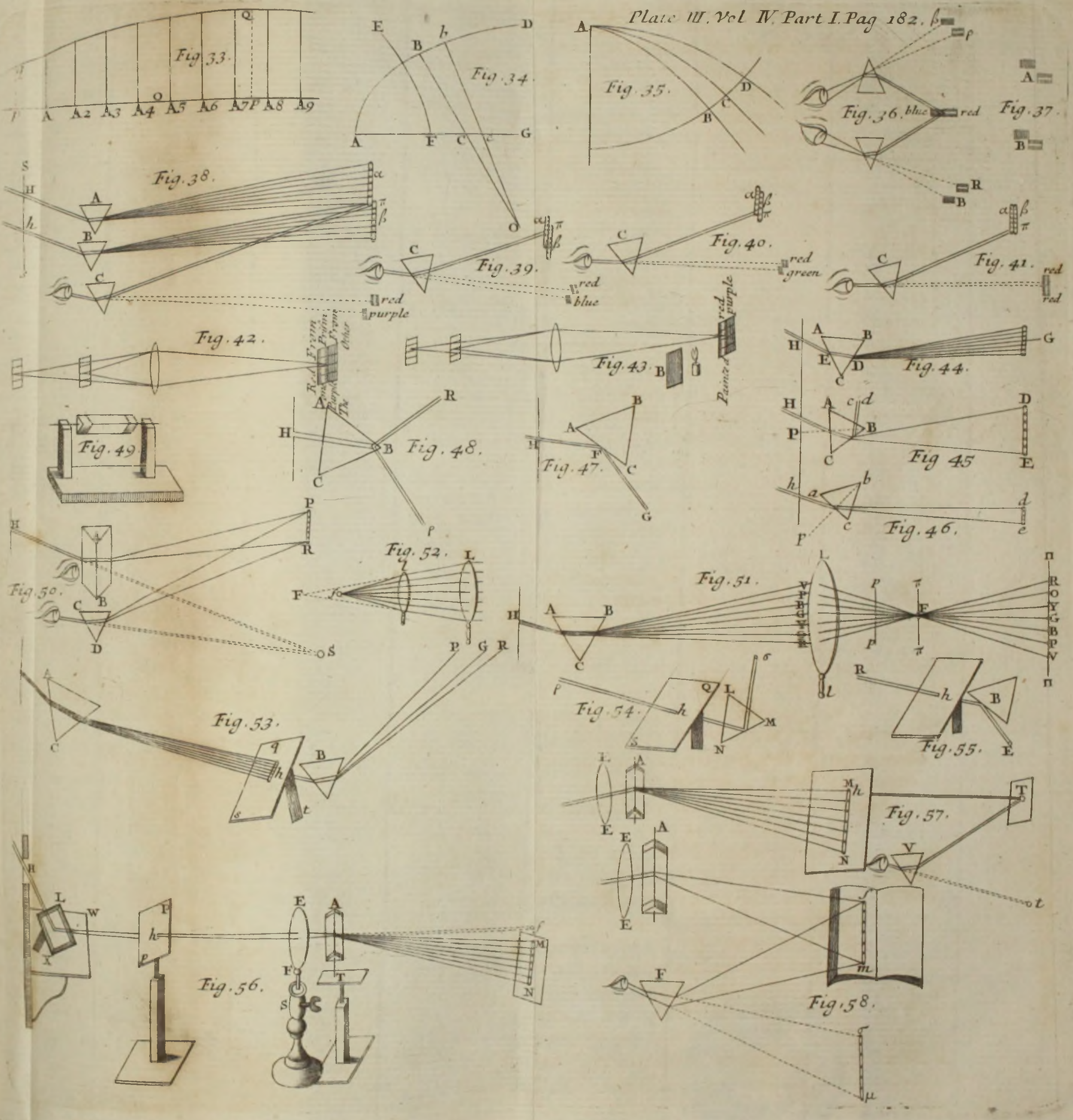
If you come nearer to *b*, ſo that the Eyes at ε 1, ε 2 receive the moſt and the leaſt refrangible Rays, there will be a double *Spectrum*, *viz.* a red and a purple one juſt touching, or upon one another: And the Phæ-Fig. 60. nomenon will anſwer as before, *Fig.* 60.

Fig. 61. If keeping both Eyes open, you direct their Axes towards *O* a Point nearer than the uſual Place of the compound *Spectrum S*, Fig. 61. which Point is in a Line from the Noſe *N* to the Point *S*; or in other Words, if you look full at *O*, or at the End of your Finger held in *O*, the red and the blue (or purple Spot) will appear to be divided from each other Fig. 62. after the Manner repreſented at *p r* in *Fig.* 62. where the red will appear to be on the Right-hand, and the blue on the Left.

To make plain what is meant by *ſeeing* the *Spectra p* and *r* whilſt we look full at *O*, I beg Leave to explain the Diſtinction between *looking* and *ſeeing*; that I may the better ſhew how this Phænomenon proves that the Senſation of different Colours is cauſed by Rays differently refracted.

Definition 1.] The *Optic Axis* is a Line which, going thro' the Center of

Plate III. Vol IV. Part I. Pag 182.



of the Convexity of all the Coats and Humours of the Eye, falls upon the Middle of the *Retina*, as *α a* or *A a*, Fig. 63. Fig. 63.

Def. 2.] To *look* at any Point, is to turn both Eyes towards it in ſuch Manner, that the *Optic Axes* making an Angle at the ſaid Point as *a*, the Rays from *a* may have the *Optic Axis* for their Axis, and (by their Convergence upon the *Retina* after Refraction in the *Eye*) may paint the Image of the ſaid Point upon the Middle of the *Retina* of each Eye, where the Optic Axis in each Eye falls.

Def. 3.] To *ſee* without *looking*, is to direct the Optic Axis to ſome other Place than to the Point which is then ſeen; and in ſuch a Caſe, the Image of the Point ſeen will be projected upon a Part of the *Retina* of each Eye, where the Optic Axis does not fall, namely, either nearer to the Noſe *N*, as in *Fig.* 61. at the Points of the *Retina* mark'd *n n*; or farther from the Noſe than the Middle of the *Retina*, as at *o o* in *Fig.* 64. Fig. 61. Fig. 64.

Whatever is *ſeen*, by being *look'd* at with both Eyes, always appears ſingle, by reaſon of the Communication between the Middle of the *Retina* in one Eye, and the Middle of the *Retina* of the other; there being no ſuch Communication between any other Part of the *Retina* in one Eye, and the correſpondent Part of the *Retina* in the other, when theſe correſpondent Parts are equally diſtant from the Noſe.

There is indeed a Communication between the nervous Fibres on the Right-ſide of the Retina *of one Eye, and the nervous Fibres on the Right-ſide of the* Retina *of the other Eye, and ſo of thoſe on the Left; but no ſingle Object can be ſo painted in each Eye, as to have its Image on the right or left Part of one* Retina *that communicates with the right or left Part of the other, of the ſame Bigneſs and at the ſame Time as in the other; becauſe in whatever Poſition the Object is, it muſt be nearer to one Eye than to the other, except it be juſt in a Line from the Noſe betwixt the two Eyes ſtreight forward.*

Hence it is, that if there be two Candles ſet before any one, the firſt at the Diſtance of one Foot, and the ſecond at the Diſtance of two Feet, from the Eyes; he that looks at the ſecond Candle at *B* will ſee it ſingle, but ſee the firſt Candle, or the Candle *A* double; one Appearance being in the Line *A D γ*, the other in *o A E*, becauſe it paints itſelf upon *o o* in the *Retina* of each Eye; which Points are not the middle Points, but farther from the Noſe than the Middles *m m*.

So if *B* be the firſt Candle, and *C* the ſecond, he that looks at *B* will ſee *C* double, becauſe it is painted in the *Retina* at the Points *n n* nearer the Noſe than *m m*; and ſo will appear to be in the ſame Poſition as *p r*, in *Fig.* 62. Fig. 62.

If *γ ρ* be two Candles ſo diſpoſed, that by the Interpoſition of a perforated Board *FF*, *γ* can paint itſelf only in the Eye *R*, and *ρ* in the Eye *L*. Upon making the Optic Axes meet at *B*, and to tend towards *ρ* and *γ*, *ρ* and *γ* will each paint an Image on the Middle of the *Retina* of each Eye, by croſſing their Rays at *B*: And thus the two Candles will appear to be but one, or rather to be in one Place, upon the Account of the Communication of the Middle of each *Retina*. But if Fig. 65.

inſtead

inſtead of the Candles, ϱ be a Piece of red Silk, and γ a Piece of green Silk, the ſame Poſition of the Eyes will make an Image at *B*, appearing like a red and green Spot together, without a Mixture of the Colours. If ϱ be a red hot Iron, and γ a Candle of Sulphur, the Phænomenon will be more diſtinct. If the Optic Axes be turned directly towards γ and ϱ, as if there was no Board *FF* in the Way, there will appear two Holes in the Board, the one having the red hot Iron in it, the other the Candle.

Fig. 60. Now if, of the refracted Rays of the Candle in the firſt Caſe, thoſe which diverge from each other, ſo as to fall into each Eye, cauſe the ſame Senſations reſpectively, as the Rays which come from a red hot Iron, and thoſe which come from a blue Candle; it is evident that the Candle in the firſt Caſe affords red-making and blue-making Rays after

Fig. 60. Refraction, and that thoſe Rays are differently refrangible; the red *bR* the leaſt refrangible, as declining leſs from the Perpendicular $\varpi\varpi$; and the purple as *bP*, declining moſt from the ſaid Perpendicular.

The ſame will *(cæteris paribus)* be found true in the intermediate Rays, and to be certain that the Experiment is as I have related it, the Planes *αf* and *fd* of the Bar may be covered with Paper.

An Univerſal Sphærico-Catoptric *Theorem, by Mr.* H. Ditton, n. 295. p. 1810.

III. The Finding of the Foci, both in Dioptrics and Catoptrics, eaſily follows from the Calculation for the Curves called Cauſtics. For nothing more is required, than that the Locus may be known in which the Radius (perpendicular to the Curve either refracting or reflecting) is a Tangent to the Diacauſtic or Catacauſtic Curve. Concerning which Method Mr. *Hayes*'s Book of Fluxions lately publiſhed may be conſulted. We ſhall undertake the Matter upon other Principles, as far as Catoptrics are concerned.

Fig. 66. Let *DEF* be a Portion of a concave Spherical Speculum, whoſe Center is *B*, Semidiameter *BE* or *BD*; alſo let *A* be a radiating Point placed in the Axis, from whence proceeds the Ray *AD*, which at the Point *D* is reflected in *DC*. Now the Diſtance of the Focus *C*, from the Vertex of the Speculum *E* is to be inveſtigated.

It is to be obſerved, that we ſuppoſe the Point *D* to be very near *E*. For the remoter Rays go beſide the Eye, which we place in the Axis *AE*, nor do they contribute any Thing to Viſion. And becauſe of the indefinitely ſmall Arch *DE*, the Angles *DAB*, *ADB*, and alſo their Sum *DBC*, are the ſmalleſt poſſible, and therefore will have the ſame ratio to one another as their oppoſite Sides. By making this the Principle of his Reaſoning, Dr. *Halley*, Profeſſor of Geometry at *Oxford*, arrived at his Dioptrical Theorem.

Theſe Things being premiſed, let $AB = b$, $BD = BE = r$, $BC = z$, $CE = r - z$, which for Brevity we will call f. The Quantities b and r are known; for the Semidiameter of the Speculum, and the Diſtance of the lucid Point from the Vertex are given; but z and f are unknown and required. Now in the Triangle *DAB*, it will be be Ang. $DAB \cdot ADB :: r \cdot b$. Alſo in the Triangle DBC, 'tis Ang. $BDC = ADB$ from the Nature of Reflection; and Ang. $DBC = DAB + ADB$, by *El. Eucl.* Therefore ſince Ang. DBC is as $r + b$, and Ang. BDC as b; it will be alſo Ang. $DBC \cdot BDC :: r + b \cdot b$. And then it follows from the Principle above-mentioned, $DC \cdot BC :: r + b \cdot b$. But becauſe the Point D is very near the Point E, DC will be eſtimated as equal to CE; and therefore it will be $CE \cdot BC :: r + b \cdot b$, that is, $f \cdot z :: r + b \cdot b$. And by comparing the Sums of the Antecedents and Conſequents to the Antecedents, $f + z \cdot f :: r + 2b \cdot r + b$. But $f + z = r$, therefore $r \cdot f :: r + 2b \cdot r + b$, whence $f = \frac{rr + br}{r + 2b}$. *Q. E. I.*

If we make $r + b = AE = d$, the Theorem will be abbreviated thus $f = \frac{rd}{2d - r}$. But in either Caſe the Theorem will ſerve for finding the Focus, whatever be the Form of the Speculum, or the Condition of the Rays.

Corol. 1. It will be $zd = df - rf$, or $AE \times BC = AB \times CE$, or which is the ſame Thing, the Line AE is harmonically divided in the Points A, B, C, E. For the foregoing Equality of Rectangles is the Property of a Line divided in harmonical Proportion. This appears to be true, becauſe $f = \frac{dr}{2d - r}$, and $z = r - f = r - \frac{dr}{2d - r}$; whence by ſubſtituting theſe Values, the Equation becomes plane. So that in every ſpherical Speculum, the Lines DA, DB, DC, DE, are Harmonicals; and the radiating Point, the Center, the Focus, and the Vertex, are Points that conſtitute an harmonical Diviſion.

Corol. 2. Firſt, if you make $d > r$, then by the Calculation f, or $\frac{rd}{2d - r}$, is alwas greater than $\frac{1}{2} r$. That is, if the Diſtance of the radiating Point is greater than the Semidiameter of the Speculum, the Diſtance of the Focus will always be greater than a Quarter of the Diameter.

Alſo it will always be $\frac{rd}{2d - r} < r$. That is, the Diſtance of the Focus will always be leſs than the Semidiameter of the Speculum.

Secondly, If you make $d = r$, it will be $f = \frac{rd}{2d - r} = r$. That is, if the radiating Point be placed in the Center of the Speculum, its Image will there be united with it.

Thirdly, if you put $d < r$, then the Expreſſion of f will either be

 poſitive,

positive, or negative, or infinite, according as the Quantity $2d$ is either greater or less than the Quantity r, or equal to it.

If $2d > r$, that is, if $d > \frac{1}{2}r$, then the radiating Point and the Focus lie on the same Side of the Speculum.

If $2d < r$, or $d < \frac{1}{2}r$, then the Image will be in the Axis of the Speculum produced beyond the Vertex.

If $2d = r$, or $d = \frac{1}{2}r$, the Image is at an infinite Distance, or the reflected Ray becomes parallel to the Axis.

Corol. 3. It may be determin'd very readily by Means of this Calculus, how the Motion of the Image corresponds with the Motion of the radiating Object in respect of the Speculum. Let the Distance of the Image from the Speculum be as before $\frac{dr}{2d - r}$, when the Distance of the Object is d. Now let the Distance of the Object be any how changed, and of d let it become nd, making n to stand for any Number Integer or Fraction. Then instead of the former Equation $f = \frac{dr}{2d - r}$, we shall have another Equation to a new Focus $F = \frac{ndr}{2nd - r}$. And if n is supposed to stand for an integer Number, this second Distance of the Object will be greater than the first; but if n be a Fraction, then it will be less than the first.

These Things supposed, if $d > r$, and n be an Integer, it will be $F < f$, that is, it will be $\frac{ndr}{2nd - r} < \frac{dr}{2d - r}$, or $2nddr - ndrr < 2nddr - drr$ as is manifest. That is, in a concave Speculum if the Distance of the Object be greater than the Semidiameter, then the Object withdrawing from the Speculum, the Image will approach nearer the Speculum. Again, let n denote a Fraction, and then it will be found, that $2nddr - ndrr > 2nddr - drr$, or $F > f$. That is, as the Object approaches nearer the Speculum, the Image at the same Time will withdraw farther from it.

Now let it be supposed that $d < \frac{1}{2}r$; and let nd be any other Distance of the Object, which is always less than $\frac{1}{2}r$. Then will $2nddr - ndrr$ and $2nddr - drr$ be negative Quantities, or $ndrr - 2nddr$ and $drr - 2nddr$ will be positive Quantities. And if n be an integer Number, then will $ndrr - 2nddr > drr - 2nddr$, or $F > f$. But if n be a Fraction, then will $ndrr - 2nddr < drr - 2nddr$, or $F < f$. That is, if in the Concave Speculum the Distance of the Object

ject

ject be lefs than a fourth Part of the Diameter of the Speculum, then the Object withdrawing from the Speculum, the Image will withdraw alfo. Or if the object approaches towards the Speculum, the Image will alfo approach towards it.

Now all thefe Conclufions which we have deduced, by tracing the Footfteps of the Calculation, are included in one *Scholium*, which Dr. *Gregory*, Profeffor of Aftronomy in the Univerfity of *Oxford*, has delivered in his Catoptricks.

Corol. 4. In the Equation $f = \frac{dr}{2d - r}$, if d be fuppofed infinite, it will be $f = \frac{1}{2} r$, which is a Rule for parallel Rays, or for a radiading Object placed at an infinite Diftance. The fame Thing will follow, if b be made infinite in the Equation $f = \frac{rr + rb}{r + 2b}$.

Corol. 5. In the Equation $\frac{dr}{2d - r} = f$, the negative Sign of the Quantity r being made pofitive, it will be $f = \frac{dr}{2d + r}$; or in the Equation $f = \frac{rr + br}{r + 2b}$, changing the pofitive Sign into a negative, it will be $f = \frac{rb - rr}{2b - r}$; which gives a Rule for a Speculum which is convex towards the radiating Object. This Change of the Sign is very plain; for as in the concave Speculum it is $d = r + b$, fo in the convex it will be $d = b - r$.

Corol. 6. In a convex Speculum, (thofe Things remaining which we have taken Notice of at *Corol.* 3. about a concave Speculum) it will appear, that if n be an integer Number it will be $2rndd + ndrr > 2rndd + drr$; and if n is a Fraction it will be $2rndd + ndrr < 2rndd + drr$. That is, if the Object withdraws from the Speculum, or if it approaches towards it, the Image in like Manner will recede or approach.

Alfo it appears in a convex Speculum, that if the Object withdraws to an immenfe Diftance, yet that its Image will not recede from the Vertex beyond a fourth Part of the Diameter, but will there ftop in the middle Point between the Center and the Vertex. For fuppofing d or b to be infinite, it will be $f = \frac{dr}{2d}$ or $\frac{br}{2b}$, in each Cafe $f = \frac{1}{2} r$.

To thefe may alfo be join'd the Solution of a Catoptrical Problem. To find fuch a Pofition of the radiating Point in refpect of a given Speculum, that the radiating Object may have a given Ratio to its Image made by the Speculum. Let the given Ratio be that of r to q, and by the Symbol O let the Magnitude of the Object be denoted, let I be the Image, d the Diftance of the Object, and f of the Image from the *Speculum*. Now as Dr. *Gregory* has demonftrated, it will be $O . I :: d . f$. That is, the

Object and the Image are directly proportional to their Distances from the Vertex of the Speculum. And because it is required that it may be $O . I :: r . q$; it must also be $d . f :: r . q$; or instead of f taking its Value, $d . \frac{dr}{2d - r} :: r . q$, whence $2dq - rq = rr$, and $d = \frac{rr + rq}{2q}$. Now because $dr = \frac{r^3 + r^2 q}{2q}$, and $2d - r = \frac{rr}{q}$; it will be f or $\frac{dr}{2d - q}$ $= \frac{r^3 + r^2 q}{2q} \div \frac{r^2}{q} = \frac{r^3 + r^2 q}{2q} \times \frac{q}{r^2} = \frac{r + q}{2}$, which is the Distance of f or of the Image from the Speculum, agreeing to the Distance of the Object. Therefore if the Object be set at the Distance $\frac{rr + rq}{2q}$, its Image made at the Distance $\frac{r + q}{2}$ being compared to it, will have the Ratio of q to r. Or it will be, $O . I :: r . q$. For it is $O . I :: d . f ::$ $\frac{rr + rq}{2q} . \frac{r + q}{2} :: r . q$.

Here we have considered the radiating Object and the Image as if they were Lines; but if we consider them as Surfaces then it will be $O . I :: dd . ff :: r . q$; so that then we shall come to this Equation $4dd - 4qdr = r^3 - qr^2$, from whence the Value of d may be found very easily by the common Methods.

A Way for Myopes *to use* Telescopes *without Eye-Glasses*, &c. *communicated by Dr.* J. T. Desaguliers, n. 361. p. 1017.

IV. *Myopes* may use *Telescopes* without Eye-Glasses, an Object-Glass alone becoming as useful to them, and sometimes more than a Combination of Glasses.

Lemma 1.] What is required of a Telescope is to give large and distinct Vision; that is, to make the Object (as in *Galilæo*'s Telescope) or its Image (as in the Telescopes made up of convex *Lentes*) appear under a great Angle, and to have all the Rays of those Pencils that enter the Eye, meet in a Point upon the *Retina* of the Eye, on their respective Axes.

Fig. 67.

The 67th Figure represents the Combination of two convex *Lentes* for the astronomical or inverting Telescope; where the abovementioned Requisites are obtain'd. *A B* is the Object supposed at a vast Distance from the objective *Lens L L*, so that Rays coming from the Extremity *A* of the Object, will fall upon the *Lens L L*, in the same Manner as if they were parallel to their Axis *A X*; and after passing the Glass unite at *a*, where they project the Image of the Point *A*, from whence diverging, they fall on the Eye-Glass *l l*, and having passed through it, go on parallel to each other, and enter the *Cornea* of a common Eye *E*, which unites those parallel Rays upon its *Retina R R R* at *a*, where the Image of *a* is projected: The same may be said of the Rays that come from *B*, and after their several Refractions thro' the two Glasses and the Coats and Humours of the Eye, meet upon the *Retina* at β, where they project the diſtinct Image of the Point *b*. The Rays that come from all the Points of the Object *A B*, being affected after the ſame Manner, give a diſtinct Image of thoſe Points upon the *Retina*, and therefore the Object does appear diſtinct.

The Object will alſo appear magnified in the ſame Proportion as the Angle $l C l =$ to $b M a$ (under which its Image is ſeen) is greater than the Angle *A C B* under which the Object *A B* would be ſeen by the naked Eye; as is more at large demonſtrated by dioptrical Writers.

Lemma 2.] If parallel Rays fall upon the *Cornea* of a *Myops*, or ſhort-ſighted Perſon, they will unite in the Eye before they come to the *Retina*, the farther from it, the more convex the Eye is; but if the Rays which fall upon the *Cornea*, diverge in Proportion to the too great Convexity of the Eye, as from *D*, ſuch Rays will be ſo refracted by the Coats and Humours of the Eye, as to meet in one Point upon the *Retina* Fig. 68, 69. *R R*, where I have in the Scheme neglected the Refraction of the Rays paſſing out of the *chryſtalline K* into the *vitreous* Humour V, as I do in the other Caſes.

This *Lemma* is alſo demonſtrated by dioptrical Writers.

Lemma 3.] If two Pencils of Rays (in each whereof all the Rays are parallel to the Axis, as *a C*) fall upon different Parts of the *Cornea*, at the greateſt Diſtance from one another that can be allow'd for thoſe Rays to enter the Pupil *P P*, their *Axes* will, after entring the *aqueous* Humour, converge, and meet either in the vitreous or chryſtalline Humour, according to the Convexity of the *Cornea* through which they paſſed, and diverge again before they come to the *Retina*; the Rays of each Pencil converging upon their reſpective Axes, to the Place where Fig. 70. the ſaid Axes croſs one another.

Demonſtration.] The Axes *a C a*, α *C* α, falling obliquely upon the *Cornea* at *C C*, and entring from *Air* into the *aqueous* Humour, will be refracted towards the Perpendicular to *K*, where ſtriking more directly upon the *chryſtalline*, they will go on to α, *a*, upon the *Retina R R R R*, decuſſating at *V* within the *vitreous* Humour. The other Rays *r*, *r*; ρ ρ, after their Refraction in the *aqueous* Humour, fall more obliquely on the *chryſtalline*, and therefore are refracted again ſo as to meet at *V*, Fig. 70. where the Axes alſo meet, and thence go on to the *Retina R R R R*.

Lemma 4.] But if the Axes of the abovementioned Pencils are parallel, the Rays that accompany them diverging from a Point ſo near the Eye, that the Divergence may be proportionable to the too great Convexity of the Eye; then only the Axes will meet in the Eye before they come to the *Retina* (by *Lemma* 3.) but the other Rays will not unite upon their reſpective Axes, till they come to the *Retina*, (by *Lemma* 2.)

Prop.] I ſuppoſe the Eye of the *Myops* ſo convex that he can ſee no farther than a common Eye, with the Eye-Glaſs of a Teleſcope before it: Then the Eye of the *Myops* being in the Place of the Eye-Glaſs, will receive the Rays diverging from the ſeveral Points of of the Image (projected by the Object-Glafs in its *Focus*,) in fuch Manner, that they will, after the feveral Refractions, meet in refpective Points on the *Retina*; and the Axes of the Pencils, which come from the Extremities of the Object, will, in the Eye, make the Angle $B V A$ = to $b c a$, under which the Image $a b$ is feen, by *Lemma* 4. The *Cornea* and *aqueous* Humours here fupply the Place of the Eye-Glafs, and the *chryftalline* and *vitreous* Humours that of a common Eye. In the Figure R is the *Retina*, V the *vitreous* Humour, and $K K$ the *chryftalline* Humour; and the Image $b a$ is fuppofed to be brought down from the 67th Figure, which reprefents the *Aftronomick Telefcope*; the too great Convexity of the Eye here being in the Place of an Eye-Glafs.

Fig. 71.

An Objection may be made to this, *viz.* that $P P$ the Pupil of the Eye being fmall, will take in but a very little Image, or a fmall Part of the Object: But then if the Eye be moved fucceffively, to all the Parts of the Space where the Eye-Glafs was, it can take any Part of the Object; and if the Object-Glafs be large, which may more eafily be made than a large Eye-Glafs, and the Tube a Foot wide or wider, as much may fucceffively be taken in, as if an Eye-Glafs might be had of a Foot Diameter. A little Practice may make any *Myops* fo ready, as to keep an Object when once found, though the Place where he ftands be fhaken. It would not be amifs to hold a *Lens* in one's Hand (for an Eye-Glafs) to find the Object at firft, till Cuftom has made it eafy without it: When once the Object is found, it may be eafily kept.

An Eye more fhort-fighted than I have fuppofed, will perform the Office of a more convex Eye-Glafs, being brought nearer to the diftinct Bafe of the Object-Glafs; and an Eye lefs convex, the Office of a lefs convex Eye-Glafs: But with this Difference, that the more convex the Eye is, the eafier may any Part of the Object be found, and the larger and more lucid it will appear.

I have feen *Saturn*'s Ring very plain with an Object-Glafs of little more than fix Foot *Radius*, without an Eye-Glafs.

I have alfo found out a Way for the *Presbitæ* to make ufe of an Object-Glafs, by placing their Eye nearer the Lens than its Focus, by fo much as their Eye is flatter than a common Eye, fo as to make (as it were) the Telefcope of *Galilæo*; the flat Eye ferving as a common Eye armed with a concave Lens. I have fo fixed the Telefcope as to make a *Presbita* read at a great Diftance a fmall Print. The Truth of this may be eafily demonftrated, if it be required.

If this Experiment be made at Sea with a very large Tube, big enough to put in the Head and move it about, and the Object-Glafs be alfo large, it may not perhaps be difficult to obferve the Eclipfes of the *Satellites* of *Jupiter*, which I would recommend to the Confideration of thofe that would try for the Longitude by fuch-like Obfervations.

Experiments upon Metals, with the D. of Orleans's *Burning-glafs, by Mr.* Geoffroy, n. 322. p. 374.

V. The Burning-Glafs is three Foot in Diameter; it collects the Rays of the Sun at ten Foot Diftance, where it forms a Focus of about three Inches over, which is again contracted by means of another Glafs-Lens

Lens to an Inch Diameter, and conſequently is rendered three Times as ſtrong.

I ſhall only relate here what I have obſerved upon the four Imperfect Metals, *viz.* Iron, Copper, Tin and Lead: I ſhall ſay nothing at preſent of Gold or Silver; becauſe as their Analyſis ſeems to me much more difficult than of the other Metals, I ſhall forbear Inquiries upon them, till I have examined as far as poſſible into the Nature and Compoſition of the former.

What was a great Hindrance to me in making theſe Experiments in the Focus of the Glaſs, was the Difficulty I had to find any Matter capable of holding the Metals in Fuſion.

Charcoal, which is commonly made Uſe of, is indeed a very proper Subſtance; but it is impoſſible with it to vitrify any one of the Metals: The Particles of the Metal, when held any long Time in Fuſion in the Focus of the Glaſs, diſſipate and fly away in Fume or ſmall Particles; and as long as any Part remains, that little that does remain, is always metallick, until the whole be quite evaporated.

The Reaſon of which I take to be this. Charcoal is a Subſtance deeply impregnated with oily or ſulphurous Parts (if I may ſo call them.) The firſt Effect that Fire has upon Metals is to ſeparate the ſulphurous Parts: Now, if in Proportion as the Sulphur is ſeparated from the Metal, the Body that ſupports the Metals furniſhes it anew with other ſulphurous Parts, the other Principles will never ſeparate, and the Metal will always remain Metal. And nothing but the greateſt Degree of Fire is able to raiſe and ſeparate the Sulphur, and that but by little and little, and in very ſmall Particles.

I had then Recourſe to another Matter, that could not any ways be ſuſpected of containing any oily Parts. *Tſchirnhaus* (to whom we are obliged for making of theſe large Glaſſes, and the firſt Experiments that have been made with them) ſays, he has vitrified Metals by holding them in *China*-Ware. It is true, this ſucceeds pretty well, provided the Pieces be very thick, and the Glazing taken off: But the Difficulty I had to find a ſufficient Quantity of thick and proper *China*-Ware to make all theſe Experiments, forced me to have Recourſe to more common Subjects, as well as ſuch, if poſſible, as were leſs capable of melting.

Of all the different Sorts of Matter that I made Trial of, what ſeemed beſt were the common Coppels and Plates of grey Fire-ſtone. The Coppels hold the Metal a long Time in Fuſion in the Focus of the Glaſs without melting; excepting Lead, which eaſily runs through them as ſoon as it vitrifies, and helps to diſſolve them. The Plates of Fire-ſtone bear the Heat of the Focus much longer than any other Matter; but great Care is to be taken in heating them without breaking, 'till they become red hot, and when they are hot the leaſt cold Air makes them melt. Neverthele ſs this is the only Subſtance that I have uſed with moſt Succeſs, to hold Metals a long Time in Fuſion, though with the greateſt Caution that was poſſible, to avoid the Inconveniencies aforementioned.

An-

Another Thing that has hinder'd me from carrying on these Inquiries upon Metals so far as I could have wished, has been the few clear Days we have had for these two Years past; for the greatest Part of these Experiments require a bright, strong, and constant Sun, to keep the Matter a long Time in perfect Fusion: And I have scarce had, for this last Year, above three or four such Days as I could wish for; the Sky having been almost every Day covered with Clouds about Noon, which is the Time of the Day fit for these Experiments.

On Iron. I placed in the Focus of the Burning-Glass a Piece of forged Iron of about a Drachm Weight: It turned red hot, and its Surface was cover'd with a black Matter like Pitch or Tar. If one withdraws the Iron out of the Focus in this State, this Matter fixes itself on the Surface of the Metal, and there forms a small Skin, or a very fine blackish Scale, which is commonly very easily separated by striking upon it; and that Part of the Iron that was covered with this Scale appeared blacker than ordinary. This Scale is some of the sulphureous Part of the Iron that rises to the Surface of the Metal when it is ready to melt, and there remains for some Time before it exhales. It is plainly this sulphureous Part that rises upon Iron and polished Steel when heated, and gives them all those different Colours, from a Yellow to a Violet, Water-colour, or Black.

If one continues to hold this Piece of Iron on the Charcoal, it intirely melts; and at the same Time casts forth very bright Sparks in a great Quantity, sometimes to above a Foot Distance from the Coal.

If one saves what flies off during this Sparkling, by holding a Sheet of Paper under the Coal; we find that they are so many very small Globules of Iron, and the greatest Part of them hollow.

All the Iron that is held in Fusion upon the Coal, flies away in Sparkles after this Manner, till none remains. Sometimes the Metal leaves off sparkling, when the Coal is in part consumed, and covered with a Bed of Cinders, upon which lies the melted Iron. For as the sparkling of the Iron seems to me to proceed from nothing but the oily Parts of the Coal acting upon those of the Metal, the Cinders hinder this Oil from passing from the Coal to the Iron, so that it remains quietly in Fusion. But if through any Shake, or the like Accident, the Cinders are so removed, that the Iron comes to touch immediately the Coal, it will begin to sparkle afresh. Sometimes the Heat, that keeps in Fusion the Metal, vitrifies also the Cinders; and this vitrified Matter mixing with the Metal makes a considerable Ebullition. If one at this Instant withdraws the Metal out of the Focus, it appears half vitrified, or reduced to a blackish friable Mass. Other Times this vitrified Matter swims on the Surface of the Metal, and there forms itself into Drops, that are sometimes clear and transparent, and other Times opaque, according as it is more or less mixed with the Metal.

Further-

Furthermore, if after having let the melted Iron cool upon the Coal, one expoſes it again to the Focus of the Glaſs upon the Stone, it ſparkles afreſh till it is all conſumed; which common Iron will not do, that has not been expoſed to the Heat of the Focus upon Charcoal. This Sparkling probably proceeds from the ſudden Rarefaction of the oily Parts of the Coal, with which the Pores of the Iron are ſo plentifully ſaturated; or perhaps it may be cauſed by the Salts of the Iron acting on the Oil of the Coal.

I expoſed to the Focus, upon a Stone-ſlate, Iron and Steel; they grew red hot, and melted without crackling or caſting off any Sparks; they ſmoaked very conſiderably, and the melted Metal turned by little and little like an Oil. After having withdrawn this melted Matter out of the Focus, it fixed in a Regulus-like, friable Maſs, and appear'd ſometimes lightly ſtriated, or ſhot into ſharp Points like Needles. Tho' this Matter does not appear at all tranſparent, yet we may look on it as the Beginning of Vitrification, or a middle State between Metal and Glaſs; for it would vitrify in the End like other Metals, if one could hold it a ſufficient Time in the Focus without melting or mixing with what ſuſtains it: But continuing it long in the Focus, the extreme Heat of the Sun, that is neceſſary to keep it in perfect Fuſion, melts likewiſe the Stone or Coppel that contains it, the Reſult of which Mixture is a brown or greyiſh Sort of Ennamel.

We may then take this Regulus Maſs to be a half vitrified Iron, by reaſon it is deprived of a great Part of its Sulphur. If one adds to this Maſs a Sulphur like that which was taken from it, from being friable it turns very hard and malleable; and the Dulneſs it had before, changes to the Brightneſs of a Metal. This is what I have experienced in expoſing again this Matter to the Focus upon Charcoal: It melts, and ſo continues a conſiderable Time in Fuſion without ſparkling, but at laſt it ſparkles with the ſame Briſkneſs as Iron itſelf; and when withdrawn from the Focus, appears nothing different from melted Iron.

It appears from theſe Experiments, that Iron contains a Sulphur or oily Subſtance, that renders it bright, malleable, and eaſy to melt.

That this Sulphur is raiſed by the Fire of the Sun, when the Metal is for ſome Time held in Fuſion in the Focus of the Glaſs.

That this ſame Sulphur may be raiſed by the Flame of common Fire, which tho' not ſtrong enough to melt the Iron, yet is able to reduce it to an Eſchar or Sort of Ruſt.

That Iron deprived of this ſulphureous Part, melts into a Regulus, or brittle and friable Maſs, in Colour much like Antimony.

That if one can hold a ſufficient Quantity of this Matter long enough in the Focus by itſelf, without melting or mixing with the Body that contains it, it perfectly vitrifies.

That this Glaſs or metallick Regulus, with the Help of a little Oil, returns to its former State of a Metal.

That it reaſſumes this metallick Form upon Charcoal, by drawing thence this oily Subſtance.

That, in ſhort, this oily Part contained in the Coal, is little different from the Sulphur of Iron. Neverthelefs we muſt imagine it to differ in ſome Particulars; in that melted Iron, that has been ſaturated with it, crackles and ſparkles very much when melted again upon the Stone or Coppel.

Iron being the only Metal in which I have obſerved this Sparkling, I take it to be a Property peculiar only to Iron, and not to any other Metal. Perhaps we may attribute it to the vitriolick Salt that this Metal ſo plentifully abounds with, which is very greedy of Sulphurs.

To this ſame Greedineſs alſo, with which the vitriolick Salt of Iron abſorbs the oily Part of the Coal, we may attribute the Eaſineſs with which Iron conſumes the Coal; for there is no other Metal that ſo ſoon waſtes the Coal in the Focus of the Glaſs, as Iron does.

Another Obſervation is, that it is the only one of the four imperfect Metals, on which vitrified Drops ariſe, while it is in Fuſion upon the Coal: The Reaſon of which I have not yet been able to diſcover.

On Copper. Copper expoſed to the Focus of the Burning-Glaſs, at firſt turns white on its Surface, and afterwards grows black, and is covered with a Kind of Skin, or black, furrowed, and uneven Scales, till at laſt it quite melts.

I have withdrawn this Metal out of the Focus, as ſoon as this white Colour has appear'd, and after it has been cold, found nothing extraordinary on its Surface, which has again by little and little recover'd very near the ſame Colour as it had before.

I have not been able to diſcover from whence this white Colour proceeds, unleſs we may attribute it to ſome volatile arſenical Salt contained in the Copper, and driven by Extremity of Heat to the Surface of the Metal; or whether it purely proceeds from the Alteration that is made in the groſſer Parts of the Surface of the Metal when it begins to melt. The black Colour that Copper afterwards takes, ſeems to be cauſed by the ſulphureous Matter that melts firſt in this Metal as well as Iron, and is raiſed to its Surface by the extreme Heat.

I placed a Piece of Copper in the Focus upon Charcoal: It melted, and emitted a very thin Fume, and by little and little diminiſhed till it was all evaporated.

I put a Piece of red Copper on a Coppel into the Focus of the Glaſs: It melted, and ſent forth ſome thin Fumes; and after it had been ſome Time in Fuſion, it turned liquid like an Oil. I withdrew this melted Matter, and as it grew cold, it fixed into a Regulus of a reddiſh brown Colour, which was hard, brittle, and not ductile under the Hammer. If one breaks it, it turns into a red Powder like Cinnabar of Antimony; and when viewed with a Microſcope, appears ſo many little, red, tranſparent Grains, like ſmall Rubies; inſomuch that one

one would readily take this Regulus to be a deep coloured red Glass.

I endeavoured to make this vitrified Copper spread abroad in melting, by mixing it with common white Glass; for which End I powder'd some of this vitrified Copper and common Glass, and mixing them, melted them together; but the Mixture when in Fusion took at first a beautiful green Colour, and continuing it longer in the Focus, it turned bluish. I believe we may attribute this Change of Colour to the Alcali Salts of the Glass acting on the Particles of Copper; for those Salts usually draw a green or bluish Tincture from this Metal.

To preserve therefore this red Colour of the vitrified Copper, when mixed with common Glass, I made Use of this Expedient. I melted in the Focus upon a Coppel a Piece of Copper, and as soon as it began to vitrify, I cast upon it some common Glass; as soon as the Glass was melted, I took them together out of the Focus without confusing them; and as soon as they were cold, separated the Regulus from the Glass as well as possible; and picked out of it some Pieces of the Glass, loaded with some very small red transparent Particles of the Regulus.

This vitrified Copper is then nothing but Copper, deprived, by Means of Heat, of the sulphureous Part, that gave it the Form of a Metal. A Proof, that this metallick Form proceeds from nothing else but this Sulphur, is, that if one exposes this vitrified Copper to the Focus upon Charcoal, it reassumes, in a little Time, the Colour and Consistence of melted Copper; and as it grows cold, fixes into a good red malleable Copper, as fine and hard as it was before it was vitrified.

It follows from these Experiments, that the Basis of Copper is a red Earth susceptible of Vitrification.

That this Earth receives its metallick Form from a sulphureous Substance, in Appearance no Ways different from the Oil of Vegetables or Animals.

That one may deprive Copper of this Oil, by holding it long enough in the Focus, or by calcinating it in the Flame of common Fire.

That Charcoal restores again this oily Part to Copper, and at the same Time its metallick Form.

It appears further, that the Oil of the Coal has not so considerable an Effect upon Copper, as it has upon Iron.

Copper exposed a long Time to the Focus upon a Stone or Coppel, fumes very much, and diminishes in Weight very considerably. I don't think that this Fume is only the sulphureous Part of the Metal, the Evaporation of which must be insensible; but I believe that with this Oil there is mixed a great deal of the earthy, vitrifiable Part of the Metal, which the Heat of the Sun sublimes and raises in Flowers.

On Tin. Tin expofed upon Coal to the Focus of the Burning-Glafs, melts and emits a grofs, white, thick Fume, until it is all confumed in Vapours.

If one melts Tin upon a Coppel in the Focus of the Glafs, it fumes very much, and its Surface is covered with a white rarefied Calx; on which, by little and little, arifes a Tuft, or Heap of fharp, Needle-like, tranfparent chryftalline Particles, confifting of an infinite Number of fmall Points.

If one continues to hold this Mafs in the Focus upon the Stone, thefe Chryftals at Length leave off fuming, and remain fixt, while the Stone melts and vitrifies.

I took Calx of Tin, which is Tin reduced to a grey Powder by Means of Fire, that has taken away by Calcination great Part of its oily Subftance, and expofed it on a Coppel to the Focus, where it fumed again very much, and was reduced into fharp chryftalline Particles confifting of other fmall Points.

In re-expofing thefe chryftalline Particles to the Focus upon Charcoal, they melted very eafily, and took again the Form of Tin; the Coal having furnifhed them with the fulphureous Part that the Fire had before taken away. Every body knows, that if one add any Fat, or the like inflammable Matter, to the Calx of Tin, when red hot in the Crucible, it reaffumes immediately the Form of Tin.

Thefe Experiments fhew, that Tin contains a Sulphur that is very eafily feparated, fince common Fire can do it fo readily; and that this Metal calcined, or deprived of its Sulphur, is eafily faturated again with it from the oily Part of any inflammable Matter whatfoever.

It proves alfo, that the metallick Earth, which is the Bafis of Tin, is a chryftalline Earth, very difficult to be melted; fince common Fire cannot vitrify this Metal by itfelf, and that the Heat of the Sun, in the Focus of this large Burning-Glafs of the *Palace-Royal*, cannot perfectly melt the Calx, into which this Metal is reduced. We may prefume, that the Chryftallization, or reducing of this Metal into fharp-pointed Particles, proceeds from the Force of the Sun's breaking and melting together into a Sodder (if I may fo fpeak) fome of thefe fmall Chryftals, by Degrees, as the fulphureous Part leaves them; it not being ftrong enough to melt them all down together in one entire Mafs.

On Lead. I took Lead, and held it in Fufion upon Charcoal in the Focus of the Glafs: It all wafted away in Abundance of Fumes.

I expofed the like Quantity of Lead upon a Stone to the Focus, where it caft forth great Quantities of Fumes, and by little and little changed into a fluid Liquor like Oil or melted Rofin. This Liquor, as it grew cold, fixed into Glafs; which has this peculiar to itfelf, that it is difpofed into Plates like *Venetian* Talk, and that it is flabby, foft soft to the Touch, transparent, and in some Parts of a greenish or reddish Yellow.

In continuing this Matter in the Focus, it spread upon the Stone like Varnish; and at last penetrating it, helped to melt it.

I placed this talky Earth in the Focus upon Charcoal: It melted, and in a little Time after reassumed the Form of melted Lead. I withdrew it from the Focus, and having let it cool, found it nothing different from Lead.

These Experiments shew, that there is in Lead, as well as the other imperfect Metals, a sulphureous Part, that is easily separated by common Fire, or the Heat of the Sun; and that this Metal has for its Basis a foliated or talky Earth.

On Quicksilver.

I shall add here some Experiments, that I have made upon Quicksilver; though I cannot yet draw any positive Conclusions from them, not having prosecuted them so far as is necessary for that Purpose.

I placed Quicksilver in the Focus of the Burning-Glass upon Charcoal, upon the Coppel, and upon the Stone: It all immediately dispersed, and exhaled in a very thick Fume.

I exposed upon the Stone to the Focus some Mercury *Precipitate per se*, in a Degree of Heat equal to that of Digestion: It seemed to melt, but presently dispersed in Vapours; there remained a small Quantity of a very rarefied Dust like a Froth or Scum; but continuing it in the Focus, it melted, and gathered into a yellowish Glass, in which one might distinguish some Particles of Metal like Silver.

I exposed some Mercury *Precipitate per se* upon Charcoal: It fumed very much; and, as it melted, one might see little Globules of Mercury unite and form themselves together upon the Coal, but they dispersed again presently in Vapours.

These Experiments seem to prove, that there is in Quicksilver a Sulphur, that may be separated by a very gentle Heat, such as that of Digestion.

That as soon as this Sulphur is taken away, it loses its Fluidity and Brightness.

That the Basis of Mercury is a Calx, or red Earth.

That this Calx does not melt into Glass as the Calx of other Metals, because it is too volatile, and as soon as it melts is evaporated by the Heat.

That if one restores to this Calx a Sulphur, by exposing it again to the Focus upon Charcoal, it reassumes immediately its metallick Brightness and Fluidity, and becomes Quicksilver.

I cannot tell whether this light Earth, that remains upon the Stone after the Evaporation of the Calx of the Mercury, be a Part of the Earth of the Mercury more exactly deprived of its Oil, and consequently more fixt and proper for Vitrification; or whether it may not be some Matter foreign to the Mercury, that fixes itself, and remains behind

behind at its Evaporation. But this I shall examine more particularly hereafter.

The Result of all these Experiments is, that these four Metals, which we call imperfect, *viz.* Iron, Copper, Tin, and Lead, are composed of a Sulphur or oily Substance, and of a metallick Earth capable of Vitrification.

That from this Sulphur proceeds the Opacity, Brightness, and Malleability of a Metal.

That this metallick Sulphur does not appear at all different from the Oil of Vegetables or Animals.

That it is the same in Mercury as in the four imperfect Metals.

That these four Metals have, for their Basis, an Earth susceptible of Vitrification.

That this Earth is different in every one of these four Metals, in that it vitrifies differently in each of them.

And that on this Difference in vitrifying depends the Difference of Metals.

It remains, that I should examine more particularly the Nature of these Earths, or metallick Vitrifications, to know if any other Principles or Substances may be separated from them: But this I shall endeavour to do hereafter, in prosecuting the Analysis of these Metals, as far as possible.

Experiments with Mr. Villette's *Burning-Glass in* June, *by Dr.* J. Harris, *and Dr.* J. T. Desaguliers, n. 360. p. 976.

VI. This Miroir is a Concave 47 Inches wide, and ground to a Sphere of 76 Inches *Radius*; so that its Focus is about 38 Inches distant from the Vertex of the Glass. The Metal of which it is made is a Mixture of Copper, Tin and Tin-Glass, whose Reflection has something of a yellow Cast. The Concave-Surface has scarce any Flaws, and those very small; but the Convex-Side, which is also polished, has some Holes in it.

Having held several Bodies in the Focus of this Miroir, we observed what happened to them whilst exposed to this great Heat; and with a half-second Pendulum, took Notice of the Time in which any material Change happened to them.

The Experiments were as follow, and made from Nine till Twelve in the Morning.

N° 1. A red Piece of a *Roman Patera*, which began to melt in 3 Seconds, was ready to drop in 100.

2. Another black Piece melted at 4, and was ready to drop at 64 Seconds.

3. Chalk taken out of an *Echinus Spatagus* filled with Chalk only, fled away in 23 Seconds.

4. A Fossile-Shell calcin'd in 7 Seconds, and did no more in 64.

5. A Piece of *Pompey*'s Pillar at *Alexandria* was vitrified in the black Part in 50 Seconds, and in the white Part in 54.

6. Copper-Oar

6. Copper-Oar, that had no Metal in it viſible, vitrified in 8 Seconds.
7. Slag, or Cinder of the ancient Iron-work ſaid to have been wrought by the *Saxons*, ready to run in 29 Seconds and an half.

Here the Glaſs growing hot, burned with much leſs Force.

8. Iron-Oar fled at firſt, but melted in 24 Seconds.
9. Talk began to calcine at 40 Seconds, and held in the *Focus* 64.
10. *Calculus humanus* in 2 Seconds was calcined, and only dropped off in 60.
11. An anonymous Fiſh's Tooth melted in 32 Seconds and an half.
12. The *Asbeſtos* ſeemed condenſed a little in 28 Seconds; but it was now ſomething cloudy: Monſ. *Villette* ſays, that the Glaſs uſually calcines it.
13. A golden Marchaſite broke to Pieces, and began to melt in about 30 Seconds.
14. A Silver Sixpence melted in 7 Seconds and an half.
15. A King *William*'s Copper Halfpenny melted in 20 Seconds, and ran with an Hole in it in 31.
16. A King *George*'s Halfpenny melted in 16 Seconds, and ran in 34.
17. Tin melted in 3 Seconds.
18. Caſt Iron in 16 Seconds.
19. Slate melted in 3 Seconds, had an Hole in 6.
20. Thin Tile melted in 4 Seconds, had a Hole, and was vitrified through in 80.
21. Bone calcined in 4 Seconds, and vitrified in 33.

An Emerald was melted into a Subſtance like a Turquois Stone.
A Diamond weighing 4 Grains loſt $\frac{7}{8}$ of its Weight.

VII. Experience (as well as the Authority of Dr. *Hook* in his Preface to his *Micrographia*) aſſures us, that ſingle Magnifying-Glaſſes (when they can be uſed) are preferable to Microſcopes, compoſed of two or more Magnifying-Glaſſes: Nor are the late Improvements, made by Magnifying-Glaſſes, ſo much owing to the making *Them*, and compoſing *Microſcopes*, as to the *Methods* of applying Objects for the *Advantage* of Light; in which, I hope, the following deſcribed *Inſtruments* will not be found inferior to any yet made, at leaſt commonly ſold. *A Pocket-Microſcope, by Mr.* J. Wilſon, n. 281. p. 1241.

This *Sett* of Microſcopes has eight different Magnifying-Glaſſes; ſeven of which may be uſed with two different Inſtruments, for the better applying them to various Objects: One of theſe Inſtruments is repreſented *Fig.* 72. *AAAA*, and is made of Ivory; it has three thin Braſs Plates *EE*, and a Spring of Braſs *H* within it; to one of the thin Plates of Braſs is fixed a Piece of *Cork F*, with a Concavity *G*..... both in the Cork and Braſs to which it is affixed: In one End of this Inſtrument Fig. 72.

Inſtrument there is a long Screw *D*, with a Glaſs *C*, ſcrewed in the End of it: In the other End there is a hollow Screw *o o*, wherein any of the Magnifying-Glaſſes are ſcrewed when they are to be made Uſe of. The 8 different Magnifying-Glaſſes are all ſet in Ivory, 7 of which are ſet in Fig. 76. the Manner of *Fig.* 76. *n.* 4. The greateſt Magnifyer is marked upon the Ivory, wherein it is ſet, with *n.* 1. the next *n.* 2. and ſo on till *n.* 7. the 8th Glaſs is not marked, but ſet in the Manner of a little Barrel Fig. 74. Box of Ivory, as in *Fig.* 74.

Fig. 73. *Figure* 73 is a flat Piece of Ivory *e e*, whereof there are 8 belonging to this Sett of Microſcopes, (though any one, who has a Mind to keep a Regiſter of Objects, may have as many of them as he pleaſes) in each of which there are 3 Holes *f f f*, wherein 3 or more Objects are placed between two thin Glaſſes or Talks, when to be uſed with the greateſt Magnifyers.

Fig. 75. The Inſtrument *Fig.* 75. is made of Braſs or Prince's Metal, with Joints *PPP*, to turn eaſily any Way with a ſmall Pair of Tongs *GG*, which open at the Points *k*, by preſſing together the two Heads of the Pins *II* for taking up of Objects: There is a round Piece of Ivory *H*, ſcrewed upon the other End of the Tongs, white on one Side for black Objects that are opaque Bodies, (ſuch as are Seeds or Sands) and black on the other Side for white ones of that Nature.

Upon the ſharp End *A*, of this braſs Inſtrument, all the 8 Glaſſes Fig. 76. may be fixed, as you ſee *Fig.* 76. *n.* 4. there being a Hole in the Ivory wherein the Glaſſes are ſet for that Purpoſe, with a thin Piece of Braſs *B* in the Manner of a Spring, that holds it firmer: So when any Object is taken up in the Points of the Tongs *k*, or laid upon the other End *H*, it may be very eaſily (as any one who ſees the Inſtrument will perceive) applied to the true Diſtance of any of the Glaſſes, by the Help of the Joints *PPPP*, as alſo the Screw *C*, and Wheel *D*, which will bring the Object to the Exactneſs of the *Center* or true Diſtance, being regulated by a Spring *E*.

Fig. 72. The Uſe of the firſt mentioned Inſtrument, *Fig.* 72. *AAAA*, is thus:
Fig. 73. Take one of the flat Pieces of Ivory *e e*, or *Regiſters*, *Fig.* 73. (as they may be called) and ſlide it in betwixt the two thin Plates of Braſs *EE*, in the Body of the Microſcope, *Fig.* 72. ſo that the Object, you intend to look upon, be juſt in the Middle, remarking, that you put that Side of the Plate *e e*, where the Ring is furtheſt from your Eye: Then you are to ſcrew into *o o* (the hollow Screw in the End of the Body of your Microſcope) the 3^d, 4th, 5th, 6th, 7th Magnifying-Glaſs; which being done, while you are looking through the Magnifying-Glaſs upon the Object, you are to ſcrew in or out the long Screw *D* in the other End of the Body of the Microſcope, till you bring the Object to the true Diſtance; which you will know by ſeeing the Object clearly, and diſtinctly: But ſince in the greater Magnifyers you can ſee but a ſmall Part of the Object, *viz.* the Legs or Claws of a Flea; while you are looking upon any Part of the Object, if you take hold of the End of the

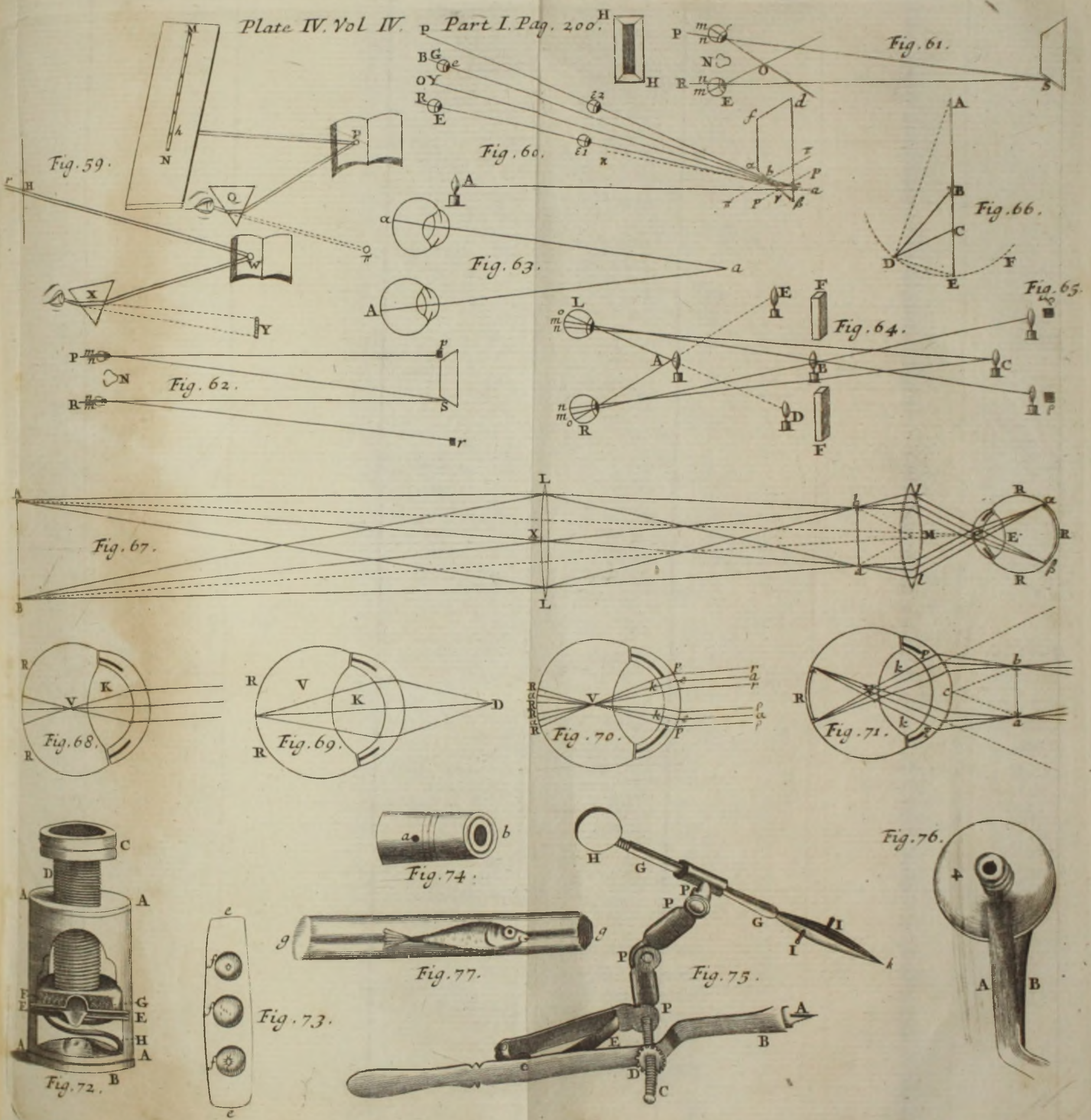


Fig. 59.
Fig. 60.
Fig. 61.
Fig. 62.
Fig. 63.
Fig. 64.
Fig. 65.
Fig. 66.
Fig. 67.
Fig. 68.
Fig. 69.
Fig. 70.
Fig. 71.
Fig. 72.
Fig. 73.
Fig. 74.
Fig. 75.
Fig. 76.
Fig. 77.

the Plate *e e*, whereon the Object lies, and move it gently, you may ſee the whole Object ſucceſſively, or any Part of the Object you pleaſe; and if that Part of the Object, you deſign to look upon, be out of the true Diſtance, remember your End-ſcrew *D* can always bring it in, by ſcrewing it nearer or farther off.

After this Manner may be ſeen all tranſparent Objects, Duſts, Liquids, Chryſtals of Salts; ſmall Inſects, ſuch as Fleas, Mites, *&c.* If they be Inſects that will creep away, or ſuch Objects as one intends to keep, they may be placed between the two *Regiſter-Glaſſes f f.* For by taking out with the Point of a Penknife the Ring that keeps in the Glaſſes *f f*, where the Object lies, they will fall out of themſelves; ſo you may lay the Object between the two hollow Sides of them, and put the Ring in as it was before: But if the Objects are Duſts or Liquids, a ſmall Drop of the Liquid, or a little of the Duſt laid on the Outſide of the Glaſs *f f*, and applied as before, will be ſeen very eaſily. Fig. 73.

As to the firſt and ſecond Magnifying-Glaſſes, being marked with a Croſs upon the Ivory wherein they are ſet, they are only to be uſed with thoſe *Regiſter-Plates*, that are alſo marked with a Croſs, wherein the Objects are placed between two thin Talks, becauſe the Thickneſs of the Glaſſes, in the other Regiſters or Plates, hinders the Object from approaching to the Centre, or true Diſtance of theſe great Magnifyers. But the Manner of uſing them is the ſame with the former; only remember when you put in or pull out the ſame Plate or Regiſter *e e*, (whereon the Object lies) or move it from one Object to another, not to let it rub your Magnifying-Glaſs, which is done by unſcrewing a little the End-ſcrew *D*, when you put in or pull out the Plate, or move it from one Object to another. Fig. 73. Fig. 72.

For ſeeing the *Circulation* of the *Blood* at the Extremities of the *Arteries* and *Veins*, in the tranſparent Parts of Fiſhes, Eels, *&c.* there are two Glaſs-Tubes, the one bigger and the other leſſer, is deſigned in *Fig.* 77. wherein the Fiſh is to be put; when this leſſer Tube is uſed, you are to unſcrew the End-ſcrew *D* in the Body of the Microſcope until the Tube *g g*, *Fig.* 77. can eaſily enter into that little Cavity *G* of the Braſs-Plate faſtened to the Cork *F*, under the other two thin Plates of Braſs *E E*; when the Tail of your Fiſh lies flat to the Glaſs Tube, ſet it oppoſite to your Magnifying-Glaſs, and by ſcrewing in or out the End-ſcrew *D*, as is ſaid before, you may eaſily bring it to the true Diſtance, and ſee the Blood circulate with great Pleaſure. Fig. 77. Fig. 72.

When the bigger Tube is to be uſed with a larger Fiſh or Frog; then you are to take out the Braſs-Plate *G F* faſtened to the Cork, by preſſing down the other two Plates *E E*, and the Spring *H*, to the End of the Microſcope *B*; and by turning the Cork and Braſs-Plate *G F* ſideways, you may eaſily take it out, and put it in again; when the Cork-Plate *G F* is out, the larger Tube will eaſily enter into the Body of the Inſtrument, and is to be uſed as the other leſſer one. Fig. 72.

If you would ſee the Blood circulate in a Frog's Foot, chuſe ſuch a Frog as will juſt go into your Tube, then with a little Stick expand the hinder Foot of the Frog, and apply it cloſe to the Side of the Tube, obſerving, that no Part of the Frog hinders the Light coming on its Foot; and when you have it at the juſt Diſtance by Means of the Screw *D*, as aboveſaid, you will ſee the rapid Motion of the Blood in its Veſſels, which are very numerous, in the tranſparent thin Membrane that is between the Frog's Toes: For this Object the 3d and 4th Magnifyers will do very well; but you may ſee the Circulation in the Tails of *Water-Newts* with the 5th and 6th Glaſs, by Reaſon the Globules of the Blood of thoſe Newts are as big again as the Globules of the Blood of Frogs or ſmall Fiſh, as has been taken Notice of by Mr. *Cowper* in N° 280, of theſe *Tranſactions*.

Vid. infra V. v. C. VI. S. iii.

N. B. The Circulation cannot be ſo well ſeen by the 1ſt and 2d Magnifyers, becauſe the Thickneſs of the Glaſs wherein the Fiſh lies, hinders the Approximation.

Fig. 74. The Glaſs placed in the Manner of a Barrel-Box, *Fig.* 74. is only to be uſed with the Braſs Inſtrument, (or in your Hand) being the leaſt Magnifyer for greater Objects, ſuch as Flies and common Inſects, *&c.* The Hole *a* in the Side of this Box is to be fixed on the Point *A* of the Braſs Inſtrument, remembring to put the End *b* next your Eye, and the other to the Object; ſo if you take up any Inſect

Fig. 75. in the Point of the Tongs *k*, or lay any opaque Object on *C*, the other End, you may approach them to the true Diſtance, by the Help of the Joints and Screw *C*, *D*, *P*, *E*, (ſpoken of before) and ſee them diſtinctly.

In the Viewing of Objects, one ought to be careful not to hinder the Light from falling upon them, by the Hat, Peruke, or any other Thing, eſpecially when they are to look upon opaque Objects: For nothing can be ſeen with the beſt of Glaſſes, unleſs the Object be in a due Diſtance, with a ſufficient Light.

The beſt Light for the Plates or Regiſters, (where the Object lies between the two Glaſſes) is a clear Sky-light, or where the Sun ſhines on any white Thing, or the Reflection of the Light from a Looking-Glaſs. The Light of a Candle is likewiſe good for the Circulation of very ſmall Objects, though it be a little uneaſy to thoſe who are not practiſed in Microſcopes to find out the Light of the Candle; but Uſe will make every Thing eaſy.

For the Conveniency of thoſe who would *draw* or make any Sketches or Deſigns after Microſcopical Objects, I have alſo made a *Pedeſtal* to fix the two Inſtruments above deſcribed, and make them ſtationary to any convenient Light. This Pedeſtal may be placed on a Table, *&c.* and after the Object, and Light are fixt, as many Perſons as pleaſe may view the Object, without any Trouble or Difficulty in finding the Light.

The

The following Figures were drawn by this Microſcope from ſeveral Objects: *Some Objects viewed with this Microſcope.*

A, B, C, D, E, F repreſent the Feathers of the Wings of *Butterflies* and *Moths*; *A, B* are the ſame, but differently magnified. *A* was expreſſed by the 4th Glaſs, and *B* as it appeared by the 1ſt. The reſt being taken from different Parts of thoſe *Inſects*, *C, D, E, F* were all viewed by the 4th Glaſs. Fig. 78.

G is one Grain of the *Farina* of the *Capilaments* of *Maloes* by the *firſt* Glaſs. Fig. 79.

H H is the *Tail* of a ſmall *Fiſh*, viewed when living by the 4th Glaſs; Fig. 80. *i i* is the Part of the Tail next the Body of the Fiſh, where the Trunks of the *Veins* and *Arteries* paſs together. *I I I I* their *Extremities* which appear united, *k k k* other *Inoſculations*, with the *Arteries* and *Veins* appearing in the tranſparent Membrane, between the Cartilages *K K*. *L L L L* the *Cartilages* compoſed of ſeveral Joints, on each Side of which the Trunk of a Vein and Artery paſſes. *M* an *Animalculum*, whereof a great Number appeared moving themſelves up and down on the Tail of the Fiſh, while the Circulation was viewing. Fig. 81.

N repreſents a Side-view of the ſame *Animalculum*. Fig. 82.

O is another *Animalculum* of a different Figure from the former, that ſtuck to the Tail of the Fiſh by its jagged Extremity, and frequently drew its long Body out and in again. Fig. 83.

P Q one of the *Lice* found on the *Beetle*, called *Scarabeus Pediculoſus* by the 4th Glaſs. *P* its *Anus*. *Q* its two Claws, not unlike thoſe of a Lobſter, *r r r r* the Extremities of its Feet, which have a remarkable Contrivance for ſticking faſt to the poliſhed Surface of the Beetle, not in the Manner of Claws, as many other Inſects, but divided into Capilaments, as expreſſed in the Figure. Fig. 84.

S the ſame *Animal* as it appeared to the naked Eye. Fig. 85.

VIII. I think that all the Microſcopes, which preceded Mr. *Leeuwenhoeck*'s, are ſo much out-done by his, that it will be proper only to take Notice of theſe and the reſt of later Invention, not deſigning to leſſen their Uſefulneſs, but only to add a few Thoughts which may be of Service. *The Manner of making* Microſcopes, &c. *by Dr.* A. Adams, n. 325. p. 24.

I had not an Opportunity of examining Mr. *Leeuwenhoeck*'s Glaſſes particularly, which is a Favour he allows to none; therefore I am not capable at this Diſtance to deſcribe either their Make or Uſe, any further than that to me they appeared to be Spherules lodged between two Plates of Gold or Braſs, in a Hole whoſe Diameter might not be bigger than that of a ſmall Pin's Head; and the Objects I ſaw through them were pretty and diverting; but ſtill their Make and Truth are unknown.

Mr. *Butterfield* is very curious in melting his Glaſs, but I ſuppoſe unſucceſsful in caſting his Spheres; for beſides that a ſufficient Quantity of beaten Glaſs cannot ſtick to the moiſtened Point of a fine Needle;

ſo neither can it run equally, hold the Needle how you will, nor the *Globule*, when run, ſtick to the Needle, but muſt unavoidably drop; and whereſoever it happens to fall, it muſt in that almoſt liquid State receive Impreſſions ſufficient to ſpoil the Figure of a Sphere.

Mr. *Gray* has ſhewn the Defect of his Method, which he uſed to recover by grinding and poliſhing his Glaſſes on a Braſs-Plane, and ſo reduce them to Hemiſpherules; but how far ſhort poliſhed Glaſſes (I ſpeak of ſmall ones) come of thoſe which are caſt, I leave to any one to judge who has ſeen both. His Water and Quickſilver Microſcopes I never ſaw, ſo can ſay little to them.

After what Manner Mr. *Wilſon*'s Glaſſes are made I know not, but ſure his greateſt Magnifyers are ill placed, they being ſunk to ſo great a Diſtance from the Eye, the Object cannot appear to that Advantage it otherwiſe would; if therefore inſtead of a hollow Cap, he would contrive a plain Plate of any Metal for the Reception of the Glaſs, then the Eye and the Object might come to their due Diſtance; neither ought there to be any Calx or Glaſs between the Object and the Spherule, when we uſe the greateſt Magnifyers; becauſe if the *Focus* of a Sphere be upon the Extremity of its Circumference, any ſmall Diſtance from that muſt ſpoil the Truth of the Object's Appearance.

I cannot ſay, that the Glaſſes I have made are without Fault, but I think they magnify more than any I have yet ſeen; and were they placed to the beſt Advantage, they would magnify much more than they do: They are made thus.

I take a Piece of fine Window-Glaſs, and I raſe it with a Diamond into as many Lengths as I think needful, not exceeding an Eighth of an Inch in Breadth; then holding one of theſe Lengths between the Fore-finger and Thumb of each Hand, over a very fine Flame, until the Glaſs begins to ſoften, I draw it out till it be as fine as a Hair, and break: Then inuring each of the Ends into the pureſt Part of the Flame, I have two Spheres preſently, which I can make larger or leſs as I pleaſe; if they ſtay long in the Flame, they will have Spots, ſo I draw them out preſently after they turn round. As for the Stem, I break it off as near the Ball as I can, and lodging the Remainder of this Stem between the Plates, and by drilling the Hole exactly round, all this Protuberance is buried between the Plates, and the Microſcope performs to Admiration; inſomuch, that the ſame Thread of very fine Muſlin appeared 3 or 4 Times bigger in one of theſe, than it did in the firſt or ſecond of Mr. *Wilſon*'s. I thought I ſaw Animals in fine old Brandy; but they were ſo nimble in their Motion, that I can give no particular Deſcription of them. *A Remark on Blood.* Human *Blood* is ſo far from ſhowing any red *Globules* ſwimming in *Serum*, that immediately after its *Emiſſion* it appears to be a Body of infinite Branches, running in no certain Order, variouſly coloured; where it lies thickeſt on the Glaſs, it is of a dull Red; where thin, inclining to Yellow; but the Whole ſo blended, as to repreſent very near the Top of a Yew-

Tree

Tree in a very fine Landskip, having its ſuppoſed Branches of a Red and Yellow confuſedly intermixt. But not ſatisfied with this Appearance, though the ſame as to Quality in eleven different Glaſſes, and as many different Sorts of Blood, I reſolved to view it another Way, which was, by diluting one Third of Thick in the *Serum* of Blood; and laying it upon my Glaſs, I could ſee the red Branches as before, and the Tranſparent filled with Particles of great Variety of Figures, which I took to be the Salts of the Blood, but feweſt globular, and they were pellucid.

If the Fluids moving in an evaneſcent Artery appear globular, I ſuppoſe it is becauſe the Canal is round, which alters the Caſe much.

I had at the ſame Time an Opportunity of viewing ſome *Pluritic* Blood; and thought that its Branches did ſpread in a different Method from the Sound, and more ſtrongly perplexed with overthwart Branches, which appeared black like Blood that had ſtood two or three Days. Whether the Attraction of Particles ariſing from the Difference of Figure, may not render the Blood incapable of paſſing through the capillary Arteries of the *Pleura* in that Caſe, let others judge; but I ſhould think, that ſince the propellent Force of the Heart is leaſt at the capillary Arteries, then there the attractive Force of the Particles of the Blood ſhould be greateſt; and ſince Spherical Bodies are the moſt attractive of any, Reſpect being had to their Solidities, were the Blood ſo plentifully ſtocked with Globules, as ſome ſay, we ſhould never be free from Obſtructions, the natural Conſequence of this attractive Force. If my Glaſſes have deceived me, and if this that I have written be found to be a Miſtake, no one ſhall more readily retract and acknowledge it. —— *on* Pleuritic *Blood.*

IX. *Account of a Book omitted.*

Linear Perſpective, or a *new* Method of *Repreſenting* juſtly all Manner of Objects, *&c.* By *Brook Taylor*, LL. D. and R. S. *Secr.* 8vo. London, 1715. N. 344. p. 300.

The Author having in his *Book* obſerved, that there might be a very good *Expedient* made Uſe of in painting of large Rooms and Churches, which is drawn from the Nature of thoſe Rays which produce the *Vaniſhing Points*, but not having mentioned it in his Book, has thought proper to take Notice of it in the *Account*. *An Addition* ibid. p. 303.

The *Expedient* is this: Having ſome Way or other found the Repreſentation of one Point of a Line that is wanted in the Picture, to find the whole Line, paſs a Thread ſtretched through the Place of the Spectator's Eye, in a Direction, parallel to the Direction the original Line ought to be in, and the Shadow of that Thread caſt by a Candle, ſo as to paſs through the given Point on the Picture, will be the Repreſentation ſought. The Reaſon of this Conſtruction is, becauſe the Rays of Light that paſs from the Candle to the Thread ſo ſtretched, make the Plane which generates the Repreſentation ſought; (*Vid.* Prop. 1.) and there may be other Expedients of the like Nature gathered from the ſame Principle.

CHAP. III.

CHAP. III.

ASTRONOMY.

Of Caſſini's *Orbit of the* Planets, *by Dr.* Gregory, n. 293. p. 1704.

I. SINCE the Time that the celebrated Mr. *Caſſini*, in his Treatiſe *of the Origin and Progreſs of Aſtronomy*, has propoſed to the Aſtronomers a certain Curve for the Orbit of a Planet; there has been much Debate among the Learned concerning the Nature of this Curve, and the Law of Gravitation required in order to its being deſcribed. As I have thought again upon the ſame Subject, not only the different Species of this Curve, but ſome of their Properties not ſufficiently examined, have occur'd to my Enquiries.

Fig. 86.

It is now well known, that the Nature of this Curve is this. If from two given Points *F* and *G* to any Point of the Curve *H* two Lines are drawn *FH* and *GH*; the Rectangle under *FH* and *GH* is equal to a given Space. The right Line *FG*, being produced each Way till it meets the Curve, will ſhew the two Vertices *A* and *B*; and *AB* is the principal Axis. And *C* the middle Point between the Vertices will be the Center of the Figure. And *DE* drawn through *C* perpendicular to *AB* will be the leſſer Axis: And the Points *F* and *G* are the Foci.

In this Figure, if the leſſer Axis is greater than the Diſtance of the Foci, the Curve terminating the Figure is every where concave towards the Center, ſuch as the Figure is commonly deſcribed. But if the Diſtance of the Foci is leſſened, while the principal Axis continues the ſame, the leſſer Axis will be increaſed, which yet remains leſs than the Axis of an Ellipſis, deſcribed with the ſame principal Axis and the ſame Foci: Till at laſt when the Foci unite, it becomes equal to the greater Axis, and the Figure changes into a Circle. But on the contrary, if the Diſtance of the Foci increaſes, the leſſer Axis will be leſſen'd, and will become equal to the ſaid Diſtance, when this is to the principal Axis, as Unity is to a mean Proportional between Unity and the Number 3.

If the Diſtance of the Foci be farther encreaſed, the leſſer Axis will be ſtill leſſen'd, and the Curve at its Extreams will be no longer concave towards the Center, but convex as in *Fig.* 87. till the Diſtance of the Foci being ſo far increaſed, that it is to the greater Axis, as the Side of a Square to its Diameter; the leſſer Axis will become nothing, and the Curve will reach on each Side to the Center.

Fig. 87.

If the Diſtance of the Foci is greater than the aforeſaid ratio, the leſſer Axis becomes impoſſible, and the Figure changes into two conjugate Figures, as in *Fig.* 88. which will be leſſen'd as the Diſtance of the Foci increaſes, till at laſt the Figures paſs into two conjugate Points only.

Fig. 88.

The

The Diſtance of the Foci ſtill increaſing, the two conjugate Figures emerge again, which increaſe in the ſame Manner as they decreaſed before, being different from the former in the Order of the Foci and Vertices, and go on increaſing till they become infinite. And afterwards this Syſtem will again approach to the Circle by the ſame Degrees as before it receded from it.

Hence it appears at the firſt View, that this Figure cannot be at all proper to conſtitute the Orbit of a Planet. For not to mention the Caſe wherein it becomes two Figures, and forſakes the Nature of an Orbit, that is, whenever its Excentricity is ſo great as the Comets require, (if they turn about the Sun like Planets, as is moſt probable) to deſcribe their reſpective Courſes : To paſs over theſe Caſes, as ſaid before, even in thoſe Caſes in which it returns into itſelf, and compleats its Orbit, ſome of its Excentricities are ſo large, that near *D* and *E* the Curve becomes convex towards the Sun, and therefore the Planet would have Occaſion for a Centrifugal Force from the Sun, that it may deſcribe this Part of its Orbit, whilſt at the ſame Time both in Places that are nearer and more remote *B* and *A* there ought to be a Centripetal Force towards the Sun. That is, it muſt be allow'd, that the Circumſolar Bodies may move by ſuch a Law, that at equal Intervals from the Sun here a Centripetal Force, and there a Centrifugal can obtain, which how contrary it is to the known Laws of Nature is eaſy to perceive. And tho' none of the Planets have ſo great Excentricity, yet ſince it is known to Geometricians, that if all the Species of a Figure beyond a certain Limit are unfit for performing a natural Function, the remaining Species of that Figure on this Side the Limit, cannot be admitted as fit for performing the ſame Function : This Curve of *Caſſini*'s muſt of Neceſſity be rejected out of Aſtronomy, not only for the Reaſons alledged, *Prop.* 8. *Lib.* 3. of the Elements of Aſtronomy, *that it neither agrees to the Obſervations of the Heavens, becauſe of the Brevity of the ſhorter Axis, nor do Phyſical Reaſons correſpond, ſince for the Deſcription of this a Centripetal Force towards the Sun is required, greatly deviating from that which Nature makes Uſe of* ; but likewiſe becauſe of the utter Impoſſibility of it. For it is impoſſible that any Species of this Figure ſhould be deſcribed by a Planet, ſo that the Angles at that Focus, which is different from the Sun, may be proportional to the Times ; for thus the Area deſcribed by the Radius tending to the Sun could not be proportional to the Times. For the Angle at one Focus being increaſed by equal Increments, the contemporary Increments of the Area to the other cannot become equal ; contrary to the Opinion I lately entertain'd of this Matter. Fig. 87.

In Figures 87, 88. the greateſt Breadth of the Figure is found, if with Center *C* a Circle is deſcribed thro' the Foci ; for this will cut the Curve in the Points *L*, *L*, which are the Points required. And the greateſt Ordinate *KL* is a third Proportional to the right Lines *GF*, and *FD* is the firſt of theſe, or a fourth Proportional to *GF*, *GA*, and *AF*, in both of them. Fig. 87, 88.

DE

DE remaining, the Ordinate *FP* from the Focus is equal to the leſſer Semiaxis *CD*, when the leſſer Axis is to the Diſtance of the Foci, as the Side of a Square is to its Diameter. If the Diſtance of the Foci is greater than in this ratio, *FP* will exceed *CD*.

The Newtonian *Solution of* Kepler's *Problem demonſtrated*, &c. *by Dr*. J. Keil, n. 337. p. 1.

II. *Kepler* was the firſt that demonſtrated, that the Planets were not carried about in Circular Orbits, but in Elliptical ones; and that they ſurrounded the Sun, placed in one of the Foci of the Ellipſis in ſuch a Manner, that a Radius extended from the Planet to the Sun's Center, alway deſcribes Elliptical Areas, which are proportional to the Times of Deſcription.

This Divine Diſcovery of the moſt ſagacious *Kepler* is owing to the very exact Obſervations of *Tycho Brahe*; and is ſo much the more to be eſteemed, that by the Help of this *Newton* has moſt happily laid open the Laws of Motion, and the Philoſophy of the Syſtem of the Univerſe.

Since therefore the Planets move about the Sun according to this Law, that their Places in their own Orbits may be determined to any given Time, there is Occaſion that the following Problem ſhould be ſolved.

To find the Poſition of a right Line, which paſſing through either of the Foci of a given Ellipſis, may cut off an Area deſcribed by its Motion, which may be to the whole Area of the Ellipſis in a given ratio.

Fig. 89.

Let the Ellipſis be *APB*, either of whoſe Foci is *S*. The Poſition of the right Line *SP* is to be found, which may cut off the Trilinear Area *ASP*, to which the Area of the whole Ellipſis has the ſame ratio, as the periodical Time of the Planet deſcribing the Ellipſis, has to any other given Time. This being found, the Point *P* will be given, where the Planet will be at that given Time. Or let *AQB* be a Semicircle deſcribed upon the greater Axis of the Ellipſis; a Line *SQ* is to be drawn thro' *S* cutting off the Area *ASQ*, to which the Area of the whole Circle is in the ſame ratio. For if from *Q* a Perpendicular *QH* is let fall upon the Axis, meeting the Ellipſis in *P*; drawing the Line *SP* it will give the Elliptic Area required, and the Point *P* will be the Place of the Planet at the given Time. For the Semiſegment of the Ellipſis *APH* is to the Semiſegment of the Circle *AQH*, as *HP* to *HQ*; that is, as the Area of the whole Ellipſis is to the Area of the whole Circle. But the Triangle *SPH* is to the Triangle *SQH* in the ſame ratio of *PH* to *QH*. Therefore the Area *ASP* is to the Area of the whole Ellipſis, as the Area *ASQ* is to the Area of the whole Circle. Therefore if we had a Method of cutting the Area of the Circle in a given ratio, by a Line drawn through the given Point *S*, it would be eaſy to cut the Elliptic Area in the ſame ratio.

Kepler himſelf, who firſt propoſed the Problem, had no direct Method of computing the Planets Place from the Time being given. He was fain to proceed thro' the ſeveral Degrees of the Semicircle *AQB*, from the given Arch *AQ*, which they call the Anomaly of the Excentrick, and to calculate as well the Time by the Area *ASQ*, which is proportional to the

the mean Anomaly, as the Angle *ASP*, that is the Place of the Planet, or the coequated Anomaly correſponding to this Time.

Therefore as the Solution of this Problem was difficult, Aſtronomers betook themſelves to other Hypotheſes, feigning ſome other Point to be that, about which the Motion was equable or proportional to the Time, and thence the mean Anomaly being given, they determined the coequated Anomaly. But Computations founded upon theſe Hypotheſes were found not to agree with the Obſervations. Therefore Geometers had Recourſe to various Approximations, by which from the given Area *ASQ*, which is analogous to the Time, the Angle *ASP*, or the Place of the Planet may be had very nearly. But the eaſieſt of all theſe, and moſt ready for Practice, ſeems to me to be that Method which is taught by Mr. *Newton* in his *Principia*, p. 111, and 112. of the firſt Edition, which is very much like that Method, by which Analyſts extract the Roots of affected Equations; and indeed is ſo much the more to be eſteemed, as that it not only exhibits the Places of the Planets, whoſe Orbits approach very nearly to the Form of Circles, but almoſt with the ſame Facility may be applied to Comets, which move in Orbits that are very excentrick.

Therefore I thought it not amiſs to explain that Method here, for the Sake of ſuch Artiſts as are deſirous of conſtructing Aſtronomical Tables, according to the true Laws of Motion, and not by any fictitious Hypotheſes.

Therefore let *AQB* be a Semicircle deſcribed upon the greater Axis Fig. 90.
of an Ellipſis, whoſe Center is *C*, and *S* is the Focus in which the Sun is placed. Let *CQ* be drawn, upon which, produced if Need requires, let fall the Perpendicular *SF*. The Area *ASQ* is equal to the Sector *ACQ* added to the Triangle $CSQ = \frac{1}{2} CQ \times AQ + \frac{1}{2} CQ \times SF$; and therefore becauſe of $\frac{1}{2} CQ$ being given, the Area *ASQ* will always be proportional to the Arch *AQ*, added to the right Line *SF*, when the Motion is from the Aphelium towards the Perihelium. But when the Planet tends from the Perihelium towards the Aphelium, as in *Fig.* 92. the Area $BSQ =$ Sector $BCQ -$ Triang. *CSQ*, and therefore it will be proportional to the Arch $BQ -$ right Line *SF*. Hence if there is taken the Arch *AN*, in *Fig.* 90, 91. and *BN* in *Fig.* 92. proportional to the Fig. 90, 91, 92.
Times, it will be $AQ + SF = AN$, and $BQ - SF = BN$. Whence *SF* will be equal to *QN*, if *AN* or *BN* are proportional to the Times in which the Areas *ASQ* or *BSQ* are deſcribed. Now that the Meaſure of the Arch in the Periphery *AQB*, which Arch is equal to the right Line *SF*, may be found in Degrees and Parts of a Degree: Let it be made as *CQ* to *CS*, ſo is the Arch of 57,29578 Degrees, (which is equal to the Radius *CQ*) to a fourth Arch, which will be equal to *CS*. Let that Arch be *B*. But it is *CS* to *SF*, as Radius to the Sine of the Angle *SCF* or *ACQ*. Therefore let it be made, as Radius to the Sine of the Angle *ACQ* or the Arch *AQ*, ſo is the Arch *B* to another *D*; that Arch *D* will be equal to the right Line *SF*; therefore if, at a given Time, the Area *ASQ* were proportional to the Time, the Arch *D* would

be equal to NQ; and taking the Arch $NP = D$, the Point P would fall upon Q. But if the Area ASQ ſhould not exactly anſwer to the Time, the Point P will fall above or below Q, according as the Area ASQ is greater or leſs than the true Area which anſwers to the Time. Let it be ASQ, and let SH fall perpendicularly upon Cq. By what has before been demonſtrated, it will be $SH = Nq$. But it is $SF = NP$, whence it will be $SH - SF$, or $SF - SH$, that is nearly $HE = qP = QP - Qq$, or $Qq - QP$. And if the Angle QCq be a ſmall one, it will be $CH . CQ :: HE . Qq :: QP - Qq . Qq$. Whence $CQ + CH . CQ :: QP . Qq$, when the Arch QA is leſs than a Quadrant. But when it is greater than a Quadrant, it will be $CQ - CH . CQ :: QP . Qq$. And in like Manner when the Arch BQ is leſs than a Quadrant, it will be $CQ - CH . CQ :: QP . Qq$.

If the Angle ACQ or BCQ is a ſmall one, that is, if the Planet is near the Apſids, it will be $CA \pm CS . CA :: QP . Qq$.

Let it be made, as CS to CQ, ſo is Radius R to a certain Length L; it will be $CQ = \frac{CS \times L}{R}$. But it is, as Radius to the Coſine of the Angle ACQ, ſo is SC to CF or CH; (for CH and CF are nearly equal) wherefore it will be $CH = \frac{SC \times \text{Coſ.}\, ACQ}{R}$; and therefore $QP . Qq :: \frac{CS \times L + CS \times \text{Coſ.}\, ACQ}{R} . \frac{CS \times L}{R} :: L + \text{Coſ.}\, ACQ . L$, when the Arch AQ is leſs than a Quadrant. But if AQ be greater than a Quadrant, it will be $QP . Qq :: L - \text{Coſ.}\, ACQ . L$.

And in this Method if an Arch AQ be any how taken, which is ſomething leſs or greater than the Truth, thence will be found an Arch Qq to be added to it, or taken from it, which will make the Area ASq very nearly proportional to the Time. And if inſtead of AQ be taken an Arch Aq, and a Proceſs be made with this like the former, another Aq will be found, which in like Manner by repeating the ſame Proceſs, will give another Aq; and thus we may approach as near the Truth as we pleaſe.

The Angle ACq being found, we ſhall eaſily have the Angle ASq, ſince in the Triangle qCS are given the Sides Cq and CS, and the Angle qCS. Thence will be given the Angle CSq, whoſe Tangent is to be leſſen'd in the ratio of the leſſer Axis of the Ellipſis to the greater, that at laſt may be had the Tangent of the Angle ASP. Or perhaps the Angle ASP may be thus found more eaſily. Let F be the Number which expreſſes the Length CS is ſuch Parts as CQ is 100000. From the Point q to the Axis let qr be a Perpendicular, which will be the Sine of the given Arch Aq, and Cr will be the Coſine of the ſame, and Sr will be equal to the Sum or Difference of the right Lines Cr and CS; that is, $Sr = F \pm \text{Coſin.}\, ACq$. Therefore in the right angled Triangle

rSq,

rSq, Sr and rq being given, the Angle rSq may be found. Hence if the Logarithm of the Sine of the Angle ACq, and the Arithmetical Complement of the Logarithm of Sr, and the Logarithm of the Ratio of the leſſer Axis of the Ellipſis to the greater, be added together into one Sum; the Logarithmical Tangent of the Angle ASP will be given.

But the Readineſs of this Method is ſo great, that it requires rather to be illuſtrated by Examples, than by any farther Explanation. Therefore we may try it in the Motion of the Planet *Mars*, in the Orbit of which, according to the *Caroline* Tables, the Excentricity is to the middle Diſtance, as 14100 to 152369, and therefore the Logarithm of the Arch B, which is equal to the right Line SC, will be 0.7244451. Alſo in this Example L will be 1080631 Parts, ſuch as the Radius is 100000: Let the Angle ACQ be found, when the mean Motion, or the Arch proportional to the Time computed from the Aphelium, is of one Degree. Becauſe CS is here nearly one tenth Part of CA, I ſuppoſe the Arch AQ to be 0.9 Degrees, that is, one tenth Part leſs than the mean Motion. Let there be added the Logarithmical Sine of the Arch AQ to the Logarithm of B, and the Sum 8.9205471 is equal to the Logarithm of the Number 0.083281, which Number expreſſes an Arch equal to the right Line $SF = NP$. And if the Arch AQ had been rightly aſſumed, it would be $AN - NP = AQ$, and $QP = 0$. But here it is $QP = 0.016719$, from whence if we take away its 11 Part, ſince AS exceeds AC by about an eleventh Part of itſelf, there will remain $Qq = 0,0152$; which being added to AQ, gives $AQ = 0.9152$, which does not differ from the true Aq by a thouſandth Part of a Degree. Secondly, let the Arch AN or the mean Motion be 2 Degrees. I make $AQ = 1,83$ almoſt double the former Aq, and to its Logarithmical Sine let be added the Logarithm of B. The Sum will be 9,2286997, which is equal to the Logarithm of the Number 0,16931. Whence it will be $QP = 0,00063$, and $Aq = 1,83063$, which does not differ from the true Aq by the ten thouſandth Part of a Degree. After the ſame Manner let the mean Motion, or the Arch proportional to the Time, be 3 Degrees. Make the Arch AQ $2,745 = 1,83 + 0,915$, and to its Logarithmical Sine adding the Logarithm of B, there will be had the Logarithm of the Number $0,25392 = NP$, and $AN - NP = 2,74608$, and therefore $QP = 0,00108$. Whence $Qq = 0,001$ nearly, and $Aq = 2,746$. Thus by one Addition of two Logarithms, the Arch Aq will be found, which will be true to the thouſandth Part of a Degree.

Now if the Angle ACq is to be found, not by proceeding by Degrees but *per ſaltum*, when the mean Motion is 45 Degrees: I make the Arch AQ to be 40 Degrees, and to its Logarithmical Sine adding the Logarithm of B, the Sum is 0,5325125, which is the Logarithm of the Number 3,4081. This Number ſubtracted from 45 leaves $AN - NP = 41,5919$, whoſe Exceſs above the Arch AQ is 1,5919. Whence if it be made, as $L +$ Coſin. ACQ to L, ſo is 1,5919 to another, the Arch Qq will be found to be 1,4865 Degrees. Therefore $Aq =$

 41,4865,

41,4865, which differs from the Truth not much above the thoufandth Part of a Degree. But without this Proportion Aq may be found, by taking a new Arch AQ, which is a little lefs than $AN - NP$, yet nearly equal to it. For Inftance, make $AQ = 41,50$, and adding the given Logarithm of B to its Logarithmical Sine, there will be had another $NP = 3,35131$, which fubtracted from AN gives 41,4869 for a new Aq. And this Arch is derived with lefs Trouble, and comes nearer the Truth than the former Aq.

After Aq is found correfponding to the mean Motion 45 Degrees, proceeding again by Steps, by one Addition of two Logarithms will be had Aq to all the fubfequent Degrees of the mean Motion. For Inftance, when the mean Motion is 46 Degrees, I make $AQ = 42,4249$; and adding its Logarithmick Sine to the conftant Logarithm of B, it will be $AN - NP = 42,4249$; to which Arch if a new AQ be put equal, there will be had Aq, which will not differ from the true Aq by the thoufandth Part of a Degree. So when the mean Motion is 47 Degrees, I make $AQ = 43,36$, equal to the former Aq added to the Increment of that Arch for one Degree of mean Motion, and adding its Logarithmick Sine to the Logarithm of B, the Sum will be the Logarithm of the Number 3,6402, which fubtracted from AN leaves $AN - NP =$ 43, 593 equal to the new Aq, which differs from the true Aq about the ten thoufandth Part of a Degree.

If omitting the intermediate Degrees, the Arch Aq is to be found when the mean Motion is 100 Degrees; make AQ 96, and adding its Logarithmical Sine to the Logarithm of B, the Sum will be equal to the Logarithm of the Number 5,273, whence $AN - NP = 94,727$. Therefore, fecondly, make $AQ = 94,72$, and adding its Sine to Log. B. there will arife the Logarithm of 5,285, which fubtracted from AN leaves $AN - NP = 94,715 = Aq$ very nearly. In like Manner, if the mean Motion be 101°. make $AQ = 95,71$, whofe Logarithmical Sine added to the Logarithm of B, gives the Logarithm of the Number 5,2756, which Number taken from 101, there will remain $AN - NP = 95,7244, = Aq$. And in this Manner the mean Motion being given by a gradual Procefs the Angle at the Center will be had, by the Addition only of two Logarithms, one of which being conftant may be preferved upon the Paper, that the Labour may be fpared of writing it down too often.

Now let us proceed to an Orbit of the other Species, fuch as the Diftance of the Aphelium may be to the Diftance of the Perihelium, as 70 to 1. Such nearly was the Orbit of that Comet, which compleats its Period in $75\frac{1}{2}$ Years; as was firft found by that fagacious Aftronomer and Geometrician Dr. *Edmund Halley*. In this Orbit AC or CQ will be 35,5 and CS 34,4, of fuch Parts as SB is one. And the Arch Bq is to be found, when the mean Motion is one hundredth Part of a Degree. Since the middle Diftance exceeds the leaft Diftance about 35 Times, I make $BQ = 0,35$, when the mean Motion is 0,01. In this Orbit the conftant ſtant Logarithm of B is found 1,7457133. Therefore this Logarithm being added to the Logarithmical Sine of the Arch 0,35, gives the Logarithm of the Number 0,34013, which added to the Arch 0,01 will make 0,35013. If this Sum had been equal to 0,35, the Arch BQ would have been rightly aſſumed; but the Difference is 0,00013. Whence becauſe CB is to SB as 35,5 to 1, let the Difference 0,00013 be multiplied by 35,5, and there will ariſe $Qq = 0{,}004615$; whence it will be Arch $Bq = 0{,}354615$, which hardly differs from the Truth by three Parts of ten Thouſand.

Secondly, let the mean Motion be 0,02, and ſuppoſe BQ to be 0,71. To its Logarithmick Sine adding the Logarithm of B, the Sum will be the Logarithm of the Number 0,68998; whence $BN + NP = 0{,}70998$, and therefore the aſſumed Arch $BQ = 0{,}71$ was too much, and the Difference is 0,00002. Which if it be multiplied by 35,5, and the Product ſubtracted from BQ, there will remain $Bq = 0{,}7092$, deviating from the Truth hardly the ten thouſandth Part of a Degree.

Let the mean Motion be 0,03. Suppoſe BQ to be 1,06 Degrees, adding its Logarithmick Sine to the Logarithm of B, the Sum will be the Logarithm of the Number 1,03008. To which if $BN = 0{,}03$ is added, the Sum will be 1,06008, which Number is greater than BQ; wherefore if the Difference 0,00008 is multiplied by 35,5, and added to BQ, it will be $Bq = 1{,}06284$. In like Manner, when the mean Motion is 0,04, I ſuppoſe $BQ = 1{,}40$ Degrees, and find $NP = 1{,}3604$; to which Number adding $BN = 0{,}04$, the Sum is 1,4004, which exceeds 1,40 by 0,004. Let this Difference be multiplied by 35,5, and the Product 0,01420 will be equal to Qq; whence $Bq = 1{,}41420$. In all theſe Inſtances the Errors are very ſmall, and ſeldom go beyond the thouſandth Part of a Degree.

Now let the Arch Bq be to be found, when the mean Motion is equal to one Degree. Suppoſe $BQ = 20°$, and adding its Logarithmick Sine to the Logarithm of B, there will be had the Logarithm of the Number 19,045; to which adding $BN = 1°$, the Sum 20,045 exceeds 20 by 0,045. And ſince in this Caſe L — Coſin. BQ is to L, as 1 to 11,5 nearly, I multiply the Difference 0,045 by 11,5, and the Product 0,5175 added to BQ makes 20,5175. Therefore I ſuppoſe ſecondly $BQ = 20{,}51$, and there will ariſe, in the ſame Manner as in the foregoing, $NP = 19{,}5092$; to which adding BN, the Sum is 20,5092, which is leſs than BQ. Wherefore if the Difference 0,0008 is multiplied by 11,5, and the Product 0,0092 is ſubtracted from BQ, there will remain $Bq = 20{,}5008$.

Laſtly, let the mean Motion be equal to 2°. I ſuppoſe BQ 30°, and there is found $NP = 27{,}84$; to which adding 2°, the Sum 29,84 is leſs than 30. And if the Difference 0,16 is multiplied by 6,3 (for L— Coſin. BQ is to L, as 1 to 6,3 nearly) it will be $1{,}008 = Qq$. Therefore this Arch ſubtracted from BQ gives $Bq = 28{,}982$. Now that Bq may be corrected, I aſſume (ſecondly) $BQ = 29$ Degrees; and by a like Proceſs we ſhall find $Bq = 28{,}9672$.

III. No

The Parallax of the Sun to be found, by seeing *Venus* between the Sun and the Earth. *by Dr.* E. Halley, n. 348. p. 454.

III. No Problem seems more difficult than that, of determining the Distance of the Sun from the Earth near to the Truth; which yet may be perform'd with no great Labour, by having certain accurate Observations, perform'd at chosen and foreseen Times. This I shall lay before this Society in the present Differtation, that I may shew a Way how it may be done to our younger Astronomers, who may have an Opportunity of observing this; so that they shall be able to measure truly the immense Distance of the Sun, within the five hundredth Part of that Distance.

Now it is known, that by different Authors of Astronomy this Distance is variously supposed, as it seems probable by Conjecture to every one. By *Ptolomy* and his Followers, as also by *Copernicus* and *Tycho Brahe*, it is made a thousand and two hundred Semidiameters of the Earth, and by *Kepler* nearly three thousand five hundred. *Ricciolus* doubles the Distance of *Kepler*, which *Hevelius* only enlarges by one Half. But when the Planets *Venus* and *Mercury* are seen in the Disk of the Sun by Means of a Telescope, and so stript of their borrow'd Splendor, it is at last found out, that the visible Diameters of the Planets are much less than they were thought to be hitherto; and that the Semidiameter of *Venus* seen from the Sun does not subtend above a Quarter of a Minute. That the Semidiameter of *Mercury*, at its mean Distance from the Sun, is seen only under an Angle of ten Seconds; and that the Semidiameter of *Saturn* is seen from the Sun under the same Angle. That the Semidiameter of *Jupiter*, the greatest of the Planets, does not subtend at the Sun an Angle of above a third Part of a Minute. Whence some of our modern Astronomers have been of Opinion, that keeping to the same Analogy, the Semidiameter of the Earth also, when seen from the Sun, subtends an Angle of an intermediate Magnitude, or greater than that of *Jupiter*, and less than that of *Saturn* and *Mercury*, being equal to that of *Venus*, or about 15 Seconds: And therefore the Sun is distant from the Earth near 14000 Semidiameters of the Earth. But with the same Authors another Argument has a little enlarged this Distance. For since the Diameter of the Moon is something greater than a fourth Part of the Diameter of the Earth, if the Parallax of the Sun is supposed to be 15 Seconds, the Body of the Moon would become greater than the Body of *Mercury*. That is, a secondary Planet would be greater than a primary one; which would seem contrary to that Concinnity which should obtain in the System of the World. And on the contrary, the same Concinnity will hardly allow, that *Venus* an inferior Planet, and destitute of Satellites, should be greater than our Earth a superior Planet, and attended with so remarkable a Companion. Therefore that we may keep a Medium, let the Semidiameter of the Earth seen from the Sun, or which is the same Thing, let the horizontal Parallax of the Sun be 12 Seconds and an Half; whence the Moon will be less than *Mercury*, and the Earth greater than *Venus*, and the Distance of the Sun from the Earth will come out 16500 Semidiameters of the Earth

very

very near. Now at preſent I give my Aſſent to this Diſtance, till it may appear more certainly how great it is, by the Experiment I ſhall now propoſe. Nor am I ſwayed by the Authority of thoſe, who enlarge the Diſtance of the Sun immenſely beyond theſe Limits, relying upon the Obſervations of the Vibrations of a Pendulum, which cannot be truſted, (as I think) in determining ſuch minute Angles. Surely to any one that trys to find the Parallax this Way, ſometimes it will come, out nothing, ſometimes negative; that is, the Diſtance will be infinite or more than infinite, which is abſurd. Moreover, to diſtinguiſh certainly to Seconds, or even to ten Seconds, by Inſtruments never ſo artificially made, is hardly allow'd to mortal Man. And therefore it is not at all to be wonder'd at, if the great Subtilty of the Thing has hitherto eluded all the many and ingenious Attempts of the greateſt Aſtriſts.

But now almoſt 40 Years ago, when I was in the Iſland of St. *Helena*, where I was employ'd in Obſervations of the fixt Stars which ſurround the South Pole; I had an Opportunity of obſerving *Mercury* paſſing through the Disk of the Sun, which I did with the greateſt Diligence. I obtain'd moſt accurately, with an excellent Tube of 24 Feet, the Moment in which *Mercury* entring the Sun's Limb was ſeen to touch it within; and in like Manner, the Moment in which at going out he touch'd the Sun's Limb, making an Angle of inward Contact. Whence I was ſure of the Interval of Time, in which the whole Body of *Mercury* appeared at that Time within the Disk of the Sun, and that without an Error of one Second of Time. For the Thread of the Solar Light, intercepted between the obſcure Limb of the Planet and the bright Limb of the Sun, ſlender as it was, appear'd to ſtrike the Eye, and in ſtriking the Eye, the Denticle made in the Limb of the Sun by the Entrance of *Mercury*, vaniſh'd, as that made by his going out began as it were in a Moment. When this was known, I was immediately aſſured, that the Sun's Parallax might be truly determin'd from this Kind of Obſervations, if only *Mercury* being nearer the Earth ſhould have a greater Parallax from the Sun. For this Difference of Parallaxes is ſo very little, that it is always leſs than that of the Sun which we enquire after. Wherefore *Mercury*, tho' often to be ſeen within the Sun, will not be thought very proper for this Buſineſs.

There remains therefore the Tranſit of *Venus* through the Sun's Disk, whoſe Parallax, almoſt four Times as big as that of the Sun, will make very ſenſible Differences between the Spaces of Time, in which *Venus* will be ſeen to paſs over the Sun, in the different Regions of our Earth. Now from theſe Differences, if obſerved after a due Manner, I ſay the Parallax of the Sun may be determin'd within a ſmall Part of a Second. Nor are other Inſtruments required than Teleſcopes and common Clocks, but good ones; and in the Obſervations nothing is required but Fidelity and Diligence, with a moderate Skill in Aſtronomy. For there is no Neceſſity that the Latitude of the Place ſhould be determin'd with much Scrupuloſity, or that the Hours, in reſpect of the Meridian, ſhould

ſhould be accurately determin'd. It will be ſufficient to have the Clocks well corrected to the Revolutions of the Heavens, and that the Times be reckoned from the total Ingreſs of *Venus* within the Sun's Disk, to the Beginning of its Egreſs from the ſame. That is, when firſt the opaque Globe of *Venus* begins to reach the lucid Limb of the Sun; which Moments, as I know by my own Experience, may be obſerved to a Second of Time.

But becauſe the Laws of Motion are greatly confined, *Venus* can be ſeen but very rarely within the Orb of the Sun; and for a Series of above 120 Years, it will not be ſeen th re once: That is, from the Year 1639, (when our *Horrox* enjoy'd this Spectacle, who was the firſt and only one from the Creation of the World to this Day) to the Year 1761, *May* 26, in the Morning, when the Planet *Venus* will again paſs under the Sun, according to thoſe Theories which hitherto we have found to agree with the Heavens; ſo that at *London*, at about Six in the Morning, ſhe may be expected to be found in the Middle of the Sun's Disk, nor will ſhe be above four Minutes more Southerly than the Center of the Sun. And the Duration of this Tranſit will be nearly eight Hours, or from Two in the Morning to almoſt Ten. Therefore the Ingreſs will not be viſible in *England*. And whereas the Sun at that Time will be in 16 Degrees of *Gemini*, declining Notherly almoſt 23 Degrees; it may be ſeen not to ſet through the whole Northern frigid Zone. So that they which inhabit the Sea-ſhore of *Norway*, beyond the City *Nidroſia*, which they call *Dronthem*, as far as its Northern Promontory, may obſerve *Venus* at its entring upon the Sun's Disk. And perhaps that Ingreſs into the riſing Sun may be ſeen by the Northern *Scots*, and the Inhabitants of the Iſland of *Shetland*. Now at the Time that *Venus* will be neareſt to the Center of the Sun, the Sun will be vertical above the Northern Shores of the Bay of *Ganges*, or rather of the Kingdom of *Pegu*. And therefore in the adjacent Countries, as the Sun at the Ingreſs of *Venus* will be diſtant almoſt four Hours to the Eaſt, and at the Egreſs almoſt as much to the Weſt; the apparent Motion of *Venus* within the Sun will be accelerated by almoſt the double of the horizontal Parallax of *Venus* from the Sun; becauſe then *Venus* moves retrograde from Eaſt to Weſt, and at the ſame Time an Eye upon the Earth's Surface has a contrary Rotation from Weſt to Eaſt.

Now if the Sun's Parallax be ſuppoſed 12 $\frac{1}{2}$ Seconds, the Parallax of *Venus* will be 43 Seconds. And taking away the Sun's Parallax, there will remain at leaſt Half a Minute for the horizontal Parallax of *Venus* from the Sun, and therefore the Motion of *Venus* will be advanced by that Parallax at leaſt three Quarters of a Minute, while it runs over the Sun's Disk; in thoſe Altitudes of the Pole as are near the Tropick, and ſtill more near the Equator. Now at that Time *Venus* will move four Minutes an Hour within the Sun pretty exactly, and therefore at leaſt eleven Minutes of Time belong to the three Quarters of a Minute, by which the Duration of this Eclipſe of *Venus* will be contracted be-

cauſe

cause of the Parallax. And from this Contraction alone we might safely conclude about the Parallax we seek, if the Diameter of the Sun and the Latitude of *Venus* were given exactly; yet to postulate these for Computation, in a Matter so subtile as this is, is hardly allowable.

Therefore another Observation is to be provided, if it may be done in those Places where *Venus* has Possession of the Middle of the Sun at Midnight, or under the Meridian which is opposite to the former; that is, at a Place that is about 6 Hours or 90 Degrees more Westerly than *London*, and where *Venus* enters the Sun a little before its Setting, and comes out a little after its Rising. This will be in the said Meridian under the North Pole's Altitude of about 56 Degrees: That is, in that called *Hudson*'s Bay, at a Place call'd *Nelson*'s Harbour. For in Places near this, the Parallax of *Venus* will protract the Duration of the Transit, and will make it at least six Minutes longer; because while the Sun seems under the Pole to proceed from East to West, those Places in the Disk of the Earth will seem to move with a contrary Motion towards the West, that is, by a Motion conspiring with the proper Motion of *Venus*. Therefore *Venus* will seem to move more slowly within the Sun, and to pass over his Disk with more Delay.

If therefore it should happen, that in each Place this Transit should be observed by proper Observers, it is plain that the Mora to be observed in *Nelson*'s Harbour would be full seventeen Minutes longer, than what is to be expected at the *East-Indies*. Neither is it much Matter whether the Observation be taken at Fort St. *George* commonly call'd *Maderas*, or at *Bencoulen* on the Western Shore of the Island of *Sumatra* near the Equator, if the *English* at that Time shall be inclined to do it. Or if the *French* shall think fit to do it, the Observer will be conveniently situated at *Poudecherry*, on the Western Shore of the *Sinus Gangeticus*, under the Altitude of the Pole about 12 Degrees. To the *Dutch*, their famous Emporium of *Batavia*, will supply an Observatory convenient enough, if they have a Mind to advance this Part of Astronomy. And indeed I could wish, that Observations of this Phænomenon might be made by several Observers in different Places, as well for the greater Confirmation of the same by their Agreement, as for Fear a single Observer might be prevented by the Interposition of Clouds, and hinder'd from a Sight which I know not whether the Men of this and the following Generation will ever see again; and on which depends the certain and adequate Solution of a most noble Problem, which is not otherwise to be attain'd. Therefore we recommend again and again, to the curious Enquirers into Sydereal Affairs, to whom these Observations are reserved, that being mindful of this our Admonition, they would apply themselves strenuously and with all their Power, to the due Performing this Observation; wishing them all prosperous Success, and that the Magnitudes of the Celestial Orbs, being then determin'd within more exact Limits, may reward them with perpetual Fame and Glory.

Now we have affirm'd above, that by this Method the Parallax of the Sun may be discover'd within a five hundredth Part of its own Magnitude, which, without doubt, to some will seem wonderful. But if in both the

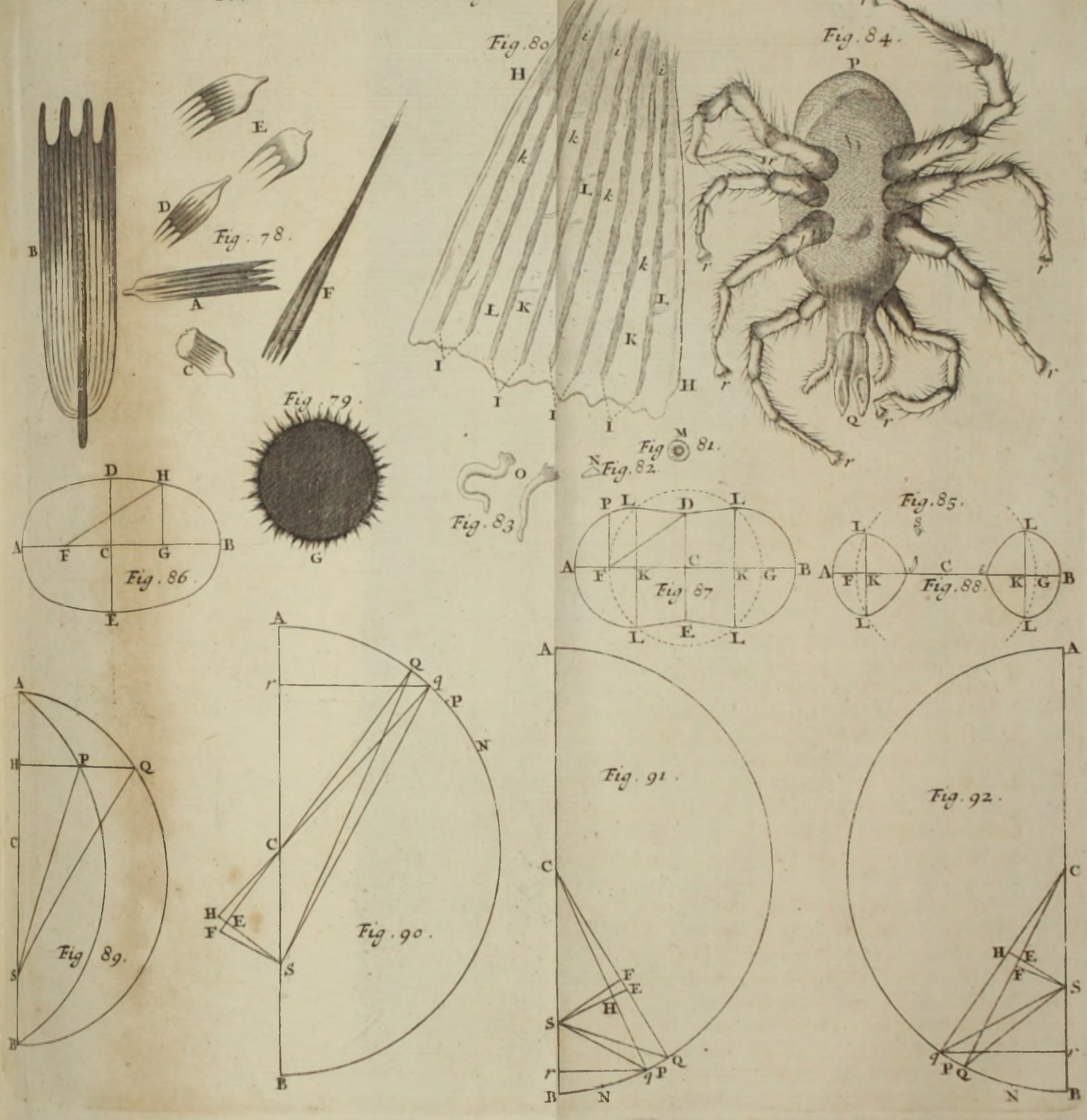
Places here mark'd out, an accurate Obſervation be made, we have already ſhew'd, that the Durations of theſe *Venereal* Eclipſes may differ from one another by full 17 Minutes, from the Hypotheſis that the Parallax of the Sun is 12 Seconds and an half. Now if this Difference be found by Obſervation to be either greater or leſs, the Parallax of the Sun will be greater or leſs almoſt in the ſame Proportion. And ſince 17 Minutes of Time correſpond to 12 $\frac{1}{2}$ Seconds of the Solar Parallax, for every Second of Parallax will ariſe a Difference of above 80 Seconds of Time. Therefore if this Difference is had true and approved within two Seconds, it will appear within a fortieth Part of a Second how great the Sun's Parallax is. And therefore his Diſtance will be determined within a five hundredth Part of itſelf, at leaſt of its Parallax, ſhall be found not leſs than we have ſuppoſed it: For 40 Times 12 $\frac{1}{2}$ make 500.

Hitherto I have explained the Matter enough, or perhaps more than enough, to Aſtronomers; whom I would alſo inform, that in this Argument I have took no Account of the Latitude of the Planet, as well to avoid the Trouble of an intricate Calculation, which would make the Concluſions leſs evident, as becauſe of the Motion of *Venus*'s Nodes not yet found out, nor to be truly determin'd but by ſuch corporal Conjunctions of the Planet with the Sun. For it is not concluded that *Venus* will paſs in four Minutes under the Sun's Center, but upon Suppoſition, that the Plain of *Venus*'s Orbit, immoveable in the Sphere of fixt Stars, will have its Nodes in the ſame Places where they were found in the Year 1639. Now if in the Year 1761, it ſhould paſs in a Path that is more Southerly, it will plainly appear that the Nodes go backwards; but if in a more Northerly, that they go forwards among the fix'd Stars; and that in the Ratio of 5 $\frac{1}{2}$ Minutes in 100 *Julian* Years, for every Minute, in which the Path of *Venus* ſhall be diſtant at that Time more or leſs from the Sun's Center, than the ſaid four Minutes. But the Difference between the Durations of theſe Eclipſes will be a little leſs than 17 Minutes, becauſe of *Venus*'s South Latitude; but it will be greater, as the Nodes go on, if it paſſes the Sun to the North of its Center.

Now for the Sake of thoſe who delight in Celeſtial Obſervations, and yet have not imbibed the whole Doctrine of Parallaxes, I ſhall farther explain the Matter with a Scheme, and with a Calculation that is ſomething more accurate.

Let us ſuppoſe therefore, that in the Year 1761, *May* 25. $17^h. 55'$. at *London*, the Sun will be in ♊ $15°. 37'$. and therefore at its Center the Ecliptic will tend towards the North in an Angle of $60°. 10'$. But at that Time the viſible Path of *Venus* within the Sun's Disk will deſcend towards the South, making an Angle with the Ecliptic of $8°. 28'$; therefore the Path of *Venus* will tend a little towards the South in reſpect to the Equator, interſecting the Parallels of Declination in an Angle of $2°. 18'$. Let us alſo ſuppoſe, that at the ſame Time *Venus* is neareſt to the Center of the Sun, and is diſtant from the ſame towards the South four Minutes, and that every Hour with a retrograde Motion it runs four Minutes within the Sun. But the

Plate V. Vol IV. Part I. Pag. 218.



the Sun's Semidiameter will be nearly 15′. 51″, and that of *Venus* 0′. 37″ ½. And let us ſuppoſe for Experiment Sake, that the Difference of the horizontal Parallaxes of *Venus* and the Sun, which we are enquiring for, to be 0′. 31″, as is derived from ſuppoſing the Sun's Parallax to be 0′. 12″ ½. Therefore with Center *C* let a little Circle *AEBD* be deſcribed, whoſe Semidiameter is 0′. 31″. repreſenting the Disk of the Earth, and in it the Ellipſes of the Parallels of 22 and 56 Degrees, North Latitude, in a Manner now uſed by Aſtronomers for the Conſtruction of Solar Eclipſes, at *DabE*, *cde*: And let *BCA* be the Meridian in which the Sun is, to which let the right Line *FHG*, denoting the Path of *Venus*, be inclined in an Angle of 2°. 18′, whoſe Diſtance from the Center *C* is 240 ſuch Parts as *BC* is 31; and from *C* let fall the right Line *CH* perpendicular to *FG*. And ſuppoſing the Planet in *H* at 17ʰ. 55′, or 5ʰ. 55′ in the Morning, let the right Line *FHG* be divided into the Horary Spaces III. IV, IV. V, V. VI, &c. equal to *CH*, that is to four Minutes. Alſo make the right Line *KL* equal to the Difference of the apparent Semidiameters of the Sun and *Venus*, or 15′. 13″ ½. And a Circle deſcribed with Radius *KL*, and its Center any Point within the little Circle of the Earth's Disk, will meet the right Line *FG* in a Point denoting the Hour which will be reckoned at *London*, when *Venus* with its Angle of Contact ſhall touch the Limb of the Sun within, in that Place of the Earth's Surface which lies under the Point aſſumed in the Disk. But if the Circle deſcribed with Center *C* and Radius *KL* ſhould meet *FG* in the Points *F* and *G*, the right Lines *FH*, *HG* = 14′, 41″, which *Venus* will ſeem to paſs over in 3 Hours and 40 Minutes. Therefore *F* will fall in 2ʰ. 15′, at *London*; and *G* in 9ʰ. 35′, in the Morning. Whence it appears, that if the Magnitude of the Earth ſhould vaniſh as it were into a Point, becauſe of its immenſe Diſtance, or if deprived of its diurnal Motion, it ſhould have the Sun always vertical to the ſame Point *C*, the entire Mora of this Eclipſe would continue for 7 ⅓ Hours. But as the Earth revolves in the mean Time with a Motion contrary to that of *Venus* through 110 Degrees of its own Longitude, and therefore the Duration of the ſaid Mora being contracted, ſuppoſe 12 Minutes, it will come out 7ʰ. 8′ nearly, or 107 Degrees. Fig. 93.

Now in the Meridian itſelf *Venus* will be next the Center of the Sun, at the Eaſtern Mouth of the River *Ganges*, where the Altitude of the Pole is about 22 Degrees. Therefore that Place will be equally diſtant from the Sun on each Side, at the Moments of Ingreſs and Egreſs of the Planet, that is at 53° ½, as the Points *a* and *b* are in the greater Parallel *DabE*. But the Diameter *AB* will be to the Diſtance *ab*, as the Square of the Radius is to the Rectangle under the Sines of 53° ½ and 68°, that is, as 1′. 02″. to 0′. 46″. 13‴. And when the Calculation is rightly perform'd, (which I omit that I may not be tedious) I find that the Circle deſcribed with Center *a* and Radius *KL*, will meet the right Line *FH* in the Point *M*, at 2ʰ. 20′. 40″; but when deſcribed with Center *b*, it will meet *HG* in *N*, at 9ʰ. 29′. 22″: That is, if the Hours are reckon'd at *London*. Therefore all *Venus* will be ſeen within the Sun upon the Banks of the *Ganges*, for 7ʰ. 8′. 42″.

Therefore we have rightly ſuppoſed its Duration will be $7^h. 8'$, ſince here a Part of a Minute is of no Conſideration.

Now the Calculation being adapted to *Nelſon*'s Harbour, I find that *Venus* will enter the Sun's Disk when it is ready to ſet; and that it will come out of the ſame preſently after its Riſing. In the mean Time that Place will be transfer'd from *c* to *d* through the Hemiſphere oppoſite to the Sun, with a Motion conſpiring with the Motion of *Venus*. Therefore the Mora of *Venus* within the Sun will be longer becauſe of the Parallax, perhaps by four Minutes, that it may be full $7^h. 24'$, or 111 Degrees of the Equator. And ſince the Latitude of the Place is 56 Degrees, it will be as the Square of the Radius to the Rectangle under the Sines of $55\frac{1}{2}$ and 34 Degrees, ſo in *AB* or $1'. 02''$ to *cd* or $28''. 33'''$. And when the Calculation is rightly perform'd it will appear, that a Circle deſcribed with Center *c* and Radius *KL*, will meet *FH* in *O*, at $2^h. 12'. 45''$; but deſcribed with Center *d*, it will meet *HG* in *P*, at $9^h. 36'. 37''$. Whence the Duration of the Mora at *Nelſon*'s Harbour will be $7^h. 23'. 52''$, that is greater than at the Mouth of the *Ganges* by $15'. 10''$, of Time. Now if *Venus* ſhould paſs without Latitude, the ſaid Difference would be $18'. 40''$; and if it ſhould be 4 Minutes more Northerly than the Center of the Sun, the ſame Difference will be increaſed to $21'. 40''$; and it would be much greater, if the Planets Northern Latitude ſhould be greater.

Now it follows from the aforeſaid Suppoſitions, that at *London Venus* will riſe after it has enter'd within the Sun; and that at $9^h. 37'$. in the Morning it will touch the inward Limb of the Sun at its Egreſs; and laſtly, that it will not leave its Orb entirely till $9^h. 56'$.

From the ſame Suppoſitions it follows alſo, that in the Year 1769, *May* 23, at $11^h. 00'$. the Center of *Venus* muſt skim by the utmoſt Northern Limb of the Sun; ſo that, becauſe of the Parallax, it may wholly appear not to immerge in the Sun, in the Northern Parts of *Norway*; whilſt upon the Shores of *Peru* and *Chili* it will be ſeen as it were riding upon the Disk of the ſetting Sun, with a very ſmall Segment immerſed: As in the *Molucca* Iſlands and the neighbouring Places, on the Disk of the riſing Sun. Now if the Nodes of *Venus* are found to go backwards, (as is ſuſpected becauſe of ſome late Obſervations) then being every where conſpicuous with its whole Body within the Orb of the Sun, by the very great Difference of theſe Eclipſes, it will ſupply a much more convincing Argument of the Sun's Parallax.

But how from Obſervations made ſomewhere in the *Eaſt-Indies*, An. 1761, of the Ingreſs and Egreſs of *Venus*, and compared with Obſervations of the Exit made here, the ſame Parallax may be ſettled, by adapting the Angles of a Triangle given in Species to the Circumferences of three equal Circles, ſhall be ſhewn upon another Occaſion.

Of the Maxima *and* Minima *in the Motions of the* Celeſtial *Bodies, by*—— n. 360. p. 948.

IV. The Theory we now receive is owing to *Kepler*, that the Heavenly Bodies ſurround the Sun, placed in the common Focus of the Elliptical Orbits, on this Condition, that by Lines drawn to the Sun, they deſcribe

describe Areas which are proportional to the Times of Description. But it requires the most sublime Geometry to shew, by what Cause this is perform'd, and that it could not be otherwise. This was reserved to be the Glory of the famous *Newton*.

Treading in his Steps, that excellent Mathematician Mr. *A. de Moivre*, F. R. S. has deliver'd certain Corollaries mentioned before, which are ready Theorems by which the Velocities or Moments both of real and apparent Motion about the Sun are determin'd, as also of the Approach or Recess to or from the Sun; in any given Points of given Orbits. Then farther to improve the Theory of the Planetary System, he has enquired after the Moments of the said Moments by Means of the same Theorems, and shews in what Points of the Orbits are the greatest Changes of these Velocities, and this by Solutions that excel in Neatness and Facility. *Vid. supra, C. I. S. I.*

Let ABP be the Elliptical Orbit of a Planet, AP the Transverse Axis, CB the Conjugate Semiaxis, S the Sun, Q the other Focus of the Ellipsis. Through S draw SM parallel to CB; and the Point M will be that in which the Distance from the Sun is increased or diminished with the greatest Velocity, $SM = AC - \frac{SC}{AC}$. Fig. 94.

But if SL is taken a mean Proportional between the Semiaxes AC, CB, the Point L will be that in which there will be the greatest Equation of the Center, as they call it, or where the Angular Motion is equal to the mean Motion. Now if the Eccentricity does not exceed that which most of the Planets obtain, it will be $BL = \frac{1}{4} BM$ very nearly. For it is $SL = \sqrt{ACqq - ACq\ SCq}$.

If the Point N be required in which is the greatest Change of Velocity of the real Motion in the Curve, the Problem will be solid. For it is $2\,NS = 4AC - 2\,NQ$ to $3\,NQ - AC$, as $ACq - CSq = CBq$ to NQq. And therefore if we make $AC = a$, $BC = c$, and $NQ = y$, we shall have the Equation $y^3 - 2ay^2 + \frac{1}{2}ccy - \frac{1}{2}acc = 0$, which being resolved y or NQ will be the Distance of the Point sought N from the other Focus of the Ellipsis. But in Orbits that are but little excentric, such as those of the Planets, if it be made $CD = Sq$, and joining AD if AK be made equal to it, the remaining Part of the Axis $KP = NS$ will be the Distance of the Point N from the Sun very nearly. But if the Orbit be parabolical, SN will be to SP, as 5 to 4, and the Angle NSP will be $53^{\circ}.\ 8'$, nearly, whose Sine is $\frac{4}{5}$ of the Radius.

But the Point O, in which is the greatest Acceleration of the apparent or angular Motion of the descending Planet, or the greatest Retardation of the ascending, will be had in this Manner. In AC let there be taken $GC = \frac{1}{4}AC$, and let CSF be made an Angle of 30 Degrees, and drawing SF let CE be made equal to it, and let GH be made equal to GE. I say, that if the Distance SO be made equal to PH, that in the Point O will be the greatest Change of the Angular Motion of the Planet moving in the Elliptical Orbit $ABOP$; for in that Place of the Orbit the second Differences of the Equations of the Center of the Planet will be found the greatest. But

it

it is $SO = \frac{7}{6} AC - \sqrt{\frac{1}{36} ACq + \frac{1}{3} SQq}$. Now if the Orbit is Parabolical, as in the Comets, it will be $SO . SP :: 8 . 7$, and the Angle OSP will be 41°. 24′ ½, or whoſe Sine is to Radius as $\sqrt{7}$ to 4.

Laſtly, The Direction of the Tangent of the Orbit will be changed with the leaſt Velocity in R, if SR be made equal to two third Parts of the greater Axis AB. But if the Eccentricity SC is leſs than $\frac{1}{3} PC$, this Minimum does not take Place, but this Velocity with which the Tangent revolves is always decreaſing, as far as the Aphelium itſelf, as it is in the Motions of all the Planets. But it does not obtain in a Parabolical Orbit, becauſe of its Axis continued *in infinitum*.

All theſe Things are demonſtrated from the foregoing Theorems of Mr. *Abr. Demoivre*, according to the Precepts of the Doctrine *de Maximis et Minimis*.

A new Star in the Swan's Neck, *by Mr.* G. *Kirch*, n. 343.p.226 *Miſcel. Berol.* p. 208.

V. Though many Varieties and Changes happen in the Heavens, among the fixt Stars, as to their apparent Magnitude, yet among all the mutable Appearances of the fixt Stars none is more wonderful, than that which *Fabricius* firſt obſerved *Ann.* 1596, in the *Whale's Neck*. For though at firſt it was accounted as ſuch a new Star as had no Exiſtence before, and after it had diſappear'd that it would return no more; yet now Experience has ſufficiently proved that it conſtantly exiſts, and that without all doubt it has always exiſted from the Beginning of the World in that Place, which it ſtill poſſeſſes. This only is wonderful in it, that it ſhews itſelf yearly of a different Magnitude, and generally at certain Times it is not at all to be ſeen by the naked Eye. For this Reaſon it is call'd by *Hevelius* the wonderful Star.

Like to this I have alſo found another in the *Swan's Neck*, but much leſs, and which may be ſeen yearly for a ſhorter Space of Time. Therefore it is no wonder that it has ſo long continued unknown. Nay, it is to be conſider'd as a ſingular Felicity, that it was viſible at that very Time, and appear'd in its greateſt Magnitude, when *Bayer* contemplated and delineated the Stars in the *Swan*, where he denoted it by the Character χ, and reckon'd it among the Stars of the fifth Magnitude which conſtantly appear. As alſo the above-mention'd Star in the *Whale's Neck*, when he conſider'd and delineated this Conſtellation, he found it of the fourth Magnitude, and mark'd it with the Letter *o*, and look'd upon it as a Star that always appears.

Fig. 95.

To find out the mutable Appearance of the Star χ in the *Swan's Neck*, Occaſion was given by a neighbouring Star in the *Swan's Head*, which *Hevelius* obſerv'd *Ann.* 1670, and 1671. For when I had entertained ſome Hope, that the ſame Star would often appear again, not otherwiſe than the Star in the *Whale's Neck*, which after its firſt Diſappearance would ſoon appear again, as was evident to *Hevelius*; I ſought for it on the 1*ſt* and 6*th* of *July* (or 11° and 16°) in the Year 1686 in clear Nights, but could not find it. I rather took Notice, that that Star of the fifth Magnitude in the Neck of the Swan was wanting, which *Bayer* marks with the Greek Letter

Letter χ. But on the *9th* (19) Day of *October*, I found it very plainly with my naked Eye. And becauſe I was eaſily perſuaded to think, that it might again diſappear to my naked Eye, I delineated ſome Stars that ſtood round it, by the Help of a two Foot Teleſcope of large Capacity, that by a Compariſon of theſe with it I might examine its Magnitude when it decreaſed, as is repreſented in the Figure *A*.

I alſo found, that that Star decreaſed by little and little, till I could no longer perceive it with a Tube of eight Feet; whereas I could always diſtinguiſh another in the *Whale's Neck*, through a Tube of four Feet, when it could no longer be perceived with the naked Eye. Fig. 96.

From that Time I ſought in vain for that Star ſeveral Nights together, yet at laſt I found it again, *Auguſt* 6, (16) 1687, by Help of an eight Foot Tube, though it was very ſmall. From thence I obſerved it to increaſe daily; and it happen'd that *Oct.* 23, (*Nov.* 2,) it firſt preſented itſelf to my naked Eye, though ſtill very ſmall. On the *2d* Day of *November*, (12°) it was very conſpicuous, and even after *Nov.* 26, (*Dec.* 6,) though on this laſt Day it was again in a State of Decreaſe. Afterwards it could not be diſtinguiſh'd but by the Tubes, and ſoon became ſo ſmall, that I could not find it again with the eight Foot Tube. And thus it was obſerved, that from one Diſappearance to another, there paſs'd about one Year, one Month, and one Week. Alſo the following Obſervations have inform'd me, that this Star kept a pretty conſtant Time in its Appearance, yet at every Period it did not arrive at an equal Magnitude. Nay it happen'd ſometimes, that it continued altogether inviſible to the naked Eye, whilſt through the Tube it was conſpicuous, and increaſed to its greateſt Magnitude. As at the End of the Year 1688, and the Beginning of 1689. On the contrary, in the Year 1690, this Star could be ſeen better, and was notably greater than its Neighbour, which *Bayer* has placed near χ, without the *Swan's Neck*, but has mark'd it with no Letter; but only for the Help of my Memory I have mark'd it with the *Hebrew* Letter ɔ. And after I had often obſerved the Appearance of this Star, I found it to be very regular, and to obſerve the Revolution of $404\frac{1}{2}$ Days.

N. B. *Whereas the* Berlin *Miſcellanies come late to our Hands, we did not obſerve this new Star, which Mr.* Kirch *has inform'd us of, before the Year laſt paſt, and that near the Ides of* July, *ſt. vet. when it appear'd much brighter than the neighbouring Star* ɔ, *and almoſt equal to the middle Star in the* Swan's Neck, *mark'd by* Bayer *η*. *But becoming inconſpicuous to the naked Eye, at laſt it vaniſh'd alſo in the Teleſcope. According to the Period in which it is ſaid to revolve, it muſt have arriv'd at its greateſt Brightneſs at leaſt in the Month of* Auguſt *of the current Year* 1715.

Now that it may be found more eaſily in the Heaven, we have added two Fig. 95, 96. *Schemes, one of which ſhews the* Swan's Neck, *with the Stars adjoining to this new one, and with two other new ones that have appear'd near it within this laſt Age; of which that before the* Swan's Breaſt *is ſtill to be ſeen as it were of the fifth Magnitude. But that which is under the Head was ſeen only for two Years, and now diſappears. The other Figure, which is Mr.* Kirch's

A, *ſhews*

A, *shews the Telescupick Stars which are near the new one, that it may be known in what Place exactly it may be look'd for, and where the diligent Observers of the Heavens, assisted by their Tubes, may expect its first Ray at its Return.*

A History of the new Stars for the last 150 Years, by .... n. 346. p. 354.

IV. Although it be said that *Hipparchus*, on Occasion of a new Star that appeared in his Time, was induced to number the Stars, and make the first Catalogue of them, which was, in the Opinion of *Pliny*, *a rash Thing to be attempted even by a God*; yet neither he nor any of the Ancients have left us the Place of that new Star, to compare with those lately seen, one of which might perhaps be the same with it, re-appearing after a long Period of Years. Now though several Authors have severally described those that have been seen nearer to our Times, it may not be amiss to give a short Recapitulation of what was principally remarkable in each of them, with the Times of their first Appearance, as far as can be collected.

And first, That in the *Chair* of *Cassiopeia* was not seen by *Cornelius Gemma* on the 8th of of *November* 1572, who says, he that Night considered that Part of Heaven in a very serene Sky, and saw it not; but that the next Night, *November* 9°, it appeared with a Splendour exceeding all the fixt Stars, and scarce less bright than *Venus*. This was not seen by *Tycho Brahe* before the 11th of the same Month, but from thence he assures us, that it gradually decreased and died away, so as in *March* 1574, after sixteen Months, to be no longer visible; and at this Day not the least Signs of it remain. The Place thereof, in the Sphere of fixt Stars, by the accurate Observations of the same *Tycho*, was $0^s. 9°. 17'.$ *à* 1^{ma} ∗ ♈^{is}, with 53°. 45′. North Latitude.

Such another Star was seen and observed by the Scholars of *Kepler*, to begin to appear on *Sept.* 30°. *st. vet. anno* 1640, which was not to be seen the Day before; but it broke out at once with a Lustre greater than that of *Jupiter*; and like the former it died away gradually, and in much about the same Time disappear'd totally, there remaining no Footsteps thereof in *January* $160\frac{5}{6}$. This was near the Ecliptick, following the Right Leg of *Serpentarius*; and by the Observations of *Kepler* and others, was in $7^s. 20°. 00'$ à 1^{ma} ∗. ♈, with North Latitude 1°. 56′. These two seem to be of a distinct *Species* from the rest, and nothing like them has appear'd since.

In the Year 1596, we have the first Account of the wonderful Star *in Collo Ceti*, seen by *David Fabricius* on the 3d of *August*, *st. vet.* as bright as a Star of the third Magnitude, which has been since found to appear and disappear periodically; its Period being precisely enough 7 Revolutions in six Years, though it return not always with the same Lustre. Nor is it ever entirely extinguished, but may at all Times be seen with a Six-foot Tube. This was singular in its Kind, till that in *Collo Cygni* was discovered. It precedes the first Star of *Aries* 1°. 40′, with 15°. 57′ South Latitude.

Another new Star was first observed by *Will. Jansonius* in 1600, *in Pectore* or rather *in eductione Colli Cygni*, which exceeded not the third Magnitude. This Star having continued some Years, became at length so small, as to be thought by some to disappear entirely; but in the Years 1657, 58, and 59, it it again arose to the third Magnitude, though soon after it decayed by Degrees to the fifth or sixth Magnitude, and at this Day is to be seen as such in $9^s. 18^\circ. 38'$. *à* 1^{ma} ∗. ♈, with $55^\circ. 29'$. North Latitude.

A fifth new Star was first observed by *Hevelius* in 1670, on *July* 15, *st. vet.* as a Star of the third Magnitude, but by the Beginning of *October* was scarce to be perceived by the naked Eye. In *April* following it was again as bright as before, or rather greater than of the third Magnitude, yet wholly disappeared about the Middle of *August*. The next Year, in *March* 1672, it was seen again, not exceeding the sixth Magnitude; since when it has been no farther visible, though we have frequently sought for its Return; its Place is $9^s. 3^\circ. 17'$. *à* 1^{ma} ∗. ♈. and has Lat. North $47^\circ. 28'$.

The sixth and last is that we described before from the *Acta Berolinensia*, discovered by Mr. *G. Kirch* in the Year 1686, and its Period determined to be of $404\frac{1}{2}$ Days; and though it rarely exceed the fifth Magnitude, yet is it very regular in its Returns, as we have found in the Year 1714. Since then we have endeavoured, as the Absence of the Moon and the Clearness of Weather would permit, to catch the first Beginning of its Appearance in a six-foot Tube, that bearing a very great Aperture discovers most minute Stars. And on *June* 15. last, it was first perceived like one of the very least Telescopical Stars; but in the rest of that Month and *July* it gradually increased, so as to become in *August* visible to the naked Eye; and so it continued all the Month of *September*. After that it again died away gradually, and on the 8*th* of *December* at Night was scarce discernible by the Tube, and as near as could be guessed, equal to what it was at its first Appearance on *June* the 15*th*; so that this Year it has been seen in all near six Months, which is but little less than half its Period; and the Middle, and consequently the greatest Brightness, falls about the 10*th* of *September*. Those that please to seek for it, may expect its first Appearance in *July* next, and find it in $9^s. 6^\circ. 30'$. *circiter à* 1^{ma} ∗. ♈, with *Lat. Bor.* $52^\circ. 40'$.

A Return of the new *Star in* Collo Cygni.

Vid. supra. §. V.

VII. Wonderful are certain luminous Spots or Patches, which discover themselves only by the Telescope, and appear to the naked Eye like small fixt Stars; but in Reality are nothing else but the Light coming from an extraordinary great Space in the Æther; through which a lucid *Medium* is diffused, that shines with its own proper Lustre. This seems fully to reconcile that Difficulty which some have raised against the Description *Moses* gives of the Creation, alledging that Light could not be created without the Sun. But in the following Instances the contrary is manifest; for some of these bright Spots discover no Sign of a Star in the Middle of them; and the irregular Form of those that have, shews them not to proceed from the Illumination of a central Body. These are six in Number, all which we will describe in the Order of Time, as they were discovered, giving also their Places in the Sphere of fixt Stars.

Lucid Spots or Nebulæ, *amongst the fixt Stars, by* —— n. 347. p. 390.

The first and most considerable is that in the Middle of *Orion*'s Sword, marked with θ by *Bayer* in his *Uranometria*, as a single Star of the third Magnitude; and is so accounted by *Ptolemy*, *Tycho Brahe* and *Hevelius*; but is in Reality two very contiguous Stars environed with a very large tranſparent bright Spot, through which they appear with ſeveral others. Theſe are curiouſly deſcribed by *Hugenius* in his *Syſtema Saturnium*, pag. 8. who there calls this Brightneſs, a monſtrous Thing, the like of which he could no where obſerve among the other fixt Stars; affirming that he found it accidentally in the Year 1656. The Middle of this is at preſent in ♊ 19°. 00′, with South Lat. $28^\circ \frac{3}{4}$.

About the Year 1661, another of this Sort was diſcovered (if I miſtake not) by *Bullialdus*, in *Andromeda*'s *Girdle*. This is neither in *Tycho* nor *Bayer*, having been omitted, as are many others, becauſe of its Smallneſs: But it is inſerted into the Catalogue of *Hevelius*, who has improperly called it *Nebuloſa* inſtead of *Nebula*; it has no Sign of a Star in it, but appears like a pale Cloud, and ſeems to ſend forth a radiant Beam into the North Eaſt, as that in *Orion* does into the South Eaſt. It proceeds in Right Aſcenſion the Northern in the Girdle, or *ν Bayero*, about a Degree and three Quarters, and has Longitude at this Time ♈. 24°. 00′ with Lat. North $33^\circ\frac{1}{3}$.

The third is near the Ecliptick between the *Head* and *Bow* of *Sagittary*, not far from the Point of the Winter Solſtice. This was found in the Year 1665, by a *German* Gentleman *M. J. Abraham Ihle*, while he attended the Motion of *Saturn* then near his *Aphelion*. This is ſmall but very luminous, and emits a Ray like the former. Its Place at this Time is ♑ $4^\circ\frac{1}{2}$ with about half a Degree South Lat.

A fourth was diſcover'd by M. *Edmund Halley* in the Year 1677, when he was making the Catalogue of the Southern Stars. It is in the *Centaur*, that which *Ptolemy* calls the Star in the Excreſcence on the Back, which he names the Cloud on the Horſe's Back, and is *Bayer*'s *ω*; It is in Appearance between the fourth and fifth Magnitude, and emits but a ſmall Light for its Breadth, and is without a radiant Beam; this never riſes in *England*, but at this Time its Place is ♏ $5^\circ\frac{3}{4}$ with $35^\circ\frac{1}{5}$ South Latitude.

A fifth was diſcovered by Mr. *G. Kirch* in 1681, preceding the Right Foot of *Antinous*: It is of itſelf but a ſmall obſcure Spot, but has a Star that ſhines through it, which makes it more bright. The Longitude of this is at preſent ♑. 9°. *circiter*, with $17^\circ\frac{1}{6}$. North Latitude.

The ſixth and laſt was accidentally hit upon by M. *Edmund Halley* in the Conſtellation of *Hercules*, in the Year 1714. It is nearly in a Right Line with *ζ* and *η* of *Bayer*, ſomewhat nearer to *ζ* than *η*: and by comparing its Situation among the Stars, its Place is ſufficiently near in ♏ $26^\circ\frac{1}{2}$. with 57°. 00. North Lat. This is but a little Patch, but it ſhews itſelf to the naked Eye, when the Sky is clear, and the Moon abſent.

There are undoubtedly more of theſe, which have not yet come to our Knowledge, and ſome perhaps bigger, but though all theſe Spots are in Appearance but ſmall, and moſt of them but of few Minutes in Diameter; yet ſince they are among the fixt Stars, that is, ſince they have no annual Parallax, they cannot fail to occupy Spaces immenſely great, and perhaps not leſs than our whole Solar Syſtem. In all theſe ſo vaſt Spaces it ſhould ſeem, that there is a perpetual uninterrupted Day, which may furniſh Matter of Speculation, as well to the curious Naturaliſt as to the Aſtronomer.

VIII.

VIII. I have compared the Declinations of the fixt Stars delivered by *Ptolemy*, in the 3d Chapter of the 7th Book of his *Almag.* as obſerved by *Timocharis* and *Ariſtyllus* near 300 Years before *Chriſt*, and by *Hipparchus* about 170 Years after them, that is about 130 Years before *Chriſt*, with what we now find, and by the Reſult of many Calculations, I concluded that the fixt Stars in 1800 Years were advanced ſomewhat more than 25 Degrees in Longitude, or that the Preceſſion is ſomewhat more than 50″ *per annum*. But that with ſo much Uncertainty, becauſe of the imperfect Obſervations of the Ancients, that I have choſen in my Tables to adhere to the even Proportion of five Minutes in ſix Years, which from other Principles we are aſſured is very near the Truth. But while I was upon this Enquiry, I was ſurprized to find the Latitudes of three of the principal Stars in the Heaven directly to contradict the ſuppoſed greater *Obliquity* of the *Ecliptick*, which ſeems confirmed by the Latitudes of moſt of the reſt; they being ſet down in the old Catalogue, as if the Plane of the Earth's Orb had changed its Situation, among the fixt Stars, about 20′ ſince the Time of *Hipparchus*. Particularly all the Stars in *Gemini* are put down, thoſe to the *Northward* of the Ecliptick with ſo much leſs Latitude than we find, and thoſe to the *Southward* with ſo much more *Southerly* Latitude. And yet the three Stars *Palilicium* or the *Bull*'s Eye, *Sirius* and *Arcturus* do contradict this Rule; for by it, *Palilicium*, being in the Days of *Hipparchus* in about 10 *gr.* of *Taurus*, ought to be about 15 Min. more *Southerly* than at preſent; and *Sirius* being then in about 15 of *Gemini* ought to be 20 Min. more *Southerly* than now; yet *Ptolemy* places the firſt 20 Min. and the other 22 more *Northerly* in Latitude than we now find them. Nor are theſe the Erors of Tranſcribers, but are proved to be right by the Declinations of them ſet down by *Ptolemy*, as obſerved by *Timocharis*, *Hipparchus* and himſelf, which ſhew that thoſe Latitudes are the ſame as thoſe Authors intended. As to *Arcturus*, he is too near the Equinoctial Colure, to argue from him concerning the Change of the Obliquity of the Ecliptick, but *Ptolemy* gives him 33′ more *North* Latitude than he now is found to have; and that greater Latitude is likewiſe confirmed by the Declinations delivered by the aboveſaid Obſervers. So then theſe three Stars are found to be above half a Degree more *Southerly* at this Time, than the Ancients reckoned them. When on the contrary at the ſame Time the bright Shoulder of *Orion* has in *Ptolemy* almoſt a Degree more *Southerly* Latitude than at preſent. What ſhall we ſay then? It is ſcarce to be believed, that the Ancients could be deceived in ſo plain a Matter, three Obſervers confirming each other. Again, theſe Stars being the moſt conſpicuous in Heaven, are in all Probability the neareſt to the Earth; and if they have any particular Motion of their own, it is moſt likely to be perceived in them, which in ſo long a Time as 1800 Years may ſhew itſelf by the Alteration of their Places, though it be intirely imperceptible in the Space of one ſingle Century of Years. Yet as to *Sirius* it may be obſerved, that *Tycho Brahe* makes him 2 Min. more *Northerly* than we now find him, whereas he ought to be above as much more *Southerly* from his Ecliptick, (whoſe Obliquity he

Change *of the* Latitudes *of ſome of the* fixt Stars, *by Dr.* E. Halley, n. 355. p. 736.

 makes

makes $2\frac{1}{2}$ greater than we esteem it at present) differing in the whole $4\frac{1}{2}$ Min. One half of this Difference may perhaps be excused, if Refraction were not allowed in this Case by *Tycho*; yet two Minutes, in such a Star as *Sirius*, is somewhat too much for him to be mistaken.

But a more evident Proof of this Change is drawn from the Observation of the Application of the Moon to *Palilicium*, *Anno Christi* 509. *Mart.* 11°. when in the Beginning of the Night the Moon was seen to follow that Star very near, and seemed to have eclipsed it, ἐπέβαλλε γὰρ ὁ ἀςὴρ τῷ παρὰ τὴν διχοτομίαν μέρει τῆς κύρτης περιφερείας τοῦ πεφωτισμένου μέρους, *i. e.* the Star was apply'd to that Part, by which the illuminated Limb of the Moon was bisected. Now, from the undoubted Principles of Astronomy, this could never be true at *Athens*, near it, unless the Latitude of *Palilicium* were much less than we at this Time find it. *Vide Bullialdi Astr. Philolaica*, pag. 172.

This Argument seems not unworthy of the *Royal Society*'s Consideration, to whom I offer the plain Fact as I find it, and would be glad to have their Opinion.

But whether it were really true, that the Obliquity of the Ecliptick was, in the Time of *Hipparchus* and *Ptolemy*, really 22 Min. greater than now, may well be questioned; since *Pappus Alexandrinus*, who lived but about 200 Years after *Ptolemy*, makes it the very same that we do. *Vide Pappi Collect. Lib.* VI. *Prop.* 35.

Mock-Suns, *and Circular Arches, seen by* Mr. E. Halley, n. 278. p. 1127. Fig. 97.

IX. On the 8*th* of *April* 1702, walking in *London* Streets about Ten in the Morning, the Air being clear, I observed the Sun to shine faintly, or as we call it waterish; whereupon casting up my Eye, I perceived several Arches of Circles about him. I made what Haste I could to get on the Top of a House, which I did at Mr. *Mordens* by the *Royal-Exchange*, and found the Appearance as is described *Figure* 97, wherein

S is the true Sun, *Z* the Zenith.

S T P P a great wide Circle passing through the Sun, and as well as I could judge, parallel to the Horizon. It was very distinct and entire, about two Degrees broad in the *Northern* Part about *T*; and held much the same Breadth in the *East* and *West*, but grew narrower towards the Sun; its Edges were not very well defined, the whole appearing like a faint white Cloud, and a Part of it would have been taken for such, but the whole Circle seen in the pure Azure Sky was a very surprizing Sight.

V N X Y a *Halo*, or rather *Iris*, that was likewise an entire Circle, having the Sun for its Center. I measured the Semidiameter of this to be much about 22 Degrees; the Breadth of this Arch, which was well defined, was by Estimate equal to the Sun's Diameter, and it was coloured with the Colours of the *Iris*, but nothing near so *vivid* as in the common Rainbow. The *Reds* were next the Sun, and the *Blews* in the outward Limb. Within this Circle the Sky appeared somewhat obscure, especially near the Arch; and I take it, that the Cause of that Obscurity was likewise the Cause that the Sun shone so faint and waterish. I expected two *Parhelia* at *X* and *Y* in the

the Interſections of this with the white Circle, having often ſeen them at that Diſtance and Poſition from the true Sun, but at this Time none ſuch appeared.

PVP, an Arch of another Circle, of which only the upper Part appeared, it was in all Reſpects, both for Breadth and Colours, like the Circle *VNXY*, which it touched in the vertical Point *V*, but its Center was below at *N*, or near it. In the Interſections of this Arch with the white Circle on both Sides, were two very bright *Parhelia*, ſo luminous, that I do not remember to have ſeen the like, which were alſo tinged with Colours, eſpecially on the Side next the Sun, where they were very red. I meaſured their Diſtance from the true Sun, and found it $31\frac{1}{2}$ Degrees. About *V* where the two Arches were coincident, it was very bright likewiſe, and the *red* on the Inſide very ſtrong, that ſome might have imagined another Sun there alſo, but the *Species* thereof was drawn out ſo in Length, that it could not properly be called a *Parhelion:* This Arch *PVP* broke off on both Sides, about five or ſix Degrees below the *Parhelia P. P.*

At *N* or the lower Part of the Circle *VNXY*, there appeared likewiſe a ſmall Piece of an Arch, which touched it there, after the ſame Manner as *PVP* touched in *V*; it ſeemed to have its Center in *V*, and about *N* there appeared another longiſh red *Species*, ſuch as at *V*, but not altogether ſo bright.

The Height of the Sun, during the Obſervation, was from 40 to 45 Degrees, when Clouds interpoſing, no more was to be ſeen; the Weather was cooler than ordinary, with a gentle *N. W.* Wind. And it was plain, that the Vapour which cauſed this Appearance, was higher than the Clouds, for they were ſeen to drive under the Circles.

X. *June* 15, 1703, between Four and Five of the Clock in the Afternoon, I ſaw a Spot in the Sun, by placing a white Paper ſo far behind the Teleſcope of ſix Foot, as to give the Image of the Sun nine Inches Diameter; the Spot was in the lower Right-hand Quadrant of the Sun's Diſk; its Form was almoſt round, inclining to an Ellipſis; it was diſtant from the Limb of the Sun about ſix or ſeven Minutes, and its Diameter I judged to be about 10 or 12 Seconds: A little before the Sun ſet I ſaw the Spot with a 16 Foot Teleſcope, and could perceive that it was environed with a Miſtineſs. On the 16*th* I ſaw the Spot again about Two in the Afternoon, and found it advanced near to the Weſtern Limb of the Sun; the 17*th* was cloudy, and ſo was the Night, which hindered me from obſerving the Eclipſe of the Moon; the 18*th* in the Afternoon it cleared up, and a little before Five, I ſaw the Spot with the 16 Foot Glaſs through thin Clouds, and found it was now very near the Limb of the Sun, little more than half a Minute; it was much contracted in its Breadth, ſo as to be four or five Times longer than broad: On the 19*th* in the Morning, I looked for it again, but could not ſee it; ſo I concluded, it was then either gone off the Diſk of the Sun, or if it adhered to the Limb, the great Tremulation of the Atmoſphere hindered me from ſeeing it.

Spots obſerved in the Sun, in June 1703. by Mr. S. Gray, n. 288. p. 1502.

Aſtro-

Aſtronomers have by theſe Spots found, that the Sun revolves on its Axis, ſo as that in 27 Days the ſame Point in the Sun's Diſk, returns to the ſame Place ſeen from the Earth; hence its Semi-revolution in 13 ½ Days, and conſequently the Spot going off the Sun's Diſk the 19*th* of *June*, may be expected to return the 2*d* of *July* next to the Eaſtern Limb of the Sun's viſible Hemiſphere, if it be not diſſolved before that Time. I have in the

Fig. 104. *Figure* endeavoured to expreſs the Appearance, but had not the Conveniency of meaſuring the Angle of the Spot's Way, with the Vertical, which is only gueſſed at.

June the 26*th* 1703. In the Evening I looked to ſee, whether there were generated any new Spots in the Sun, but found none; but on the 27*th*, about half an Hour after Eight in the Morning, by receiving the Sun's Image on white Paper from the ſix Foot Glaſs, I ſaw a Spot near the Vertical of the Sun towards the lower Limb; betwixt Nine and Ten I elevated the 16 Foot Tube, the Clouds now being of a convenient Thickneſs to let me ſee the Sun without Prejudice to my Eyes, and found that this Spot was of a triangular Form, and that it was accompanied with two other

Fig. 98. leſſer ones, as is expreſt in the *Figure*; the Sides of the great Spot were curvilinear, this with two leſſer ones, made an Equicrural Triangle; at Four in the Afternoon the triangular Spot had a ſmall Fragment ſeparated from it, and itſelf was now become Elliptical, the Spot *b* was much augment-

Fig. 99. ed, but the Spot *c* diminiſhed, and become longiſh, as in *Figure* 99; at half an Hour after Five the Fragment from the great Spot was itſelf divided into two, and the Spot *c* was ſo narrow as ſcarce to be ſeen; as at

Fig. 100. *Fig.* 100. at Six a Clock, and 30 Minutes, there was a ſmall Fragment ſe-

Fig. 101. parated from the lower End of the great Spot, as at *Fig.* 101; at Seven a Clock the Spot *b* was much encreaſed, but *c* was vaniſhed; the Obſervations made this Afternoon with the 16 Foot Glaſs, were when the Air was clear, and ſo to ſecure my Eye, the Eye-Glaſs was ſmoaked with a Wax-Candle.

The 28*th*, about Seven in the Morning, I ſaw that the great Spot was much augmented, but the leſſer ones that Yeſterday attended it, were vaniſhed, and that there were two new ones generated at about 1 ½ Minutes Diſtance from the great one below, and towards the Left-hand of it the

Fig. 102. great one was a Paralellogram, with a black Diagonal croſſing it *Fig.* 102;

Fig. 103. at Ten a Clock there was another Diagonal croſſing the former, *Fig.* 103. and the two leſſer Spots which before were longiſh, had now taken a round Form, the Spot *c* being much larger than the other at *b*.

I am not yet furniſhed with proper Inſtruments to find the Poſition of the Sun's Spots, with reſpect to Longitude and Latitude on the Sun's Disk, ſo I contented myſelf with obſerving the Poſition and Variation of the Spots among themſelves, which afforded me a moſt ſtrange and wonderful Variety.

On the ſame by Mr. W. Derham, *ibid.* p. 1504. Fig. 105, 106.

2.] The two Circles *Fig.* 105, 106. repreſent the Sun's Disk, and N. the Northern Part thereof, S. the Southern, E. the Eaſtern, and W. the Weſtern Part.

The

The Place of the Spots, and the Manner of their Appearance every Day, is repreſented with the Day of the Month on the Sun's Disk.

But I deſire it may be obſerved, that altho' the Figures of the Spots are done pretty exactly, yet their Places on the Sun are not ſo, for being unprovided with convenient Inſtruments for the Purpoſe, I could not exactly ſet off their Delineations, nor their Diſtances from the Sun's Limb, but was forced to repreſent them only as well as I could, by taking the Species of the Sun upon Paper, through a Teleſcope, and ſo marking out their Places.

But ſince the laſt Appearance of the Spots, I have invented, and have provided myſelf with an exceeding nice Micrometer, and a Watch that beateth half Seconds, hoping to have been able to have ſeen another Revolution of them.

My Micrometer is not, as uſually, to be put into a Tube, but is to meaſure the Species of the Sun on Paper, (of any *Radius*) or to meaſure any Part of it, which I am inclined to think is more exact than the common Way. By this Means I can eaſily, and very exactly, with the Help of a fine Thread, take the Declination of a Spot, at any Time of the Day; and by my half Second's Watch, and a fine croſs Hair, (which latter Way I learnt from my Friend Mr. *Flamſtead*) I can meaſure the Diſtance of the Spot from the Sun's Eaſtern or Weſtern Limb.

This croſs fine Hair, I adviſe, from my own Experience, ſhould be ſet, not at the exact focal Diſtance from the Eye-Glaſs, (as uſually) but a little out of that Diſtance, nearer towards the Object-Glaſs, becauſe the Shadow of the Hair will be thereby much narrower, and more ſtrongly appear croſs the Species of the Sun received on the Paper, which I take this Occaſion to note, not only becauſe I believe it hath ſcarcely ever been before obſerved, but becauſe it may be of good Uſe in taking the Sun's Altitude, meaſuring his Diameter, *&c.* this being a more eaſy, and perhaps a more exact Way, than by looking through the Tube.

Being thus provided, if I could have ſeen another Revolution of the Sun's Spots, I ſhould have been able to have given a more accurate Account of their Poſition and Motion. They ſeemed ſtrong enough to have laſted another, or more Revolutions, but none have been viſible ſince the ſixth of this Month, on which Day I think I had a Glimpſe of a Spot on the Sun's Weſtern Limb, about Seven of the Clock in the Morning.

The Appearances of the Spots, being in the Figures above, ſet with every Day of the Month, I need ſay but little, only take Notice of a few Things, that the Figures do not ſo well expreſs.

The Spot in *Fig.* 105, was as repreſented, *viz.* 1ſt round and ſtrong, afterwards long, and with a *Nucleus*. The very ſame Spot (I doubt not) I ſaw again on the Sun's Eaſtern Side on *July* 5, but very faint, ſmall and long, (as in *Fig.* 106.) ſo as to be but juſt diſcernable. On *July* 6, it quite diſappeared, both through my Tubes, and on Paper, which is better.

The Spots in *Fig.* 106, had theſe remarkable Appearances and Variations. On *June* 28, viewing the Sun towards Evening, I eſpied a large, ſtrong dark Spot, with two or more glaring *Nubeculæ* behind it, ſomewhat like the Repreſentation in the Figure. Theſe the next Day were become four ſtrong dark

dark Spots, the foremoſt with a Tail to it, conjoining the little Spot next it, as in the Figure. On *June* 30, I ſaw Spots; but it being a cloudy Morning, and I abſent from my Tubes in the Afternoon, the Repreſentation of them in the Figure is not exactly as they were. *July* 4, between two long Spots appeared ſomething like a round *Nubecula*, as in the Figure. The reſt as in the Figures.

Thus I have given the beſt Account I could of the late Solar Spots. The ſingle Spot in *June* may be ſeen to have paſſed above half over the Diſk, before a Friend of mine gave me Notice of it: And that and ſome others were, I hear, ſeen in *May*; but it was not my Fortune to ſee them ſooner; which if I had, I ſhould have been able to have made my Account better.

Spots obſerved in the Sun, *in* 1704. *by Capt.* Stannyan, n. 294. p. 1756.

On *Saturday*, *May* the 15th, 1703, As I was obſerving the Setting of the Sun, in order to examine my Clocks, there appeared two Suns, the Mock-ſun ſeemed above the Real one, which was then only five Degrees above the Horizon. Whereupon I took a good ſeven-Foot Teleſcope, with a ſmall Aperture, and ſoon diſcover'd a Solar Spot near the Sun's Center, which I deſign'd to obſerve more exactly the Day following, but it proved cloudy.

May

Sunday no Sun-ſhine.

Monday, *May* the 17th, At Six a Clock in the Morning I took the ſame Teleſcope, armed with a clouded Eye-Glaſs, and immediately perceived that the Spot was advanced conſiderably towards the Sun's Weſtern Limb; it ſeemed of a ſtrong Conſiſtence, very compact, reſembling a Face, and was diſtant by Noon from the anterior Limb of the Sun's Diſk 61 Seconds of

Fig. 107. Time. See *Fig*. 107.

Tueſday, *May* the 18th, At Noon I found the Spot diſtant from the pre-

Fig. 107. ceding Limb 46 Seconds of Time. *Fig*. 107.

Wedneſday, *May the* 19th, At Noon I obſerved the Solar Spot to be mo-

Fig. 107. ved within 33 Seconds of Time of his Weſtern Limb. *Fig*. 107.

Thurſday, *May* the 20th, At Noon the Spot was arrived within 21 Seconds of Time of the preceding Limb, and moving nearly in a ſtrait, Line; inter-

Fig. 107. ſecting the Parallel of Declination paſſing thro' the Sun's Center. *Fig*. 107.

Friday, *May* the 21ſt, We had no Sun-ſhine.

Saturday, *May* the 22d, At Seven o'Clock in the Morning I obſerved the

Fig. 107. Solar Spot was advanced very near the Limb of the Sun's Diſk. *Fig*. 107.

Sunday, *May* the 23d, At Six in the Morning I ſaw the Spot, which by that Time was got to the very Edge of the Sun's Diſk, reſembling a Barley Corn, lean and ſlender, and of a duskiſh Colour, wanting only its own ſhorteſt Diameter of the Sun's Limb. At Eight a-Clock I obſerved it again: Alſo at Ten, and at Twelve. At Two I perceived it was ſlid into the very Circumference, and hardly viſible, had I not had an Eye upon it all the Day long. At Four I examined the Sun's Body with my eighteen Foot Glaſs, which is a good one, but could not perceive the leaſt Glimpſe of it; ſo that

Fig. 107. about Three in the Afternoon it totally diſappeared. *Fig*. 107.

June On *Thurſday*, *June* the 3d, About Six in the Evening I obſerved with my eighteen Foot Glaſs four Spots in the Sun's Diſk, environed with a Miſtineſs,

thicker

thicker on the Right-hand than on the Left, ſituated in the upper Left-hand Quadrant, about the 12th Part of the Sun's Diameter diſtant from his neareſt Limb. From the Cloud about them proceeded both Ways five long curve Rays, of a yellower Colour than the Sun's Body. Theſe Spots I could never ſee more, though I watched them for ſeveral Days together. *Fig.* 108. Fig. 108.

On *Monday*, *June* the 7th, 1703, At Three a-Clock in the Afternoon, I diſcovered the ſame Spot (to my thinking) that I ſaw go off the Sun's Disk on *May* the 23d, re-entring the Sun's Face juſt at the Time and Place that I expected it.

At Four of the Clock, the Sun being extremely clear, I mounted my eighteen Foot Teleſcope, through which the Spot appeared diſtinct, but ſlender like a Spider, with an Elliptical ſpeckly Miſt about it, and 5 or 6 light colour'd Streaks. It ſeemed to me to be as it were divided near the Top, as in *Fig.* 109. Fig. 109.

Tueſday, *June* the 8th, At Six in this Morning the Spot was very viſible, and I ſaw it trace again its former Path, coming in exactly where I expected; it kept its Shape, but thoſe Limon coloured Streaks diſappeared, though itſelf and the Miſt about it grew bolder and broader viſibly, as it re-entred the Sun's Disk.

Wedneſday, *June* the 9th, At Five of the Clock this Evening I obſerved the Spot with the eighteen Foot Glaſs, but could not perceive it had altered its Shape, but advanced gradually over the Sun's Disk, as it had formerly done.

Thurſday, *June* the 10th, At Noon the Sun ſhining very bright, I had an Opportunity of being aſſured it was the ſame Spot; I plainly ſaw it move over its former Path, and was then diſtant from its neareſt Limb 29 Seconds of Time. At Five in the Evening I obſerved its Shape (with my eighteen Foot Tube) to be altered, appearing bigger and blacker than ever, as in *Fig.* 110. Fig. 110.

Friday, *June* the 11th, was an ill Day for Obſervations, but I had a Sight on't with the eighteen Foot Glaſs; it continued black and bold, as before.

Saturday, *June* the 12th, At Seven a-Clock in the Morning, the Sun's Body being very clear, I ſaw the Spot through the eighteen Foot Glaſs, retaining its former Shape.

Sunday, *June* the 13th, By this Day Noon the Spot was arrived at the ſame Point of the Sun's Disk, that I found it in on *Monday* at Noon, *May* the 17th, which makes me inclinable to believe it was the very ſame Spot.

Monday, *June* the 14th, according to Rules received Yeſterday from Mr. *Flamſtead*, I meaſured the Diſtance of the Spot from the next Limb of the Sun's Disk, which I found to be 45 Seconds of Time from the anterior Edge of the Sun's Body: And upon *Tueſday*, *May* the 18th, it was obſerved to be in the very ſame Place of its Path, within a ſingle Second of Time. At Four I obſerved it with my 18 Foot Glaſs, and perceived that it had alter'd its Shape, appearing as at *Fig.* 111. I received it on the Scheme, and it was diſtant from the preceding Limb 612 ſuch Parts, as the Sun's Semidiameter is 900. Fig. 111.

Tueſday, *June* the 15th, At Noon the Solar Spot was diſtant 32 Seconds of Time, from the leading Limb of the Sun's Disk, and covered the very Place, where the ſame Spot had been obſerved on *Wedneſday* the 19th of *May*.

Wednesday, *June* the 16th, No Sun-ſhine.

Thurſday, *June* the 17th, No Sun-ſhine.

Friday, *June* the 18th, At Noon I obſerved the Solar Spot waxing very ſlender, but notwithſtanding that, it was black and bold to Appearance, the Miſtineſs about it on the Right-hand perceiveable, and that on the Left grown ſlender, in Proportion with the Spot itſelf, and found it diſtant 5 Seconds of Time.

Saturday, *June* the 19th, At Five this Morning, it being clear Weather, I ſaw the Spot diſtinctly with my ſeven Foot Tube: At Nine a-Clock I mounted my eighteen Foot Glaſs, obſerving once in half an Hour all the Morning: At Twelve I perceived, that all the Cloud or miſty Matter, that uſed to ſurround the Spot, was inviſible, and the Spot itſelf reduced to little or no Breadth, in Compariſon to what it had been towards the Sun's Center, and ſo cloſe to the Limb of the Disk, that I could only perceive a ſmall Streak of the Sun's Light between it, and the Limb of the Sun's Body: At Two a-Clock I could juſt perceive it, but grown extremely ſlender.

The firſt Revolution, I ſaw the Spot half in the Circumference of the Sun's Limb at Two a-Clock on *Sunday*, *May* the 23d: And the ſecond Revolution, I juſt perceived it with the eighteen Foot Glaſs, at half an Hour after Two a-Clock on *Saturday* the 19th Day of *June*.

On *Sunday*, *June* the 27th, About Six a-Clock in the Evening, I obſerved ſeveral Spots in the Sun's Disk, but had not the Conveniency to uſe my longeſt Teleſcope, becauſe of ſome Trees that were in my Way to Weſtward, ſo that I made no Obſervation till *Tueſday* following.

Tueſday, *June* the 29th, About Seven in the Morning I counted 16 remarkable Spots in the Sun's Body, and near his Center they appeared as in *Fig.*
Fig. 112. 112, through the eighteen Foot Glaſs; then I took my ſeven Foot Teleſcope and Frame, and obſerved, that the foremoſt Center of ſix, that looked on the Paper as one Spot, was diſtant from the Sun's anterior Limb 81 Seconds of Time, and the laſt Cluſter 87.

This Day the foremoſt Spot was diſtant from the following Limb, according to the Path of the Spot, juſt 55 Seconds of Time. The Sun's Diameter was always 136 Seconds in the Tranſit, and the Spot was 126: So that the Spots Path was 10 Seconds ſhorter than the Sun's Diameter.

Wedneſday, *June* the 30th, At Eight a-Clock this Morning, obſerving the Solar Spots with my eighteen Foot Teleſcope, I perceived very plain, that they had wonderfully increaſed in Number, and ſtrangely changed their Places. The Cluſter of ſeven Spots ſeemed to me to move gradually, as the ſingle Solar Spot did in *May*, but the Cluſter 4 went too faſt forward, the
Fig. 113. 12 Spots without a Miſt about them ſtraggled all Manner of Ways, and the 9 Spots and the 5 black little ones went backward and unbent itſelf at the ſame Time as it were into a ſtreight Line. I am apt to believe it went backward, as that the other went too faſt, or faſter than ordinary forward; for in 24 Hours, the foremoſt Cluſter advanced 21 Seconds of Time, which is more by 6 Seconds than ever the ſingle Spot moved in that Time, even when nearest

neareſt the Sun's Center; and the Diſtance in Time between the firſt and the laſt Cluſter this Day was greater by three Seconds than the Day before.

The foremoſt Cluſter of 4 Spots was diſtant from the advancing Limb of the Sun 60 Seconds of Time.

At half an Hour paſt Four the advancing Cluſter paſſed the Interſection in 55 Seconds of Time, after the Sun's foremoſt Limb had paſſed conformable to the Spots Path; and the laſt Spot paſſed in 63 Seconds of Time, the laſt Limb paſſing the Interſection, according to the Path of the Spot, in 126 Seconds of Time, the Sun's largeſt Diameter paſſing in 136 Seconds; the Spots by this Time appeared ſtrangely black, and of very odd Shapes, as in the upper Part of the Circle. Fig. 113.

Thurſday, *July* the 1ſt, At Eight a-Clock in the Morning, I obſerved the Solar Spots with my eighteen Foot Teleſcope, the Weather being good, and ſaw that they had ranged themſelves in reſpect of one another, as is repreſented in the upper Part of the Scheme: The leading and largeſt Spot being diſtant from the anterior Limb 44 Seconds of Time, the laſt Cluſter lying a little awry, paſſed in 53 Seconds: After the anterior Limb had ſo done, the following Limb alſo paſſed the Interſection, according to the Path of the Spot, in 125 Seconds of Time. *July.* Fig. 114.

Friday and *Saturday*, No Sun-ſhine.

Sunday, *July* the 4th, This Morning at Eight a-Clock the leading Spot was diſtant from the advancing Limb 10 Seconds of Time, the Spots and Cluſters retaining nearly the ſame Shape, but beginning to contract themſelves, the foremoſt methought looked ſtrong enough to make another Revolution, and paſſed in 127 Seconds.

Monday, *July* the 5th, At Seven a-Clock I found the Spots had quite altered their Shape, appearing dull and ſlender, as in the lower Part of the Scheme, and diſtant about 4 Seconds, being all included in a Cloud.

Tueſday, *July* the 6th, At Ten a-Clock the Sun's Disk, viewed with my eighteen Foot Teleſcope, was found clear of all Spots.

On the 17th Day of *July*, about Four a-Clock in the Afternoon I obſerved ſome Spots in the Sun's Body, reſembling thoſe I ſaw on *Thurſday* the 3d of *June*, only with this Difference, that theſe appeared to me as if they had been heated red hot; they ſeemed to be in the ſame Part of the Sun's Disk. I obſerved them above an Hour together that Day, but could never afterwards ſet Eye on them, nor diſcover whether they were coming in, or going off his viſible Disk. I continued to obſerve the Sun as often as was poſſible, with my eighteen Foot Glaſs, till the End of the Month, but without farther Succeſs.

4.] I have endeavoured to render this Account of the Spots of the Sun more complete than my former; few of thoſe Appearances having eſcaped my Sight; and being alſo better provided with competently good Inſtruments to take their Places on the Sun, *viz.* a *Micrometer* (after Mr. *Gaſcoign*'s Manner) to take their Diſtance from the Sun's Northern or Southern Limb, which is parallel with the Pole of the Earth; and a *Half-Second's Movement*, to meaſure their Diſtance from the Sun's Eaſtern or Weſtern Limb. Spots in the Sun from 1703 to 1708, by Mr. W. Derham, n. 330. p. 270.

A Table of all the Spots *and* Faculæ *on the* Sun, *visible at* Upminster *since* July 1703.

1703.	*Jun.* 23. *none*	8	30	*Feb.* 14	*Nov.* 1
Octob. 9	*Some* 24	* 10	*April* 2	15	4
10	*Vanished* 25	* 22	*Vanished* 3	16	8
11	*July* 18	* 23	*June* 7	18	10
Nov. 19	20	*June* 22	8	24	15
22	21	23	*July* 24	*March* 6	16
1704.	* 23	24	25	9	17
Jan. 16	*Sept.* * 10	26	*Sept.* 4	11	18
17	*nothing* 14	27	5	12	19
18	18	28	6	14	20
19	* 21	30	7	18	22
21	*Nov.* 17	*July* * 1	8	21	N. B. *This* Nov. 15. *another Spot arose on the Eastern Side of the Disk whilst this on the Western.*
22	18	* 3	10	*June* * 29	
23	21	24	*Feint* 11	*Extinct* 30	
Jan. 30	*Dec.* * 2	*Sept.* 30	*Extinct* 12	*July* 1	
Feb. 23	1705.	*Octob.* 2	*Oct.* * 29	*Languid* 2	
25	*Jan.* 1	3	*Nov.* 5	*Scarce visible* } 4	
Mar. 7	2	5	*Faint* 6	*More visible* } 5	
8	3	6	* 8	*Extinct* 6	
9	5	7	28	*Appears* 8	
10	* 25	25	29	* 10	
11	*Feb.* * 19	26	30	*Aug.* * 31	
13	*March* 14	30	*Dec.* 1	*Sept.* * 12	
April 11	16	31	2	16	
12	*April* 1	*Nov.* 2	3	*Spot* & * 17	
13	*May* 5	4	4	19	
May 1	6	* 21	5	21	
* 11	7	1706.	* 22	27	
		Feb. 7	* 31	*No Spot nor* * } 28	
		Mar. 7	1707.	*Octob.* 31	
		27			

In

In this Table the *Faculæ* are noted with an Aſteriſk; and the Duration of every Appearance of the ſame Spots or *Faculæ*, or the Time they diſappeared, with a Line.

Out of many Things that I took Notice of in viewing the Spots and *Faculæ*, I ſhall ſelect only ſome few Obſervations, which are the moſt remarkable.

And firſt, as to the *Figure* of the Spots. They are well known to change frequently; and therefore I think it of little Uſe to give their Figures every Time I obſerved them. But it is ſomewhat remarkable, that the Spots generally appear longiſh near the extreme Parts of the Disk. If they are never ſo round near the Middle of the Disk, they become longer and longer towards the Extremes, till (at going off) they ſeem to be nearly a ſtrait Line, nearly parallel to the Sun's Limb. Which is a manfeſt Argument, that the Sun is a Globe, and that theſe Spots are on, or very near its Surface.

Another Thing remarkable is, the *Mutability* of the *Shape* of the Spots. I have more than once manifeſtly perceived them to change in the very Time I have been looking upon them. Thus *Nov.* 19, 1703, I ſaw three or more Spots not far off the Middle of the Disk; and whilſt I was looking upon them, they ſeemed to vary, both as to their Shape and Strength; ſometimes ſeeming longer, ſometimes ſhorter; ſometimes ſpiſs, ſometimes languid. And this they ſeemed to do, not only through my ſixteen Feet Tube, (which I thought at firſt was from the different Diſpoſition of my Eye) but alſo when I received the Sun's Image through a ſix Feet Teleſcope, on a white Paper, in a darkened Room. Theſe mutable Spots the Weather hindred me from ſeeing again till *November* the 22d following; and then they were become only like a thin Smoak, or *Nebula*.

So again *April* 11, 1704, there were divers Spots with *Umbræ* about them. Theſe *Umbræ*, or *Nebulæ*, I could plainly perceive, whilſt I was looking on them, to be ſometimes very faint and thin, and ſometimes much darker and thicker. Theſe *Maculæ* and *Umbræ* I obſerved ſuddenly brake out in the Sun: For on *April* 9, the Disk was free. But this *April* 11. laſt mentioned, I perceived them advanced near a quarter Part on the Disk: And conſequently they brake out in the Sun within 48 Hours before. On *April* 13, the Spots were become *Umbræ* in the Morning; and at Four of the Clock in the Afternoon, there were no Remains of either *Maculæ* or *Umbræ*.

From this *ſhort Continuance* of theſe Spots on the Sun, it is more than probable, they were in a perpetual Flux and Change; and that thoſe Mutations, which I perceived in them, whilſt I was looking on them, were real, not imaginary.

Alſo it may be farther remarked, (which I have frequently obſerved, and which as I remember *Scheiner* obſerved long ago) That thoſe *Spots* and *Umbræ*, which *ſuddenly* ariſe, do as *ſuddenly* decay, and are ſoon *extinct*.

extinct. And ſuch Spots, I have farther obſerved, do ſeldom turn to *Faculæ*, as they commonly do when longer on the Sun.

Again, *May* 5, 1705, I could perceive two Spurs or Branches (running from a Spot) to change, and be ſometimes darker, ſometimes thinner.

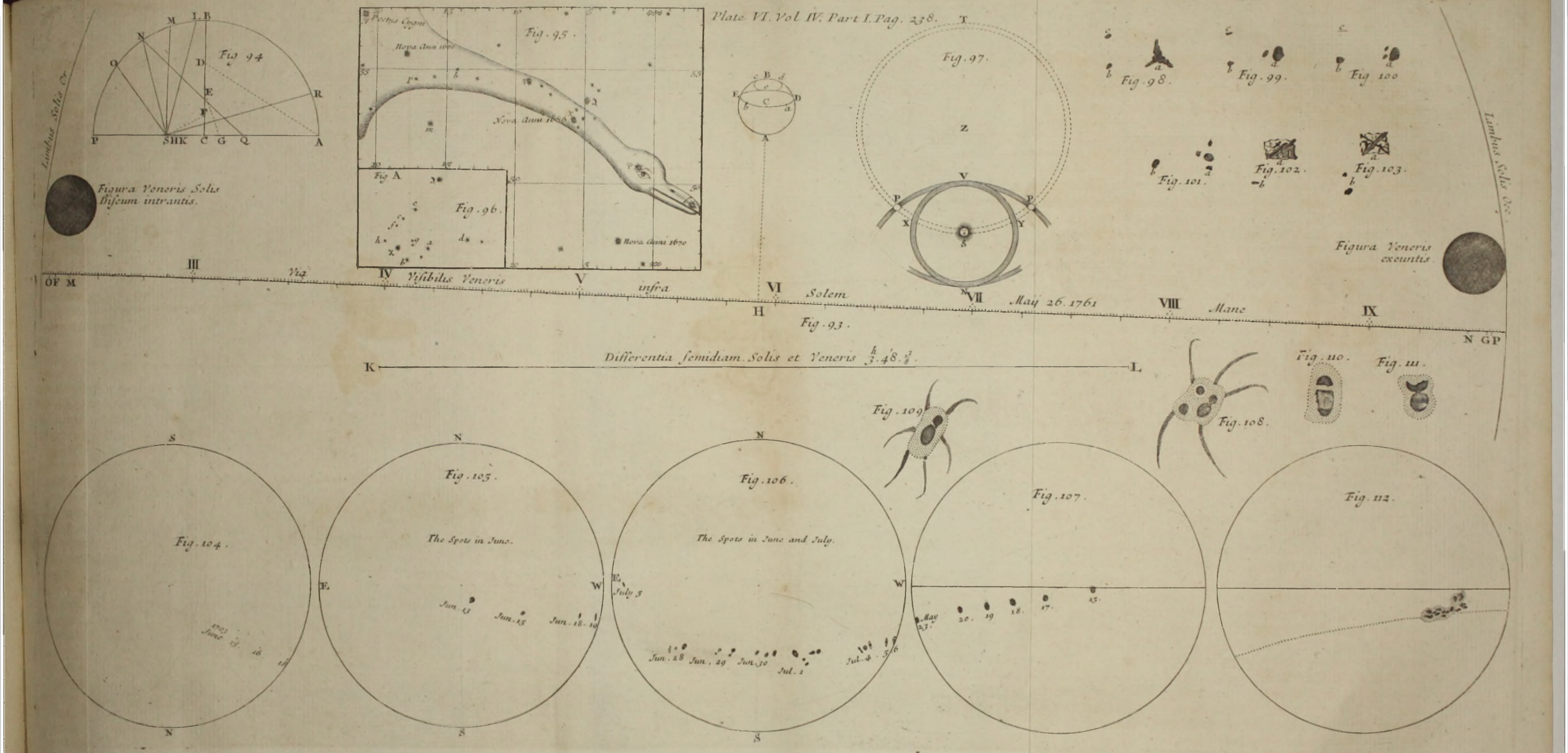
So *March* 30, 1706, I obſerved ſuch another Variation. This Day, or but a little before, *Spots* with *Faculæ* aroſe in the Sun, which remained not above three Days on him. One of theſe Spots I could manifeſtly perceive to be ſometimes quite extinct, and then again immediately to appear: And the *Faculæ* alſo, in half an Hour's Time, had plainly altered their Shapes.

October 29, the ſame Year, I could plainly perceive the *Maculæ* and *Faculæ* both to change: And whilſt I was carefully viewing them, I ſaw a Spot ariſe in one of the brighteſt *Faculæ*, and again nearly diſappear; and then again appear ſtrong and ſpiſs. I ſhould have been glad to have ſeen how they appeared next Day; but the Weather was ſtormy, cloudy, and wet for ſeveral Days after.

Another Thing I have obſerved (and not having the Book by me, I forget whether *Scheiner* obſerved the ſame or not) is, That the *Maculæ* do generally, if not always, become *Nebulæ* or *Umbræ*, before they quite vaniſh; and after that, very frequently turn to *Faculæ*, or bright golden Spots, more illuſtrious and fulgid than the other Parts of that glorious Globe. If the Spots are of a ſhort Duration, *Faculæ* ſeldom enſue: Or if they do, they are commonly the Remains of ſome Spots that had been on the Sun, and vaniſhed perhaps on the Side oppoſite to us. But Spots that *long continue*, if they vaniſh, before that Part of the Sun revolveth out of our Sight, do very often become *Faculæ*: Of which the Table affordeth ſeveral Inſtances, particularly *July* 3, 1705.

From theſe preceding Particulars, and their Congruity to what we perceive in our own Globe, I cannot forbear to gather, *That the Spots on the Sun are cauſed by the Eruption of ſome new Vulcano therein*; which at firſt, pouring out a prodigious Quantity of Smoak, and other opacous Matter, cauſeth the Spots: And as that fuliginous Matter decayeth and ſpendeth itſelf, and the Vulcano at laſt becomes more torrid and flaming, ſo the *Spots* decay and grow to *Umbræ*, and at laſt to *Faculæ*; which *Faculæ* I take to be no other than more flaming brighter Parts than any other Parts of the Sun. Theſe *Faculæ* I have obſerved *never* continue *long* on the Sun: And the Reaſon I conceive is, becauſe the *Vulcano*, after its Smoak is over, doth not long emit its Flames, by reaſon the fiery *Pabulum* is then near ſpent, when once it begins to flame: After which the torrid *Vulcano* ſoon returneth to the natural Temperature of the Sun, ſo nearly at leaſt, as to eſcape our Sight, at ſo vaſt a Diſtance as the Sun is from us.

Another Thing, that may be accounted for, and indeed doth in ſome Meaſure confirm alſo what I have ſaid, is the *Nuclei*, or darker Part of


Limbus Solis &c.
Fig. 94
Fig. 95
Pectus Cygni
Nova Anni 1600
Nova Anni 1686
Nova Anni 1670
Fig. 96
Fig A
Fig. 97
Fig. 98
Fig. 99
Fig. 100
Fig. 101
Fig. 102
Fig. 103
Figura Veneris Solis Discum intrantis.
Figura Veneris exeuntis.
Limbus Solis &c.
Via Visibilis Veneris infra Solem May 26. 1761 Mane
Fig. 93.
Differentia semidiam. Solis et Veneris 3.48
Fig. 109
Fig. 108.
Fig. 110.
Fig. 111.
Fig. 104.
Fig. 105.
The Spots in June.
Jun. 13
Jun. 15
Jun. 18-19
Fig. 106.
The Spots in June and July.
July 3
Jun. 28
Jun. 29
Jun. 30
Jul. 1
Jul. 4
Fig. 107.
May 23
Fig. 112.

of the Spots; generally in moſt Spots, and towards the Middle of them. Now it is very uſual in culinary Fires in this our Globe, when they emit Smoak, that the Middle is the darkeſt Part. If, for Inſtance, we were from aloft in the Air, to ſee a thick Smoak come tumbling out of a *Chimney*, or the Mouth of a *Vulcano* juſt kindled, we ſhould find the middle Part, juſt over the Mouth of the *Chimney*, or *Vulcano*, to be the more ſpiſs and dark, and towards the Extremes clearer and thinner. And ſo I take it to be in the Eruptions of the Sun; that the *Nucleus* is juſt over the Mouth of the ignivomous Cavern, and that the miſty Parts of the Spot are the thinner Parts of the Smoak, ſwimming about in that Fluid, or *Atmoſphere*, which I ſuppoſe doth ſurround the Sun, as well as our Globe, and the Moon manifeſtly; yea, and in all Probability, every Planet of this our Solar Syſtem.

From what hath been ſaid, we may give a Reaſon, why there are ſometimes Spots *frequently* on the Sun, and ſometimes *none* in many Years. One Thing I believe there is in this, That there may be Spots, but not always ſeen. But there are doubtleſs great Intervals ſometimes, when the Sun is free; as between the Years 1660, and 1671, 1676, and 1684. In which Time Spots could hardly eſcape the Sight of ſo many curious Obſervers of the Sun, as were then perpetually viewing him with their Teleſcopes in *England*, *France*, *Germany*, *Italy*, and all the World over; whatever might be before, from *Scheiner*'s Time. The Reaſon of this long Diſappearance of the Spots, I take to be from the Want of extraordinary Eruptions in that fiery Globe. The ſulphureous, or other Matter, or *Pabulum* of thoſe Eruptions, is ſpent or diſſipated, and that Globe continues in its natural ordinary burning State, till there happens to be a freſh Collection of ſmoaking, diſploſive, and extraordinary Matter, that cauſeth a new Eruption: Which Eruptions generally happen between what we may call the Sun's *Tropicks*, or in his Torrid Zone: For I never obſerved any Spots to be near the Sun's *Poles*. And if I miſremember not, the Spots in *Scheiner*'s Cuts are all about the middle *Zone* of the Disk. The greateſt Evagation I ever obſerved of them was *March* 8, 170$\frac{3}{4}$. On which Day, beſides the dark Spots in the uſual Zone, I perceived ſome faint Spots, ſcarce viſible, much nearer the Southern Pole, than I ever had ſeen them. But this was, no doubt, in ſome Meaſure owing to the Poſition of the Earth in reſpect of the Sun, as well as to the Southerly Place of the Spots on him: For, about the Equinoxes, the Spots ſeem to march pretty far towards the Poles of the Sun, as may be ſeen by the *Schemes*. Fig. 115, 116.

Having thus obſerved, what *Part* of the Sun the Spots commonly poſſeſs, I ſhall next take Notice of their *Stages* and *Path* over the Sun. That the Sun moveth round his own *Axis*, is manifeſt, beyond Doubt, from the Motion of the Spots. And that the Spots ſeem to traverſe the Sun, ſometimes in ſtrait Lines, ſometimes in Curve Lines,

curved

curved this Way, and that Way, is as manifeſt alſo, and well known, and is ſet forth in the Figures: Which Figures ſhew the Stages of the Spots, every Day that I obſerved them, and the Lines they deſcribe in ſeveral Months of the Year. The daily Stages in both Figures are exact; or if they ſeem otherwiſe, it is by reaſon the Obſervations were made at different Times of the Day; as One in the Morning, the other ſome following Day in the Evening, or Afternoon. But the Declinations of the Spots, or their Diſtances from the Sun's Northern or Southern Limb, are leſs exact in the 116th Figure than the 115th, in which latter they are very near the Truth.

And the Cauſes of the Defects in the 116th Figure I ſhall mention, to prevent the ſame Errors in others I myſelf ran into.

1. The Diminution of the Sun's vertical Diameter by the Refractions was the principal Cauſe of my Errors. This, although I was ſufficiently aware of, yet I did not think had been ſo conſiderable, for Want of experimenting, or well conſidering the Matter: For I have ſometimes found the perpendicular or vertical Diameter of the Sun diminiſhed, from 32′, 21″ on the Meridian, to 26′, 3″ at the Horizon, in one and the ſame Day.

2. For the ſame Reaſon I was not aware of the Time being ſo long, before the Sun goes round, as I found it.

3. Another Error was meaſuring the Sun's Image on the Scene of white Paper, with the Shade of the Micrometer; and not by looking through the Tube, and ſo claſping the Limb of the Disk with the parallel Edges of the Micrometer. The former, although practiſed by ſome eminent Aſtronomers, is a far more eaſy and indulgent, than accurate Way.

—Spots--from 1707, to 1711, by the ſame, *ibid.* p. 278.

5.] I have ſince ſeen other Spots on the Sun, whoſe Times are expreſſed in the *following Table.*

From my Obſervation of theſe Spots, I am farther confirmed in the Opinion I expreſſed in the foregoing Paper: Particularly in viewing the Spots of *Auguſt* 1, 1708, (repreſented *Fig.* 117.) where ſome were large and dark, others leſs and thinner, and all encompaſſed with *Nebulæ*. In viewing theſe, I obſerved great *Alterations* at the very Time I was looking on them. Sometimes the *Nuclei* were very dark and black, ſometimes leſs ſo; and the ſame Thing I obſerved alſo in the *Nebulæ* encompaſſing them. One of the leſſer Spots *b* in *Fig.* 117, which the Day before was ſufficiently viſible and ſtrong, was, this Day now thick and ſtrong, and Anon languid and leſs viſible. And from the two Spots *a* and *d* I could plainly ſee a Smoak iſſuing out to *c* and *f*, ſometimes viſible for 5 or 6 Minutes, and then diſappearing for a Quarter of an Hour, or more; and then again ſmoaking out, and again diſappearing, as before. All which Particulars I ſaw over and over again repeated, for a good While together, till I was weary of the Obſervation.

(Margin notes: Fig. 117. — Fig. 117.)

Theſe

1707.		1709.	
Decemb.	4	*Jan.*	15
	10		21
			22
	* 29		
	* 30	*August*	13
			* 17
1708.		*Octob.*	8
July	31		
August	1	*Novemb.*	1
	5		2
	6		4
			5
	22		6
	23		
	24	1710.	
	28		
Septemb.	1	*Jan.*	22
Novemb.	5	*April*	* 6
Dec.	14	*Octob.*	14
	26		* 18

These Spots I was hindred from viewing until *August* 5, following: And then I found the Spot *b* quite extinct, (as I expected) as also some of the other Spots; together with the *Nebulæ* grown less. But the great Spot *a* continued dark and strong, only sometimes fainter, and then again stronger; and sometimes like a half, or horned Moon; sometimes roundish, or rather of an oval Figure; of which latter Figure they commonly are, when they are near the Sun's Limb, which this Spot was not far off at this Time. Fig. 117.

These Particulars are Confirmations of what I said, *That the Solar Spots are no other than a Smoak rising out of the Body of the Sun.* Of which Opinion I have been, almost ever since I first observed them, and find that I am not singular in this Opinion, as I shall shew from Part of a Letter (which with some others is lately fallen into my Hands) from Mr. *Crabtrie* to Mr. *Gascoigne*, the Inventor of the Micrometer.

Mr. Crabtrie's *Opinion of these* Spots, *in a* Letter *to Mr.* Gascoigne, *of* August 7, 1640.

'I writ to Mr. *Townley* my Opinion in Brief of the Sun's Spots, '(which you conceive to be Stars) and it seems he, or Mr. *Kay*, writ 'to the same Purpose to you, desiring your Opinion: Which you freely deliver: Yet give me Leave to speak my Mind likewise freely concerning these Appearances. I do not value the Authority of *Galilæus*, 'or *Kepler*, further than either demonstrative, or the most probable 'Reasons confirm their Opinions. I acknowledge you say more for 'the Stellifying of these Solar Obscurities, than I have heard before; 'yet I conceive not sufficient, either demonstratively or probably to 'countermand those, which *Galilæus*, *Kepler*, and others have produced 'to the contrary; nor yet such as can be cleared from such Objections, 'as Reason, Demonstration, and Observation may lay against them. 'My Occasions will not admit a full Disquisition hereof at this Time; 'yet something I would say for the present, the better to furnish you 'where to object, when I see you.

'I have often obſerved theſe Spots; yet from all my Obſervations 'cannot find one Argument to prove them other than *fading Bodies*. 'But that they are no Stars, but unconſtant (in regard of their Gene- 'ration) and irregular Excreſcences ariſing out of, or proceeding from 'the Sun's Body, many Things ſeem to me to make it more than 'probable.

'For firſt, for their *Form*; they are ſeldom round, but of irregu- 'lar Shapes, and, as I have often ſeen, one Side, or End of the Spot 'more thin than the reſt, like to a certain miſty Darkneſs, and by De- 'grees thicker, groſſer, and darker, nearer to the main Body of the 'Spot; juſt as the Smoak of ſome pitchy Fire, which is in one Part 'very groſs, and in another more rare and thin, turning at laſt into 'mere Air: Or like a Cloud, Fog, or Miſt, more thick, dark, and 'groſs in the Midſt; and more thin, fluid, penetrable, and tranſparent 'towards the Sides; which I ſuppoſe is not compatible with any of the 'Stars.

'Secondly, for their *Colour*: The Lightneſs thereof differenceth them 'from Stars, or Planets; they being never of ſuch abſolute Darkneſs, 'as I obſerved *Venus* the 24th of *November* laſt: Though I have ſeen 'Spots ſometimes little leſs than ſhe, yet always of a far paler and 'whiter Colour, looking (at leaſt in ſome Parts) like ſome thin diſſipa- 'ted Subſtance.

'Thirdly, for the *Manner* of their *Appearance*. I have ſeen many 'Spots, which in the Middle of the Sun appear of a round Body, but 'coming towards the Side of the Sun, appear long. Which is a de- 'monſtrative Argument, that they are not Globes, as all the Planets 'and Stars are: For Globes always appear of one Form (round) in 'every Poſition; but Exhalations, or ſuch like fluid Subſtances, ex- 'tended to a broad flat Form, like our Clouds, which being over our 'Heads, and ſo in their full Breadth, appear large and broad; but 'driven with the Wind, till they turn one Edge upon us, ſeem of a 'long Shape. So theſe Solar Clouds, being turned about the Sun, may 'in the Middle ſhew their full Breadth to us, and about both Edges 'of the Sun, turn their Edges to us: Which anſwereth to the Appear- 'ance.

'Fourthly, for their *Continuance*. Some of theſe Spots, ariſing at the 'Eaſt-ſide of the Sun, vaniſh before they come to the Midſt of the Sun. 'Others appear firſt in the Middle of the Sun, and vaniſh before they 'come to the Weſtern Limb; and for the moſt Part they vaniſh, be- 'fore they have made a full Revolution about the Sun. Which argues 'them to be but thin, vaniſhing, fading Subſtances, not like the per- 'manent Bodies of the Stars.

'But to take off theſe Reaſons, you anſwer, That you conceive 'theſe Spots to be Stars moving regularly in their own Orbs, which

'are

‘ are many, though none of greater Extent than about $\frac{1}{10}$ of the ⊙ Semidiameter from its Circumference; and that the ſwifter Movers in the lower Orbs, overtaking the ſlower in the higher Orbs, cauſe an Appearance. You ſeem therefore to think, that they being ſo thin Bodies, the Sun's Rays paſs through them, and ſo one cannot be ſeen alone, till more being together, one heaped behind another, they ſtop the Light of the Sun's Rays, and ſo cauſe an Appearance. This I conceive is your Meaning: Or elſe (as you ſeem to inſinuate afterwards) that the higher reflects the Sun's Rays ſtrongly enough upon the lower, (when they come within the Angle of Reflection) to make the interjacent Planet indiſcernible.

‘ But to theſe I anſwer, that

1. ‘ If it be by their coming within the Angle of Reflection, that the Light of the Sun reflected from the outer Planet upon the inner, doth make it (as you ſpeak) indiſcernible, then that Light ſo reflected, is reflected either upon all Places, as the Moon and Planets Light; or but upon one, as is the Reflection of a plain Looking-Glaſs. If the *Firſt*, there would never be many ſeen, (ſeldom above one or two) becauſe the outermoſt would continually make the inner undiſcernible. But *Gaſſendus* affirms, there are ſeen ſometimes 40 at once in the Sun's Body. If the ſecond, there would always be many ſeen, becauſe the reflected Light would but occupy a little Room, and that but for a ſmall Time, till the ſwifter were paſt the Place of Reflection: Whereas many Days there are none at all ſeen in the Sun's Hemiſphere: And in both theſe Caſes, the outermoſt Planet of all would always in the Space of 27 Days, be ſeen in the ſame Place, being never obſcured, none of the Inferior being able to reflect Light upon it. Add hereunto, if any Kind of Reflection ſhould make them to appear bright like the Sun, and ſo not diſtinguiſhable from the Light of the Sun, what ſhould [a] hinder, but we ſhould ſee them alſo bright Bodies by the Side of the Sun, when they are paſſing either by the Weſt or Eaſt-ſide of the Sun's Body? The Light being then reflected upon them by the inferior Planets, as well as at other Times, and that alſo upon much of that Side of them which we ſhould behold.

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[a] N. B. *Mr.* Gaſcoigne *having, againſt theſe Words, inſerted a rough-drawn Figure in the Margin of Mr.* Crabtrie's *Letter, I have alſo repreſented it in* Fig. 118. *imagining it may ſomewhat explain Mr.* Gaſcoigne's *Hypotheſis, and what Mr.* Crabtrie *ſaith againſt it.* Fig. 118.

'But if you wave this Conceit, as insufficient, and fly to your former, That the swifter Movers in the lower Orbs, overtaking the slower in the higher Orbs, cause an Appearance. To this I answer: 1. The Thing you suppose seems to me neither necessary nor probable, nor do I conceive, why they should not be seen, being themselves alone, as well as conjoined, seeing all other Stars and Planets are so. 2. If it be, because they are of a thin, transparent Substance, till many, being one behind another, make them to seem grosser; then they are not of the Nature of other Planets, as is proved in ☿ and ♀, who of themselves appear dark Bodies, when they come between us and the Sun; nay, they must be more thin than our Clouds, which will easily be seen between us and the Sun, and hide it from us. 3. If it be, because they are so little, that the Imperfection of our Glasses cannot discover one alone, there must be, without doubt, many Millions of them; which how they can be included within the Compass of $\frac{1}{10}$ of the ☉ Semidiameter, we shall consider anon. I have seen one of an ordinary Darkness, (yea darker than many greater) yet not above 5″ Diameter. If this consist of two, or many, of themselves invisible, how many were in those which *Gassendus* saw of 1′ $\frac{1}{2}$ Diameter? 4. The Figure of these great ones (being necessarily composed of Stars of such different Orbs and Motions) would quickly vary, by reason of the Diversity of their Motions; like as we see in a Flock of small Birds. But 5thly, You say the furthest of these Orbs is not above $\frac{1}{10}$ of the Sun's Semidiameter from its Circumference. But there would not, in that small Space, be Room enough for so many Orbs of Planets, as have been seen at once. Which I prove thus: 1. *Gassendus* affirms, there are sometimes some of about the $\frac{1}{10}$ Part of the ☉ Semidiameter; which is the whole Space allowed by you for them all. And myself have seen of the $\frac{1}{15}$ of the ☉ Semidiameter: And yet you must confess, these great ones could only be Conjunctions of some, not all. 2. There are many Times seen in the Sun's Superficies, a great Number of Spots, whose Diameters added together, would do more than twice fill the Space you speak of. I myself have seen it, and so I believe have you. *Gassendus* affirms, there are sometimes 40 seen at once: If this was by Conjunction of Planets, in every Appearance, there were at least 80 Bodies at once on this Side the ☉; it may be as many on the other Side, besides those unseen, (by your Reflection or otherwise) which doubtless must be far more than seen. For it is a most rare, and I think unheard of Thing, to see but three (which is less than the Half) of our Planets, conjoined in visible ☌ at once: So that without Question, if they be Planets, they are many Hundreds; which must have so many several Orbs, and which certainly cannot be done in so narrow a Compass, as the $\frac{1}{10}$ of the ☉ Semidiameter. And that they cannot have any larger (I suppose not so

'large

'large an) Extent from the ☉ Superficies, may be proved by their Motion through the visible Hemisphere of the Sun's spherical Body, by comparing the Swiftness of their Motion towards the Middle and Sides together. 6. If one of these (imagined) Planets be swifter than another, as they must needs be, then the ☌ of 2 or 3 swifter ones would make a Spot of speedier Motion than the ☌ of 2 slower ones: But the Motion of all about the ☉ Center, is always equal; yea, and the Spots retain the same Position one to another, (considering the Sun's Sphericity, and the Angle of their Appearance to us) just like the fixed Stars. So affirms *Gassendus*, They all move with the same uniform Motion, so that when there are several of them, no one overtakes another, but they all preserve the same Distances in the Sun's Disk, as the fixt Stars do in the Firmament.

'As for that other *annual Motion* of the Spots, you speak of, from West to East, upon their Axis inclined above 8 Degrees to the Ecliptick; I suppose it is not any real Motion of the Orbs of those solar Planets or Spots, but only a visible Motion so appearing, caused (in *Kepler*'s System) by the Sun's rolling upon its own Center in the Midst of all the Orbs, and not exactly in the Way of the *Temporary* Ecliptick, but in the *Via regia* (as *Kepler* calls it) inclined certain Degrees to the *Temporary*; thereby turning about with him, the same Way, his adventitious, or excrementitious Parts, the Spots, by his *magnetical* or *sympathetical Rays*. And hence may be demonstrated the Appearance of that annual Motion in the Sun's Spots you speak of. See *Galilæus*, *Syst. Cosm. p.* 339, *& seq.* So also in *Ptolemy*'s and *Tycho*'s System, the same Appearance may be demonstrated, supposing the ⊖ fixed in the Middle of the Universe, and the ☉ rolling round upon the same Poles of that *Via regia* (or way of the Spots) and keeping his Axis in Parallelism continually towards one and the same Part of the Universe. This may be certainly demonstrated, although *Galilæus* there affirms the contrary. Other *Hypotheses* of that Motion may be feigned, as by the annual Conversion of the Poles of the *Via regia* about the Poles of the Ecliptick in the Sun's Body: But none I conceive so compendious, as the one of the former. For my Part, I incline to the first.

'Thus you have, what for the present, I conceive of these *Maculæ Solares*. *Fromundus* mentions one *Jo. Tarde Gallus*, who thinks them to be secondary Planets; who hath written a Book of that Subject, and calls them *Astra Borbonia*: But I could never yet see it. What you, or he, or others may alledge for that Opinion, I know not. In the mean Time I cannot acknowledge them Stars, unless I see at least some Possibility how they may be so, or some Probability why they should not rather be Spots. Which when you, or they do produce from better grounded Reasons, Optical Experiments, or Demonstrations, I shall willingly recant my Opinion.

It

Of the Diameter *of* Venus.

' It is true which you ſay, That I found *Venus*'s Diameter much leſs ' than any Theory extant made it. *Kepler* came neareſt, yet makes her ' Diameter five Times too much. *Tycho*, *Lansberge*, and the Ancients, ' about 10 Times greater than it was. So alſo they differ in the Time ' of the ☌ as far from the Truth. By *Lansberge* the ☌ ſhould have ' been 16^h 3′ before we obſerved it: By *Tycho* and *Longomontane* 1^d 8^h ' 25′ before. By *Kepler* (who is ſtill neareſt the Truth) 9^h 40′ before. ' So that had not our own Obſervations, and Study, taught us a better ' Theory than any of theſe, we had never attended at that Time for ' that *rare Spectacle*. You ſhall have the Obſervation of it, when we ' ſee you. The Clouds deprived me of Part of the Obſervation, but ' my Friend Mr. *Jeremiah Horrox*, being near *Preſton*, obſerved it ' clearly from the Time of its coming into the Sun, till the Sun's ſet- ' ting; and both our Obſervations agreed, both in the Time and Dia- ' meter, moſt preciſely.

' *Langsberge* in Eclipſes, 'eſpecially of the ☽, comes often nearer the ' Truth than *Kepler*, yet it is by packing together Errors; his Diame- ' ter of the ☉ and ☽ being falſe, and his Variation of the Shadow be- ' ing quite repugnant to Geometrical Demonſtration. His circular ' Hypotheſis Mr. *Horrox* (before I could perſuade him) aſſayed a long ' Time with indefatigable Pains and Study to correct, and amend; ' changing and turning them every way (ſtill amazed and amuſed with ' thoſe lofty Titles of Perpetuity and Perfection, ſo impudently im- ' poſed upon them) until we found, by comparing Obſervations in ſe- ' veral Places of the Orbs, that his Hypotheſes would never agree with ' the Heavens for all Times, as he confidently boaſts; no, nor ſcarce ' for any one whole Year together, alter the equal Motion, Proſtha- ' phæreſes, and Excentricity howſoever you will.

' *Kepler*'s *Elliptick* is undoubtedly the Way which the Planets deſcribe ' in their Motions: And if you have read his *Comment. de motu* ♂*is*, and ' his *Epit. Aſtron. Copern.* I doubt not you will ſay his Theory is the ' moſt rational, demonſtrative, harmonious, ſimple, and natural, that ' is yet thought of, (or I ſuppoſe can be;) all thoſe ſuperfluous Fictions ' being rejected by him, which others are forced ſo abſurdly to intro- ' duce. And although in ſome Reſpects his Tables be deficient, yet be- ' ing once corrected by due Obſervations, they hold true in the reſt: ' Which *Lansberge*'s and all others want.

' Your Conceit of turning the Circle into 100,000,000 Parts, were ' an excellent one, if it had been ſet on foot, when Aſtronomy was ' firſt invented. Mr. *Horrox* and I have often conferred about it. But ' in reſpect that all Aſtronomy is already in a quite different Form, and ' the Tediouſneſs of reducing the Tables of Sines, Tangents, and all ' other Things we ſhould have occaſion to uſe, into that Form; as alſo ' ſome Inconveniences, which we foreſaw would follow in the compoſing ' the Tables of Celeſtial Motions, together with the Greatneſs of the

' Innova-

'Innovation, deterred us from the Conceit. Only we intend to use 'the Centesmes or Millesmes of Degrees, because of the Ease in Calcu- 'lation. I have turned the *Rudolphine* Tables into Degrees and Mil- 'lesmes, and altered them into a far more concise, ready, and easy 'Form, than they are done by *Kepler.*

Forasmuch as every Thing of Mr. *Crabtrie*'s is valuable, I have taken this Occasion from my own Observations of the solar Spots (for the most Part drawn up near four Years ago) to give Mr. *Crabtrie*'s Letter at large (which I saw not till about a Month ago) containing as well some Things of another Nature, as what relates to the Spots; I have two other of his Letters concerning the Spots (with Mr. *Gascoigne*'s Answers.) One contains his Theory of their Motion and Appearances; the other his Way of observing them. But being long, I shall omit them for the present.

Fig. 115. *Shows the Stages and Lines described by the* Spots *upon the* Sun *in* Sept. *and* Novemb. 1706, *and in* Feb. *and* March, 170$\frac{6}{7}$, *and in* Sept. *and* Novemb. 1707. Fig. 115.

Fig. 116. *Shows the Stages and Lines described by the* Spots *upon the* Sun *in* Jan. 170$\frac{3}{4}$, *and in* May, June, *and* Octob. 1705. Fig. 116.

Eclipse of the Sun, *June* 12, 1694, *in* New-England, *by Mr.* T. Brattle, n. 292. p. 1630.

XI. On the 12*th* of *June* 1694, in the Morning, I went to the College at *Cambridge*, about four Miles from *Boston*, and observed with the Brass Quadrant there, with Telescopick Sights, the Rays of the Sun being transmitted through one of the said Sights, on a clean Paper, pasted on a plain Piece of Board, and fastned at right Angles at about a Foot distance from the said Sight, on which Paper I had drawn a Circle between 2 and 3 Inches Diameter, equal to the Sun's Disk, and within that several Concentric Circles dividing the Diameter into 24 equal Parts, whereby I could observe to $\frac{1}{2}$ a Digit: The Room in which the Observation was made, was darkened with Blankets, and in order to render the Observation more exact (Mr. *Henry Newman* assisting me all the while) I took the Altitude of the Sun with the foresaid Quadrant, as followeth.

Observations made of the Sun's Altitude before the Eclipse began, in order to rectify the Watch.

	By the Watch.				Comp. Altit.		Time by Calcul.			Differ.	
	h	′	″								
at	8	26	37.	Mane	49	31	8	16	40	9	57
		31	27		48	26		21	40	9	47
		38	26		47	20		28	32	9	54

The Eclipse was first perceived at 9^h 25′ by the Watch, at which Time the Sun had scarcely been eclipsed 1 Minute, so that

By

	By the Watch.			True Time.		
	h	′		h	′	
At	9	24	———	9	14	It began
	9	32	———	9	22	about 1 digit eclipsed
	9	48	———	9	38	full 3 digits
	9	57¾	———	9	48	about 4
	10	06	———	9	56	near 5
	10	15	———	10	05	full 6
	10	33	———	10	23	about 8
	10	43	———	10	33	full 9
	10	47	———	10	37	full 9½
	10	53	———	10	43	full 10
	10	59	———	10	49	about 10½
	11	03	———	10	53	better than 10½
	11	06	———	10	56	much the same
	11	09	———	10	59	rather decreasing
	11	10½	———	11	00½	sensibly decreased near ¼ of a digit
	11	14½	———	11	04½	nearest to 10 digits
	11	25	———	11	15	full 9 digits, *i. e.* full 3 digits restored, or the Shadow rather within 9 digits
	11	29	———	11	19	8½ compleat
	11	34½	———	11	24½	full 8 digits
	11	44	———	11	34	full 7
	11	48	———	11	38	full 6 ½ digits
	11	52½	———	11	42½	just 6
	0	02½	P. M. —	11	32½	just 5
	0	13	———	0	03	P. M. full 4
	0	26	———	0	16	full 2½
	0	32	———	0	22	better than 2
	0	41	———	0	31	better than 1
	0	48	———	0	38	ended.

Observations made after the Eclipse was done, of the Sun's Altitude, in order to rectify the Watch.

	Time by the Watch				Comp. Altit.			True time.			Differ.	
	h	′	″									
At	3	31	30	P.M.	45	52	3	21	36		9	54
		36	15		46	23		26	16		9	59
		38	10		46	45		28	16		9	54
		46	50		48	19		36	48		10	02
		48	10		48	30		38	20		9	50

Hence

Hence it appears, that the Watch went about 10 Minutes too fast during the whole Eclipse, as we have all the Way allowed.

So that the Eclipse

	h	′
Began at	9	14 Mane.
Ended	0	38 P. M.
Lasting in all	3	24.

Note, That in the Calculation, the Latitude of *Boston* was allowed to be 42 . 25′.

XII. At half an Hour past Eight in the Morning, I set my Clock exactly by my Ring-Dial, and at half an Hour past Nine they nicely agreed, at *Eclipse of the Sun, Nov. 23. 1703. in* New England, *by the* same, *ibid.* p. 1634.

h	′	
10	00	The Sun was not touch'd.
	06	The Moon enter'd on the S. S. W. Point as near as I could judge.
	15	The Eclipse was considerably advanc'd:
	20	Seem'd to be about half a Digit eclipsed, rather more than less, and the Section to be a small Matter more Westwardly.
10	25	Much the same, and near the same Point.
	30	Seem'd to be less.
	$33\frac{1}{2}$	The Middle of the Section nearer the S. W. and the Diameter of the Section less every way.
	$37\frac{1}{2}$	Much less and nearer the West.
	$44\frac{1}{2}$	It ended, and was just over, going off near the S. W. so that all the while it was within a Point or two of the Place where it first came on, or between the S. S. W. and the S. W.

I judg'd, when it was at the Height, that the Chord of the eclipsed Part was nearest equal to the Side of an inscrib'd Decagon, or subtended about $\frac{1}{10}$ of the Periphery of the Sun's Disk.

I observ'd this Eclipse with a Telescope of one Joint, 4 Foot and a half in Length, and had only two Glasses, so that it inverted the Object; and I had a red Glass, which suited it, so that I could screw it in just before the Eye-Glass, and was not fain to hold it in my Hand, as when I observ'd the Sun's Altitude with the Brass Quadrant, which was a great Convenience.

XIII. 1.] The Morning was cloudy and moist, till about Eight a Clock, when the Clouds began to break, and we had sometimes a Sight of the Sun thro' the Spaces betwixt them. A seven-foot Telescope was fitted up with a Scene to receive the Species of the Sun cast through it, and on which it was about seven Inches Diameter, divided *Eclipse of the Sun, May $\frac{1}{12}$. 1706. at* Greenwich, *by Mr.* Flamsted, n. 306. p. 2237.

divided into Digits by ſix concentrick Circles. But Clouds covering the Sun frequently rendred this way of obſerving inconvenient, and therefore laying aſide the *Apparatus* of the Scene, I viewed him through the Teleſcope with ſmoaked Glaſſes, to ſave my Eyes, and noted

Correct Time by the Pend. Clock.

h	′	″			
8	21	30	A very ſmall Part of the ⊙ Diameter was eclipſed.		
	28	00	The Chord of the Arch of the ⊙ Periphery eclipſed was 14′. 40″. then followed frequent Clouds through the Spaces betwixt; then ſome Zenith Diſtances of the Sun were taken for correcting the Clock, and afterwards near the Middle of the Eclipſe		
9	21	46	The Parts of the Diameter remaining clear	5	00
	26	20	———————	4	30
			Frequent large Clouds again, till the Sun appeared		
10	31	50	through the Breaks, and we ſaw the Eclipſe was not ended. Clouds again till		
10	33	50	When the Sun ſhone out again, we ſaw his Limb intire, and the Eclipſe certainly over.		

— *at* Canterbury, *by Mr.* S. Gray, *ibid.*

2.] Mr. *S. Gray* had prepared a Scene placed behind his ſeven Foot Glaſs, ſo that the Species of the Sun projected on it was ſeven Inches over; but having the ſame Sort of Weather that was at *Greenwich*, he ſaw not the Beginning, by reaſon of Clouds, but other Phaſes with the End he noted, as follows.

Correct Time by the Pend. Clock.

h	′	
8	53	digits 5½ darkned
9	08	——— 7
	31	——— 10 or more
	36	——— The Sun ſhining for a ſhort Time, the Eclipſe ſeem'd to decreaſe.
	55	——— 7½ a little clearer.
	57	——— 6¼
10	02	——— 6
	04	——— 5¾
	14	——— 4
	16	——— 3¾
	20	——— 2½
	30	——— 1
	31	——— 0¾
10	36½	The End accurately with a Tube of 16 Foot.

3.] Mr. *Abr. Sharp* caſt the Species of the Sun on a Scene-plate, behind his ſeven Foot Glaſs, ſo as it appeared ſeven Inches over. By reaſon of cloudy Weather, he ſaw neither the Beginning nor End : But other Phaſes near the Middle, as follows.

— *at* Horton *near* Bradford *in* Yorkſhire, *by Mr.* A. Sharp, *ibid.*

Time correct by the

h	′	″			
8	35	00	digits dark	3	by Ocular Eſtimation.
9	01	00	———	7	
	4	54	———	8 $\frac{3}{10}$	Eclipſed on the Scene.
	6	33	———	8 $\frac{1}{2}$	
	7	53	———	8 $\frac{6}{10}$	
	12	50	———	9	
	16	08	———	9 $\frac{4}{10}$	
	18	48	———	9 $\frac{1}{2}$	exactly the ☉ ſhining out clear.
	20	45	———	9 $\frac{1}{2}$	the ☉ ſtill ſhining clearly. Greateſt Obſcurity.
	21	48	———	9 $\frac{1}{2}$	ſtill clear.
	28	46	———	9	
	44	45	———	7	
	54	42	———	5 $\frac{1}{4}$	
10	06	10	———	3 $\frac{1}{2}$	
	19	55	———	1	preciſely.
	24	00	The ☉ ſeen thro' Clouds, the Eclipſe not ended.		
	30	00	The ☉ ſeen again perfectly round and intire.		

4.] Captain *Stannyan*, from *Bern* in *Switzerland*, writes ‘ That the ‘ Sun was totally darkned there for 4 $\frac{1}{2}$ Minutes of Time ; that a fixed ‘ Star and a Planet appeared very bright ; and that *his getting out of* ‘ *the Eclipſe was preceded by a Blood-red Streak of Light, from its Left* ‘ *Limb ; which continued not longer than* 6 *or* 7 *Seconds of Time* ; then ‘ Part of the Sun's Disk appear'd, all on a Sudden, bright as *Venus* ‘ was ever ſeen in the Night ; nay, brighter ; and in that very Inſtant ‘ gave a Light and Shadow to things, as ſtrong as Moon-light uſes ‘ to do.

— *at* Bern *in* Switzerland, *by Capt.* Stannyan, *ibid.*

The Captain is the firſt Man I ever heard of, that took Notice of a red Streak of Light preceding the Emerſion of the Sun's Body from a total Eclipſe : And I take Notice of it, becauſe it infers that *the Moon has an Atmoſphere* ; and its ſhort Continuance of only 6 or 7 Seconds of Time, tell us, that *its Height is not more than the* 5 *or* 6 *hundredth Part of her Diameter.*

—*at* Geneva, *by Mr.* J. C. *Facis Duillier*, ibid.

5.] A little after the Sun's riſing, the Sky did ſeem clear; tho' the Air was thick already with ſome Vapours. Many little Clouds did afterwards ariſe here and there, and the Vapours did much increaſe. For want of a Pendulum Clock, in a convenient Place, the Moment of the *total* Immerſion, the Moment of the *firſt* Emerſion, and that of the *End* of the Eclipſe, could not be accurately obſerved. Tho' the Sky was ſomewhat overcaſt, the Heat of the Sun was already felt, when the Eclipſe did begin: But a very ſenſible Coldneſs took Place, as the Moon did by degrees cover a greater and greater Part of the Sun, and the Light decreaſe. The Eclipſe was obſerved only with ſome Glaſſes, either darkned with Smoak, or but little tranſparent; and by receiving the Sun's Image through a ſix Foot Teleſcope, which repreſented the Objects inverted, upon a white Paper, placed at ſome Diſtance, from the Eye-Glaſs. When the Sun was near being totally dark, the bright Creſcent, which did remain, was ſeen to diminiſh more and more, upon the Paper, where its Image was received. And when that Creſcent was reduced to a very narrow Breadth, and to a very little Length, it was ſeen on a Sudden to diſappear: And in that Moment the whole Sun was eclipſed. At the ſame Inſtant of Time, the Darkneſs, which was already very conſiderable, did become much greater. The Clouds did change on a Sudden their Colour, and became red, and then a pale Violet. There was ſeen, during the whole Time of the total Immerſion, a Whiteneſs, which did ſeem to break out, from behind the Moon, and to encompaſs it on all Sides equally. The ſame Whiteneſs was but little determined, in its outward Side, and was not broad the twelfth Part of the Diameter of the Moon. This Planet did appear very black, and her Disk very well defined, within the Whiteneſs, which encompaſſed it about, and whoſe Colour was the ſame with that of a white Crown, or *Halo*, of about four or five Degrees in Diameter, which accompanied it, and had the Moon for its Center. The Star of *Venus* was ſeen, at the ſame Time, at ſome Diſtance, without that Crown, between the Eaſt and N. E. in reference to the Sun. The Planets of *Saturn* and *Mercury* were ſeen alſo by many, Eaſtward from the Sun's Place. And if the Sky had been clear, many more Stars might have been ſeen, and with them the Planets of *Jupiter* and *Mars*; that towards the Eaſt, and this towards the Weſt: And ſo the ſeven Planets might have been ſeen, almoſt all at once. Accordingly ſome Gentlewomen being in the Country, did tell, as it is ſaid, more than ſixteen Stars. And many People, which were on the Neighbouring Mountains, did ſee the Sky ſtarry, in ſome Places, where it was not overcaſt, as during the Night, in the Time of the full Moon. The total Emerſion did begin about three Quarters paſt Nine. The Duration of the total Darkneſs was preciſely three Minutes, or 180 Seconds, to the Moment that the firſt Ray of the Sun did begin to appear again, with much Brightneſs.

Brightneſs. And this Time was obſerved, with a ſimple Pendulum; which was afterwards compared with a Pendulum Clock, ſhewing the Seconds, and regulated upon the mean Motion of the Sun. A little after the Sun had begun to appear again, the Whiteneſs and the Crown, which did encompaſs the Moon, did entirely vaniſh. The Sun did then ſhew itſelf more and more; appearing at firſt as a little Creſcent, which did ſtill encreaſe; and whoſe concave Side did ſeem terminated, as by an Arch deſcribed with the Compaſs. A little before the total Obſcuration, the Country, on the Weſt-ſide, did already ſeem overcaſt with Darkneſs; and after the total Obſcuration, the Darkneſs was ſeen to leave us more and more, and to fly Eaſtward. According to Mr. Profeſſor *Gautier*'s Obſervations, from the firſt Emerſion of the Sun, to the End of the Eclipſe, there was $1^h\ 9'\ 30''$. As to the accurate Times they are uncertain, the Pendulum Clock having been ſet only by a ſmall Sun-Dial. I ſend you alſo the following Account, which the ſame Gentleman did communicate to me.

	h	′	″	
6.] ‘ At *Marſeilles* the Eclipſe did begin at ———	8	28	40	— *at* Marſeilles *by Mr.* Chazelles, *and Father* Laval, *ibid.*
‘ It did reach the Sun's Center at ———	9	6	11	
‘ It was total at ———	9	34	15	
‘ The Sun did begin to appear again at ———	9	37	9	
‘ The Eclipſe did come again to the Center at ———	10	12	23	
‘ It did entirely end at ———	10	47	50	

‘ Three Stars were diſtinctly ſeen; and during three Minutes it was ‘ not poſſible to read. And there did remain one bright Digit, all ‘ about the Globe of the Moon.

The Manor-Houſe of *Duillier* is in the Latitude of 46° 24′. In Longitude it is 4° 13′ 45″ to the Eaſtward of the *Royal Obſervatory* at *Paris*. And St. *Peter*'s Church at *Geneva* is, in Latitude, 0° 12′ to the Southward, and in Longitude, 0° 5′ 2″ to the Weſtward of *Duillier*.

Before I make an End, I muſt take Notice, that, according to theſe Obſervations, the Altitude of the Moon's Atmoſphere cannot well be ſuppoſed leſs than of 130 Miles in perpendicular Height: Of which Miles 60 go to one Degree of Earth. Neither could that Atmoſphere be diſcovered before the Time of this Eclipſe, by any Refraction of the Stars; probably becauſe of this Refraction's Smallneſs, and for Want of proper Obſervations. And though it was very plain that the Atmoſphere of the Moon muſt needs ſhew itſelf in the Time of a total Eclipſe of the Sun; yet I do not know that any Body did think of this, till, in the laſt Month of *May*, many Perſons did actually ſee it.

Some

Some particular Obſervations, which are intended to be made publick, do evince that our Atmoſphere is ſometimes viſible, all along, from the Surface of the Earth to the perpendicular Height of one Semidiameter of the Terreſtrial Globe. And the continued Appearance of a Crown, of only four or five Degrees Diameter, about the Sun, during the whole Time of the total Obſcuration, does ſhew, that the Æthereal Matter, in which that Crown was produced, muſt be at a very great Height above the Surface of the Earth. But if that Crown was to be ſeen, ſo far as the Weather did permit, in all the Places where the Eclipſe was total, it muſt be concluded, that the Cauſe of it was not in our Air, but in ſome Vapours encompaſſing the Sun: And probably, in thoſe very Vapours, which produce that pointed Light, that has been obſerved lying in a Manner along the Ecliptick, and that has the Sun for Center. Now either of theſe Concluſions, *viz.* concerning the great Height of the Parts of our Atmoſphere, capable of producing that Crown, or elſe concerning a Meteor obſerved, not in our Air, but in the Vapours that encompaſs the Sun, is very ſingular, and deſerves a great deal of Attention. If ever ſuch another Appearance ſhould be ſeen, in the Time of a total Eclipſe, it would be proper to obſerve accurately the leaſt Diameter of the Crown, from Inſide to *Inſide:* And to take Notice, whether, during the whole Time of the total Immerſion, the inward Circle be every where continued, and of an uniform Figure. The leſs the ſaid Diameter, and the greater the Exceſs of the Moon's apparent Diameter above that of the Sun; as alſo the greater the apparent Altitude of the Sun is above the Horizon, the higher the Cauſe, which produces the Crown, muſt be above the Surface of the Earth. And the Poſition upon the Moon's Disk, in Reference to the Zenith, of the Points of Contact, where the Sun diſappears, or begins to ſhew itſelf again, is here alſo of ſome Conſideration.

— *at* Zurich, *by Dr.* J. J. Scheuchzer.

7.] We have had here, *May* 12. both a total and Annular Eclipſe of the Sun; total, becauſe the whole Sun was cover'd by the Moon; Annular, though not properly ſo called, but by Refraction, for a ruddy Brightneſs appear'd about the Moon, ariſing from the Rays refracted by the Moon's Atmoſphere.

Fig. 119.

The Beginning of the Eclipſe was in the Morning	8h.	54′.
The Middle	9.	58′.
The End	11.	12′.
The Mora of the mean and full Obſcuration		4′.

Both the fixt Stars and Planets might be ſeen. The Birds betook themſelves to their Neſts. The Bats came out of their Holes, and the Fiſhes ſwam upon the Water. We ourſelves perceived a ſenſible Degree of Cold, and the Dew fell down upon the Plants.

XIV. The

Eclipſe of the Sun, *Sept.* 3. 1708. *at* Upminſter, *by Mr.* Derham, n. 320. p. 312.

The Corr. App. Time. h	′	″	
			XIV. The Beginning of the Eclipſe we could not ſee for Clouds.
6	44	15	The Sun peeped out of the Clouds, and I judged, by my Eye, that about one Tenth of a Digit was eclipſed.
			Then Clouds nearly all the Time of the Eclipſe. But at
8	31	15	A little Obſcuration appeared through the Teleſcope.
8	32	45	A *very* little Obſcuration through the Teleſcope.
			Then Clouds. And at
8	35	45	We could diſcern no Remains of the Eclipſe through the Teleſcope.

From theſe Obſervations I imagine the End of this Solar Eclipſe was much about 8 h. 33′ in the Morning.

Obſervations of the Total Eclipſe *of the* Sun, *April* 22. 1715, *at* London, *by Dr.* E. Halley, n. 343. p. 245.

XV. 1. Though it be certain from the the Principles of Aſtronomy, that there happens neceſſarily a Central Eclipſe of the Sun, in ſome Part or other of the Terraqueous Globe, about twenty-eight Times in each Period of eighteen Years; and that of theſe, no leſs than Eight do paſs over the Parallel of *London*, three of which eight are total with Continuance: Yet, from the great Variety of the Elements, whereof the *Calculus* of Eclipſes conſiſts, it has ſo happened, that ſince the 20th of *March*, *Anno Chriſti* 1140, I cannot find that there has been a total Eclipſe of the Sun ſeen at *London*, though in the mean Time the Shade of the Moon has often paſt over other Parts of *Great Britain*.

Having found, by comparing what had been formerly obſerved of Solar Eclipſes, that the whole Shadow would fall upon *England*, I thought it a very proper Opportunity to get the Dimenſions of the Shade aſcertained by Obſervation; and accordingly I cauſed a ſmall Map of *England*, deſcribing the Track and Bounds thereof, to be diſperſed all over the Kingdom, with a Requeſt to the Curious to obſerve what they could about it, but more eſpecially to note the Time of Continuance of total Darkneſs, as requiring no other Inſtrument than a *Pendulum Clock*, and as being determinable with the utmoſt Exactneſs, by reaſon of the momentanous Occultation and Emerſion of the luminous Edge of the Sun, whoſe leaſt Part makes Day. Nor did this fail of the deſired Effect, for the Heavens having proved generally favourable, we have received from ſo many Places ſo good Accounts, that they fully anſwer all our Expectations, and are ſufficient to eſtabliſh ſeveral of the Elements of the *Calculus* of Eclipſes, ſo as for the Future we may more ſecurely rely on our Predictions: Though it muſt be granted, that in this our Aſtronomy has loſt no Credit.

Having

Having received the Orders of the Society to provide for the Observation to be made at their House in *Crane-Court*, I procured a *Quadrant* of near 30 Inches *Radius*, exceedingly well fixt with Telescope Sights, and moved with Screws, so as to follow the Sun with great Nicety; as also a very good *Pendulum Clock* well adjusted to the mean Time, and several Telescopes to accommodate the other Observers.

In order to examine both Clock and Quadrant, I, on the 20th of *April*, observed the Distance of the upper Limb of the Sun from the Zenith 36° 16′, and the next Day 35°. 58′; by which it appeared, that the Distances from the *Zenith*, taken by this Quadrant, ought to be increased by about one Minute: And that Allowance being made, by many Observations taken before and after Noon on the said 21st Day, the Clock was found to answer the apparent Time or Hour of the Sun with sufficient Exactness, as not going above 10″. too fast. The next Day, *April* 20°, just before the Eclipse began, we took the Distances of the Sun, from the Zenith, *viz.* at 7^h. 42′. 52″ A. M. the correct Distance of the Sun's Center *à vertice* was 62°. 1′. 40″. at 7^h. 45′. 48″. it was 61°. 34′. 40″. And again at 7^h. 48′, 55″ it was 61°. 6′. 40″: Which, with the given Declination of the Sun and Latitude of the Place, shew the true Times respectively to have been 7^h. 42′. 38″, 7^h. 45′. 35″. and 7^h. 48′. 39″; all agreeing, that the Clock was only 14 Seconds too fast, and had gained scarce any Thing sensible in a Day's Time: So that it might be entirely depended upou during the Continuance of the Eclipse.

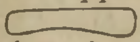
Having computed that the Eclipse would begin at 8^h. 7′, I attended soon after Eight, with a very good Telescope of about six Foot, without stirring my Eye from that Part of the Sun whereat the Eclipse was to begin: And at 8^h. 6′. 20″. by the Clock, I began to perceive a small Depression made in the Sun's Western Limb, which immediately became more conspicuous; so that I concluded the just Beginning not to have been above five Seconds before, that is, exactly at 8^h. 6′. 00″. correct Time.

From this Time the Eclipse advanced, and by Nine of the Clock was about ten Digits, when the Face and Colour of the Sky began to be changed from perfect serene azure Blue, to a more dusky livid Colour, having an Eye of Purple intermixt, and grew darker and darker till the total Immersion, which happen'd at 9^h. 9′ 17″ by the Clock, or 9^h. 9′. 3″ true Time. This Moment was determinable with great Nicety, the Sun's Light being extinguish'd at once; and yet more so was that of the Emersion, for the Sun came out in an Instant with so much Splendor, that it surprized the Beholders, and in a Moment restored the Day, *viz.* at 9^h. 12′. 26″. true Time, after he had been totally obscured for 3′. 23″ of Time. And as near as I could estimate the Points on the Moon's Limb, where the last Particle of the Sun vanished was about the Middle of the *South East* Quadrant of her Limb, or about 45 Degrees from her *Nadir* to the Left-hand: And the first Emersion was about ten Degrees below the Horizontal Line through the Moon's

Center

Center on the Weſt-ſide and at 14 Minutes paſt Nine, correct Time, I judged the Horns of the Eclipſe to have been exactly perpendicular, and by Conſequence, the Centers of the Sun and Moon to be in equal Altitude.

It was *univerſally* obſerved, that when the laſt Part of the Sun remained on his Eaſt-ſide, it grew very faint, and was eaſily ſupportable to the naked Eye, even through the Teleſcope, for above a Minute of Time before the total Darkneſs; whereas on the contrary, my Eye could not endure the Splendour of the emerging Beams in the Teleſcope from the firſt Moment. To this perhaps two Cauſes concurred; the one, that the Pupil of the Eye did neceſſarily dilate itſelf during the Darkneſs, which before had been much contracted by looking on the Sun. The other, that the Eaſtern Parts of the Moon, having been heated with a Day near as long as thirty of ours, muſt of Neceſſity have that Part of its Atmoſphere replete with Vapours, raiſed by the ſo long continued Action of the Sun; and by conſequence, it was more denſe near the Moon's Surface, and more capable of obſtructing the Luſtre of the Sun's Beams. Whereas at the ſame Time the Weſtern Edge of the Moon had ſuffered as long a Night, during which there might fall in Dews all the Vapours that were raiſed in the preceding long Day; and for that reaſon, that Part of its Atmoſphere might be ſeen much more pure and tranſparent.

About two Minutes before the total Immerſion, the remaining Part of the Sun was reduced to a very fine Horn, whoſe Extremities ſeemed to loſe their Acuteneſs, and to become round like Stars. And for the Space of about a Quarter of a Minute, a ſmall Piece of the Southern Horn of the Eclipſe ſeemed to be cut off from the reſt by a good Interval, and appeared like an oblong Star rounded at both Ends, in this Form : Which Appearance could proceed from no other Cauſe, but the Inequalities of the Moon's Surface, there being ſome elevated Parts thereof near the Moon's Southern Pole, by whoſe Interpoſition, Part of that exceedingly fine Filament of Light was intercepted.

A few Seconds before the Sun was totally hid, there diſcovered itſelf round the Moon a luminous Ring, about a Digit or perhaps a tenth Part of the Moon's Diameter in Breadth. It was of a pale Whiteneſs, or rather Pearl Colour, ſeeming to me a little tinged with the Colours of the *Iris*, and to be concentrick with the Moon; whence I conclude it the Moon's Atmoſphere. But the great Height of it, far exceeding that of our Earth's Atmoſphere; and the Obſervations of ſome who found the Breadth of the Ring to increaſe on the Weſt-ſide of the Moon, as the Emerſion approached; together with the contrary Sentiments of thoſe, whoſe Judgment I ſhall always revere, makes me leſs confident, eſpecially in a Matter whereto I gave not all the Attention requiſite.

Whatever it was, this Ring appeared much brighter and whiter near the Body of the Moon, than at a Diſtance from it; and its outward Circumference, which was ill defined, ſeemed terminated only by the extreme Rarity of the Matter it was compoſed of; and in all Reſpects reſembled the Appearance of an enlightned Atmoſphere viewed from far: But whether it belonged to the Sun or Moon, I ſhall not at preſent undertake to decide.

During the whole Time of the total Eclipſe, I kept my Teleſcope conſtantly fixt on the Moon, in order to obſerve, what might occur in this uncommon Appearance, and I ſaw perpetual Flaſhes or Coruſcations of Light, which ſeemed for a Moment to dart out from behind the Moon, now here, now there, on all Sides, but more eſpecially on the Weſtern Side, a little before the Emerſion: And about two or three Seconds before it, on the ſame Weſtern Side, where the Sun was juſt coming out, a long and very narrow Streak of a dusky, but ſtrong red Light, ſeemed to colour the dark Edge of the Moon, though nothing like it had been ſeen immediately after Immerſion. But this inſtantly vaniſhed upon the firſt Appearance of the Sun, as did alſo the aforeſaid luminous Ring.

As to the Degree of Darkneſs, it was ſuch, that one might have expected to have ſeen many more Stars than were ſeen at *London:* The three Planets, *Jupiter*, *Mercury* and *Venus* were all that were ſeen by the Gentlemen of the Society from the Top of their Houſe, where they had a free Horizon: And I do not hear that any one in Town ſaw more than *Capella* and *Aldebaran* of the fixed Stars. Nor was the Light of the Ring round the Moon capable of effacing the Luſtre of the Stars, for it was vaſtly inferior to that of the full Moon, and ſo weak, that I did not obſerve it caſt a Shade. But the under Parts of the Hemiſphere, particularly in the *South Eaſt* under the Sun, had a crepuſcular Brightneſs; and all round us, ſo much of the Segment of our Atmoſphere as was above the Horizon, and was without the Cone of the Moon's Shadow, was more or leſs enlightned by the Sun's Beams; and its Reflection gave a diffuſed Light, which made the Air ſeem hazy, and hindred the Appearance of the Stars. And that this was the real Cauſe thereof, is manifeſt by the Darkneſs being more perfect in thoſe Places, near which the Center of the Shade paſt, where many more Stars were ſeen, and in ſome, not leſs than twenty, though the Light of the Ring was to all alike.

During the Time whilſt the Sun recovered his Light, ſeveral Altitudes were taken to examine the Regularity of the Clock's Motion, and tho' the Sun now roſe much ſlower than at the Beginning, yet they all conſpired within a very few Seconds, that the Clock went ſtill one Quarter of a Minute too faſt. And the End of the Eclipſe approaching, I attended the Moment thereof, with all the Accuracy I could, and concluded the complete Separation of the Sun and Moon at $10^h.\ 20'.\ 15''$ by the Clock, or exactly $10^h.\ 20'$ correct Time.

Hitherto

Hitherto I exhibit only what myſelf ſaw, but there were with us a great many Members of the Society; and the Right Honourable the *Earl of Abingdon*, and the Lord Chief Juſtice *Parker* were of the Number: The latter of which ſhewed an uncommon Curioſity and Deſire of Exactneſs, his Lordſhip doing us the Honour to aſſiſt at moſt of the Obſervations made for determining the Error of the Clock, and did himſelf, at the Moment of the Emerſion from total Darkneſs, obſerve the Diſtance of the Planet *Jupiter* from the Zenith 48°. 29′. by which the Time thereof is verified.

There were alſo preſent ſeveral Gentlemen of other Nations, and among them Monſieur *le Chevalier de Louville* and Mr. *Monmort*, both of them Members of the *Royal Academy* of *Sciences* at *Paris*: The firſt whereof came purpoſely to obſerve this Eclipſe with us, and having ſeen the Beginning applied, himſelf to take Digits with his Micrometer, and to obſerve the Occultations of three Spots at that Time ſeen in the Sun; and communicated the following Notes, *viz.*

	h	′	″	
At	8	28	20	Four Digits were eclipſed.
	8	32	57	The firſt and bigger Spot touched the Moon.
	8	33	18	The ſame was wholly hid.
	8	34	08	The firſt of the two leſſer Spots was hid.
	8	34	58	The ſecond of them was hid.
	9	36	01	Emerſion of the greater Spot.
	9	38	06	Emerſion of the firſt leſſer Spot.
	9	40	25	Emerſion of the ſecond leſſer Spot.
	10	20	04	The End of the Eclipſe.

And he determined the Time of the total Darkneſs 3′. 22″, or one Second leſs than by my Account.

The Heavens were all the while very favourable to us, and there was very little or no Wind, and not ſo much as one Cloud interrupted our View from the Beginning to the End; but no ſooner was the Eclipſe over, but a great Body of Clouds hid the Sun for many Hours after.

Theſe Obſervations having been made with all the Care we could, are not, 'tis hoped, far from the Truth.

What we have received from other Places is as follows:

The Reverend Mr. *James Pound*, Rector of *Wanſted* in *Eſſex*, gives the following Account of the principal Phænomena obſerved there; he being furniſhed with very curious Inſtruments, and well skilled in the Matter of Obſervation, and having rectified his Clock by ſeveral Altitudes of the Sun taken both before and after, *viz.*

h.	′	″	
At 8	6	37	The Eclipse first perceived.
9	9	28	The total Immersion.
9	12	48	The Emersion.
10	20	32	The just End of the Eclipse.
0	3	20	The Continuance of total Darkness.

The near Agreement of this Observation with our own (the Difference being only what is due to the Difference of the Meridians) makes us the less solicitous for what was noted at the *Royal Observatory* at *Greenwich*; from whence we can only learn, that the Duration of total Darkness was 3′. 11″.

The Reverend Mr. *William Derham*, Rector of *Upminster* in *Essex*, assisted by *Samuel Molineux*, Esq; Secretary to his *Royal Highness* the *Prince*, and other Persons of Quality, made the following Observations there, *viz.*

h.	′	″	
At 8	7	41	The Eclipse began.
8	33	46	The Moon touched the greater Spot.
8	34	36	She touched the middle Spot.
8	35	41	She touched the third Spot.
9	10	58	The total Darkness began on a sudden, and *Aldebaran* appeared.
9	14	6	The Emersion, or the End of total Darkness.
0	3	8	Continuance of total Darkness.
9	42	41	The third and last Spot discovered.
10	21	45	The End of the Eclipse, by a 13 ½ Foot Glass.

And a little before the Beginning of the Eclipse, he found the greater and preceding Spot to be more Northerly than the Sun's Center 373 ½ such Parts as the Sun's Diameter was 1647, and that it followed his Western Limb 0′. 43″ of Time: By which *Data* the Situation of that Spot is well determined.

The Professors of Astronomy in both Universities were not so fortunate: Dr. *Keill*, by reason of Clouds, saw nothing distinctly at *Oxford*, but the End, which he observed at 10^{h}. 15′ 10″. As to the total Darkness, he could only estimate it by the sudden Change of the Light of the Sky; and reckoned its Continuance but 3′. 30″; which was certainly too little, the Center of the Shadow having without doubt past very near *Oxford*. And the Reverend Mr. *Cotes* at *Cambridge*, had the Misfortune to be opprest by too much Company; so that though the Heavens were favourable, yet he miss'd both the Time of the Beginning of the Eclipse, and that of total Darkness. But he observed the Occultations of the three Spots, *viz.* of the first and greatest

greateſt at $8^h. 34'. 11''$. of the ſecond at $8^h. 35'. 15''$. and of the laſt at $8^h. 36'. 55''$. He noted alſo the End of total Darkneſs at $9^h. 14'. 37''$, and the exact End of the Eclipſe at $10^h. 21'. 57''$.

We have received ſeveral Accounts from ſome Places, which lay near the Track of the Center of the Shade, and which might have been very proper to determine the greateſt Continuance of the Darkneſs; as from *Plymouth*, *Exeter*, *Weymouth*, *Daventry*, *Northampton* and *Lynn-regis*, all agreeing that the whole Sun was obſcured at thoſe Places full four Minutes, and ſome of them rather more. But as theſe Obſervers give us no Account how they meaſured this Time, it may well be ſuppoſed they took it in a round Number, and perhaps from Pocket Minute-Watches. What I think may beſt be relied on for this Purpoſe, are two correſponding Obſervations made, the one at *Barton* near *Kettering* in *Northamptonſhire*, where by the Obſervation of *John Bridges*, Eſq; Treaſurer of his Majeſty's Revenue of Exciſe, with a good Pendulum Clock and all due Care, the whole Sun was hid no more than $3'. 53''$. The other was by Mr. *John Whiteſide*, A. M. Keeper of the *Aſhmolean Muſeum* at *Oxford*, and a skilful Mathematician, who obſerved after the ſame Manner, at *King's Walden* in *Hertfordſhire* near *Hitchen*, that the total Eclipſe continued but $3'. 52''$. Hence it follows, that the Center of the Shade paſt near the Middle between theſe two Places, which are but 30 Geographical Miles diſtant from one another, and ſituate near at right Angles to the Way of the Shade, and therefore that the total Obſcurity, where longeſt, could laſt but about $3'. 57''$, or perhaps a Second or two more at *Lynn*, and leſs at *Plymouth*, the Velocity of the Progreſs of the Shade gradually decreaſing, and its Diameter increaſing as it paſt on to the Eaſtward. And this Situation of the middle Line is confirmed by an Obſervation made at the Seat of the Right Honourable the Lord *Foley* at *Witley*, eight Miles beyond *Worceſter*, by his Order, and communicated by his Lordſhip to the Society; whereby it appears, that the total Darkneſs laſted there $3'. 15''$. Hence it follows that *Witley* was about three or four Miles farther from the Center of the Shade on the North-ſide than *London* on the South: And *Witley* being, by *Ogilby*'s Menſuration, 118 Meaſured Miles from *London*, it is plain that the Center paſt over *Iſlip*, which is, by the ſame Admeaſurement, 57 ſuch Miles on that Road, and about five Miles almoſt due North from *Oxford*; ſo that the Center of the Shade left *Oxford* but very little upon the Right-Hand. This Situation agrees perfectly well with the former between *Barton* and *King's Walden*, and as far as the Geography of our Country may be depended on, I conclude the Center to have entred upon *England* about *Plymouth*, and to have paſt over *Exeter*, the *Devizes*, *Iſlip*, *Buckingham*, and *Huntington*, leaving *Oxford* and *Bedford* on the Right, and *Lynn* on the Left, and to have quitted the Coaſt of *Norfolk* about *Wells* and *Blakeney*.

As

As to the Southern Limit or Term, where the Eclipse ceased to be total on the South-side of the Sun, we have received an Account of an Observation made at *Norton-court*, about Ten Miles on this Side *Canterbury*, by the Reverend Dr. *Harris*, assisted by that accurate Observer Mr. *Stephen Gray*, by which we learn that the Eclipse began there at $8^h. 8'. 55''$. and ended at $10^h. 24'. 47''$; and that the total Darkness continued but about one Minute or rather less, the Middle thereof being at $9^h. 13'. 52''$. From this Duration it will follow, that *Norton-court* was but about three or four Miles within the Shade. And that it was really so, is confirmed by the Relation of the Inhabitants of *Bocton*, about Midway between *Norton-court* and *Canterbury*, who assured Mr. *Gray*, that the Eclipse was not total there, but, as one of them exprest it, before the Sun had quite lost his Light on the East-side, he recovered it on the West: And that there was a small Light left on the lower Part of the Sun that appeared like a Star. And from *Cranbrook* in *Kent* we are informed, by the Relation of *William Tempest*, Esq; that he observed there the Sun to be extinguished but for a Moment, and instantly to emerge again: So that the Limit past exactly over this Town, which is about 38 Geographical Miles from *London*, and very near the right Angle, where the Perpendicular from *London* falls on the Line of the Limit, being $3'. 00''$ of Time to the Eastward of *London*, in the Latitude of $51°. 6'$, as near as I can gather.

How it past over *Sussex* we have no authentick Relations, but have learnt that it was total at *Wadhurst* beyond *Tunbridge-wells*, as also for some short Time at *Lewis*; but that it was not so at *Brightling*, which Place being situated on an Eminence, all the Country to the Northward was seen in Darkness, whilst they there had some Benefit of a small Remainder of the Sun.

From these Observations we may conclude, that this Limit came upon the Coast of *England*, about the Middle between *Newhaven* and *Brighthelmston* in *Sussex*, and passing by *Cranbrook* and *Bocton*, about four Miles on the Right-hand, quitted the Coast of *Kent* not far from *Hern* toward the ancient *Regulbium*, now called *Reculver*. So that scarce one third Part of *Kent*, and not so much of *Sussex*, out of all the South Coast of *Great-Britain*, escaped being involved in this Darkness.

The Northern Limit, having past over a much greater Space, has had more Observers, and is not less curiously determined than the other. By the Account given by the Reverend Mr. *Roger Prosser*, Rector of *Haverford-West*, the Eclipse was total there a Minute and a half; whence it follows, that *Haverford* was but about 6 Miles within the Shade, and therefore that it entred on *Pembrokeshire* about the Middle of St. *Bride*'s Bay, leaving St. *David*'s and *Cardigan* on the left Hand; and having traversed those two Counties and *Montgomeryshire*, it entred *Shropshire*,

Shropſhire, leaving the Town of *Shrewsbury* 1′. 40″ in the Shadow, as was obſerved there by Dr. *Hollings*; whereby it appears that *Shrewsbury* was about 8 Miles within the Limit. Thence it proceeded by the Eaſt-ſide of *Cheſhire*, leaving *Whitchurch* and *Nantwich* a very little without, and paſſing by *Congleton*, went over the *Peak* of *Derbyſhire* into *Yorkſhire*, and croſs'd the great Northern Road between *Pontefract* and *Doncaſter*, ſomewhat nearer the former than the latter. For by the Obſervations of *Theophilus Shelton*, Eſq; at *Darrington*, about two Miles on this Side *Pontefract*, (in Lat. 53°. 40′ and Long. Weſt from *London* 4′. 40″. of Time, as may be concluded from *Norwood*'s Meaſure of a Degree) the Sun at 9h. 11′. was reduced almoſt to a Point, which both in Colour and Size reſembled the Planet *Mars*; but while he watched for the total Eclipſe, that Point grew bigger, and the Darkneſs diminiſhed; whence he argued the Limit to have been very little more Southerly. And ſince has been informed, that it was juſt total in *Barnſdale*, three Miles South from thence. And that it was ſo at *Badſworth*, about the ſame Diſtance from *Darrington*, we are told by a Letter of the Reverend Mr. *Daubuz*, that he has a certain Account from that Place, that the luminous Ring round the Moon was ſeen there, which was no where viſible but while the Eclipſe was total. From theſe *Data* we may ſecurely determine the Remainder of this Track, and that the Edge of the Shadow, having paſt over the reſt of *Yorkſhire*, paſſed off to Sea about *Flamborough* Head.

So that of the forty Counties, into which *England* is ſubdivided, only the five moſt Northerly have not had the Sun wholly hid from them; and ſix others have eſcaped but in Part, *viz.* *Shropſhire*, *Cheſhire* and *Yorkſhire*, and the extreme Part of *Derbyſhire* on the *North*, and *Kent* and *Suſſex* on the *South*; all the reſt of the Kingdom having more or leſs ſuffered an Interval of total Darkneſs.

I ſhall not at preſent conſider this Eclipſe as univerſal, but only as it related to *England*; and it ſhall ſuffice to ſay, that the Shadow came out of the *Atlantick* Ocean, having paſt over the Iſlands *Azores*; and that the Southern Limit of it reach'd the Iſle of *Uſhant*, and the North-weſt Coaſts of *Britanny* between *Breſt* and *Morlaix*; and dividing our Iſlands of *Guernſey* and *Jerſey*, juſt touched upon the Promontory of *Normandy* called *Cape de Hague*. And that after it had quitted *England*, and traverſed the *German Ocean*, it fell on *Jutland* on the South-ſide, and *Norway* on the North; and thence proceeded to the Eaſtward over *Sweden*, *Finland*, &c.

It remains now to conſider the Figure, Poſition, Direction, Velocity and Magnitude of the Shadow as it paſſed over us. As to the Figure, 'tis obvious that the Shadow of the Moon being a Cone, and the Earth's Surface ſufficiently Spherical, the apparent Shadow on the Earth will be the common Interſection of a Cone and Sphere, which is a Figure hitherto little conſidered by Geometers; and not being *in Plano*, is not to to be exactly described but in the spherical or conical Surface. How to find the Points of this Curve, in all Cases, is shewn by *P. Cousier*, in a very scarce *Latin* Book printed at *Dijon* in *Burgundy*, and published at *Paris* in the Year 1663: Nor do I know of any other Author, that has handled the same Subject since, though capable and worthy of further Improvement. By what he there delivers, *Prop.* 11, 12. *Lib.* I. it will be easily understood, that the Convexity of so small a Part of the Earth's Surface, as the Shadow commonly occupies, can produce only an inconsiderable Effect; so that without sensible Error we may take it for a Plane, and the Section for a true *Apollonian Ellipsis*, whose transverse *Axis*, by reason of the Smallness of the Angle of the Cone, will be to its Conjugate nearly as *Radius* to the Sine of the Sun's Altitude at its Center, especially if he be considerably elevated. But when he is near the Horizon, it will be necessary to have Regard to the true Figure, by reason of the great Length, to which the transverse Axe is extended, and particularly when the Shade is entring upon, or leaving the Earth's Disk.

As to the Position of the *Axis* of the Shadow, it is manifest, that it must always lie in the Plane of a great Circle of the Earth, passing through the *Axis* of the Cone of the Shade; and therefore all that is required, is, to obtain the Azimuth and Altitude of the Sun, at the Place where the Center of the Shade at any Time is found, to determine the Situation of the Axe and *Species* of the Ellipse required. Thus the Middle of the Eclipse at *London* having been observed at $9^h. 10'. 45''$, by the given Latitude and Declination, we find his Azimuth about $59^\circ. 00'$. and Altitude $40^\circ. 46'$. that is just 40 Degrees high, at the Center of the Shadow. Wherefore the transverse Axe of the Ellipse was to its Conjugate very near as *Rad.* to the *Sine* of 40°, or as 1000 to 643 *proximè*; and did make an Angle of 59°, or very little more, with the Meridian passing at that Time through the Center of the Shade.

The Direction and the Velocity of the Motion, wherewith the Center of the Shade past over *England*, is next to be considered, wherein it is to be observed, that the Shadow passes in a very compound Curve, which as the former is not *in Plano*, and only describable on the Surface of the Sphere: Nor is its Motion equable, but compounded of very many Elements, producing a great Variety. By what Method its Points, and its Tangents in those Points, are to be obtained, I reserve to another Opportunity; only I observe, that for so small a Part of the Curve, as went over *England*, it may be esteemed a right Line, with more Exactness than we usually find in most of our Geographical Charts. And the same may be said for the Velocity, which, though in our present Instance, it was continually decreasing, may, for so short a Time, be supposed to have been the same without sensible Error.

By

By a careful Calculation, I have determined the Velocity of the Motion, at the Time of the Middle of the Eclipse at *London*, to have been 29 Geographical Miles in a Minute of Time *quam proximè:* And that its Way made an Angle of 52°. 45′. with the Meridian towards the Eastwards of the North; wherefore the said Way made an Angle with the Axis of the Ellipsis of 68°. 15′. And the greatest Duration of total Darkness having been 3′. 57″, it will follow, that that Diameter of the Elliptick Figure, according to which the Shade past, was no less than 114½ Geogr. Miles. And from the Elements of the Conicks 'tis easy to be proved, that supposing the Figure of the Shade a true Ellipse, whose Axes are as *Radius* to the *Sine* of 40 Degrees, the greater Axis would be 171 Geographical Miles, and the lesser 110; and the nearest Distance between the Limits supposed Parallel 164 such Miles.

And this Length of the *Axis* of the Shade, derived purely from the Duration of total Darkness, is fully confirmed by the observed Distance of the parallel Limits; the one passing by *Badsworth* in *Yorkshire*, the other by *Cranbrook* in *Kent*. For by the two Latitudes 53°. 37′ and 51°. 6′, with the Difference of Longitude 7′. 40″. of Time, or 1°. 55′, the Distance of these two Places is given 166½ Geogr. Miles; with the mean Angle of Position 25 Degrees from the North Westwards; wherefore this Arch makes an Angle with the Track of the Shade of 77° ¾, and hence the nearest Distance of the Parallels becomes 163 such Miles, which by the other Way was found 164.

If then we conclude the *Axis* of the Shadow, when the Sun was just 40 Degrees high, to have extended over 2°. 50′ of a great Circle, we may securely determine the Difference of the Sun and Moon's Diameters at this Time. For the Difference of the Horizontal Parallaxes of the Sun and Moon being found to be 60′. 38″. (as shall be hereafter shewn, but is not required with extreme Exactness for this Purpose) the Difference of the Parallaxes in Altitude at both Ends of the Axis, will be found to be 1′. 56″, and by so much did the Diameter of the Moon, when forty Degrees high, exceed that of the Sun: Hence the Horizontal Diameter of the Moon, in this Anomaly is found 33′. 27″, which may serve for a Rule in all other Cases.

I forbear to mention the *Chill* and *Damp*, with which the Darkness of this Eclipse was attended, of which most Spectators were sensible, and equally Judges; or the Concern that appear'd in all Sorts of Animals, *Birds*, *Beasts* and *Fishes* upon the Extinction of the Sun, since ourselves could not behold it without some Sense of Horror.

Lastly, I have added the following *Synopsis* of such Observations as have hitherto come to my Hands.

Place.	Observers.	Beginn. h. ′ ″	Immers. h. ′ ″	Emers. h. ′ ″	Tot. ′ ″	End. h. ′ ″
Barton	M. *Bridges*				3. 53	
Bell-bar	M. *Jones*	8. 6. 25	9. 9. 45	9. 13. 27	3. 42	
Broadway } *Carmarth.*			8. 47. 00	8. 49. 30	2. 30	10. 21. 57
Cambridge	M. *Cotes*			9. 14. 37		10. 24. 30
Canterbury	M. *Gray*	8. 10. 00				
Chester	M. *Ward*	7. 57. 40				10. 6. 35
Crew	M. *Wright*		9. 2. 8		2. 00	10. 9. 00
Dublin	L. *Archbish.*	7. 42. 11				9. 49. 40
Dublin	M. *Hawkins*	7. 41. 30				9. 48. 45
Exon	L. *Bishop*		8. 55. 0	8. 59. 0	4. 00	10. 0. 00
Exon	M. *Hudson*	7. 47. 30			3. 30	10. 0. 30
Greenwich	M. *Flamsteed*				3. 11	
King's Wald.	M. *Whitside*				3. 52	
Llanidan } *Anglesey*	M. *Rowland*	7. 52. 30				
London	R. *Society*	8. 6. 00	9. 9. 3	9. 12. 26	3. 23	10. 20. 00
Northampt.	M. *Hawkins*		9. 5. 22	9. 9. 24	4. 2	10. 15. 35
Norton-court	D. *Harris*	8. 8. 55	9. 13. 23	9. 14. 22	0. 59	10. 24. 47
Oxon	D. *Keill*				3. 30	10. 15. 10
Paris	R. *Academy*	8. 11. 00				10. 28. 00
Plymouth	M. *Heines*	7. 41. 00	8. 45. 30	8. 50. 00	4. 30	9. 54. 30
Portchester	C. *Chandler*		9. 2. 25	9. 6. 15	3. 50	
Salop	D. *Hollings*	7. 58. 00			1. 40	10. 6. 00
Upminster	M. *Derham*	8. 7. 41	9. 10. 58	9. 14. 6	3. 8	10. 21. 45
Wansted	M. *Pound*	8. 6. 37	9. 9. 28	9. 12. 48	3. 20	10. 20. 32
Weymouth	M. *Hobbs*		8. 53. 00	8. 58. 00	4. 00	
Witley	M. *Baxter*	7. 59. 0			3. 15	10. 13. 00

Accounts of the same from abroad, n. 345. p. 314.

2. Since the Publication of the former Account of what was observed in *England*, and particularly at *London*, of this Eclipse, we have received from Foreign Parts the following Observations.

— near the Island Forte ventura, *by Mr.* J. Edens.

Mr. *J. Edens*, being on his Voyage to the Pike of *Teneriff*, observed the Eclipse at Sea, in Latitude by Observation 34°, 20′, and Longitude 0^h 54′, West from *London*, as he concluded by their Distance and Position from the Island *Forte Ventura*, which they soon after fell in with. He writes, that it began at VI^h. 49′, and ended at $VIII^h$. 47′. this latter very exactly, though not quite so nice as to the Beginning.

Had

Had this Gentleman ſignified, what Difference of Meridians there was found between the Place of Obſervation, and the Weſt End of *Forte Ventura*, we might, without ſenſible Error, have concluded the true Longitude, not only of that Iſland, but alſo of the Pike of *Teneriff*, where *our Geographers* and the *Dutch* have fixed their firſt Meridian. He adds, that the greateſt Darkneſs was about $\frac{3}{4}$ of the Sun's Diameter, or nine Digits on the North Side.

From *Germany* we have received the following Accounts.

At *Nurenburg*, The Beginning and greateſt Obſcurity could not be ſeen for Clouds, but the End happen'd at XI^h. 10$'$. $\frac{1}{5}$. — *at* Nurenburgh.

At *Hamburg*, The Beginning was obſerved at VIII^h 57$'$. the greateſt Obſcurity at X^h. 5$'$. 30$''$. when XI $\frac{1}{2}$ *Digg.* were darkned. The End could not be obſerved for Clouds. — *at* Hamburgh.

At *Keil* in *Holſtein*, The Beginning IX^h. 14$'$. The greateſt Obſcurity X^h. 19$'$. 20$''$, and the Quantity then eclipſed XI *Digg.* 20$'$. The End was at XI^h. 29$'$. — *at* Kiel *in* Holſtein.

At *Berlin*, The Beginning could not be obſerved for Clouds, but the greateſt Obſcurity was at 22 *min.* paſt Ten, when XI *Digg.* were eclipſed. The juſt End was at XI^h. 34$'$. — *at* Berlin.

At *Frankfort* on the *Meine*, The Eclipſe began at VIII^h. 50$'$. The greateſt Darkneſs at X^h. 11$'$, *but perhaps ſhould be* X^h. 01 *min.* the Digits being X. and 34 *min.* The End was obſerved at 10 *min.* paſt Eleven. — *at* Franckfort *on the* Meine.

By whom theſe Obſervations were made, and with what Inſtruments, we are not as yet informed, but hope they may be exact enough to confirm the *Longitudes* of thoſe ſeveral Places, which are at preſent reaſonably well known.

In a Book entituled, *Nouvelles Literaires*, publiſhed at the *Hague*, *pag.* 404, 405, there is an Account of the Obſervation of this Eclipſe at *Upſal* in *Sweden*, made by M. *Jo. Waller*, Profeſſor of Mathematicks in that Univerſity, who was very careful to obſerve it exactly; the Times being verified by three Clocks perfectly agreeing with one another and with the Sun: But more eſpecially by a Quadrant of five Foot Radius for taking the Sun's Altitude. By this Inſtrument he determined the Height of the Pole at *Upſal* 59°. 51$'$. 54$''$. And by the ſame, a little before the Beginning of the Eclipſe, he found the Height of the Sun 39°. 36$'$. 42$''$. his Clocks then ſhewing the Hour IX^h. 47$'$. 50$''$, which proves that they were very near the true Time. At X^h. 58$'$. 15$''$. the Altitude of the Sun being 44°. 17$'$. 29$''$, was the Beginning of the total Darkneſs, and at XI^h. 2$'$. 24$''$. was the End thereof, *alto ſole* 44°. 29$'$. 13$''$. ſo that here the Duration of the total Eclipſe was 4$'$. 9$''$, and the Middle thereof but one Third of a Minute after Eleven. And laſtly, the End is ſaid to have happen'd about 4 Minutes before Noon, the Sun being 45°. 42$'$. 6$''$. high: But in this is a manifeſt Error, for it makes the Time of Emerſion, or from the Middle to the End, but 55$'$. 20$''$; whereas being ſo near the Meridian, 'tis certain that this Emerſion was the greater Part of the Duration of the — *at* Upſal *in* Sweden. *by Mr.* J. Waller.

whole Eclipſe, and conſequently more than an Hour. Perhaps the Times might be deduced from the Altitudes only, and then the Miſtake might be in ſuppoſing the End ſo much before Noon, as it was really after it. However, to prevent all Doubts, we have compared this Obſervation with what we obſerved of this Eclipſe at *London*, and find, that in the Latitude of 59°. 50′, the Place where the Middle of total Darkneſs was at XI^h. 0′. 20″, was near 19 Degrees more Eaſterly than *London*, (that is exactly in the Meridian of *Dantzick*) and that the Eclipſe began there at XI^h. 52′$\frac{1}{2}$, and ended at XII^h. 10″, wherefore the Duration could not be 2^h. 7′. 50″, as the *Editor* of the ſaid *Nouvelles* has publiſhed; not conſidering, that the Beginning could not be ſeen for Clouds, as in the very next Words he aſſures us.

As to the Darkneſs, it was ſuch, that they could ſcarce diſtinguiſh one another; and beſides *Jupiter*, *Mercury* and *Venus*, of the fix'd Stars, *Caſſiopea*, *Capella*, *Oculus Tauri* and *Orion* (Sirius *not being yet riſen*) were viſible.

Eclipſe of the Sun, Feb. 19, 1718. O. S. *at* Norimberg, *by Mr.* Wurtzelbau, *and at* Berlin, *by Mr.* G. Kirch, n. 357. p. 822.

XVI. From the *Literary News* of *Berlin* we have obtain'd two Obſervations of a ſmall Solar Eclipſe, *Feb.* 19, *St. Vet.* 1718. One at *Norimberg*, by Mr. *Wirtzelbau*; the other obſerved at *Berlin* by Mr. *Kirch*.

At *Norimberg* the Sun roſe ſomething deficient in his upper Limb, which Defect increaſed to 3 full Digits. The Eclipſe ended at 8^h. 8′. 48″, about 60 Degrees from the Vertex of the Sun to the left Hand. But at *Berlin* the Sun began to be eclipſed preſently after his Riſing, or at 6^h. 49′ or 49′$\frac{1}{2}$. About the Middle of the Eclipſe, or at 7^h. 35′, the lucid Parts remaining in the Sun were 24′. 40″. Whence the Digits obſcured were 2^d. 50′. The End happen'd at 8^h. 28′. 10″.

Eclipſe of the Moon *at* Cambridge *in* New-England, *Feb.* 11, 1700, *by Mr.* T. Brattle, n. 202. p. 1633.

XVII. On *Feb.* 11, the Moon roſe eclipſed, and the Horizon was ſo overcaſt, that I deſpaired of having any Obſervations; but at $\frac{1}{2}$ an Hour paſt 6. ſhe came from under the Cloud, and at 6^h. 25′. I had juſt a Sight of her, and judged her eclipſed about 5 Digits; at

h.	′	
6	29	The Section equidiſtant from *M. Ætna* and *Horminius*.
	32	*Palus Maræotis* begins to be ſeen.
	34$\frac{1}{2}$	*Palus Maræotis* and *Mons Apollonius* $\frac{1}{2}$ out.
	37$\frac{1}{2}$	*Palus Maræotis* quite free, and *Palus Maræotis* and *Palus Mæotis* in the Perpendicular.
	42$\frac{1}{2}$	The Shadow near an Inch from *Palus Maræotis*, *Mons Horminius*, and *Mons Hercules*.
	46$\frac{1}{4}$	*Palus Maræotis* in the *Nadir*, and that Part of *Palus Mæotis* to my Right-hand in the Prime Vertical.
	57	The upper Part of the Section is now, and has been for a long Time, in *Inſula Major in Mare Caſpio*, (and the Section now perpendicular,) and the lower Part wheeling about from *Palus Maræotis*.

7^h

h	′	
7	20	*Mount Sinai* firſt appears at 22′ wholly free.
	$25\frac{1}{2}$	*Palus Maræotis* and *Mons Horminius* near perpendicular.
	43	The Eclipſe over in the Teleſcope, and at 49 to the naked Eye.

My Clock was ſet by my Ring-Dial about Nine of the Clock in the Morning, as exactly as I could judge; and the Obſervation was made with my $4\frac{1}{2}$ Foot Teleſcope, with all four Glaſſes in it.

XVIII. 1.]

Eclipſe of the Moon, Dec. 12. 1703. *at* Cambridge *in* New England, *by Mr.* T. Brattle, n. 292. p. 1656.

Time by the Clock.

h	′	
11	45	In the Morning, that Part of the Moon's Diſk near *Alabaſtrinus*, look'd ſomewhat duskiſh, and the Eclipſe beginning to enter between *Palus Meræotis* and *M. Porphyritis.*
11	53	The true Shadow was well entred.
	58	*M. Porphyritis* juſt cover'd.
12	$03\frac{1}{2}$	Near 3 Digits darkened.
	$7\frac{1}{2}$	*Mount Ætna* begins.
	$9\frac{1}{2}$	Quite cover'd.
	$14\frac{1}{2}$	*Lacus Niger Major* and *M. Sinai* almoſt equidiſtant from the Section of the Shadow, *Lacus Niger Major* being ſomewhat the nearer of the two.
	$18\frac{1}{4}$	*Lacus Niger Major* begins $19\frac{1}{2}$ quite cover'd.
	$21\frac{1}{4}$	*Mount Sinai* begins.
	$21\frac{3}{4}$	Quite cover'd, and the Moon about 6 Digits eclipſed.
12	$24\frac{1}{2}$	*Besbicus* begins.
	26	Quite cover'd.
	$28\frac{3}{4}$	*Byſantium* begins.
	$29\frac{1}{2}$	Cover'd, and *Mount Horminius* begins.
	32	*Apollonia* begins.
	33	Cover'd.
	37	The Shadow equidiſtant from *M. Corax* and *Mount Paropamiſus*, or ſomewhat nearer to *M. Corax.*
	$39\frac{1}{2}$	Between 9 and 10 Digits eclipſed.
	43	*M. Corax* begins.
12	$44\frac{3}{4}$	*Palus Mæotis* begins, and at $45\frac{3}{4}$ the Inner of *M. Paropamiſus* begins.
	50	*Palus Mæotis* quite cover'd.
	$51\frac{1}{2}$	The Moon not quite eclipſed.
	52	Nor yet.
	53	Nor yet.
	54	Scarce.
	$54\frac{1}{2}$	Quite immerged and the Mora begins.
14	39	Preciſely, ſhe emerged between *Palus Maræotis* and *Mons Porphyritis.*

h ′

42 *Palus Maræotis* begins.
43 Quite clear.
47 *M. Porphyritis* quite clear.
55 About 3 Digits restored.
59 Mount *Ætna* begins.
15 02 That and *Lacus Niger Major* at the same Time clear.
8½ *Mount Sinai* about half free.
9½ Quite free, and about 6 Digits restored.
15 *Besbicus* free.
19¼ *Bysantium* free.
29½ About 9 Digits seem'd to be restored.
30¼ *Mons Herculis* free.
32¾ *Palus Mæotis* begins.
38½ Quite free.
41½ *Insula Major in Mare Caspio* free, and in the Middle of the Section.
42½ Not yet wholly clear.
45 Fully over in the Telescope, tho' a Kind of a Smoak remained some little after to the naked Eye.

In order to the Adjusting of the Time, I set my Clock with the greatest Exactness I could the Morning preceding, both from my Ring-Dial and the Rising of the Sun, which I very narrowly watch'd and observed, and found it to agree with the Sun's setting the following Evening; so that it went all the Time the Eclipse was, very steadily and regularly; but for the greater Certainty and Satisfaction, I took the Altitudes of the following Stars with the Brass Quadrant with Telescope Sights out of my Chamber Window, the Lowness whereof would not permit me to take them, when they were at all higher elevated.

*. in dextro humero Orionis.

By the Watch.		Comp. Alt.					Differ.	
h	′			h	′	″	′	″
6	15	78	18	6	13	40	1	20
6	21½	77	03	6	20	28	1	02
	26¼	76	11	6	25	08	1	07
Procyon								
8	9¼	77	20	8	08	04	1	11
	14½	76	20	8	13	32	0	58
	21	75	13	8	19	36	1	24
Regulus 10	8½	77	46	10	07	18	1	12
	17¼	76	11	10	15	58	1	17

So that my Clock went by these Observations nearest 1′ too fast.

2.] I had the good Fortune at *London* near the *Exchange* to make some few Observations of the Eclipse of the *Moon* of *December* the 11th, 1703. (of which I gave an Account to the Society some Time since) as follows:

— the same observ'd at London, *by Mr.* J. Hodgson, n. 291. p. 1594, n. 292. p. 1637.

The Heavens being cloudy most Part of the Night, it was 35′ after Four in the Morning following, before I could perceive that the Moon was eclipsed; and then as near as I could judge, she had been so about three or four Minutes at most, from whence we may conclude it began at *London* about 31 or 32 Minutes after Four the same Morning.

Mr. *Brattle* found, that at 44 Minutes after Eleven at Night, Part of the Moon's Disk look'd somewhat duskish, and that at 52 Minutes, the Shadow was well enter'd, *so that from hence, as well as from a Comparison of the Ingress and Egress of the principal Spots, it probably began there about 49 Minutes after Eleven, whence it follows, that* Cambridge *in* New-England *lies* $4^h\ 42' \frac{1}{2}$, *or* 70 37′ *to the Westward of the Meridian of* London.

Difference of Longitude *between* London *and* Cambridge *in* New-England.

I happen'd to see the Moon the same Morning at 35 Minutes after Five, when she wanted at most but three Minutes of being totally eclipsed; so that at *London* she immerged at 38 Minutes past Five.

Mr. *Brattle* saw her immerge exactly at 54 Minutes after Twelve, whence it follows, that the Difference of the Meridians found by comparing these Observations, is $4^h\ 43' \frac{1}{2}$, or 70° 52′, agreeing very well with the former; so that by taking a Mean between them, the Difference of Longitude of the two Places $4^h\ 43'$, or 70° 45′.

I saw no more of the Eclipse that Morning, by reason of the Clouds, and should be very glad to meet with some other Observations to confirm these; but their mutual Agreement gives great Reason to believe that the Deductions are good, and may be rely'd upon; for during the Eclipse I had a View of the Moon at least twenty Times, tho' Clouds frequently intervening have made this Account of mine less accurate and certain than otherwise it would have been.

Eclipse of the Moon, Apr. 5, 1707, *at* Boston *in* New-England, *by Mr.* T. Brattle, n. 312. p. 2471.

XIX. *Immersions.*	Time corrected by Altitude. h	′	″
A very notable *Penumbra*, (Evening)	6	52	
Palus Marœotis is cover'd	6	58	20
Mons Porphyrites begins	7	8	15
Is cover'd	7	9	20
Mount *Ætna* begins	7	16	
Is quite covered	7	17	15
Mount *Sinai* begins	7	21	40
Quite cover'd	7	22	40
The Isle of *Corsica* is cover'd	7	24	
The greater *black Lake* is cover'd	7	31	40
The Island *Besbicus*	7	33	
Bizantium	7	36	30

Immer-

			h	'	"
Immersions.					
Mount *Horminius*			7	37	20
Mount of *Apollo*			7	40	30
Mount *Hercules*			7	44	30
Mount *Corax*			7	51	30
Palus Mæotis begins			7	52	45
The great Iſland in the *Caſpian* begins			7	54	45
Is cover'd			7	56	
Palus Mæotis is quite cover'd			7	57	30
The Moon quite immerſed			8	1	15
Emerſions.					
The Complement of the Altitude of *Arcturus*	53°	34'	8	28	
	51	30½	8	39	15
The Complement of the Altitude of the Star that follows the bright Star in the Northern Crown					
Latitude 44° 33'	60°	2'	9	0	30
	56	57	9	17	15
The Moon plainly began to emerge			9	46	30
Mount *Ætna* was wholly illuſtrated			10	9	30
Mount *Sinai* wholly appear'd			10	10	15
The Iſle *Besbicus*			10	25	
Bizantium			10	28	30
The Mount of *Apollo*			10	33	
Mount *Hercules*			10	36	30
Palus Mæotis begins			10	44	
The greater Iſle in the *Caſpian* is reſtor'd			10	47	
Palus Mæotis wholly uncover'd			10	49	
The Moon is fully illuminated			10	54	

Eclipſe of the Moon, April 11, 1707, *at* Zurich, *by the two Doctors* Scheuchzer's, n. 310. p. 2394.

XX.

h	'	"	
12	9 ad	18	The *Penumbra* on the Side of *Maræotis*.
	18	40	The true Shadow within the Disk.
	20	15	*Palus Maræotis* in the Shadow.
	23		The Beginning of *Mare Eoum*.
	25	20	The Mount of *Alabaſter*. The Middle of *Mare Eoum*.
	27	40	The Beginning of the Bay of *Sirbon*.
	29		The Middle of the Bay of *Sirbon*, and *Mare Ægyptiacum*.
	29	20	The Beginning of the Iſland *Cercinna*.
	29	30	The Southern Lake.
	30	30	The Middle of *Cercinna*.
	31	40	The End of *Cercinna*.
	33		The utmoſt Promontory of Mount *Sopher*.
	34		The Iſlands between *Sicily* and *Cercinna*.
	35		The Beginning of *Mauritania*, and of *Sirus Hyperboreus*.
	36	40	The Middle of *Sinus Hyperboreus*.

h	′	″	
	37	20	*Mare Pamphilium.*
	37	30	*Crete.*
	37	40	The Beginning of *Ætna.*
	38	40	The Middle of *Ætna.* *Melos.* *Carpathos.*
	39	30	The End of *Ætna.*
	41		*Rhodes.*
	43	40	The Beginning of *Sinai.*
	45		The Middle of *Mare Adriaticum* and *Sinai.*
	45	20	The End of *Sinai.*
	46	30	The Middle of *Adriaticum.*
	48	40	The Beginning of *Propontis*, and of *Mare Hyperboreum.*
	51	30	The Middle of *Propontis*, and the End of *Adriaticum.*
	52	40	The Beginning of the *Greater Black Lake.*
12	53	10	*Lacus Thrasumenus.*
	53	30	The Middle of the *Greater Black Lake.*
	54	30	The Beginning of the Iſland *Besbycus.*
	55	10	The Beginning of *Pontus Euximus* in the Bay *Salmydeſſus.*
	56		The End of *Propontis.*
	57	30	The Beginning of the *Lower Euxine Sea.*
	58		The Beginning of *Byzantium.*
	59	10	——— The End.
	59	40	The *Acheruſian* Promontory.
1	1		The Beginning of *Boryſthenes.* *Apollonia.*
	1	30	The Middle of the *Euxine Sea.*
	3	20	The Middle of the *Athenian* Bay.
	4	40	The Marſh of *Byce.*
	5	40	The Promontory of *Heracleum.*
	7	40	The Middle of *Cochlis:* The End of the farther Bay of *Pontus.*
	8	30	The Middle of the Lake *Coroconda.*
	9	20	The Beginning of the Promontory of *Hercules*, and of the *Caſpian Sea.*
	11	40	The Beginning of *Palus Amadoca.*
	16	20	The Middle.
	18	10	The End.
	20		The bitter Marſhes and the Leſſer Lake.
	20	40	The Greater Lake.
	22	10	A very ſlender lucid Margin.
	23	20	The whole Body of the Moon in the Shadow.
	24	40	The Moon's Disk was almoſt entire, except the *Mediterranean Sea.* It ſhin'd with a Kind of dilute Brightneſs, ſo that the Seas might be diſtinguiſh'd thro' the Tube.
	40		No other Spot but *Palus Mæotis* could be diſtinguiſh'd through the Tube.
	45		The middle Disk of the Moon is obſcured more and more, the Circumference continuing brighter.

h	′	″	
2	12		The Disk of the Moon ſhined to the naked Eye with a reddiſh Colour, nor could any Spot be diſtinguiſhed through the Teleſcope.
	15		The whole Disk grew more and more obſcure, the Circumference remaining a little brightiſh.
	28		The Disk was brighter over againſt *Palus Maræotis*, and there was a very denſe Shadow towards *Palus Mæotis.*
	33		By Degrees the whole Disk became brighter; but a greater Obſcurity cover'd *Palus Mæotis*, and the neighbouring Places.
	51	40	By Degrees the Images of the Seas return.
	56	30	The *Euxine Sea* and the *Caſpian* continue in the Middle of Obſcurity, as it were cover'd by a thick Cloud.
3	5		*Mare Eoum* and the neighbouring Places might be diſtinguiſh'd, tho' the Moon had not yet emerged out of the Shadow.
	9	40	The true Beginning of the Emerſion.
	11	30	*Palus Maræotis* begins to emerge.
	13		It is emerged.
	15	40	*Mare Eoum* begins.
	21	30	*Sinus Sirbonius* and the *Egyptian Sea* emerged.
	26		*Caſſiotis Reggio* comes out, and the Iſle *Circinna* ſome Minutes before.
	27	40	*Mount Athos* and the Iſland *Malta* come out.
	31		*Mauritania* emerges.
	38		——— *Corſica* and *Sicily.*
	44		——— The *Adriatick Sea.*
	45	20	——— The Middle of *Propontis.*
	49		——— *Besbycus.*
	52		——— *Byzantium.*
	57	30	——— Promont. *Acheruſ.*
4	5	20	The *Euxine Sea*, and the Middle of the *Caſpian* came out.
	6		*Palus Mæotis* begins.
	2		The *Caſpian* emerged, and the Middle of *Mæotis.*
	11	20	*Mæotis* emerged.
	13	40	The *Penumbra.*
	14	20	All the Moon entire.
			The Shadow ſeem'd to me more diſtinct in the Emerſion than it was in the Immerſion.
12	18	40	The Beginning of the Eclipſe in the true Shadow.
1	23	20	The greateſt Obſcuration.
3	9	40	The Beginning of Emerſion.
1	46	30	The Duration of total Obſcuration.
4	14	20	The End of the Emerſion.
3	55	50	The whole Duration.

h	′	″	
1	5	40	From the Beginning to the total Immersion of the Moon.
1	5	40	From the Emersion of the total Eclipse to the End.

XXI. As I was coming from *London*, *Sept.* 18, in the Evening, I observed, for Half an Hour or more, a thin Shade to possess that Part of the Disk where the Eclipse began, which remain'd a good While after the Eclipse was over. After I got Home, I got all Things in Readiness before the Eclipse began. The principal Observations were as follow:

Eclipse of the Moon, *Sept.* 18, 1708. *at* Upminster, *by Mr.* W. Derham, n. 320. p. 312.

The Corr. App. Time.			
h	′	″	
7	56	30	A thin Penumbra.
7	57	40	A darker Penumbra.
7	59	00	Yet darker, which may pass for the Beginning of the Eclipse.
8	00	00	The Eclipse no Doubt begun.
9	01	00	The lucid Parts of the Moon, not long before the Middle of the Eclipse, were 925 Parts of my Micrometer.
9	16	40	Diameter of the Moon 1634 Parts of the Micrometer.
10	23	11	The End of the Eclipe draws nigh.
10	25	00	A little Obscuration.
10	26	00	Less.
10	28	15	A very little, excepting the Duskishness before mentioned.

XXII. In the last *Lunar* Eclipse on *Feb.* 2, 1709-10, the Time of the End, (which was what alone the Want of a proper *Apparatus*, and a favourable Sky would give me Leave exactly to determine) I found to be the same (with but a very inconsiderable Difference) which the *Calculation*, according to Sir *Isaac Newton*'s admirable Theory, promised me to expect.

The Account of the Moon's *Eclipse*, Feb. 2, 1709-10. Compared *with the* Calculation, *by Mr.* H. Cressener, n. 325. p. 16.

I have added the Calculation from Mr. *Flamstead*'s Tables according to Mr. *Horrox*'s Theory, as I find them publish'd in Mr. *Whiston*'s Astronomical Lectures, with the *Radix's* or the mean Motions, corrected according to their first Author's later Observations, which are the same with those assumed in Sir *Isaac Newton*'s Theory.

By comparing these two Calculations, we may observe, that tho' most of the additional Equations in Sir *Isaac Newton*'s Theory be very small in this Situation of the Moon, yet they all conspire so as to make its Place considerably more agreeable to Observation, than those of *Horrox*'s System.

The Observation was made at *Streatham*, about six Miles near direct South of *London*, with a very good eight Foot Telescope. To correct the Clock, (for Want of an Instrument) I carried with me next Day two Watches, that were before adjusted to the Clock, and compared them with Mr. *Flamstead*'s at the Royal Observatory, having first noted its Error by an Observation of the Sun's Transit of the Meridian his Assistant

ſiſtant communicated to me. Upon my Return I found my Watches ſtill to agree together, and to my Clock, which proved them to have gone true, and gave me the exact Error of my Clock, and the true Time at Obſervation.

Mr. *Flamſtead* has ſince been pleaſed to acquaint me, that by his Obſervation of the meridional Tranſit of the *Lyon's Heart* during the Eclipſe, his Clock needed a yet farther Correction of one Minute, which I have here accounted for.

$17\frac{09}{10}$ *Feb.*	*D.*	*H.*	*M.*	*Sec.*
The mean Time of the mean Oppoſition	2	4	9	42
The mean Time of the true Oppoſition	2	10	54	48
At which the true Place of the Sun is	10	24	55	50
And its Æquation to be added.				

The Place of the Moon from Sir Iſaac Newton's *Theory.*

	S.	*D.*	′	″
Mean Motion of the Moon	4	26	57	37
Annual Æquation Subtr.			8	34
The correct mean Motion	4	26	49	03
Mean Motion of Apog.	11	18	13	54
Annual Æquation of Apog. Ad.			14	31
Correct mean Motion of Apog.	11	18	28	25
Second Æq. from the Diſt. of Ap. from Sun Ad.			2	57
Place of the Moon the 2ᵈ Time Æquat.	4	26	52	00
Mean Motion of Node	11	01	34	25
Æquation of Node Subt.			06	54
Correct mean Motion of Node	11	01	27	31
The 3ᵈ Æquation of the Moon from Node's Aſpect with the Sun Subt.				10
Place of the Moon the 3ᵈ Time Æquated	4	26	51	50
Second Æquation of Apog. Subtr.		7	45	41
True Place of Apog.	11	10	42	44
Mean Anomaly	05	16	09	06
Æquation of Center Sub.		1	53	31
Moon's Place the 4th Time æquated	4	24	58	19
The Variation Ad.				11
Moon's Place the 5th Time æquated	4	24	58	30
The 6th Æquation from the Diſtance of Luminaries and Apog. Ad			1	20
Moon's Place 6th Time æquated	4	24	59	50
The 7th Æquation. Ad.				34
True Place of the Moon in its Orbit	4	25	00	24
True Place of the Sun	10	24	55	50
Moon beyond the Oppoſition			4	34
Which divided by the Horary Motion of Moon from Sun, gives			7	42
	D.	*H.*		
The mean Time therefore of Oppoſit. *Feb.*	2	10	47	06
And the true Time	2	10	32	20

Mean

The Place of the Moon according to Mr. Horrox's *Theory.*

Mean Motion of the Moon	4	26	57	37
Physical Parts Sub.			8	21
Correct mean Motion	4	26	49	16
Mean Motion of Apog.	11	18	13	54
Æquation of Apog. Sub.		7	25	00
Mean Anomaly	5	16	00	22
Æquation of the Center Sub.		1	53	53
Place of Moon in its Orbit	4	24	55	23
Distance from the Opposition				27
That is in Time to be added				45½
	D.	*H.*		
The mean Time therefore of true Opposition is exactly	2	10	55	33
The apparent Time	2	10	40	41
Place of Moon in Ecliptick	4	24	57	27
Reduction between the true Opposition and Middle of Eclipse. Ad.			2	47
	D.	*H.*		
Middle of Eclipse	2	10	43	34
Continuance of Eclipse		2	55	06
Digits eclipsed		9	55	
Beginning of Eclipse	2	9	16	01
End of Eclipse		12	11	07
End of Eclipse by the Moon's Place from Sir *Isaac Newton*'s Theory		12	02	00
End by Observation		12	01	30
End by Calculation from *Horrox*'s Theory		12	11	08

The Error therefore of Sir *Isaac Newton*'s Thereory is by this Observation but half a Minute, or none; of *Horrox*'s System, nine Minutes and a Half.

XXIII. The Evening being clear, gave me a good Opportunity of observing the Lunar Eclipse. The Times are very nice, and the Observations made with an excellent Six-foot Telescope, as followeth.

Eclipse of the Moon, Jan. 12. 1711-12, *at* Upminster, *by Mr.* W. Derham, n. 336. p. 522.

h	′	
6	15	A Duskishness upon the N. East-side of the Moon.
6	36	A thick *Penumbra* on the Moon.
6	37	The *Penumbra* so dense, that it may be taken for the Beginning of the Eclipse.
6	39	The Eclipse undoubtedly is begun.
6	41	The Shadow so dark, that it nearly hid the Moon's N. Easterly Limb.

7 12

h.	′	
7	21	Moon's Diameter by the Micrometer 1612 equal Parts, equal to 31′ 25″.
7	25	The Diſtance of the Shadow from the oppoſite luminous Limb of the Moon, repreſented by the Line *l. u.* was 1025 Parts of the Micrometer, equal to 20 Minutes.
8	31	End of the Eclipſe is very near.
8	32	End of the Eclipſe.
8	32	45″ Eclipſe is undoubtedly ended.
8	36	A *Penumbra* is left.

It unluckily fell out, that I diſordered my Micrometer at the Beginning of the Eclipſe; ſo that I could not take with any Exactneſs the Inclination of the Cuſps, and ſome other Matters I had a Mind to have obſerved; to ſupply which Defect in ſome Meaſure, I have ſent a *Type* of the Eclipſe, as well as I could, by Gueſs. And from the ſame Defect I cannot warrant the Micrometrical Meaſures of the Moon's Diameter, and her eclipſed Parts to be otherwiſe, than ſomewhat near the Truth; perhaps not exactly true.

Fig. 120. A *Type* of the *Lunar Eclipſe*, Jan. 12, 1711-12.

m. i. c. r. repreſents the two Claſpers of the Micrometer, parallel to the Equator.

N. The Northern, *S.* the Southern Part of the Moon's Disk, running between the Claſpers of the Micrometer.

l. u. The enlightened Part of the Moon, being 1025 Micrometrical Parts, or 20′.

I am ſorry I had not *Hevelius*'s Map of the Moon, to have noted the Spots the Shadow paſſed over.

XXIV.
Eclipſe *of the Moon*, Oct. 30, 1715, *at* Wanſted, *by Mr.* J. Pound.

Obſ.	App. Time. h	′	″		′	″
1	15	09	00	The Eclipſe had been for ſome Time begun		
2		17	00	The Moon's Diameter meaſured by a Micrometer was	34	04
3		22	25	The Chord connecting the Horns	30	28
4		35	45	The enlightened Part of the Diameter continued to the Chord between the Horns	19	58
5		43	24	The enlightened Part of the Diameter	13	52
6		49	50	The ſame repeated	12	02
7		52	43	The ſame repeated	11	44
8		56	51	The enlightened Part of the Diameter continued to the Chord between the Horns	15	22
9		59	27	The enlightened Part of the Diameter	10	35
10	16	04	04	The ſame repeated	9	43

	h	′	″		′	″
11		18	34	The ſame again repeated	09	07
12		23	45	The Chord between the Horns	32	35
13		25	30	The ſame repeated	33	07
14	16	31	16	The ſame again	33	19
				At which Time alſo the Shade paſſed thro' the Middle of *Schikardus*		
15		37	15	The Chord between the Horns, agreeing with the ☽'s Diameter	33	57
16		40	45	The inlightned Part of the Diameter	11	56
17		43	40	The ſame produced to the Chord between the Horns	16	13
18		46	55	The ſame repeated	17	28
19		47	57	The inlightned Part of the Diameter	13	38
20		52	57	The ſame	15	30
21		55	27	The Edge of the Shadow paſſed through the Middle of *Gaſſendus*		
22		56	12	The inlightned Part produced to the Chord between the Horns	19	58
23	17	02	45	The Chord between the Horns	32	12
24		8	20	The ſame repeated	30	28
25		10	39		29	56
26		13	00		28	31
27		15	29	The ſame again ——— ———	27	33
28		17	37		26	35
29		19	35		25	36
30		21	47	The ſame again ——— ———	24	38
31		23	24		23	39
32		24	54		22	40
33		26	27	The ſame again ——— ——— ———	21	41
34		27	57		20	42
35		29	08		19	43
36		30	20	The ſame again ——— ——— ———	18	44
37		31	07		17	45
38		32	04		16	46
39		32	50	The ſame again ——— ———	15	47
40		34	12		13	48
41	17	35	20	The ſame again repeated	11	42

At $17^h. 39'$. the Eclipſe was thought to be ended; and was viſibly ſo at $17^h. 41'$: But by comparing the laſt Obſervations of the Chords between the Horns, it follows, that the true End of the Eclipſe was at $17^h. 38'. 20''$. At $17^h. 43'$. the Moon's Diameter was $33' 40''$.

The

The middle cannot be suppoſed to be very accurately determined by theſe Obſervations, which are not ſufficiently diſtant from the Time of the greateſt Obſcuration. However by comparing ſeveral of them together, the Middle will be obtained, *viz.*

	h	′	″
By Obſ. 3. compared with Obſ. 24. at	16	15	21
By Obſ. 4. compared with Obſ. 22. at	16	15	58
By Obſ. 5. compared with 19. and 20. at	16	16	00
By Obſ. 6. and 7. compared with 16. at	16	15	48

By reaſon of Clouds I could not ſee the Beginning of the Eclipſe, nor make ſuch Obſervations of the Moon's immerging into the Shadow, as I did of her emerging out of it.

By Obſervation 11. compared with Obſervation 15. the Digits eclipſed were $8\frac{3}{4}$.

The Angles were meaſured by a *Micrometer* in a 15 Foot Teleſcope: I have not conſidered how far they are conſiſtent with one another; they being ſet down here exactly, as they were firſt taken.

This Eclipſe is the more conſiderable, as happening very near the Moon's *Perigee*, and therefore uſeful to verify her *Anomaly*; as alſo to limit the greateſt Diameter of the Shadow of the Earth, and conſequently the Parallax of the Moon. This may very properly be compared with that of the 19*th* of *October*, 1697, whoſe middle was at 7ʰ. 41′. P. M. at *London*, and Quantity the ſame as now.

The Times by the Clock were 17′. 45″. ſooner than the apparent Time, as were found by the following Obſervations of *Cor Leonis* and *Arcturus*, which through the Clouds were but juſt diſcernible.

Apparent Zenith Diſtance		Time by the Clock			Apparent Time by Calculat.			The Difference		Mean Diff.	
of Cor ♌											
°	′	h	′	″	h	′	″	′	″		
70	16 ½	13	32	43	13	50	35	17	52		
69	38		36	50		54	44	17	54	′	″
69	09		40	06		57	51	17	45	17	50
68	40		43	09	14	00	59	17	50		
68	08		46	37		04	26	17	59		
of *Arctur.*											
65	19	17	37	40	17	55	24	17	44		
65	06		39	12		56	48	17	36		
64	41		41	49		59	29	17	40	17	40
63	47		47	40	18	05	17	17	37		
								Clock too ſlow		17	45

The

The Latitude of *Wansted* is 51°. 34′. Its Longitude is 8″ in Time Eastward from the *Observatory* at *Greenwich*.

N. B. *The Account given of this Eclipse by the Reverend Mr.* William Derham, *who observ'd it at* Upminster, *is agreeable to this, as far as Clouds would permit him to observe.*

XXV. Observations *on the* Heavens *at* Greenwich, 1711, 1712, *by Mr.* J. Flamstead, n. 337. p. 65. — *of* Saturn.

Observations of SATURN. 1711. 1712.

Time by the Clock.	Time corrected.	*Friday, Jan.* 5, 1711.	Distance from the Zenith.
10 14 26	10 14 41	The Heel of *Castor* μ ♊ pass'd over	28 50 10
11 11 38	11 11 53	In the Groin of *Pollux* the Star δ pass'd over ——	28 59 00
12 4 45	12 5 00	*Saturn* pass'd over ——	30 23 20
		Saturn's Right Ascension 119.01.00	
		Distance from the North Pole 68.55.20	
		Sunday, Jan. 7.	
11 2 53	11 0 31	δ in *Gemini* pass'd over —	28 59 20
11 50 36	11 48 14	μ in the Northern Foot of ♋ pass'd over	29 5 20
11 55 22	11 53 00	*Saturn* pass'd over ——	30 21 50
12 15 41	12 13 19	" in *Cancer* (according to *Bayer*) pass'd over ——	30 4 30
12 26 18	12 23 56	γ in *Cancer*, the Northern Ass pass'd over	28 59 50
		Saturn's Right Ascension 118.52.00	
		Distance from the North Pole 68.53.50	
		Monday, Jan. 8.	
10 58 35	10 57 49	δ in *Gemini* pass'd over ——	28 59 15
11 50 46	11 50 00	*Saturn* pass'd over ——	30 20 35
12 11 25	12 10 30	" in *Cancer* pass'd over ——	30 4 25
		Saturn's Right Ascension 118.46.30	
		Distance from the North Pole 68.52.35	
		Thursday, Jan. 25.	
9 45 53	9 46 26	δ in *Gemini* pass'd over ——	28 59 10
10 32 27	10 33 00	*Saturn* pass'd over ——	30 4 00
11 10 54	11 11 27	δ in *Cancer*, the Southern Ass pass'd over	32 17 50
		Saturn's Right Ascension 117.23.00	
		Distance from the North Pole 68.36.00	

Time by the Clock.	Time corrected.	*Saturday, Jan.* 25, 1711.	Distance from the Zenith.
9 37 25	9 38 2	δ in *Gemini* pass'd over ——	28 59 20
10 13 11	10 13 48	*l* under the Side of *Pollux* pass'd over	30 51 50
10 23 23	10 24 00	*Saturn* pass'd over ——	30 2 30
10 40 59	10 41 36	*d* behind the Tail of ♋ pass'd over	32 14 10
10 50 16	10 50 53	*η* in *Cancer* pass'd over ——	30 4 35
		Saturn's Right Ascension 117.14.00	
		Distance from the North Pole 68.34.30	
		Thursday, Jan. 30.	
9 29 45	9 26 55	δ in *Gemini* pass'd over ——	28 59 40
10 14 50	10 12 00	*Saturn* pass'd over —— ——	30 0 00
10 33 20	10 30 30	*d* in *Cancer* pass'd over ——	32 14 10
10 42 35	10 39 45	*η* in *Cancer* pass'd over —— ——	30 4 45
		Saturn's Right Ascension 117.00.30	
		Distance from the North Pole 68.32.00	
		Wednesday, Feb. 28.	
8 18 32	8 15 00	*Saturn* pass'd over ——	29 43 00
8 27 11	8 23 39	*μ* pass'd over in the Northern Foot of the *Crab* —— ——	29 5 40
9 2 50	8 59 18	The *Northern Ass*, *γ* of the *Crab*, pass'd over —— ——	29 0 20
10 39 59	10 36 27	*γ* the bright Star of the *Lion's Neck* pass'd over —— ——	30 12 05
		Saturn's Right Ascension 115.31.30	
		Distance from the North Pole 68.15.00	
		Tuesday, March 1.	
8 14 5	8 11 00	*Saturn* pass'd over —— ——	29 42 50
8 22 47	8 19 42	*μ* in *Cancer* pass'd over —— ——	29 5 35
8 58 28	8 55 23	The *Northern Ass* pass'd over ——	29 0 15
10 35 36	10 32 31	The bright Star of the *Lion's Neck* pass'd over —— ——	30 12 00
		Saturn's Right Ascension 115.30.30	
		Distance from the North Pole 68.14.50	
		Friday, Nov. 9.	
17 21 12	17 21 23	The Star following *π* in the *Crab* pass'd over —— ——	35 21 10
17 29 49	17 30 00	*Saturn* pass'd over —— ——	34 9 50
18 13 25	18 13 36	The South Star *η* in the *Lion's Neck* pass'd over —— ——	33 18 50
		Saturn's Right Ascension 136.58.00	
		Distance from the North Pole 72.42.00	

Time by the Clock.	Time corrected.	*Monday, Nov.* 19, 1711.	Distance from the Zenith.
16 46 20	16 41 00	*Saturn* pass'd over —— —	34 8 00
16 59 31	16 54 11	A Telescopic Star *a* pass'd over —	33 45 30
17 29 59	17 24 33	The South Star *κ* in the *Lion's Neck* pass'd over ——	33 18 30
		Saturn's Right Ascension 136.58.30	
		Distance from the North Pole 72.40.05	
		Thursday, Nov. 22.	
16 24 36	16 19 30	The Star following *π* in *Cancer* pass'd over —— ——	35 21 30
33 06	16 28 00	*Saturn* pass'd over ——	34 6 50
46 27	16 41 21	The aforesaid Telescopic Star pass'd over	33 45 10
17 16 51	17 11 45	The South Star in the *Lion's Neck* pass'd over —— ——	33 18 30
		Saturn's Right Ascension 136.55.45	
		Distance from the North Pole 72.39.00	
		Sunday, Dec. 30.	
11 9 43	10 52 42	*γ* in the Right Foot of *Gemini* pass'd over —— ——	34 51 30
13 29 25	13 12 24	The Star before *o* in *Cancer* pass'd over	35 4 00
13 49 01	13 32 00	*Saturn* pass'd over —— —	33 30 00
14 39 50	14 22 49	The bright Star in the *Lion's Neck* pass'd over —— ——	33 18 40
		Saturn's Right Ascension 134.10.00	
		Distance from the North Pole 72. 2. 5	
		Saturday, Jan. 12, 1712.	
11 39 52	11 33 06	*ζ* at the Tail of *Cancer* pass'd over —	32 59 10
12 40 46	12 34 00	*Saturn* pass'd over ——	33 12 00
12 43 16	12 36 30	The Star following *π* in *Cancer* pass'd over —— —	35 22 20
13 36 44	13 29 58	The *Lion's Heart* pass'd over —	38 6 55
		Saturn's Right Ascension 134.12.00	
		Distance from the North Pole 71.44.00	
		Saturday, Jan. 19.	
11 45 31	11 36 55	The *Southern Ass* pass'd over — —	32 17 10
11 58 14	49 38	The Southern Star at *o* in *Cancer* pass'd over —— —	35 3 50
12 11 36	12 3 00	*Saturn* pass'd over — —	33 2 5
12 20 00	11 24	The Telescopical Star *b* pass'd over —	32 33 20
		Saturn's Right Ascension 133.37.00	
		Distance from the North Pole 71.34.10	

Time by the Clock.	Time corrected.	*Sunday, Jan.* 27, 1712.	Distance from the Zenith.
10 57 50	11 4 31	The Southern Ass pass'd over —	32 17 10
11 10 54	11 17 35	The Northern Star at *o* of *Cancer* pass'd over — —	34 48 10
11 21 19	12 28 00	*Saturn* pass'd over —	32 50 20
11 32 20	39 01	The Telescope Star *b* pass'd over —	32 33 25
		Saturn's Right Ascension 132.58.00	
		Distance from the North Pole 71.22.20	
		Monday, March 31.	
7 8 46	7 7 41	The Southern Ass pass'd over —	32 16 50
7 20 35	7 19 30	*Saturn* pass'd over —	32 03 30
7 36 50	7 35 45	The preceding Star at *π* in *Cancer* pass'd over —	35 19 10
7 39 36	7 38 31	The Subsequent at *π* pass'd over —	35 21 20
8 31 51	8 30 46	The Southern Star in the *Lion's Neck* pass'd over —	33 18 40
		Saturn's Right Ascension 130.02.00	
		Distance from the North Pole 70.35.30	
		Friday, Nov. 7.	
18 17 13	18 22 00	*Saturn* pass'd over —	37 57 30
18 26 13	18 31 00	Star in the *Lion* mark'd 40 in *British Catal.* pass'd over —	41 14 10
18 28 35	18 33 22	Star mark'd 41 in the same, pass'd over	40 15 20
		Saturn's Right Ascension 150.15.30	
		Distance from the North Pole 76.29.40	
		Monday, Nov. 17.	
17 33 19	17 34 59	The *Lion's Heart* pass'd over —	38 7 20
17 42 20	17 44 00	*Saturn* pass'd over —	38 00 50
17 57 48	17 59 28	*ρ* near the *Lion's Breast* pass'd over	40 41 45
18 14 19	18 15 59	*l* in the *Lion's Belly* pass'd over —	39 24 50
		Saturn's Right Ascension 150.31.00	
		Distance from the North Pole 76.33.00	

—*Of* Jupiter.

Observations of JUPITER, 1711.

		Saturday, May 26.	
12 44 28	12 39 44	The Star mark'd 58 in *Serpentarius* of the *British* Catalogue pass'd over	76 12 00
52 40	47 56	The Nebulous Stars mark'd *a* in *Sagittarius* pass'd over —	75 39 20
13 8 44	13 4 00	*Jupiter* pass'd over —	74 37 00
15 43	10 59	Star mark'd 11 of *Sagittarius* pass'd over	72 5 20
		Jupiter's Right Ascension 270.19.00	
		Distance from the North Pole 113.11.50	

Sunday,

Time by the Clock.	Time corrected.	*Sunday, May* 27, 1711.	Distance from the Zenith.
10 47 24	10 43 2	The middle Star of the Forehead of ♏, δ pass'd over ——	73 12 00
12 57 46	53 24	The Telescopial Star *c* preceding *Jupiter* pass'd over ——	75 7 50
13 4 22	13 00 00	*Jupiter* pass'd over —	74 36 50
11 53	7 31	The Star mark'd 11 of *Sagit.* pass'd over	72 5 20
		Jupiter's Right Ascension 270.11.00	
		Distance from the North Pole 113.11.40	
		Sunday, June 3.	
10 15 41	10 10 45	The middle Star in the Front of *Scorpio* pass'd over ——	73 12 5
12 14 11	12 9 15	The Nebulous Stars *b* of *Sagit.* pass'd over	75 10 20
12 17 09	12 12 13	The Nebulous Stars *a* of *Sagit.* pass'd over	75 39 30
12 28 56	12 24 00	*Jupiter* pass'd over ——	74 38 00
13 8 36	13 03 40	The preceding Star in the Eye of *Sagit.* pass'd over —	74 29 15
		Jupiter's Right Ascension 269.15.00	
		Distance from the North Pole 113.12.50	
		Monday, June 4.	
10 11 51	10 7 19	The middle Star in the Front of *Scorpio* pass'd over ——	73 12 00
12 10 18	12 5 46	The Nebulous Stars *b* of *Sagit.* pass'd over	75 10 20
12 24 32	12 20 00	*Jupiter* pass'd over ——	74 38 10
13 4 45	13 0 13	The preced. Star at *v* in *Sagit.* pass'd over	74 29 10
13 5 41	13 1 09	The subsequent Star at *v* pass'd over —	74 24 50
		Jupiter's Right Ascension 269.07.00	
		Distance from the North Pole 113.13.00	
		Saturday, June 9.	
9 52 16	9 49 5	The middle Star in the Front of *Scorpio* pass'd over ——	73 11 55
11 50 44	11 47 33	The Nebulous Stars *b* pass'd over —	75 10 15
12 2 11	11 59 00	*Jupiter* pass'd over —	74 38 25
12 45 8	12 41 57	The preced. Star at *v* in *Sagit.* pass'd over	74 29 10
		Jupiter's Right Ascension 268.25.00	
		Distance from the North Pole 113.13.15	
		Sunday, June 10.	
9 48 22	9 44 38	The middle Star in the Front of *Scorpio* pass'd over — —	73 12 00
11 46 51	11 43 07	The Nebulous Stars *b* pass'd over —	75 10 15
11 57 44	11 54 0	*Jupiter* pass'd over ——	74 38 35
12 41 14	12 37 30	The preced. Star at *v* in *Sagit.* pass'd over	74 29 15
		Jupiter's Right Ascension 268.16.45	
		Distance from the North Pole 113.13.25	

Time by the Clock.	Time corrected.	*Saturday, July* 14, 1711.	Distance from the Zenith.
8 58 10	8 50 4	48 of *Serpentarius*, or C pass'd over —	75 7 20
9 10 32	9 2 26	54 of the same, or D pass'd over —	72 56 00
9 21 6	9 13 00	*Jupiter* pass'd over ———	74 37 50
9 40 49	9 32 43	μ in the Bow of *Sagittary* pass'd over	72 31 30
		Jupiter's Right Ascension 264.12.30	
		Distance from the North Pole 113.12.40	
		Sunday, July 15.	
8 54 8	8 46 24	C of *Serpentarius* pass'd over ———	75 7 20
9 6 28	8 58 44	D of *Serpentarius* pass'd over ———	72 56 00
9 16 44	9 9 00	*Jupiter* pass'd over ———	74 37 50
9 36 46	9 29 2	μ of *Sagittarius* pass'd over ———	72 31 35
		Jupiter's Right Ascension 264.07.45	
		Distance from the North Pole 113.12.40	
		Tuesday, July 3, 1712.	
11 36 3	12 25 14	σ under the Eye of *Capricorn* pass'd over	71 24 40
12 44 6	12 35 17	π in *Capricorn*'s Nose pass'd over —	70 33 00
12 45 42	12 34 53	ρ in *Capricorn* pass'd over ———	70 09 50
12 57 49	12 47 00	*Jupiter* pass'd over ———	71 26 20
13 16 24	13 5 35	20 in *Capricorn* by the *British* Catalogue pass'd over ——	71 33 10
		Jupiter's Right Ascension 306.09.20	
		Distance from the North Pole 110. 0.30	
		Thursday, July 15.	
11 39 34	11 39 33	σ in *Capricorn* pass'd over ———	71 24 45
55 1	11 55 00	*Jupiter* pass'd over ———	71 49 20
12 19 53	12 19 52	20 in *Capricorn* pass'd over ———	71 33 10
24 38	12 24 57	ν in the Body of *Capricorn* pass'd over	72 23 25
		Jupiter's Right Ascension 304.34.35	
		Distance from the North Pole 110.23.30	
		Wednesday, Sept. 17.	
7 32 37	7 31 56	The Telescopic Star preceding *Jupiter* pass'd over — —	72 40 40
7 38 41	7 38 00	*Jupiter* pass'd over ———	72 51 10
7 42 28	7 41 37	The Telescopic Star following *Jupiter* pass'd over —	73 14 40
		Jupiter's Right Ascension 299.43.00	
		Distance from the North Pole 111.25.30	
		Friday, Sept. 19.	
7 30 55	7 32 00	*Jupiter* pass'd over ———	72 50 45
8 19 47	8 20 52	ν in *Capricorn* pass'd over ———	72 23 40
		Jupiter's Right Ascension 299.45.00	
		Distance from the North Pole 111.25.05	

Monday,

Time by the Clock.	Time corrected.	*Monday, Octob.* 6, 1712.	Distance from the Zenith.
6 25 35	6 31 30	*Jupiter* pass'd over ———	72 39 50
6 36 22	6 42 17	*o* in *Capricorn's Nose* pass'd over ——	70 56 25
6 46 36	6 52 31	*υ* in *Capricorn's Neck* pass'd over ——	70 33 30
		Jupiter's Right Ascension 300.39.00	
		Distance from the North Pole 111.14.10	
		Observations of MARS, 1711.	
		Sunday, Jan. 7.	
12 40 35	12 38 56	The Southern Star at *o* in *Cancer* pass'd over —— ——	35 04 00
12 40 56	12 39 17	The Northern Star at *o* in *Cancer* pass'd over ——— ——	34 48 00
12 58 45	12 57 6	The subsequent Star at *π* in *Cancer* pass'd over ——— —	35 21 20
13 27 15	13 25 36	⚴ in the *Lyon* pass'd over ——	36 8 50
13 50 52	13 49 13	The Southern Star at *n* in the *Lion's Neck* pass'd over ———	33 19 00
14 00 39	13 59 00	*Mars* pass'd over ———	34 51 30
14 30 14	14 28 35	*k* the Northern Star in the *Lion's Belly* pass'd over ——— ——	35 45 40
		Mars's Right Ascension 150.20.00	
		Distance from the North Pole 73.23.35	
		Saturday, Jan. 27.	
10 40 59	10 41 30	20 of *Cancer*, or the first at *d* pass'd over	32 14 10
10 50 16	10 50 47	*n* in *Cancer* pass'd over ———	30 4 35
12 8 29	12 9 00	*Mars* pass'd over ———	32 14 00
12 25 27	12 25 58	*n* the Southern Star in the *Lion's Neck* pass'd over —— ——	33 19 00
12 37 59	12 38 30	*γ* the bright Star in the *Lion's Neck* pass'd over ——— —	30 11 50
		Mars's Right Ascension 143.37.00	
		Distance from the North Pole 70.36.00	
		Monday, Jan. 29.	
12 1 50	11 59 00	*Mars* pass'd over ——— ——	31 58 50
12 21 58	12 19 08	*n* in the *Lion* pass'd over ———	33 19 05
12 34 31	12 31 41	*γ* the bright Star in the *Lion's Neck* pass'd over ——— ——	30 11 50
		Mars's Right Ascension 142.49.30	
		Distance from the North Pole 70.30.50	

Time by the Clock.	Time corrected.	*Tueſday, Jan.* 30, 1711.	Diſtance from the Zenith.
10 33 30	10 30 25	20 of *Cancer*, or firſt at *d* paſs'd over	32 14 10
10 42 35	10 39 30	η of *Cancer* paſs'd over	30 4 45
11 56 5	11 53 00	*Mars* paſs'd over	31 51 35
12 17 49	12 14 44	η in the *Lion* paſs'd over	33 19 5
12 30 20	12 27 15	γ in the *Lion* paſs'd over	30 11 50
		Mars's Right Aſcenſion 142.25.00	
		Diſtance from the North Pole 70.23.35	
		Wedneſday, Feb. 28.	
8 27 11	8 23 58	μ in the Northern Foot of *Cancer* paſs'd over	29 5 40
9 2 50	8 59 37	γ the *Northern Aſs* paſs'd over	29 00 20
9 31 13	9 28 00	*Mars* paſs'd over	30 5 10
10 27 28	10 24 15	η in the *Lion* paſs'd over	33 19 55
10 39 39	10 36 26	γ in the *Lion* paſs'd over	30 12 5
		Right Aſcenſion of *Mars* 133.45.00	
		Diſtance from the North Pole 68.37.10	
		Thurſday, March 1.	
8 22 47	8 19 27	μ in *Cancer* paſs'd over	29 5 35
8 58 28	8 55 8	The *Northern Aſs* paſs'd over	29 00 15
9 26 20	9 23 00	*Mars* paſs'd over	30 6 20
10 23 04	0 19 44	η in the *Lion* paſs'd over	33 19 50
10 35 36	10 32 16	The bright Star in the *Lion's Neck* paſs'd over	30 12 00
		Right Aſcenſion of *Mars* 133.37.30	
		Diſtance from the North Pole 68.38.20	
		Monday, Nov. 17, 1712.	
18 21 7	18 19 52	55 in the *Lion* (*Britiſh* Catalogue) paſs'd over	43 45 40
18 25 53	18 24 38	*c* under the *Lion's Belly* paſs'd over	43 49 40
18 30 14	18 28 59	χ in the *Lion* paſs'd over	42 35 10
18 43 15	18 42 00	*Mars* paſs'd over	43 7 25
		Right Aſcenſion of *Mars* 165.48.00	
		Diſtance from the North Pole 81.39.45	
		Thurſday, Nov. 20.	
18 13 8	18 6 40	55 in the *Lion* paſs'd over	43 45 40
18 17 53	18 11 25	58 or *c* in the *Lion* paſs'd over	43 49 50
18 22 14	18 15 46	χ in the *Lion* paſs'd over	42 35 15
18 38 18	18 31 50	σ in the *Lion*'s *Ham* paſs'd over	43 52 10
18 40 28	18 34 00	*Mars* paſs'd over	43 36 55
		Mars's Right Aſcenſion 167. 6.30	
		Diſtance from the North Pole 82. 9.15	

Observations of the SUN, 1711.	Distance from the Zenith.
Saturday, Jan. 6.	
The Sun's Center passing through the Plain of the Meridian, his remote and southern Limb was distant from the Zenith ——— ———	72 31 10
Friday, Jan. 26.	
The nearest Limb of the Sun in the Meridian from the Zenith ——— ——— ———	66 35 10
Tuesday, Jan. 30.	
The nearest Limb of the Sun from the Zenith ———	65 38 40
Monday, June 4.	
The remote Limb of the Sun from the Zenith ———	28 24 18
Saturday, July 14.	
The remote Limb of the Sun ——— ———	31 55 20
Tuesday, Nov. 20.	
The remote Limb of the Sun ——— ———	73 30 40
Tuesday, Dec. 4.	
The remote Limb of the Sun from the Zenith ———	74 59 30
Wednesday, Jan. 2, 1712.	
The Sun's remote Limb from the Zenith ———	73 17 30
Saturday, Jan. 12.	
The Sun's remote Limb ——— ———	71 18 00
Friday, March 7.	
The Sun's remote Limb ——— ———	52 31 00
Friday, May 9.	
The Sun's remote Limb ——— ——— ———	31 40 50
Tuesday, October 7.	
The Sun's remote Limb from the Zenith ———	61 29 50

—*of the* Sun.

Observations of the MOON, 1711.

—*of the* Moon.

Time by the Clock.	Time corrected.	*Saturday, May* 19, 1711.	Distance from the Zenith.
10 22 12	10 18 32	γ in *Scorpio* or 15 of *Libra* pass'd over	75 32 50
10 50 55	10 47 15	The *Moon's* Limb pass'd over, the Center being from the Zenith —	75 38 40
10 52 10	10 48 30	The *Moon's* Center pass'd over, the nearest Limb being distant ——	75 22 00
11 8 30	11 4 50	*b* in *Scorpio* pass'd over ———	76 15 20
11 11 10	11 7 30	A in *Scorpio* pass'd over ———	75 50 55
		The *Moon's* Right Ascension 229.18.30	
		Distance from the North Pole 114.13.40	

Time by the Clock.	Time corrected.	*Monday, Nov.* 19, 1711.	Distance from the Zenith.
16 41 28	16 36 8	The Center of the Moon pass'd over, the remote Limb being ———	37 6 30
17 42 16	16 36 56	The Limb of the Moon pass'd over, the Center being distant ———	36 52 30
16 46 20	16 41 00	*Saturn* pass'd over —— ———	34 8 00
16 59 31	16 54 11	A Telescopic Star *a* pass'd over ———	33 45 30
17 29 53	17 24 33	η in the South Neck of the *Lion* pass'd over	33 18 30
		Moon's Right Ascension 135.41.40	
		Distance from the North Pole 75.23.20	
		Saturday, Jan. 12, 1712.	
7 40 50	7 34 00	The Middle of the Lunar Eclipse, at what Time the Chord of the eclipsed Part of the Moon was 24. 30: but the greatest Defect on the Northern Side was 8. 30. The Moon's Diameter 30.48	
11 39 52	11 33 6	ζ in *Cancer* (accord. to *Bayer*) pass'd over	32 59 10
12 13 46	12 7 00	The Center of the Moon pass'd over, the remote Limb being ———	34 14 10
40 46	12 34 00	*Saturn* pass'd over —— ——	33 12 00
43 16	12 36 30	The Subsequent to π in *Cancer* pass'd over	35 22 20
13 36 44	13 29 58	The *Lion*'s *Heart* pass'd over ——	38 6 55
		Moon's Right Ascension 127.25.30	
		Distance from the North Pole 71.31.00	
		Thursday, March 6.	
7 39 8	7 38 8	80 in *Gemini* (*Brit.* Catal.) pass'd over	30 29 30
7 49 42	7 48 42	86 or *l* in *Gemini* pass'd over ———	30 51 45
7 54 45	7 53 45	The Limb of the Moon pass'd over, the Center being distant ———	30 50 40
7 55 48	7 54 48	The Moon's Center pass'd over, the nearest Limb being ——	30 35 30
8 1 41	8 0 41	μ or 9 of *Cancer* pass'd over —	29 4 55
		Moon's Right Ascension 116.14.40	
		Distance from the North Pole 69.22.50	
		Wednesday, May 7.	
9 47 38	9 46 2	70 of *Virgo* (*Brit.* Cat.) pass'd over———	66 9 50
9 48 48	9 47 12	71 of *Virgo* pass'd over —— ———	65 45 30
9 52 51	9 51 15	75 of the same pass'd over ——	68 7 40
10 9 36	10 8 00	The Moon's Limb pass'd over, the Center being ——— ———	69 12 30
10 10 43	10 9 7	The Center of the Moon pass'd over, the remote Limb being ———	69 28 35
		Moon's Right Ascension 208. 4.30	
		Distance from the North Pole 107.45.50	

Time by the Clock.	Time corrected.	*Thursday, May* 8, 1712.	Distance from the Zenith.
10 52 2	10 50 00	8 in *Libra* (*Brit.* Cat.) pass'd over —	74 51 50
11 1 35	10 59 33	γ in *Scorpio*, or 15 in *Libra*, pass'd over	75 32 30
11 8 2	11 6 00	The Moon's Limb pass'd over, the Center being — —	73 42 30
11 09 16	11 7 14	The Moon's Center pass'd over, the remote Limb being distant —	73 58 20
11 50 33	11 48 31	A in *Scorpio* pass'd over —	75 51 10
11 57 34	11 55 32	δ or the middle Star in the Front of *Scorpio* pass'd over —	73 12 10
		Moon's Right Ascension 223.45.00	
		Distance from the North Pole 112.16.10	
		Saturday, May 10.	
12 18 05	12 16 30	*Antares* or the *Scorpion*'s *Heart* pass'd over	77 10 00
13 3 48	13 2 13	A in *Serpentarius* pass'd over —	77 32 50
13 18 13	13 16 38	The Moon's Center pass'd over, the Limb being distant —	78 1 30
13 19 30	13 17 55	The Limb of the Moon pass'd over, the Center being distant —	77 46 05
13 35 25	13 33 50	*p* the first in *Sagittary* pass'd over —	79 4 20
14 16 8	14 14 33	λ in *Sagittary* pass'd over —	76 57 00
		The Moon's Right Ascension 258.03.20	
		Distance from the North Pole 116.20.15	

N. B. *In the Copy of the Observations,* Ann. 1712 *sent by Mr.* Flamsteed *to the Royal Society, the Name of the Observer seem'd to be erased out of the Title; and by his own Order these Words were subscribed at the Bottom of the Page.*
– *Observed and transcribed by* Jos. Crosthaite.

Observations of SATURN, 1713.

XXVI. Observations *on the* Heavens, 1713. *at* Greenwich, n. 344. p. 285. — *of* Saturn.

Time by the Clock.	Time corrected	*Sunday, Jan.* 25.	Distance from the Zenith.
h. ′ ″	h. ′ ″		° ′ ″
8 30 15	8 28 5	The Star before *Gemini* pass'd over —	28 13 20
8 41 7	38 57	*Castor's Foot* or *n* pass'd over —	28 54 50
8 49 8	8 46 58	His Heel or μ pass'd over —	28 50 40
12 10 49	12 8 39	♃ in the *Lion* (accord. to *Bayer*) pass'd over	36 9 30
12 25 24	23 14	ν in the *Lion* pass'd over —	37 40 30
12 34 21	32 11	The Center of *Saturn* pass'd over —	36 51 45
12 35 44	12 33 34	The *Lion*'s *Heart* pass'd over —	38 7 00
		Saturn's Right Ascension 147.55.10	
		Distance from the North Pole 75.23.55	
		Longitude of ☊ — 25. 8.15	
		Latitude North — 1.31.27	

Time by the Clock.	Time corrected.	*Thursday, Feb.* 5, 1713.	Distance from the Zenith.
h. ′ ″	h. ′ ″		° ′ ″
11 20 52	11 14 10	ξ in the *Lion* (*Bayer*) pass'd over ——	38 55 20
30 9	23 27	ο in the *Lion* pass'd over ——	40 17 30
35 18	28 36	16 of the *Lion* (Cat. *Brit.*) pass'd over	38 21 20
47 8	40 26	ν in the *Lion* pass'd over ——	37 40 30
52 42	46 00	The Center of *Saturn* pass'd over ——	36 32 50
11 57 25	50 43	The *Lion's Heart* pass'd over ——	38 7 5
12 0 35	53 53	31 in the *Lion* pass'd over ——	36 42 50
5 40	11 58 58	34 in the *Lion* pass'd over ——	36 19 30
12 10 50	12 4 8	38 in the *Lion* pass'd over ——	35 3 50
		Saturn's Right Ascension 147. 4.45	
		Distance from the North Pole 75. 5.00	
		Longitude of ♌ 24.14. 8	
		Latitude North 1.32.16	
		Friday, Feb. 6.	
8 22 53	8 14 43	γ in *Gemini*, or the bright Foot of *Pollux* pass'd over ——	34 52 00
11 29 22	10 21 12	ψ in the *Lion* pass'd over ——	36 9 15
11 43 58	11 35 48	ν in the *Lion* pass'd over ——	37 40 30
11 49 10	11 41 0	The Center of *Saturn* pass'd over ——	36 31 00
		Saturn's Right Ascension 146.59. 0	
		Distance from the Pole 75. 3.10	
		Longitude of ♌ 24. 8.17	
		Latitude North 1.32. 8	
		Wednesday, Feb. 18.	
11 0 15	10 53 0	The Center of *Saturn* pass'd over ——	36 12 00
11 8 53	11 1 38	The *Lion's Heart* pass'd over ——	38 7 5
11 17 7	11 9 52	The 34 in the *Lion* (*Brit.*Cat.) pass'd over	36 19 35
11 22 16	11 15 1	38 of the same pass'd over ——	35 3 55
		Saturn's Right Ascension 146. 5.00	
		Distance from the Pole 74.44.10	
		Longitude of ♌ 23.12.47	
		Latitude North 1.32 41	
		Monday, March 2.	
10 6 30	9 52 3	ψ in the *Lion* pass'd over ——	36 9 30
19 27	10 5 00	Center of *Saturn* pass'd over ——	35 55 20
31 21	16 54	*Lion's Heart* pass'd over ——	38 7 5
34 31	20 4	31 of the *Lion* pass'd over ——	36 42 50
10 39 35	10 25 8	34 of the *Lion* pass'd over ——	36 19 40
		Saturn's Right Ascension 145.16.00	
		Distance from the Pole 74.27.30	
		Longitude of ♌ 22.22.40	
		Latitude North 1.23.52	

Tuesday

Time by the Clock.	Time corrected.	*Tuesday, April* 7, 1713.	Distance from the Zenith.
h. ′ ″	h. ′ ″		° ′ ″
7 43 1	7 41 15	ψ of the *Lion* paſs'd over ———	36 9 25
50 46	7 49 00	The Center of *Saturn* paſs'd over —	35 31 10
57 36	55 50	ν of the *Lion* (*Bayer*) paſs'd over —	37 40 20
8 6 39	8 4 53	ι in the *Lion's Neck* paſs'd over ——	33 19 25
16 7	14 21	34 of the ſame (*Brit*. Cat.) paſs'd over	36 19 30
8 21 17	8 19 31	38 of the ſame paſs'd over ———	35 3 50
		Saturn's Right Aſcenſion 143.57.45	
		Diſtance from the Pole 74. 3.15	
		Longitude of ♌ 21. 3.32	
		Latitude 1.21.20	
		Wedneſday, April 8.	
7 47 43	7 46 00	The Center of *Saturn* paſs'd over —	35 31 15
54 34	52 51	ν in the *Lion* paſs'd over ———	37 40 20
8 3 37	8 1 54	ι in the *Lion* paſs'd over ——	33 19 20
13 4	11 21	34 of the *Livn* paſs'd over ———	36 19 35
8 18 14	8 16 31	38 of the *Lion* paſs'd over ———	35 3 45
		Saturn's Right Aſcenſion 143.57.30	
		Diſtance from the Pole 74. 3.20	
		Longitude of ♃'s ♌ 21. 3.20	
		Latitude 1.31.10	
		Saturn was almoſt Stationary.	
		Thurſday, Nov. 5.	
18 22 00	18 15 37	π in the ſubſequent Knee of the *Lion* paſs'd over ———	42 4 00
30 8	23 45	The *Lion's Heart* paſs'd over ——	38 7 5
54 35	48 12	ρ in the *Lion's Shoulder* paſs'd over —	40 42 0
19 11 5	19 4 42	*l* in the *Lion's Belly* paſs'd over ——	39 25 15
19 26 23	19 20 00	The Center of *Saturn* paſs'd over —	42 10 40
		Saturn's Right Aſcenſion 162.23.20	
		Diſtance from the Pole 80.43.00	
		Lontitude, *Virgo* 10.13.40	
		North Latitude 1.39.37	

Obſer-

—*Of* Jupiter.

Observations of JUPITER, 1713.

Time by the Clock.	Time corrected.			Distance from the Zenith.
		Sunday, Aug. 9.		
h. ′ ″	h. ′ ″			° ′ ″
12 40 4	12 37 27	λ in the Running Water of *Aquarius* pass'd over	——	60 32 50
12 48 37	12 46 0	The Center of ♃ pass'd over	——	60 48 35
12 52 36	12 49 59	73 of *Aq.* (*Brit.* Cat.) 1 at *b* pass'd over		60 46 20
13 4 21	13 1 44	χ in the Water of *Aquarius* pass'd over		60 49 10
		Jupiter's Right Ascension	341.33. 5	
		Distance from the North Pole	99.21.40	
		Longitude ♓	9.26.00	
		Southern Latitude	1.25.08	
		Monday, Aug. 10.		
12 36 21	12 33 55	λ in *Aquarius* pass'd over	——	60 32 50
12 44 26	12 42 00	The Center of *Jupiter* pass'd over	——	00 52 00
12 48 53	12 46 27	73 of *Aquary* pass'd over	——	
		Jupiter's Right Ascension	341.26. 5	
		Distance from the Pole	99.25. 5	
		Jupiter's Longitude ♓	9.18.17	
		Southern Latitude	1.25.40	
		Monday, Oct. 26.		
7 29 16	7 28 42	σ in the Buttock of *Aquary* pass'd over		63 34 40
7 36 34	7 36 0	*Jupiter*'s Center pass'd over	——	63 00 5
8 14 34	8 14 0	80 of *Aquary*, first at ↓, pass'd over	—	62 5 20
8 17 45	8 17 11	84 of *Aquary* subsequent at ↓, pass'd over		62 37 5
		Jupiter's Right Ascension	335.41.30	
		Distance from the Pole	101.33.20	
		♃ Longitude ♓	3.16.00	
		South Latitude	1.19. 8	
		Tuesday, Oct. 27.		
7 25 40	7 23 34	σ in the Buttock of *Aquary* pass'd over		63 34 35
7 33 6	7 31 00	*Jupiter*'s Center pass'd over	——	62 59 15
8 11 00	8 8 54	First at ↓ of *Aquary* pass'd over	——	62 5 15
8 14 10	8 12 4	Subsequent at ↓ pass'd over	——	62 37 10
		Jupiter's Right Ascension	335.43.20	
		Distance from the Pole	101.32.30	
		Jupiter's Longitude ♓	3.17.58	
		Southern Latitude	1.19.00	

Thursday,

Time by the Clock.	Time corrected.	*Thurſday, Oct.* 29, 1713.		Diſtance from the Zenith.
h. ′ ″	h. ′ ″			° ′ ″
7 18 29	7 15 19	σ in *Aquarius* paſs'd over ——		63 34 40
26 10	7 23 00	*Jupiter*'s Center paſs'd over ——		62 57 20
8 3 47	8 0 37	The firſt at ↓ paſs'd over ——		62 5 20
6 55	8 3 45	The Subſequent at ↓ paſs'd over ——		62 37 10
		Jupiter's Right Aſcenſion	335.47.45	
		Diſtance from the Pole	101.30.35	
		Jupiter's Longitude ♓	3.22.41	
		Southern Latitude	1.18.49	

Obſervations of MARS, 1713.

— *of* Mars.

		Wedneſday, Feb. 18.		
12 28 38	12 21 20	τ in the *Lion*'s *Ham* paſs'd over ——		47 2 15
12 51 6	12 43 48	β in the Bending of ♍ Wing paſs'd over		48 5 40
13 10 21	13 3 3	10 of *Virgo* (*Brit.* Cat.) *r* paſs'd over		47 57 40
13 13 18	13 6 00	The Center of *Mars* paſs'd over ——		47 2 5
13 21 8	13 13 50	*c* in the *Virgin*'s *Neck* paſs'd over——		46 33 30
		Mars's Right Aſcenſion	179.29.20	
		Diſtance from the Pole	85.34.35	
		Longitude, *Virgo*	27.46.00	
		Northern Latitude	3.51.10	

		Tueſday, March 3.		
11 55 52	11 53 2	ν in the *Virgin*'s *Neck* paſs'd over——		43 20 00
12 ~~4~~ 50	12 2 00	The Center of *Mars* paſs'd over ——		45 14 20
10 54	8 4	π in the *Virgin*'s *Face* paſs'd over ——		43 15 25
20 8	17 18	11 of the *Virgin s* paſs'd over ——		44 3 30
12 30 25	27 35	16 of *Virgo c* in the Neck paſs'd over		46 33 25
12 32 37	12 29 47	17 of *Virgo* (*Brit.* Cat.) paſs'd over —		44 33 45
		Right Aſcenſion of *Mars*	175. 1.15	
		Diſtance from the North Pole	83.46.45	
		Longitude, *Virgo*	22.57.33	
		Northern Latitude	3.43.37	

Tueſday,

Time by the Clock.	Time corrected.	*Tuesday, April* 7, 1713.	Distance from the Zenith.
h. ′ ″	h. ′ ″		° ′ ″
9 4 47	9 3 10	χ under the *Lion*'s *Belly* pass'd over	42 35 25
9 17 37	9 16 00	The Center of *Mars* pass'd over ——	42 42 50
9 38 10	9 36 33	The first of *Virgo* (*Brit.* Cat.) ω pass'd over —— ——	41 44 50
9 45 00	9 43 23	The Northern Star in the Head of *Virgo* ξ pass'd over ——	41 37 15
		The Right Ascension of *Mars* 165.45.40	
		Distance from the Pole 81.15.10	
		Longitude of *Mars*, ♍ 13.30.40	
		Northern Latitude 2.26.31	
		Wednesday, April 8.	
9 1 44	9 0 29	χ of the *Lion* passes over ——	42 35 30
9 14 15	9 13 00	The Center of *Mars* passes over——	42 43 40
9 35 7	9 33 52	ω of the *Virgin* passes over ——	41 44 55
9 41 58	9 40 43	ξ of the *Virgin* passes over ——	41 37 15
		The Right Ascension of *Mars* 165.41.00	
		Distance from the Pole 81.16.00	
		The Longitude of *Mars*,*Virgo* 13.26.45	
		Northern Latitude 2.23.58	
		Friday, May 1.	
7 55 9	7 50 00	The Center of *Mars* passes over——	44 17 30
8 18 12	8 13 3	ν in the *Virgin*'s *Head* passes over ——	43 20 00
8 33 14	8 28 5	π in the *Virgin*'s *Face* passes over ——	43 15 30
		The Right Ascension of *Mars* 166.59.40	
		Distance from the Pole 82.49.50	
		Longitude, *Virgo* 15.15.00	
		Northern Latitude 1.27.40	
		Saturday, May 2.	
7 52 45	7 47 00	The Center of *Mars* passes over ——	44 24 20
8 15 7	8 9 22	ν in *Virgo*'s *Head* passes over — ——	43 20 5
8 30 8	8 24 23	π in *Virgo*'s *Face* passes over —— ——	43 15 25
		Mars's Right Ascension 167.10.00	
		Distance from the Pole 82.56.40	
		Longitude, *Virgo* 15.27. 5	
		North Latitude 1.25.20	

—*of the* Moon.

Observations of the MOON, 1713.

Time by the Clock.	Time corrected.	*Sunday, Jan.* 25, 1713.	Distance from the Zenith.
h. ′ ″	h. ′ ″		° ′ ″
8 9 33	8 7 23	The Telescopic Star *a* passes over ——	28 26 20
8 15 5	8 12 55	123 of *Taurus* (in *Brit.* Cat.) passes over	27 2 30
8 20 20	8 18 10	The preceding Limb of the Moon passes over, the Center being distant from the Zenith —— ——	27 32 40
8 21 23	8 19 13	The Center of the Moon passes over, the Limb being distant from the Zenith	27 47 40
8 22 52	8 20 42	The Northern Cusp of the Moon from the Zenith —— —	27 17 40
8 30 15	8 28 5	The Star before *Gemini* passes over —	28 13 20
8 41 7	8 38 57	η the Foot of *Castor* passes over ——	28 45 50
8 49 8	8 46 58	μ the Heel of the same passes over ——	28 50 40
		The Right Ascension of the Moon's Center 84.26.55	
		The visible Dist. from the Pole 66. 4.40	
		But taking in the Parallax 65.39.50	
		The Longitude of the Moon, ♊ 24.56.30	
		North Latitude 0.57.00	
		Monday, Jan. 26.	
8 26 41	8 24 36	The Star before *Gemini* passes over —	28 13 30
8 37 31	8 35 26	η the Foot of *Castor* passes over ——	28 54 50
8 45 31	8 43 26	μ the Heel of *Castor* passes over ——	28 50 40
9. 9 43	9 7 38	The preceding Limb of the Moon passes over, the Center being from the Vertex —— —	28 45 00
9 10 50	9 8 45	The Center of the Moon passes over, the remote Limb being from the Vertex	29 0 5
9 12 30	9 16 25	The Northern Cusp of the Moon is from the Vertex —— —	28 30 10
9 27 50	9 25 45	The 46 of *Gemini* (*Brit.*Cat.) passes over	28 26 10
9 42 44	9 40 39	δ in the Groin of *Gemini* passes over --	28 59 30
		The Right Ascension of the Moon's Center 97.43.50	
		Dist. from the Pole observed 67.17. 5	
		Taking in the Parallax 66.51.15	
		Longitude of the Moon, ♋ 7. 6.18	
		South Latitude 0. 8.48	

— *of* Jupiter's *Satellites.*

Observations of JUPITER's Satellites.

		Friday, Oct. 30.
6 56 30	6 52 35	The fourth Satellite is seen emerging out of the Shadow, being distant from the Third near it on the Right Hand, by a Diameter of *Jupiter*; with a Tube of eight Foot.
7 4 00	7 00 0	It shined bright, and drawing a Line from that nearest one through the Center of *Jupiter*, it left the emerging one to the South; but in an inverted Situation.
7 36 31	7 32 30	The Star in μ in *Pegasus* pass'd over the Arch of the Meridian.
		Saturday, Nov. 7.
7 13 2	7 5 00	The second Satellite emerged, or rather began to emerge; in a Tube of eight Feet.
9 5 11	8 57 00	δ of the Constellation *Pisces* in the Southern String pass'd over.

N. B. *That Telescopic Star* a, *which* Jan. 25. *went before the Moon, then had its Right Ascension* 81°. 28′$\frac{1}{2}$, *and was distant from the Pole* 66°. 58′. 20″; *whence its Longitude was* ♊ 22°. 9′$\frac{2}{3}$, *with Southern Latitude* 9°. 13′$\frac{1}{2}$. *Now this is the very Star to which* Jupiter *applied in his second Station*, Feb. 6, 1634, *and left it to the South not more than three Diameters of his own Body, according to the Observation of* Gassendus; *as is deliver'd in his Book*, p. 174. *And* Mars *was observed near the same*, Sept. 6, 1644, *in the Morning, as may be seen in the Prolegomena of* Hevelius *before his Selenographia*, p. 65. Fig. 1. *Now it will much conduce to the accurate Determination of* Jupiter's *Node, and his Motion, if the Plain of his Orbit among the fixt Stars does not remain immoveable. For after the Term of* 83 *Years, in which* Jupiter *compleats seven Periods enough exact, that is*, An. 1717, Jan. 10, *in the Morning, the Planet will either cover that Star with his Body, or at least will pass very near it; a Sight very rare, nor yet granted, as I know on, to any Astronomer in regard to* Jupiter.

Vid. Infr. S. XXXI.

Now the Star itself, tho' called a Telescopic *Star, in a clear Sky and in the Absence of the Moon, may be seen by the naked Eye, and has a Companion following it to the South, and distant about the Sun's Diameter; near which* Jupiter *will be seen closely joined*, July 20, *in the Morning, the preceding Year* 1716.

Observations *of the* Occultations *of the* fix'd Stars *by the* Moon, *useful for finding the* Longitude, *by* —— n. 354. p. 693.

XXVII. Of all the Methods hitherto proposed for finding the *Longitudes* of Places for Geographical Uses, none seems more adapted to the Purpose, than that by the *Occultations* of the fixed Stars by the *Moon*

Moon observed in distant Places: For those Immersions of the Stars, which happen on the dark Semicircle of the Moon, and their Emersions from the same, are perfectly momentaneous, without that Ambiguity, to which the Observations of the Eclipses of the *Moon*, and those of *Jupiter*'s *Satellites* are subject. Besides, whilst the Moon is horned, and her weaker Light less dazzling, an ordinary short Telescope, such as is found to be manageable on Ship-board, suffices to observe those Moments, even in the Occultations of very minute Stars: On which Account, this Way seems to bid fairest for finding the Longitude at Sea. But since it would be needless to enquire exactly what Longitude a Ship is in, when that of the Port to which she is bound is still unknown; it were to be wish'd, that the Princes of the Earth would cause such Observations to be made in the Ports, and on the principal Head-Lands of their Dominions, as might once for all settle truly the Limits of the Land and Sea. This Work however, being likely to be left to the Care of private Persons, it may not be amiss to give Notice of the present Opportunity of performing it in this our Northern Hemisphere, by Means of the frequent Appulses of the Moon, to the more Southerly of the *Hyades*, many of which she eclipses in each monthly Revolution, and will continue so to do, during the Years 1718, 1719 and 1720.

These Stars are but three or four in all former Catalogues, but the *British* of Mr. *Flamsteed* encreases them to Sixteen; to them we have added three others somewhat smaller, *viz. c*, *i*, and *n* in the Figure of the *Hyades* hereto annexed. Fig. 121. In it the principal Stars are mark'd with *Bayer*'s Marks, and the rest with the Letters of the *Italick* Alphabet; their Longitudes are fitted to the Beginning of the Year 1718, and being truly laid down, may serve to instruct the Observer, when and where to look for them, when the Moon is among them.

It appears by this Scheme, that the Distance between *a* and α or *Palilicium*, is about nine Hours Motion of the Moon, in which Time, supposing her to pass one to the other, she must eclipse γ and ε, and four or five of those about θ, and must apply very close, with her Southern Limb, to all those, which have about six Degrees South Latitude; which would be a very entertaining Sight. But if the Times of the Occultations of any one of these Stars, or even of any two of them in the same Night, be accurately observed under distant Meridians, the Difference of those Meridians may be truly obtained thereby; especially since the *Moon's Parallax*, and all other Parts of her Theory thereto required, are at present sufficiently stated and known.

For the Sake of such, as are willing to make use of this Method, we have added the Places of all the *Hyades* fitted to the present Time, and chiefly taken from the *British* Catalogue, which being faulty in the Stars we call *k* and *l*, we have here rectified them.

A Catalogue of the Hyades, *for the Beginning of the Year* 1718.

Names of the Stars.		Long. ♊			South Lat.			Mag.
Which goes before γ in the Bull ——	a	0	51	3	5	50	14	7
In the Bull's Nostrils, —— *Bayer*	γ	1	50	54	5	46	22	3
That under γ —— ——	b	1	56	31	6	19	57	7
In the End of the Bull's Nose ——	c	2	54	25	4	47	5	7
Between the Nostrils and the North Eye of the Bull ——	δ	2	54	47	4	0	34	$\frac{3}{4}$
Joyning this to the South ——	d	3	10	33	4	9	4	6
The Northern one of those before θ ——	e	3	17	21	5	41	50	8
The bright Southern one of the same —	f	3	25	32	6	2	44	6
That which follows, δ —— ——	g	3	35	2	3	43	27	5
The Northern of the near ones between the Nostrils and the Bull's Eye ——	θ	3	59	45	5	47	16	4
The more Southern one of the same —	θ	4	0	11	5	52	55	4
The Northern of the two above θ ——	h	4	2	32	5	23	43	7
The Southern one of the same ——	i	4	7	44	5	36	40	8
The first of the 3 in a right Line under θ	k	4	19	27	6	9	45	$\frac{4}{7}$
The Middlemost of the same ——	l	4	26	55	6	7	35	$\frac{4}{7}$
The Northern one of the two following θ	m	4	30	26	5	37	49	7
The Northern Eye of the Bull ——	ε	4	30	31	2	35	58	3
The Southern of those following θ ——	n	4	32	35	5	41	00	8
The Subsequent of the 3 under θ ——	o	4	45	55	6	0	35	7
Palilicium, or the Bull's Eye, or α according to *Bayer*. ——	α	5	50	20	5	29	50	1
The next following this ——	p	6	17	35	6	3	20	7
The Southern of the contiguous ones following —— ——	q	6	30	34	6	19	19	6
The Northern one the brighter of the two	σ	6	33	12	6	12	35	7

An Account of the Cause *of* Venus *being* seen *in the* Day-time, *by* *Dr*.E. Halley. n. 349. p.466.

XXVIII. The late Appearance of *Venus* in the *Day-time*, for many Days together, was generally taken Notice of about *London*, and elsewhere; and by some reckoned to be prodigious. This put me upon the Enquiry, how it came to pass, that at that Time the *Planet* should be so plainly seen by Day; whereas she rarely shews herself so, unless to those, who know exactly where to look for her. To resolve this, the following Problem arose, *viz.* To find the Situation of the *Planet* in respect of the *Earth*, when the *Area* of the illuminated Part of her Disk is a *Maximum*.

To

To investigate this *Maximum*, I found it requisite to assume the following *Lemmata*. I. That the visible *Areas* of the Disk of the same *Planet*, at differing Distances, are always reciprocally as the Squares of those Distances; which is evident from the first Principles of Opticks. II. That the *Area* of the whole Disk of the *Planet* is to the *Area* of the enlightned Part thereof, as the Diameter of a Circle to the Versed-Sine of the exteriour Angle at the *Planet*, in the Triangle, at whose Angles are the *Sun*, *Earth*, and *Planet*. III. That in all plain Triangles, four Times the Rectangle of the Sides containing any Angle, is to be the Excess of the Square of the Sum of the Sides above the Square of the Base, as the Diameter is to the Versed-Sine of the Complement of the contained Angle to a Semicircle, which I call the exteriour Angle: This is a new *Theorem* of good Use in *Trigonometry*, and is easily to be proved from the 12*th* and 13*th* of the II. *Elem. Euclid.*

This premised, putting m for the Distance of the *Sun*, and *Earth*, and n for that of the *Sun* and *Venus*, and x for the Distance of the *Earth* and *Venus*, or the third Side of the Triangle which we seek; by the third *Lemma*, $4nx$, will be to the Excess of the Square of $n+x$ above the Square of m, as the *Area* of the whole Disk of *Venus* to the *Area* of the Part enlightned; and by the *first Lemma*, the *Area's* of her whole Disk, are at all times as the Squares of x reciprocally; whence the Quantity $\frac{nn+2nx+xx-mm}{4nx^3}$ will in all Cases be proportional to the Area of the enlightned Part.

Now that this should be a *Maximum*, it is required that the Fluxion thereof be equal to o, or that the negative Parts thereof be equal to the Affirmative; that is, that $\overline{2n\dot{x}+2x\dot{x}} \times 4nx^3 = 12nx^2\dot{x} \times \overline{nn+2nx+xx-mm}$; and dividing all by $4nx^2\dot{x}$, the Equation becomes $2nx+2xx=3nn+6nx+3xx-3mm$. Consequently $3nn+4nx+xx=3mm$, and therefore $x=\sqrt{3mm+nn}-2n$.

From hence a ready and not inelegant Geometrical Construction becomes obvious; for with the Center S, and Radius $ST=m$, describe the Semicircle TDA; and with the same Center and Radius $SE=n$, Fig. 121.
the Semicircle EVB; which two Semicircles shall represent the Orbs of the *Earth*, and *Venus*. Then make the Chord $AD=$ to the Radius ST, and from D towards A, lay off $DF=SE$; draw TF, and thereon place $FG=BE=2n$, and with the Center T and Radius TG describe the Arch GV, cutting the Semicircle BVE in V; and

and draw the Lines *SV*, *TV*; I say, the Triangle *STV* is similar to that, at whose Angles are the *Sun*, *Earth*, and *Venus*; at the Time when the *Area* of the inlighted Part of that Planet's Disk, as seen from the *Earth*, is greatest. How this Geometrical Effection follows from the Equation is too evident to need repetition.

In consequence then of this Solution, I find this *Maximum* always to happen, when the *Planet* is about 40 Degrees distant from the *Sun*; and the Times thereof, about the Middle between her greatest Elongations on both Sides from him, and her retrograde Conjunctions with him; when little more than a Quarter of her visible Disk is luminous, and resembling the Moon of about 5 Days old; and tho' her Diameter is at that Time but 50 Seconds, yet she shines with so strong a Beam, as to surpass the united Light of all the fixt Stars that appear with her, and casts a very strong Shade on the Horizontal Plain, whereon they all shine; an irrefragable Argument to prove, that the Disks of the fixt Stars are inconceivably small, and next to nothing; since shining with a *native* Light, so many of them do not equal the *reflex* Light of one quarter of a Disk of less than a Minute Diameter.

In this Situation *Venus* was found in *July* last, on the tenth Day; about which Time, when the *Sun* grew low, she was very plainly seen in the Day Time, for several Days together; as she might have been in the Mornings, about the latter End of *September*. But this, arising from the Causes we have now shewn, is nothing uncommon; for every eighth Year it returns again, so that the *Planet* may be seen on the same Day of the Month and Hour, very nearly in the same Place.

Lastly, It may not be amiss to note, that the Equation $x = \sqrt{3mm+nn} - 2n$ has a Limit; for if n be equal to $\frac{1}{4}m$, the Point V will fall on B; and the whole Disk of a Planet at that Distance from the *Sun* would be the *Maximum*, *viz.* when in its superior Conjunction with the Sun. And the like if n were less than $\frac{1}{4}m$; the Arch GV in such Case not intersecting the Semicircle BE.

XXIX. *The* Occultation *of a* Star *by the* Moon; *and an* Eclipse *of the* Moon *following it.* Nov. 21, O.S. 1713. *by Mr.* F. Blanchin, n. 340. p. 88.

Hours after Noon. h. ′ ″	
12 53 34	A Star in *Taurus*, by *Bayer*, mark'd τ, nearly apply'd it self to the Limb of the Moon; observ'd by a Telescope of twelve Palms.
12 54 34	It was now hid by that Part of the Moon's Limb, which is almost in the Middle between the Spots of *Aristarchus* and *Galileus*. The diurnal Parallel describ'd by the Moon's Center appears more to the South than the Star τ by $7\frac{1}{2}$ Parts of the Micrometer, such as the Diameter of the Moon subtends 37. Therefore the Declination of the Star τ is more to the North than the apparent Declination of the Moon's Center, by about $5\frac{1}{2}$ Minutes of a great Circle.
14 0 14	*Sirius* came to the Meridian, by which the Times were verified.

The

h.	′	″	
14	32	57	The Star τ, which had emerged some Minutes out of the Moon's Limb, in its diurnal Revolution precedes the Western Limb of the Moon by 33 Seconds of Time, and it precedes the Center of the Moon by 103 Seconds, or 1′. 43″.
14	42	50	The same precedes the Limb of the Moon by 48″, and the Center by 1′. 58″.
14	50	37	The Difference of Right Ascension of the Star and of the Limb is 1′. 03″. But of the Center of the Moon and the same Star is 2′. 13″.
15	0	0	The Penumbra in the Limb of the Moon, which before was pretty dilute, is now become sensibly denser.
15	2	20	The Penumbra is more apparent, but the true Shadow does not yet appear.
15	4	20	The Beginning of the Incidence of the Moon into the true Shadow, on that Part of the Limb which is next to the Spot *Schickard.*
15	5	21	The true Shadow now covers one Part, such as the Diameter of the Moon in the Micrometer makes 37.
15	7	20	Now two such Parts of the Moon's Diameter are cover'd as make 37.
15	16	20	Now $\frac{5}{37}$ Parts of the Moon's Diameter are cover'd.
15	31	20	Now $\frac{9}{37}$ of the Moon's Diameter are hid.
16	12	0	Now $\frac{12}{37}$ Parts are conceal'd.
16	17	20	The latent Parts are 15 as before.
16	50	20	Now the latent Parts are $\frac{17}{37}$.
16	54	35	The first Limb of *Tycho* begins to emerge.
16	56	9	Now all *Tycho* emerges.
17	13	30	Five Parts of 37 of the Moon's Diameter lye hid.
17	27	45	The true Shadow comes out of the Limb of the Moon, in a Place mark'd out by drawing a Diameter between *Aristarchus* and *Plato.*

N. B. *This Observation is so much the more to be valued, because the Occultation of the Star τ happens so near to the Sun's opposite Point, that his Place among the fixt Stars may from hence be truly examin'd.*

The Occultation of Jupiter *by the Moon,* July 14. *in the Morning,* 1715, *at* Wansted, *by Mr.* J. Pound, n. 347. p. 401.

XXX. Having after Midnight carefully corrected the Clock by no less than ten Observations of the Altitude of the bright Star in *Aries*, the Error thereof was found 5′. 13″ too fast, the Extremes not differing above 6″: And in the Morning about 7^h, by as many Altitudes of the *Sun*, with a like Agreement, the same Error was found 5′. 14″ to be deducted from the Times shewn by the Clock.

July

July 13. P. M. N.	Time by the Clock.			Time corrected.		
	h.	′	″	h.	′	″
The third Satellite of *Jupiter* was hid by the Moon	13	27	33	13	22	20
The first Satellite was hid	13	32	35	13	27	22
The second Satellite was hid	13	34	25	13	29	11
The first Contact of the Limbs of ♃ and ☾	13	34	54	13	29	41
Jupiter wholly hid	13	36	23	13	31	10
The third Satellite came out from behind the dark Side of the Moon	14	7	25	14	2	12
The first Satellite	14	12	25	14	7	12
The second Satellite	14	14	38	14	9	25
The first Limb of *Jupiter* came out	14	14	45	14	9	32
The following Limb of ♃ or last Contact	14	16	15	14	11	2
The fourth Satellite emerged	14	18	49	14	13	36

Jupiter and the Satellites were to the Northward of the visible way of the Moon's Center.

This Occultation was observed through a Telescope, in which the focal Length of the Object-Glass was 14 $\frac{1}{2}$ Feet, and of the Eye-Glass 2 $\frac{1}{4}$ Inches. And the Aperture of the Object-Glass was 1 $\frac{1}{10}$ Inch.

I could perceive no Colours on *Jupiter*'s Limb, either at his Immersion or Emersion, when the Axis of the Tube was directed to him.

The Occultation *of a fixt* Star *in* Gemini *by* Jupiter, *Jan.* 11, O. S. 1717. &c. *by* —— n.351. p.546. * *Vid.* Supr. p. 298.

XXXI. Two Years * ago we gave Notice that *Jupiter* would cover a certain fixt Star with his Body, mentioning the tenth Day of *January* of this Year. But as *Jupiter* was almost Stationary, and something farther advanced towards the East than by our Tables, the foresaid Occultation did not happen till the eleventh Day; which because of Clouds we could not observe at *London* as we wish'd.

Our Astronomers did not watch for it in vain. *Martin Folkes*, Esq; at *London*, with some others of the Royal Society, on *Jan.* 11, at Eight at Night, saw the Center of *Jupiter* to follow the fixt Star at the Distance of one Diameter of his Body, which Star was more to the North of the said Center by about $\frac{3}{4}$ of *Jupiter*'s Semidiameter. Afterwards Clouds intercepted *Jupiter*. Now taking an Account of *Jupiter*'s Motion he concluded, that the Star was in Conjunction with *Jupiter* after Midnight, and was cover'd by the Northern Part of his Disk.

The Reverend *J. Theoph. Desaguliers*, and Mr. *Stephen Grey*, at *Westminster*, saw the fixt Star at Six in the Evening, when it was distant from *Jupiter*'s Limb his whole Diameter, towards the North West. Whence, and from the Observations of the following Days, it appears that the Conjunction happen'd about Midnight.

Also

Also the Rev. Dr. *J. Pound* at *Wanstead* made the following very accurate Observations, which were taken by a very long Tube with a Micrometer.

Jan. 5. at $5^h. 6'$, equal Time, the Center of *Jupiter* was distant from the said fixt Star $31'. 49''$: which at $5^h. 38'$ it follow'd with $34'. 12''$ of Right Ascension. And at the same Time the Southern Limb of *Jupiter* had the same Declination with the Star.

On the ninth Day following at $6^h. 6'$ the Center of *Jupiter* was distant from the Star $10'. 49''$; and after 8 Minutes the Difference of Right Ascensions was $11'. 32''$. And then the Center of the Planet was so little more Southerly than the Star, that the Difference could hardly be perceived.

On the eleventh Day at $5^h. 30'$, equal Time, the Distance of their Centers was $1'. 24''$, and at the same Time the Star was seen to be about a Quarter of *Jupiter*'s Diameter more to the North than his Center. Now the least Diameter of *Jupiter* is found to be $0'. 43''$. Then Clouds came on.

But upon the twelfth Day at $5^h. 17'$. the Distance of the Centers was $3'. 7''$. And at $5^h. 50'$. *Jupiter* preceded the Star by $3'. 30''$ of Right Ascension. And at the same Time the Northern Limb of *Jupiter* had the same Declination as the fixt Star exactly.

Now by comparing these Observations it appears, that this fixt Star was in Conjunction with *Jupiter* on *Jan.* 11, at 13^h nearly, and was not more Northerly than his Center than $17''$ or $18''$, and therefore underwent an Occultation by him.

This fixt Star, tho' yet enter'd in no Catalogue, had its Place then in ♊ $22°. 13'$, with South Latitude $0°. 13'\frac{1}{2}$, and has a Companion more Northerly by 7 Minutes, which precedes him by 17 Minutes, or is in ♊ $21°. 56'$, with South Latitude $0° 6'\frac{1}{2}$, to which *Jupiter* was seen to be joined *Jan.* 16, at $6^h. 30'$, in the Evening.

Thus in a Space of less than two Months *Jupiter* has corporally eclipsed two fixt Stars, of which Thing we have not one Instance since the Invention of the Telescope. Wherefore these Observations are to be laid up for the Use of Posterity, among the most precious Curiosities of *Urania*.

Now our Star in the Year 1634, *Feb.* 6, was in Conjunction with *Jupiter* then stationary, and was more to the South than him by three of his Diameters; as *Gassendus* observed. Whence it will appear, by a due Calculation, that the Nodes of *Jupiter* as to Sense have continued immoveable for the 83 Years last past; and that at two Signs, $8°. 35'$, from the first Star of *Aries*.

The same Astronomers have been watchful of another Observation of a Transit of *Mars* near the Northern Star of the Forehead of *Scorpio*, which was not less remarkable. For *Mars* on *Feb.* 5, in the Morning, or the 4^{th}. 16^h. was seen so near the same Star, that it could not be perceived with the naked Eye; but by the Telescope it was found above towards

A Transit of Mars below the Northern Star in the Forehead of Scorpio, Feb. 5, 1717.

towards the East, and therefore *Mars* was not yet join'd with it. At $16^h. 10'$, apparent Time, *Mars* was in a right Line with the Northern Star in the Forehead, and the Telescopic Star which follows it to the North, at the Distance of about 8 Minutes. At $16^h. 35'$ *Mars* was intermediate in the right Line with the Northern Star, and that in the Middle of the Forehead; and after a Quarter of an Hour, with that in the Southern Part of the Forehead; so that at $16^h. 54'$, apparent Time, it was estimated to be the very Conjunction as to Longitude, at what Time *Mars* with sufficient Exactness was only two Minutes more to the South than the Star. Also Dr. *Pound* observed the Conjunction in respect of Right Ascension to be $17^h. 25'$, apparent Time, with the Distance of the Centers $2'. 7''$. It was a pleasant Sight to see *Mars* gradually approaching the Star, and manifestly to discover his Motion, tho' a very slow one.

With this let the Observation of our *Horrox* be compared, *An.* 1638, *Feb.* 7, in the Morning, as may be seen in his Letters, p. 304. For then *Mars* had an Appulse to the same Star, and came much nearer it, but the Conjunction was over before his rising.

To these add the Observation of *Saturn*, Jan. 25. $12^h. 25'$, equal Time, made by Dr. *Pound*. Then the Planet was distant from the Star, which is the 58 of *Virgo* in the *British* Catalogue, $13'. 16''$ towards the South, and follow'd it, with $2'. 30''$. Right Ascension. The Star was in ♎ $19^\circ. 21'. 52''$, with North Latitude $2^\circ. 47'. 25''$.

Emersions *of the first* Satellite *of* Jupiter, *at* Rome, *&c.* 1713, *by Mr.* F. Blanchin, n. 340. p. 89.

XXXII. *Sept.* $\frac{11}{22}$, Afternoon $8^h. 38'. 20''$, the inmost Satellite of *Jupiter* began to emerge over-against the Space which is extended between the two Belts of *Jupiter*. The Observation was made with a Telescope of Mr. *Andrew Chiarelli*, of the Length of 40 *Roman* Palms. Afterwards at $8^h. 44'$ the third Satellite appeared to be so united with the fourth, that both seem'd to be but one Satellite. They were distant from the Center of *Jupiter* about $5\frac{1}{2}$ of his Diameters. But at $9^h. 4'$ they were again disjoined. The Fourth in an inverse Situation appear'd something more depress'd than the Third, and something more distant from *Jupiter*. Therefore it was more Northerly than the Third.

Sept. $\frac{18}{29}$, at $10^h. 36'. 23''$, the first or the inmost Satellite began to emerge from the Shadow, in a Tube of 25 Palms, of Mr. *Campani*.

Nov. $\frac{12}{23}$, at $7^h. 32'. 22''$, the first Satellite began to emerge, when seen thro' a Tube of Mr. *Chiarelli* of 40 Palms. Afterwards the same Night at $7^h. 46'$ the First and Second are very near, and at $7^h. 53'$ the same were so near, that they could scarce be distinguished from one little Point.

Dec. 9, *n. st.* or *Nov.* 28, old st. at $5^h. 45'. 45''$, the first Satellite began to emerge from the Shadow of *Jupiter*.

Dec. 21, old st. at $5^h. 50'. 22''$, the first Satellite was again seen as beginning to emerge out of the Shadow.

N. B. *From these Observations reduced to an accurate Calculation, it is very plain, that the second Equation, which we suppose to arise from the progressive Motion of Light, must necessarily be admitted. For after 57 Revolutions*

lutions of the inmost Satellite, in which Jupiter *has withdrawn from the Earth more than by a* Radius *of the* Orbis Magnus, *the last Eclipse is seen almost 9 Minutes later than it ought, according to the Tenor of the first Observation: Which agrees with the Hypotheses of Mr.* Cassini

From the same it is also confirm'd, (which we also have observed before) that the Motion of the inmost Satellite of Jupiter *is a little quicker than in the most elaborate Tables of Mr.* Cassini, *communicated to the Publick now* 20 *Years ago. Now that little Error seems hardly to exceed two Minutes of Time in each Revolution of* Jupiter, *or in* 12 *Years; by which the Heavens anticipate the Calculation of Mr.* Cassini. *But when this Correction is taken in, the Agreement will be sufficiently accurate.*

Transit *of the Body and Shade of* Jupiter's *fourth* Satellite *over the* Disk *of the* Planet, *communicated by Mr.* J. Pound, n.359. p. 900.

XXXIII. On the 16th of *February* 1719, at $6^h\frac{1}{4}$, thro' a short Tube, we saw all the 4 Satellites, the 3 outermost on the *East* Side of *Jupiter*, and the innermost near the *Western* Limb approaching to an Eclipse. The Fourth at that Time was about half a Semidiameter of *Jupiter* from the Eastern Limb. Then it proved cloudy till about 8^h, at which Time (thro' the *Hugenian* Telescope) we could see only the second and third Satellites, the first being behind *Jupiter* in the Shadow, and the fourth entred upon the Disk. We saw at this Time a dark Spot, a little *Northward* of the great *Northern* Zone, and near the *Eastern* Limb, where the Satellite was to enter on the Disk; which Spot we took for the Shade of the Satellite. The Clouds then again intercepted our View, till $8^h. 53'$. *Æq. T.* at which Time the first Satellite was lately emerged out of the Shadow, and the Spot advanced so far, that we perceived it would arrive at the Middle of *Jupiter*, near two Hours sooner than the Shade ought to have done by our Computation; but not imagining, that this dark Spot could be any Thing else but the Shade, we concluded there had been some Error in the Calculation, which we thought to re-examine afterwards. On this Presumption we left off observing till $9^h. 35'$. at which Time we were surprized to see a Notch in the Limb of *Jupiter*, near the Place where the former Spot entred. This last Appearance agreeing well with the Time, that the Shade of the Satellite ought to have entred the Disk, soon made us alter our former Opinion, and conjecture, that this, and not the other Spot, was the said Shade. At $9^h. 39'\frac{1}{2}$ *Æq. T.* the Notch vanishing, a round black Spot appeared within the Limb, but in Contact with it. At $9^h. 45'$. we judged the first Spot, and at $11^h. 45'$. the second, to be in the Middle of *Jupiter*.

At $11^h. 50'$. the first Spot touched the Limb, being within the Disk; soon after which the Limb in that Place seemed a little protuberant. At $12^h. 5'$. appeared the fourth Satellite just come out of the Disk, and touching the Limb in the Place where the Protuberancy was. At $12^h. 7'$. we could perceive the Satellite separated from the Limb. At $13^h. 56'$. the second black Spot, still within the Disk, just touched the *Western* Limb; soon after which there appear'd a Notch in this Part of the Limb, as it did on the other at the coming on of this Spot. At $14^h. 6'$. the Spot was

was all gone off, and the Limb appear'd clear and entire. The first Spot, when in the Middle of *Jupiter*, was almost as black as the second when near the Limb, but somewhat less and a little more *Northerly*.

At the Time that the first Spot was in the Middle of the Disk, the three innermost Satellites appear'd to the *East* of *Jupiter*; the first (as aforesaid) having lately emerged out of the Shadow; the second being almost at its greatest Distance; and the third having passed the Axis of the Shade about twelve Hours before, and appearing at this Time about three Diameters of *Jupiter* from his Limb. The Times that these Spots arrived at the Middle of the Disk are agreeable to the Times found by Calculation, in which the fourth Satellite and its Shade ought to have appeared there. From all which 'tis very plain, that the first of these Spots was the fourth Satellite itself, and the second its Shadow.

We have seen the first and second Satellites appearing not as dark Spots, but as bright ones (somewhat different from the Light of *Jupiter*) for some little Time after they entred his Disk, but as they approached nearer the Middle we lost Sight of them. And we have frequently observed, that the same Satellites appear brighter at some Times than at others; and that when one of them hath shined with its utmost Splendour, the Light of another hath been considerably diminished. From whence 'tis very probable at least, not only that the Satellites revolve upon their proper Axes, but also that some Parts of their Surfaces do very faintly (if at all) reflect the Solar Rays to us.

All which hath for some Time since been observed and taken Notice of by Mess. *Cassini* and *Miraldi*, as may be seen in the *Memoirs* of the *Academie Royale* for the Years 1707 and 1714.

Tables *for computing the* Eclipses *of the* first Satellite *of* Jupiter, *by Mr.* J. Pound, n. 361. p. 1021.

XXXIV. It being now 26 Years since Mr. *Cassini*'s Tables were published, Length of Time hath discovered, that the Motion of the first *Satellite* is a small Matter swifter than M. *Cassini* hath supposed it; and Mr. *Pound* has of late applied himself to rectify by frequent Observation what he found amiss in his *Calculus*; and has put it into a Form more easy and compendious, by bringing what M. *Cassini* had given us in odd Numbers, to the Millesimals of a Circle, both as to *Numb.* I. which he calls *Numb.* A. being the mean Anomalie of *Jupiter* in such Parts; as also to *Numb.* II. or our *Numb.* B. which is the Distance of the mean Place of *Jupiter*, from the true Place of the *Sun*, and which, with the Addition of the Equation of *Numb.* B. gives the true Angle of *Commutation* in the same Millesimals of a Circle. And having deducted from the *Epoches* the greatest Equations both of *Numb.* A. and B. he restores them again by adding as much to the Equations themselves, by which Means they all become affirmative, so that the whole Computation is performed by *Addition* only.

* *Vid.* supr. V. I. C. IV. §. XCII.

In N° 214. of the *Phil. Transf.* * there is an Epitome of M. *Cassini*'s Tables, where the Method of his *Calculus* is explained at large, for which Reason this shorter Description may at present suffice.

Epochs

Epochs of the Conjunctions of the first Satellite with Jupiter.

Current Julian Year.	Conjunction. D.	H.	′	″	Num. A.	Num. B.
1719	1	6	11	13	872	396
1720	0	20	22	40	956	310
21	1	5	2	44	40	229
22	0	19	14	11	125	143
23	0	9	25	38	209	57
1724	1	18	5	42	293	971
1725	0	8	17	10	377	889
26	1	16	57	13	462	808
27	1	7	8	41	546	722
28	0	21	20	8	630	636
1729	1	6	0	12	715	554
1730	0	20	11	39	799	468
31	0	10	23	7	883	382
32	0	0	34	34	967	296
33	0	9	14	38	52	215
1734	1	17	54	41	136	133
1735	1	8	6	9	220	47
36	0	22	17	36	305	961
37	1	6	57	40	389	880
38	0	21	9	7	473	794
1739	0	11	20	35	557	708
1740	0	1	32	2	642	622
41	0	10	12	6	726	540
42	0	0	23	33	810	454
43	1	9	3	37	895	373
1744	0	23	15	4	979	287
1745	1	7	55	8	63	205
46	0	22	6	35	148	119
47	0	12	18	3	232	33
48	0	2	29	30	316	947
1749	0	11	9	34	400	866

Current Julian Year.	Conjunction. D.	H.	′	″	Num. A.	Num. B.
1749	0	11	9	34	400	866
1750	0	1	21	1	485	780
51	1	10	1	5	569	698
52	1	0	12	33	653	612
53	1	8	52	37	738	531
1754	0	23	4	4	822	445
1755	0	13	15	32	906	359
56	0	3	27	0	990	273
57	0	12	7	3	75	191
58	0	2	18	30	159	110
1759	1	10	58	34	243	24
1760	1	1	10	1	328	938
61	1	9	50	35	412	856
62	1	0	1	2	496	770
63	0	14	13	0	580	684
1764	0	4	24	27	665	598
1765	0	13	4	13	749	517
66	0	3	15	58	833	431
67	1	11	56	2	918	349
68	1	2	7	29	2	263
1769	1	10	47	33	86	182
1770	1	0	59	0	171	96
71	0	15	10	28	257	10
72	0	5	21	56	339	924
73	0	14	2	0	423	842
1774	0	4	13	27	508	761
1775	1	12	53	31	592	675
76	1	3	4	58	676	589
77	1	11	45	1	761	507
78	1	1	56	28	845	421
1779	0	16	7	56	929	335

Revolutions of the fixt Satellite of Jupiter in Months of

January.				N.	Nu.
D.	h.	′	″	A.	B.
1	18	28	36	0	5
3	12	57	12	1	9
5	7	25	48	1	14
7	1	54	24	2	18
8	20	23	0	2	23
10	14	51	36	2	27
12	9	20	12	3	32
14	3	48	48	3	37
15	22	17	24	4	41
17	16	46	0	4	46
19	11	14	36	4	51
21	5	43	12	5	55
23	0	11	47	5	60
24	18	40	23	6	64
26	13	8	59	6	69
28	7	37	35	7	73
30	2	6	11	7	78
31	20	34	47	7	82
February.					
0	20	34	47	7	82
2	15	3	23	8	87
4	9	31	59	8	92
6	4	0	35	9	96
7	22	29	11	9	101
9	16	57	47	9	105
11	11	26	23	10	110
13	5	54	59	10	114
15	0	23	35	11	118
16	18	52	11	11	123
18	13	20	47	11	128
20	7	49	23	12	132
22	2	17	59	12	137
23	20	46	35	13	141
25	15	15	11	13	146
27	9	43	47	13	150

March.				N.	Nu.
D.	h.	′	″	A.	B.
1	4	12	23	14	155
2	22	40	59	14	159
4	17	9	35	15	164
6	11	38	10	15	168
8	6	6	46	16	173
10	0	35	22	16	177
11	19	3	58	16	182
13	13	32	34	17	186
15	8	1	10	17	190
17	2	29	46	18	195
18	20	58	22	18	199
20	15	36	58	18	204
22	9	55	34	19	208
24	4	24	10	19	213
25	22	52	46	20	217
27	17	21	22	20	221
29	11	49	58	20	225
31	6	18	34	21	230
April.					
0	6	8	34	21	230
2	0	47	10	21	235
3	19	15	46	22	239
5	13	44	22	22	244
7	8	12	58	22	248
9	2	41	34	23	252
10	21	10	10	23	257
12	15	38	46	24	261
14	10	7	22	24	265
16	4	35	58	25	270
17	23	4	33	25	274
19	17	33		25	279
21	12	1	45	26	283
23	6	30	21	26	287
25	0	58	57	27	292
26	19	27	33	27	296

April.				N.	Nu.
D.	h.	′	″	A.	B.
28	13	56	9	27	300
30	8	24	45	28	304
May.					
0	8	24	45	28	304
2	2	53	21	28	309
3	21	21	57	29	313
5	15	15	33	29	317
7	10	90	9	29	322
9	4	47	45	30	326
10	23	16	21	30	330
12	17	44	57	31	335
14	12	13	33	31	339
16	6	42	9	31	343
18	1	10	45	32	348
19	19	39	21	32	352
21	14	7	57	33	356
23	8	36	33	33	361
25	3	5	9	33	365
26	21	33	45	34	369
28	16	2	21	34	373
30	10	30	57	35	378
June.					
0	10	30	57	35	378
1	4	59	32	35	382
2	23	28	8	36	386
4	17	56	44	36	391
6	12	25	20	36	395
8	6	53	56	27	399
10	1	22	32	37	403
11	19	51	8	38	408
13	14	19	44	38	412
15	8	48	20	38	416
17	3	16	56	39	420
18	21	45	32	39	425
20	16	14	8	40	429
22	10	42	44	40	433

June.				N.	Nu.
D.	h.	′	″	A.	B.
24	5	11	20	40	438
25	23	39	56	41	442
27	18	8	32	41	446
29	12	37	8	42	450
July.					
1	7	5	44	42	455
3	1	34	20	42	459
4	20	2	56	43	463
6	14	31	32	43	468
8	9	0	8	44	472
10	3	28	44	44	476
11	21	57	20	45	480
13	16	25	55	45	485
15	10	54	31	45	489
17	5	23	7	46	493
18	23	51	43	46	498
20	18	20	19	47	502
22	12	48	55	47	506
24	7	17	31	47	510
26	1	46	7	48	515
27	20	14	43	48	519
28	14	43	19	49	523
31	9	11	55	49	528
August.					
0	9	11	55	49	528
2	3	40	31	49	532
3	22	9	7	50	536
5	16	37	43	50	541
7	11	6	19	51	545
9	5	34	55	51	549
11	0	3	31	51	554
12	18	32	7	52	558
14	13	0	43	52	562
16	7	29	19	53	567
18	1	57	55	53	571
19	20	26	31	54	575

Revolutions of the fixt Satellite of Jupiter in Months of

August.				N.	Nu.
D.	h.	′	″	A.	B.
21	14	55	7	54	580
23	9	23	43	54	584
25	3	52	18	55	588
26	22	20	54	55	593
28	16	49	30	56	597
30	11	18	6	56	602
September.					
1	5	46	42	56	606
3	0	15	18	57	610
4	18	43	54	57	615
6	13	12	30	58	619
8	7	41	6	58	624
10	2	9	42	58	628
11	20	38	18	59	632
13	15	6	54	59	637
15	9	35	30	60	641
17	4	4	6	60	646
18	22	32	42	60	650
20	17	1	18	61	655
22	11	29	54	61	659
24	5	58	30	62	663
26	0	27	6	62	668
27	18	55	42	62	672
29	13	24	18	63	677
October.					
1	7	52	54	63	681
3	2	21	30	64	686
4	20	50	6	64	590
6	15	18	41	65	595
8	9	47	17	65	699
10	4	15	53	65	704
11	22	44	29	66	708
13	17	13	5	66	713
15	11	41	41	67	717
17	6	10	17	67	721
19	0	38	53	67	726
20	19	7	29	68	730
22	13	36	5	68	735
24	8	4	41	69	739
26	2	33	17	69	744
27	21	1		69	749
29	15	30	29	70	753

October.				N.	Nu.
D.	h.	′	″	A.	B.
31	9	59	5	79	758
November.					
0	9	59	5	70	758
2	4	27	41	71	762
3	22	56	17	71	767
5	17	24	53	71	772
7	11	53	29	72	776
9	6	22	5	72	781
11	0	50	41	73	785
12	19	19	17	73	790
14	13	47	53	74	794
16	8	16	29	74	799
18	2	42	5	74	804
19	21	13	40	75	808
21	15	42	16	75	813
23	10	10	52	76	817
25	4	39	28	76	822
26	23	8	4	76	827
28	17	36	40	77	831
30	12	5	16	77	836
December.					
0	12	5	16	77	836
2	6	33	52	78	840
4	1	2	28	78	845
5	19	31	4	78	849
7	13	59	40	79	854
9	8	28	16	79	859
11	2	56	52	80	863
13	21	25	28	80	868
14	15	54	4	80	873
16	10	22	40	81	877
18	4	51	16	81	882
19	23	19	52	82	886
21	17	48	28	82	891
23	12	17	4	82	897
25	6	45	40	83	900
27	1	14	16	83	905
28	19	42	52	84	909
30	14	11	28	84	914

First Equations of the Conjunctions of the first Satellite with Jupiter.

Num. A.	Æquat. Conjunct. Adde. ′	″	Æq Nu. B.	Num. A.	Æquat. Conjunct. Adde. ′	″	Æq Nu. B.	Num. A.	Æquat. Conjunct. Adde. ′	″	Æq Nu. B.	Num. A.	Æquat. Conjunct. Adde. ′	″	Æq Nu. B.
0	39	8	15	128	12	7	26	256	0	1	31	384	11	52	26
4	38	12	16	132	11	27	26	260	0	0	31	388	12	37	26
8	37	16	16	136	10	47	26	264	0	1	31	392	13	23	25
12	36	21	16	140	10	9	27	268	0	3	31	396	14	11	25
16	35	26	17	144	9	31	27	272	0	7	31	400	14	59	25
20	34	30	17	148	8	45	27	276	0	12	31	404	15	48	24
24	33	35	17	152	8	19	27	280	0	19	31	408	16	38	24
28	32	40	18	156	7	44	28	284	0	28	30	412	17	30	24
32	31	45	18	160	7	10	28	288	0	38	30	416	18	22	23
36	30	50	19	164	6	38	28	292	0	50	30	420	19	15	23
40	29	56	19	168	6	7	28	296	1	3	30	424	20	9	23
44	29	3	19	172	5	37	28	300	1	17	30	428	21	4	22
48	28	10	20	176	5	8	29	304	1	33	30	432	22	59	22
52	27	16	20	180	4	41	29	308	1	50	30	436	22	55	22
56	26	23	20	184	4	15	29	312	2	8	30	440	23	53	21
60	25	30	21	188	3	49	29	316	2	28	30	444	24	51	21
64	24	38	21	192	3	24	29	320	2	51	30	448	25	49	21
68	23	47	21	196	3	1	29	324	3	15	29	452	26	48	20
72	22	56	22	200	2	40	30	328	3	40	29	456	27	48	20
76	22	5	22	204	2	20	30	332	4	6	29	460	28	48	19
80	21	15	22	208	2	1	30	336	4	34	29	464	29	49	19
84	20	26	23	212	1	42	30	340	5	3	29	468	30	50	19
88	19	37	23	216	1	25	30	344	5	34	29	472	31	51	18
92	18	48	23	220	1	10	30	348	6	5	28	476	32	53	18
96	18	0	24	224	0	58	30	352	6	38	28	480	33	55	17
100	17	14	24	228	0	47	30	356	7	13	28	484	34	57	17
104	16	28	24	232	0	36	30	360	7	50	28	488	35	59	17
108	15	42	24	236	0	26	30	364	8	27	27	492	37	1	16
112	14	57	25	240	0	18	30	368	9	6	27	496	38	5	16
116	14	11	25	244	0	12	31	372	9	46	27	500	39	8	15
120	13	30	25	248	0	7	31	376	10	27	27	504	40	11	15
124	12	48	26	252	0	4	31	380	11	9	26	508	41	15	14
128	12	7	26	256	0	1	31	384	11	52	26	512	42	17	14

First Equations of the Conjunctions of the first Satellite with Jupiter.

Num. A.	Æquat. Conjunct. Adde.		Æq Nu. B.	Num. A.	Æquat. Conjunct. Adde.		Æq Nu. B.	Num. A.	Æquat. Conjunct. Adde.		Æq Nu. B.	Num. A.	Æquat. Conjunct. Adde.		Æq Nu. B.
	′	″			′	″			′	″			′	″	
512	42	17	14	640	70	26	3	768	77	40	0	896	61	48	6
516	43	19	14	644	71	3	3	772	77	29	0	900	61	2	7
520	44	21	13	648	71	38	3	776	77	18	0	904	60	15	7
524	45	23	13	652	72	11	2	780	77	6	0	908	59	28	7
528	46	25	13	656	72	42	2	784	76	51	1	912	58	39	8
532	47	26	12	660	73	13	2	788	76	34	1	916	57	50	8
536	48	27	12	664	73	42	2	792	76	15	1	920	57	1	8
540	49	28	11	668	74	10	2	796	75	56	1	924	56	11	9
544	50	28	11	672	74	36	1	800	75	36	1	928	55	20	9
548	51	28	11	676	75	1	1	804	75	15	1	932	54	29	9
552	52	27	10	680	75	25	1	808	74	52	1	936	53	38	10
556	53	25	10	684	75	48	1	812	74	27	1	940	52	46	10
560	54	23	9	688	76	8	1	816	74	1	2	944	51	53	10
564	55	21	9	692	76	26	1	820	73	35	2	948	51	0	11
568	56	17	9	696	76	43	0	824	73	8	2	952	50	6	11
572	57	12	8	700	76	59	0	828	72	39	2	956	49	13	11
576	58	7	8	704	77	13	0	832	72	9	2	960	48	20	12
580	59	1	8	708	77	26	0	836	71	38	3	964	47	26	12
584	59	54	7	712	77	38	0	840	71	6	3	968	46	31	12
588	60	46	7	716	77	48	0	844	70	32	3	972	45	36	13
592	61	38	6	720	77	57	0	848	69	57	3	976	44	41	13
596	62	28	6	724	78	4	0	852	69	21	3	980	43	46	13
600	63	17	6	728	78	9	0	856	68	45	4	984	42	50	14
604	64	5	5	732	78	13	0	860	68	7	4	988	41	55	14
608	64	53	5	736	78	15	0	864	67	29	4	992	41	0	14
612	65	39	5	740	78	16	0	868	66	49	4	996	40	4	15
616	66	24	5	744	78	15	0	872	66	9	5	1000	39	8	15
620	67	7	4	748	78	12	0	876	65	28	5	1004	38	12	16
624	67	49	4	752	78	9	0	880	64	46	5	1008	37	16	16
628	68	30	4	756	78	4	0	884	64	3	5	1012	36	21	16
632	69	10	4	760	77	58	0	888	63	19	6	1016	35	26	17
636	69	49	3	764	77	50	0	892	62	34	6	1020	34	30	17
640	70	26	3	768	77	40	0	896	61	48	6	1024	33	35	17

Second Equations of the Conjunctions of the first Satellite with Jupiter.

Addendæ.

Num. B. Æqu.	0		100		200		300		400		500		600		700		800		900	
	Æqu.		Æqu.		Æq.		Æq.		Æq.		Æq.		Æq.		Æq.		Æqu.		Æqu.	
	′	″	′	″	′	″	′	″	′	″	′	″	′	″	′	″	′	″	′	″
0	14	0	12	52	9	45	5	30	1	37	0	0	1	37	5	30	9	45	12	52
4	14	0	12	46	9	36	5	20	1	30	0	0	1	44	5	40	9	54	12	58
8	13	59	12	40	9	26	5	9	1	23	0	1	1	52	5	51	10	3	13	2
12	13	59	12	35	9	17	4	59	1	16	0	2	1	59	6	1	10	12	13	7
16	13	58	12	29	9	7	4	48	1	9	0	3	2	7	6	11	10	21	13	11
20	13	57	12	23	8	58	4	38	1	3	0	4	2	15	6	22	10	31	13	16
24	13	56	12	17	8	48	4	28	0	57	0	5	2	24	6	33	10	40	13	20
28	13	54	12	11	8	38	4	18	0	52	0	7	2	32	6	44	10	49	13	25
32	13	53	12	4	8	28	4	8	0	46	0	10	2	41	6	55	10	57	13	29
36	13	51	11	56	8	17	3	58	0	40	0	13	2	50	7	5	11	5	13	33
40	13	49	11	49	8	7	3	58	0	35	0	16	2	59	7	16	11	13	13	36
44	13	47	11	42	7	57	3	38	0	31	0	19	3	9	7	26	11	20	13	38
48	13	44	11	34	7	47	3	29	0	27	0	23	3	19	7	36	11	27	13	41
52	13	41	11	27	7	36	3	19	0	23	0	27	3	29	7	47	11	34	13	44
56	13	38	11	20	7	26	3	9	0	19	0	31	3	38	7	57	11	42	13	47
60	13	36	11	13	7	16	2	59	0	16	0	35	3	48	8	7	11	49	13	49
64	13	33	11	5	7	5	2	50	0	13	0	40	3	58	8	17	11	56	13	51
68	13	29	10	57	6	55	2	41	0	10	0	46	4	8	8	28	12	4	13	53
72	13	25	10	49	6	44	2	32	0	7	0	52	4	18	8	38	12	[illegible]	13	54
76	13	20	10	40	6	33	2	24	0	5	0	57	4	28	8	48	12	[illegible]	13	56
80	13	16	10	31	6	22	2	15	0	4	1	3	4	38	8	58	12	[illegible]	13	57
84	13	21	10	21	6	11	2	7	0	3	1	9	4	48	9	7	12	29	13	58
88	13	7	10	12	6	1	1	59	0	2	1	16	4	59	9	17	12	35	13	59
92	13	2	10	3	5	51	1	52	0	1	1	23	5	9	9	26	12	40	13	59
96	12	58	9	54	5	40	1	44	0	0	1	30	5	20	9	36	12	46	14	0
100	12	52	9	45	5	30	1	37	0	0	1	37	5	30	9	45	12	52	14	0

Third Equations to be added.				*Half Durations of the Eclipses of the first Satellite of* Jupiter.																
Nu. A.	*Equations.*		*Nu.* A.	*Nu.* A.	*Half Durations.*			*Nu.* A.	*Half Durations.*			*Nu.* A.	*Half Durations.*			*Nu.* A.	*Half Durations.*			
	′	″			h.	′	″		h.	′	″		h.	′	″		h.	′	″	
0	3	30	1000	0	1	5	9	250	1	7	0	500	1	5	9	750	1	7	46	
20	3	29	980	10	1	4	56	260	1	7	15	510	1	4	53	760	1	7	57	
40	3	28	960	20	1	4	44	270	1	7	31	520	1	4	39	770	1	8	7	
60	3	25	940	30	1	4	33	280	1	7	45	530	1	4	26	780	1	8	15	
80	3	19	920	40	1	4	23	290	1	7	57	540	1	4	15	790	1	8	22	
100	3	12	900	50	1	4	13	300	1	8	7	550	1	4	7	800	1	8	26	
120	3	4	880	60	1	4	7	310	1	8	15	560	1	4	3	810	1	8	28	
140	2	56	860	70	1	4	4	320	1	8	22	570	1	4	1	820	1	8	30	
160	2	46	840	80	1	4	2	330	1	8	27	580	1	4	0	830	1	8	28	
180	2	34	820	90	1	4	0	340	1	8	28	590	1	4	3	840	1	8	26	
200	2	22	800	100	1	4	2	350	1	8	29	600	1	4	7	850	1	8	22	
220	2	10	780	110	1	4	3	360	1	8	27	610	1	4	13	860	1	8	16	
240	1	57	760	120	1	4	6	370	1	8	24	620	1	4	23	870	1	8	8	
260	1	44	740	130	1	4	12	380	1	8	17	630	1	4	35	880	1	8	0	
280	1	30	720	140	1	4	21	390	1	8	9	640	1	4	49	890	1	7	50	
300	1	17	700	150	1	4	31	400	1	7	58	650	1	5	4	900	1	7	37	
320	1	5	680	160	1	4	42	410	1	7	46	660	1	5	19	910	1	7	22	
340	0	53	660	170	1	4	55	420	1	7	31	670	1	5	36	920	1	7	8	
360	0	41	640	180	1	5	9	430	1	7	14	680	1	5	54	930	1	6	55	
380	0	31	620	190	1	5	23	440	1	6	58	690	1	6	10	940	1	6	40	
400	0	22	600	200	1	5	39	450	1	6	40	700	1	6	28	950	1	6	23	
420	0	14	580	210	1	5	55	460	1	6	20	710	1	6	46	960	1	6	8	
440	0	8	560	220	1	6	11	470	1	6	2	720	1	7	2	970	1	5	54	
460	0	4	540	230	1	6	26	480	1	5	45	730	1	7	17	980	1	5	37	
480	0	2	520	240	1	6	43	490	1	5	26	740	1	7	33	990	1	5	22	
500	0	0	500	250	1	7	0	500	1	5	9	750	1	7	46	1000	1	5	9	

The Eclipses of the *first* Satellite of *Jupiter* afford the best Means of determining the Longitude of Places on the Land, where Telescopes of a convenient Length may be used; thirteen of these Eclipses happening every 23 Days; and that the Observer may know near the Matter, when these Opportunities offer themselves, he may readily compute the Times of the Immersions or Emersions of this Satellite with great Exactness, by following very short Precepts, which admit of no Exception or Caution, *viz.* *The Use of the Tables.*

Out of the first Table take the *Epoche* for the Year, with its corresponding *Numb.* A and *Numb.* B; and to them add out of the Tables of Months, the Day, Hour, Minute and Second, nearest less than the Time of the Eclipse you seek for, together with its *Numb.* A and B; the Sum of the Times is the mean Time of the Middle of the Eclipse. 2. With *Numb.* A thus collected take out the first Equation of the Conjunctions; as also the Æquation of *Numb.* B always to be added to *Numb.* B before found. 3. With *Numb.* B so equated, take out the second Æquation of the Conjunctions; and in the last Table, the third Æquation, as also the Semi-duration of the Eclipse answering to *Numb.* A. 4. To the mean Time of the Middle of the Eclipse, add all those three Æquations; the Sum shall be the true equated Time of the Middle of the Eclipse sought. 5. If *Numb.* B equated be less than 500, subtract the Semiduration, and you will have the Time of the Immersion, or if it be more than 500, adding the same, it will give the Time of the Emersion.

But Note, the Times thus found are equal Time, still to be reduced to the Apparent: And that in the *Bissextile* Year, after *February*, one Day is to be deducted from the Day of the Month.

Let it be required to find the Time of the *Immersion* of this Satellite into *Jupiter*'s Shadow, *November* the 9*th* 1719, in the Morning. The Work stands thus,

	D.	h.	′	″	Nu. A.	Nu. B.
1719.	1	6	11	13	872	396
Novem.	7	11	53	29	72	776
Conj. Med.	8	18	4	42	944	172
Æquat. I.			51	53		10 Æq. B.
Æquat. II.			10	26		
Æquat. III.			3	26		182 B. Æquat.
	8	19	10	27		
		1	6	33	*Semidur. Subst.*	
Novem.	8	18	3	54		

So

So that by this *Calculus*, on the *9th* of *November*, at four Minutes after Six in the Morning, equal Time, may be seen the *Immersion* of this Satellite into *Jupiter's* Shadow.

Another Example shall be of the *Emersion* on the *5th* of *April* 1720, *viz.*

	D. h. ′ ″		Nu. A.	Nu. B.
1720,	0 . 20 . 22 . 40		956	310
April	4 . 13 . 44 . 22	Biss.	22	244
Conj. Med.	5 . 10 . 07 . 02		978	554
Æquat. I.	44 . 13			13 Æq. B.
Equat. II.	0 . 45			
Æquat. III.	3 . 29			567 B. Æquat.
	1 . 5 . 40	*Semidur. Add.*		
April	5 . 12 . 01 . 09			

Hence it appears, that at one Minute after Midnight following the 5th of *April*, equal Time, will happen the Emersion required.

It may not be amiss to inform the Reader, that we have learnt, by the Experience of many Years Observation, that the second Inequality of this Satellite proceeds from the progressive Propagation of Light, and is common to all the rest of the Satellites: Light being found to proceed, in about seven Minutes of Time, as far as from the Sun to the Earth, whether with an equable Motion or otherwise is still a Question. For this Reason we have added a *third Equation*, whereby to account for the greater Distance of *Jupiter* from the *Earth* in *Aphelio* than in *Perihelio*, as the *second Æquation* answers to the greater Distance of the Planet, when near the Conjunction of the Sun, than when near his Opposition.

Observations *on some of the primary* Planets; *and the* Occultation *of a* fixt Star, *by* Jupiter; *by Mr.* J. Pound, n. 350. p. 506.

XXXV. The Reverend Dr. *Pound*, second to none in Art or Industry, has offered the following Observations to be communicated to the Publick, made by him at *Wanstead* with great Accuracy, with very long Tubes and a Micrometer.

Anno 1715, *Aug.* 21, 8^h. $25'\frac{1}{2}$ equal Time, *Mars* preceded in Right Ascension the middle Star in the Forehead of *Scorpio* (*Bayer* δ) by 6′. 54″, being more to the North than the Star by 9′. 47″.

Sept. 18, 7^h. 30′, *Mars* preceded the bright Star in the Foot of *Serpentarius*, (*Bayer* θ) by 17′. 48″, and had the same Declination exactly.

November 30, 18^h. 8′, *Saturn* preceded γ, or the Second in the Wing of *Virgo*, by 23′. 19″, and was more South than the fixt Star by 25′. 3″. But *December* 4, 17^h. 25′. it preceded it by 10′. 50″, and was more South by 29′. 00″.

Anno 1716, *Feb.* 22, 7^h. 23′, equal Time, *Mars* preceded ζ in *Pisces*, or the subsequent of the three bright Stars in the Southern String of *Pisces*, by by 3′. 25″, and was more to the South than the ſame by 1′. 23″, which therefore he ought to cover before two Hours, perhaps corporally.

June 22, 8^{h}. 52′, equal Time, *Venus* follow'd the *Lion's Heart* at 34′. 50″, and was more South than the fixt Star by 7′. 23″.

Aug. 14, 15^{h}. 0′, *Jupiter* preceded *Propus* by only one Minute, with Northern Declination leſs than 14′. 26″.

Aug. 19, 13^{h}. 2′, *Jupiter* preceded the Teleſcopic fixt Star which is call'd *b*, by 50′. 08″, having the ſame Declination exactly.

Aug. 24, 12^{h}. 19′, *Jupiter* in the Micrometer was diſtant from the aforeſaid *b*, 5′. 54″, and at the ſame Time from another brighter fixt Star *a*, 7′. 17″. The Diſtance of the fixt Stars was 12′. 31″. Then the leſſer Diameter of *Jupiter* was 0′. 38″.

Sept. 12, 17^{h}. 0′, *Venus* newly come from her ſecond Station follow'd a Teleſcopic Star at 17′. 40″, and was more Southerly than it by 5′. 30″. This fixt Star was then in ♌, 27°. 44′ $\frac{1}{2}$, with South Latitude 5°. 39′.

Octob. 15, 17^{h}. 12′ $\frac{1}{2}$, *Venus* in the Micrometer was diſtant 27′. 55″, from a fixt Star *τ* in the Leg of the *Lion*.

Nov. 20, 6^{h}. 18′ $\frac{1}{2}$, *Jupiter* went back to the Stars *a* and *b*, at which he was obſerv'd *Aug.* 24, and was diſtant from *b* 6′. 21″, from *a* 11′. 36″.

Nov. 21, 7^{h}. 38′, *Jupiter* was diſtant from *b* 9′. 19″, and from *a* 3′. 48″. The Stars were diſtant from one another 12′. 30″. *Jupiter*'s Axis or leſſer Diameter was 0′. 44″. Afterwards at 18^{h}. 50′. the Star *a* ſeem'd to adhere as it were to *Jupiter*'s Limb, and was about $\frac{2}{3}$ of his Semidiameter, or 0′. 15″, more to the North than the Center of *Jupiter*. Now according to theſe Obſervations it appears, that the Middle of the Occultation of the fixt Star, by the Body of *Jupiter* interpoſing, happen'd *Nov.* 21, 19h. 55″, very nearly. Afterwards

Nov. 30.	5^{h}. 41′	♃ preceded *Propus*	12′. 36″	more Southerly	7′. 36″
Dec. 4.	6. 0	♃ follow'd it ——	22. 49	————	7. 47
Dec. 5.	6. 0	Repeated ——	31. 35	————	7. 50
Dec. 6.	6. 0	Again ——	40. 30	————	7. 52
Dec. 7.	6. 0	Again ——	49. 15	————	7. 54

From theſe laſt Obſervations it appears, that *Jupiter* and *Propus* had the ſame Longitude *Dec.* 1, 15^{h}. 29′, at what Time *Jupiter* was more to the South than the Star by 7′. 40″. From the ſame it will alſo appear, that *Jupiter* was in Oppoſition to the Sun, as to Right Aſcenſion, *Dec.* 6, 12^{h}. 46′; but as to Longitude *Dec.* 6, 12^{h}. 34′ $\frac{1}{2}$.

N. B. Thoſe Teleſcopic Stars, call'd *a* and *b*, are had in Mr. *Flamſteed*'s *Britiſh* Catalogue of the fixt Stars, wherein, at the Beginning of the Year 1690, the Place aſſign'd to *a* is ♊ 27°. 54′. 29″, with South Latitude 0°. 21′. 55″. The Place of the other *b* is ♊ 28°. 5′. 24″, with South Latitude 28′. 5″. Neither do we know any other fixt Star hid by the Body of *Jupiter*, and obſerved ſince the Invention of the Teleſcope, except the aforeſaid Star *a*; to which heretofore *Jupiter* apply'd himſelf very cloſely 83 Years ago, *Dec.* 9, ſt. n. in the Year 1633,

in

in the Evening. Then at *Dinia*, *Gassendus* saw *Jupiter* in Conjunction with this fixt Star, and not above five Semidiameters of his Body above it. Whence a Calculation being duly made, it will appear that the Nodes of this Planet, and the Plain of its Orbit, keep their Situation immoveable in the Sphere of fixt Stars, or at least are moved with a very slow Motion. See *Gassendus's* Observations, Tom. IV. p. 162.

Rectification *of the* Motions *of the five* Satellites *of* Saturn; *with* Observations *by Mr.* J. Pound, n. 355. p. 768.

XXXVI. M. *Cassini* above 30 Years communicated to the World his Discovery of the two new *Satellites* of *Saturn*, which made their Number Five. Much about the same Time Mr. *Huygens* made the Society a Present of the Glasses of a *Telescope* of 125 Foot Length, with the *Apparatus* for using them without a Tube; by Help whereof we might have satisfied ourselves of the Reality of these Discoveries. But those here, that first tried to make use of this Glass, finding, for want of Practice, some Difficulcies in the Management thereof, were the Occasion of its being laid aside for some Time.

In the mean Time the *French* Astronomers, giving us in their yearly *Memoirs* no Observations of these *Satellites* till very lately, nor having seem'd willing to shew them in their Glasses to such as requested it, occasioned in some Persons a Suspicion of the Reality of this Discovery: And Mr. *Derham* having borrowed of the Society their long Glass, could not thereby assure himself, that the small Stars he sometimes found about *Saturn*, were really his Satellites, their Situation not agreeing with their Places derived from the Tables of their Motions exhibited in N° 187. * of *Phil. Transact.* besides that he wanted a sufficient Height to raise the Object Glass, so as to view *Saturn* to Advantage, above the Vapour of the Horizon. But in the *Memoirs* for 1714, M. *Cassini*, the worthy Successor of his great Father, has given us some Observations, which clear up the Point, and by shewing the Errors of those first Tables, has enabled us to be assured, that we have seen the whole *Satellitium* of *Saturn* ourselves.

* *Vid.* Supr. V. I. C. IV. S. LXXXII.

The Substance of these Observations is as follows:

Anno 1714, *May* 6, *St. N.* about Mid-night, *Saturn* being then Stationary in ♍ 4°. 27′, the fifth and outermost Satellite was in its superior Conjunction with the Planet, and at the same Time, the Earth was nearly in the Plain of this Satellite's Orbit, so that it appeared to pass very near the Center of *Saturn*: From hence, and from some other preceding Observations, M. *Cassini* concludes, that the Nodes of this Satellite's Orb are in four Degrees of ♍ and ♓, and that its Inclination to the Ecliptick is not much more than half that of the other Satellites. Hence it should follow that the Ellipses it describes by its apparent Motion about *Saturn*, when in ♊ and ♐ are much flatter and nearer to his Body, than those of the other four, which he allows to move in the Plain of the *Ring*, and to have their Nodes in 21 gr. of ♍ and ♓, with an Inclination to the Ecliptick of 31 Degrees. To confirm this Discovery, he produces another Observation of his Father's, ther's, near thirty Years before, *viz.* that *Anno* 1685, *May* 31, *St. N.* about Noon, the ſame Satellite was obſerved in ſuperiour Conjunction with *Saturn*, with leſs than one Diameter of the Ring *North* Latitude, *Saturn* being then in ♍ 11°. 48′. So that the Satellite wanted but 7°. 21′ of completing 134 Revolutions, in the Interval of Time between them. From theſe *Data* it was eaſy to ſettle the Theory of this Satellite.

As to the *Fourth*, or the *Hugenian* Satellite; in the *Memoirs* for 1715, we find a very curious Obſervation of it, and the firſt of its Kind, *viz.* that *Mart.* 25°. *S. N.* about 11ʰ. *P. M.* this fourth Satellite, then in *Apogeo*, did immerge behind the Body of *Saturn*. With this Emendation the Place of this Satellite may for the future be computed with a ſufficient Exactneſs.

The third Satellite, by an original Miſtake in the Letters in N°. 187, * is all wrong; its daily Motion being there printed 2ˢ. 18°. 41′. 50″ inſtead of 2ˢ. 19°. 41′. 50″; as may be perceived by the Period thereof being determined in the aforeſaid *Memoirs* of 1714, to be 4ᵈ. 19ʰ. 25′. 12″. that is, that it makes 400 Revolutions in 1807 Days. This Satellite was obſerved by M. *Caſſini*, *April* 4ᵗᵒ *St. N.* 10ʰ. *P. M.* to have newly paſt its inferior Conjunction with *Saturn*, and a Perpendicular from it fell on the Extremity of the Weſtern *Anſe*, ſo that at about 5ʰ. *P. M.* it was with the Center of the Planet then in ♍. 5°. 23′. and conſequently in ♓ 5°. 23′. But at the Beginning of the *Gregorian* Year 1686, the *Epoche* thereof was ♍ 9°. 39′. So that from the Noon of the laſt of *December* 1685, to *April* 4°. 6ʰ. 18′. *anno* 1714, that is, in 10320 Days 6ʰ. 18′, there have been made 2284½ Revolutions of this Satellite to the Equinoctial; from which *Data*, the Tables of its Motion are readily derivable.

* *Vid.* Supra, V. I. C. IV. S. LXXXII.

The Radix of the Penintime or ſecond Satellite, according to the aforeſaid Letter, at the Beginning of the *Gregorian* Year 1686, was in ♍ 9°. 10′. But by the Obſervations of M. *Caſſini* made the Nights before and after, this Satellite was in its ſuperior Conjunction *anno* 1714, *April* 4ᵈ. 21′½. *St. N.* that is, in ♍ 5°. 21′, where *Saturn* then was: So that *April* 4ᵈ. 22ʰ. 12′, an entire Number of Revolutions were performed ſince the *Epoche* of 1686, that is, in 10320 Days 22ʰ. 12′: Which Number can be no other than 3771, according to the Period thereof given in this *Memoire*, *viz.* 2ᵈ. 17ʰ. 41′. 22″.

Laſtly, The innermoſt or firſt Satellite, at the ſame Time, *viz.* 1714, *April* 4°. 21ʰ. 30′. *St. N.* was in its inferior Conjunction *proxime*, and conſequently in ♓ 5°. 21′. But the *Epoche* thereof for 1686, is ♑. 24°. 50′. which Place the Satellite had paſt 40 gr. 31′ at the Time of the Obſervation. This Arch it moves in 5ʰ. 6′: Wherefore from the Time of the *Epoche* to *April* 4ᵈ. 16ʰ. 24′, 1714, or in 10320 Days 16ʰ. 24′. the Satellite has performed 5467 Revolutions, its Period being determined to be 1 Day, 21 Hours, 18′. 27″, in this *Memoire*.

Having by the Help of these late Observations corrected the Motions of the Satellites, and having fixed their *Epoches* for the present Year, we were enabled to know where to expect them with more Certainty, and to distinguish them one from another, and from the small fixt Stars appearing with them. And Mr. *J. Pound* having, by Means of his Steeple of *Wansted*, provided a *Gnomon* high enough for the Purpose, and having fitted a very commodious *Apparatus* for using the *Society*'s aforesaid long Telescope, soon discovered by it all these five Satellites; and lately communicated to them the following Observations.

1718. *April* 21^{d}. 10^{h}. 40$'$. The third and fourth Satellites of *Saturn* were in *Apogæo*, a little past their Conjunction with *Saturn:* A Perpendicular from the fourth to the transverse *Axis* of the Ring (or Line of the *Ansæ*) fell a little without the Eastern *Ansa*; and a Line through the
Fig. 132. fourth and third touched the Eastern Limb of *Saturn*, *Fig.* 132.

The first was Northward of the Line of the *Ansæ* (and therefore in the *Apogæon* Semicircle also) distant from the said Line about as far as the End of the conjugate Axis of the Ring was from the Center of ♄, *viz.* nearly $\frac{3}{4}$ of *Saturn*'s Semidiameter; and it was about a Semidiameter of the Ring from the Westetn *Ansa*.

The second was a very little Southward of the Line of the *Ansæ* (and therefore in the *Perigæon* Semicircle) above a Semidiameter of the Ring + the Semidiameter of ♄) from the Western *Ansa*. And the third, first and second were in a strait Line.

At 10^{h}. 50$'$. A Perpendicular from the third to the Line of the *Ansæ* fell almost on the Middle of the bright Part of the Eastern *Ansa*, but somewhat nearer the Center than the said Middle.

April 22^{d}. 11^{h}. 5$'$. The four innermost Satellites were all Eastward of ♄. The 2^{d} and 4th in the *Apogæon*, and the 1st and 3^{d} in the *Perigæon* Semicircle. A Line thro' the 2^{d} and 4th touched the *South-East* Limb of ♄. A Line passing through the 3^{d} and the End of the conjugate *Axis* of the Ring, was parallel to the Line of the *Ansæ*.

At 11^{h}. 10$'$. A Perpendicular from the first to the Line of the *Ansæ*,
Fig. 133. fell on the Eastern Extremity of the Ring, *Fig.* 133.

These Distances and Directions were taken only by Estimation, and not by any actual Measurement.

The fifth (or outermost) Satellite being at this Time near its greatest Elongation Eastward, among several very small Telescopick Stars, he could not determine its Position. But by observing the Motion of this some other Nights before, he was now fully satisfied, from the Motions rectified as above, that there are five Satellites of *Saturn*, as M. *Cassini* had long since asserted.

In the bright Part of each *Ansa* was a darkish Ellipse nearer to the Outside than the Inside of the Ring, as if it was composed of two Rings near to one another.

On the Body of ♄, beside the Ring on the *South*-side, there appeared on the *North*-side a Zone not so far from the Center as the Ring, and not

not much unlike the smallest of *Jupiter*'s Belts. Which Appearances were first taken Notice of by M. *Cassini*, in *Phil. Trans.* * N° 128. *Vide Fig.* 134.

* *Vid.* supra. V. I. C. IV. S. LXXVI. 14.
Fig. 134.

It is not to be expected that these Satellites, exceedingly minute in themselves, and so faintly illuminated, should appear when the Air is but ordinarily serene, they requiring not only the *Medium* to be *summo modo* defecate and limpid, but withal in perfect Darkness. For which Reasons it may well be understood why the Gentlemen of the *Parisian* Observatory may have sometimes made a Difficulty to undertake to shew them upon Demand.

Correct Tables *of the* Motions *of* Saturn's *five* Satellites, *by* —— n. 356. p. 776.

Vid. Supr. V. I. C. IV. S. LXXXII.

XXXVII. By Help of the late Observations, and making some Correction in the Motions, we owe the whole System of *Saturn*'s Satellites to Mr. *Huygens*'s Telescope: And taking in the accurate Observations of the Reverend Dr. *James Pound*, we have obtain'd the following Tables, which very well agree with the Heavens. That is, by adding 2°. 9′ to the Motion of the Inmost, and 3°. 25′ to that of the Inmost but one, retaining the *Epochs* of M. *Cassini* to the Year 1686. Also increasing the annual Motion of the Outmost by 9′, and taking away 16° from the *Epoch*, which in the Epistle N° 187, was faultily wrote ♓ 16°. 19′ for ♓ 0°. 16′, we found that of *Huygens* to be yearly 6′ slower. But we have been obliged intirely to reform the Tables of the Third, yet still retaining the *Epoch*, because the daily Motion deliver'd in that Epistle was utterly false.

A Table of the mean Motions of the inmost of Saturn's *Satellites, discover'd by M.* Caſſini, *An.* 1686.

Years of Chriſt beginning.	*Epochs.* s	′	″	*In Years.*	*Mean Motion.* s	°	″	*In Days.*	*Mean Motion.* s	°	″	*H M*	*Mean Motion.* Sex. °	′	″	*Min.*	*Mean Motion* °	′
1681	♒	-8	48	1	4 .	4	43	1	6	10	42	1	0	7	57	31	4	6
1686	♉	13	4	2	8	9	25	2	0	21	24	2	0	15	53	32	4	14
1701	♍	6	34	3	0	14	8	3	7	2	6	3	0	23	50	33	4	22
1714	♎	9	57	4	10	29	33	4	1	12	47	4	0	31	47	34	4	30
1715	♒	14	39	5	3	4	16	5	7	23	29	5	0	39	44	35	4	38
1716	♊	19	22	6	7	8	59	6	2	4	11	6	0	47	40	36	4	46
1717	♉	4	47	7	11	13	42	7	8	14	53	7	0	55	37	37	4	54
1718	♍	9	30	8	9	29	6	8	2	25	35	8	1	3	34	38	5	2
1719	♑	14	13	9	2	3	49	9	9	6	17	9	1	11	31	39	5	10
1720	♉	18	55	10	6	8	32	10	3	16	59	10	1	19	28	40	5	18
Months of the Com. Year.	*Mean Motion.* s	°	″	11	10	13	15	11	9	27	41	11	1	27	24	41	5	26
				12	8	28	40	12	4	8	22	12	1	35	21	42	5	34
				13	1	3	23	13	10	19	4	13	1	43	18	43	5	42
Jan.	0	0	0	14	5	8	5	14	4	29	46	14	1	51	15	44	5	50
Febr.	5	1	38	15	9	12	48	15	11	10	28	15	1	59	11	45	5	58
Mar.	3	1	10	16	7	28	13	16	5	21	10	16	2	7	8	46	6	5
Apr.	8	2	48	17	0	2	56	17	0	1	52	17	2	15	5	47	6	13
May	6	23	44	18	4	7	39	18	6	12	34	18	2	23	1	48	6	21
June	11	25	22	19	8	12	21	19	0	23	16	19	2	30	58	49	6	29
July	10	16	19	20	6	27	46	20	7	3	57	20	2	38	55	50	6	37
Aug.	3	17	57	40	1	25	32	21	1	14	39	21	2	46	52	51	6	45
Sept.	8	19	35	60	8	23	19	22	7	25	21	22	2	54	49	52	6	53
Octob	7	10	31	80	3	21	5	23	2	6	3	23	3	2	45	53	7	1
Nov.	0	12	9	100	10	18	51	24	8	16	45	24	3	10	42	54	7	9
Dec.	11	3	5	120	5	15	37	25	2	27	27	25	3	18	39	55	7	17
In Leap-Year after February add the Motion of one Day.								26	9	8	9	26	3	26	35	56	7	25
								27	3	18	50	27	3	34	32	57	7	33
								28	9	29	32	28	3	42	28	58	7	41
								29	4	10	14	29	3	50	25	59	7	46
								30	10	20	56	30	3	58	22	60	7	57

A Table

A Table of the Mean Motions of the Inmost but one of the Satellites of Saturn, *discover'd by M.* Cassini, *Anno* 1686.

Years of Jul. beginning.	*Epochs.* s	°	′	*In Years.*	*Mean Motion.* s	°	′	*In Days.*	*Mean Motion.* s	°	″	*H M*	*Mean Motion.* Sex. °	′ °	″ ′	*Min.*	*Mean Motion.* ′	″
1681	♓	3	23	1	4	10	2	1	4	11	32	1	0	5	29	31	2	50
1686	♉	5	25	2	8	20	4	2	8	23	4	2	0	10	58	32	2	56
1701	♓	22	3	3	1	0	6	3	1	4	36	3	0	16	26	33	3	1
1714	♑	7	5	4	9	21	40	4	5	16	8	4	0	21	55	34	3	7
1715	♉	17	7	5	2	1	42	5	9	27	40	5	0	27	24	35	3	12
1716	♍	27	9	6	6	11	44	6	2	9	12	6	0	32	53	36	3	17
1717	♊	18	43	7	10	21	46	7	6	20	44	7	0	38	22	37	3	23
1718	♎	28	45	8	7	13	20	8	11	2	16	8	0	43	51	38	3	28
1719	♓	8	47	9	11	23	22	9	3	13	49	9	0	49	19	39	3	34
1720	♋	18	49	10	4	3	24	10	7	25	21	10	0	54	48	40	3	40
Months of the Com. Year.	*Mean Motion.* s	°	′	11	8	13	26	11	0	6	53	11	1	0	17	41	3	45
				12	5	5	0	12	4	18	25	12	1	5	46	42	3	51
				13	9	15	2	13	8	29	57	13	1	11	15	43	3	56
Jan.	0	0	0	14	1	25	4	14	1	11	29	14	1	16	44	44	4	1
Febr.	3	27	34	15	6	5	6	15	5	23	1	15	1	22	13	45	4	7
Mar.	6	20	32	16	2	26	40	16	10	4	33	16	1	27	42	46	4	12
Apr.	10	18	6	17	7	6	42	17	2	16	5	17	1	33	11	47	4	17
May	10	4	7	18	11	16	44	18	6	27	37	18	1	38	39	48	4	23
June	2	1	41	19	3	26	46	19	11	9	9	19	1	44	8	49	4	28
July	1	17	43	20	0	18	20	20	3	20	41	20	1	49	37	50	4	34
Aug.	5	15	17	40	1	6	40	21	8	2	13	21	1	55	6	51	4	39
Sept.	9	12	51	60	1	25	0	22	0	13	45	22	2	0	35	52	4	45
Oct.	8	28	53	80	2	13	20	23	4	25	17	23	2	6	4	53	4	50
Nov.	0	26	27	100	3	1	4	24	9	6	49	24	2	11	32	54	4	56
Dec.	0	12	28	120	3	20	0	25	1	18	22	25	2	17	1	55	5	1
								26	5	29	54	26	2	22	30	56	5	7
In Leap-Year after February								27	10	11	26	27	2	27	59	57	5	12
add the Motion of one Day.								28	2	22	58	28	2	33	27	58	5	18
								29	7	4	30	29	2	38	56	59	5	23
								30	11	16	2	30	2	44	25	60	5	29

A Table of the Mean Motion of the Middlemost of Saturn's *Satellites, discover'd by M.* Cassini, *Anno* 1671.

Years of Jul. begin-ning.	Epochs. s	°	′	In Years.	Mean Motion. s	°	′	In Days.	Mean Motion. s	°	′	H M	Mean Motion. Sex. °	′	″	M	Mean Motion. ′	″
1681	♍	12	16	1	9	17	2	1	2	19	41	1	0	3	19	31	1	43
1686	♏	27	6	2	7	4	3	2	5	9	23	2	0	6	38	32	1	46
1701	♎	1	17	3	4	21	5	3	7	29	4	3	0	9	58	33	1	49
1714	♎	11	43	4	4	27	48	4	10	18	46	4	0	13	17	34	1	53
1715	♋	28	45	5	2	14	50	5	1	8	27	5	0	16	36	35	1	56
1716	♉	15	47	6	0	1	52	6	3	28	9	6	0	19	55	36	2	0
1717	♉	22	30	7	9	18	53	7	6	17	50	7	0	23	15	37	2	3
1718	♓	9	32	8	9	25	36	8	9	7	31	8	0	26	34	38	2	6
1719	♐	26	34	9	7	12	38	9	11	27	13	9	0	29	53	39	2	10
1720	♎	13	35	10	4	29	40	10	2	16	54	10	0	33	12	40	2	13
Months of the Com. Year.	Mean Motion. s	°	″	11	2	16	42	11	5	6	36	11	0	36	31	41	2	16
				12	2	23	25	12	7	26	17	12	0	39	51	42	2	19
				13	0	10	26	13	10	15	59	13	0	43	10	43	2	23
Jan.	0	0	0	14	9	27	28	14	1	5	40	14	0	46	29	44	2	26
Febr.	10	10	24	15	8	14	30	15	3	25	21	15	0	49	48	45	2	29
Mar.	0	21	44	16	7	21	13	16	6	15	3	16	0	53	8	46	2	33
Apr.	11	2	9	17	5	8	15	17	9	4	44	17	0	56	27	47	2	36
May	6	22	51	18	2	25	16	18	11	24	26	18	0	59	46	48	2	39
June	5	3	16	19	0	12	18	19	2	14	7	19	0	3	5	49	2	43
July	0	23	58	20	0	19	1	20	5	3	49	20	1	6	24	50	2	46
Aug.	11	4	23	40	1	8	2	21	7	23	30	21	1	9	44	51	2	49
Sept.	9	14	47	60	1	27	4	22	10	13	11	22	1	13	3	52	2	53
Oct.	5	5	30	80	2	16	5	23	1	2	53	23	1	16	22	53	2	56
Nov.	3	15	54	100	3	5	6	24	3	22	34	24	1	19	41	54	2	59
Dec.	1	6	37	120	3	24	7	25	6	12	16	25	1	23	1	55	3	3
								26	9	1	57	26	1	26	20	56	3	6
In Leap-Year after February								27	11	21	39	27	1	29	39	57	3	9
add the Motion of one Day.								28	2	11	20	28	1	32	58	58	3	13
								29	5	1	1	29	1	36	17	59	3	16
								30	7	20	43	30	1	39	36	60	3	19

A Table

A Table of the Mean Motion of the Outmost but one of the Satellites of Saturn, *discovered by Mr.* Huygens, *Anno* 1655.

Years of Jul. begin-ning.	Epochs. s	°	'	In Years.	Mean Motion. s	°	'	In Days.	Mean Motion. s	°	″	H M	Mean Motion. ° '	' ″	Min.	Mean Motion. '	″
1641	♑	2	48	1	10	20	35	1	0	22	35	1	0	56	31	29	10
1661	♓	13	23	2	9	11	10	2	1	15	9	2	1	53	32	30	6
1681	♈	27	58	3	8	1	45	3	2	7	44	3	2	49	33	31	3
1686	♏	3	28	4	7	14	55	4	3	0	18	4	3	46	34	31	59
1701	♊	12	33	5	6	5	30	5	3	22	53	5	4	42	35	32	55
1714	♓	17	53	6	4	26	5	6	4	15	28	6	5	39	36	33	52
1715	♒	8	28	7	3	16	40	7	5	8	2	7	6	35	37	34	48
1716	♐	29	3	8	2	29	50	8	6	0	37	8	7	31	38	35	45
1717	♐	12	13	9	1	20	25	9	6	23	12	9	8	28	39	36	41
1718	♏	2	48	10	0	11	0	10	7	15	46	10	9	24	40	37	38
1719	♍	23	23	11	11	1	35	11	8	8	21	11	10	21	41	38	34
1720	♌	13	58	12	10	14	45	12	9	0	55	12	11	17	42	39	31
1721	♋	27	8	13	9	5	20	13	9	23	30	13	12	14	43	40	27
Months of the Com. Year.	Mean Motion. s	°	'	14	7	25	55	14	10	16	5	14	13	10	44	41	24
				15	6	16	30	15	11	8	39	15	14	7	45	42	20
Jan.	0	0	0	16	5	29	40	16	0	1	14	16	15	3	46	43	17
Febr.	1	9	54	17	4	20	15	17	0	23	48	17	16	0	47	44	13
Mar.	8	12	2	18	3	10	50	18	1	16	23	18	16	56	48	45	10
Apr.	7	21	55	19	2	1	25	19	2	8	58	19	17	52	49	46	6
May	6	9	14	20	1	14	35	20	3	1	32	20	18	49	50	47	3
June	5	19	7	40	2	29	10	21	3	24	7	21	19	45	51	47	59
July	4	6	26	60	4	13	45	22	4	16	42	22	20	42	52	48	56
Aug.	3	16	18	80	5	28	20	23	5	9	16	23	21	38	53	49	52
Sept.	2	26	12	100	7	12	55	24	6	1	51	24	22	35	54	50	49
Oct.	1	13	30	120	8	27	30	25	6	24	25	25	23	31	55	51	45
Nov.	0	23	34	140	10	12	5	26	7	17	0	26	24	27	56	52	42
Dec.	11	10	42	160	11	26	40	27	8	9	35	27	25	24	57	53	38
								28	9	2	9	28	26	20	58	54	35
In Leap-Year after February								29	9	24	44	29	27	17	59	55	31
add the Motion of one Day.								30	10	17	18	30	28	13	60	56	27

A Table of the mean Motion of the outward Satellite of Saturn, *discovered by M.* Caſſini, *Anno* 1671.

Years of Jul. beginning.	*Epochs.* s	°	′	*In Years.*	*Mean Motion.* s	°	′	*In Days.*	*Mean Motion.* s	°	″	*H M*	*Mean Motion.* ° ′	″	*Min.*	*Mean Motion.* ′	″
1681	♈	8	40	1	7	6	32	1	0	4	32	1	0	11	31	5	51
1686	♈	15	50	2	2	13	3	2	0	9	5	2	0	23	32	6	3
1701	♉	11	53	3	9	19	35	3	0	13	37	3	0	34	33	6	14
1714	♓	20	20	4	5	0	35	4	0	18	9	4	0	45	34	6	25
1715	♎	26	52	5	0	7	10	5	0	22	42	5	0	57	35	6	37
1716	♊	3	23	6	7	13	42	6	0	27	14	6	1	8	36	6	48
1717	♑	14	27	7	2	20	13	7	1	1	44	7	1	19	37	7	0
1718	♌	20	58	8	10	1	17	8	1	6	18	8	1	31	38	7	11
1719	♓	27	30	9	5	7	49	9	1	10	51	9	1	42	39	7	22
1720	♏	4	2	10	0	14	20	10	1	15	23	10	1	53	40	7	34
Months of the Com. Year.	*Mean Motion.* s	°	″	11	7	20	52	11	1	19	55	11	2	5	41	7	45
				12	3	1	56	12	1	24	28	12	2	16	42	7	56
				13	10	8	27	13	1	29	0	13	2	27	43	8	8
Jan.	0	0	0	14	5	14	59	14	2	3	32	14	2	39	44	8	19
Febr.	4	20	41	15	0	21	30	15	2	8	5	15	2	50	45	8	30
Mar.	8	27	46	16	8	2	34	16	2	12	37	16	3	1	46	8	42
Apr.	1	18	27	17	3	9	6	17	2	17	9	17	3	13	47	8	53
May	6	4	37	18	10	15	37	18	2	21	42	18	3	24	48	9	4
June	10	25	18	19	5	22	9	19	2	26	14	19	3	35	49	9	16
July	3	11	27	20	1	3	13	20	3	0	46	20	3	47	50	9	27
Aug.	8	2	9	40	2	6	26	21	3	5	18	21	3	58	51	9	38
Sept.	0	22	50	60	3	9	38	22	3	9	51	22	4	9	52	9	50
Oct.	5	8	59	80	4	12	51	23	3	14	23	23	4	21	53	10	1
Nov.	9	29	41	100	5	16	4	24	3	18	55	24	4	32	54	10	12
Dec.	2	15	50	120	6	19	17	25	3	23	28	25	4	43	55	10	24
								26	3	28	0	26	4	55	56	10	35
In Leap-Year after February *add the Motion of one Day.*								27	4	2	32	27	5	6	57	10	46
								28	4	7	5	28	5	17	58	10	58
								29	4	11	37	29	5	29	59	11	9
								30	4	16	9	30	5	40	60	11	21

The

The mean Motion of the Satellites being thus ſettled, their Revolutions very near the Truth will be theſe following.

	D.	h.	′	″
Of the firſt or inmoſt	1	21	18	26½
Of the ſecond or inmoſt but one	2	17	41	10½
Of the third or Middlemoſt	4	12	25	10
Of the fourth or that of *Huygens*	15	22	41	28
Of the fifth or outmoſt	79	7	46	00

Now according to the univerſal Law of Nature, at leaſt in this our Syſtem, and which obtains as well in the Motions of *Jupiter*'s Satellites and of the Moon, as in thoſe of the Primary Planets about the Sun; if we ſuppoſe their Centripetal Forces towards *Saturn* to be in a reciprocal duplicate Ratio of their Diſtances, and therefore the Cubes of their Diſtances from the Center to be as the Squares of their Periodical Times: From the given Diſtance and Period of that of *Huygens*, the Diſtances of the others will come out as follows.

		Semidiam. Saturn's Ring.	*Semidiam. Saturn's Body.*
Diſtance of the	Firſt	1.9289	4.3400
	Second	2.4708	5.5593
	Third	3.4508	7.7643
	Fourth	8.0000	18.0000
	Fifth	23.3146	52.4578

And theſe Diſtances very well agree with M. *Caſſini*'s Obſervations. Now the four inmoſt Satellites deſcribe their Orbits nearly in the Plain of *Saturn*'s *Ring*, that is, in a Plain which to Senſe is parallel to the Plain of our Equator, whatever ſome may alledge to the contrary. But as to the Fifth, Mr. *James Caſſini*, Son to the former, and Heir of his Talents, has lately found, that the Situation of its Orbit is ſomething different from the others. See the *Memoirs of the Academy of* Paris, *for the Year* 1714.

XXXVIII. The following Obſervations, of which we have made a Collection, are extremely accurate, the Meaſures being taken with very long Tubes and Micrometers made with unuſual Nicety.

A Collection of Aſtronomical *Obſervations for* 1717, 1718. *by* —— n. 357. p. 840. — *of the* Planets.

An. 1717, *April* 15. 9ʰ. 49″, equal Time, Dr. *Pound* obſerved at *Wanſtead*, that *Jupiter* was returned to that Star, which *Nov.* 22, 1716, in the Morning, he cover'd with his Body; concerning which ſee *Phil. Tranſ.* n. 350, (or before, p. 319. Now the Center of * *Jupiter* at that Time was diſtant from that Star, (which is the third of *Gemini* in the *Britiſh* Catalogue) 23′. 22″ towards the North; and at the ſame Time from another near it, (which is the fourth in *Gemini* in the ſame Catalogue) 27′. 11″, and the Planet was very nearly joined to this.

* *Vid.* ſupra, p. 319. n. 354. p. 723.

April 25 following, by the ſame Obſerver and Place, at 10ʰ. 3′, equal Time, *Jupiter* was ſeen at four ſmall fixt Stars, going before them all, and at the very Beginning of *Cancer*. The Center of the Planet was diſtant from *e* 13′. 00″, from *h* 11′. 32″, from *f* 19′. 53″, and from *g* 9′. 27″.

The Day after, *April* 26, at 9^h. 7′, the Center of *Jupiter* was distant from *e* 8′. 35″, from *f* 9′. 0″, from *g* 4′. 5″, and from *h* 13′. 50″. And he had now gone beyond them all, except *f* to which he was going, and which he ought to leave very little below him the next Day.

Almost at the same Moment, that is, at Nine a-Clock at *London*, the Star *g* was seen in the Vertex of an Equicrural Triangle, or almost Equilateral, with the Center of *Jupiter* and his third Satellite, then distant six Diameters of *Jupiter* towards the West; unless that the Leggs of the Triangle were something longer than the Base. And within a Quarter of an Hour the Angle at the Center of *Jupiter*, which before was greater than that at the Satellite, grew sensibly less.

But the three Stars *h*, *g*, *e*, are the 10, 11, and 12 of *Gemini* in the *British* Catalogue, according to which they had at that Time this Situation; *h* in ♋ 0°. 22′. 15″, with Northern Latitude 0°. 11′. 25″. And *g* in ♋ 0°. 28′. 25″, with Northern Latitude 0°. 3′. 40″. Lastly *e* in ♋ 0°. 29′. 20″. with Lat. 0°. 8′. 5″. Nor. But the fourth *f* is distant from the Star *g* 11′. 40″, from *e* 12′. 15″, and lastly from *h* 20′ 36″, whence its Place is given. From hence it appears, that *Jupiter* had very little North Latitude, not greater than Half a Minute, at least if Credit may be given to the aforementioned Places of the Stars. This may be of Use to Posterity, in determining the Motion of *Jupiter*'s Nodes, if they have any Motion at all.

The same Year, *June* 18, at *London*, in the House of the Royal Society, *Saturn* was seen very near a Telescopic fixt Star, from whence it was distant towards the South hardly one Diameter of the Ring, and a Perpendicular let fall from the Star upon the Line of the *Ansæ*, fell upon the Middle of the Eastern *Ansa*. This little fixt Star, inserted in no Catalogue, then was in ♎ 12°. 58′$\frac{1}{2}$, with Northern Latitude 2°. 33′, veary nearly; and had a Companion joined with it of equal Magnitude four Minutes distant from it towards the East, but something more Southerly; whence it may easily be distinguished, and its Places verified at Pleasure.

The same Night at 10^h. 30′, *Mars* was seen near the Star which precedes 35 of *Scorpio*, from which it was distant 7′. 16″. as measured by a Tube of 24 Feet; and that in a right Line drawn through the bright Star θ in the Foot of *Ophiuchus*, and the said fixt Star. Now this Star precedes 35 of *Scorpio* 30′. 27″, of Right Ascension, and is more Southerly than it by 2′. 28″. Whence its Place at that Time was *Sagittary* 15°. 24′. 20″. with South Latitude 3°. 59′. 25″. But θ in *Ophiuchus* was then *Sagittary* 17°. 28′. with North Latitude 1°. 47′. 38″. So that *Mars* preceded that Star in Longitude 4′. 58″. and was more Southerly than it by 5′. 30″.

Afterwards, *Sept.* 13, at 8^h. 30′. equal Time, *Mars* was seen by Dr. *Pound* to precede the bright Star σ in the Shoulder of *Sagittary* 11′. 54″. At 8^h. 25′. the Distance of the Planet from the Star was 25′. 00″. exactly.

Dec. 5, at 18^h. 30′. equal Time, by the Agreement of Observations several Times repeated, Dr. *Pound* found *Saturn* to precede the bright Telescopic Star that was near it 27′. 19″. Right Ascension, and that it was more Southerly than the Star 1′. 59″. At the same Time *Saturn* preceded κ in the *Virgin*'s *Garment* 1°. 25′. 21″. and was more Southerly than it 4′. 05″.

05″. Hence the Place of *Saturn* was *Libra* 29°. 16′. 21″. Its Northern Latitude was 2°. 22′. 21″. The Telescopic Star was then *Libra* 29°. 40′. 56″. Its North Latitude 2°. 33′. 43″.

An. 1718, *Jan.* 7, at 5h. 30′. equal Time, *Venus* was observed near two Stars, which are omitted in the *British* Catalogue. Now the Planet was more to the South than either of the fixt Stars, being distant from the preceding 32′. 30″. and from the subsequent 17′. 30″. The preceding Star was then in *Pisces*, 14°. 42′. 20″. with South Latitude 40′. 10″. The other subsequent Star was in *Pisces* 15°. 21′. 55″. with South Latitude 27′. 15″. as may be collected from Mr. *Flamsteed*'s Observations.

Jan. 15, at 8h. 0′. equal Time, *Jupiter* preceded η in the Breast of *Cancer*, 3°. 30′. 50″ Right Ascension, and was more Southerly than the fixt Star 14′. 15″. Hence *Jupiter*'s Place was *Cancer* 28°. 20′. with North Latitude 36′. 45″.

March 11, at 10h. 36′. equal Time, *Saturn* preceded κ in the *Virgin*'s *Garment* 18′. 51″. and was more Southerly than that Star 5′. 23″. Hence the Place of *Saturn* was *Scorpio* 18°. 34′. with North Latitude 2°. 44′. 8″. That is supposing, according to the *British* Catalogue, that κ in *Virgo* was ♍ 0°. 34′. 10″. with Latitude 2°. 55′. 40″. The same Night at 17h. 00′. at *Westminster* Dr. *Desaguliers* and Mr. *Gray* observed *Saturn* to precede the Star 19′. 00″. with a greater Declination Southerly 4′. 45″.

April 8, at 11h. 30′. at *London*, *Saturn* was lately seen Acronych very little more Westerly than a bright Telescopic Star, and more Northerly than the same by 5 Minutes. Whence the Place of the fixt Star was *Libra* 28°. 18′. 30″. its North Latitude 2°. 41′. Now a great Circle drawn through this Star and *Saturn* seem'd to be directed to a Star of the fifth Magnitude omitted in the *British* Catalogue, but which according to *Hevelius* is in the Point of the Northern Wing of *Virgo*, whose Place he assigns *Libr.* 26°. 10′. with North Latitude 14°. 43′.

The same Night at 13h. 20′. at *Wansted*, a Perpendicular let fall from the said Telescopic Star upon the Line of *Saturn*'s *Ansæ* preceded the Center of the Planet about one and a half of the Diameters of the Ring; and the Star was distant towards the South 4′. 30″. from the *Anis* of the *Ansæ*. Also the Extremity of the Eastern *Ansa* was found in a right Line between this Star and another joyn'd to it as it were in Longitude, which was then distant from *Saturn* 24′. 48″. towards the North. But the Place of the former Star was then *Libra* 28°. 18′. 3″. with North Latitude 2°. 41′. nearly.

Sept. 7. about Noon, there happen'd a very near Conjunction of *Jupiter* and *Venus*, the View of which was obstructed by the Clouds from our Astronomers. But on the 6th Day aforegoing in the Morning, or 5d. 22h. 57′ 30″. equal Time, at *Wanstead*, *Venus* was distant from *Jupiter* 1°. 3′. 28″. to the West. But on 7d. 17h. 21′. *Venus* now to the East was distant from *Jupiter* 43′. 18″. and at 17h. 34′. *Venus* was more Southern than *Jupiter* by the Difference of Declinations 14′. 23″. And at 17h. 39′. the Distance of the Planets was taken 44′. 4″. Hence by the Calculation of a very accurate Observer, they were in Conjunction *Sept.* 7. 0h. 9′. equal Time; the Center of *Venus* being more South than *Jupiter*'s only 1′. 42″.

Sept. 18, in the Morning, at *Wanstead*, *Jupiter* was seen near *Cor Leonis*, with

with which he had been in Conjunction the Day before, *Sept.* 17. at 16ʰ. 51′. equal Time. *Jupiter*'s Center was distant from *Cor Leonis* 24′. 22″; and at 17ʰ. 6′. 20″. the Difference of Declination was 12′. 43″. Then an Hour after, or at 17ʰ. 54′. the Distance became 24′. 44″, and at 18ʰ. 7′. the Difference of Declinations was found 12′. 35″. Hence by Dr. *Pound's* Calculation, on *Sept.* 17, at 18ʰ. 0′. equal Time, the Place of *Jupiter* was *Leo* 26°. 11′. 7″. with 45′. 32″. North Latitude.

— *of the* Moon *and* Eclipses.

Anno 1717, *Jan.* 12. at *Westminster*, Mr. *Stephen Gray* observed an Appulse of the Moon to four contiguous Stars under the South Horn of *Taurus*, near which the Moon was observed *An.* 1683, *March* 23, old Stile, by *Hevelius* and *Flamstead.* Therefore at 9ʰ. 45′. apparent Time, the Moon being gibbous, was seen as in Conjunction with the preceding Star of the four, which is 107 of *Taurus* in the *British* Catalogue, and which was then more Southern than the Southern Limb of the Moon, by a Minute and half; at 11ʰ. 29′. another which is less, and therefore omitted in the Catalogue, was hid a little below the Middle of the obscure Limb. At 12ʰ. 24′. the third and brightest (110 of *Taurus*) almost in Conjunction, was distant 6′ from the Northern Limb. Lastly, At 12ʰ. 54′. the last of the four (111 of *Taurus*) was higher than the Northern Limb by 3′. 30″. Now the Place of the preceding, or 107 of *Taurus*, by the said Catalogue was then *Gemini* 18°. 12′. with Southern Latitude 5°. 18′. And 110 of *Taurus* was *Gemini* 19°. 26′$\frac{1}{4}$. with South Latitude 4°. 44′. And the Consequent, or 111 of *Taurus*, was in *Gemini* 19°. 45′. with South Latitude 4°. 48′$\frac{1}{6}$. The second little Star, as appear'd from other Observations, had then its Place in *Gemini* 19°. 17′. and its Latitude nearly 5°. 5′.

The same Year, *March* 16, in the Morning, there was a partial Eclipse of the Moon, not conspicuous with us, because of the cloudy Weather. But at *Cambridge* in *New England*, Mr. *Robie*, a very skilful Astronomer, saw the Beginning of the Eclipse about Nine a-Clock. And the End near *Palus Mæotis*, at 11ʰ. 42′. 30″. exact enough. But *Cambridge* is under the Altitude of the Pole 42°. 25′. more Western than *London*, 71°. or 4ʰ. 44′. as appears from many former Observations.

Vid. Infra p. 338.

Sept. 9, in the Evening, at the House of the Royal Society at *London*, some observed the End of the *Lunar* Eclipse at 7ʰ. 26′. But the Moon arose at the Middle of the Eclipse, nor had freed herself from the Clouds about the Horizon till a little before the End.

Sept. 14, in the Evening, now for the first Time, after a long Interval, the Moon returned to eclipse the *Bull's Eye.* The Sky at *London* was clearer than usual, so that the Moon and the Star were seen as it were rising together in the Horizon. The Immersion of the Star happen'd at 9ʰ. 6′. 20″, the Moon being not yet 3° high, in the very Middle as it were of the Eastern Limb, that is, over-against the Northern Part of that little Spot, which *Hevelius* calls the Lake of *Mæris*, and which *Ricciolus* has denoted by his own Name. But it emerged a little below the Middle of the obscure Limb, at 9ʰ. 58′. 20″, and in the twinkling of an Eye shone forth with its whole Brightness. Hence also is proved, that even this remarkable Star has next to no Diameter.

Sept.

Sept. 23, in the Evening, there happen'd an Eclipse of the Sun, that was hardly to be seen in any Part of *Europe.* But from our Parts of *America* we have obtain'd two Observations of it, one by a Letter from the worthy Mr. *Keith*, Governour of the Province of *Pensylvania*, who at *Philadelphia*, under the Altitude of the Pole 40°. 0′. nearly, saw the Eclipse already begun, (but which was not begun a Minute before,) at 11ʰ. 55′. About the Middle there were about ten Digits. The End was seen exactly at 2ʰ. 46′. 35″.

But the other Observation of this was made at *Cambridge*, the University of *New England*, by Mr. *Robie.* The Beginning of the Eclipse was observed there at 0ʰ. 23′. 0″. after Noon. At 1ʰ. 47′. nine Digits were eclipsed. At 3ʰ. 5′. 10″. the Eclipse ended, the whole Sun being seen through a Telescope of 24 Feet. Mr. *W. Derham* communicated this from the Letters of this exact Observer.

Dec. 5. The Moon pass'd a little above the *Bull's Eye.* This near Transit was observed by that Learned Youth Mr. *Ja. Bradley*, M. A. (the Moon being now almost at full,) who compared the Star with that remarkable Spot, which *Ricciolus* calls *Tycho*, but *Hevelius*, *Sinai*; and from several equal Distances, taken with the Micrometer before and after, he concluded that the Star approached near to the Center of the said Spot at 11ʰ. 15′. 8″. equal Time, at *Wanstead.* At 11ʰ. 15′. 42″. the *Bull's Eye* was distant from the nearest and Southern Limb of the Moon 5′. 55″. But the Spot *Tycho* was distant from the same Limb 4′. 16″. At 11ʰ. 18′. 42″. the Star was in a right Line with the Spots *Tycho* and *Copernicus*, or *Sinai* and *Ætna*; and at 11ʰ. 25′. 27″. equal Time, it was in a right Line with *Tycho* and *Kepler.* Among these Observations the Moon's Diameter was found 32′. 45″.

Anno 1718, *Jan.* 29, in the Evening, Dr. *Desaguliers* and Mr. *Gray* at *Westminster*, expected another Occultation of the *Bull's Eye*; but by the Interposition of Clouds they could only see, that at 5ʰ. 52′. the Star was not yet immersed; but afterwards the Clouds growing thinner, the Emersion was concluded to be at 7ʰ. 20′, over-against *Hevelius*'s Spot called the Promontory of *Asiatic Sarmatia.*

Feb. 19, in the Morning, the same Observators in the same Place, variously struggling with the Clouds, could hardly see the Eclipse of the Sun. Yet at 6ʰ. 59′. two Digits were seen to be eclipsed, and after a Minute of Time the Chord between the *Cuspids* seem'd to be equal to the Semidiameter of the Sun.

But at *Wanstead* Dr. *Pound* observed, that at 6ʰ. 54′. 7″. apparent Time, the Chord between the *Cuspids* was 18′. 30″. At 7ʰ. 17′. 0″. it was 10′. 18″. At 7ʰ. 19′. 30″. the same was found to be 8′. 5″. The Eclipse ended at 7ʰ. 23′. 20″.

Feb. 25, in the Evening, at 6ʰ. 44′. at *Westminster*, the first Star of the *Hyades* in the *Bull's Snout*, (γ according to *Bayer*,) was seen in a right Line through the *Cuspids* of the Moon, and therefore nearly in Conjunction. Now its Distance from the Southern Limb of the Moon was 5′. 51″. The Diameter of the Moon measured with a Micrometer was 31′. 45″.

Feb. 28, at 8ʰ. 36′. apparent Time, also at *Westminster*, an Immersion was seen of the Star in the Knee of *Pollux*, (according to *Bayer* λ of *Gemini*,) under the

the obſcure Limb of the Moon, on that Side which is ſomething more Northerly than the Spot which *Hevelius* calls *Crete*. The Emerſion was not ſeen, becauſe the Sky was not clear enough. But at $9^h. 51'$. the Star was come out, over-againſt the Northern Part of the greater *Caſpian* Iſland.

Aug. 8, The Moon aroſe a little below the *Bull's Eye*, but becauſe of the Clouds, could not be compared with it. But at *Wanſtead*, at $13^h. 2'. 0''$. apparent Time, the preceding Star of the Contiguous ones at σ of *Taurus* according to *Bayer*, (or the laſt but one in our catalogue of the *Hyades*, in N. 354. of the *Philoſophical Tranſactions*, mark'd with the Letter *q*,) was ſeen in a right Line through the *Cuſpids* of the Moon, diſtant from the Southern $4'. 36''$. At $13^h. 7'. 15''$. the Star *p* of the ſame Catalogue emerged a little below the Middle of the obſcure Limb. At $13^h. 19'. 4''$. the Conſequent of the ſaid contiguous Stars emerged, as much diſtant from the Southern Horn, as thoſe contiguous Stars were diſtant from one another, that is, 7 Minutes.

Vid. ſupra, p. 300.

Aug. 29, in the Evening, the Moon almoſt in her Apogæ, ſuffer'd almoſt a total and central Eclipſe; but ſhe aroſe after the Eclipſe was begun. The Reverend Dr. *Pound* exhibited very perſpicuous Obſervations of this Eclipſe, in the Order they are here deſcribed.

Obſerv.	Apparent Time. h	'	"	An *Eclipſe* of the Moon, obſerved at *Wanſtead*, *Aug.* 29, 1718.	'	"
1	6	53	38	The Chord between the *Cuſpids*, meaſured with a Micrometer	22	37
2		55	8	The ſame repeated	21	14
3		56	31	Repeated	19	51
4		57	49	Again	18	28
5		59	38	Once more	15	00
6	7	2	41	The total Immerſion into the Shadow		
7	8	36	13	A bright Star omitted in the Catalogues was hid by the Moon, below the *Palus Mareotis* of *Hevelius*	10	2
8	8	48	18	The Moon began to emerge out of the Shadow		
9		51	13	The Limit of the Shadow through the Middle of *Marectis*; as alſo the Chord between the *Cuſpids*	15	0
10		53	7	The Chord between the *Cuſpids*	18	28
11		54	16	The ſame repeated	19	51
12		54	59	Again	21	14
13	8	56	18	Once more	22	37
14	9	0	48	*Porphyrites* emerged out of the Shadow		
15		8	3	Mount *Sinai* began to emerge		
16		9	17	The Shadow through the Middle of *Sinai*		
17		10	6	All *Sinai* was now out of the Shadow		
18		11	20	The Shadow paſs'd through the Middle of *Ætna*		
19		17	23	Through the Middle of *Corſica*		
20		20	0	Through the Middle of the greater black Lake		
21		27	54	Through the Middle of *Beſbicus*		
22		28	45	The aforeſaid Star now emerged		
23		32	34	*Byzantium* and *Horminius* emerge together		
24		33	58	The Star had the ſame Declination with the Southern *Cuſpid* of the Eclipſe		
25		43	28	The Chord between the *Cuſpids*	18	28
26		47	2	The ſame repeated	15	00
27	9	53	0	The Eclipſe ſeem'd to finiſh		

At

At $10^h. 30'$. the Diameter of the Moon was taken $29'. 45''$. Now the Observations being compared with one another, where the Chords of the deficient Parts were found to be equal, the Middle of the Eclipse is discover'd.

	Middle. h.	′	″
From the first and thirteenth Observations ——	7	54	58
From the second and twelfth —— ——	7	55	3
From the third and eleventh —— ——	7	55	24
From the fourth and tenth —— ——	7	55	28
From the fifth and ninth —— ——	7	55	25
From the sixth and eighth —— ——	7	55	29
Of all which the Middle is —— —— ——	7	55	18

Martin Folkes, Esq; with some other Members of the Royal Society, observed the same Eclipse with like Diligence at *London* in *Fleet-street*, with the Instruments and an excellent Telescope of Mr. *George Graham*, a very skilful Watch-maker; which Observations are as follow.

h.	′	″	
6	38	0	The Moon was hardly seen through the Smoke and Vapours of the City.
6	54	13	The Chord between the Cuspids $21'. 17''$. or thereabouts.
7	2	0	The total Immersion into the Shadow.
7	42	15	A pretty bright Star was distant from the Eastern Limb of the Moon $19'. 21''$.
8	35	18	The same fixt Star was hid, about 10 Minutes more to the South than the Center of the Moon.
8	45	50	Now, or as some thought a Minute later, the Moon began to emerge.
8	49	38	*Palus Mareotis* emerged with its first Margin.
8	50	14	The whole *Palus* was without the Shadow.
9	0	5	The Middle of Mount *Porphyrites* emerged.
9	7	39	The first Margin of *Sinai* emerged.
9	9	8	Mount *Sinai* was quite without the Shadow.
9	10	35	The Shadow pass'd through the Middle of *Ætna*.
9	12	0	All Mount *Ætna* was without the Shadow.
9	18	51	The Shadow pass'd through the Middle of the *Greater Black Lake*.
9	27	35	The Island *Besbicus* wholly emerged.
9	42	21	The Chord between the Cuspids was $19'. 9''$.
9	51	25	The End of the Eclipse as was judged by some.
9	52	45	The End concluded from the foregoing Distance of the Cuspids.
9	56	45	The Diameter of the Moon was $29'. 54''$, and again $29'. 48''$.

The Shadow was very thin, whence arose some Difficulty to distinguish the Moments of Emersion and the End. And even the obscurer Spots were plainly seen several Minutes before they reach'd the Margin of the Shadow. The Star which was hid during the Eclipse was then in ♓ $17^\circ. 16'\frac{1}{4}$. with Southern Latitude $1^\circ. 6'. 30''$. very near.

We

We have also received Observations of this Eclipse from the Rev. Mr. *Derham*, made at *Upminster* in the County of *Essex*; from Mr. *Wright* at *Crew* in *Cheshire*; and from Mr. *Hawkins* at *Wakefield* in the County of *York*; every where almost agreeing with the foregoing, having Regard to the Difference of Meridians: That is, supposing *Upminster* to be $1\frac{1}{2}$ Minutes more Easterly than *London*, *Crew* to be 10 Minutes, and *Wakefield* 5 Minutes, more to the West.

Lastly, to sum up all, we will add a very notable Observation, and the first of its Kind that we know of, since the Discovery of the Telescope, and which we owe to the unwearied Application of Dr. *James Bradley*. For on the 5th of *September* in the Morning, the Sun being nearly 30 Degrees high, he saw at *Wanstead* a most near Transit of the Moon below the *Bull*'s *Eye*, the Distance of which from the next Limb he found with a Micrometer to be 5′. 38″. at 7^h. 59′. 0″. of equal Time. At 8^h. 17′. 5″. it was distant from the Limb 1′, 25″. And at 8^h. 33′, 15″. the Star was in a right Line through the Cuspids of the Moon, which at that Time were something blunted, nor was it distant from the Northern above 0′. 13″. and at 8^h. 41′. 0″. it had left that Cuspid 3′. 42″. And at 8^h. 45′. 37″. it was distant from the same 5′. 36″. The Moon's Diameter taken at 8^h. 58′. was 31′. 7″.

A Collection of Astronomical Observations for 1719, by — n. 263. p. 1109. — of the Planets. * *Vid. infra*, §XLI.

XXXIX. Our late Observations are these; 1718, *Octob.* 10, in the Morning, *Jupiter* applied to the Telescopic fixt Stars, the Places of which the Rev. Dr. *Pound* has carefully enquired into, on Occasion of the first Appearance of the Comet of the Year 1680, (of which see *Phil.* * *Trans.* n. 342.) and having verified them lately, has communicated them to us, together with an accurate Observation of a Transit of *Jupiter* near them at this Time, and afterwards another *Feb.* 11. presently after the Opposition of *Jupiter* and the Sun. Now at the Beginning of *January* 1719, the Places of the Stars were thus.

	Longitude.	*North Latitude.*
d	♌ 29°. 59′. 43″	1 . 7 . 50
e	♍ 0 . 6 . 13	1 . 10 . 18
c	♍ 0 . 3 . 13	0 . 32 . 50
a	♍ 0 . 25 . 41	1 . 28 . 54
x	♍ 0 . 5 . 43	0 . 51 . 56

Where it is to be observed, that the Stars *d* and *e* have the same Declination exactly in this our Age; but *x* is a very little Star, which because of its Smallness was omitted in the former Description.

Now *Octob.* 9°, at 17^h. 5′. equal Time, the Eastern Limb of *Jupiter* reach'd the Line joyning the Stars *e* and *c*, and at the same Time his Center was distant from *e* 21′. 20″. and from *c* 16′. 25″. and was presently distant from *d* 19′. 35″. The little Star *x* being very near *Jupiter* was hid, or overpower'd with his Light.

Dec. 11. at 18^h. 30′. equal Time, the Center of *Saturn* was distant from μ in *Libra* (according to *Bayer*) 28′. 32″. and was more to the North than the fixt Star by 4′. 31″. Hence the Observer Dr. *Pound* concluded the Place of *Saturn* to be ♏ 1°. 41′. 10″. with Northern Latitude 2°. 16′. 43″.

1719,

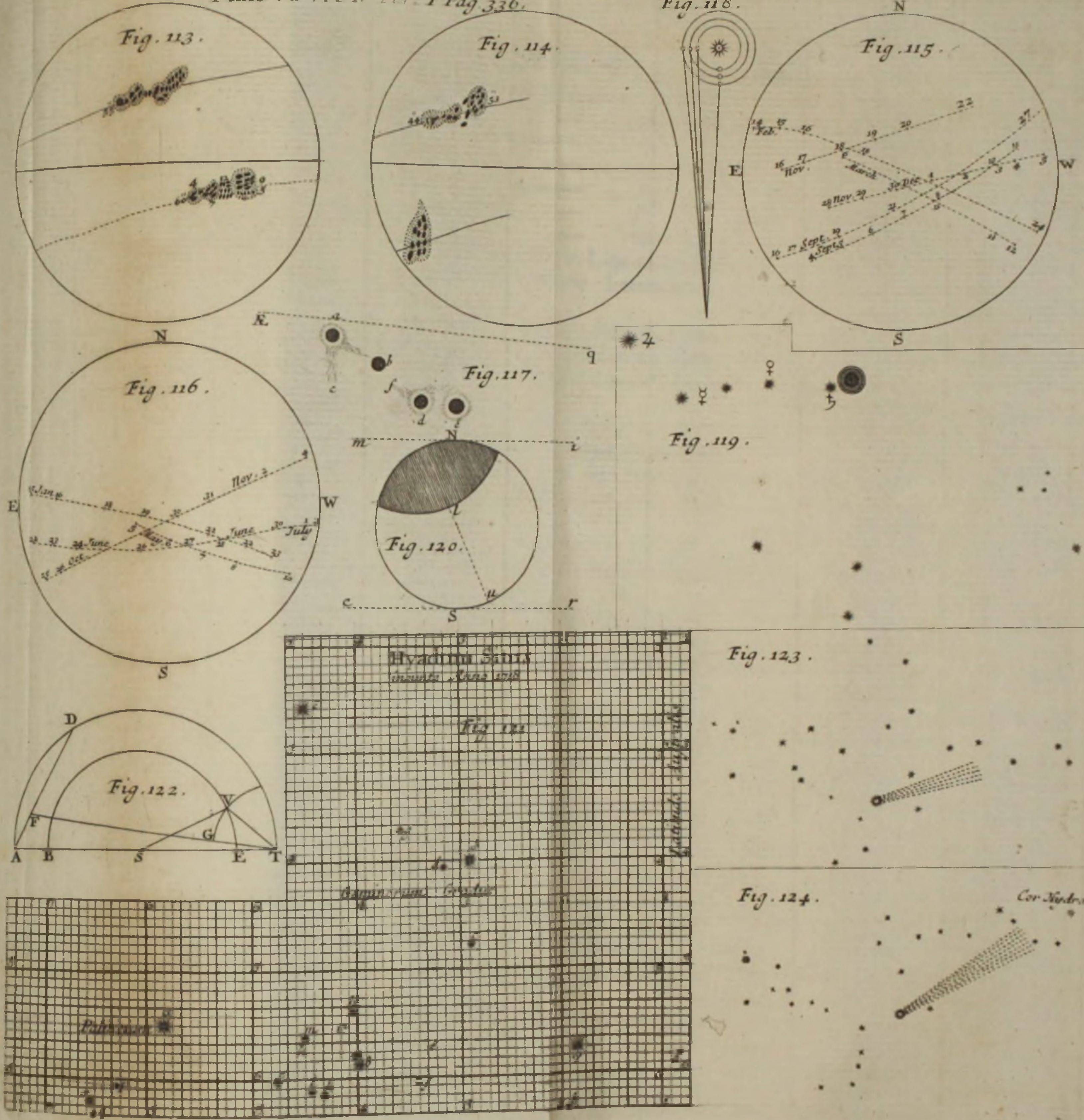


Fig. 113.
Fig. 114.
Fig. 118.
Fig. 115.
Fig. 116.
Fig. 117.
Fig. 119.
Fig. 120.
Fig. 121
Fig. 122.
Fig. 123.
Fig. 124.
Hyadum Sinus

1719, *Feb.* 11. at 6^h. 56'½. equal Time, the Center of *Jupiter* being retrograde was distant from the above described Star —— 10'. 42"
6 . 58¼ The same Center was distant from *e* —— 6 . 7
9 . 37⅓ The Distance taken again from *d* — —— 10 . 9
9 . 43½ Again from *e* —— —— 6 . 11
9 . 49½ The Center of *Jupiter* was distant from *a* —— 25 . 21
9 . 58½ The same Center from the small Star *x* —— 24 . 38

About Seven a-Clock the Eastern Limb of *Jupiter* reach'd the Line extended through *x* and *e*; *Jupiter* then was in ♍ 0°. 6'. with North Latitude 1°. 16'. 30". Then

Feb. 13, at 8^h. 0', equal Time, the Declination of *Jupiter*'s Center, measured by the Micrometer, was more Northerly than that of either of the Stars *d* and *e*, by 11'. 37". and at 8^h. 20'. the same Difference was found 11'. 36". But at 8^h. 48'. the Center of *Jupiter* was distant from *e* 17'. 40".

April 22, at 10^h. 45'. equal Time, the Center of *Saturn* follow'd μ in *Libra* 4'½ Minutes of Time, or 1'. 8". of Right Ascension. Being measured with the Micrometer, he was found more Northerly than the fixt Star 35'. 25". Now in the *British* Catalogue the Star was then in ♏ 10°. 16'. 8". North Latitude 2°. 3'. 54".

May 16, at 8^h. 0'. equal Time, *Jupiter* follow'd *Cor Leonis* at 1°. 34'½. of Right Ascension; but was 0'. 41"½. more to the North than that Star. In Time this is 10'. 7". of an Arch of the Heavens.

The same Night at 15^h. 18'. apparent Time, Mr. *Stephen Gray* observed *Mars*, in respect of Right Ascension, to follow the Eastern Star in the Tail of *Capricorn* at 16'. 15". and at the same Time it was more to the South than the fixt Star only 0'. 11".

June 7, at 10^h. 15'. apparent Time, *Jupiter* being direct, return'd again to the said Telescopic Stars, and then follow'd the Star *d* at 0'. 35". of Right Ascension, and at 10^h. 30'. the Star was distant from the nearest Limb of *Jupiter* 4'. 18".

The next Day, *June* 8, at 10^h. 20'. *Jupiter* follow'd the other Star *e* at 1'. 30". of Right Ascension, and immediately the Distance of the nearest Limb of *Jupiter* from the Star was taken with the Micrometer 7'. 30".

July 5, at 8^h. 26'. apparent Time, *Jupiter* and *Venus* were in close Conjunction; she more to the North, then preceded *Jupiter* according to Right Ascension 1'. 20". But the Distance of their Centers being taken, or the middle Distance of ten repeated Distances was 13'. 36". *Martin Folkes*, Esq; a great Cultivator of these Sciences, communicated these three Observations made at *London*.

August 3, at 12^h. 20' equal Time, *Mars* nearly Acronich follow'd the Star τ of *Aquary* (according to *Bayer*) at 10'. 58". of Time, or 2°. 44'. 57". of Right Ascension. *Mars* was only 0'. 36". more to the North than the Star. Whence the Place of the Star being granted as in the *British* Catalogue, the observed Place of *Mars* will be ♓ 7°. 10'. 11". with South Latitude 6°. 38'. 10".

August 10, at 11^h. 50', equal Time, *Mars* follow'd the lesser Star which precedes τ in *Aquary* 1°. 39′. 30″. in respect of Right Ascension, but more Southerly than the fixt Star 10′. 42″.

August 16, at 7h. 18′. equal Time, *Spica Virginis* preceded the Center of *Venus* 5″¼ of Time, or 1′. 20″. of Right Ascension, more Southerly than the Planet 18″¼ of Time, or 4′. 35″.

August 17, *Mars* the Day before Achronich, and next the Earth, was observed at two contiguous little Stars, for the Sake of investigating his Parallax, according to the Method exhibited by M. *Cassini*, in his Book concerning the Comet of the Year 1680. Whence we shall endeavour to deduce the Parallax of *Mars* in the next Transaction. Now the more Northerly of these little Stars was then in ♓ 3°. 5′. 50″. with South Latitude 6°. 6′¼ nearly. But the more Southerly one was in ♓ 3°. 5′. 30″. with South Latitude 6°. 10′¼ nearly. But at 10h. 40″. equal Time, *Mars* follow'd that to the South 41′. 40″. of Right Ascension, and was more to the South than it 7′. 50″.

Sept. 18, at 9h. 20′. equal Time, *Mars* was seen to precede the Star which in the *British* Catalogue is the 33 of *Aquary*, by 3′. 45″. of Time, or 56′. 24″. of Right Ascension; and at the same Time the Star was more Northerly than the Northern Limb of *Mars* only by one Diameter of the Planet. The Place of the Star was ♒ 29′. 57″½. South Latitude 4°. 48′½.

Oct. 30, in the Evening, at 5h. 45′. apparent Time, *Mars* was near two contiguous Stars at *h* ♒ by *Bayer*, which are ♒ the 73 and 74 in the *British* Catalogue. It had pass'd by the right Line drawn through the same, and the Angle at the Center of *Mars* was a right one according to Sense. The more Northern of the Stars had the same Declination with the Southern Limb of the Planet. At 5h. 53′. the Distance of the Star from the Center of *Mars* was 2′. 30″. At 5h. 56′. the Center of *Mars* was distant from the third and more Southerly one at *h*, or the 75 of *Aquarius*, 17′. 4″. At 6h. 18′. the Distance of the Center from the more Northerly, or the 73, was 3′. 5″. Hence we may conclude, that at 3h. 30′. nearly, *Mars* was in Conjunction with the Northerly Star, and left it only one Minute to the North. But the Place of the fixt Star from the *British* Catalogue was then ♓ 10°. 29′. 0″. with South Latitude 1°. 40′½. And the 74 was then in ♓ 10°. 29′. 50″, with South Latitude 1°. 44′½.

Nov. 16, at 19h. 18′. equal Time, *Venus* preceded the Southern Scale of *Libra* 3′. 13″. of Time, or 48′. 23″. of Right Ascension, and at the same Time the Center of *Venus* was more Northerly than the fixt Star 7′. 45″. *Venus* was stationary as it were at her ascending Node.

Dec. 3, at 19h. equal Time, *Saturn* preceded the third at ζ of *Libra*, or *Libra* 29, by the *British* Catalogue 0′. 46″. of Time, or 11′. 32″. of Right Ascension. It was more Southerly than the fixt Star 15′. 29″. the Difference being taken by a Micrometer. Whence the Place of Saturn was ♏ 20°. 25′½. with North Latitude 2°. 5′¼.

— *of the* Moon *and* Eclipses. We have given above * an Observation of the Lunar Eclipse, *An.* 1717, *March* 15, Afternoon, *Old Style*, made at *Cambridge* in *New-England*, which because of Clouds was not seen by us. There the Eclipse ended at 11h.

**Vid.* sup. p. 332.

42′½,

42′′, nor at that Time had we any other Obſervation of it. But afterwards Mr. *Chandler*, a Captain of one of the King's Ships, brought us from *America*, and communicated to us the Phaſes of the ſame Eclipſe, as they were obſerved at *Lima* in *Peru*, by D. *Peter Peralta* the King's Mathematician, and there printed. He makes the Beginning of the Eclipſe at *Lima*, at $8^h. 4'. 8''$. and the End at $11^h. 19'. 55''$. At the ſame Time the ſaid Mr. *Chandler* communicated his own Obſervation, made at an Iſland which they call *Virgin Gorda*, where the Eclipſe ended at $12^h. 13'$. Afternoon. Becauſe of a clear Sky the End was ſeen very diſtinctly. Among the Acts of the Royal Academy of Sciences for that Year, we alſo find two Obſervations of this Eclipſe agreeing very well with ours; one made by M. *Caſſini*, the other by M. *de la Hire* in the Royal Obſervatory. This makes the Beginning at $13^h. 54'$. But the End more ſurely at $16^h. 38'. 10''$. The other puts the Beginning at $13^h. 55'$. and the End at $16^h. 38'. 25''$. The greateſt Obſcuration with this was $7\frac{1}{2}$ Digits, with the other $7\frac{1}{2}$ Digits.

Hence from the End, which ſeems to be taken more accurately in each Place, the Difference of Longitude between *Paris* and *Lima* is $5^h. 18'. 20''$. Between *Paris* and *Cambridge* $4^h. 55'. 50''$. Between *Paris* and the *Iſland Virgin Gorda* $4^h. 25'. 20''$. From which if you ſubtract $9'. 40''$. there will come out the Longitudes to the Weſt of *London*; that of *Lima* $77°. 10'$. of *Cambridge* in *New England* $71°\frac{1}{2}$. And laſtly of the Iſland *Virgin Gorda* $63°. 55'$. Whence the Geographers may correct with Certainty the Situation of the neighbouring Iſlands.

The other Eclipſe of the Moon of the ſame Year, *Sept.* 9, in the Evening, was ſeen by the ſame Obſervers, and M. *Maraldi* at *Paris*. In the Houſe of the Royal Society at *London*, we obſerved the End at $7^h. 26'$. At *Paris* the End was obſerved by M. *Caſſini* at $7^h. 34'. 50''$. by M. *Miraldi* at $7^h. 35'. 30''$. and by M. *de la Hire* at $7^h. 34'. 15''$. Alſo M. *Wurtzelbaur* at *Norimburg* ſaw the ſame End at $8^h. 10'. 45''$. Hence the Difference of Meridians is confirm'd between *London* and *Paris*, eſpecially from the Obſervation of M. *Maraldi*, to be $9'. 30''$. as alſo between *London* and *Norimburg* $44'. 45''$. The ſame we have often found before. Now on the fifth Day after the Eclipſe, *Sept.* 14, in the Evening, at *Paris* the Moon eclipſed the *Bull*'s *Eye*, as M. *Maraldi* and M. *Deliſle* the Younger obſerved ſeparately. The Star diſappear'd over againſt the Spot *Grimaldi*, or *Palus Mareotis*, at $9^h. 11'. 35''$. And it emerged from the obſcure Limb of the Moon at $10^h. 3'. 55''$. We have given above * an Obſervation of this Occultation at *London*. * *Vid.* ſupra, p. 332.

We are obliged to the Rev. Dr. Pound *for thoſe Obſervations, in which equal Time is made Uſe of; being taken with a Teleſcope of* 15 *Feet, they may be eſteemed as very accurate.*

Obſervations *at* Rome *on the Comet* 1664. *by the late Mr* J. Ray, *communicated by Mr* S. Dale n. 309. p. 2350.

XL. *December* the 20th, 1664, *N. S.* about Three a-Clock this Morning, I obſerved the *Comet*; it was in the Conſtellation of *Hydra*, not far from the Foot of *Crater*. It appeared about the Bigneſs of a Star of the firſt Magnitude, but nothing ſo lucid and bright. It had a very long Tail, which pointed almoſt directly towards the Heart of *Hydra*: The Tail ſhewed ſomewhat like Rays of a Candle burning in a Miſt; the Figure of it was conical; the

the Length of it 5 or 6 Degrees; the Breadth at the Baſe not above a Degree and an Half. The Body of this *Comet* was about 3 Degrees to the
Fig. 123. South-Eaſt of the moſt Southerly Star in the Foot of *Crater*; it ſtood very near in a Right Line with the two lowermoſt Stars in the Foot of *Crater*, which are common to it and *Hydra*.

December 21, In the Morning, about the ſame Hour, it was removed about a Degree and an Half from the Place where it ſtood, Weſtward, and
Fig. 124. a little to the South. The Tail pointed ſtill towards the Heart of *Hydra*, and appeared 10 Degrees long at the leaſt.

December 22, At the ſame Time it was removed from the Place where it ſtood the Day before, to the ſame Point, and about the ſame Diſtance as
Fig. 125. the Night before. The Tail of it ſtill pointed to *Cor Hydræ*, or a little Thought above it, as the two former Days, and was rather longer than ſhorter: It alſo, to my Thinking, appeared brighter and larger; the Body of it being bigger than any fixt Star, except *Sirius*.

December 23, It was removed to the ſame Point, and about the ſame
Fig. 126. Diſtance as the Day before; the Tail of it was as long as ever, and the *Comet* brighter. The Tail pointed almoſt directly to *Cor Hydræ*.

December 24, 25, 26, All theſe three Nights were cloudy, ſo that I could make no Obſervations.

December 27, We found it ſtrangely removed from the Place where it
Fig. 127. was: It was ſtill Weſtward, and a little to the South, as before. The Body of the Star was ſtill brighter, and the *Cauda* about it greater, and more buſhy, and yet as long as before; it pointed almoſt directly againſt *Canis major*. The Body of it was amongſt the Stars of *Argo*.

December 28, The ſame Time it was removed above two Degrees towards
Fig. 128. the ſame Point, and come within four or five Degrees of the moſt Eaſtern Stars in the bright Triangle in the Buttocks of *Canis major*. The *Moon* ſhining, we could not ſo well judge, either of the Bigneſs of the Body, or the Length and Buſhineſs of the Tail.

December 29, It was ſtrangely removed, and got before, not the Eaſtern Star only of the mentioned bright *Triangle*, but alſo the moſt Northern. I
Fig. 129. think, at leaſt, in this laſt twenty-four Hours, it had moved four Degrees. The *Moon* ſhining bright, the Tail could not well be obſerved, yet ſtill it ſeemed to point directly to *Canis minor*.

Obſervations *on the* Comet 1680. *ſeen in* Saxony *by Mr.* G. Kirch, *by* — n. 342. p. 170.

XLI. The Comet, which was ſeen at the End of the Year 1680, for many Reaſons is to be conſider'd as the principal of its Kind; as well on Account of its Courſe for four Months, in which it paſs'd through nine whole Signs, as becauſe of the immenſe Magnitude and Brightneſs of its Tail: But chiefly of the remarkable Curvity of its Orbit, by the Help of which the Theory of Comets is at laſt diſcover'd by the illuſtrious *Newton*, who firſt of all Mortals proved that Comets deſcribe Orbs, which very nearly approach to the Form of Parabolic.

Now it happen'd, I know not by what Fate, that this Comet,(which in the Evening was ſo much attended to by Aſtronomers,) in the Morning before it reach'd the Sun, was not once obſerv'd either at *Paris* or at *Greenwich*. And thoſe that ſaw it and obſerv'd it, deliver'd incongruous and contradictory Things about it, and but little ſuitable to the Nicety of the Affair. Nor was it ſeen by any intelligent Obſerver till *Nov.* 17, in the Morning. Hence it was that that Part of the Orbit, in which the Comet approach'd towards the Sun, could not be determin'd but with ſome uncertainly. But we lately happen'd upon a Book of a very deſerving Aſtronomer Mr. *Gottfried Kirch*, a *German*, printed at *Norimberg*, *An.* 1681, called Neue Himmels zeitung, that is, a new Celeſtial Meſſenger; in which the very diligent Author explains to us, by what good Fortune he diſcover'd this Comet, being as yet obſcure without a Tail, and hardly viſible to the naked Eye: While he was taking his View in order to obſerve the Moon and *Mars* that was near her, *Nov.* 4, old Stile, in the Morning, at *Coburg* in *Saxony*, which Town is 11 Degrees more Eaſterly than *London*, under the Pole's Altitude 50°. 20′. nearly; he was urged, as he ſays, by the Rumour of a Comet ſeen in *Germany*, and ſat up at Night with his Face towards the Eaſt, that if any Thing new ſhould ariſe in the Heavens, which were then very ſerene, he might take Notice of its Situation. Now as the Moon approach'd to a certain Star, not taken Notice of by *Tycho*, (but which is put down in *Flamſtead's Britiſh* Catalogue, and is 44 in *Leo*,) he had a Mind to determine the Place of the ſaid Star by thoſe which were near it; and as he turn'd about his Tube which was capable of receiving three Degrees, he fell upon a Kind of cloudy Light, making an unuſual Appearance, and which he immediately concluded to be either the new Comet, or a nebulous Star like that which is in *Andromeda*'s *Girdle*.

Now he firſt ſaw the Comet at $4^h \frac{1}{2}$ in the Morning, being a little higher than the two little Teleſcopic Stars, which are mark'd with the Letters *a* and *c*, with which however at 6^h. it was exactly in a right Line. Whence it was plain that it moved, and with a direct Motion. Between the Hours of Five and Six he view'd this Phenomenon with a 10 Foot Tube, and ſaw two other little Stars contiguous, but leſs than the former, mark'd with the Letters *e* and *d*, and above thoſe a third *g*. Now the Diſtance of the Comet from *e* was ſomething leſs than that from *a*, but greater than the Diſtance *d e*. At 6^h. 38′. the Diſtance of the Comet from *e* was the Double of the Interval between them *d e*, and the Line *d e* produced left the Comet * below it, yet ſo that it reach'd its upper Margin. At 6^h. 45′. the Comet was now inſenſibly more remote from *e* than from *a*, and was diſtant from *a* ſomething more than half the Diſtance of the little Stars *a* and *g*. Fig. 130.

* By a Tube inverting the Objects.

Now it is to be obſerved, that the Clock was before the Heavens full 14 Minutes, as appeared from the Altitudes of *Cor Leonis* which were then taken.

This is truly a noble Obſervation, and therefore we have inquired into the Places of the little Stars then adjoining to the Comet by more than one Method; Mr. *James Pound* affording us his moſt expert Hand and very excellent Inſtruments. Whence it appear'd, that at that Time thoſe little Stars had the following Situations.

		Longitude.			*Latitude.*			
a	♌	29°	54	20″	1	29	20	North.
b		29	27	20	1	8	00	
c		29	34	30	1	10	45	

Now

Now a great Circle drawn through *a* and *c* was found to pass through the last Star of the Tail of the greater Bear, and therefore that the Angle with a Circle of Longitude at *a* was 15°. 36′½. And whereas the Distance of the Comet from *a* towards *c* was something greater than half the Distance *a g*; (which with a Tube of 16 Feet, and a Micrometer, we find to be 21′½,) we may suppose it to have been 12 Minutes. And from what is given the Place of the Comet will come out ♌ 29°. 51′. with North Latitude 1°. 17′¾. The Hour of the Clock being 6, but at *London* 5ʰ. 2′. apparent Time.

Again, *Nov.* 6, in the Morning, at 4ʰ. 42′. with a two Foot Telescope he found the Comet to be just in a right Line between *Mars* and the little Star *N*; which in the *British* Catalogue is the 45 of *Leo*, and was then in ♍ 2°. 42′. with South Latitude 0°. 16′½. but *Mars* at that Time, (by comparing Observations made before and after) was in ♍ 3°. 46′½. with North Latitude 1°. 56′. Whence, because of its Part being given, the Comet was in ♍ 3°. 23′. with North Latitude 6′, at *London* at 3ʰ. 58′. apparent Time, in the Morning.

Fig. 131. Also *Nov.* 11, at 5ʰ. 15′. in the Morning, the Comet was equally distant from the Stars σ and τ of the *Lion*, according to *Bayer*, but had not yet reach'd the right Line joining the same, but was at a little Distance from it. In the *British* Catalogue σ was then in ♍ 14°. 15′. with North Latitude 1°. 41′. nearly; and τ was in ♍ 17°. 3′½. with South Latitude 0°. 34′. Therefore the Latitude of the Comet was something less than a Mean between them, that is, than 0°. 33′½ Northerly; and its Longitude than ♍ 15°. 39′. But this is not to be much rely'd on, since it depends upon the estimated Equality of the Distances, which is a slippery Matter. Now the Tail was not yet begun, except by the Length of half a Degree, view'd by a ten Foot Tube.

He that would know more must have recourse to the Book itself, wrote in the *German* Language.

Observations on a Comet *seen in* 1718, *at* Berlin, *by* *Mr.* G. Kirch, n. 357. p. 820.

XLII. Mr. *Chr. Kirch*, *Jan.* 18, new Stile of the present Year, as he was diligently observing the Motions of the Heavenly Bodies, in the Evening in the Middle of the Week, by chance perceived a Comet towards the North. It was near and to the Right Hand of *Bayer*'s Stars γ and β in the lesser *Bear*, and appear'd much more distinct to the naked Eye than β in *Ursa minor*, tho' that be a remarkable Star of the second Magnitude; it being indeed much paler, yet of a greater Diameter, and of a pretty bright Light, especially towards the Center. When seen through the Tube it shew'd as a bright round little Cloud; but no Foot-steps of a Tail could be observed, nor any Nucleus. It went on with a very swift Motion from the Hour Seven to Eleven, and compleated 4½ Degrees, as was concluded by Observation.

Jan. 19 and 20, the Heavens were cover'd with Clouds. But on the 21*st* the Comet had departed far from its former Place, and was found in *Cassiopea*, where it made a Triangle (was it equicrural?) with the Stars ε and δ, at 5ʰ. 45′. in 17°. 34′. ♉, under Northern Latitude 49°. 54′. Afterwards at 9ʰ. 15′. it was seen in 16°. 38′. ♉, under Northern Latitude 49°. 2′.

2′. But it was much decreaſed, and came ſhort of its former Velocity; as alſo it appear'd paler than before, and being ſeen with the naked Eye, ſeem'd hardly to exceed in Magnitude a Star of the fourth Dignity; and had proceeded in its Orbit not above a Degree and a Half in four Hours and a Half. By the Aſſiſtance of the Tube its Diameter was found to be ſeven Minutes.

Jan. 23, at Four in the Morning, the Comet made an equicrural Triangle with δ and φ in *Caſſiopea*, being diſtant 2°. 41′½. from each. This Morning it hardly moved half a Degree in the Space of two Hours. At Ten in the Evening it was ſeen in a right Line with δ of *Caſſiopea* and φ of *Perſeus*, and was diſtant from the former 3°. 38′. from the latter 3°. 9′. Its Diameter was five Minutes, and view'd with the naked Eye ſeem'd a Star of the fifth Magnitude.

Jan. 24, at Six in the Morning, it had not reach'd φ of *Perſeus*, but made an equicrural Triangle with υ and *g* of the ſame Conſtellation, and was diſtant from each not quite 3° ½. This very accurate Perſon will inform us of more Things from his Obſervations, in that more copious Hiſtory of this Comet, which he is now preparing.

Hitherto the Treatiſe call'd the Literary Journal, *p.* 43, 44, *in which are wanting the Obſervations of the* 18*th Day, when the Comet was neareſt to the Earth, and moved with greateſt Velocity, whence we might make a ſurer Judgment of its true as well as apparent Courſe. Now it is evident that on* Jan. 19, *it paſſed neareſt to the Northern Pole of the Equator. If any one ſhould have a Mind to bring theſe Obſervations to a Scrutiny, and to make them ſubmit to a ſtricter Calculation; for this Purpoſe here are ſubjoined the Places of the fixt Stars here mention'd, taken out of the* Britiſh *Catalogue; whence it will plainly appear, that ſome Things cannot be right in the Deſcription of the Motion of this Comet, which we hope will be corrected in that fuller Account which he promiſes.*

The Places of the fixt Stars at the Beginning of An. 1718.

According to *Bayer.*		Longitude.				North Lat.		
			°	′	″	°	′	″
Of *Urſa Minor*	β	♌	9	18	0	72	58	10
	γ	♌	17	35	15	75	13	15
Of *Caſſiopea*	δ	♉	14	00	35	46	23	25
	ε	♉	20	50	8	47	31	50
	φ	♉	11	36	35	45	4	5
Of *Perſeus*	υ	♉	8	32	0	35	23	45
	ο	♉	10	41	35	36	49	15
	g	♉	12	15	20	36	18	37

XLIII. That

A small Telescopical Comet *seen* June 10, 1717, *by Dr.* E. Halley, n. 354. p. 721.

XLIII. That the Number of Comets is much greater than some, on Account of the late Rareness of their Appearance, have supposed, may be collected from several small ones, which have within few Years been described in the Memoirs of the *French* Royal Academy of Sciences; those Observers assuring us, that they discovered one in *Sept.* 1698. another in *Feb.* 1699, a third in *April* 1702, and again a fourth in *Nov.* 1707, none of which, as far as I can learn, were ever seen in *England*; all of them having been very obscure and without Tails, by Means whereof Comets usually first shew themselves. And besides these, two other Comets with remarkably long Tails, the one in *Nov.* 1689. the other in *Feb.* 1702. past by unobservable in these our Northern Climates, they having great Southing Latitude, and their Motions directed toward that Pole. Hence we may justly conclude, that the Returns of Comets are much more frequent than is vulgarly reckoned, and that it is only contingent, that for these Thirty Five Years no one of them has been seen and observed by our Astronomers.

But there may be still a much greater Number of these Bodies, which by reason of their Smallness and Distance are wholly invisible to the naked Eye; so that unless Chance do direct the Telescope of an Observer, almost to the very Points where they are, it will not be possible for them to be discovered: And that this is not barely a Conjecture, take the following Instance.

On *Monday June* 10, in the Evening, the Sky being serene and calm, directing my Twenty-four Foot Telescope towards *Mars*, I accidentally fell upon a small whitish Appearance near the Planet, resembling in all Respects such a *Nebula* as I described in *Phil. Trans.* N°. 347. but smaller. It seemed to emit from its upper Part a very short Kind of Radiation directed towards the East, but Northerly withal; which, considering its Situation, was nearly towards the Point opposite to the Sun. The great Light of the Moon, then very near it, and also near full, hindered this *Phænomenon* from being more distinctly seen; but its Place in the Heavens was sufficiently ascertained from the Neighbourhood of *Mars*, from whom it was but about half a Degree distant towards the Southwest, the Difference of Latitude being somewhat more than that of Longitude; and *Mars* being at Time in ♐ 17°. 30′ with 3°. 48′ South Latitude. I concluded the Place thereof in ♐ 17°. 12′. with 4°. 12′. Latitude South, or thereabouts; the which may yet be more securely determined by Help of two small fixt Stars I found near it, the more Northerly of which I judged to have the same Latitude with it, and to follow it at about the Distance of six Minutes; the other Star was about four Minutes more Southerly than the former, and about one Minute in consequence thereof; the Angle at the Northern Star was a little obtuse, as of about 100 Degrees, and the Distance of our *Nebula* from it *Sesqualter* to the Distance of the two Stars, or rather a little more. The Reverend Mr. *Williams*, Mr. *Thomas*, and myself, contemplated

Vid. Supr. p. 224.

Fig. 125.

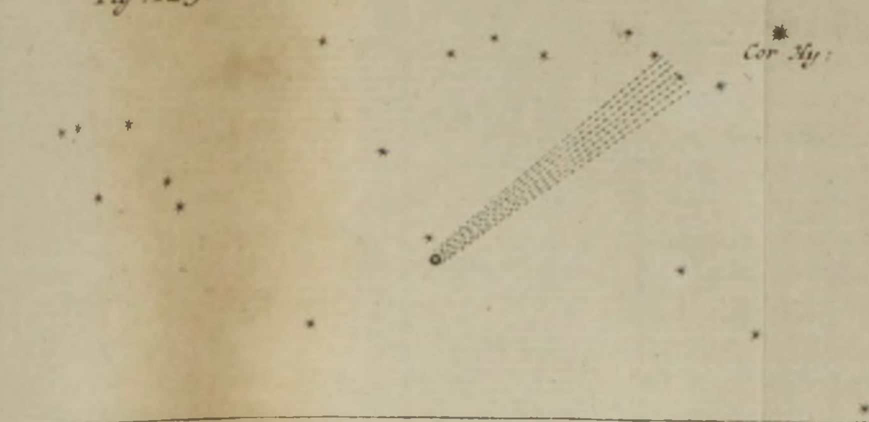


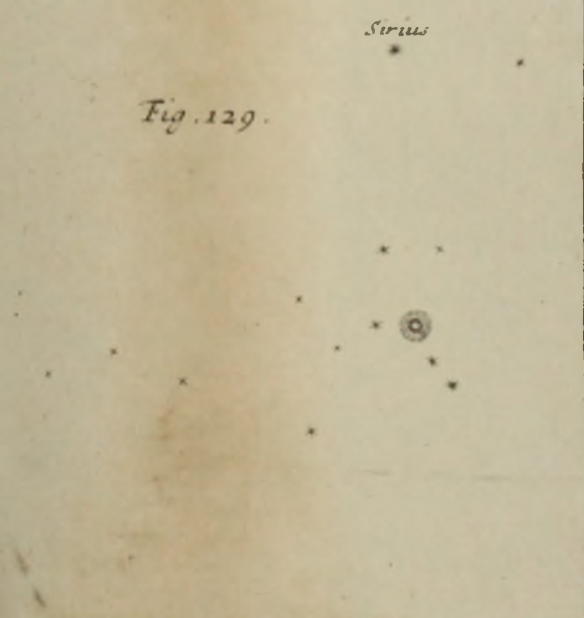
Fig. 126.

Fig. 127.

Hoc schema vero situi Stellaru: propius

Fig. 128.

Fig. 129.



Cursus Cometæ Anni 1680 Observationes primæ Coburgi Saxoniæ a D.no Gottfried. Kirch. habitæ.

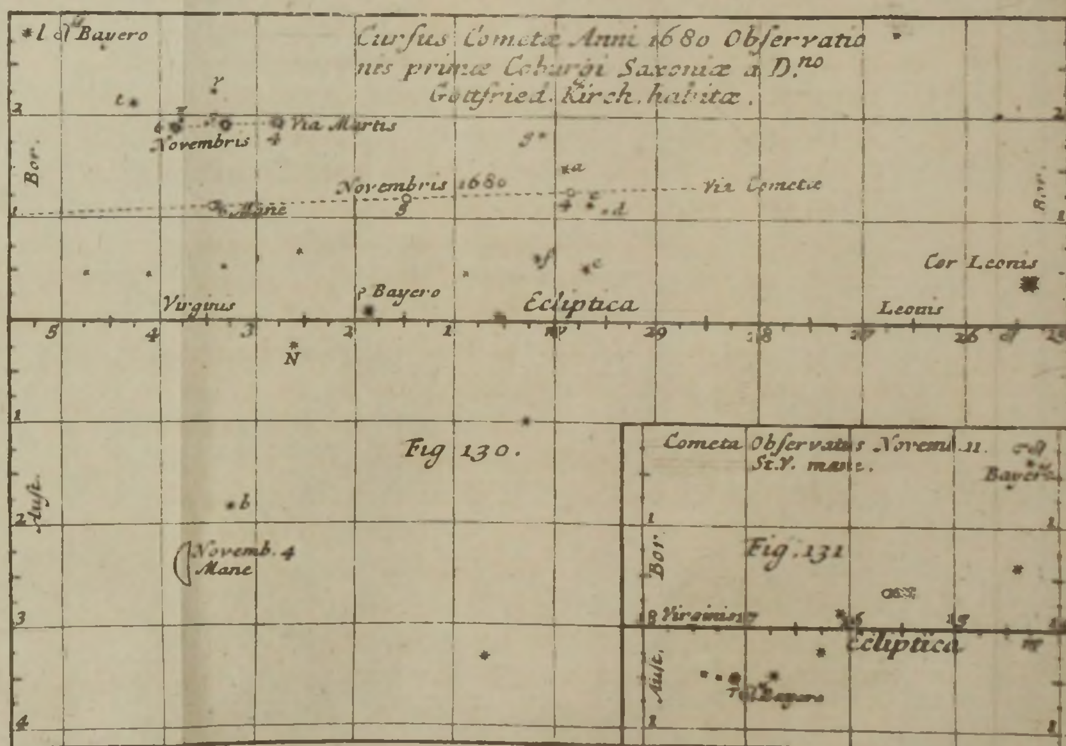


Fig. 130.

Fig. 131.

plated this Appearance for above an Hour, *viz.* from half an Hour past 10 to near 12, and we could not be deceived as to its Reality; but the Slowness of its Motion made us at that Time conclude, that it had none, and that it was rather a *Nebula* than a *Comet.*

However, suspecting that it might have some Motion, I attended the next Night at the same Hours, and in the same Company; when with some Difficulty, by Reason of the Thickness of the Air, we found the two little Stars; but the *Nebula* could not at that Time be seen, which we then imputed to the Want of a clearer Sky: But on *Saturday June* 15, the Moon being absent, and the Air perfectly clear, we had again a distinct View of the two Stars, with an entire Evidence that there remained no Footstep or Sign of it in the Place where we had first seen this *Phænomenon*, which we therefore now found to be a Comet, and that being far without the Orb of the Earth, and in itself a very small Body, it appeared only like a little Speck of a Cloud, such as would scarce have been discerned in an ordinary Telescope, much less by the naked Eye.

XLIV. *Papers omitted.*

1. Extracts from Mr. *Gascoigne*'s and Mr. *Crabtrie*'s Letters, proving Mr. *Gascoigne* to have been the Inventer of the *Telescopick Sights* of Mathematical Instruments, and not the *French*, by the Reverend Mr. *W. Derham*, Prebend of *Windsor*, and F. R. S. n. 352. p. 603.

Mr. *De la Hire*, in the first Part of his *Tabulæ Astron.* published in 1687, having ascribed to Mr. *Picard* the *Application of Telescopick Sights to Astronomical Instruments* (which also was in Effect claimed as his, by Mr. *Auzout*, in a Letter in the *Ph. Trans.* N. 21. in the Year 1666.) Mr. *Derham*, from these Letters of Mr. *Gascoigne* and Mr. *Crabtrie*, proves that Mr. *Gascoigne*, as early as the Year 1640, made Use of these *Telescopick Sights* in two or more Sorts of his *Micrometers*, and in his *Quadrant* and *Sextant.*

2. Astronomiæ *Cometicæ* Synopsis, Autore *Edmundo Halleio* apud *Oxonienses* Geometriæ Professore *Saviliano*, & R. S. S. n. 292. p. 1882.

XLV. *Account of a Book omitted.*

Astronomiæ Physicæ & Geometricæ Elementa, Auctore *Davide Gregorio*, M. D. Astronomiæ Professore *Saviliano*, & R. S. S. *Oxoniæ* 1702. *Folio.* n. 283. p. 1312.

CHAP. IV.

Mechanics. Acoustics.

To find a Solid of the least Resistance, &c. by Mr. J. Craig, n. 268. p. 747. *Dec.* 21.1700.

I. 1. THE Problem of finding a Round Solid, which moving in a Fluid shall suffer the least Resistance, which was formerly solved by the Illustrious *Newton*, has been lately attempted by those great Men the Marquiss of *Hospital* and Mr. *J. Bernoulli*; because Mr. *Newton* thought fit to suppress his Analysis.

Our Solution of the same Problem is as follows.

Lemma. To find the Ratio between the Resistance which the right angled Triangle AIG suffers, and the Resistance which the circumscrib'd Rectangle $AIGg$ suffers, when both are moved in a Fluid according to the Direction of the Line IA, from I towards X.

Fig. 135.

From any Point B let BC be drawn perpendicular to AG; and Bb parallel to AI, also BM perpendicular to AI. Then in Bb take $bH = \frac{CMq}{BC}$, and $bE = BC$; and through the Points H, E, draw the right Lines HA, EA, which produced may cut Gg in K and F. I say the Resistance of the Triangle AIG is to the Resistance of the Rectangle $AIGg$, as the Area of the Triangle AKG is to the Area of the Triangle AFg. Also the Resistance upon any Part of the Line AG is to the Resistance on the corresponding Part of the Line Ag, for Example, on AB and Ab, as the Area AHb to the Area AEb. The Demonstration depends on the general Theorem, which I easily derived from *Prop.* XXXV. *p.* 324. of *Newton*'s *Principia*.

Corol. 1. Now let BG, bg, be infinitely small Parts of the Lines AG, Ag, and let bB be produced to L. I say the Resistance upon BG, (which we may call e) is to the Resistance upon bg, (which we may call E) as GLq to GBq.

For $e.E :: KHbg.FEbg$, that is, $e.E :: bg \times bH . bg \times bE$, (by the foregoing *Lemma*;) therefore $e.E :: bH.bE$, that is, $e.E :: \frac{CMq}{BC}.BC$, (by the Construction of the foregoing *Lemma*;) therefore $e.E :: CMq.BCq$. But $CMq.BCq :: GLq.GBq$, (because of the similar Triangles BMC, GLB,) therefore $e.E :: GLq.GBq$. *Q. E. D.*

Corol. 2. The Resistance upon an infinitely small Part GB is equal to the Cube of the Line GL divided by the Square of the Line GB. For if all the infinitely little Parts in the Line Ag, as bg, are supposed equal, then the Resistance upon bg may be express'd by bg itself, that is, $E = bg$, and therefore $E = GL$. Therefore by the first Corollary, $e.GL :: GLq.GBq$; whence $e = \frac{GL\,cub.}{GBq}$. *Q. E. D.*

Corol. 3.

Corol. 3. Let r be the Radius and c the Circumference of any Circle; I say that the Resistance upon a conical Superficies produced by the Rotation of the little Line GB about AI, is equal to the Product of $\frac{c \times BM}{r}$ into $\frac{CL\ cub.}{GBq}$. For the Resistance upon that Conical Superficies is equal to all the Resistances upon the little Line GB, that is, to all the e's; which is equal to the Circumference of a Circle whose Radius is BM, multiply'd into e. That is, the Resistance upon that Conical Superficies is equal to $\frac{c \times BM}{r} \times e$; and therefore by *Cor.* 2. is equal to $\frac{c \times BM}{r} \times \frac{CL\ cub}{GBq}$. *Q. E. D.*

Problem. To find the Curve Line, by the Rotation of which a round Solid is produced, which if moved in a fluid Medium, according to the Direction of its Axis, shall suffer the least Resistance possible.

Let OG, GB, be two infinitely small Particles in the Curve required, which revolving about AQ may produce the round Solid of least Resistance. Draw BM, GP, perpendicular to AQ, also BL, GN, parallel to AQ, and ON parallel to BM. Now $\frac{c \times BM \times GL\ cub.}{r \times GBq}$ is the Resistance upon the Superficies produced by the Rotation of the little Line GB about AQ, and $\frac{c \times GP \times ON\ cub.}{r \times OGq}$ is the Resistance upon the Superficies produced by the Rotation of OG, by *Cor.* 3. Now these two Resistances taken together, ought to be the least possible, that is, $\frac{c \times BM \times GL\ cub.}{r \times GBq} + \frac{c \times GP \times ON\ cub.}{r \times OGq} = \text{Min.}$ Therefore in the Line RS parallel so to AQ, that $ON = GL$, the Point G is to be sought that this may happen. Now supposing the Points O and B to be fix'd, this will be easily found, by the known Method of *Maxima* and *Minima*. By pursuing the Calculation we shall come at last to $\frac{BM \times BL}{BGqq} = \frac{GP \times NG}{OGqq}$; whence it appears that $\frac{BM \times BL}{BGqq} =$ a constant Quantity. Thus if the Abscifs AM is called x, and the Ordinate BM is y, it will be $BL = \dot{x}$, $LG = \dot{y}$, (which I have supposed to be constant in all this Calculation) and therefore $BGq = \dot{x}\dot{x} + \dot{y}\dot{y}$, whence $\frac{y\dot{x}}{\dot{x}^2 + \dot{y}^2} =$ constant Quantity. Let a be any constant Line, and therefore (that the Law of Homogeneity may be observed) it will be $\frac{y\dot{x}}{\dot{x}^2 + \dot{y}^2} =$ Fig. 136.

$\frac{a}{\dot{y}^3}$, as is found by the illuſtrious M. *Hoſpital* and the famous Mr. *J. Bernoulli.*

And here by the bye I take this Opportunity to ſignify to Mr. *Bernoulli*, that I am very much pleaſed with his Method of conſtructing Curves from Fluxional Equations, in which one of the indeterminate Quantities x or y is wanting; as it is publiſhed in the *Leipſic Acts* for the Month of *May*, *An.* 1700. And from whence he has deduced the Conſtruction of the Curve here required, *Nov.* 1699. *p.* 515.

—— *on the ſame, by Mr.* N. Facio, n. 337. p. 172. *May* 17, 1712.

2. The celebrated Mr. *John Bernoulli* has wrote many Things about my Solution (printed at *London*) of the Problem for finding a round or taper Solid, againſt which the leaſt Reſiſtance ſhall be made. He denies that it is in my Power, tho' I am quite unknown to him, from ſuch a Solution involved in ſecond Fluxions, to go back to *Newton*'s Solution, a like to which *Bernoulli* himſelf has found. And inſinuates by ſuch Aſſertions, that at the Time *Bernoulli* was writing this, ſuch a Regreſs would be eaſy to him. But the Letters of the celebrated *James Bernoulli* are not a little contrary to this, from which it appears, that at the Time he wrote, neither he nor his Brother were acquainted with that Transformation of ours, of Equations in which Fluxions are involved; in which they are multiply'd for duly determining a Product, ſuppoſe for Inſtance $x^n y^\lambda$, or any other complex Quantity. Now under Multiplication alſo Diviſion is contained. Now with this Transformation, found by me in the Years 1687 and 1688, I acquainted Mr. de *Moivre* and Mr. *Huygens*, by whom perhaps the Knowledge might be communicated to others. And upon Inquiry I found, that our moſt worthy Preſident *Newton* at that very Time was not ignorant of it, or rather had found it the firſt of all.

But though I could have anſwered many Ways the Cavils of the famous Mr. *John Bernoulli*, yet I choſe to inſert in the *Leipſic Acts* that Inveſtigation which is far the ſimpleſt of all, and which Mr. *John Bernoulli* could not reject: And from which he might farther underſtand, that he had vainly accuſed of Falſity what I had wrote beſides, of finding the *Line of Quickeſt Deſcent*; duly admitting the Conſideration of the Motion (as it were) of a Ray of Light continually refracted, according to *Fermat*'s Doctrine of Refractions.

What I am to perform is this; I am to commit to Writing an Equation involving only firſt Fluxions, which is rightly deduced, from that Equation which I have exhibited in pag. 16. This is what Mr. *John Bernoulli* required, and aſſerts I cannot find any ſuch. Nor can theſe be committed to Writing, but at the ſame Time a Way will be opened to Mathematicians, for the farther Advancement of the more abſtruſe Geometry.

Our little Treatiſe may be conſulted, as to what belongs to the ſecond Figure of the ſame.

Fig. 137, 138.

In the adjoin'd Figure let C be the Center of the equicurved Circle

A

AEF, which in the Point A may coincide as intimately as may be with the Section of the Solid required, whose Axis is SY. And the Radius of this Circle will be CA, or $u = \frac{3psx}{tt} - \frac{px}{s}$, which was our Solution.

Make as before $AS = x$, perpendicular to the Axis of the Solid YS, the Fluxion of which $AB = \dot{x}$ let it be of an invariable Magnitude, and let $BE = y$ be parallel to the Axis; and again raise the Perpendicular $EG = \dot{x}$, and $GF = \dot{y} + \ddot{y}$ will be parallel to the Axis.

Now it will be $p \,.\, t :: u = \frac{3psx}{tt} - \frac{px}{s} \,.\, \frac{3sx}{t} - \frac{tx}{s} = \frac{3\dot{y}x}{\dot{x}} - \frac{\dot{x}x}{\dot{y}}$; which will be equal to n, or AD, parallel to the Axis, supposing CD to be perpendicular to the same Axis. Make $CD = m$.

Again it will be $p \,.\, s :: u = \frac{3psx}{tt} - \frac{px}{s} \,.\, \frac{3ssx}{tt} - x = \frac{3\dot{y}\dot{y}x}{\dot{x}\dot{x}} - x$; which will be equal to m or CD. The Value of this is indeed unnecessary here, but it will be of Use in what follows.

Now from the Property of the equicurved Circle AEF, producing BA and BE to the other Part of the Circumference, we shall have $\dot{x} \times \overline{2m + \dot{x}} = \dot{y} \times \overline{2n - \dot{y}}$.

And again from the Property of the same Circle, producing GE and GF to the other Part of the Circumference, we shall have $\dot{x} \times \overline{2m + 3\dot{x}} = \overline{\dot{y} + \ddot{y}} \times \overline{2n - 3\dot{y} - y}$.

Therefore substracting the former Equation from the latter, it will be $2\dot{x}\dot{x} = -2\dot{y}\dot{y} - \dot{y}\ddot{y} + 2n\ddot{y} - 3\dot{y}\ddot{y} - \ddot{y}\ddot{y}$.

And those Terms being expunged which are infinitely less than the others, it will be $2\dot{x}\dot{x} = -2\dot{y}\dot{y} + 2n\ddot{y}$

And the Value of n being substituted, it is $\dot{x}\dot{x} = -\dot{y}\dot{y} + \frac{3x\dot{y}\ddot{y}}{\dot{x}} - \frac{x\dot{x}\ddot{y}}{\dot{y}}$; that is $\underset{*}{\dot{x}^3}\dot{y} + \underset{*}{\dot{x}}\,\underset{*}{\dot{y}^3} - 3x\underset{*}{\dot{y}}\underset{*}{\dot{y}}\ddot{y} + x\dot{x}\underset{*}{\dot{x}}\ddot{y} = 0$.

This Equation is composed only of the indeterminate Quantities x, $\dot{y}$, and their Fluxions $\dot{x}$, $\ddot{y}$, and the invariable Quantity $\dot{x}$, and of given Coefficients. And there are two Pairs of Terms in which the same Letters occur, and Powers of Letters, except that the flowing Quantity express'd by one Letter is converted into a Fluxion, or a Fluxion into a Fluent.

a Fluent. Thoſe Pairs of Terms are $\dot{x}^3\dot{y} + x\dot{x}^2\ddot{y}$ and $\dot{x}\dot{y}^3 - 3x\dot{y}^2\ddot{y}$, ariſing only from two generating Terms. For nothing hinders in the whole Equation, but that it may be transform'd, that is, Multiplication being made into $x^{\kappa}\dot{y}^{\lambda}$; the Indices κ and λ being rightly determin'd, that by that Means the new Equation ariſing may become manageable.

Therefore according to our Theory of ſuch Transformations, in the generating Term, from whence ariſes the firſt Pair of Terms mark'd with one Aſteriſk, the Number of Dimenſions of the Indeterminate x, to the Number of Dimenſions of the Indeterminate $\dot{y}$, that is, $1 + \kappa$ will be to $1 + \lambda$, as the Coefficient 1 in the Term $\dot{x}^3\dot{y}$ is to the Coefficient 1 in the Term $x\dot{x}^2\ddot{y}$. Again in the generating Term, from whence ariſes the other Pair of Terms mark'd with two Aſteriſks, the Number of Dimenſions of the Indeterminate x will be to the Number of Dimenſions of the Indeterminate $\dot{y}$, that is, $1 + \kappa$ will be to $1 + \lambda$, as the Coefficient 1 in the Term $\dot{x}\dot{y}^3$ is to the Coefficient -3 in the Term $-3x\dot{y}^2\ddot{y}$; whence it is $\kappa = -\frac{3}{2}$, and $\lambda = -\frac{3}{2}$, and therefore the Multiplier $x^{\kappa}\dot{y}^{\lambda} = x^{-\frac{3}{2}} \times \dot{y}^{-\frac{3}{2}}$.

Therefore it will be $-x^{-\frac{1}{2}}\dot{x}^2\dot{y}^{-\frac{1}{2}} - x^{-\frac{1}{2}}\dot{y}^{\frac{3}{2}} = \pm q$, which is the generating Equation of the former Equation multiply'd by $x^{-\frac{3}{2}}\dot{y}^{-\frac{3}{2}}$; where q is a determinate Quantity. Now if you ſquare this generating Equation, (others call it a Fluent) that the Roots may be taken away, there will ariſe $x^{-1}\dot{x}^4\dot{y}^{-1} + 2x^{-1}\dot{x}^2\dot{y} + x^{-1}\dot{y}^3 = qq$, or $\frac{\dot{x}^4 + 2\dot{x}^2\dot{y}^2 + \dot{y}^4}{x\dot{y}} = qq$. Which is the very Equation of *Newton*, which *J. Bernoulli* has alſo found, and I myſelf have derived formerly, by the moſt eaſy Inveſtigation of all, that can be hoped for this Equation. Now the Quantity qq may be determined, either from the Poſition of the indefinite Axis *YS*, the Point *A*, and the Tangent of the Solid in *A* being given; or from the Poſition of the Point *A*, the Center of the equicurved Circle *C*, and *AD* parallel to the Axis of the Solid being given.

II. *Problem.*

II. *Problem.* To find the Line of swiftest Descent.

To find the Line of Quickest Descent, by Mr. J. Craig, n. 268. p. 750. Dec. 21. 1700.

Fig. 139.

Let BC, CD, be two infinitely small Particles in the Curve required. Now this Curve must be such, that the Passage from B to D, after the Fall from the Horizontal Line AQ, may be perform'd in the shortest Time. Therefore a Point is to be found in the Line RS, (parallel to a AQ in such a Manner, as that the Differences of the Ordinates GC, DE, may be equal) such a Point C that this may happen.

Now the Velocity in the Point C is such, as will be represented by $\sqrt{LC}$, and the Velocity in the Point D will be $\sqrt{QD}$. Therefore $\frac{BC}{\sqrt{LC}}$ is the Time of Descent along BC, and $\frac{CD}{\sqrt{QD}}$ is the Time of Descent along CD; (by *Prop.* 54. *p.* 158. of *Newton.*) Therefore the Point C must be such, that $\frac{BC}{\sqrt{LC}}+\frac{CD}{\sqrt{QD}}=Minim.$ Suppose B and D to be fixt, and make the constant Lines $GC=DE=m$, $LC=b$, $QD=p$; the indeterminate Lines $BG=u$, $CE=z$; whence $\sqrt{\frac{m^2+u^2}{b}}+\sqrt{\frac{m^2+z^2}{p}}=Min.$ Therefore $\frac{u\dot{u}}{b^{\frac{1}{2}}\sqrt{m^2+u^2}}+\frac{z\dot{z}}{p^{\frac{1}{2}}\sqrt{m^2+z^2}}=0$. But $\dot{u}=-\dot{z}$, because $u+z$ is equal to a constant Quantity. Therefore $\frac{u}{b^{\frac{1}{2}}\sqrt{m^2+u^2}}=\frac{z}{p^{\frac{1}{2}}\sqrt{m^2+z^2}}$; whence it appears that $\frac{u}{b^{\frac{1}{2}}\sqrt{m^2+u^2}}$ is equal to a constant Quantity. Now make the Abscis $AL=x$, the Ordinate $LG=y$, and therefore $BG=\dot{x}$, $GC=\dot{y}$, $BC=\sqrt{\dot{x}^2+\dot{y}^2}$, and let a be any constant Line. Then it will be $\frac{\dot{x}}{y^{\frac{1}{2}}\sqrt{\dot{x}^2+\dot{y}^2}}=\frac{1}{\sqrt{a}}$; whence $\dot{x}\sqrt{a}=\sqrt{y}\times\sqrt{\dot{x}^2+\dot{y}^2}$. But in every Curve 'tis $\dot{x} \,.\, \sqrt{\dot{x}^2+\dot{y}^2}$ :: Subtangent. Tangent. Therefore the Nature of the Curve required is such, that the Subtangent is to the Tangent as $\sqrt{a}$ to $\sqrt{y}$. But all know that this is a Property of the Cycloid, who know that the Tangent of the Cycloid is parallel to the Chord of the conterminate Arch in the generating Circle, whose Diameter is a, and whose Vertex points downwards.

And with the same Facility we may find the Curve of swiftest Descent, in any other Hypothesis of Gravity.

By Mr. J. Machin, n. 358. p. 860.

III. To find a Curve which a falling Body would describe in the shortest Time, being urged by a Centripetal Force tending to a given Point;

Point; which Force increases or decreases according to any Power of the Distance from the Center; when the lowest Point of the Curve is given, and the Altitude in the Beginning of the Fall.

Let the Center of Force be C, from which Center, with the Distance CB equal to the Altitude from whence the Body falls, let a Circle BEG be described, and let the Angle BCG be a right Angle. Let A be the lowest Point of the Curve, where it meets the Axis CB at the given Distance CA. 'Tis required to find a Point Q, where the Curve of quickest Descent EQA meets the Circle QF at another given Distance CF. This Problem has two Cases, one of which depends on the Hyperbola and Circle, the other on the Ellipsis and Circle.

Fig. 140. *Cas.* 1. If the Centripetal Force be reciprocally as the Distance from the Center, let KLM be any rectangular Hyperbola, described with Center C and Asymptote CB, which meets the Perpendiculars BK, AM, erected upon BC, in the Points K, M; but any intermediate Ordinate FL, erected at F, in the Point L. Let it be CD to CG as $\sqrt{AFLM}$ to $\sqrt{ABKM}$, and let DH be perpendicular to CG. Then let the Sector RCB be taken to the Area $HDCB$, as the given Hyperbolical Area $ABKM$ to the given Rectangle $CA \times AM$. Then the right Line RC will meet the Circle FQ in the Point Q, which will be in the Curve of swiftest Descent EQA.

Now the Point E may be had, from whence the Fall of the Body should begin, by taking the Sector BCE to the Area of the Quadrant BCG, in the same Ratio as is the Hyperbolic Area $ABKM$, to the Rectangle contained under CA and AM.

Corol. Hence if the right Line RC, revolving about the Center C, makes the Sectors RCB proportional to the Areas $HDCB$, in which the Squares of the Bases CD are taken in Arithmetical Progression; then the right Lines CR will intersect the Curve EQA at Distances from the Center CQ, which will decrease in Geometrical Progression.

Cas. 2. Now if the Centripetal Force should be reciprocally as any other Power of the Distance from the Center, let $n+1$ be the Index of that Power, (where n may be any Number Integer or Fraction, affirmative or negative) and let $H = CB$ be the greatest Altitude of the Curve required EQA, $b = CA$ be the least Altitude of the same, and $A = CF$ be any other intermediate Altitude.

In the right Line CG let there be taken CD to CB as $\sqrt{b^n}$ to $\sqrt{H^n}$,

Fig. 141. and also CH to CD as $\sqrt{A^n - b^n}$ to $\sqrt{H^n - b^n}$. Then Center C, and with the Semiaxes CD, CB, let the Ellipsis BLD de described, which let the Ordinate HL meet in L. And draw the right Line LK, touching the Ellipsis in L, and meeting the lesser Axis CD produced in K. Then draw NM parallel to the Tangent KL, touching the Circle $BEMG$ in M, and meeting CD in N. Lastly, take the Sector RCB, which may be to the Area NMB, LKN, comprehended between the Circle and Ellipsis and the Tangents of each and the right Line NK, in the Ratio of the Number 2 to the Number n. Then the right Line RC will intersect the

the Circle FQ in the Point Q, which will be in the Curve of the quickest Descent EQA.

Now if the Sector BCE be to the Area BDG, intercepted between the Quadrants of the Ellipsis and Circle, in the aforesaid Ratio of 2 to n, that is, the Points L, D, as also M, G, coinciding (because of $A^n = H^n$) the Point E will be that from whence the Fall of the Body should begin which descends to A in the shortest Time, and which by its Motion describes the Curve EQA, which the right Line CE touches in E, and which CB cuts at right Angles in A.

The Demonstrations of these Constructions, which are derived from the Quadratures of the celebrated *Newton*, and from his Principles of natural Philosophy, (*Prop.* 39, *&c.*) shall be given on some other Occasion. Now it is a Problem of another Kind, to describe Curves through which Bodies would move from the highest Point E, which is the Beginning of the Fall, with the swiftest Descent to lower Points Q which are given, when urged by any centripetal Force; the Solution of which Problem I have in my Power. At present it may suffice to have given a general Idea of these Curves, and to shew their Relation to the Quadratures of the Circle and Hyperbola, without which it will hardly be very easy to construct them Geometrically.

IV. We must lay down these three Principles as a Foundation, on which the whole Science of Physicks is to be built. 1. That there is a *Vacuum*, or empty Space. 2. That Quantity is divisible *in infinitum*. 3. That Matter has an attractive Force: That there is a *Vacuum* is evident from the Motion of Bodies: From the Nature of continued Quantity, Geometricians have demonstrated the infinite Divisibility of Matter: And Experience informs us, that Matter has an attractive Force. Now from the two first Principles it follows: *The Laws of Attraction, &c. by Dr.* John Keill, n. 315. p. 97.

Theor. 1. That any the smallest Particle of Matter may so occupy any Space tho' ever so large, that the Diameters of all the Pores or Interstices shall be less than any given right Line, or that all the Particles shall be at a Distance from one another, which shall be less than any given Interval.

Theor. 2. Two Bodies may be given equal in Bulk, but as unequal as you please in Weight or Density, (that is, in Quantity of Matter) in which the Aggregates of the Pores or Interstices shall be nearly equal.

For Instance, let there be a cubical Inch of Gold, and another of Air; tho' the Matter in the Cube of Gold be 20000 Times more than the Matter in the Cube of Air; yet it may be so order'd, that the empty Spaces in the cubick Inch of Gold shall be nearly equal to the empty Spaces in the cubick Inch of Air: That is, that the Vacuities in the Gold shall be to the Vacuities in the Air, as 999999 is to 1000000.

Theor. 3. The Particles which constitute Water, or Air, or any other such Fluid, (if they touch one another) are not absolutely solid, but are composed of other Particles, containing many Pores and Interstices within themselves.

The least Particles of Bodies being absolutely solid, that is without any *Vacuum*, may be call'd Particles of the first Composition. Little Parts arising from several of these Particles growing together may be call'd Particles of the second Composition. Lumps of these made by several of these Parts compounded together may be call'd Particles of the third Composition; and so on, till we at last arrive at

Particles, of which the last Composition of Bodies is made, and into which they are resolved again by their first Resolution.

That there is an attractive Force in Matter, by which every Particle of Matter attracts to itself every other Particle of Matter, and is mutually attracted, was first discover'd by Sir *Isaac Newton* from observing the *Phenomena*. In a given Parcel of Matter this Force is, at different Distances, reciprocally proportional to the Squares of those Distances. From hence arises that Force which we call Gravity, by which all Terrestrial Bodies are urged directly towards the Earth, and is the Weight of Bodies always proportional to the Quantity of their Matter. By extending this attractive Force of Matter, of which he was the first Discoverer, he has most beautifully explain'd all the Motions of the Planets, and the Appearances of Comets.

After frequently revolving in my Mind the Divine Discoveries of this most sagacious Man, I fell at last upon this Thought, that a certain Principle might be apply'd, not unlike to this of *Newton*'s, to the Explaining of the Terrestrial *Phenomena*. After Experiments often repeated, I perceived there was a certain attractive Force in terrestrial Matter; from whence the Reason of many *Phenomena* is to be derived. And these Thoughts of mine about five Years ago I open'd to Mr. *Newton*, and I understood from him, that he had long ago observed the same Things that I had found. Mr. *Newton* proposed some Queries relating to this attractive Force, at the End of his *Opticks*, published in *Latin* about two Years ago. Now as it cannot be expected that that great Man should proceed still to improve these Studies, both by reason of his Age and other Business; I thought it would not be amiss if I should pursue his Steps herein, tho' at a great Distance from him. At present I shall barely propose some *Theorems*, which I may hereafter farther enlarge upon, and give their Demonstrations in a just Volume.

Theor. 4. Besides that attractive Force, by which the Bodies of the Planets and Comets are retain'd in their proper Orbits, there is also another Power in Matter, by which the several Particles of which they are composed attract one another, and are mutually attracted. Which Power decreases in a greater than a duplicate Ratio of the increasing Distance.

This *Theorem* may be proved by a Multitude of Experiments. But the Ratio in which this Power is diminish'd, when the Particles recede from one another, whether it be a triplicate, quadruplicate, or any other Ratio of the increasing Distances, which is greater than the Duplicate, cannot so well be known by Experiment. Yet perhaps a Time will come, when by a more accurate and diligent Enquiry it may be discover'd.

Theor. 5. If a Body consists of Particles, each of which is endued with an attractive Force decreasing in a triplicate, or more than a triplicate Ratio of the Distances; the Force by which a Corpuscle is urged by that Body, in Contact itself, or at an infinitely small Distance from Contact, will be infinitely greater, than if that Corpuscle were placed at a given Distance from the same Body.

See *Prop*. 80, and 91, of *Newton*'s Principles.

Theor. 6. The same Things being supposed, if that attractive Force at an assignable Distance has a finite Ratio to Gravity, the same will be infinitely greater than the Force of Gravity in Contact, or at an infinitely small Distance.

Theor. 7.

Theor. 7. But if in Contact the attractive Force of the Bodies has a finite Ratio to Gravity, the same at every assignable Distance is infinitely less than the Force of Gravity, and will therefore vanish.

Theor. 8. The attractive Force wherewith all the Particles of Matter are endued in Contact, exceeds the Force of Gravity almost immensely; yet is not infinitely greater than the Force of Gravity; therefore in a given Distance that Force will vanish.

Therefore this Force being superadded to Matter, is only diffused thro' Spaces that are vastly little. At greater Distances it is nothing at all. Therefore the Motions of the Heavenly Bodies, which are separated from one another at very great Distances, will not at all be disturbed by this attractive Force, but will continually perform their Courses in the same Manner, as if Bodies had no such Force.

Theor. 9. If any Corpuscle touch a Body, the Force by which that Corpuscle will be urged, that is, the Force by which it adheres to the Body, will be proportional to the Quantity of Contact. For the Parts that are any Thing remote from the Contact will contribute nothing to the Coherence.

Therefore various Degrees of Coherence will arise according to the various Contact of the Particles. But those Forces of Coherence will be greatest of all, when the Superficies are plain by which the Bodies touch one another. In which Case, when other Things are alike, the Force by which a Corpuscle coheres with others, will be as the Parts of the Superficies that touch one another.

Hence appears the Reason, why two Marble-stones that are exactly polished, and touching one another according to plain Superficies, cannot be pull'd directly from each other, unless by a Weight that much exceeds the Gravity of the incumbent Air.

Hence may be derived a Solution of that most celebrated Problem, concerning the Cohesion of Matter.

Theor. 10. Those Corpuscles are most easily separated from one another, the Contacts of which with others are the fewest and the least. Such are the Contacts of Globular Corpuscles that are of an infinitely little Magnitude.

Hence is given the Reason of Fluidity.

Theor. 11. The Force by which any Corpuscle is attracted to another Body which is very near it, does not change its Quantity, whether the Matter of the attracting Body is increased or diminished, the Density of the Body remaining the same, and also the Distance of the Corpuscle.

For since the attracting Forces of the Particles are diffused only through very small Spaces; it is plain that the remoter Parts at *C*, *D*, and *E*, contribute nothing to the Attraction of the Corpuscle *A*. Therefore the Corpuscle will be attracted towards *B* with the same Force, whether these Parts are present or not, or others are annexed to them. Fig. 142.

Theor. 12. If the Texture of any Body be such, that Particles of the last Composition are a little removed from their original Contact by any external Force, such is a Weight compressing them, or a Blow proceeding from another Body, nor do they pass into new Contacts; the Particles mutually approaching by the attractive Force will soon return to their original Contacts. But the same Contacts and Positions returning of the Particles that compose any Body, the same Figure also of the Body will return; and therefore by the attractive Force Bodies may again recover their former Figures which they have lost.

Hence a Reaſon may be given for Elaſticity. For whereas Bodies impinging againſt each other, by their elaſtic Force mutually rebound from one another; from the attractive Force of Bodies, (as is demonſtrated in my Phyſical Lectures) a Reſilition from one another ought thence to ariſe.

Theor. 13. Now if the Texture of a Body be ſuch, that the Particles being removed from their former Contacts by an impreſſed Force immediately come into others which are of the ſame Degree, that Body will not reſtore itſelf to its former Figure.

Hence it may be underſtood, of what Texture ſoft Bodies ſhould be, or in what the Softneſs of Bodies conſiſts.

Theor. 14. The Particles of Matter may be endued with different attractive Forces, according to their different Structure and Compoſition; the Attraction for Inſtance will not be ſo ſtrong when a Particle of a given Magnitude is perforated with ſeveral Pores and Paſſages, as If it were entirely ſolid and without any Vacuity.

Theor. 15. The attractive Forces of perfectly ſolid Particles very much depend upon their Figures.

For if any ſmall Particle of Matter were form'd into a circular Plate of an indefinitely ſmall Thickneſs, and a Corpuſcle were placed in the right Line paſſing through the Center, and were put at a Plain of a perpendicular Circle, and the Diſtance of the Corpuſcle was equal to to a tenth Part of the Semidiameter of the Circle; the Force by which the Corpuſcle is urged will be thirty Times leſs than if the attracting Matter ſhould put on the Form of a Sphere, and the Virtue of the whole Particle were diffuſed as it were from one Phyſical Point. Alſo the ſame circular Plate would attract a Corpuſcle more ſtrongly to itſelf than another Particle of the ſame Weight, which ſhould be formed into a thin and long Cylinder.

Theor. 16. Salts are Bodies whoſe Particles of the laſt Compoſition are endued with a great attractive Force, among which Particles however many Vacuities are interſperſed, which are pervious to the Particles of Water of the laſt Compoſition: Which therefore being ſtrongly attracted by the ſaline Particles, ruſh upon them with Violence, and disjoin them from their mutual Contact, and diſſolve the Coherence of the Salts.

Theor. 17. If two Corpuſcles mutually approach to each other, with attractive Forces decreaſing in a triplicate or more than triplicate Ratio of their Diſtances; their Velocity when they impinge upon one another will be infinitely greater than when they are at a given Diſtance from one another. *See* Newt. *Prin. Pr.* 39.

Theor. 18. The Magnitude of a Body which is heavier than Water may be ſo far diminiſh'd, that at laſt it may float ſuſpended in the Water, and not deſcend by vertue of its Gravity.

Hence appears the Reaſon, why the Particles of Salt, Metal, and others of the like Kind, being reduced to the ſmalleſt Particles, hang as it were ſuſpended in their proper *Menſtrua*.

Theor. 19. Greater Bodies approach to one another with leſs Velocity than leſſer Bodies.

Fig. 143, 144. For the Force by which the Bodies *A* and *B* approach to one another, belongs only to the Particles that are neareſt; for the more remote have no Force at all. Therefore

Therefore a greater Force is not applied to the moving of the Bodies *A* and *B*, than to the moving of the Particles *c* and *d*. But the Velocities of Bodies that are moved with the ſame Force, are reciprocally proportional to thoſe Bodies. Whence the Velocity with which the Body *A* tends towards *B*, will be to the Velocity with which the Particle *c*, not connected with the Body, would tend towards the ſame *B*, as the Particle *c* to the Body *A*. Therefore the Velocity of the Body *A* is much leſs than the Velocity of the Particle *c* would be, if at Liberty from the Body.

Hence it comes to paſs, that the Motion of the greater Bodies is ſo ſlow and languid of its own Nature, that it is commonly hinder'd by an ambient Fluid, and other Bodies that are round about it. But in the ſmaller Corpuſcles the Virtue is vigorous, and very many Effects are produced by them. So much is the Energy greater in the ſmaller Bodies, than in the greater.

Hence appears the Reaſon of that Chemical Axiom, that Salts do not act but when in Solution.

Theor. 20. Two Corpuſcles not touching one another may be placed ſo near, that the Force by which they affect one another, may much exceed the Force of Gravity.

Theor. 21. If a Corpuſcle placed in a Fluid is equally attracted on all Sides by the ſurrounding Particles, no Motion will thence ariſe to the Corpuſcle. But if it be urged more by ſome Particles, and leſs by others, the Corpuſcle will tend that Way where the greateſt Attraction is. And the Motion produced will anſwer the Inequality of Attraction, or in a greater Inequality the Motion will be greater, but leſs in a leſſer Inequality.

Theor. 22. Corpuſcles floating in a Fluid, and attracting one another more than the intermediate Particles of the Fluid; having diſpel'd the Particles of the Fluid they will approach to one another with that Force, by which their mutual Attraction exceeds the Attraction of the Particles of the Fluid.

Theor. 23. If any Body is placed in a Fluid, whoſe Parts more attract the Particles of the Fluid, than the Particles of the Fluid are attracted by one another; and if in the Body there are many Paſſages and Pores that are pervious to the Particles of the Fluid; that Fluid will ſoon diffuſe itſelf thro' theſe Interſtices. And if the Connexion of the Parts of the Body is not ſo firm, but that it may be overcome by the Violence of the ruſhing Particles, a Diſſolution of the immerged Body will thence ariſe.

Hence three Things are required, that a *Menſtruum* may be fit for diſſolving a given Body. 1. That the Parts of the Body may more attract the Particles of the *Menſtruum*, than they are attracted by one another. 2. That the Body may have Pores and Interſtices open and pervious to the *Menſtruum*. 3. That the Coherence of the Particles conſtituting the Body may not be ſo great, but that it may be overcome by the Violence of the ruſhing Particles of the *Menſtruum*. Hence it is alſo plain, that the conſtituent Particles of Spirit of Wine are more attracted by one another, than by the Particles of a ſaline Body immerſed in Spirit of Wine.

Theor. 24. If the Corpuſcles floating in a Fluid, and attracting one another, are Elaſtic, after Congreſs they will rebound from one another. And then other Corpuſcles impinging again, will be again reflected. From whence there will be innumerable Conflicts with other Corpuſcles, and continual Reboundings. But by the attractive Force the Velocity of the Corpuſcles will continually increaſe, and the inteſtine Motion of the Parts will be apparent even to Senſe.

Now

Now as the Corpuſcles may attract one another either more ſtrongly or more weakly, according to the different Elaſticity they are endued with, theſe Motions will be various, and will become ſenſible in different Degrees and at different Times.

Theor. 25. If Corpuſcles that attract touch one another, no Motion will ariſe; for they cannot come nearer than Contact. If they are ſeparated from one another at a very ſmall Diſtance, a Motion will ariſe. But if they are farther from one another, they will not attract one another with a greater Force than the intermediate Particles of the Fluid, and ſo no Motion will be produced.

On theſe Principles depend all the *Phenomena* of Fermentation and Effervescence. Hence the Reaſon appears, why Oyl of Vitriol ferments and bubbles, by the Infuſion of a little Water. For the ſaline Corpuſcles are ſomething removed from mutual Contact by the Affuſion of the Water; and as they attract one another more forcibly than the Particles of Water, and as they are not equally attracted on all Sides, a Motion muſt thence neceſſarily ariſe.

Hence alſo the Reaſon appears, why ſo great an Ebullition is produced, when Filings of Steel are ſuperadded to the foreſaid Mixture. For the Particles of Steel are endued with a great Elaſticity, whence a ſtrong Reflexion ariſes. And hence we may ſee, why certain *Menſtrua* act with a greater Force, and diſſolve any Body ſooner, if they are ſomething diluted with Water.

Theor. 26. If Corpuſcles that attract one another are without any Elaſtic Force, they will not rebound from one another, but will compoſe Lumps or Congeries of their Particles. Hence a *Coagulum* or Curdling is produced. And if the Gravity of the accumulated Particles exceeds the Gravity of the Fluid, a Precipitation or Subſiding will ſucceed. Alſo a Precipitation may ariſe from the Gravity of the *Menſtruum* being either increaſed or diminiſh'd, in which the Corpuſcles floated.

Theor. 27. If the Figures of the attracting Corpuſcles, and thus floating in a Fluid, be ſuch, as that in ſome of their given Parts they are endued with a greater attractive Force than in others, and alſo a greater Contact is in the ſame, thoſe Corpuſcles will go together into Bodies having given Figures, whence Chryſtallizations will be produced. And the Figures of the compounding Corpuſcles may be determin'd by Geometry, from the Figure of the Chryſtal being given.

Theor. 28. If the Corpuſcles are more attracted by the Particles of the Fluid than by one another; they will be made to recede as if they fled from one another, and they will ſoon be diffuſed through all the Fluid.

Theor. 29. If any Corpuſcle intervenes between two Particles of a Fluid, the two oppoſite Faces of which are endued with very great attractive Forces; this intervening Corpuſcle will conjoin the Particles of the Fluid to one another; and many Corpuſcles of this Kind diſperſed through the Fluid will fix all the Particles into a firm Body, or will reduce the Fluid into ſolid Ice.

Theor. 30. If any Body ſhould emit a very great Plenty of *Effluvia*, the attractive Forces of which are very ſtrong; when theſe *Effluvia* approach to any light Body, their attractive Virtues will at laſt overcome the Gravity of the lighter Body; and the *Effluvia* will attract that Body upwards to themſelves; and as the *Effluvia* are much more denſe in leſſer Diſtances from the emitting Body than in greater, the light Body will always be urged towards the denſer *Effluvia*, till at laſt it adheres to the Body itſelf that emits the *Effluvia.* Hence many of the *Phenomena* of Electricity may be explained.

Perhaps it may be objected againſt this Doctrine of ours concerning attractive Forces, that if this Power of Attraction was in all Matter of every Kind, the more ponderous Bodies having more Matter within a given Space ought to attract moſt, or more than lighter Bodies, which is contrary to Experience. But this Objection may eaſily be anſwer'd. For Particles of the laſt Compoſition, (to which only the attractive Virtue is imputed) being placed very thick near one another, may make a Body ponderous, tho' they may be more rare among themſelves than thoſe Particles, which conſtitute a light Body of the laſt Compoſition, being more remote from one another, and having more and wider Vacancies.

There are many other *Phenomena* of Nature, which ſeem to me capable of being explain'd from the ſame Principles; as the Aſcent of the Sap in Plants and Trees, the conſtant and determinate Figures of Leaves and Flowers, and their ſpecifick Virtues, *&c.* And many Things alſo, which daily occur in the Animal Body, eſpecially which have a Relation to the Motion of the Fluids and the Secretions, and depend on the ſame Properties of Matter; and hence the Theories of Diſeaſes and the Effects of Medicines may be eaſily derived.

V. The learned Dr. *Halley* communicated a Theorem to me, by which the Law of the Centripetal Force may be exhibited in finite Quantities, which was ſhew'd him by M. *de Moivre*, who ſaid that Sir *Iſaac Newton* had before found a Theorem which was like this. Now as the Demonſtration of this Theorem is very eaſy, I had a Mind to communicate it to the Publick, with ſome other Thoughts on the ſame Subjects. *The Laws of the* Centripetal Force, *by Dr.* John Keill, n. 317. p. 174.

Theorem. *If a Body moves in any Curve by Means of a Centripetal Force tending to any Center; in every Point of the Curve that Force will be in a Ratio compounded of the direct Ratio of the Diſtance of the Body from the Center of Force, and the reciprocal Ratio of the Cube of a Perpendicular let fall from the Center upon the right Line touching the Curve in the ſame Point, drawn into the Radius of Curvature belonging to the ſame Point.*

Demonſtration. Let QAO be any Curve, which is deſcribed by the Motion of a Body, attracted by a Centripetal Force tending to the Point S. And let AO be an Arch which is deſcribed in any the leaſt Time, P*m* its Tangent, AR the Radius of a Circle of equal Curvature, that is, the leaſt Part of whoſe Periphery coincides with the Arch AO. And let SP be a right Line falling perpendicularly from the Point S upon the Tangent. Draw O*m* and O*n* parallel to SA and SP. And let O*m* denote the Force by which the Body in A is urged towards S. The Force by which Body recedes perpendicularly from the Tangent will be as O*n*; that is, the Force tending towards R, and cauſing the Body, moving with the ſame Velocity as before, to deſcribe a Circle equicurved to the Arch AO, will be to the Force tending towards S, by which the Body moves in the Curve AO, as O*n* to O*m*, or becauſe of equiangled Triangles, as SP to SA. But the Centripetal Forces of Bodies moving in Circles are as the Squares of the Velocities applied to the Radii, by Cor. of Theor. 4. of *Newton*'s Principles. But the Velocity is reciprocally as SP, or directly as $\frac{1}{SP}$ and therefore the Square of the Velocity will be as Fig. 145.

$\frac{1}{SPq}$. Therefore the Force at O*n*, or the Force with which the Body could move in an equicurved Circle, will be as $\frac{1}{SPq \times AR}$. Now it has been ſhewn, that it is SP to SA, ſo is the Force tending towards R, by which the Body may move in an equicurved Circle, to the Force tending towards S. But the Force tending towards R is as $\frac{1}{SPq \times AR}$, and therefore ſince it is SP . SA :: $\frac{1}{SPq \times AR}$. $\frac{SA}{SPc \times AR}$, the Force tending toward S, will be as $\frac{SA}{SPc \times AR}$. *Q. E. D.*

Fig. 146. *Corol.* If the Curve QAO is a Circle, the Centripetal Force tending towards S will be as $\frac{SA}{SPc}$. And therefore if the Centripetal Force tends towards S a Point ſituate in the Circumference, it will be (by 32 of the Third) Ang. PAS = AQS; and therefore becauſe of ſimilar Triangles ASP, ASQ, it will be AQ . AS :: AS . SP, whence SP $= \frac{ASq}{AQ}$, and SP$c = \frac{AScc}{AQc}$, whence $\frac{SA}{SPc} = \frac{SA \times AQc}{AScc} = \frac{AQc}{ASqc}$, that is, becauſe of AQ being given, the Force will be reciprocally as ASqc.

Fig. 147. Let DAB be an Ellipſis, whoſe Axis is DB, and Foci F and S, AR, OR, two Perpendiculars to the Curve very near each other. Draw KL, OT, perpendicular to SA, and KM perpendicular to OR. Becauſe 'tis SA . SK :: FA + SA . FS, (by *Prop.* 3. *El.* 6.) that is, in a given Ratio, the Fluxions of SA, SK, that is, AT, Kk, will be proportional to SA, SK. And it is (by *Propoſition* 6. of Part 4. of *Milns*'s Conic Sections) AL = ½ the Latus Rectum = ½ L. Moreover becauſe of KA parallel to SP, 'tis Ang. ASP = KAL = TOA, becauſe the Angle TAO is the Complement of each to a right Angle. Wherefore KA . AL :: SA . SP, whence SP $= \frac{L}{2} \times \frac{SA}{KA}$, and KA $= \frac{L \times SA}{2\,SP}$. Again, becauſe of equiangled Triangles KMk, GPS, and OTA, SPA.

It is KM . Kk :: GP . GS :: AP . SK,
Alſo Kk . AT :: SK . SA,
Alſo AT . AO :: AP . SA,

It will be KM . AO :: APq . SAq :: SAq — SPq . SAq :: SAq — $\frac{Lq \times SAq}{4\,AKq}$. SAq :: 4AKq — Lq . 4AKq. Whence Lq . 4AKq :: (AO—KM . AO ::) AK . AR; and therefore AR $= \frac{4\,AKq}{Lq}$. And by the ſame Way of Reaſoning in the Hyperbola, the Radius of Curvature will be found $\frac{4\,AKq}{Lq} = \frac{L \times SAc}{2\,SPc}$.

Fig. 148. But in the Parabola the Calculation will be eaſier. For becauſe of the Subnormal being given, 'tis always Kk = AT equal to the Fluxion of the Axis: and the Triangles KkM, ATO, SPA, ALK, are equiangular, whence KM . Kk :: AP . SA; alſo it is AT (Kk) . AO :: AP . SA, whence KM . AO :: APq . SAq :: SAq — SPq . SAq, whence it will be SPq . SAq :: AO — KM . AO :: KA . AR, and therefore AR $= \frac{SAq \times AK}{SPq}$. But AL = ½ the Latus Rectum =

$= \frac{1}{2} L$, and $AK . AL :: SA . SP$; wherefore it will be $\frac{L}{2} \times \frac{SA}{AK} = SP$, and $SPq = \frac{Lq \times SAq}{4AKq}$. Therefore it is $AR = \frac{4AKc}{Lq}$, or because it is $AK = \frac{L \times SA}{2SP}$, it will be $AR = \frac{L \times SAc}{2SPc}$.

And hence arises a most easy Construction, for determining the Radius of Curvature in any Conic Section. For let A K be perpendicular to the Section meeting the Axis in K; from K upon A K let the Perpendicular H K be erected, meeting A S produced in H. From H let H R be erected perpendicular upon A H, and A R will be the Radius of Curvature. In the Parabola the Construction becomes still something more simple. For because by the Nature of the Parabola 'tis $SA = SK$, and the Angle A K H is a right one, S will be the Center of a Circle passing through A K H. Whence the Radius of Curvature is found by producing S A to H, that $SH = SA$, and at H erecting the Perpendicular HR. Then R will be the Center of the Circle that coincides most intimately with the Parabola in A. Fig. 149.

The Centripetal Force tending to the Focus of the Conic Section, in which the Body moves, is reciprocally proportional to the Square of the Distance. For because $AR = \frac{L \times SAc}{2SPc}$ it will be $\frac{AR}{SPc \times AR} = \frac{SA \times 2SPc}{SPc \times L \times SAc} = \frac{2}{L \times SAq}$. That is, because of $\frac{2}{L}$ being given, the Centripetal Force will be as $\frac{1}{SAq}$.

Let B A D be an Ellipsis, which the right Line G E touches in A. And let S P passing through the Center of the Ellipsis, and K A passing through the Point of Contact, be both perpendicular to the Tangent. $SP \times KA$ will be equal to a fourth Part of the Figure of the Axis, or it will be equal to the Square of the lesser Semiaxis, $= BO \times DE$. For because of equiangular Triangles G B O, G L A, G A K, G P S, and G D E, Fig. 150.

$$SP . SG :: \quad BO . GO.$$
$$SG . DG :: BG . LG :: GO . GA.$$
$$DG . DE :: \quad GA . AK.$$

Whence $SP . DE :: BO . AK$, and $SP \times AK = DE \times BO = \frac{1}{4} L \times SB$.

Hence if a Body moves in an Ellipsis with a Centripetal Force tending to the Center of the Ellipsis, that Force will be directly as the Distance. For it is $\frac{SPc \times 4AKc}{Lq}$ = to a given Quantity, because $SP \times AK$ is a given Quantity. Therefore a Force as $\frac{SA}{SPc \times AR}$ will be as the Distance S A.

In Fig. 147. from the other Focus F letting fall a Perpendicular F I upon the Tangent; because of similar Triangles S A P, F A I, it will be Fig. 147.

$SA . SP :: FA . FI = \frac{SP \times FA}{SA}$. Whence it will be $SP \times FI = \frac{SPq \times FA}{SA}$ equal to the Square of the lesser Semiaxis. Whence if the greater Axis be call'd b, and the lesser $2d$, it will be $SPq = \frac{dd \times SA}{b - SA}$, and $SP = \frac{d \times \sqrt{SA}}{\sqrt{b - SA}}$.

But in the Hyperbola it is $SP = \frac{d \times \sqrt{SA}}{\sqrt{b + SA}}$.

In the Parabola it is $SP = \sqrt{d \times SA}$, supposing its Latus rectum $4d$. Because it is $TAq . TOq :: APq . SPq :: SAq - SPq . SPq :: SAq - \frac{ddSA}{b - SA} . \frac{ddSA}{b - SA} :: SA - \frac{dd}{b - SA} . \frac{dd}{b - SA} :: bSA - SAq - dd . dd$, it will be $\sqrt{bSA - SAq - dd} . d :: TA . TO$. And as it is $TA = \dot{SA}$, it will be $TO = \frac{d\dot{SA}}{\sqrt{bSA - SAq - dd}}$.

Now let Q A O be any Curve, one of whose least Arches is A O, the Tangents in the Points A and O are A P and Op, the Radius of Curvature A R, and Perpendiculars upon the Tangents are S P and S p. It will be $\frac{SA \times TA}{fP} = AR$. For because of similar Triangles it is $fP . AO :: PA . RA$. And $AO . TA :: SA . PA$; whence *ex æquo* it will be $fP . TA$ or $\dot{SA} :: SA . RA$. But it is $fP = \dot{SP}$; where-

Fig. 151. fore it will be $RA = \frac{SA \times \dot{SA}}{\dot{SP}}$.

Hence if the Distance S A be drawn into its Fluxion, and divided by the Fluxion of the Perpendicular, we shall have the Radius of Curvature. By which Theorem the Curvature is easily determined in Radial Curves or Spirals. For Example. Let A Q be the Nautical Spiral; because the Angle S A P is given, also the Ratio of S A to S P will be given. Let that Ratio be a to b. Then it will be $SP = \frac{bSA}{a}$, and $\dot{SP} = \frac{b\dot{SA}}{a}$, and $AR = \frac{SA \times \dot{SA}}{\dot{SP}} = \frac{aSA}{b}$. Whence it plainly appears, that the Evolute of the Nautical Spiral is the same Spiral in another Position.

Because $AR = \frac{SA \times \dot{SA}}{\dot{SP}}$, it will be $\frac{SA}{SPc \times AR} = \frac{\dot{SP}}{SPc \times \dot{SA}}$.

And

And from hence again from the given Relation of S A and S P, the Law of the Centripetal Force will easily be found.

Example. Let V A B be an Ellipsis whose Focus is S, the greater Axis $VB = b$, the lesser Axis $2d$, the Latus rectum $2R$. And let V*a*Q be another Curve so related to this, that the Angle V S A may be always proportional to the Angle V S *a*, and let $Sa = SA$. The Law of the Centripetal Force tending to S is required, by which the Body may move in the Curve V *a* Q. Fig. 152.

Because the Angle V S A is to V S *a* in a given Ratio; the cotemporary Increments of these Angles will be in the same Ratio, and let this be the Ratio of *m* to *n*. Whence it will be $ot = \frac{n \times OT}{m}$. But it is $OT = \frac{d\dot{SA}}{\sqrt{bSA - SAq - dd}}$; whence $ot = \frac{nd\dot{SA}}{m\sqrt{bSA - SAq - dd}}$.

Now because it is $SAq + SPq . SPq :: ta \times ta + ot \times ot . ot \times ot :: SAq + \frac{n^2 d^2 \dot{SA}q}{m^2 \, in \, bSA - SAq - d^2} . \frac{n^2 d^2 \dot{SA}q}{m^2 \, in \, bSA - SAq - d^2} :: 1 + \frac{n^2 d^2}{m^2 \, in \, bSA - SAq - dd} . \frac{n^2 d^2}{m^2 \, in \, bSA - SAq - d^2} :: m^2 bSA - m^2 SAq - m^2 d^2 + n^2 d^2 . n^2 d^2$. Whence it will be $\sqrt{m^2 bSA - m^2 SAq - m^2 d^2 + n^2 d^2} . nd :: SA . SP$, and therefore $SP = \frac{ndSA}{\sqrt{m^2 bSA - m^2 SAq - m^2 d^2 + n^2 d^2}}$. Now that the Fluxion of this may be had, for $m^2 bSA - m^2 SAq - m^2 d^2 + n^2 d^2$ let x be wrote, and it will be $SP = \frac{ndSA}{\sqrt{x}}$, and $SPc = \frac{n^3 d^3 SAc}{x^{\frac{3}{2}}}$; and then $\dot{x} = m^2 b\dot{SA} - 2m^2 SA \times \dot{SA}$, and $\dot{SP} = nd\dot{SA} \times x^{-\frac{1}{2}} - \frac{1}{2} \times \frac{ndSA\dot{x}}{x^{\frac{3}{2}}}$. And reducing the Fractions to the same Denominator it will be $\dot{SP} = \frac{nd\dot{SA}x - \frac{1}{2} ndSA\dot{x}}{x^{\frac{3}{2}}}$. And in the Numerator instead of x and $\dot{x}$ putting their Values, and reducing to Order, it will be $\dot{SP} = \frac{nd\dot{SA} \times \frac{1}{2} m^2 bSA - m^2 d^2 + n^2 d^2}{x^{\frac{3}{2}}}$. Whence it will be $\frac{\dot{SP}}{SPc \times \dot{SA}} = \frac{\frac{1}{2} m^2 bSA - m^2 d^2 + n^2 d^2}{n^2 d^2 SAc}$. But $\frac{\dot{SP}}{SPc \times \dot{SA}}$ is as the centripetal Force; therefore this Force will be as $\frac{\frac{1}{2} m^2 bSA - m^2 d^2 + n^2 d^2}{n^2 d^2 SAc}$;

or becauſe of the given $n^2 d^2$ in the Denominator, this Force will be as $\frac{\frac{1}{2}m^2 b\,SA - m^2 d^2 + n^2 d^2}{SA\,c}$. Or inſtead of d^2 putting $\frac{bR}{2}$, the Force will be as $\frac{\frac{1}{2}m^2 b\,SA - \frac{1}{2}m^2 bR + \frac{1}{2}n^2 bR}{SA\,c}$, or becauſe of the given $\frac{b}{2}$, as $\frac{m^2\,SA - R m^2 + R n^2}{SA\,c} = \frac{m^2}{SA\,q} + \frac{R n^2 - R m^2}{SA\,c}$. All which exactly coincide with what is deliver'd by Sir *Iſ. Newton*, in *Prop.* 44. of his *Principia*, concerning the Centripetal Force of a Body moving in the ſame Curve.

Becauſe the Centripetal Force tending to the Point S, by which a Body may move in a Curve, is always as $\frac{S\dot{P}}{SP\,c \times S\dot{A}}$; hence from the Law of the Centripetal Force being given, the Relation of S A and S P may be found, and therefore by the Inverſe Method of Tangents the Curve may be exhibited, which ſhall be deſcribed by a given Centripetal Force.

For Inſtance, let the Force be reciprocally as any Dignity m of the Diſtance; that is, let $\frac{S\dot{P}}{SP\,c \times S\dot{A}} = \frac{b}{a^2 \times SA^m}$. It will be $\frac{S\dot{P}}{SP\,c} = \frac{b\,S\dot{A}}{a^2\,SA^m}$; and taking the Fluents of theſe Fluxions, it will be $\frac{1}{2}\,SP^{-2} = \frac{b\,SA^{1-m} \mp e}{m-1 \times a^2}$. Whence it will be $\frac{\frac{m-1}{2} \times a^2}{b\,SA^{1-m} \mp e} = SP\,q$, and multiplying both the Numerator and the Denominator of the Fraction by SA^{m-1}, and inſtead of $\frac{m-1}{2}\,a^2$ putting d^2, it becomes $\frac{d^2\,SA^{m-1}}{b \mp e\,SA^{m-1}} = SP\,q$. Therefore $SP = \frac{d\sqrt{SA^{m-1}}}{\sqrt{b \mp e\,SA^{m-1}}}$.

Now if the conſtant Quantity $e = 0$, it will be $SP = \frac{d\sqrt{SA^{m-1}}}{\sqrt{b}}$.

Thus if the Force be reciprocally as the Square of the Diſtance, it may be put $SP = \frac{\sqrt{d^2\,SA}}{\sqrt{b}}$, and the Curve will be a Parabola, whoſe Latus Rectum is $\frac{4dd}{b}$. Or it may be $SP = d \times \frac{\sqrt{SA}}{\sqrt{b - SA}}$, and the Curve will be an Ellipſis. Or laſtly, it may be $SP = d \times \frac{\sqrt{SA}}{\sqrt{b + SA}}$, and the Curve becomes an Hyperbola.

If the Force is reciprocally as the Cube of the Distance, it may be ſuppoſed that $\overline{SP} = \frac{d\,SA}{b}$, and the Curve becomes the Nautical Spiral. Or it may be $SP = \frac{d\,SA}{\sqrt{b - e\,SA^2}}$, and the Curve will be the ſame as that, whoſe Conſtruction Sir *Iſ. Newton* has derived from the Sector of the Hyperbola. Or it may be $SP = \frac{d\,SA}{\sqrt{b + e\,SA^2}}$, the Conſtruction of which Curve *Newton* has deliver'd by the Sectors of the Ellipſis, *Cor.* 3. *Prop.* 1. *Lib.* 1. *Princip.*

If the Centripetal Force be reciprocally as the Diſtance, the Relation between S A and S P cannot be determined by an Algebraical Equation; yet the Curve may be conſtructed by the Logarithmic Line, or by the Quadrature of the Hyperbola. For it is then $SP = \frac{d}{\sqrt{b - L.SA}}$, where L . S A denotes the Logarithm of S A.

All theſe Things follow from the now ſo much celebrated Method of Fluxions, of which Sir *Iſaac Newton*, without any Doubt, was the firſt Inventer, as will eaſily be evident to any one, that ſhall read thoſe Epiſtles of his, which were firſt publiſhed by Dr. *Wallis*. Yet afterwards the ſame Arithmetick was publiſh'd by Mr. *Leibnitz* in the *Acta Eruditorum*, with only changing the Name, and the Manner of Notation.

Now let a Body move in the Curve Q A O, by Means of a Centripetal Force tending to S; and let the Velocity of the Body in A be called C. Now the Velocity with which a Body at the ſame Diſtance, and with the ſame Centripetal Force would deſcribe a Circle, may be call'd c. It is plain from the firſt Theorem, that if S A repreſents the Centripetal Force tending to S, the Centripetal Force tending to R, by Means of which the Body with the Velocity C will deſcribe a Circle whoſe Radius is A R, will be repreſented by S P. But the Centripetal Forces of Bodies deſcribing Circles, are as the Squares of the Velocities apply'd to the Radii of the Circles. Therefore it will be $SP . SA :: \frac{C^2}{AR} . \frac{c^2}{SA}$; whence $SP \times AR . SAq :: C^2 . c^2$, and therefore $C . c :: \sqrt{SP \times AR} . SA$. Fig. 145.

If S P coincides with S A, as it is in the Vertices of the Figures, it will be $C . c :: \sqrt{AR} . \sqrt{SA}$. Now if the Curve be a Conic Section, A R the Radius of Curvature at its Vertex will be equal to half the Latus rectum, or $\frac{1}{2}$ L, and therefore the Velocity of the Body in the Vertex of the Section will be to the Velocity of the Body deſcribing a Circle at the ſame Diſtance, in a ſubduplicate Ratio of the Latus rectum to the double Diſtance.

Becauſe

Because it is $A R = \frac{S A \times S \dot{A}}{S \dot{P}}$, it will be $C^2 . c^2 :: \frac{SP \times SA \times S\dot{A}}{S\dot{P}}$. $S A q :: \frac{SP \times S\dot{A}}{S\dot{P}} . S A :: S P \times S\dot{A} . S A \times S\dot{P}$; and therefore from the given Relation of S P to S A, the Ratio of C to c will be given. For Example, if the Centripetal Force be reciprocally as the Dignity m of the Diſtance, that is, if it be $\frac{S\dot{P}}{S P c \times S\dot{A}} = \frac{b}{a^2 S A^m}$; it will be $S\dot{P} = \frac{b S P c \times S\dot{A}}{a^2 S A^m}$, and therefore it will be $C^2 . c^2 :: S P \times S\dot{A} . \frac{b S P c \times S A \times S\dot{A}}{a^2 S A^m} :: a^2 S A^{m-1} . b S P c$. Whencc if we put $S P q = \frac{d^2 S A^{m-1}}{b} = \frac{\frac{m-1}{2} a^2 S A^{m-1}}{b}$, it will be $C^2 . c^2 :: a^2 S A^{m-1} . \frac{m-1}{2} a^2 S A^{m-1} :: m-1 . 2$; and therefore $C . c :: \sqrt{2} . \sqrt{m-1}$.

Now if it be put $S P q = \frac{d^2 S A^{m-1}}{b - e S A^{m-1}} = \frac{\frac{m-1}{2} a^2 S A^{m-1}}{b - e S A^{m-1}}$, it will be $C^2 . c^2 :: a^2 S A^{m-1} . \frac{\frac{m-1}{2} a^2 b S A^{m-1}}{b - e S A^{m-1}}$, that is, as $b - e S A^{m-1}$ to $\frac{m-1}{2} b$; but the Ratio of $b - e S A^{m-1}$ to $\frac{m-1}{2} b$ is leſs than the Ratio of b to $\frac{m-1}{2} b$, or than the Ratio of 2 to $m-1$; whence C will be to c in a leſs Ratio than that of $\sqrt{2}$ to $\sqrt{m-1}$.

In like Manner if there be taken $S P = \frac{d^2 S A^{m-1}}{b + e S A^{m-1}}$ it will be found that C will be to c in a greater Rttio than that of $\sqrt{2}$ to $\sqrt{m-1}$.

Cor. If a Body move in a Parabola, and the Centripetal Force tends to the Center S; the Velocity of the Body will be to the Velocity of the Body deſcribing a Circle at the ſame Diſtance, every where as the $\sqrt{2}$ to 1. For in that Caſe it is $m = 2$, and $m - 1 = 1$. The Velocity of the Body in an Ellipſis is to the Velocity of the Body moving in a Circle at the ſame Diſtance, in a leſs Ratio than that of $\sqrt{2}$ to 1. And the Velocity in an Hyperbola is to the Velocity in the Circle, in a greater Ratio than that of $\sqrt{2}$ to 1.

If a Body moves in the Nautical Spiral, its Velocity is every where equal to the Velocity of a Body deſcribing a Circle at the ſame Diſtance; for in this Caſe 'tis $m = 3$, and $m - 1 = 2$. *Problem.*

Problem. Supposing that the Centripetal Force, (whose absolute Quantity is known,) be reciprocally as the Square of the Distance, and a Body be projected according to a given right Line with a given Velocity; to find the Curve in which the Body will move. Fig. 153.

Let a Body be projected according to a given right Line A B, with a given Velocity C. Now because the absolute Quantity of the centripetal Force is known, the Velocity will thence be given, with which a Body can describe a Circle at the Distance S A, by the same Centripetal Force. For it is equal to that which is acquired while the Body falls through $\frac{1}{2}$ S A, if urged by the same Force. Let that Velocity be *c*. From A upon A B let the Perpendicular A K be erected, and in that take A R, a fourth Proportional to c^2, C^2, and $\frac{SAq}{SP}$; and A R will be the Radius of Curvature in A. From R upon A S let fall the Perpendicular R H, and from H upon A R the Perpendicular H K, and drawing the right Line S K, it will give the Position of the Axis. Make the Angle F A K equal to the Angle S A K; and if F A is parallel to S K, the Figure in which the Body moves will be a Parabola. But if S K meets the Axis in F, and the Points S and F fall on the same Side of the Point *K*, the Figure will be an Hyperbola. If the Points S and F fall on the contrary Sides, the Figure will be be an Ellipsis. Then with the Foci S and F, and with the Axis = S A $\pm$ F A, the Section in which the Body will move may be described.

VI. To find the Curve described by a Body, which is urged by a given Law of Centripetal Force, when projected with a given Velocity from a given Place, according to a given right Line; is a Problem of the greatest Dignity. *Newton*, in his *Mathematical Principles of Natural Philosophy*, gave us long since a compleat Solution of it, granting the Quadratures of Curvilinear Figures. Since which the celebrated Mr. *John Bernoulli* has again undertaken the same Problem, in the *Memoirs of the Academy of* Paris *for Ann.* 1710. I have compared his Solution with that of *Newton*, and made the following Remarks upon them.

The Inverse Problem *of Centripetal Forces; with Remarks on* Bernoulli, *by Dr.* John Keil, n. 340. p. 91. *Nov.* 24. 1713.

Mr. *Bernoulli* premises the same Proposition which *Newton* makes use of, for demonstrating his Problem; which is the XLth in his *Principia*, and is no less elegant than easy to be demonstrated. It is as follows.

If a Body moves any how by Means of a Centripetal Force, and another Body ascends or descends directly, and their Velocities are equal in any Case of equal Altitudes; their Velocities will be equal in all equal Altitudes.

Bernoulli says the Demonstration of this Proposition is deliver'd by *Newton* in too perplex a Manner; and therefore he substitutes his own in its Room, which he calls a more simple one. But give me Leave to say, without offending so great a Man, that if there be any Difference between the Demonstrations of *Newton* and *Bernoulli*, it is this, that *Newton's* Fig. 154.

ton's ſeems to be much the eaſier, and leſs perplex than the other. For if with Center *C* the two Circles *D I*, *E K*, are deſcribed, the Diſtance of which *D E* is as ſmall as may be, and the Velocities of the Bodies in *D* and *I* are equal, and if from *N* to *I K* the Perpendicular *N T* is let fall; *Newton* fully ſhews, that the accelerating Force according to *D E* is to the accelerating Force according to *I K*, as *I N* to *I T*. For if the Force according to *D E* or *I N* be repreſented by the right Lines *D E* or *I N*, that Force, according to *I N*, may be reſolved into two, *T I*, *T N*, of which that only which is as *T I* accelerates the Motion according to the Direction *I K*. But the Accelerations or Increments of the Velocities are as the Forces and the Times in which they are generated conjunctly. But becauſe of the equal Velocities in *D* and *I*, the Times are as the Ways deſcribed *D E*, *I K*. Wherefore the Accelerations in the Motion of the Bodies along the Lines *D E* and *I K*, are as *D E* and *I T*, and *D E* and *I K* conjunctly; that is, as DEq or INq to the Rectangle $IT \times IK$, and therefore, becauſe of $INq = IT \times IK$, the Increments of the Velocities are equal. Therefore the Velocities in *E* and *K* are equal, and by the ſame Way of arguing they will always be found equal at equal Diſtances. This is the Sum of *Newton*'s Demonſtration, which is explain'd ſo clearly by him, that we ſhall find but few eaſier even among the Elementary Propoſitions. But *Bernoulli* does not proceed thus. He is contented to ſay, that Mechanicks ſhew the Force according to *D E* is to the Force according to *I K*, as *I K* to *D E*. Alſo that Mechanicks ſhew the Increments of the Velocities to be in the Ratio of the Forces and Times conjunctly. And that at the Beginning of the Motion ſuppoſing the Velocities to be equal, the Times are as the Ways deſcribed *DE*, *I K*. And hence he concludes, by an Argumentation altogether like that of *Newton*, that the Increment of the Velocity acquired by the Body whilſt it deſcribes *I K*, is to the Increment of Velocity while *D E* is deſcribed, as $DE \times IK$ to $IK \times DE$, and therefore that the Increments of the Velocities will always be equal in equal Diſtances.

But if he had had a Mind to give an eaſy Demonſtration for the Sake of Novices, he ought to have cited the Mechanical Propoſition, and have accommodated it to the preſent Caſe. And indeed there was Occaſion for many Words, that this may become the Theorem which he ſeems to hint at, in which is treated of the Deſcent of heavy Bodies along inclined Plains. For here no Plain is given, which may hinder the direct Deſcent of Bodies. Nay, the Body is ſo far from being hinder'd by a Plain, that on the contrary it is continually attracted by a certain Force from the Plain or Tangent. Therefore without Doubt the Force of his Reaſoning would have been more manifeſt, if forbearing to introduce his Propoſitions of Mechanicks, he had demonſtrated the whole Matter from its own genuine Principles, as *Newton* has done. For by reſolving the right-angled Triangle *K N I* into two ſimilar Triangles, it is *K I* to *I N* as *I N* to *I T*, and therefore inſtead of the Ratio *I N* to *I T*, he might have put the Ratio of *K I* to *I N* or *D E*.

If

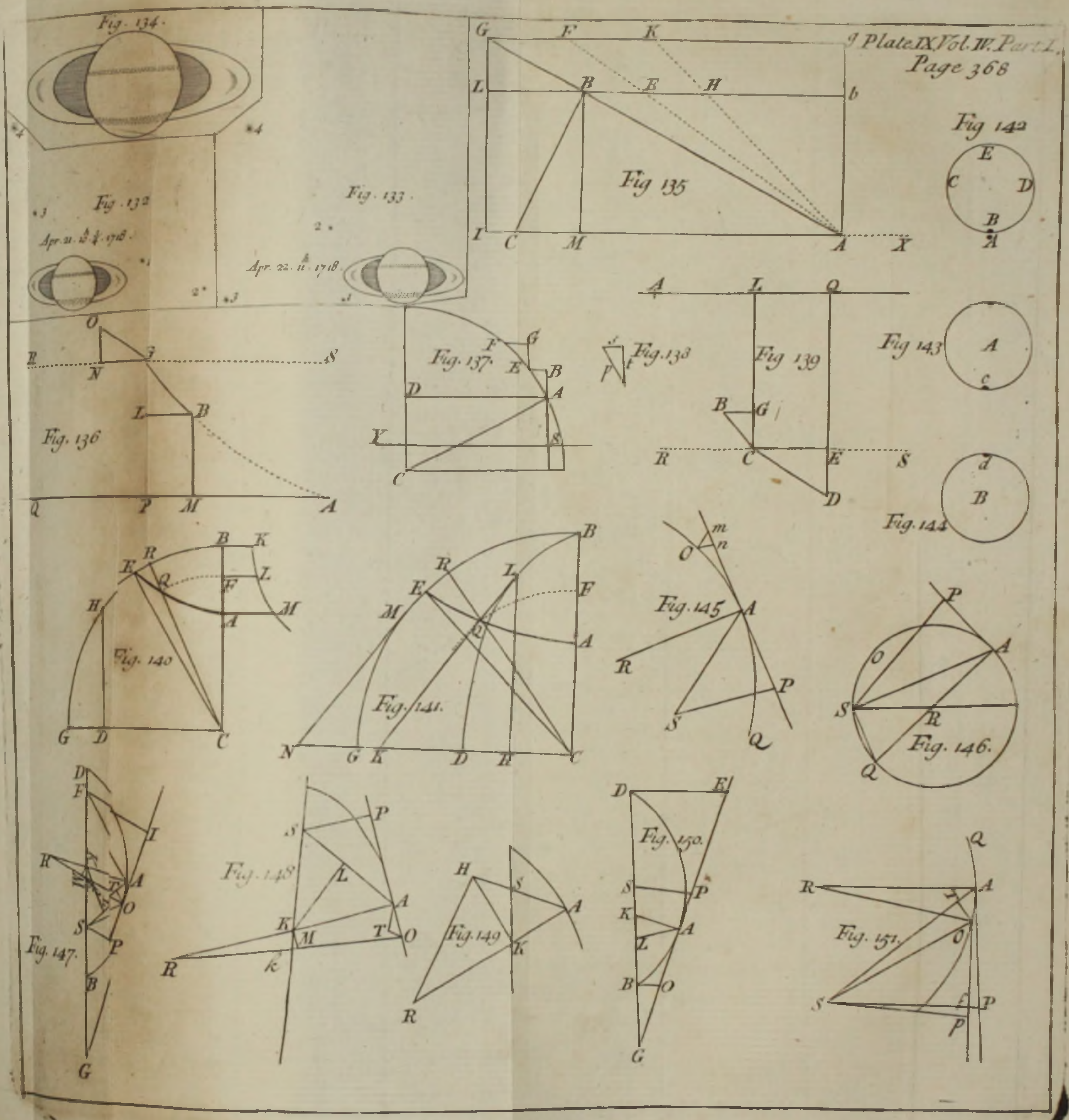


Plate IX Vol. IV. Part I.
Page 368
Fig. 134.
Fig. 132
Apr. 11. 1718.
Fig. 133.
Apr. 22. 11. 1718.
Fig 135
Fig 142
Fig. 136
Fig. 137.
Fig. 138
Fig 139
Fig 143
Fig. 144
Fig. 140
Fig. 141.
Fig. 145
Fig. 146.
Fig. 147.
Fig. 148
Fig. 149
Fig. 150.
Fig. 151.

If the Body falls from any Place A in the right Line AC, and from its Place E a Perpendicular EG be always raiſed, which may be proportional to the Centripetal Force, and if BFG be the Curve Line which the Point G always touches; *Newton* demonſtrates (*Prop.* 39. and 40. of his *Principia*,) that the Velocity of the Body in any Place E is as the Square-root of the Curvilinear Area $ABGE$. Therefore if the Velocity be called v, then v^2 will be as the Area $ABGE$. And if P be the greateſt Altitude to which the Body revolving in the Trajectory can aſcend, when projected upwards from any Point of it with the Velocity which it has there; and if A be the Diſtance of the Body from the Center in any other Point of its Orbit; and if the Centripetal Force be always as any Dignity of A, ſuppoſe as A^{n-1}; the Velocity of the Body in every Altitude A will be as $\sqrt{nP^n - nA^n}$.

In like Manner Mr. *Bernoulli* ſhews, that if the Diſtance from the Center be called x, the Velocity v, and the Centripetal Force φ, it will be $v = \sqrt{ab - Flu{:}\ \varphi\dot{x}}$; where it is plain from Quadratures, that the Area $ABGE = ab - Flu{:}\ \varphi\dot{x}$. Therefore it is all one, whether the Square of the Velocity is expreſs'd by the Area $ABGE$, or by the Quantity $ab - Flu{:}\ \varphi\dot{x}$ which is equal to it. And if the Centripetal Force φ be as nA^{n-1} or nx^{n-1}, it will be $ab = P^n$, and $Flu{:}\ \varphi\dot{x} = A^n$; ſo that $ab - Flu{:}\ \varphi\dot{x}$ is as the Quantity $P^n - A^n$.

Let the Body deſcribe the Curve VK with a Centripetal Force tending to C, and let the Circle VXY be given, deſcribed with Center C and any Radius CV. Let Q be a conſtant Quantity, and make $\frac{Q}{A} = z$. And let KI be an Element of the Curve, IN or DE an Element of the Altitude, XY an Element of the Arch. *Newton* demonſtrates, that the Element of the Arch or XY may be expreſs'd by this Formula $\frac{Q \times IN \times CX}{AA\sqrt{ABGE - zz}}$. Likewiſe from the Premiſes Mr. *Bernoulli* making the Arch $VX = z$, and the Altitude or Diſtance $= x$, reduces the Element of the Arch to this Form $\dot{z} =$

$$\frac{a^2 c\dot{x}}{\sqrt{abx^4 - x^4 \times Flu{:}\ \varphi\dot{x} - a^2c^2x^2}}.$$

Now even at firſt View the Formula of *Newton* ſhould ſeem ſomething more ſimple than that of *Bernoulli*, as conſiſting of fewer Terms. But when I had examin'd the Matter carefully, I ſaw that *Bernoulli*'s Formula exactly coincided with that of *Newton*, and differ'd from it only in the Manner of denoting the Quantities. For if for $ab - Flu{:}\ \varphi\dot{x}$ we put $ABGE$, and for ac

we put $\mathcal{Q}$, and for x we put A, and for a we put CX, and for $\dot{x}$ we put IN; then $\frac{a^2 c \dot{x}}{\sqrt{abx^4 - x^4 \times Flu: \varphi \dot{x} - a^2 c^2 x^2}} = \frac{\mathcal{Q} \times CX \times IN}{\sqrt{A^4 \times ABGE - \frac{\mathcal{Q}^2 Aqq}{Aq}}}$ $= \frac{\mathcal{Q} \times CX \times IN}{A^2 \sqrt{ABGE - \mathcal{Q}^2}}$; or putting zz inſtead of $\frac{\mathcal{Q}^2}{A^2}$, (which *Newton* does for the Sake of a more commodious Notation) *Bernoulli*'s Form becomes $\frac{\mathcal{Q} \times CX \times IN}{A^2 \sqrt{ABGE - zz}}$. Whence it appears, that his Formula does not differ from that of *Newton*, any otherwiſe than as any Thing written in *Latin* Characters would differ from the ſame Thing if written in *Greek* Characters.

After having deliver'd the general Formula, Mr. *Bernoulli* deſcends to a particular Caſe, in which the Centripetal Force is reciprocally as the Square of the Diſtance. And through various Reductions and troubleſome Operations, he ſhews the Conſtruction of Curves which may be deſcribed by means of that Centripetal Force, and by reducing them to Equations he proves they are Conic Sections. Then he complains, that *Newton* ſuppoſes, without any Demonſtration, that Curves deſcribed with ſuch a Force would be Conic Sections.

It is impoſſible he ſhould think, that *Newton* was not acquainted with the Demonſtration of this Matter. For he knew very well that *Newton* was the firſt and only one that had treated of this Doctrine about Centripetal Forces in a Geometrical Manner, and had brought it to ſuch Perfection. *Bernoulli* alſo knew, that beſides giving the general Solution of the Inverſe Problem, *Newton* had ſhew'd how Curves might be conſtructed, which are deſcribed by a Centripetal Force decreaſing in a triplicate Ratio of the Diſtance, and therefore he could not be ignorant of that other Caſe. Nor indeed can I underſtand for what Reaſon *Bernoulli* objects to *Newton*, that he had omitted the Demonſtration of this Caſe; ſince he himſelf has often propoſed Theorems, whoſe Demonſtrations he has no where given. And why may not *Newton* do the ſame, when in Haſte to proceed to other Matters? But now in the new Edition of the *Principia*, he has a Demonſtration of this very Thing, which, though very ſhort, is yet much eaſier and clearer than that of *Bernoulli*.

Laſtly, that *Bernoulli* might ſhew the Neceſſity of his Demonſtration of the Inverſe Problem in this particular Caſe, he thus adds. It muſt be conſider'd, ſays he, that the Force which cauſes a Body to move in the Logarithmic Spiral muſt be reciprocally as the Cube of the Diſtance from the Center. But it does not follow from thence, that ſuch Curves muſt always be deſcribed with ſuch Forces, ſince the like Forces may alſo be the Cauſe, that the Body may move in the Hyperbolical Spiral.

I

I wonder truly how this great Man could imagine, that *Newton* ever drew ſuch a Conſequence. For beſides the Logarithmic Spiral *Newton* ſhews, how other Curves, infinite and different in Number, may be form'd, all which may be deſcribed with the ſame Centripetal Force as the Logarithmic Spiral. And among theſe this very Hyperbolical Spiral may be reckon'd, as we ſhall ſhew hereafter.

Now from hence *Newton* concludes, that only Conic Sections can be deſcribed by a Centripetal Force which is reciprocally proportional to the Square of the Diſtance; becauſe the Curvature of any Orbit is given, by having given the Velocity, the Centripetal Force, and the Poſition of the Tangent. But the Focus being given, the Point of Contact, and the Poſition of the Tangent, a Conic Section may always be deſcribed, which ſhall have a given Curvature. This I have ſhewn above. Therefore, by Virtue of this Force, the Body ſhall move in this Curve, and no other. Since a Body ſetting out from the ſame Place, according to the ſame Direction, with the ſame Velocity, and urged by the ſame Centripetal Force, cannot deſcribe different Courſes. Vid. Supra S. V.

In Imitation of Mr. *Bernoulli*, let me attempt to reſolve this Problem of the Inverſe Method of Centripetal Forces, but after a very different Manner; and alſo to apply it to a particular Caſe, in which the Force is reciprocally as the Cube of the Diſtance; and at the ſame Time produce a Demonſtration of *Cor.* 3. *Prop.* 41. of *Newton*'s *Principia*.

In Order to do this I muſt premiſe a few Things which I have already explain'd above. Vid. Supra S. V.

Let VIL be any Curve which a Body deſcribes by Means of a Centripetal Force tending to the Center C. Let this Curve be touch'd by the right Lines IP, Kp, in two Points I and K which are infinitely near; to which from the Center let fall the Perpendiculars CP, CF, and with Center C let the Circles KE and ID be deſcribed, and draw CI. Fig. 155.

The Centripetal Force will be as the Quantity $\frac{Pp}{PCc \times IN}$; which Theorem, though we have demonſtrated it before, yet here is another Demonſtration. From K draw Km parallel to CP, and Kn to CI. Then becauſe of like Triangles ICP, IKN, and nKm, alſo IKm and IpP, it will be

$$Ip \text{ or } IP \,.\, IK :: pP \,.\, Km,$$
$$PC \,.\, IP :: Km \,.\, mn,$$
$$IN \,.\, IK :: mn \,.\, nK. \quad \text{Therefore } ex\ \textit{æquo},$$
$$PC \times IN \,.\, IKq :: pP \,.\, nK,$$
$$\text{Therefore } nK = \frac{pP \times IKq}{PC \times IN}.$$

Now the Time in which the Arch IK is deſcribed, is as the Area or Triangle ICK, or its double $PC \times IK$; therefore if the Time be given, $PC \times IK$ will be a conſtant Quantity. But in a given Time the Centripetal Force will be as the little Line Kn, which is deſcribed by that Force; and

and therefore the Centripetal Force is as that little Line Kn drawn into the constant Quantity $\frac{1}{PCq \times IKq}$; that is, the Centripetal Force will be as $\frac{1}{PCq \times IKq} \times \frac{Pp \times IKq}{PC \times IN}$, or as the Quantity $\frac{Pp}{PCc \times IN}$. *Q. E. D.*

The Velocity of the Body in any Place is as the Path described in any the least Time directly, and as that Time inversely; and therefore is as $IK \times \frac{1}{PC \times IK}$; that is, the Velocity will be reciprocally as the Perpendicular from the Center to the Tangent.

If the Distance of the Body from the Center be called x, and the Perpendicular upon the Tangent p; it will be $IN = \dot{x}$, and $Pp = \dot{p}$, and the Centripetal Force may be expounded by the Quantity $\frac{f^4 \dot{p}}{p^3 \dot{x}}$, by assuming any Quantity for f^4.

Therefore if with Mr. *Bernoulli* we call the Centripetal Force φ, then $\frac{f^4 \dot{p}}{p^3 \dot{x}} = \varphi$, and $\frac{f^4 \dot{p}}{p^3} = \dot{x}\varphi$; and taking the Fluents of these Quantities, it will be $\frac{f^4}{2p^2} = Flu\!: \dot{x}\varphi$.

Now since the Velocity of the Body is reciprocally as the Perpendicular p, its Square may be expounded by $\frac{f^4}{2p^2}$. If therefore the Velocity is call'd v, it will be $v^2 = \frac{f^4}{2p^2}$, which is equal to the Fluent of the Quantity $\dot{x}\varphi$. And if A be the Place from whence a Body should fall, in order to acquire the Velocity v in D or I, and from the Place of the Body D the Perpendicular $DF = \varphi$ be erected, then the Rectangle $DE \times DF = \dot{x}\varphi$. Now let the Curve BFG be the Curve-line, whose Ordinates expound the Centripetal Forces, or the Quantities φ. The Flowing Quantity of $\dot{x}\varphi$ will be the Curvilinear Area $ABFD = v^2 = \frac{f^4}{2p^2}$, and therefore v will be as the Square-root of the Area $ABFD$. Now if the Velocity be such as is acquired by falling from an infinite Distance, then v^2 or the Fluent of $\dot{x}\varphi$ will be equal to the Area $ODFO$ indefinitely extended.

Hence

Hence the Quantity p will always be given in finite Terms, when that Curvilinear Area can be expreſs'd in finite Terms. For Inſtance, let the Centripetal Force be reciprocally as the Dignity m of the Diſtance, that is, let $\dot{x}\,\varphi = \frac{g\dot{x}}{x^m}$. If the Velocity of the Body be ſuch as is acquired by falling from an infinite Diſtance, then $v^2 = \frac{g}{m-1 \times x^{m-1}} = \frac{f^4}{2p^2}$.

And in all theſe Caſes the Area indefinitely extended will be a finite Quantity. But the Body may revolve in a Trajectory with a Velocity, the Square of which may be either greater or leſs than the Quantity $\frac{g}{m-1 \times x^{m-1}}$, or what is equal to it. Therefore it will be $v^2 = \frac{f^4}{2p^2}$

$$= \frac{g}{m-1 \times x^{m-1}} \pm e^2.$$

Hence by the Action of theſe Forces, three kinds of Curves may be deſcribed, according as e^2 is a poſitive, or negative, or no Quantity.

For Example, if the Velocity is greater than that which is acquired by falling from an infinite Diſtance, it will be $\frac{f^4}{2p^2} = \frac{g}{m-1 \times x^{m-1}} + e^2$.

If the Velocity be leſs, it will be $\frac{f^4}{2p^2} = \frac{g}{m-1 \times x^{m-1}} - e^2$. If equal, it will be $\frac{f^4}{2p^2} = \frac{g}{m-1 \times x^{m-1}}$.

Make $\frac{1}{2}f^4 = a^2e^2$, and $\frac{1}{m-1}g = b^2e^2$, and if the Velocity of the Body be ſuch as is acquired by falling from an infinite Diſtance, it will be $p^2 = \frac{a^2x^{m-1}}{b^2}$, or $p = \frac{ax^{\frac{m-1}{2}}}{b}$.

But if the Velocity be either greater or leſs than this Velocity, it will be as has been ſhewn, $\frac{f^4}{2p^2} = \frac{g}{m-1 \times x^{m-1}} \pm e^2 = \frac{\frac{1}{m-1}g \pm e^2x^{m-1}}{x^{m-1}}$.

Where for $\frac{1}{2}f^4$ and $\frac{g}{m-1}$ putting their Values a^2e^2 and b^2e^2, it will be $\frac{a^2e^2}{p^2} = \frac{b^2e^2 \pm e^2x^{m-1}}{x^{m-1}}$, or $\frac{a^2}{p^2} = \frac{b^2 \pm x^{m-1}}{x^{m-1}}$; and $p^2 = \frac{a^2x^{m-1}}{b^2 \pm x^{m-1}}$.

Therefore if the Centripetal Force be reciprocally as the Cube of the Diſtance,

Distance, that is, if it be $m = 3$, and $m - 1 = 2$; then $p^2 = \frac{a^2 x^2}{b^2}$, or $p^2 = \frac{a^2 x^2}{b^2 + x^2}$, or lastly $p^2 = \frac{a^2 x^2}{b^2 - x^2}$.

In the first Case it is plain, that the Curve will be a Logarithmic Spiral; for let $p = \frac{ax}{b}$, or $b \,.\, a :: x \,.\, p$, and therefore because of the constant Ratio b to a, the Angle CIP will be constant.

Now let us suppose it is $p^2 = \frac{a^2 x^2}{b^2 + x^2}$, from which Supposition three different Species of Curves will arise, according as a is greater, or equal to, or less than b.

Fig. 156. And first, let a be greater than b. With Center C, at any given Distance, let the Circle HYX be described, which let the right Lines CK, CI, produced meet in Y and X. And it is $INq \,.\, KNq :: IPq \,.\, PCq$, and therefore $CIq - PCq \,.\, PCq :: x^2 - p^2 \,.\, p^2 :: x^2 - \frac{a^2 x^2}{b^2 + x^2} \,.\, \frac{a^2 x^2}{b^2 + x^2} :: 1 - \frac{a^2}{b^2 + x^2} \,.\, \frac{a^2}{b^2 + x^2} :: b^2 + x^2 - a^2 \,.\, a^2$. Therefore it will be $\sqrt{x^2 + b^2 - a^2} \,.\, a :: IN \,.\, KN :: \dot{x} \,.\, \frac{a\dot{x}}{\sqrt{x^2 + b^2 - a^2}} = KN$. And because a is greater than b, the Quantity $b^2 - a^2$ will be negative. Let it be $-c^2$, whence $KN = \frac{a\dot{x}}{\sqrt{x^2 - c^2}}$. Let the Radius of the Circle YH be called b, and it is $CK \,.\, KN :: CY \,.\, YX$; that is $x \,.\, \frac{a\dot{x}}{\sqrt{x^2 - c^2}} :: b \,.\, \frac{ba\dot{x}}{x\sqrt{x^2 - c^2}} = XY = \dot{y}$, if the Arch HY be called y.

Let it be $x = \frac{c^2}{z}$, whence $\dot{x} = -\frac{c^2 \dot{z}}{z^2}$, and $\frac{\dot{x}}{x} = -\frac{\dot{z}}{z}$. Also it will be $x^2 - c^2 = \frac{c^4}{z^2} - c^2 = \frac{c^4 - c^2 z^2}{z^2} = \frac{c^2}{z^2} \times \overline{c^2 - z^2}$. Whence $\sqrt{x^2 - c^2} = \frac{c}{z} \times \sqrt{c^2 - z^2}$. Which Values being substituted, it will

will be $\frac{ba\dot{x}}{x\sqrt{x^2-c^2}} = \frac{-ba\dot{z}}{c\sqrt{c^2-z^2}}$. Let it be $a \,.\, c :: n \,.\, 1$, that is, let $a = nc$, and it will be XY or $\dot{y} = -\frac{nb\dot{z}}{\sqrt{c^2-z^2}}$. But it is $\frac{nb\dot{z}}{\sqrt{c^2-z^2}} \,.\, \frac{c\dot{z}}{\sqrt{c^2-z^2}} :: nb \,.\, c$, that is, in a given Ratio. And therefore their Fluents, if they begin together, will be in the ſame Ratio. That is, it will be HY or y to the Fluent of the Quantity $\frac{c\dot{z}}{\sqrt{c^2-z^2}}$, as nb to c.

Now if Center C and Radius $CV = c$ a Circle VL be deſcribed, and if $CG = z$, and $no = \dot{z}$, then the Arch $mn = \frac{c\dot{z}}{\sqrt{c^2-z^2}} =$ Fluxion of the Arch Qm when the Fluxion is a poſitive Quantity; but when it is negative, its Fluent is the Arch Vm the Complement of the former. For an Arch and its Complement have the ſame Quantity denoting its Fluxion, but affected with contrary Signs; for when one increaſes, the other decreaſes.

Hence it is $HY \,.\, Vm :: nb \,.\, c$; but it is $CV \,.\, CH :: Ve \,.\, HY$, that is, $c \,.\, b :: Ve \,.\, \frac{b \times Ve}{c} = HY$. Therefore it will be $\frac{b \times Ve}{c} \,.\, Vm :: nb \,.\, c$. Whence $Ve \,.\, Vm :: n \,.\, 1$.

Moreover from the Nature of the Circle it will be $CG \,.\, CV :: CV \,.\, CT$, when mT touches the Circle; that is, it will be $z \,.\, c :: c \,.\, \frac{c^2}{z} = CT = x$. Hence if the Angle VCe be taken to the Angle VCm as n to 1, and Ce be produced to K, ſo that CK may be equal to the Secant CT, K will be a Point in the Curve required.

Here it may be obſerved by the Way, that if n be a Number, that is, if it be a to c, or a to $\sqrt{a^2-b^2}$ as Number to Number, the Curve VI will be an Algebraical one. For in this Caſe the Relation of mC to the Sine of the Angle VCe is defined by an Equation, and thence will be had the Relation of the Sine of the Angle VCe to CT, or CK, by ſome determinate Equation; and then at laſt an Equation will be given, which will expreſs the Relation between the Ordinate and Abſciſs, beginning from the Point C.

The Orders and Degrees of theſe Curves, in the Algebraick Scale of Equations, will be different according to the Magnitude of the Number

n.

n. In all these Curves thus described, the Position of the Asymptote may be thus determined. Make the Angle VCL to a right Angle as n to 1. In that Angle the Distance of the Body from the Center becomes infinite. Now the Square of the Perpendicular upon the Tangent PC is equal to $\frac{a^2 x^2}{b^2 + x^2}$, when x is infinite, it becomes $PCq = \frac{a^2 x^2}{x^2}$, or $PC = a$. Therefore let CR be drawn perpendicular to CL, and equal to the right Line a; and if through R be drawn RS parallel to the right Line CL, this will touch the Curve at an infinite Distance, or will be an Asymptote to the Curve.

If in any of these Curves the Body by descending should arrive at the lowest Apsid; it will again ascend from hence *in infinitum*, and will describe another Curve similar to the former, or rather a like Portion of the same Curve by its Ascent.

These Curves may wind about the Center with many Circumvolutions, before they begin to converge towards their Asymptote, and the Angular Motion of the right-Line CK will be equal to so many right Angles as the Number n consists of Units. For Instance, if $n = 100$, twenty-five intire Revolutions will be compleated before the Distance from the Center becomes infinite.

If the Number n is increas'd, a remaining the same, c will be diminish'd. For it is $\frac{a}{n} = c$, and $\frac{a^2}{n^2} = c^2 = a^2 - b^2$; whence $\overline{n^2 - 1} \times a^2 = n^2 b^2$, and therefore $a^2 . b^2 :: n^2 . n^2 - 1$. Therefore if b^2 approaches to Equality with a^2, also $n^2 - 1$ will approach to a Ratio of Equality with n^2, and therefore n will be increased, and c will be diminished in the same Ratio. Therefore let b^2 be supposed nearly equal to a^2, so that when the Difference is infinitely little, let the Number n be infinitely great, and let the Radius of the Circle c be infinitely little, or let the Circle be contracted into its Center. But although c vanishes thus, yet CT will not vanish in like Manner, if the Angle VCM be nearly a right Angle. For in every Circle, though never so little, the Secant of a right Angle will be an infinite Quantity. Therefore because of the infinite Number n, this Curve will wind about the Center with infinite Revolutions, before it will begin to converge towards its Asymptote.

But when c vanishes, it is $b = a$, and $p = \frac{ax}{\sqrt{x^2 + a^2}}$. And because in every Case it is $\dot{y} = \frac{ba\dot{x}}{x\sqrt{x^2 + c^2}}$, when c vanishes it will be $\dot{y} = \frac{ba\dot{x}}{x^2}$. Whence taking the Fluents it will be $y = \frac{ba}{x}$, or $xy = ba$, which is a given Quantity.

This

This Curve is the Hyperbolic Spiral, which has many remarkable Properties. If any Radius CIY be drawn, meeting the Curve in I, and the Periphery of the Circle in Y, and if from C a Perpendicular $C\mathcal{T}$ is raised to CI, and $I\mathcal{T}$ touches the Curve in I, and meets the right Line $C\mathcal{T}$ in $\mathcal{T}$; then $C\mathcal{T}$ will be a constant right Line, which will be equal to the Arch VE. In which Property it resembles the Logarithmic Curve, since $C\mathcal{T}$ may be called the Subtangent of the Curve. For let the Radius of the Circle CE be b, the Arch VE be a, and let CI be called x, and VY be y. Because it is $ba = xy$, it will be $\frac{ba}{x} = y$, and $\frac{ba\dot{x}}{x^2} = \dot{y}$. Also it is $CY \,.\, CI :: YX \,.\, NK$, that is, $b \,.\, x :: \frac{ba\dot{x}}{x^2} \,.\, NK$, which therefore is $\frac{a\dot{x}}{x}$. And because it is $IN \,.\, NK :: CI \,.\, C\mathcal{T}$, that is $\dot{x} \,.\, \frac{a\dot{x}}{x} :: x \,.\, C\mathcal{T} = a$.

If with Center C, and any Distance CG, an Arch of a Circle GF is described, this Arch being intercepted between the right Line CV and the Curve, will always be equal to the constant right Line $C\mathcal{T}$ or a. For because it is $VL \times CF = CV \times VE$, it will be $VL \,.\, VE :: CV \,.\, CF :: VL \,.\, GF$; whence $VE = GF$. If from C be raised $CR = VE = FG = a$ perpendicular to CG, and through R be drawn RS parallel to CV, RS will be the Asymptote to the Curve. For the right Line MS is equal to the Arch GF, and therefore FS, the Distance of the Curve from RS, is always equal to the Excess by which the Arch exceeds its Sine. But when the Distance increases *in infinitum*, that Excess will be diminish'd *in infinitum*, and at last will be less than any given Line, and therefore RS will be an Asymptote to the Curve.

Now let b be greater than a; and in like Manner it will be found, as in the former Case, that $KN = \frac{a\dot{x}}{\sqrt{x^2+b^2-a^2}}$. But because b exceeds a, therefore $c^2 = b^2 - a^2$ will be a positive Quantity, and $KN = \frac{a\dot{x}}{\sqrt{x^2+c^2}}$. Then making the Radius of the Circle $HY = b$, we shall find $XY = \frac{ba\dot{x}}{x\sqrt{x^2+c^2}}$. Make $x = \frac{c^2}{z}$, and it will be $\dot{x} = -\frac{c^2\dot{z}}{z^2}$, and $\frac{\dot{x}}{x} = -\frac{\dot{z}}{z}$. It will be also $x^2 = \frac{c^4}{z^2}$, and $x^2 + c^2 = \frac{c^4}{z^2} + c^2 = \frac{c^4 + c^2z^2}{z^2} = \frac{c^2}{z^2} \times \overline{c^2+z^2}$. Whence $\sqrt{x^2+c^2} = \frac{c}{z} \times \sqrt{c^2+z^2}$.

Therefore these Values being substituted, it will be $\frac{ba\dot{x}}{x\sqrt{x^2+c^2}} = -\frac{ba\dot{z}}{c\sqrt{c^2+z^2}} = -\dot{y}$. For the Beginning of the Arch HY may be taken such, that it may increase and decrease together with the Fluent of the Quantity $\frac{-ba\dot{z}}{c\sqrt{c^2+z^2}}$. Make $nc = a$, and it will be $\frac{nb\dot{z}}{\sqrt{c^2+z^2}} = \dot{y}$, and $\frac{\frac{1}{2}nb^2\dot{z}}{\sqrt{c^2+z^2}} = \frac{1}{2}b\dot{y} =$ the Sector CXY.

But it is $\frac{\frac{1}{2}nb^2\dot{z}}{\sqrt{c^2+z^2}} . \frac{\frac{1}{2}c^2\dot{z}}{\sqrt{c^2+z^2}} :: nb^2 . c^2$; that is, in a given Ratio; and therefore the Sector CXY will be to $\frac{\frac{1}{2}c^2\dot{z}}{\sqrt{c^2+z^2}}$ always in a given Ratio. Therefore the Fluents of these Quantities will be always in the same Ratio, since they are supposed to begin together. But the Fluent of the Sector CXY is the Sector CVY; and the Fluent of the Quantity $\frac{\frac{1}{2}c^2\dot{z}}{\sqrt{c^2+z^2}}$ is the Sector of the Hyperbola, which is thus shewn.

Fig. 158. With Center C, and half the transverse Axis $CV = c$, let an equilateral Hyperbola be described, and from the two Points D and F, which are very near each other, let the Ordinates DB and EF be drawn to the Conjugate Axis; also draw CD and CF. Now the Increment or Fluxion of the Triangle BCD will be equal to $BE \times DB$ — Sector DCF. Whence the Sector DCF (which is the Fluxion of the Sector CVD) will be equal to $BE \times DB$ — the Increment of the Triangle BCD. And if BC be called z, because of the Hyperpola it is $BDq = BCq + CVq = z^2 + c^2$. Whence $BD = \sqrt{c^2+z^2}$, and $BE \times BD = \dot{z} \times \sqrt{c^2+z^2}$. But the Triangle BCD is $\frac{1}{2}z \times \sqrt{c^2+z^2}$, whose Fluxion is $\frac{1}{2}\dot{z} \times \sqrt{c^2+z^2} + \frac{\frac{1}{2}\dot{z}z^2}{\sqrt{c^2+z^2}}$. Let this Quantity be subtracted from $\dot{z}\sqrt{c^2+z^2}$, and there will remain the infinitely little Hyperbolic Sector $CDF = \frac{1}{2}\dot{z}\sqrt{c^2+z^2} - \frac{\frac{1}{2}\dot{z}z^2}{\sqrt{c^2+z^2}} = \frac{\frac{1}{2}\dot{z}\times c^2+z^2 - \frac{1}{2}\dot{z}\times z^2}{\sqrt{c^2+z^2}} =$

$=\frac{\frac{1}{2}c^2\dot{z}}{\sqrt{c^2+z^2}}$. Therefore the Fluent of the Sector CDF is equal to the Fluent of the Quantity $\frac{\frac{1}{2}c^2\dot{z}}{\sqrt{c^2+z^2}}$. Therefore the Sector CVD will be the Fluent of the Quantity $\frac{\frac{1}{2}c^2\dot{z}}{\sqrt{c^2+z^2}}$. Moreover let the right Line DT touch the Hyperbola, and meet the Conjugate Axis in T. From the Nature of the Hyperbola it is $BC.CV::CV.CT$, that is, $z.c::c.\frac{cc}{z}=CT=z$. And hence arises this following Construction.

With Center C, and half the transverse Axis CV, let an Equilateral Hyperbola Vm be described, and also a Circle Ve. Let the Circular Sector CVe be taken in Proportion to the Hyperbolic Sector CVm, as n to 1; let the right Line Tm touch the Hyperbola in m, meeting the Conjugate Axis in T, let Ce be produced to k, so that $Ck=CT$, and the Point k will be in the Curve required. For that Curve is such, that if Ck be called x, the Perpendicular let fall from C upon the Tangent will always be equal to $\frac{ax}{\sqrt{b^2+x^2}}$. When x is infinite b^2 vanishes, and the Perpendicular becomes equal to a, and then CR coincides with CV. If therefore in the Conjugate Axis be taken $CR=a$, and RS be drawn parallel to CV, this will be the Asymptote of the Curve. Fig. 159.

If a be so far increased that the Quantity b^2-a^2 may become infinitely little, then c^2 will vanish, and the Quantity $\frac{ba\dot{x}}{x\sqrt{x^2+c^2}}$ will be $\frac{ba\dot{x}}{x^2}=\dot{y}$. Whence if the Fluents of these Quantities are taken, we shall have $\frac{ba}{x}=y$, and $ba=xy$. That is, the Rectangle under the Circular Arch and the Distance of the Curve from the Center will always be a given Quantity. And upon this Account the Curve will pass into the Hyperbolic Spiral. Therefore the Hyperbolical Spiral is a Kind of intermediate Limit between such Curves as are constructed by Circular Sectors, and those that are constructed by Hyperbolical Sectors. Therefore that Hyperbolical Spiral may be conceived to be formed, either by a Sector of a Circle or Ellipsis, or by a Sector of an Hyperbola, whose transverse Axis is diminish'd *ad infinitum*, and the Number n is increased in the same Ratio.

Now we come to that Case, in which the Velocity of the Body is less than that which is acquired by falling from an infinite Distance, and wherein $p^2=\frac{a^2x^2}{b^2-x^2}$. And here by a like Reasoning as in the former

Case we shall find $KN = \frac{a\dot{x}}{\sqrt{b^2 - a^2 - x^2}}$, where b^2 must necessarily be greater than a^2. Hence if $b^2 - a^2$ is called c^2, it will be $KN = \frac{a\dot{x}}{\sqrt{c^2 - x^2}}$, and therefore XY or $\dot{y} = \frac{ba\dot{x}}{x\sqrt{c^2 - x^2}}$.

Now let it be $x = \frac{c^2}{z}$, and it will be $\frac{\dot{x}}{x} = -\frac{\dot{z}}{z}$, or $\frac{ba\dot{x}}{x} = -\frac{ba\dot{z}}{z}$, and it will be $c^2 - x^2 = \frac{c^2}{z^2} \times \frac{z}{z^2 - c^2}$; which Values being substituted it will be $-\frac{ba\dot{z}}{c\sqrt{z^2 - c^2}} = \frac{ba\dot{x}}{x\sqrt{x^2 - c^2}} = -\dot{y}$. For such a Beginning is to be assigned to the Arch VX, that it may begin together with the Fluent of the Quantity $\frac{ba\dot{z}}{c\sqrt{z^2 - c^2}}$. Whence it will be $\frac{\frac{1}{2}b^2 a\dot{z}}{c\sqrt{z^2 - c^2}} = \frac{1}{2}b\dot{y} =$ Sector $CXY = \frac{\frac{1}{2}nb^2\dot{z}}{\sqrt{z^2 - c^2}}$, by making $nc = a$. But it is $\frac{\frac{1}{2}nb^2\dot{z}}{\sqrt{z^2 - c^2}} \cdot \frac{\frac{1}{2}c^2\dot{z}}{\sqrt{z^2 - c^2}} :: nb^2 . c^2$, that is, in a constant Ratio. Wherefore the Fluents of these Quantities are in the same Ratio. That is, the Fluent of the Quantity $\frac{1}{2}b\dot{y}$ or $\frac{\frac{1}{2}nb^2\dot{z}}{\sqrt{c^2 - z^2}}$, will be to the Fluent of the Quantity $\frac{\frac{1}{2}c^2\dot{z}}{\sqrt{z^2 - c^2}}$, as nb^2 to c^2. But the Fluent of the Quantity $\frac{1}{2}b\dot{y}$ is the Sector CVX, and the Fluent of the Quantity $\frac{\frac{1}{2}c^2\dot{z}}{\sqrt{z^2 - c^2}}$ is the Sector of the Hyperbola, which is thus proved.

Fig. 160. With Center C, and transverse Semiaxis $CV = c$, let an Equilateral Hyperbola be described, and from two Points B and D that are infinitely near, let the two right Lines BE and DF be drawn as Ordinates to the Axis. Also draw CB, CD. And the Fluxion o Increment of the Triangle $CBE =$ to the Triangle $CBD + BE \times EF$. Whence the Triangle CBD, or the least Sector CBD, will be equal to the Increment of the Triangle $CBE - BE \times EF$. Let CE be called z, and it will be $BE = \sqrt{z^2 - c^2}$, and $BE \times EF = \dot{z}\sqrt{z^2 - c^2}$. Also the Triangle $CBE = \frac{1}{2}z\sqrt{z^2 - c^2}$, whose Fluxion is $\frac{1}{2}\dot{z}\sqrt{z^2 - c^2} +$

$\frac{\frac{1}{2}\dot{z}z^2}{\sqrt{z^2-c^2}}$; from whence if the Quantity $\dot{z}\sqrt{z^2-c^2}$ is subtracted, there will remain the least Sector $CBD = \frac{\frac{1}{2}\dot{z}z^2}{\sqrt{z^2-c^2}} - \frac{1}{2}\dot{z}\sqrt{z^2-c^2}$ $= \frac{\frac{1}{2}\dot{z}z^2 - \frac{1}{2}\dot{z}\times z^2 - c^2}{\sqrt{z^2-c^2}} = \frac{\frac{1}{2}c^2\dot{z}}{\sqrt{z^2-c^2}}$. Whence it is plain, that the Sector CBE is the Fluent of the Quantity $\frac{\frac{1}{2}c^2\dot{z}}{\sqrt{z^2-c^2}}$. Moreover if BT the Tangent of the Hyperbola meets the transverse Axis in T, from the Nature of the Hyperbola it will be $CE.CV::CV.CT$; that is, $z.c::c.\frac{c^2}{z} = CT = x$.

Hence we deduce the following Construction. With Center C, and transverse Semiaxis $CV=c$, let an Equilateral Hyperbola VB be described, and a Circle CeG from the same Center C. To the Hyperbola draw the right Line CB, and let the Tangent of the Hyperbola BT meet the transverse Axis in T. Let a Sector of the Circle CVe be taken, which may be to the Hyperbolical Sector CVB as n to 1. In Ce take $CK = CT$, and K will be a Point in the Curve required; whose Perpendicular let fall from the Center C to the Tangent at K, if CK be called x, will be equal to $\frac{ax}{\sqrt{b^2-x^2}}$. Fig. 161.

And a Body will move in this Curve, if acted upon by a Centripetal Force which is reciprocally as the Cube of the Distance, if it proceeds with a due Velocity according to the Direction of the Tangent. Now what this Velocity must be, which shall make the Body describe any of these Curves, will be thus found.

Since the Velocity with which a Body moves in any Trajectory is reciprocally as the Quantity p, assuming any constant Quantity a, that Velocity may always be expounded by $\frac{a}{p}$. And if to the Axis CV Ordinates are drawn, which are reciprocally as the Cubes of the Distances from the Center, or as the Centripetal Forces, and by this Means a Curvilinear Figure is described; its Area indefinitely extended may always be expounded by $\frac{b^2}{x^2}$, as is manifest from the Doctrine of Quadratures. But that Area is as the Square of the Velocity which is acquired by falling from an infinite Distance, and therefore the Velocity acquired in this Case will be as $\frac{b}{x}$. Hence if this Velocity is called y, and the Velocity

with

with which the Body moves in the Trajectory be called v; and a and b be assumed such Quantities, that in any one Distance from the Center it may be $y \,.\, v :: \frac{b}{x} \,.\, \frac{a}{p}$; it will be every where and at all Distances $y \,.\, v :: \frac{b}{x} \,.\, \frac{a}{p} :: p \,.\, \frac{ax}{b}$. Whence if $y = v$, it will be $p = \frac{ax}{b}$, and the Curve described with this Velocity will be the nautical Spiral, or a Circle if $p = x$, and $a = b$.

If y is greater than v, then p will be greater than $\frac{ax}{b}$, and it will be equal to $\frac{ax}{\sqrt{b^2 - x^2}}$, as appears from the foregoing. Now the Curve will be constructed by the Hyperbolical Sector, as was shewn in the last Case, where the Distance of the Body from the Center is determined by the Concourse of the Tangent of the Hyperbola with the transverse Axis. If y be greater than v, but in so small a Ratio that b continues greater than a; the Curve will be form'd by the same Hyperbolical Sector. But the Distance of the Body from the Center is taken from the Concourse of the Tangent with the conjugate Axis.

If it be $y \,.\, v :: p \,.\, x$, in this Case it will be $a = b$, and the Curve becomes an Hyperbolical Spiral, in which it is $p = \frac{ax}{\sqrt{a^2 - x^2}}$. Hence if the Body be projected from any Place according to a given right Line, with such a Velocity as may be to the Velocity acquired by falling from an infinite Distance, as the Distance of the Body from the Center to a Perpendicular let fall from the Center to the Line of Direction; that Body will move in an Hyperbolical Spiral. Lastly, if v be so much greater than y, that a may also be greater than b, the Curve will be constructed by Circular Sectors. And thus from the given Velocity the Relation of the Quantities a and b may always be determin'd, and therefore the Curve will be described in which the Body will move with that Velocity. And on the contrary the Curve being given, or the Quantities a and b being given, the Velocity may be found with which that Curve will be described.

The Areas of all these Curves, excepting the Circle, which can be described by the Action of this Centripetal Force, are perfectly Quadrable. For first in the Logarithmic Spiral, because it is $p = \frac{ax}{b}$, it will be $KN = \frac{a\dot{x}}{\sqrt{b^2 - a^2}} = \frac{a\dot{x}}{c}$, supposing $b^2 - a^2 = c^2$. Fig. 155.

and

and therefore the Triangle $CKI = \frac{\frac{1}{2}ax\dot{x}}{c}$, whose Fluent is $\frac{ax^2}{4c}$, which is the Area of the Curve.

If it is $p = \frac{a\dot{x}}{\sqrt{b^2+x^2}}$, and a be greater than b, it has been shew'd that $KN = \frac{a\dot{x}}{\sqrt{x^2+c^2}}$; whence $KN \times \frac{1}{2}CI = \frac{\frac{1}{2}ax\dot{x}}{\sqrt{x^2+c^2}}$, whose Fluent is $\frac{1}{2}a\sqrt{x^2-c^2}$ = Area of the Curve. But if a be less than b, it is $KN = \frac{a\dot{x}}{\sqrt{x^2+c^2}}$, and $KN \times \frac{1}{2}CI = \frac{\frac{1}{2}ax\dot{x}}{\sqrt{x^2+c^2}}$, whose Fluent is $\frac{1}{2}a\sqrt{x^2+c^2} - Q$ = Area of the Curve. Suppose $x = 0$, and it will be $\frac{1}{2}ac - Q = 0$, whence $Q = \frac{1}{2}ac$, and the Area is $\frac{1}{2}a\sqrt{x^2+c^2} - \frac{1}{2}ac$.

In the Hyperbolical Spiral the Quantity c vanishes, and the Area of the Curve becomes $\frac{1}{2}ax$.

If $p = \frac{a\dot{x}}{\sqrt{b^2-x^2}}$, it has been shewn that $KN = \frac{a\dot{x}}{\sqrt{c^2-x^2}}$; whence $\frac{1}{2}CI \times NK = \frac{\frac{1}{2}ax\dot{x}}{\sqrt{c^2-x^2}}$, whose Fluent is $Q - \frac{1}{2}a\sqrt{c^2-x^2}$ = Area. Make $x = 0$, and it will be $Q - \frac{1}{2}ac = 0$, or $Q = \frac{1}{2}ac$. Whence the Area will always be $\frac{1}{2}ac - \frac{1}{2}a\sqrt{c^2-x^2}$. Make $c^2 - x^2 = 0$, or $x = c$, and the Area of the Curve will be $\frac{1}{2}ac$. Whence if the Beginning of the Area is not taken from the Beginning of x, or where $x = 0$, but where $x = c$ is the greatest, that is, if the Area begins from V, (*See Fig.* 160.) the Area will always be equal to $\frac{1}{2}a\sqrt{c^2-x^2}$.

The most skilful Dr. *Halley* has observed what follows, concerning the Areas which are described by Bodies, by Means of a Centripetal Force which is as the Cubes of the Distances reciprocally. Which is, if Bodies by this Law describe different Circles, or different Hyperbolical Spirals; the Areas of the Sectors, as well in Circles as in all those Spirals, will always be equal when described in equal Times. For the Velocities of Bodies moving in Circles by this Law, ought to be reciprocally proportional to the Radii or Distances, and therefore the Arches described in the same Time will also be in the same reciprocal Ratio of the Radii; whence it easily appears, that the Sectors described in the same Time will be equal.

In all other Curves, since the Velocity is to the Velocity of a Body moving in a Circle at the same Distance, as $\frac{a}{b} \times x$ to p, (*Fig.* 156.) or as

$\frac{a}{x}$

$\frac{a}{x}$ and IK to KN; while the Body in its Trajectory describes the little Line IK, another Body moving at the same Distance will describe an Arch $= \frac{b}{a} \times KN$. And the Area of the Circular Sector, and that of the Trajectory described in the same Time, will be $\frac{b}{a} \times KN \times \frac{1}{2} CN$, and $KN \times \frac{1}{2} CN$, which two Areas are in the given Ratio of b to a. Wherefore when it is $a = b$, as it is in the Hyperbolical Spiral, the Area so described will always be equal to the Area of the Circular Sector, described in an equal Time.

To find the Center of Oscillation; by Dr. Br. Taylor, n. 337. p. 11.

VII. *A Definition.* The Center of Oscillation is a certain Point in a pendulous Body, all the Vibrations of which are perform'd after the same Manner and in the same Time, as if that only were suspended on a Thread, at the same Distance from the Point of Suspension.

It is hardly plain enough of itself, that there is such a Point in a Body, the Acceleration of which, by this Definition, ought to be the same in all Inclinations of the pendulous Body to the Horizon, as if it was actuated only by its own Gravity; the other Particles of the whole Body giving no Impediment to its Motion. Therefore in order to the Investigation of this Center, a Proposition or two must be premised, whence it may appear that there is such a Point.

Prop. 1. *Prob.* 1. In any given Inclination of a vibrating Body to the Horizon, to find a Point the Acceleration of which shall be the same, as if it were urged only by its own Gravity.

Fig. 162.

Let A B D be a Section of the proposed Body in a Plain perpendicular to the Horizon, in which the Center of Gravity G is moved, C being the Center of Suspension. Let the Body be distinguished into Prismatical Elements perpendicular to the Plain A B D, and therefore always parallel to the Horizon; as will easily appear from the Motion of the Center of Gravity G in that Plain A B D. And because of this Situation, any such Element may be considered as a Physical Point p, placed in the same Plain A B D at the Point z. Therefore let the Body proposed be reduced to the Physical Plain A B D, consisting of such Particles p.

In this Plain that the Point O may be found, whose proper Acceleration is not changed by the Actions of the other Particles, we must give Attention to the Force of every single Particle p situate in the Point z. For from these Forces being conjoin'd, arises the absolute Motion of the whole Plain. By means of this is given the Motion of every Point proposed, whence in its Turn is found that Point, the Motion of which is given.

But the Particle p will be urged by the Force of its own Gravity, which in a given little Time, if the Cohesion of the Particles were dissolved,

ſolved, would produce a given Acceleration of Motion in the Perpendicular to the Horizon $z\,y$. Draw $y\,x$ perpendicular to $C\,z$, and the Acceleration $z\,y$ will be reſolved into the Parts $z\,x$ and $x\,y$. Becauſe of the Rigidity of the Body the Force $z\,x$ will be taken away by the Reſiſtance of the Point C. But by the other Force $x\,y$ the Space ABD is drawn round about the Point C. And drawing an Horizontal Line C o, and a Perpendicnlar $z\,s$, it will be as $\frac{C\,s}{C\,z}$; becauſe of the given Force of Gravity, and the ſimilar Triangles $x\,y\,z$ and $s\,C\,z$. Therefore the Force of the Particle p, to move the Space A B D, will be as $\frac{C\,s}{C\,z} \times p$.

To collect theſe Forces together, let O be an invariable Point, in a Line drawn at Pleaſure, and at a Diſtance C O, which is yet unknown. Then the Force of the Particle p to move the Point O, will be as $\frac{C\,z}{C\,O} \times \frac{C\,s}{C\,z} \times p$, that is, as $\frac{C\,s}{C\,O} \times p$. But the Acceleration which p contributes to the ſame Point O, will be as $\frac{C\,O}{C\,z} \times \frac{C\,s}{C\,z}$. Therefore the Force $\frac{C\,s}{C\,O} \times p$ being apply'd to the Acceleration $\frac{C\,O \times C\,S}{C\,z\,q}$, the Quotient will be $\frac{C\,z\,q}{C\,O\,q} \times$ Particle p, which if it be ſuppoſed to move in the Point O with the ſame Acceleration $\frac{C\,O \times C\,s}{C\,z\,q}$, would produce the ſame Motion intirely, as the Particle p produces in the ſame Point O. Thus at laſt the Problem is reduced to a well known Theorem of Motion; for the Sum of the Forces $\frac{C\,s}{C\,O} \times p$ being apply'd to the Sum of the Particles $\frac{C\,z\,q}{C\,O\,q} \times p$, the Quotient will be the abſolute Acceleration of the Point O. Then drawing the Perpendicular O o, and ſuppoſing this Acceleration to be equal to the given Acceleration $\frac{C\,o}{C\,O}$ of the Point O, the Diſtance C O will be given. For let $\frac{C\,o}{C\,O} = d$, and by the Method of Fluxions 'tis $C\,s \times p = \dot{M}$, and $C\,z\,q \times p = \dot{C}$. Then becauſe of C O being invariable, the Sum of all the Forces will be $\frac{C\,s}{C\,O} \times p = \frac{M}{C\,O}$, and the Sum of all the Particles $\frac{C\,z\,q}{C\,O\,q} \times p = \frac{C}{C\,O\,q}$. Whence applying the

Sum of the Moments to the Sum of the Bodies, it will be $\frac{M}{C} \times CO = d$, and therefore $CO = \frac{dC}{M}$. Therefore C and M being found, C O will be given by the inverse Method of Fluxions. *Q. E. I.*

Cor. From the Center of Gravity G draw Gg perpendicular to the Horizontal Line Co, and let the Body itself A B C be called A. Then from the well known Property of the Center of Gravity, it will be $M = Cg \times A$. Whence it is $CO = \frac{dC}{Cg \times A}$.

Prop. 2. *Theor.* 1. The same Things being supposed, let the Point O be sought, in the right Line C G passing through the Center of Gravity G; then will O be the Center of Oscillation of the Body A.

Fig. 163. For in this Case it is $\frac{Co}{CO} = \frac{Cg}{CG} = d$. Whence $CO = \left(\frac{dC}{Cg \times A}\right.$, by *Cor.* of *Prop.* 1. $\left.=\right) \frac{C}{CG \times A}$. But A is given, and the Point C being given, C G and the Quantity C are given. Whence C O is given, whatever be the Inclination of the vibrating Body to the Horizon. Therefore by the Definition, and by *Prob.* 1. O is the Center of Oscillation of the Body A. *Q. E. D.*

Prop. 3. *Theor.* 2. The same Things being supposed, let D be the Aggregate of all the $Gz^2 \times p$. Then it will be $CO = CG + \frac{D}{CG \times A}$.

Draw zF perpendicular to C G, and it will be $Czq = CGq + Gzq - 2\,CG \times GF$, when F falls between C and G. But when F falls in C G produced, it will be $Czq = CGq + Gzq + 2\,CG \times Gf$. Therefore $C =$ (Aggregate of all the $Czq \times p =$) Aggregate of all

Fig. 164. the $CGq \times p + Gzq \times p - 2\,CG \times GF \times p + 2\,CG \times GF \times p$. But because of G the Center of Gravity, the Aggregate of all the $2\,CG \times GF \times p =$ Aggregate of all the $2\,CG \times Gf \times p$. Wherefore it is $C =$ the Aggregate of all the $CGq \times p + Czq \times p = CGq \times \overline{A + D}$. But by *Theorem* 1. it is $CO = \frac{C}{CG \times A}$. Therefore $CO = CG + \frac{D}{CG \times A}$. *Q. E. D.*

Cor. Hence the Parallelogram $CG \times GO$ is given. For it is $GO = \frac{D}{CG \times A}$. But A and D are given. Therefore $CG \times GO = \frac{D}{A}$ is given.

Prop. 4.

Prop. 4. *Theor.* 3. The same Things being supposed, if in the Point O the Physical Particle $\frac{CG \times A}{CO}$ is constituted, which being actuated by its own Gravity shall vibrate about the Point C; the Motion of the Space A B C shall be just the same, as if it were agitated by the Oscillation of the Body A.

It is evident, as well from the Nature of the Center of Gravity, as by *Prob.* 1. For $\frac{CG \times A}{CO}$ is the Aggregate of all the $\frac{Czq \times p}{COq} = \frac{C}{COq}$.

Prop. 5. *Prob.* 2. The Magnitude of any Body A, the Center of Gravity G, and the Point of Suspension C being given; to find O the Center of Oscillation of the same.

It is perform'd by *Theor.* 1. by finding the Quantity C; or by *Theor.* 2. by seeking the Quantity D.

Scholium. For performing the Calculation in a particular Case, the Quantity C or D is to be made choise of, according as the Nature of the proposed Figure shall suggest. Then either of them being given, the other also will be given by the Equation (*Prop.* 3.) $C = CGq \times \overline{A + D}$. Whence also will be given the Parallelogram $CG \times GO = \frac{D}{A}$, (*Cor. Prop.* 3.) $= \frac{C}{A} - CGq$, by Help of which, from the Center of Gravity and the Point of Suspension being given, the Center of Oscillation is given by Division only. Therefore in every Example it will always be most convenient to find this Parallelogram first, either by the Computation of D, or by the Quantity C, by a proper Assumption of the Center of Suspension.

What remains is, to illustrate this by some Examples.

Ex. 1. Let the Figure proposed be the Pyramid A D C, whose Base is the Parallelogram A D, and let the Motion of the Center of Gravity be in a Plain passing through the Vertex C, and the Diameter of the Base E F parallel to the Side A B. Fig. 165.

To perform the Calculation most conveniently, let the Vertex itself C be the Point of Suspension. Then in the Manner of *Prob.* 1. let the Figure be reduced to a Physical Plain of the Isosceles Triangle C E F, in which ef parallel to E F represents a Physical Line composed of Particles p. Let $CH = a$, $HF = b$, and $Ch = x$. Then from the Nature of the Figure it will be $eh = \frac{bx}{a}$, and the Particle p placed at the Point z will be as $\dot{x}$. Or rather, making $hz = v$, then $\dot{v}\dot{x}$ will be the Base of the Elementary Prism, and p will be as $\dot{v}xx$. Fig. 166.

 Whence

Whence it will be $\dot{C} = C z q \times \dot{v} \dot{x} x = \dot{v} \dot{x} x^3 + \dot{x} \dot{v} v^2 x$. Therefore the Sum of all the $C z q + p$ in the Line $b z$ will be $v \dot{x} x^3 + \frac{1}{3} \dot{x} x v^3$; and in the Line $e f$, (putting $\frac{b x}{a}$ for v) that Sum will be $\frac{6 b a^2 + 2 b^3}{3 a^3} \times \dot{x} x^4$. Whence again taking the Fluent, and writing a for x, it will be $C = \frac{6 b a^2 + 2 b^3}{15} \times a^2$. But the Pyramid itself is $A = \frac{3 b a a}{3}$, and the Distance of the Center of Gravity G from the Vertex C is $C G = \frac{3}{4} a$. Whence $\frac{C}{A} - C G q = \frac{D}{A} = C G \times G O = \frac{3 a^2 + 16 b^2}{80}$.

Ex. 2. Let the Figure proposed be an erect Cone, described by the Rotation of the Isosceles Triangle $E C F$ about the Perpendicular $C H$.

Here again taking the Vertex C for the Center of Suspension, and making $C H = a$, $H E = b$, $C b = x$, $b z = v$, as above; it will be $p = 2 \dot{x} \dot{v} \sqrt{\frac{b b}{a a} x x - v v}$; whence $\dot{C} = 2 \dot{v} \dot{x} \times \overline{x x + v v} \times \sqrt{\frac{b b}{a a} x x - v v}$. Let B be the Segment of a Circle described with the Diameter $e f$, which adjoins to the Abscis $b z = v$, and to the Ordinate $\sqrt{\frac{b b}{a a} x x - v v}$. Then the Sum of all the $C z q \times p$ in the right Line $b z = 2 \dot{x} \times \frac{4 a^2 + b^2}{4 a^2} \times x^2 B - \frac{1}{2} \dot{x} v \times \overline{\frac{b^2}{a^2} x^2 - v^2}\Big|^{\frac{3}{2}}$. And since $v = e b$, this Sum will be $2 \dot{x} \times \frac{4 a^2 + b^2}{4 a^2} x^2 B$, the Double of which $\frac{4 a^2 + b^2}{a^2} \times \dot{x} x^2 B$ is a Part of C in $e f$. But the Area B is as x^2; therefore $B = c x^2$. And that Part of C will be $\frac{4 a^2 + b^2}{a^2} \times c \dot{x} x^4$. Whence taking the Fluent it will be $C = \frac{4 a^2 + b^2}{5} \times c a^3$. But the Cone itself $A = \frac{4}{3} c a^3$, and $C G = \frac{3}{4} a$. Wherefore $\frac{C}{A} - C G q = \frac{D}{A} = \frac{3 a^2 + 12 b^2}{80}$.

And in the same Manner the Calculus proceeds in other Figures, wherein the Ratio's of $C b$ to $b e$, and of $b z$ to p, are still more compounded.

Ex.

Ex. 3. That the Manner of the Calculation of the Quantity D may appear, let the propofed Figure be a Parallelepiped, whofe Face perpendicular to the Horizon is A B D, parallel to the Plain of the Motion of the Center of Gravity. Draw the Diameters E F and H I, and let the Altitude of the Elements be p, and draw $t\,r$ parallel to HI. Make G F $= a$, Fig. 167.

G H $= b$, G $s = x$, and $s\,z = v$. Then it will be $\dot{\mathrm{D}} = v\,\dot{x}\,x\,x + \dot{x}\,v\,v\,v$.

Whence the Part of D in the right Line $t\,r$ will be $2\,b\,\dot{x}\,x^2 + 2\,b^3\,\dot{x}$;

and again taking the Double of the Fluent, it will be $\mathrm{D} = \dfrac{4\,b\,a^3 + 4\,b^3\,a}{3}$.

But A $= 4\,a\,b$. Whence $\dfrac{\mathrm{D}}{\mathrm{A}} = \dfrac{a^2 + b^2}{3} = \frac{1}{12}\,\mathrm{DB}\,q$.

Ex. 4. Let the laft Example be in the Sphere, whofe greateft Circle is B $t\,r$, Diameter A B, and Center G. Then drawing Lines as in the Fig. 168.

Scheme, it will be $\dot{\mathrm{D}} = \mathrm{G}\,s\,q \times p + \mathrm{GM}\,q \times p$. But the Sum of all the G $s\,q \times p$ in the right Line $t\,r$ is G $s\,q$, drawn into the Area of the Circle defcribed with the Diameter $t\,r$. Alfo the Sum of all the G M $q \times p$ in the right Line $k\,i$ is G M q, drawn into the Area of the Circle defcribed with the Diameter $k\,i$. whence it eafily appears, that D is equal to four Times the Fluent of G $s\,q$ into the Area of the Circle whofe Diameter is $t\,r$. Let therefore c be the Area of the Circle, the Square of whofe Radius is 1, and let it be G A $= a$, and G $s = x$. Then it will

be $\dot{\mathrm{D}} = 4\,\dot{x}\,x^2 \times c\,a^2 - c\,x^2 = 4\,c\,a^2\,\dot{x}\,x^2 - 4\,c\,\dot{x}\,x^4$. Whence taking the Fluent, and making $x = a$, it will be $\mathrm{D} = \dfrac{8}{15}\,c\,a^5$. But A $=$

$\dfrac{4\,c\,a^3}{3}$ Whence $\dfrac{\mathrm{D}}{\mathrm{A}} = \dfrac{2}{5}\,a\,a$.

Becaufe of the Affinity of Solution, I have a Mind to add here a Problem concerning the finding the Center of Percuffion.

Prop. 6. *Prob.* 3. To find the Center of Percuffion of any Body, having a Rotation about a given Point, which Point muft be fuch, that a Body ftriking againft it, and at the fame Time being let loofe from the Center of Sufpenfion, fhall incline neither this Way or that Way.

Firft it appears, that this Point muft be fought for in the Plain of Motion of the Center of Gravity. For if the Body is refolved into Prifmatic Elements perpendicular to that Plain, they will be carried by a Motion parallel to one another; whence the Moments on each Side of that Plain will be equal: Therefore by the Refiftance made in this Plain, no Point of the Body will be driven out of it. Therefore let that Plain be A B, to which let the Body be reduced, by a Contraction of the Prifmatic Elements into Particles p fituated at the Points z, as in *Prob.* 1. In this Plain let C be the Center of Rotation, or at leaft its Projection Fig. 169.

made

made by a Line let fall perpendicularly upon this Plain; and let Q be the Point sought. Through C draw $C\xi$ at Pleasure, in which take two Points z and ξ, so that drawing zQ and ξQ, the Angle CzQ may be obtuse, and the Angle $C\xi Q$ acute; and in the Points z and ξ let there be Particles p and π. Then drawing zr and ξr perpendicular to $C\xi$, which may be to each other as Cz to $C\xi$, by these will be represented the absolute Velocities of the Particles p and π. But certain Parts of these Velocities, which are in the Directions of zQ and ξQ, are taken away by the Resistance of the Point Q. Draw CD and Cd perpendicular to Qz and $Q\xi$, and because of equal Angles $zCD = rzQ$, and $\xi Cd = r\xi Q$, the other Parts of the Velocities, which are in Directions perpendicular to Qz and $Q\xi$, will be as zD and ξd. So that in respect of the Distances Qz and $Q\xi$, the Forces of the Particles p and π, to move the Space AB the contrary Way, will be as $Dz \times zQ \times p$, and $d\xi \times \xi Q \times p$. Now by the Conditions of the Problem these Sums of contrary Forces ought to be equal..

Because of the right Angles at D and d, the Points D and d are in the Circumference of a Circle described with the Diameter CQ. Let E be the Center of this Circle. Then drawing Ez and $E\xi$, meeting the Circle in F and I, f and i, it will be $Dz \times zQ = Fz \times zI = EFq - Ezq = EQq - Ezq$; and $d\xi \times \xi Q = E\xi q - EQq$. Then the Sum of all the $EQq \times p - Ezq \times p$ will be equal to the Sum of all the $E\xi q \times \pi - EQq \times \pi$. And transposing the Terms, the Sum of all the $EQq \times \overline{p + \pi}$ will be equal to the Sum of all the $Ezq \times p + E\xi q \times \pi$: That is, if p be put as well for the Particle p within the Circle, as for the Particle π without the Circle; the Sum of all the $EQq \times p$ will be equal to the Sum of all the $Ezq \times p$. Draw zs perpendicular to CQ. Then it will be $Ezq = Czq + ECq - QC \times Cs$. Now this Value of Ezq being substituted instead of it, and the Equation being rightly managed, you will find at last the Sum of all the $CQ \times Cs \times p =$ to the Sum of all the $Czq \times p$. Whence it will be $CQ = \frac{\text{Sum of all the } Czq \times p}{\text{Sum of all the } Cs \times p}$. But the Sum of all the $Czq \times p$ is the Quantity C itself in the Calculation of the Center of Oscillation; and if the Center of Gravity be G, and Gg be drawn perpendicular to CQ, and the Body itself be called A, the Sum of all the $Cs \times p$ will be equal to $Gg \times A$. Whence it is $CQ = \frac{C}{Cg \times A}$. Let the Center of Oscillation be O; then by *Theor.* 1. $CO = \frac{C}{CG \times A}$. Whence it is $Cg . CG :: CO . CQ$. Wherefore a Perpendicular to CO being drawn through O, it will pass through the Point Q. *Q. E. I.*

VIII. A

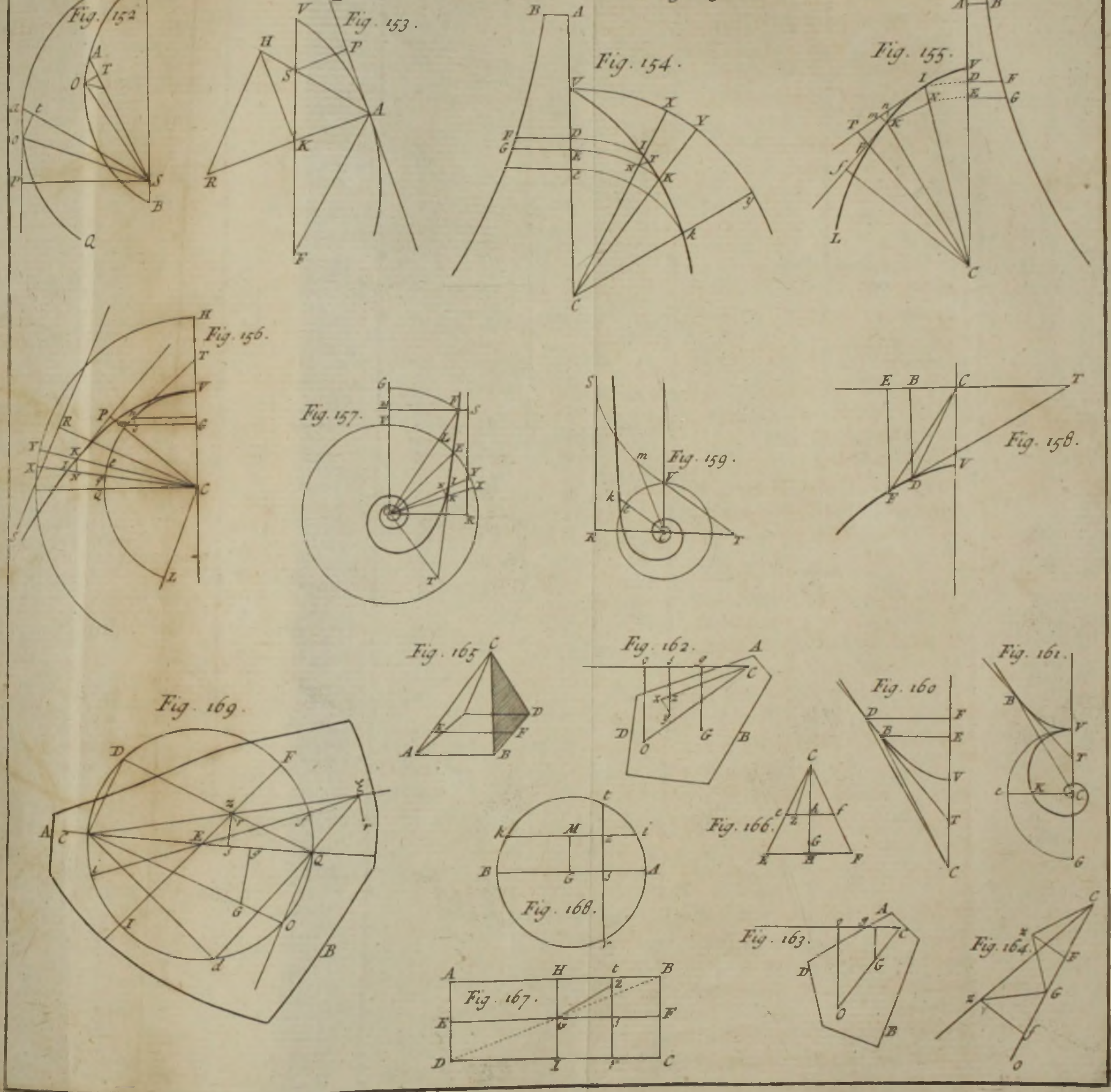


Fig. 152
Fig. 153
Fig. 154
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Fig. 156
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Fig. 163
Fig. 164
Fig. 165
Fig. 166
Fig. 167
Fig. 168
Fig. 169

VIII. A *Lemma*. Let A D F B, A Δ Φ B, be two Curves, the Relation of which is such, that the Ordinates C Δ D, E Φ F, being drawn, it may be C Δ . C D :: E Φ . E F. Then the Ordinates being diminish'd *ad infinitum*, so that the Curves may coincide with the Axis A B; I say that the ultimate Ratio of the Curvature in Δ will be to the Curvature in D, as C Δ to C D. *Of the Motion of a Stretch'd String, by Dr.* B. Taylor, n. 337. p. 26.

Demonst. Draw the Ordinate $c\,\delta\,d$ very near to C D, and at D and Δ draw the Tangents D t and Δ θ, meeting the Ordinate $c\,d$ in t and θ. Then because of $c\,\delta\,.\,c\,d$ :: C Δ . C D, (by Hypothesis) the Tangents being produced will meet one another and the Axis in the same Point P. Whence because of similar Triangles C D P and $c\,t$ P, C Δ P, and $c\,\theta$ P, it will be $c\,\theta\,.\,c\,t$ :: C Δ . C D :: $c\,\delta\,.\,c\,d$ (by Hyp.) :: $\delta\,\theta\,.\,(c\,\theta - c\,\delta)\,.\,d\,t\,(c\,t - c\,d.)$ But the Curvatures in Δ and D are as the Angles of Contact θ Δ δ and t D d; and because δ Δ and d D coinciding with c C, those Angles are as their Subtenses $\delta\,\theta$, $d\,t$; that is, by the Proportion above, as C Δ, C D. Therefore, *&c.* *Q. E. D.* Fig. 170.

Lem. 2. In some Instant of its Vibration, let a String stretch'd between the Points A and B put on the Form of any Curve A $p\,\pi$ B, I say that the Increment of the Velocity of any Point P, or the Acceleration arising from the Force of the Tension of the String, is as the Curvature of the String in the same Point. Fig. 171.

Demonst. Conceive the String to consist of equal rigid Particles, which are infinitely little, as p P, P π, *&c.* and at the Point P erect a Perpendicular P R, equal to the Radius of Curvature in P, which let the Tangents $p\,t$, $\pi\,t$, meet in t, the Parallels to them $\pi\,s$, $p\,s$, in s, the Chord $p\,\pi$ in c. Then by the Principles of Mechanicks, the absolute Force by which the two Particles p P and P π are urged towards R, will be to the Force of the Tension of the String, as $s\,t$ to $t\,p$; and half this Force, by which one Particle p P is urged, will be to the Tension of the String, as $c\,t$ to $t\,p$, that is, (because of similar Triangles $c\,t\,p$, $t\,p$ R) as $t\,p$ or P p to R t or P R. Wherefore, because of the Force of Tension being given, the absolute accelerating Force will be as $\frac{\mathrm{P}\,p}{\mathrm{PR}}$. But the Acceleration generated is in a compound Ratio of the Ratios of the absolute Force directly, and of the Matter to be moved inversely, and the Matter to be moved is the Particle itself P p. Wherefore the Acceleration is as $\frac{1}{\mathrm{PR}}$, that is, as the Curvature in P. For the Curvature is reciprocally as the Radius of Curvature in that Point. *Q. E. D.*

Prob. 1. To determine the Motion of a stretch'd String.

In this and the following Problems I suppose the String to move from the Axis of Motion through an indefinitely little Space; that the Increment of Tension from the Increase of the Length, also the Obliquity of the Radii of Curvature, may safely be neglected.

Therefore let the String be stretch'd between the Points A and B, and with a Bow let the Point z be drawn to the Distance C z from the Axis Fig. 172.

Axis A B. Then taking away the Bow, becauſe of the Flexure in the Point C alone, that will firſt begin to move, (by *Lem.* 2.) But no ſooner will the String be bent in the neareſt Points *c* and *d*, but theſe Points alſo will begin to move; and then E and *e*, and ſo on. Alſo becauſe of the great Flexure in C, that Point will firſt move very ſwiftly, and thence the Curvature being increaſed in the next Points, D, E, *&c.* they will immediately be accelerated more ſwiftly; and at the ſame Time the Curvature in C being diminiſh'd, that Point in its Turn will be accelerated more ſlowly. And in general, thoſe Points which are ſlower than they ſhould be, being accelerated more, and the quicker leſs, it will be brought about at laſt, that the Forces being duly attemper'd one with another, all the Motions will conſpire together, and all the Points will at the ſame Time approach to the Axis, going and returning alternately *ad infinitum.*

Now that this may be done, the String muſt always put on the Form of the Curve A C D E B, the Curvature of which in any Point E is as the Diſtance of the ſame E *n* from the Axis; the Velocities of the Points C, D, E, *&c.* being alſo in the Ratio of the Diſtances from the Axis C *z*, D θ, E *n*, *&c.* For in this Caſe the Spaces C *x*, D δ, E ε, *&c.* deſcribed in the ſame infinitely little Time, will be as the Velocities, that is, as the Spaces deſcribed D *z*, D ϑ, *&c.* Wherefore the remaining Spaces *x z*, δ ϑ, ε *n*, *&c.* will be to each other in the ſame Ratio. Alſo by *Lemma* 2. the Accelerations will be to one another in the ſame Ratio. By which Means the Ratio of the Velocities always continuing the ſame with the Ratio of the Spaces to be deſcribed, all the Points will arrive at the Axis at the ſame Time, and always depart from it at the ſame Time. And therefore the Curve A C D E B will be rightly determined. *Q. E. D.*

Moreover the two Curves A C D E B and A *x* δ ε B being compared together, by *Lemma* 1. the Curvatures in D and δ will be as the Diſtances from the Axis D ϑ and δ ϑ; and therefore by *Lemma* 2. the Acceleration of any given Point in the String will be as its Diſtance from the Axis. Whence, (by *Sect.* 10. *Prop.* 51. of *Newton*'s *Principia*,) all the Vibrations both great and ſmall will be perform'd in the ſame periodical Time, and the Motion of any Point will be ſimilar to the Oſcillation of a Body vibrating in Cycloid. *Q. E. I.*

Cor. Curvatures are reciprocally as the Radii of Circles of the ſame Degree of Curvature. Therefore let *a* be a given Line, and the Radius of Curvature in E will be equal to $\frac{aa}{\mathrm{E}n}$.

Prob. 2. The Length and Weight of a String being given, together with the Weight that ſtretches the String, to find the Time of a ſingle Vibration.

Fig. 173. Let the String be ſtretch'd between the Points A and B by the Force of the Weight P, and let the Weight of the String itſelf be *N*, and its Length L. Alſo let the String be put in the Poſition A F *p* C B, and at the middle Point C let C S a Perpendicular be raiſed, equal to the Radius of the Curvature in C, and meeting the Axis A B in D; and taking a Point *p* near to C, draw the Perpendicular *p c* and the Tangent *p t*.

Therefore

Therefore it appears, as in *Lemma* 2. that the absolute Force by which the Particle p C is accelerated, is to the Force of the Weight P, as $c\,t$ to $p\,t$, that is, as p C to C S. But the Weight P is to the Weight of the Particle p C, in a Ratio compounded of the Ratio's of P to N; and of N to the Weight of the Particle p C, or of L to p C; that is, as $P \times L$ to $N \times p\,C$. Therefore compounding these Ratio's, the accelerating Force is to the Force of Gravity, as $P \times L$ to $N \times CS$. Let therefore a *Pendulum* be constructed, whose Length is C D; then by *Sect.* X. *Prop.* 52. of *Newton's Principia*, the periodical Time of the String will be to the periodical Time of that *Pendulum*, as $\sqrt{N \times CS}$ to $\sqrt{P \times L}$. But by the same Proposition, the Force of Gravity being given, the Longitudes of the *Pendula* are in a duplicate Ratio of the periodical Times. Whence $\frac{N \times CS \times CD}{P \times L}$, or writing $\frac{aa}{CD}$ for CS, by *Cor. Prob.* 1. $\frac{N \times aa}{P \times L}$ will be the Length of a *Pendulum*, the Vibrations of which are isochronous to the Vibrations of the String.

To find the Line a, let the Abscis of the Curve be $AE = z$, and the Ordinate $EF = x$, and the Curve itself $AF = v$, and $CD = b$. Then by *Cor. Prob.* 1. the Radius of Curvature in F will be $\frac{aa}{x}$. But $\dot{v}$ being given, the Radius of Curvature is $\frac{\dot{v}\,\dot{x}}{\ddot{z}}$. Whence $\frac{aa}{x} = \frac{\dot{v}\,\dot{x}}{\ddot{z}}$, and therefore $aa\ddot{z} = \dot{v}\,x\,\dot{x}$, and taking the Fluents $aa\dot{z} = \frac{\dot{v}\,x^2}{2} - \frac{\dot{v}\,b^2}{2} + \dot{v}\,a^2$. Here the given Quantity $-\frac{\dot{v}\,b^2}{2} + \dot{v}\,a^2$ is added, that it may be $\dot{z} = \dot{v}$ in the middle Point C. And hence the Calculus being compleated, it will be $\dot{z} = \frac{a^2\dot{x} - \frac{1}{2}b^2\dot{x} + \frac{1}{2}x^2\dot{x}}{\sqrt{a^2b^2 - a^2x^2 - \frac{1}{4}x^4 - \frac{1}{4}b^4 + \frac{1}{2}b^2x^2}}$.

Now let b and x vanish in respect of a, that the Curve may coincide with the Axis, and it will be $\dot{z} = \frac{a\dot{x}}{\sqrt{bb - xx}}$. Now with Center C, and Radius $DC = b$, a Quadrant of a Circle D P E being described, and making $CQ = x$, and erecting the Perpendicular Q P; then the Arch D P being $= y$, it will be $\dot{y} = \frac{b\dot{x}}{\sqrt{bb - xx}} = \frac{b}{a}\dot{z}$. Fig. 174.

Whence $y = \frac{b}{a}z$, and $z = \frac{a}{b}y$. And making $x = b = CD$, in which Case it is also $y =$ Quadrantal Arch D P E, and $z = AD = \frac{1}{2}L$; it will be $\frac{1}{2}L = a \times \frac{DE}{CD}$, and $a = L \times \frac{CD}{2DE}$. Let it be therefore C D . 2 D E :: Diameter of a Circle . Circumference :: $d . c$; and it will be $aa = LL \times \frac{dd}{cc}$. Therefore this Value being substituted for aa; $\frac{N}{P} \times L \times \frac{dd}{cc}$ will be the Length of a *Pendulum* which will be isochronous to the String. Therefore let D be the Length whose periodical Time is 1, and $\frac{d}{c}\sqrt{\frac{N}{P} \times \frac{L}{D}}$ will be the periodical Time of the String. *Q. E. I.* For the Periodical Times of *Pendulums* are as the Square-roots of their Lengths.

Cor. 1. The Number of Vibrations of the String in the Time of one Vibration of the *Pendulum* D, is $\frac{c}{d}\sqrt{\frac{P}{N} \times \frac{D}{L}}$.

Cor. 2. Because $\frac{d}{c} \times \sqrt{\frac{1}{D}}$ is given, the periodical Time of the String is as $\sqrt{\frac{N}{P} \times L}$. And the Weight P being given, the Time is as $\sqrt{N \times L}$. And the Strings being made of the same Thread, in which Case 'tis N as L, the Time will be as L.

The Invention *of making* Clocks *to keep* Time *with the* Sun's apparent Motion, *asserted by Mr.* J. Williamson, n. 363. p. 1080.

IX. In a *French* Book lately published, the Author speaks of making Clocks to agree with the Sun's apparent Motion; and supposed that it was a Thing never thought of by any before himself: I shall therefore give this short Account of what I have performed in that Matter myself.

And in the first Place I must take Notice of the Copy of a Letter in this Book, wrote by one *P. Kresa* a *Jesuit*, to one Mr. *Williamson*, Clockmaker to his *Imperial* Majesty, of a Clock found in the late King *Charles* the *Second* of *Spain*'s Cabinet, about the Year 1699 or 1700. which sheweth both equal and apparent Time according to the Tables of the Equation; and which went 400 Days without winding up. This I am well satisfied is a Clock of my own making; for about six Years before that Time, I made one for Mr. *Daniel Quare*, which agrees with the Description he gives of it, and went 400 Days as he saith. This Clock Mr. *Daniel Quare* sold, soon after it was made, to go to the said King *Charles* the *Second* of *Spain*: And it was made so, that if the *Pendulum* was adjusted to the Sun's mean Motion, the Hands would show equal Time on two fixed Circles, on one the Hour, and on the other the Minute. But there were other two moveable Circles of the same kind, that moved forwards and backwards, as the Time of of the Year required; on which the ſame Hands ſhew apparent Time likewiſe, according to the Equation Tables. This Method the Author owns he knew of, and applied the ſame Motion to Pocket Watches 12 or 14 Years ago, which I confeſs I never did; being well ſatisfied that Watches with Springs and Balances are very unfit to ſhew the minute Difference, as it increaſeth and decreaſeth, between equal and apparent Time.

Soon after this Clock was ſent to *Spain*, I made others for Mr. *Quare*, which ſhewed apparent Time by lengthening and ſhortening the *Pendulum*, in lifting it up and letting it down again, by a Rowler ſomewhat in the Form of an *Ellipſis*, through a Slit in a Piece of Braſs, which the Spring at the Top of the *Pendulum* went through. By this Means every Vibration of the *Pendulum*, would agree to a ſecond of Time of the Sun's apparent Motion; that Rowler, which lifted up the *Pendulum*, and let it down again, being continually moving about all the Year; ſo that it may ſeem very ſtrange, that this Author never heard of it ſo many Years after they were made: For one of thoſe, and not the firſt, made with the riſing and ſetting of the Sun, Mr. *Quare* ſold to the late King *William*, and it was ſet up at *Hampton-Court*, where it hath been ever ſince. This Contrivance of lengthening and ſhortening the *Pendulum*, I thought of ſeveral Years before I made any of them. Since then I have made others for Mr. *Quare* likewiſe, which ſhewed the Difference between equal and apparent Time according to the Equation Tables, by a Hand moving both Ways from the Top of a Circle; on one Side ſhewing how much a Clock, keeping equal Time, ought to be faſter than the Sun; on the other Side how much ſlower.

But theſe Clocks that I then made to agree with the Sun's apparent Time, were done according to the Equation Tables, which I found not to agree very exactly with the Sun's apparent Motion: Neither can any other be made to keep equal Time, that will gain and loſe all the Year agreeable to the ſaid Tables; for though the Tables themſelves may be true, yet ſome Difference in Motion does proceed, in both Sorts of Clocks, from Cold or Heat altering the Length of their *Pendulums*. This Difference, by ſome Obſervations I have made, I ſuppoſe to be about the $\frac{1}{100}$ Part of an Inch, in the Length of a *Pendulum* vibrating Seconds, which will alter the Motion of the Clock about 12 Seconds in 24 Hours. But to make my Clocks of keeping apparent Time, to go as exact as poſſible, I made a Table my ſelf by Obſervation: For obſerving the Sun, as often as it was ſeen, when it came on the Meridian, for ſeveral Years together, always ſetting down the Difference between its coming to the Meridian and the Time, by a Clock I had adjuſted as well as I could to equal Time, and always taking Notice how much my Equal-Time Clock gain'd or loſt at the End of every Year, I compleated my Table in the Year 1711. Since then I have made many of theſe Clocks: So that I think I may juſtly claim the greateſt Right to this Contrivance, of making Clocks to go with apparent Time; and I have never yet heard of any ſuch Clock ſold in *England*, but what was of my own making, though I have made of them ſo long.

Experiments and Observations on the Motion of Sound, by Mr. W. Derham. n. 313. p. 2.

X. 1. The Difagreement among the moft famous Authors, concerning the Velocity of Sound, may be feen at one View in the following Table, in which, in *Englifh* Feet, the Space is exhibited, which they afcribe to the Progrefs of a Sound in one Second of Time.

	Feet.	
Sir *Ifaac Newton*	968	Princip. Nat. Phil. L. 2. Prop. 50.
Hon. Mr. *Robarts*	1300	Phil. Tranf. n. 209.
Hon. Mr. *Boyle*	1200	Effay on Languid Motion, p. 24.
Mr. *Walker*	1338	Phil. Tranf. n. 247.
Merfennus	1474	Baliftic. Prop. 39.
Mr. *Flamfteed* and *Halley*	1142	
Florentine Philofophers	1148	Exp. per Acad. *del Ciment.* p. 141.
French Philofophers	1172	*Du Hamel*, Hift. Acad. Reg.

There is no great Difagreement of Opinion among the three laft, but of the reft there is. The Reafon of which is manifeftly this, either becaufe of the Infufficiency of the Inftruments, or becaufe of the Diftance, or from the Winds.

1. The Inftrument by which fome of them have meafured, was not a Watch or Clock, but a fufpended Plummet, which vibrated Seconds. But it is plain, that a Plummet is much lefs convenient, nor can be fo accurate as a Clock; becaufe it is neceffary that the Eye muft firft be employ'd in obferving the Corufcation, and then muft obferve the Plummet or Pendulum. This waftes Times, and caufes Confufion. But efpecially if,

2. The Diftance between the Thing founding and the Obferver be but fmall. Now it is evident that moft of thefe made their Experiments at the Diftance of only a few Feet, and meafured by the Return or Echo of the Sound. Some of thefe extended their Meafure hardly beyond fix or feven hundred Feet, others not above a fingle Mile. But I have always obferved, that in fo fmall a Diftance an Uncertainty would neceffarily arife, even tho' the beft Inftrument was made ufe of. And a very fmall Error in fuch fhort Diftances is to be accounted a great one. For perhaps the *Pendulum* has already pafs'd over half its Swing or Arch from the laft Pulfation, when the Sound was firft emitted. But we reckon that Pulfe as if the Vibration were fully compleat, or perhaps we anticipate the Vibration. And after the Sound has reached us, perhaps we count more or lefs than we fhould do.

Or if the Diftance be long enough, yet an Error may thence arife, if

3. We take not the Winds into our Reckoning.

Thefe are certain Inconveniencies, which attend the Menfuration of the Progrefs of Sounds.

Yet it may be obferved, that the Spaces affign'd by the three laft Obfervations of the Table, agree pretty well with one another. Doubtlefs this proceeds from hence, that the Obfervers were furnifh'd with good Clocks. In the Ufe of thefe the Ear alone is employ'd in catching the Vibrations of the

the *Pendulum*, while the Eye attends to the Corufcation, or fome other Emiffion of the Sound. Alfo thefe Obfervations were made at great Diftances, in which a fmall Error could be of no great Moment. The Obfervations of *Flamfteed* and *Halley* were made at an Interval of almoft three Miles, within a few Perches, from the Royal Obfervatory, upon *Shooter's Hill*; and the Sound return'd in 13½ Seconds of Time. The noble and celebrated *Florentines*, of the Academy *del Cimento*, made their Experiments at nearly the fame Diftance. And fome at the Diftance of one Mile only. And *Caffini*, *Picard*, and *Roëmer* at the Diftance of 1280 *French* Toifes, which is more than a Mile and half *Englifh* Meafure.

I my felf have made very many Experiments, at various Diftances, from one Mile to twelve and more. And for meafuring of Time I have a moft accurate Watch, with a *Pendulum* that vibrates half Seconds.

I propofed to my felf to determine the following Queries.

1. How much Space a Sound paffes through in a Second of Time, or any other Interval of Time?

2. When a Gun is difcharged towards the Obferver, whether it fends its Sound in the fame Interval of Time, as when it is difcharged the contrary Way?

3. In every State of the Atmofphere, when the Mercury in the Barometer afcends or defcends, whether Sounds defcribe the fame Space in the fame Time?

4. Whether Sounds fly fwifter in the Day-time than in the Night?

5. Whether the Sound is accelerated by a favourable Wind, or retarded when the Wind is contrary? Or how the Wind affects Sound, if it affects it at all?

6. Whether Sound moves fwifter in calm Weather, than when the Wind blows?

7. Whether a violent Wind, blowing crofs to the Courfe of the Sound, accelerates or retards the Motion of the Sound?

8. Whether Sounds have the fame Motion in Winter and Summer, by Day and by Night?

9. Whether they are the fame in Snowy Weather, and in fair?

10. Whether a great and fmall Sound have the fame Motion?

11. Whether in all Elevations of the Gun, a Horizontal, at 10, 20, as far as 90 Degrees, the Sound arrives at the Ear of the Obferver at the fame Diftance of Time?

12. Whether all Kinds of Sounds, thofe of Guns, Bells, Hammers, and fuch like, have the fame Motion?

13. Whether the different Forces of Gunpowder vary the Motion of Sound?

14. Whether at the Tops of high Mountains, or in Vales, or in the higheft and loweft Parts of the Atmofphere, Sounds pafs over the fame Spaces in the fame Intervals of Time?

15. Whether a Sound afcending or defcending obliquely has the fame Motion?

tion? Or whether it aſcends from the Bottom to the Top of a Mountain with the ſame Velocity, as it deſcends from the Top to the Bottom?

16. Whether a Sound moves ſwifter at the Beginning, and ſlower towards the End, as happens to many other violent Motions?

17. Or is it not rather equable, that is, does it not deſcribe Half its Courſe in half the Time, one Fourth in a Fourth of the Time, and ſo on?

18. Has it not the ſame Motion in all Countries, Northern or Southern, in *England*, *France*, *Italy*, *Germany*, &c.

19. Does Sound paſs from Place to Place in a Right Line, or the ſhorteſt Way, or according to the Surface of the Ground between?

For determining theſe Inquiries I requeſted my Friends, that they would diſcharge Guns from Towers and other Eminencies, at the Diſtance of 1, 2, 3, as far as 8 Miles, which I found to be the greateſt Diſtance at which I could hear the Report of a Gun, in this Country which is ſo thick ſet with Trees and other Things. Theſe Guns were of great Uſe to me. But thoſe great Guns, call'd Sakers, were moſt for my Purpoſe, upon *Blackheath*, with which the young Ingeniers of the Train are there exerciſed. The Flaſhes of theſe Guns I could ſee from the Steeple of my Church, and I could hear their Report in almoſt all Weathers; nay even in the Day-time with the Help of my Teleſcope I could ſee the Flaſhes. Therefore I apply'd my ſelf with the utmoſt Care and Diligence to the Obſervation of theſe Guns, from *February* 1704-5.

After a few Obſervations made upon theſe Explotions, I procured a certain ſpecial Experiment to be made. Two Cannons or Sakers were placed near one another, the Mouth of one of which was towards me, and of the other from me. Theſe two Pieces were diſcharged *February* 13, 1704-5, every half Hour, from Six in the Afternoon to Midnight, a gentle Gale blowing directly againſt the Sound. The Diſtance of Time between the Flaſhing of every Gun, (which I could ſee with my naked Eye,) and the coming of the Sound, was always about 120 or 122 Half-ſeconds of Time. I ſay 120 or 122, becauſe the Sound came double; that is, the firſt Sound within 120 Half-ſeconds, (which was the Fainter,) and the ſecond within 122, which was more intenſe. And in the ſame Manner during the whole Time of Obſervation, the Noiſe of every Gun came doubled.

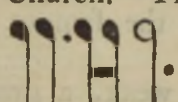
This Reduplication of the Sound to me ſeems an Echo, which was reflected, as I imagine, by the Mill upon *Blackheath*, or by the neighbouring Houſes. Of which I ſhould have no Reaſon to doubt, if it was not for the Opinion of a certain learned Friend, who believes that no Echo can be heard, but which is made by reflecting Objects not far from the Obſerver, and not by thoſe which are near the ſonorous Body, or other diſtant Objects.

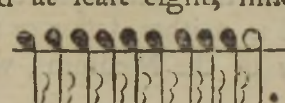
Of an Echo at a great Diſtance.

2. But this I think not to be contrary to the Nature of an Echo. Then it is to be obſerved, that this double Sound came directly from *Blackheath*. Nor did the firſt Sound come from thence, and the other (like an Echo) from elſewhere, either beyond me, from the Right Hand or Left, or from any other Side. And the ſame Thing I have frequently obſerved, when great Guns were diſcharged from the Ships in the River *Thames*, eſpecially if the Air

Air was calm and ſerene, either in the Evening or Morning, when thoſe they call the *Watch-guns* were diſcharged. After the Sound of the Gun had reached the Ear, I heard it ſtill running along the River, and echoing from the Shores, the Hills, and the Rocks, (which are in great Plenty along the *Kentiſh* Shore) and that for many Miles together.

All theſe Things, according to my Friend's Opinion, proceeded from the Repercuſſion of the Houſes and other Objects that were near me. But to ſay nothing of the Weakneſs of the Sound, after it has paſs'd on for ſeveral Miles, and of its Incapacity to produce ſuch an Effect, if it had come ſo far, and was then repell'd by reflecting Objects near the Obſerver, rather than by reflecting Objects that were near the ſonorous Body; I ſhall give an Example or two, whence it will appear, that an Echo made by reflecting Objects near the ſounding Body may be heard for ſeveral Miles, as well as the primary Sound, and ſometimes more intenſely than the ſame.

I have often obſerved, that Cannons diſcharged in the Evening on the River *Thames*, about *Deptford* and *Cuckhold's-Point*, often made a Report which was double, treble, four-fold, and ſtill more multiply'd; and that the later Reports are ſtill the louder. And when I have gone croſs-wiſe this Way or that Way perhaps a Furlong, or a Quarter or Half a Mile, yet ſtill the Sound was the ſame. I remember that on the 8*th* of *March* laſt paſt, ſeveral great Guns were diſcharged ſome where between *Deptford* and *Cuckhold's-Point* aforeſaid, from a Ship which I then ſaw upon the *Thames* from my Church. Their Sound was repeated five or ſix Times, after this Manner,  I counted 122 Half-ſeconds between the Flaſh and the Sound, the Wind blowing obliquely. Therefore at that Time the Guns were diſtant from me above 13 Miles. The two firſt Cracks were fainter than the third; but the laſt Cracks were louder than any of the reſt. And going a Quarter of a Mile to my Right Hand, the multiply'd Sound was the ſame, and likewiſe when I went to my Left Hand. And beſides in ſome of my Stations, beſides the multiply'd Sound, I plainly heard a faint Echo, which was reflected by my Church and the Houſes adjacent: Which I obſerved then very often, when ever the Guns were diſcharged.

Another Obſervation of a like Kind was made on a certain *Sunday*, about two or three Years ago, on the Sound of a large Cannon diſcharged ſomewhere in the River *Thames*, on this Side or beyond the Town of *Graveſend*. The Sound of this Gun was multiply'd at leaſt eight, nine, or ten Times, according to this Meaſure of this Time  Many thought this multiply'd Sound was the Noiſe of many Guns belonging to the Ship; but I conceive it to be nothing elſe but a reiterated Echo, from the Sound of one or two Guns, which was reverberated from ſeveral Ships, or the neighbouring Shore. I did not only hear this my ſelf, but many others who were far

far off. Mr. *Barret* heard the ſame repeated Sound at his own Houſe, which is near four Miles diſtant from *Upminſter* where I heard it.

Of an Echo *in the* Air.

3. I will add an Example alſo of the Repercuſſion of Sound in the Air, by Aerial Particles.

When I heard the Reports of great Guns, eſpecially when the Air was calm and ſerene, I often obſerved Murmuring to go before the Crack, in the Air above. And in a thin Cloud, I often heard the Sound of Guns above, juſt over my Head, which ran along for ſeveral Miles in the Air; ſo that this Murmuring continued for 15 Seconds of Time. This continual Murmuring, in my Judgment, proceeds from Particles of Vapour ſuſpended in the Atmoſphere, which oppoſe the Courſe of the Undulations of Sounds, and beat them back to the Ears of the Obſerver, after the Manner of an indefinite Number of Echos; which we call a Murmuring in the Air.

Theſe Things being duly conſider'd, it will be evident that an Echo may be heard which is made a far off; and that the aforeſaid Reduplication of the Report of the Guns upon *Blackheath* proceeded without doubt from *Blackheath* it ſelf, as I juſt now affirm'd.

No Variation *of* Sound *from the* Different *Elevation, or Direction of the* Gun.

4. Now to go on to my Obſervations concerning the Progreſs of Sounds. What I have ſuggeſted about the Sound of the Guns at *Blackheath*, I have found to be true in all others; that the Motion of Sound is neither ſwifter or ſlower, whether the Gun be diſcharged towards the Obſerver or from him.

Alſo in all Poſitions of the Gun, Horizontal, or upright, and in all Elevations of the ſame, whether 10 Degrees, or 20, &c. there is no Variation of the Sound. So true is the Obſervation of the famous *Florentine* Academy *del Cimento* in this Matter.

Alſo the Force of Gunpowder, whether ſtrong or weak, a greater or a ſmaller Quantity, tho' it may increaſe or leſſen the Intenſity of the Sound, yet it neither accelerates or retards its Motion.

The Motion *of* Sound *not* alter'd *by the* Alterations *in the* Air.

5. *Kircher* affirms, that he always found a different Velocity of Sound, at different Times, in the Morning, at Noon, in the Evening, in the Night-Time. But as I had the Convenience of a better Time-keeper, and a more commodious Diſtance, I never found there was any Diverſity in the Motion of Sound at theſe Seaſons. But in all Kinds of Weather, whether the Sky was clear and ſerene, or cloudy and turbid; whether Snow fell, or it was miſty; (for both of theſe ſtrongly abate the Audibility of Sound,) whether it Thunders or Lightens, whether it be Hot or Cold, Day or Night, Summer or Winter; whether the Mercury in the Barometer aſcends or deſcends. In all Changes of the Atmoſphere whatever, (Winds only excepted,) the Motion of Sound is neither faſter or ſlower, but is only more or leſs loud from that Variety of the Medium; which perhaps has deceived the ſagacious *Kircher*.

Hence it will follow, that the Concluſions are erroneous, which *Walker* has deduced from the Obſervations of Dr. *Plot*, *Kircher*, and thoſe of his own.

Sounds *produced from* different *Bodies* move *with the ſame Velocity.*

6. Though *Kircher* is of a contrary Opinion, yet I do not at all doubt but that the Sounds of all Bodies, of Guns, Bells, Hammers, *&c.* have the ſame Velocity.

Velocity. In the Year 1704, I compared together the Beats of a Hammer and the Report of a Gun, at the Diſtance of a Mile, which is the fartheſt that I could hear the Sound of the Hammer; and I found that the Sound of both came to me in the ſame Time. And that they paſs'd $\frac{3}{4}$, $\frac{1}{2}$, and $\frac{1}{4}$ of the ſame Space in $\frac{3}{4}$, $\frac{1}{2}$, and $\frac{1}{4}$ of the ſame Time.

As for what belongs to intenſe and languid Sounds, I doubt not but that they paſs over the ſame Space in the ſame Interval of Time; as may partly appear from theſe Experiments.

Jan. 13, 1704-5, The Maſter Gunner of *Tilbury Fort*, at my Requeſt, diſcharged a Gun or two, and a great Mortar in which he ram'd the Gunpowder very well. The Noiſe of all theſe came to me in the ſame Time, being diſtant about three Miles.

Alſo *Sept.* 11, 1705, the Head Gunner of *England*, on my Account, upon *Black-Heath* after Sun-ſet, diſcharged ſome Muskets, Sakers, and Mortars. The Muskets I could not hear, becauſe of the great Diſtance, or becauſe the Air was not very clear. But I heard the Sound of the Sakers and Mortars in the ſame Interval of Time, tho' the Noiſe of the Mortars was much more dull and remiſs than that of the Sakers.

The Motion of Sound uniform.

7. As to the Equability of the Motion of Sounds, I found it to be ſo as the Academy *del Cimento* had determin'd long ago. That Sounds paſs half their Space in Half the Time, a fourth Part in a fourth Part of the Time, and ſo on. Which will appear from the Examples in the following Table.

The Place in which the Difcharge was made.	The Number of the Vibrations of the *Pendulum.*	The Diftance of Places. Trigonometrically	By the Sound.	The Courfe of the Winds.
		Miles.	Miles.	
The Church at *Hornchurch*	9	0, 9875		Oblique.
North Okendon Church	$18\frac{1}{2}$	2, 004	2, 0	Oblique.
Upminfter Mill	$\begin{cases} 22\frac{1}{2} \\ 23 \end{cases}$	2, 4	$\begin{cases} 2, 4 \\ 2, 48 \end{cases}$	Favourable. With Snow, oblique.
Little Warley Church	$27\frac{1}{2}$	3, 0	2, 97	A ftrong fair Wind.
Rainham Church	$33\frac{1}{4}$	3, 58	3, 59	Oblique.
Alveley Mill	33	3, 58	3, 57	Oblique.
Dagenham Church	35	3, 85	3, 78	Favourable.
South Weal Church	45	4, 59	4, 86	Oblique.
Eaft Thorndon Church	$46\frac{1}{2}$	5, 09	5, 03	Little favourable.
Barking Church	$70\frac{1}{2}$	7, 7	7, 62	Favourable.
Blackheath Guns	116	12, 5	12, 55	Oblique.

The Diftances of Places from *Upminfter*, (the Place in which I obferved) as fet down in this Table, were meafured with all the Exactnefs I was able, either by a meafuring Rod, or by Trigonometry. And from the great Agreement there is found between the Diftances meafured in this Manner, and likewife by the Motion of Sound, the Excellence of my Inftruments, as alfo the Truth of my Obfervations and Calculations, appear very plainly. For the Difference between the Diftances meafured, and the fame taken by Sound, is either none at all, or only a few hundredth Parts, unlefs when the Wind was fair (excepting that at the Church of *South Weal*, of which I fhall fpeak afterwards.) Thus in the Obfervations made from the Churches of *Dagenham*, *Warley*, *Thorndon*, and *Barking*, the Diftances taken from the Sound feem fomething fhorter than they fhould be, becaufe the Wind accelerated the Sound. But in forming this Column of Diftances by the Sound, I allow'd nothing for the Acceleration of the Winds; but I only divided the Number of Vibrations, or of Half-feconds, by $9\frac{1}{4}$ or 9,25, the Number of Semi-feconds in which Sound defcribes one Mile.

Alfo the Equability of the Motion of Sound is evident from this Table; as will appear from comparing the Vibrations and the Diftances, or from the Column of Diftances from the Sound only.

Now that nothing might be wanting for the Confirmation of this Matter, I made a Journey to the Sands at *Foulnefs* on our *Effex* Shore. Thefe Sands, which are continually cover'd and wafh'd over by every Day's Tide, make a large and exact Plain of many Miles. Upon this Plain I meafured out only fix Miles; for neither the Tide nor my own Time would permit me to make Ufe of a longer Diftance. At the End almoft of every Mile I made Expeiments by difcharging Guns. From which Experiments I found, that all my former Obfervations were juft and true, that is, that Sound paffes a Mile in

$9\frac{1}{4}$

$9\frac{1}{4}$ Half-Seconds, two Miles in $18\frac{1}{2}$ Half-Seconds, three Miles in $27\frac{3}{4}$ Half-Seconds, and so on.

8. As to the 15 and 19 Queries, I confess I could never satisfy myself in these Matters, by any of the Experiments I made.

Of the Ascending and Descending of Sounds, &c.

And first as to the Progress of Sound by the shortest Way, as in Query 19. The Reason of my doubting of this was the Difference between the Space between the Village of *Weal* and *Upminster* measured Trigonometrically and by the Sound, as exhibited in the foregoing Table. The Trigonometrical Mensuration was taken in so many Manners, and with such good Angles, that I could have no Scruple about it. But because by the Motion of the Sound the Distance seems greater, and the Superficies of the intermediate Soil puts on such a Figure as is exhibited in *Fig.* 175; therefore I had some Suspicion whether the Sound might not move with a crooked Motion? or whether the Acclivity intervening in A might not oppose the Undulations of the Sound, and thereby retard it. Fig. 175.

That I might somehow untye this Knot, I caused an Experiment to be made, by the Sound of a Gun from the Top of *Langdown* Hills into the Valley beneath, at the Distance of 3,79 Miles. The Interval was carefully measured Trigonometrically, by Means of Angles and a Base that was large enough; and the Experiment was made when a gentle Gale a little opposed the Sound. I counted $35\frac{1}{2}$ Half-Seconds between the Flash and the Report; which Number so well agrees with the Distance, and approaches so near to the other Experiments, that there can be no Doubt but that Sound descends from the Top of a Hill strait down into the Vale beneath, (through the Air) and not according to the uneven Surface of the intermediate Ground.

Therefore I imagine there was some Error in the foregoing Observations at *Weal*, because I have not observed any such Thing, either in the last Experiment at *Langdown*, nor in any other.

As to the Motion of Sound up and down; that is, whether it is carried after the same Manner, and in the same Degree, from the Top of a Mountain to the Bottom, and back again? I can hardly hope to satisfy myself or any other in this Matter. Here in *Essex* and the Parts adjacent we have no Hills high enough to make the Experiments requisite for this Purpose. For the highest of all I have yet seen, such as those call'd *Langdown Hills*, do not much exceed Half a Furlong. For I have measured the highest Summit of the same, both Trigonometrically as also with my portable Barometer, and find the same by the first Method to be 363 Feet high.

But the last Summer, when I took a Journey to the Western Coasts of the Kingdom, I had a Mind to try from a certain Hill, the Height of which I measured a few Years ago, and found it, (if my Memory fails me not) to be about three Furlongs. At which Time the Wind blew obliquely, but so gently, that it would hardly have extinguish'd a lighted Candle. Then I order'd some Muskets to be discharged at the Foot of the Hill and at the Top, and I found the Sound to come both Ways nearly in the same Space of Time. If there was any small Difference it seem'd to consist in this, that the Sound ascended something sooner up the Mountain than it descended down the same.

But I could hardly meaſure the Time with the Exactneſs that was neceſſary, becauſe it happen'd unluckily, that my Time-keeper was a little out of Order by the Carriage. Therefore I muſt leave this Experiment to be made by others with better Succeſs. And indeed I could wiſh it might be tried at the *Alpes*.

Motion *of Sounds in* Italy.

9. Some Obſervations and Experiments made at my Deſire in *Italy*, by the moſt learned Dr. *Newton*.

Richard Townley, Eſq; inform'd me by Letter, *An.* 1704, " That Sounds " were ſeldom heard at *Rome* ſo far as in *England*, and in our Northern Cli- " mates. Particularly he ſaid, that while he was at *Rome* ſome Guns were " diſcharged at the Caſtle of St. *Angelo*, while he was upon Mount *Trinidad*, " and that he obſerved the Sound was much more languid in that Place, than " in any other at the ſame Diſtance. And after his Death his Brother wrote " to me, that in the Year 1688, leaving *Rome* he came to the Caſtle " *Gendolphe*, (being a higher Situation near the Lake *Albanus*, about twelve " *Italian* Miles from *Rome*) he obſerved the Sound of great Guns diſcharged " from the aforeſaid Caſtle St. *Angelo*, which ſeem'd to him to be very weak " and faint. Alſo at another Time, when his Chariot paſs'd near the Walls " of the aforeſaid Caſtle, and great Guns were diſcharged from thence, they " did not ſeem to make ſo loud a Noiſe there as he expected.

As this was obſerved by theſe two Gentlemen, and the Phenomenon itſelf ſeem'd to be new and unknown before, I had a great Mind to enquire into the Cauſe of it. Therefore I wrote a Letter to the learned Dr. *Newton*, who ſent me Word back in *October* 1706, what himſelf and his Friends had obſerved about it.

He tells me, that in his Journey from *Bononia* towards *Florence*, at the City of St. *Michael* in *Boſco* (near *Bononia*) he heard the Report of Guns that were diſcharged; which Guns were diſcharged at *Mirandula*, and were diſtant 40 Miles: For at that Time the *French* Army were then beſieging this Place. And the Night following lodging upon the *Apennines*, being 20 Miles farther off, he heard the ſame Sound.

When he received my Letter at *Florence*, he acquainted a certain Nobleman with the Contents, who afterwards communicated my Requeſt to the Great Duke. He ſays, " The Great Duke immediately gave Orders, that " Experiments might be made for my full Satisfaction." He appointed *Joſeph Averrani*, a noted Philoſopher at *Piſa*, to ſuperviſe and direct theſe Experiments. The Reſult was this.

" In the lower Tower at *Florence* a great Gun was often diſcharged between " the Hours of One and Three at Night, and certain Men at *Leghorn* were " appointed, to obſerve diligently whether they could hear the Report. Some " of theſe who were placed at *Lanterna* and *Marzocco* did not hear it; poſſibly " becauſe the Claſhing of the Waves of the Sea might diſturb the Sound. " But others who ſtood upon the Fortifications of the old Tower, which is " call'd *Denjon*, and thoſe that were ſent to that call'd *Mount Rotondo*, (which " is about five Miles diſtant from *Leghorn* towards *Mount Nero*) could hear " it well enough. And as often as it was diſcharged, ſo often the Report

" was

" was plainly heard in thoſe Places. Now the Diſtance of this *Florentine* " Tower from *Mount Rotondo* in a right Line, is thought to be not leſs than " 55 Miles. And it is worth obſerving, that the intermediate Country " abounds with Hills, which of Neceſſity muſt ſomething impede the Paſ- " ſage of the Sound. To which may be added, that the ſame Evening there " was a moderate Weſterly Wind, which may be fairly ſuppoſed to hinder " ſomething the Expanſion of the Sound, ſince *Leghorn* is ſituate to the South- " Weſt in reſpect of *Florence*.

" Now that an open Place might be had, that Tract of Sea was made " Choice of, that lies between *Leghorn* and that call'd *Porto Ferraio*, the Di- " ſtance of which is found to be 60 Miles, according to the Calculation of " skilful Navigators. And the Sound of Guns of War is often heard from " *Leghorn* to the aforeſaid *Porto Ferraio*, and the neighbouring Places; nor " is there any Occaſion for the Aſſiſtance of a fair Wind, to help the Pro- " greſs of the Sound, that it may be heard the better. For any Wind what- " ever, whether for it or againſt it, is a Hindrance to Sound, and makes it leſs " ſonorous. Perhaps becauſe the Noiſe that is thence made in the Sea is a " greater Impediment, than the Courſe of the Air ruſhing thither would be " an Aſſiſtance. Wherefore the Sound is then only heard, when the Wind " is quite ſtill, or whiſpers very ſoftly, and when the Air is ſerene, and the " Sea calm. Neither even then is it heard indifferently from all Places, but " only from thoſe which are ſomething lofty; ſuch are thoſe two Forts which " are call'd the *Star* and the *Falcon*, and the Place call'd *Mulini*. Beſides, it " is neceſſary that the Obſerver ſhould keep himſelf very attentive, nor ſhould " be incumber'd with the Noiſe of any near him. And then he may hear as " well by Day as Night, if the Atmoſphere be ſerene and calm; except that " in the Night-time Sound ſeems to be ſomething ſtronger and ſharper, when " no other Noiſes mix with it, which by Day continually aſſault the Ears.

" It has alſo been told us by Witneſſes very deſerving of Credit, that many " Years ago, when there were Tumults at *Meſſina*, and the City itſelf was " beſieged, that the Reports of the great Guns reached the Ears of the Inha- " bitants of *Auguſta* and *Syracuſe*.

" Likewiſe when the *French* bombarded *Genoa*, it is certain that the Sound " reach'd as far as the *Black Mountain*, which is near *Legora*.

" From theſe Obſervations we are inclined to believe, that there is no Dif- " ference in this Matter between *Italy* and the Northern Climates.

" As to the other Query, Whether a Wind that blows with or againſt a " Sound accelerates or retards its Motion? As yet we cannot anſwer this " with any Certainty. Yet we will produce the following Experiments.

" A great Gun (60) was planted upon the Curtain of the lower Fort at " *Florence*, and ſo fixt, that its Mouth might be directed towards *Artemino*, " which is a Country Palace of the Great Duke of *Tuſcany*, ſtanding upon a " pretty high Hill, looking towards the Weſtern-ſide of the ſaid Fort, from " whence it is diſtant about 12 Miles. We choſe a Day when a Weſterly " Wind blew pretty ſtrong, that the Motion of the Sound might be hin- " der'd by the oppoſite Wind. But this was of little Uſe, for at Evening

" the

" the Air was quite calm, or at moſt had ſo little Motion, that it would " hardly diſſipate the Flame of a lighted Candle. Then leaving ſome skilful " Perſons here, we retired to the aforeſaid Palace *Artemino*, and between the " firſt and third Hours of the Night, the Cannon was diſcharged ſeveral " Times; and we conſtantly counted 49 Seconds between the Flaſhing and " the Report. And we alſo at *Artemino* fired ſome Bombs, between the " Flaſhing and Report of which the Obſervers aforeſaid, which we left be- " hind in the Fort, counted only 48 Seconds. Whence it appear'd, that " the Sound went ſwifter from *Artemino* to *Florence* by only one Second, than " the contrary Way.

" We dare not truſt our Obſervation ſo much as to affirm, that this little " Difference of Velocity muſt be imputed to the Force of conſpiring or op- " poſite Wind. For poſſibly it may ariſe from the Miſtake of the Obſerver, " who counted the Vibrations of the *Pendulum*, which may eaſily happen. It " may ſo fall out, that he may not ſee the Flaſh till after the Vibration of " the *Pendulum* was begun, or may hear the Sound before the Vibration is " compleated; ſo that by this Means he may make his Reckoning one more " than it ſhould be, whilſt the Space of Time in both Caſes is the ſame.

" When it was Day we order'd the Gun to be diſcharged again; but the " Wind was neither favourable to our Work or our Wiſhes. For a lit- " tle before it had gone about to the North. So that the Difference of the " Time and of the Velocity of the Sound could hardly be perceived in ſo " ſmall a Change of the Wind: So that we counted 49 Vibrations of the " *Pendulum*, as before.

As to the Space, which Sounds paſs over in any given Time, they are not yet ſatisfied about this Matter; but from ſome Experiments they apprehend it to be, as has been determin'd by the Experiments of the Academy *del Cimento*.

From theſe Obſervations it abundantly appears, that Sounds may be heard much farther in *Italy* than my aforeſaid ingenious Friend has inform'd us. For the excellent Dr. *Newton* has heard the Exploſion of great Guns at the Diſtance of 60 Miles. Thoſe that were diſcharged at *Florence* at his Requeſt, were heard 55 Miles. The Guns diſcharged at *Leghorn* were heard at the Diſtance of 60 Miles. Thoſe diſcharged at *Meſſina* were heard by ſome at nearly the Diſtance of 100 *Italian* Miles, as appears by the Maps. Thoſe diſcharged at the Siege of *Genoa* were heard at above 90 Miles by the Maps.

All which Things conſider'd, I cannot but think, that Sounds are as freely propagated in the Southern Countries as in theſe to the North. Though Examples are not wanting of a farther Progreſs of Sounds in ſome Northern Countries of the Earth. A certain *Dane* aſſured me, that when he lived in *Denmark* he heard very plainly the Sound of ſome Bombs, which were diſcharged at *Carelſcroon*, when he was diſtant 80 Miles, if I rightly remember. The very skilful Dr. *Hearn*, Phyſician to the King of *Sweden*, ſent an Account to our Royal Society concerning great Guns diſcharged at *Ulm*, *A. D.* 1685, the

the Noiſe of which travel'd 30 *Swediſh* Miles, which are nearly equal to 180 *Engliſh* Miles. Alſo in that Sea-fight between *England* and *Holland*, *A. D.* 1672, the Noiſe of the Men of Wars Guns was heard by ſome that were diſtant above 200 Miles: For it paſs'd over our Iſland as far as *Shropſhire* and *Wales*.

Therefore what was obſerved by the two Brothers *Townleys*, is peculiar to the foreſaid Caſtle of St. *Angelo*, or at leaſt to *Rome*. That Diminution of Sound, which they took Notice of, muſt be owing either to the Situation of the aforeſaid Caſtle, or to the intermediate Houſes riſing high and unequally in that City, or to the foreign Noiſes intermingling, or to contrary Winds, or to ſome other Cauſe of a like Nature: Or perhaps they made their Obſervations in ſuch a Conſtitution of the Air, in which Sounds are much more faint, even though they have favourable Winds, than at other Times when they are quite contrary.

10. I have often obſerved in Summer, when the Air was hot, that Sounds ſeem'd to be fainter than uſual, and to come very weak to our Ears; whereas in another Seaſon, eſpecially in Winter, in froſty Weather, they were much more ſhrill and ſtrong. Alſo when the North or Eaſt Winds blew, though the contrary Way, I perceived Sounds to be lowder than when the Winds came from the contrary Quarters. This *Kircher* alſo took Notice of at *Rome*; tho' it is not conſtant and perpetual. *Of the* Remiſſneſs *and* Intenſeneſs *of* Sounds, *according to the* Alteration *of the* Air.

Nor could I conclude any Thing with more Certainty, from the Inſpection of the Quickſilver aſcending or deſcending in the Barometer. For ſometimes when it roſe to the very Top, Sounds were louder and ſtronger, and ſometimes weaker. When on the contrary they were more noiſy, when the Mercury fell to the Bottom.

There is a like Uncertainty when the Air is ſerene or cloudy. In rainy and moiſt Weather I have often obſerved that Sounds were blunt and dull. But after Showers that were violent, they have acquired much Strength, as *Kircher* has obſerved at *Rome*. But the Contrary alſo has often happen'd. *May* 31, 1705, the Air was here much clearer and freer from Vapours than ever I remember to have ſeen it before. The Sky was ſo very clear and limpid, that I could eaſily perceive the moſt diſtant Objects. Yet I could not hear the great Guns which were then diſcharged on *Blackheath*, (excepting one, whoſe Report I heard but very faintly) though I could eaſily ſee the Flaſh of every one. And at the ſame Time the Motion of the Clouds and Wind conſpired with that of the Sound. For a very mild Air then breathed, and every Thing then ſeem'd to concur to aſſiſt the Sound. And on the contrary, when the Conſtitution of the Air and Heavens was entirely changed, when all Things were turbid, and the Atmoſphere was full of Vapours, I have obſerved that Sounds were loud, and as often dull and heavy.

I muſt leave the Cauſes of theſe Variations to be enquired into by others, becauſe I confeſs they are as far above my Capacity as it is to aſſign what is the proper *Medium* or Vehicle of Sound. Whether it be the purer and more

ethereal

ethereal Part of the Air, or the denſer and more vaporous Part, or both together?

As to what concerns groſs Clouds, it is certain they very much blunt and deaden Sounds; for then Sounds ſeem to be generally very languid and dull. This certainly ariſes from thoſe groſs Vapours and cloſe Particles that conſtitute a Cloud.

The ſame I have obſerved of ſnowy Weather: For when Snow has juſt fallen upon the Ground, Sounds immediately grow dull. But as ſoon as its Superficies becomes frozen and icy, they are again more ſhrill and ſtridulous. I have heard Bells and Guns tinkling and bellowing as loud as before the Snow fell. Mr. *Townley* lately aſſured me, he had obſerved, (as indeed I have taken Notice of myſelf) as he was riding on Horſeback through ſome Town, that the Noiſe of the Bells, which were ringing not far off, could hardly be heard by him when a Houſe cover'd with Snow interpoſed. So that entering the Town, he very much wonder'd to find the Bells ſtop on a ſudden, as he paſs'd by ſome Houſes between, and again to ring out when he came to a Vacuity free from Houſes. And this he obſerved all the Way paſſing through the Town, that the Noiſe of the Bells came to his Ears or not, according as the Buildings did or did not interpoſe.

Of the Influence of Winds on the Motion of Sounds.

11. The moſt illuſtrious Academy *del Cimento* found by Experiment, that the Motion of Sounds was not hinder'd by contrary Winds, nor promoted by fair Winds. But that however the Winds blew, the ſame Space was always deſcribed in the ſame Time. *Gaſſendus* was of the ſame Opinion and almoſt all other Philoſophers.

Yet for all that, the Contrary appears from Experience. They ſeem to have fallen into this Miſtake, becauſe they made their Experiments at too ſhort a Diſtance: For it is very probable that theſe Philoſophers made their Obſervations from the Diſtance of one Mile only, or two or three at the moſt: Therefore it cannot be wonder'd at, that they are ſo faulty. But if they had tried with good Inſtruments, at ten or twelve Miles, which I have often done, they would ſoon have perceived their Error.

I myſelf have often fallen into this common Miſtake, ſeduced by the Authority of theſe Gentlemen; till after the Obſervation of great Guns upon *Blackheath*, for three Years and more, I at laſt very happily diſcover'd and retracted it. Now when firſt I perceived that Sounds arrived at my Ears ſometimes ſooner and ſometimes later, I entertain'd a Suſpicion that I muſt have made ſome Miſtake; either that I had miſcounted the Vibrations of my *Pendulum*, or had not rightly obſerved the Flaſh of the Powder, or had fallen into ſome other Error of the like Kind. But when the Guns were diſcharged on Purpoſe for me, every Half Hour, from Six a-Clock to Midnight, and I found the Sound to come always without any notable Variation,

riation, in the Space of 120 or 122 Half-Seconds, even though the Wind was quite contrary; and at other Times when the Wind was fair, whether direct, or a-crofs, or oblique, the Sound of the fame Guns was obferved to come in the Space of 111, 112, 113, 114, 115, 116, or at moft 117 Semi-Seconds: Then I was fully perfuaded that there was fome material Difference, which produced that Variety in the Obfervations.

Nor do fair or foul Winds only accelerate or retard the Motion of Sounds, but alfo according to their various Degrees, whether they blow more vehemently or more gently, fo much the more or lefs they promote or hinder. Concerning which I fhall fubjoin fome particular Obfervations in the following Table. Firft taking Notice, that the Guns upon *Blackheath* are diftant from the South about 60 Degrees; that is, that they decline to a Point fomething more remote from S W b S.

A Table of the Sounds *of great Guns upon* Blackheath, *according to the Variety of Winds, and Forces, with which they are agitated.*

The Day of the Month and Year.	The Hour of the Day.	The Numb. of Vibrations	The Point of the Wind.	The Courſe of the Clouds.	The Height of the Mercury.
1704.					
Feb. 13	The 6 Hour.	120	N E b E 1	N E b E	29 99
	At Midnight.	122			
21	At 11 $\frac{1}{2}$ Morn.	119	E 2	E	30 22
1705.					
Mar. 30	10 in the Morn.	113	S W 7	S W	29 30
Apr. 2	8 $\frac{1}{2}$ Afternoon.	114 $\frac{1}{2}$	S b W 1		
3					
3	10 in the Morn.	116 $\frac{1}{2}$	S 4	Lower Clouds S, Up. ditto W b N	29 80
5	1 in the Aftern.	111	S W b W 7	S W by W	29 70
13	8 $\frac{1}{2}$ in the Morn.	120	N b E 2		29 26
24	5 Afternoon.	116	S W b W o	N W	29 59
Sept. 11	6 $\frac{1}{2}$ Afternoon.	115	W 2	W b N	Saker.
	7 Afternoon.	115 $\frac{1}{2}$	W b N 2		Mortar.
29	10 $\frac{1}{2}$ Morning.	112	S S W 6	S S W	29 38
Octob. 6	10 in the Morn.	117	E S E 1 & 2	S E	29 34
Nov. 30	At Noon.	115	S S W 4	S S W	29 10
Febr. 15	11 in the Morn.	116	S b W 1	S W	29 60
1706.					
Nov. 29	11 $\frac{1}{2}$ Morn.	116	S W o	S W b W	30 06
	At Noon.	118	S W b S 1		
Febr. 7	At Noon.	113	S W b W 4	W	29 83

I have choſe theſe Obſervations out of many, which were all carefully made, and repeated twice, thrice, or oftner. Thus from the Experiments made *April* 5, and *Sept.* 29, it appears, that violent Winds preſs forwards and haſten Sounds. For on *April* 5, when the Motions of the Wind and Sound nearly conſpired together, and the Wind was pretty ſtrong, (as is denoted by the

the Figure annexed [7] and in the ſame Manner the Cipher [0] denotes a calm Air, and the Figures 1, 2, 3, 4, &c. ſignify the various Strength of the Wind) at that Time, I ſay, the Sound travel'd its Journey in the Space of 111 Semi-ſeconds. But on *April* 24, when the Wind blew the ſame Way, and the Air was calm, it perform'd the ſame Journey in the Space of 116 Semi-ſeconds. So *Feb.* 7, 1706, when the Wind blew from the ſame Quarter, and brought the Sound along with it, but with a Strength that was not above Half as much, there paſs'd 113 Semi-ſeconds before the Sound compleated its Journey. So again *Sept.* 29, 1705, the Wind blowing pretty ſtrong, and not ſo fair, the Sound perform'd its Courſe within 112 Semi-ſeconds. From which, and from the other Examples in the Table, it plainly appears, that ſtrong Winds aſſiſt the Propagation of Sound, and that weak ones do not promote it ſo much.

The ſame is evident alſo from thoſe Winds, or Torrents of Air, which directy favour or hinder the Progreſs of Sound; that is, that they make its Motion either ſwifter or ſlower. And thoſe Fluxions of the Atmoſphere which are intermediate, in like Manner cauſe an intermediate Progreſs of Sound, or make an intermediate Number of Vibrations of the *Pendulum*.

The greateſt Difference which I have yet obſerved, in the Progreſs of Sound, for the Space of almoſt 13 Miles, is equal to about nine or ten Half-ſeconds, and that is when ſtrong Winds help the Sound, and gentle ones only hinder it. But when thoſe that promote or obſtruct are very mild or none at all, then the Difference hardly exceeds two or three Half-ſeconds.

12. That I might know to a Certainty how much Space is paſs'd over by Winds in any aſſign'd Time, I made Uſe of certain light Bodies in my Experiments, ſuch as Down, ſoft Feathers, &c. which ſeem'd to be better adapted to the Purpoſe than that Inſtrument, which is deſcribed in the *Philoſophical Tranſactions*, n. 24, or that other more convenient one in the Shape of a Windmill, invented (if I miſtake not) by the the moſt ingenious Dr. *Hook*.

Of the Velocity *of* Winds.

Vid. ſupra, V. II. c. I. S. XVII.

From the many Experiments which I have made, by the Help of thoſe light Bodies, when the Strength of the Wind was very different, I found that the moſt vehement Wind hardly paſs'd over ſixty Miles in the Space of an Hour. For Example, *Auguſt* 11, 1705, there was ſuch a Storm of Wind, that it almoſt overturn'd the Windmill itſelf, near the Place where I made my Obſervations. [I generally denoted the Force of the Wind, as I have ſaid already, by the Figures, 0, 1, 2, 3, &c. as far as 10 or 15, or more Degrees.] I eſtimated the Force of the aforeſaid Wind to anſwer to about 12 or 14 of theſe Degrees; and obſerved from many repeated Experiments, that that Whirlwind deſcribed about 33 Feet in the Space of one Half-ſecond, or 45 Miles in an Hour. Whence I collect, that the moſt furious and moſt ſtormy Wind, not excepting that Tempeſt which raged in *November*, 1703, does not paſs above fifty or ſixty Miles in the Space of an Hour.

Having thus meaſured the Velocity of the moſt rapid Winds, it will not be difficult to gueſs what is the Velocity of thoſe which are leſs violent: For I have obſerved their Courſes alſo, and am certified by various Experiments, that ſome of theſe traverſe fifteen Miles in an Hour, ſome thirteen, ſome much more, and ſome much leſs. Some creep with ſo ſlow a Motion, that they hardly go a Mile an Hour. Again, ſome Winds move ſo very ſlow, that a Man on Foot or on Horſeback may eaſily overgo them. This appears to our Senſes, whenever we ſtop, and feel a gentle Air fanning and overtaking us. But if we go along with it, we ſhall not perceive it at all; but if we ſtep forwards quick, inſtead of an Air that accompanies us, we ſhall find it contrary and blowing in our Faces. Thus when the Atmoſphere is quite at Reſt, and ſtagnating as it were, if we walk or ride, we then perceive a gentle Air as it were meeting us, and of juſt ſuch a Strength as anſwers our own Motion. And an Air of Wind ſeems to move with the ſame Motion or Velocity, as we ourſelves move the contrary Way.

We may make many uſeful Inferences from theſe Obſervations about the Velocity of the Winds. Particularly we may aſſign one Reaſon why the Mercury aſcends and deſcends ſo long before fair Weather or Rain follows.

But I ſhall omit ſuch Things as theſe, as being foreign from my Purpoſe. I ſhall only obſerve this as to Sounds, that when their Motion is ſwifter than the Wind, it appears that thoſe Parts of the Atmoſphere on which Sounds are impreſs'd, or by which they are carried, are not the ſame as thoſe of which Winds are compoſed, but ſome others more etherial or volatile, as far as may be conjectured. For the ſwifteſt Winds hardly fly above 60 Miles in an Hour, whereas Sounds can paſs above 700 Miles in the ſame Space of Time.

Now if it ſhould be objected, that Winds make Sounds ſwifter or ſlower, it might be anſwer'd, that this does not proceed from the proper Flux or Tendency of the windy Particles alone, but rather from the conjunct and conſpiring Motion of all the Particles of the Atmoſphere, as well the groſs as the ethereal. Which Direction of the Courſe or Motion, if it favours the Undulations of the Sounds, their Motions will be thence accelerated, but if it is contrary to them, they will be retarded, as is very probable.

Of the Velocity *of* Sounds

13. Therefore I conclude very ſtrongly from what has been now ſaid, and many other Things before taken Notice of, that Sounds are propagated according to theſe Degrees of Velocity; that the Diſtance of a Mile, or 5280 *Engliſh* Feet, is deſcribed in the Space of $9\frac{1}{4}$ Half-ſeconds, or which is the ſame Thing, they deſcribe 571 Feet in one Semi-ſecond, or 1142 Feet in the Space of one Minute.

Now Sounds paſs over the aforeſaid Space if the Courſe of the Atmoſphere is oblique, and is their mean Progreſs or Motion. But if the Wind increaſes the Rapidity of the Sound, it is poſſible they may paſs over 600 Feet or or more in the Space of a Half-ſecond. But on the contrary if it hinders, they may not paſs over above 560 Feet in the ſame Time.

Now the aforeſaid Obſervations and Experiments may not be a little ſerviceable

To a *Philoſopher*, for explaining the abſtruſe Phenomena of Sound.

To a *Mariner*, who may learn from hence, how far Ships are from him, which he ſees fluctuating or lying at Anchor before him, or how far he is from Land which he ſees at a Diſtance. Theſe Things may be known by the Diſcharge of Guns made on Purpoſe, or from a Signal given, and that very ſurely and exactly.

To a *Soldier*, to find at what Diſtance the Camp of the Enemy is from him, or a City beſieged. For the Elevation of great Guns upon Fortifications, for directing of Bombs, *&c.*

To a *Geographer*, for the eaſy and exact Menſuration of the Diſtances of Places. For any one in the Space of a few Hours may make a Map of a whole Country very exactly by this Means. For Guns diſcharged will ſhew the Diſtances, and any Mathematical Inſtrument for meaſuring Angles, or the common Inſtrument of Surveyors call'd *the Plain Table*, or only a Ruler provided with Sights, will ſhew the Situation of the ſeveral Places: From whence it will be no difficult Matter to delineate them.

Laſtly, this Method of Obſervation may be applied very conveniently to the Meaſuring of inacceſſible Places, eſpecially very wide Rivers, and ſuch Places whoſe Diſtances are otherwiſe very difficult to meaſure.

To the *Meaſurer of Echo's*. Of this ludicrous and pleaſant Phenomenon of Sounds, the Echo, though many learned Men have formerly and lately made very anxious Enquiries, yet they are not well agreed about many Things concerning it. Particularly about the Space neceſſary for the Repetition of 1, 2, 3, or more Syllables, or which comes to the ſame, of the Space deſcribed by the Echo in a certain Space of Time. *Merſennus* allows . . . . . Paces to the Repeating of a Word of one Syllable: *Blancanus* 24 Paces, with whom Dr. *Plott* agrees. But *Kircher* affirms, that nothing certain can be determin'd about it, becauſe different Winds, and the different Intention and Remiſſion of the Force of the Sound, and many other Circumſtances, produce an immenſe Variety.

Of the Motion *of* Echo's.

But it is not difficult to aſſign a Reaſon for all this Diſagreement. For it may ariſe from many Cauſes: From the Dullneſs or different Diſpoſition of our Senſes: From the various Audibility of the Sounds: From the grave or acute Tone of the Syllables themſelves, or their contracted or prolonged Pronunciation; or from any other Cauſe, that prolongs the Interval of Time. For I make no Doubt, for Example's Sake, but that if any Object that reflects Sound could return all the Syllables of this Verſe,

Vocalis Nymphæ, quæ nec reticere loquenti,

it could hardly return all the Syllables of the Verſe following, becauſe their Pronunciation muſt be ſomething longer,

Corpus

Corpus adhuc Echo, non Vox erat, & tamen usum.

And much leſs could it repeat all the harſh and long Syllables of the following Verſe, tho' their Number is much fewer, *viz.*

Arx, tridens, roſtris, ſphynx, præſter, torrida, ſeps, ſtryx.

But we may conclude from the foregoing Obſervations about the Motion of Sound, that in the ſame Manner as Sounds, ſo Echo's deſcribe certain determinate Spaces in certain Times. This I have often been convinced of by Experience, that the Echo returns in twice the Time in which the Voice reach'd the reflecting Object. For Example, if the Obſtacle reflecting the Sound was diſtant a Furlong, then the Return of the Echo was made in the ſame Interval of Time, in which the primary Sound would have deſcribed two Furlongs, if it had not been interrupted.

And this in meaſuring the Diſtances of Places was often of great Uſe to me. For Example, as I ſtood upon the Bank of the *Thames*, over-againſt *Woolwich*, the Echo of a Monoſyllable was reverberated by the oppoſite Houſes in ſix Half-ſeconds of Time. Whence I collect, that the Breadth of the River *Thames* in that Place was 1712 *Engliſh* Feet from Bank to Bank, or above a Quarter of a Mile: For as 9,25 Semi-ſeconds is to 5280 the Feet in one Mile; ſo is 6 Semi-ſeconds to 3424,8 Feet. The Half of which is 1712,4 Feet.

Laſtly, by this Means the Height of Thunder-clouds, and the Diſtance of Thunder and Lightning itſelf, may eaſily be known.

Of the Nature *and* Properties *of* Sound, *by* G. Grandus, n. 319. p. 270.

XI. The moſt learned Archbiſhop of *Armagh* compares the Science of Hearing with that of Seeing, or with the Science of Viſion, and divides it into Direct, Reflected, and Refracted, in the ſame Manner as the other; ſo that he conſiders not only direct and reflected Sounds, as others had done before him, but alſo refracted Sounds. He obſerves, that as Viſion has been perfected in a great Meaſure by our Anceſtors, by excellent Optical, Catoptrical, and Dioptrical Inventions; ſo he doubts not but the Hearing may be greatly improved by Acouſtick, Catacouſtick, and Diacouſtick Inſtruments, or by Phonicks, Cataphonicks, and Diaphonicks, (for he denominates them both Ways) as well in reſpect of the Object as of the *Medium*, or of the Organ. He propoſes Problems thereto belonging, which are exhibited in this Diſcourſe, but not only without any Demonſtration, but alſo without Determination or Conſtruction. But there are many Differences, by which the Propagation of Light is diſtinguiſh'd from the Diffuſion of Sound. Among which this is a notable one, that Light is diſperſed always according to right Lines, whilſt Sound is ſcatter'd every Way according to Curves, or any crooked Paths, and becomes ſenſible, though the Obſtacle of any opague Body is interpoſed.

Vid. ſupr. V. I. C. V. S. XXII.

And thoſe very Things which the very learned Author treats of concerning the Diffuſion of Sound, plainly manifeſt its Difference from the Propagation of Light. For he teaches, that Sounds very eaſily run along Walls, or ſmooth Arches, which have an Elliptical or Cycloidal Flexure rather than a Circular,

Circular, and that with a kind and expeditious Courſe; and readily moves along the ſoft Superficies of Water, complying with the ſonorous Tremors with which the Air is ruffled. Now this I fear is not ſo generally obſerved in the Propagation of Light: For in the Ellipſis we have this only demonſtrated in Catoptricks, that the Rays of Light proceeding from one *Focus* D, Fig. 176. and impinging upon the Elliptical Curve ABC, being thence reflected will be collected in the other Focus E. But if the Rays proceed from any other Point G, except the Foci, they will no longer meet in the ſame Point, but will be ſo reflected as by their Contact to form the Cauſtick Curve fFf; ſo that being upon its Convexity, they can have only a reflected Ray or two, and not more, but lying in the Curve itſelf, they will coincide with thoſe that are neareſt. But ſuch as are within the Concavity of the ſame, will have no reflected Rays, nor can hope for any Advantage from them.

As to the Cycloid, the learned Mr. *John Bernoulli* has ſhewn, in the *Acts* Fig. 177. *of Leipſick* for 1697, that a Ray of Light, if it were to paſs through *Media*, whoſe Denſities varied in every Point according to a ſubduplicate Ratio of the Altitudes, would be ſo continually refracted, as that it would be bent into the Curve of a Cycloid. But I cannot perceive what the Figure of a Cycloid would contribute to the better Diffuſion of Light. For this Curve is without any Foci, ſo that it cannot recollect the Rays to a Point, but the reflected Rays will paſs from it into irregular Curves, unleſs when Rays PM, QN, parallel to the Axis KL fall upon the Cycloid EMKNH; for then the Cauſtick Line form'd by the Contact of the reflected Rays MR, NS, would be compoſed of the two Cycloids ERL, HSL, generated by a Circle of Half the Diameter, and would exhibit very denſe reflected Rays about L the Confine of each, at the Middle of the Baſe of the reflecting Cycloid. But as well in theſe as in other Cauſticks, reſulting from any Poſition of the luminous Point and the Rays, the ſame Obſervations would take Place, which we have already ſhew'd to belong to Cauſticks form'd by an Ellipſis.

I have nothing to add concerning the plain Superficies of Waters, ſince it appears, that the Rays of Light will paſs through it, either altogether refracted, or will be ſent back the contrary Way by Reflexion, juſt as by the Surface of a ſolid Chryſtal, nay ſomething more ſtrongly by this than by that; ſo far are they from creeping eaſily along its Side, that they may the more expeditiouſly be ſent directly forwards, and obtain that ready Progreſs, which the Author attributes to the Harmonic Tremors creeping along the ſoft Surface of the Waters, and by its waving Motion accommodating itſelf to their Flexure. Nay we may juſtly doubt, whether the moſt ſmooth Superficies of Specula, as well as of Light, would much conduce to the Reflexion of Sound, ſince the Echo itſelf ſeems to inhabit the very rough Receſſes of Caves, rather than poliſh'd Walls, and ſuch as are lined with a thin fine Mortar; ſince it often returns an Anſwer from uncultivated Vallies, from uneven Caverns, and from the Ruins of old Buildings.

Yet I would not be underſtood as deſigning to detract any Thing from the Credit, or the Praiſe due to what is advanced by the learned Author, who

I

I think is rather to be encouraged to the Publication anew of this Theory of Sounds, that we may know by what Law theſe ſonorous Tremors are propagated through the Air, Water, and all Kind of Bodies of any Denſity, both fluid and ſolid: And in what conſiſts that Congruity between Light and Sound, which as yet is unknown to us. Hence the Foundations of Acouſticks being confirm'd, this Science may hereafter be wonderfully improved, after the Diſcovery of proper Inſtruments for congregating, increaſing, promoting, multiplying, and diſtinguiſhing Sound. I ſhall endeavour in ſome Manner to explain his Acouſticon, or the Phonick Sphere propoſed by him, and that rather by divining than interpreting; firſt giving his own Words, that they may be compared with my Conjectures to be added afterwards, and that every one may be able to judge how exactly they anſwer.

I ſhall here add, ſays the Author, *a Semiplane of an Acouſtick or Phonical Sphere, as an Attempt to explicate the great Principle of this Science, which is, the Progreſſion of Sounds. You are to conceive this rude Semiplane as parallel to the Horizon, for if it be perpendicular thereunto, I ſuppoſe the Extremity will be no longer Circular, but Hyperbolical, and the lower Part of it ſuited to a great Circle of the Earth. So that the whole Phonical Sphere, if I may ſo call it, will be a ſolid Hyperbola, ſtanding upon a concave Spherical Baſe.* The Diagram
Fig. 178. tranſmitted from *London* was after this Manner, but without any Notes or Explication by which it might be illuſtrated.

Therefore ſubſtituting this other Figure, I ſhall attempt to explain the Mind
Fig. 179. of the Author. Let the Globe of the Earth be CGFE, and at the Point C of its Superficies let a Sound be excited. This will be propagated round about by the Earth itſelf, and alſo by the Air; ſo that at what Time it arrives at the great Circle of the Earth deſcribed with the Pole C, tho' perhaps inſenſibly, or at leaſt might arrive at it, if it were forcible enough, being diffuſed through the Air it would fill up a certain Space, according to the different Degrees of eaſy Paſſage, not extended altogether ſpherically, but unequally, and circumſcribed by the Perimeter of the Hyperbola GLAKE, about the Axis CAO, which is perpendicular to the ſonorous Body C: Or rather determin'd by the Superficies of an Hyperbolical Conoid, which is generated by the Rotation of the Hyperbola ALG about its Axis. Therefore the entire Phonical Sphere, through which the Sound is extended in a given Time, will be the ſolid Space comprehended by the Hyperbolical Conoid GAEB, which ſtands upon the great Circle of the Earth GBE, and is terminated below by the concave hemiſpherical Superficies GCEB. Which Space being cut any where by a Plain parallel to the Horizon will exhibit the Semicircle LIK, ſuch as the Author's Figure ſhews, which he calls a Semiplane, becauſe the View of his Diagram exhibits only one Half of it, the other Half remaining conceal'd beyond the vertical Hyperbola, which itſelf cuts the Phonick Sphere through the Axis into two equal Parts. But what is the Species of this Hyperbola, or by what Principles this Doctrine is ſupported, neither the Author himſelf ſhews, nor have I any Foundation to build Conjectures upon.

Therefore

Therefore proceeding in an indirect Order of Inveſtigation, I ſhall firſt explain, through what Species of Lines the ſonorous Tremors muſt be propagated, that they may be expanded into ſuch an Hyperbola in a given Time; ſecondly, what we muſt ſuppoſe the Variation of Rarity to be at different Altitudes of the Air, that admitting the common Law of Refraction obſerved by the Rays of Light, ſo that it may bend the Directions of Sounds according to the Species of Lines ſo found; and thirdly, what muſt be the Law of Refraction on the other Hand, which the ſonorous Tremors obſerve in thoſe Curves, ſuppoſing the Variation of the Air's Denſity to be ſuch as moſt Philoſophers and Mathematicians allow, according to the reciprocal Ratio of the Weight of the incumbent Atmoſphere, preſſing the lower Parts downwards; which they contend to be confirm'd by Experiment.

For which let us conſider, that the ſonorous Body C communicates its Tremors every Way according to the Directions C*n*, C*m*, C*h*, or certainly according to thoſe Lines by which the Impulſe was made, that by reſtoring itſelf it repel'd the Air, and urged it on by the frequent Oſcillations by which it is agitated, and is put into a tremulous Motion when diffuſed according to the ſame Directions. Therefore in a certain little Time let theſe Tremors be conceived to arrive at the Points *m*, *n*, *h*, whence purſuing their Way after another given Time they will be ſucceſſively propagated, the firſt to the Point N, the Second to M, and the Third to H. Again, after another given Time they will arrive together, the firſt at G, the other at L, the laſt at A. Therefore I ſhall now call the Lines C*n*NG, C*m*ML, C*h*HA, the ſonorous Rays being thoſe through which every Tremor is ſucceſſively diffuſed; but the Lines *n m h*, NMH, GLA, which the aforeſaid ſonorous Rays, and all other that are ſynchronous and intermediate to them, together approach to in any given Time, I ſhall call *ſonorous Waves.* Fig. 180.

And indeed in a *Medium* which is every where quite uniform, the Cauſe ceaſing which compels the ſonorous Tremors to go out of their Direction, it is plain the ſonorous Rays muſt always proceed ſtrait on, or go directly the ſhorteſt Way from one Term to another, which will make the Waves to be perfectly circular, and concentrical to the ſonorous Body. For ſince they find no greater Difficulty to paſs here than elſewhere, they will be promoted at equal Diſtances in every given Time. Now every Ray will cut its Wave perpendicularly, and all the Waves will be concentrical and ſimilar, as is plain from the Elements.

But in a *Medium* of unequal Denſity, as in the Air ſurrounding the Earth, which according to its different Diſtance has a different Degree of Rarity, (for now we ſhall abſtract from its Viciſſitudes of Heat, Cold, Humidity, and Dryneſs, which cannot be reduced to any certain Rule) the Ray CHA alone paſſing perpendicularly through all the aerial Layers, or Superficies concentrical to the Earth, will continue direct and unrefracted; but others falling obliquely upon the ſame Superficies, will be refracted at every Point by a certain continual Flexure, and will be bent into the Curves C*m*ML, C*n*NG; and according to the different Facility of Paſſage will not proceed every where to the ſame Diſtance in the ſame Time. Wherefore the Points A, L, G, or H, M, N, which the Sound emitted through any Rays at the ſame Moment of Time, will be unequally diſtant from the ſonorous Body C, and thence the Lines ALG, HMN, *hmn*, will by no Means be Circles concentrical to the ſonorous Body, but Curves of another Kind, which however will be ſimilar to one another, and ſimilarly poſited. Wherefore in the Hypotheſis of our Author, who will have that extream Wave ALG to be Hyperbolical, which ſurrounds the ultimate Limits of the Terraqueous Globe; it muſt follow, that any other intermediate ones HMN, *hmn*, muſt be like Hyperbola's, and ſimilarly poſited to the different Vertices A, H, *h*, but deſcribed with the ſame Center, to the ſame Axis, and under like Figures of their Latera. For by whatever Method it may be ſhewn, that becauſe of the ſimultaneous Appulſe of the Sound to the Points A, L, G, through the ſynchronal Paſſages CHA, CML, CNG, the Wave ALG will become a Curve of ſuch a Species, ſuppoſe Hyperbolical; the ſame will prove from the ſame Foundations, becauſe alſo of the ſimultaneous Appulſe of the Sound to the Points H, M, N, by the ſynchronous Lines C*h*H, C*m*M, C*n*N, the Wave HMN will paſs into a Curve of the ſame Species, which in this Caſe will be a ſimilar Hyperbola, and ſimilarly poſited, as appears of itſelf. Nor can there be any Doubt but that the ſonorous Rays CHA, CML, CNG, muſt always cut thoſe ſimilar Waves ALG, HMN, *hmn*, perpendicularly or at right Angles, as happens in the circular Waves. And as the learned Mr. *Huygens*, in his *French* Treatiſe concerning Light, *p*. 44, has proved this in a like Subject concerning Lucid Waves, we need not ſpend any more Time in confirming this Obſervation here.

Therefore the Inveſtigation of the Paſſage according to which the Propagation of the ſonorous Rays are perform'd, in the Hypotheſis of our Author, is reduced to this purely Geometrical Problem; to find the Nature of thoſe Curves, which cut perpendicularly any ſimilar Hyperbola's which are deſcribed about the ſame Axis, and the ſame Center in a ſimilar Manner. Let ALG,
Fig. 181. HMN, *hmn*, and innumerable others intermediate, be ſimilar Hyperbola's and ſimilarly poſited either above or below theſe, having the ſame common Center O, and deſcribed with the ſame Axis OAH, to which the other OS is conjugate. Through the Point C the Curve C*m*ML or C*n*NG is to be drawn, which may cut perpendicularly all the Hyperbola's propoſed. Thro' the given Point C, between the Aſymptotes OA, OS, let an Hyperbola C*m*ML of ſuch a Nature be deſcribed, that ſuppoſing the Ratio of the tranſverſe Diameter of the former Hyperbola's AL, HM, &c. to the *Latus Rectum* of the ſame to be equal to the Ratio of t to r; the Powers of the Ordinates LQ denominated by the Exponent r may be reciprocally proportional to the Powers of the Abſciſſes from the Center OQ, denominated by the Exponent t. That is, making OQ $= x$, and QL $= y$, it may be $y^r = \frac{1}{x^t}$. Or drawing any other Ordinate mi, MI; ſo that the Ratio of the

Diſtances

Diſtances from the Center O Q, O I, may be ſuch a Multiple of the Ratio of the Ordinates I M, Q L, reciprocally, as the Fraction $\frac{r}{t}$ is a Multiple of Unity. I ſay, that this will ſatisfy the Demand. For drawing L P a Tangent to any Hyperbola A L in the Point where it is cut by the Curve C M L, as alſo S L R a Tangent of the Hyperbola C M L in the ſame Point; it appears from what we have ſhewn in the Demonſtration of *Huygens*'s Theorems, *c.* 7. *n.* 9. that it will be O Q to Q R as the Exponent of the Power of the Diſtances O Q, to the Exponent of the Power of the Ordinates Q L, that is, as *t* to *r*. But as *t* to *r*, or the *Latus tranſverſum* to the *rectum*, ſo is (by 37 *L. Conic.*) the Rectangle O Q P to the Square of Q L. Therefore it is as O Q to Q R, or taking a common Altitude Q P, as the Rectangle O Q P to the Rectangle P Q R, ſo is the Rectangle O Q P to the Square of QL, which therefore will be equal to the Rectangle P Q R. Therefore the Angle P L R will be a right one. And hence the Curve C M L cuts the Hyperbola A L G perpendicularly in the Point L. And in the ſame Manner it will be proved to be perpendicular to the other Hyperbola's H M N, *h m n*, in the Points M *m*, in which it cuts them. *Q. E. D.*

Hence we may infer, firſt, that if the Hyperbola A L G, determining the Phonick Sphere of the Author, and the other like concentrick ones H M N, *h m n*, are equilateral; then becauſe of the Equality of the Sides *t* and *r*, the Hyperbola C M L will be that of *Apollonius*, and alſo equilateral. For its Equation before exhibited will be changed into this $y = \frac{1}{x}$, where the Ratio of the Ordinates is ſimply reciprocal to the Ratio of the Diſtances from the Center. Therefore alſo the ſonorous Rays, as alſo the ſonorous Waves, according to this Hypotheſis would be Hyperbola's of the ſame Species, but only in a different Poſition. I remember that *Newton* ſhews in his *Opticks*, *l.* 3. *p.* 287. *Ob.* 10. that when Rays of Light received into a dark Room are made to paſs over the Edges of two Knives, they are alike bent into Hyperbolical Frindges, ſuch as C M L; of which Phenomenon, if the phyſical Cauſe could be aſſign'd, the ſame perhaps might prevail with us to believe, that the Rays of Sound are alſo Hyperbolical, ſuch as the Syſtem of the Archbiſhop of *Armagh* ſeems to require.

Secondly, it is to be obſerved, that if many ſuch Curves were deſcribed, or Hyperbolical Rays *m* M L, *n* N G, *&c.* perpendicularly cutting the Hyperbolical Waves A L G, H M N, *&c.* they could not meet exactly in one Point C, though they might approach nearer and nearer towards C and might come to a Diſtance leſs than any aſſignable. Therefore thoſe Hyperbolical Rays muſt be conceived to proceed from a Corpuſcle C of ſome Extenſion, and not from a Mathematical Point, which is moſt agreeable to it. For Sound is produced by the Colliſion of Bodies, and cannot be generated by the Tremor of what is ſtrictly a Point, or one Form of Extenſion.

Nay, ſince all the Waves propagated from a ſonorous Body, as we have ſeen above, ought to be ſimilar Hyperbola's; it is proper we ſhould conceive the ſonorous Body C to be as it were a very ſmall Fibril vibrating very ſwiftly, whoſe leaſt and as it were initial Wave, being almoſt infinitely ſmall 2, 3, 4, is itſelf truly Hyperbolical, or rather the phyſical Apex of ſome Hyperbola. So that the vibrating Fibril of the ſonorous Body C, for Example, while it is ſtruck, being diſturbed from its direct Situation 2 C 4, by the Force of Percuſſion into the concave Situation 2 5 4, then being reſtored by the Force of its very vehement Elaſticity and alſo of its proper Tenſion, it ſwells into the convex Hyperbola 2 3 4, and again reduced by alternate Vibrations, and fluctuating on each Side expands its Tremors into Hyperbolical Waves always ſimilar to the initial ones 2 3 4, 2 5 4, upwards and downwards of their own Nature, in a *Medium* without Reſiſtance. But perhaps being hinder'd by the Obſtacle of the Earthly Globe C E, whoſe Center is T, it may propagate its Hyperbolical Waves only upwards, and deſcribe the Phonick Sphere imagin'd by our Author, interrupted and limited in its lower Part by the Terreſtrial Hemiſphere.

Now if the Doctrine of *P. Pardies* is true, which is propoſed Artic. 81. of his *Staticks*, that Strings ſtretch'd by the Force of their own Weight aſſume an Hyperbolical Figure, ſuch as 2 5 4, whoſe Center is the ſame as that of the Earth; every one may ſee this would be moſt congruous for confirming the Syſtem of our Author. Hence alſo a Reaſon would be ſupplied, why every Fibril of a ſonorous Body C, while it is agitated by harmonical Vibrations, would put on the Form of the Hyperbola 2 5 4, having its Center in the Center of the Earth T, and in like Manner would ariſe to another equal to it 2 3 4, and thence would diffuſe the Tremor through other and larger Hyperbola's, the Center of all which would be O, equally diſtant from the ſonorous Body C, as the ſonorous Body itſelf is diſtant from the Center of the Earth. Wherefore the Diſtance C O, equal to the Diameter of the Terraqueous Globe, would determine the Limit beyond which no ſonorous Wave would be propagated, and no Sound could be heard. And the Line O S, as being the Aſymptote of any of the Hyperbolical Rays, through which the Sound is convey'd, would be the Confine of that happy Region, in which Men might philoſophize in the utmoſt Tranquility, ſecure from all Noiſe of Earthly Affairs.

Now that no one may think this Speculation is to be deſpiſed on this Account, becauſe every Fibril of a ſonorous Body, as being very ſhort and very much diſtended, ſhould ſeem always to remain in a ſtrait Situation 2 C 4, nor can ever be bent into the concave or convex Hyperbola's 2 5 4, 2 3 4; it is to be conſider'd, that Hyperbola's are ſo much the more enlarged, and approach ſo much the nearer to a right Line, the longer their Axes become. Therefore becauſe of the vaſt Diſtance of the Centers T or O, like Lines which falling Bodies deſcribe, are eſteem'd as Parallels tho' directed to the

Center

Center of the Earth, and the Arches of a horizontal Circle are not diftinguifhed from a right Line that is a Tangent; fo thofe initial Hyperbola's 254, 234, may be faid almoft to coincide with the right Line 2C4. Whence the Curvature of the vibrating Fibrils in a fonorous Body is not fenfible, nor does the Species of the Hyperbolical Waves difcover itfelf, till they are dilated into a larger Space G L A L G, and approach nearer to their Center.

Yet here it is to be obferved, that it will follow from thefe Principles, the Sound will not pafs at the Sides beyond the Space comprehended by the extream Hyperbolical Rays 298g, 476g, which the right Lines T 2, T 4, would touch, drawn from the Center of the Earth through the Terms of the vibrating Fibril. And indeed the Tremors of that Fibril would not proceed according to any other Direction than by T 2, T 3, T 4, and other intermediate ones comprehended by the Angle 2T 4, correfponding to the feveral Particles of the fame Fibril. Therefore the Space without the faid Hyperbola's 298g, 476g, would remain without any harmonical Tremor, nor according to the Meaning of the Author could the Phonical Sphere be extended to the whole Hemifphere of the Earth. Therefore no one Fibril of the fonorous Body muft ever tremble, but that at the fame Time it muft draw the Terms of the other Fibrils with which it is connected, and between which it is diftended, and muft likewife excite them to an harmonical Tremor. Thefe muft bring others with which they are connected, and make them tremble likewife; juft as a mufical String ftretch'd on a wooden Inftrument evidently communicates its Tremors to it. Hence harmonical Ofcillations are prefently transfufed into other Bodies, with which they are mediately or immediately connected, tho' always more and more weakened, and at laft becoming infenfible are fpread through the Superficies of the Earthly Hemifphere, and creep on farther and farther. This the Ear itfelf can teftify, if applied to the Earth, and may diftinguifh any great Noife, tho' raifed afar off. Therefore alfo from other Places other fonorous hyperbolical Rays emerge through the whole Hemifphere of the Earth, by which the Phonick Sphere of the Archbifhop of *Armagh* may be fufficiently replenifh'd.

I intend to be fhorter in difcuffing the two remaining Problems which I have propofed above. And yet I fhall endeavour to folve the fecond Queftion after a more general Manner, that its Ufe may be the more extenfive. Let NnG be fuppofed to be any Ray, either lucid or fonorous, changed Fig. 182.
into a Curve of any Kind, by a continual Refraction. 'Tis enquired after what Law the Denfity muft be fuppofed to be varied, or the Rarity of the *Medium* at its different Altitudes; that admitting the Theory of Refraction, which fuppofes the Sine of Refraction to be always proportional to the Rarity of the refracting *Medium*, that Ray may become a Curve of fuch a Nature? Let the Axis of the Curve NnG made by the refracted Ray, be the right Line C O, in which taking any Point C, with any Radius C L let the circular Quadrant L Pp be defcribed, and drawing any where the Tangent N R, nr, of the refracted Ray, from C let there be drawn a Ray parallel to the faid Tangent, meeting the Circle in P, and drawing P F parallel to the Axis, let it meet the Ordinate N Q perpendicular to the Axis in the Point

Point F, I ſay that the Curve thence ariſing F f F by its Ordinates FQ, fq, will expreſs the Rarity of the *Medium* at its different Altitudes. For becauſe C P is parallel to R N, the Angle P C B will be equal to that Angle which the refracted Ray N n makes with the Perpendicular at the Point N. And therefore B P or F Q will always be the Sine of Refraction, the whole Sine being C P. Wherefore ſince the Law of Refraction is ſuppoſed to be ſuch, that the Sine of the ſame is proportional to the Rarity of the *Medium*, the ſame Line F Q will denote the Rarity of the *Medium* at the Altitude Q, or at the Point N of the ſame Altitude, through which the Ray paſſes. *Q. E. D.*

Now in our Propoſition, in which $QN = \frac{1}{x^{\frac{t}{r}}}$, becauſe of $y^r = \frac{1}{x^t}$, if FQ denoting the Rarity of the Air be called z, it will be $z = \frac{t}{\sqrt{x^{2r+2t} + tt}}$; or taking r and C P for Unity, 'tis $z = \frac{t}{\sqrt{x^{2+2t} + tt}}$. And in the Caſe in which the hyperbolical Wave is equilateral, and therefore the Ray alſo is a like equilateral Hyperbola, 'tis $y = \frac{l}{x}$, becauſe $t = 1$, and therefore it will be $z = \frac{1}{\sqrt{x^4 + 1}}$.

Now becauſe as well *Jacobus Hermannus* in the Acts of *Leipſick* 1706, as Dr. *D. Gregory* in his Aſtronomy, *l.* 5. prove the Curve which determines the Degrees of the Rarity of the Air to be the Logarithmic Curve, ſo that the Altitudes O Q, oq, or x, will be the Logarithms of the Numbers that expound the Rarities of the Air in the Points Q, q; it is plain that the Curvature of the continually refracted Rays N n, N G, proceeds in ſuch a Manner, that the Sines of the Complement of Incidence and of Refraction being raiſed to the Power $\frac{r}{r+t}$, ſhall have a Ratio compounded of the Ratio of the Right Sines raiſed to a like Power, and of the Ratio of the Logarithms of the Rarities.

But tho' I might grant that the ordinary Law of the Refraction of Light gives the Sines of Incidence and Refraction proportional to the Rarities of the *Medium*; yet I muſt not diſſemble, that perhaps this may not be very exact. For the Ratio of the Sines in the Refraction out of Air into Glaſs is about ſeſquialter, yet Air is above a thouſand Times rarer than Glaſs. But when Geometricians perceived, that the Sine of Refraction in the Paſſage into another *Medium* became greater, according to the greater Facility with which Light could penetrate it in the common Hypotheſis, or according to the greater Difficulty in the Opinion of *Carteſius*, who ſuppoſes on the contrary, that Light is more refracted, becauſe of the greater Difficulty in a rarer *Medium* than in a denſer, as heavy Bodies becauſe of a greater Difficulty in penetrating denſer Bodies, in theſe are more refracted by rebounding from the Perpendicular;

cular; and that both Laws agree in this, that according to the greater Rarity of the *Medium* there would be a greater Refraction. Hence it has obtain'd that the Sines are said to be proportional not to the Facility or Difficulty of the Passage, either of which by some is call'd in Question, but to the Rarity of the *Medium* in which all agree, tho' the true Proportion does not altogether answer to it in the same Geometrical Ratio. Therefore where-ever Mention is made of Rarity, perhaps we should substitute Facility of Passage in the common Hypothesis, or Difficulty in that of *Cartesius*, except where we say, that the Rarity arising from the Weight of the incumbent Atmosphere corresponds to the Altitudes, as Numbers correspond to their Logarithms. For this is most exactly agreeable to Truth.

A Paper omitted.

XII. Some Theorems concerning the infinite Divisibility of Matter, which demonstrate its great Rarity, and the Tenuity of its Composition, by Means of which many Difficulties in Physicks are removed. By Dr. *John Keill.* n. 339. p. 82.

CHAP. V.

Hydrostatics. Hydraulics.

I. THE following Experiment seems to be of Use in discovering the Proportions of the Attractions of Fluids; I shall give what Account I can of it, though I have not here Conveniencies to make it in so succesful a Manner as I could wish. Ascent of Water between two Glass Planes, by Dr. B. Taylor, n. 336. p. 538. June 25. 1712.

I fastened two Pieces of Glass together, as flat as I could get; so that they were inclined in an Angle of about two Degrees and a Half, then I set them in Water with the contiguous Edges perpendicular. The upper Part of the Water, by rising between them, made this *Hyperbola*, *Fig*. 183. which is as I copied it from the Glass. Fig. 183.

I have examin'd it as well as I can, and it seems to approach very near to the common *Hyperbola*. But my Apparatus was not nice enough to discover this exactly.

The perpendicular *Asymptote* was exactly determin'd by the Edge of the Glass; but the *Horizontal* one I could not so well discover.

II. Some Days ago a Method, proposed to me by an ingenious Friend, for making a perpetual Motion, which seem'd plausible, and easily demonstrable from an Observation of the late Mr. *Hawksbee*, said to be grounded upon Experiment, was tried; which (though not succeeding) has given Occasion not only to rectify some Mistakes, into which we had been led by the late Mr. *Hawksbee*, The Cause of the Ascent and Suspension of Water in Capillary Tubes, by Dr. J. Jurin, n. 355. p. 739.

Hawksbee, but likewise to detect the real Principle by which Water is raised and suspended in capillary Tubes above the Level.

Fig. 184. *My Friend's Proposal was as follows.*] Let *A B C* be a capillary Siphon, composed of two Legs *AB*, *B C*, unequal both in Length and Diameter; whose longer and narrower Leg *A B* having its Orifice *A* immerst in Water, the Water will rise above the Level, till it fills the whole Tube *A B*, and will then continue suspended. If the wider and shorter Leg *B C* be in like Manner immerst, the Water will only rise to some Height, as *F C*, less than the entire Height of the Tube *B C*.

This Siphon being fill'd with Water, and the Orifice *A* sunk below the Surface of the Water *D E*, my Friend reasons thus:

Since the two Colums of Water *A B* and *F C*, by the Supposition, will be suspended by some Power acting within the Tubes they are contain'd in, they cannot determine the Water to move one Way or the other. But the Column *B F* having nothing to support it, must descend, and cause the Water to run out at *C*. Then the Pressure of the Atmosphere driving the Water upward through the Orifice *A* to supply the Vacuity, which would otherwise be left in the upper Part of the Tube *B C*, this must necessarily produce a perpetual Motion, since the Water runs into the same Vessel, out of which it rises. But the Fallacy of this Reasoning appears upon making the Experiments.

Exp. 1. For the Water, instead of running out at the Orifice *C*, rises upwards towards *F*, and running all out of the Leg *BC*, remains suspended in the other Leg, to the Height *A B*.

Exp. 2. The same Thing succeeds upon taking the Siphon out of the Water, into which its lower Orifice *A* had been immerst, the Water then falling in Drops out of the Orifice *A*, and standing at last at the Height *A B*. But in making these two Experiments, it is necessary that *A G*, the Difference of the Legs, exceed *F C*, otherwise the Water will not run either Way.

Exp. 3. Upon inverting the Siphon full of Water, it continues without Motion either Way.

The Reason of all which will plainly appear, when we come to discover the Principle by which the Water is suspended in capillary Tubes.

Mr. *Hawksbee*'s Observation is as follows.

Fig. 185. Let *A B F C* be a capillary Siphon, into which the Water will rise above the Level, to the Height *C F*, and let *B A* be the Depth of the Orifice of its longer Leg below the Surface of the Water *D E*. Then the Siphon being fill'd with Water, if *B A* be not greater than *C F*, the Water will not run out at *A*, but will remain suspended.

This seems indeed very plausible at first Sight. For since the Column of Water *F C* will be suspended by some Power within the Tube, why should not the Column *B A* being equal to, or less than the former, continue suspended by the same Power?

Exp. 4.

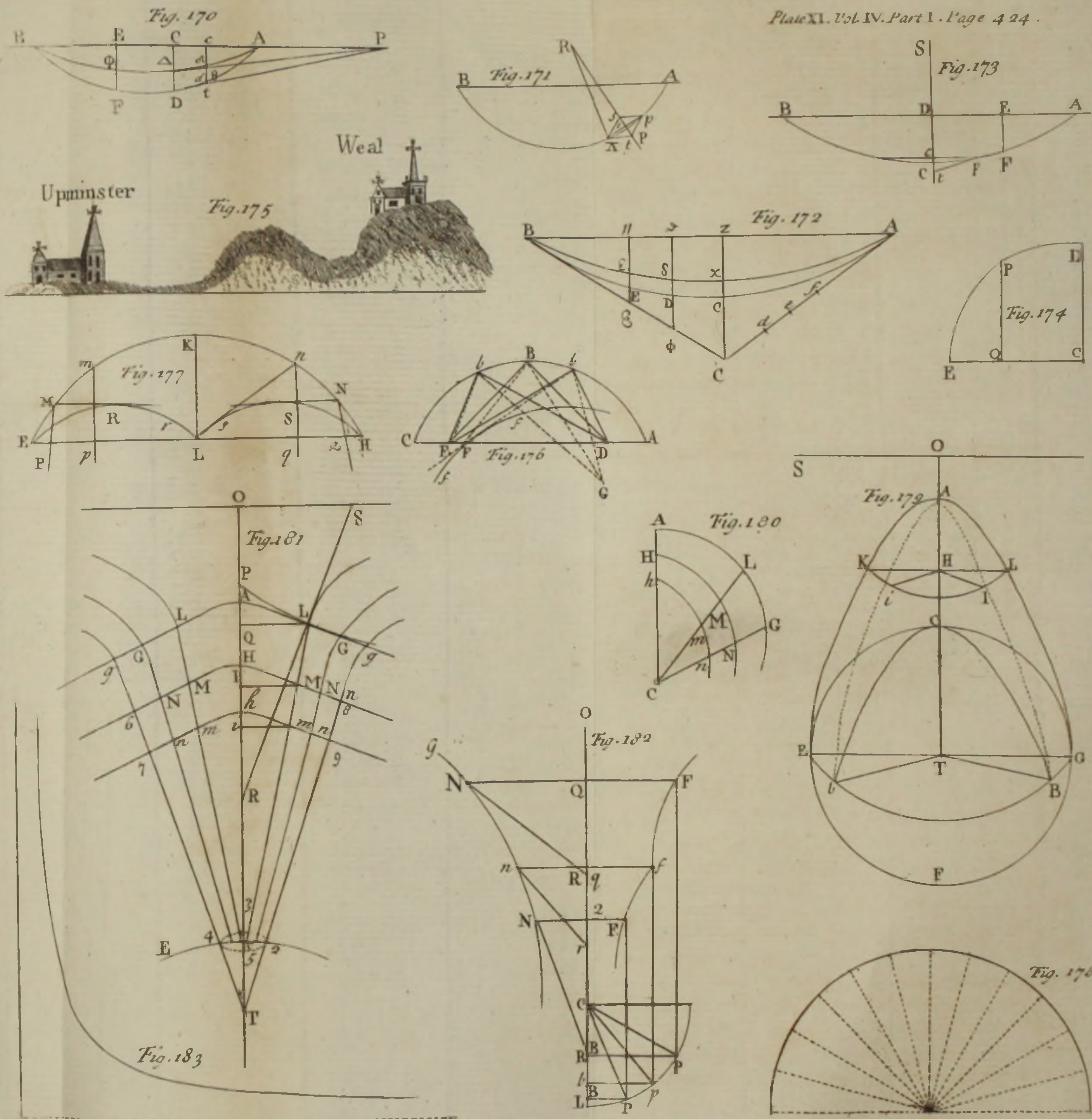


Fig. 170
Fig. 171
Fig. 173
Weal
Upminster
Fig. 175
Fig. 172
Fig. 174
Fig. 177
Fig. 176
Fig. 181
Fig. 180
Fig. 179
Fig. 182
Fig. 178
Fig. 183

Exp. 4. In fact, if the Orifice *C* be lifted up out of the Water *D E*, the Water in the Tube will continue suspended, unless *B A* exceed *F C*.

Exp. 5. But when *C* is never so little immers'd in the Water, immediately the Water in the Tube runs out in Drops at the Orifice *A*, tho' the Length *A B* be considerably less than the Height *C F*.

Mr. *Hawksbee*, in his Book of Experiments, has advanced another Observation, namely, that the shorter Leg of a Capillary Siphon, as *A B F C*, must be immers'd in the Water to the Depth *F C*, which is equal to the Height of the Column, that would be suspended in it, before the Water will run out at the longer Leg.

Exp. 6. From what Mistake this has proceeded, I cannot imagine; for the Water runs out at the longer Leg, as soon as the Orifice of the shorter Leg comes to touch the Surface of the stagnant Water, without being at all immers'd therein.

I proceed now to enquire into the Cause of the Ascent and Suspension of Water in capillary Tubes.

That this Phænomenon is no Way owing to the Pressure of the Atmosphere, has been, I think, sufficiently prov'd by Mr. *Hawksbee*'s Experiments.

And that the Cause assign'd by the same Person, namely, the Attraction of the concave Surface, in which the suspended Liquor is contain'd, is likewise insufficient for producing this Effect, I thus demonstrate.

Since in every capillary Tube the Height, to which the Water will spontaneously ascend, is reciprocally as the Diameter of the Tube, it follows, that the Surface containing the suspended Water in every Tube is always a given Quantity: but the Column of Water suspended is, as the Diameter of the Tube. Therefore, if the Attraction of the containing Surface be the Cause of the Water's Suspension; it will follow, that equal Causes produce unequal Effects, which is absurd.

To this it may perhaps be objected, that, in two Tubes of unequal Diameters, the Circumstances are different, and therefore the two Causes, tho' they be equal in themselves, may produce Effects that are unequal. For the lesser Tube has not only a greater Curvature, but those Parts of the Water, which lie in the middle of the Tube, are nearer to the attracting Surface, than in the wider. But from this, if any thing follows, it must be, that the narrower Tube will suspend the greater Quantity of Water, which is contrary to Experiment. For the Columns suspended are as the Diameters of the Tubes.

But as Experiments are generally more satisfactory in Things of this Nature, than Mathematical Reasonings, it may not be amiss to make use of the following, which appear to me to contain an *Experimentum Crucis*.

The Tube *C D* is composed of two Parts, in the wider of which the Water will rise spontaneously to the Height *B F*, but the narrower Part, if it were of a sufficient Length, would raise the Water to a Height equal to *C D*. Fig. 186.

Exp. 7. This Tube being fill'd with Water, and the wider End C immers'd in the ſtagnant Water $A B$, the whole continues ſuſpended.

Fig. 187. *Exp.* 8. The narrower End being immers'd, the Water immediately ſubſides, and ſtands at laſt at the Height $D G$ equal to $B F$.

From which it is manifeſt, that the Suſpenſion of the Water in the former of theſe Experiments is not owing to the Attraction of the containing Surface: ſince, if that were true, this Surface being the ſame, when the Tube is inverted, would ſuſpend the Water at the ſame Height.

Having ſhewn the Inſufficiency of this Hypotheſis, I come now to the real Cauſe of that Phænomenon, which is the Attraction of the Periphery, or Section of the Surface of the Tube, to which the upper Surface of the Water is contiguous and coheres.

For this is the only Part of the Tube, from which the Water muſt recede upon its ſubſiding, and conſequently the only one, which by the Force of its Coheſion or Attraction, oppoſes the Deſcent of the Water.

This likewiſe is a Cauſe proportionable to the Effect which it produces; ſince that Periphery, and the Column ſuſpended, are both in the ſame Proportion as the Diameter of the Tube.

Tho' from either of theſe Particulars it were eaſy to draw a juſt Demonſtration, yet to put the Matter out of all Doubt, it may be proper to confirm this Aſſertion, as we have done the former, by actual Experiment.

Fig. 188. Let therefore $E D C$ be a Tube, like that made uſe of in the 7th and 8th Experiments, except that the narrower Part is of a greater Length; and let $A F$ and $B G$ be the Heights, to which the Water would ſpontaneouſly riſe in the two Tubes $E D$ and $D C$.

Exp. 9. If this Tube have its wider Orifice C immers'd into the Water $A B$, and be fill'd to any Height leſs than the Length of the wider Part, the Water will immediately ſubſide to a Level with the Point G; but if the Surface of the contain'd Water enter never ſo little within the ſmaller Tube $E D$, the whole Column $D C$ will be ſuſpended, provided the Length of that Column do not exceed the Height $A F$.

In this Experiment it is plain, that there is nothing to ſuſtain the Water at ſo great a Height, except the Contact of the Periphery of the leſſer Tube, to which the upper Surface of the Water is contiguous. For the Tube $D C$, by the Suppoſition, is not able to ſupport the Water at a greater Height than $B G$.

Fig. 189. *Exp.* 10. When the ſame Tube is inverted, and the Water is rais'd into the lower Extremity of the wider Tube $C D$, it immediately ſinks, if the Length of the ſuſpended Column $D H$ be greater than $G B$; whereas in the Tube $D E$ it would be ſuſpended to the Height $A F$. From which it manifeſtly appears, that the Suſpenſion of the Column $D H$ does not depend upon the Attraction of the Tube $D E$, but upon the Periphery of the wider Tube, with which its upper Surface is in contact.

For the ſake of thoſe who are pleas'd with ſeeing the ſame Thing ſucceed in different Manners, we ſubjoin the two following Experiments, which are in Subſtance the ſame with the 9th and 10th.

A B C is a Siphon, in whoſe narrower and ſhorter Leg *A B*, if it were of a ſufficient Length, might be ſuſpended a Column of Water of the Height *E F*; but the longer and wider Leg *B C* will ſuſpend no more than a Column of the Length *G H*. Fig. 190.

Exp. 11. This Siphon being fill'd with Water, and held in the ſame Poſition as in the Figure, the Water will not run out at *C* the Orifice of the longer Leg, unleſs *D C*, the Difference of the Legs *A B* and *B C*, exceed the Length *E F*.

Exp. 12. If the narrower Leg *B C* be longer than *A B*, the Water will run out at *C*, if *D C*, the Difference of the Legs, exceed *E F*; otherwiſe it will remain ſuſpended. Fig. 191.

In theſe two Experiments it is plain, that the Columns *D C* are ſuſpended by the Attraction of the Peripheries at *A*, ſince their Lengths are equal to *E F*, or to the Length of the Column, which by the Suppoſition thoſe Peripheries are able to ſupport; whereas the Tubes *B C* will ſuſtain Columns, whoſe Lengths are equal to *G H*.

Though theſe Experiments ſeem to be concluſive, yet it may not be improper to prevent an Objection, which naturally preſents itſelf, and which at firſt View may be thought ſufficient to overturn our Theory.

For ſince a Periphery of the Tube *E D* is able to ſuſtain no more than a Column of the Length *A F*, contain'd in the ſame Tube, how comes it to ſuſtain a Column of the ſame Length in the wider Tube *D C*, which is as much greater than the former, as the Section of the wider Tube exceeds that of the narrower? Fig. 188.

Again, if a Periphery of the wider Tube *D C* be able to ſuſtain a Column of Water in the ſame Tube, of the Length *B G*; why will it ſupport no more than a Column of the ſame Length in the narrower Tube *E D?* Fig. 189.

Which Queries may likewiſe be made with regard to the 11th and 12th Experiments.

The Anſwer is eaſy; for the Moments of thoſe two Columns of Water are preciſely the ſame, as if the ſuſtaining Tubes *E D* and *C D* were continued down to the Surface of the ſtagnant Water *A B*; ſince the Velocities of the Water, where thoſe Columns grow wider, or narrower, are to the Velocities at the attracting Peripheries, reciprocally as the different Sections of the Columns.

Exp. 13. From which Conſideration ariſes this remarkable Paradox, That a Veſſel being given of whatſoever Form, as *A B C*, and containing any aſſignable Quantity of Water, how great ſoever; that whole Quantity of Water may be ſuſpended above the Level, if the upper Part of the Veſſel *C* be drawn out into a capillary Tube of a ſufficient Fineneſs. Fig. 192.

But whether this Experiment will succeed, when the Height of the Vessel is greater than that, to which Water will be rais'd by the Pressure of the Atmosphere, and how far it will be alter'd by a *Vacuum*, I shall give an Account some other time.

Having discover'd the Cause of the Suspension of Water in capillary Tubes, it will not be difficult to account for the seemingly spontaneous Ascent of it; For since the Water that enters a capillary Tube, as soon as its Orifice is dipt therein, has its Gravity taken off by the Attraction of the Periphery, with which its upper Surface is in contact, it must necessarily rise higher, partly by the Pressure of the stagnant Water, and partly by the Attraction of the Periphery immediately above that, which is already contiguous to it.

The Action *of* Glass Tubes *upon* Water *and* Quick-silver, *by the* same. n. 363. p. 1083.

III. In a former Discourse, I maintain'd, that the Suspension of Water in a capillary Tube was owing to the Attraction of a small annular surface on the inside of the Tube, which touch'd the upper Part of the Water. Among the several Experiments made use of to prove this Assertion, was that of a Glass Funnel of several Inches Diameter, having its small End drawn out into a very fine Tube, which Funnel being inverted and fill'd with Water, the whole Quantity of Water therein contain'd was sustain'd above the Level by the Attraction of that narrow *Annulus* of Glass, with which the upper Surface of the Water was in contact.

Soon after that Discourse was printed, came out a Book publish'd by a Learned and Ingenious Member of this Society, in which that Experiment was accounted for in the following Manner.

Fig. 193.

If there be a Funnel, as A B C, full of Water, and whose wide End stands in a Vessel of Water as B C; and the Top of the Funnel A ends in a Capillary Tube open at A, the whole Water will be sustain'd; the Pillar A a by the Attraction of the Circle of Glass within the Tube immediately above it; and all the rest of the Pillars of Water, as F f, D d, E e, G g, &c. in some measure by the Attraction of the Parts of the Glass above them, as F, D, E, G: And that the small Pillars or Threads of Water, D d, and E e, do not slide down to F f, and G g, and so go quite down, seems to be owing to their Cohesion with the Pillar A a, which is sustain'd by the Capillary Tube A: For if you break off the said Tube at D E, the whole Water will presently sink down.

As this Solution was different from what I had before given, and the Reputation of that Gentleman was sufficient to give Weight to any of his Opinions; I thought myself under an Obligation to examine his Account of the Experiment, in order either to demonstrate its Insufficiency, or to retract my own Solution. Accordingly at the next Meeting of the Society, I produced the following Experiment.

Fig. 194.

The Funnel *A F G B C*, whose lower Part *B C F G*, was cylindrical to a considerable Height, and whose Top was drawn out into a fine Tube at *A*, being fill'd with Water to the Height *B F*, so that the Surface of the Water *F G*, did not reach to the arched Part of the Funnel; I touch'd the End *A* with a wetted Finger, whereby a small Quantity of of Water being inſinuated into the Capillary Tube at *A*, the Water contained in the Funnel was ſuſpended above the Level of the Water in the Ciſtern *D E*, as in the former Experiment.

In this Experiment it is manifeſt, that the little Columns, into which we may ſuppoſe the Cylinder of Water *F G B C*, to be divided, are no way ſuſtain'd by the Attraction of the arched Part of the Glaſs above them, ſince they have no Contact with it. Nor is there any ſuch middle Pillar of Water, which, by its Contact with the Tube at Top, is both ſuſtain'd itſelf, and helps to ſupport the Pillars about it. Upon the Suppoſition of which two Particulars, that Gentleman's Solution was founded.

This Experiment may be thus accounted for: The Cylinder of Water *F G B C*, by its Weight balances a Part of the Preſſure of the Atmoſphere, which is incumbent on the Water in the Ciſtern, and endeavours to force that Cylinder upwards. The reſt of that Preſſure is balanced by the Spring of the Air *A F G*, which is included between the Cylinder of Water *F G B C*, and the little Column of Water in the Capillary *A*. But as this Air by its Spring preſſes equally every way, it muſt balance as much of the Preſſure of the Atmoſphere upon the little Column of Water at *A*, as it does of that upon the Water in the Ciſtern. The Remainder of the Preſſure of the Atmoſphere upon the Column of Water at *A*, is ſuſtain'd by the Force, with which that Column adheres to the Capillary Tube, which therefore does exactly balance the Weight of the Cylinder of Water *F G B C*, and is the real, though not the immediate, Cauſe of its Suſpenſion.

The Experiment ſucceeds in the ſame Manner, when a Column of Quickſilver is raiſed into the Funnel, inſtead of the Column of Water *F G B C*, the Top of the Tube being touch'd with a wet Finger as before. But then the Height of the Quickſilver in the Funnel muſt be as much leſs than that of the Water, as its ſpecifick Gravity is greater.

I proceed now, according to Promiſe, to examine whether the Experiments therein contain'd, would ſucceed *in Vacuo*; and whether Water could be ſuſpended in a wide Tube by means of a Capillary at Top, at a greater Height than what it can be rais'd to by the Preſſure of the Atmoſphere.

In order to this, I boil'd ſome Water, and afterwards purged it of its Air, by means of the Air-pump; which being done, thoſe Experiments all ſucceeded in the exhauſted Receiver, in the ſame Manner as in the open Air.

The 13th Experiment in particular, was made with a Tube of about 35 Inches in Length, and a Quarter of an Inch Diameter, the Top of it being drawn out into a fine Capillary; which being fill'd with Water purged of its Air, as before-mention'd, the whole Quantity continued ſuſpended in the exhauſted Receiver.

This plainly ſhews that the Succeſs of that Experiment does not depend

depend upon the Preſſure of the Air, ſince the ſmall Quantity of Air left in the Receiver, was by no Means capable of ſuſtaining the Water at ſo great a Height, and conſequently that the Height at which Water may be ſuſpended in this Manner, is not limited by that Preſſure.

But here I muſt not omit taking Notice of a conſiderable Difficulty, which preſents itſelf to thoſe who attentively conſider this Experiment In order to make which the better appear, it will be proper to obſerve, what happens, when a ſimple Capillary Tube is fill'd with Water purged of Air, and inclos'd in the exhauſted Receiver.

Fig. 195. In this Caſe, the whole Column of Water contained in the Tube *A C B*, is ſuſpended by the Attraction of the *Annulus* at the Top of the Tube *A*: And though that *Annulus* does not immediately act upon any Part of the Water, except what is either contiguous to it, or ſo near as to be within the Sphere of its Attraction, which extends but to a very ſmall Diſtance; yet it is impoſſible that any other Part of the Water, as for Inſtance, that at *C*, ſhould part from the Water above it, and ſink down; becauſe its Deſcent is oppos'd by the Attraction of the contiguous *Annulus* at *C*. For this being equal to the upper *Annulus* at *A*, is capable of ſuſtaining a Column of Water of the Length *A B*, and conſequently is more than ſufficient for ſupporting the Column of Water below it, *C B*. From which it is plain, that no Part of the Water contain'd in the Tube can poſſibly deſcend, unleſs the upper Part, aſſiſted by the Weight of the Water below it, be ſufficient to overcome the Attraction of the *Annulus* of Glaſs at *A*.

Fig. 196. But in ſuch a compound Tube, as that made uſe of in our Experiment, *A C B*, the Caſe is very different, and it does not eaſily appear, why in a *Vacuum* any Part of the Water in the wider Part of the Tube, as for Example at *C*, ſhould not leave that which is above it, and deſcend, ſince the *Annulus* at *C* is by much too wide to ſuſtain a Column of Water of ſo great a Length as *C B*.

The beſt Anſwer I can give to this Difficulty, is, that the Coheſion between the Water contained in the Capillary and that below it, is ſufficient to balance the Weight of the Column ſuſpended. But how far this Coheſion may depend upon the Preſſure of a Medium ſubtile enough to penetrate the Receiver, is worthy of Conſideration. For though ſuch a Medium will pervade the Pores of the Water, as well as thoſe of the Glaſs, yet it will act with its intire Preſſure upon all the ſolid Particles, if I may ſo call them, of the Surface of the Water in the Ciſtern; whereas ſo many of the ſolid Particles of the Water in the Tube, which happen to lie directly under the ſolid Particles of the Water above them, will thereby be ſecured from this Preſſure; and conſequently there will be a leſs Preſſure of this Medium upon any Surface of the Water in the Tube below the Capillary, than upon an equal Surface of the Water in the Ciſtern. So that the Column of Water ſuſpended in the Tube may be ſuſtain'd by the Difference be-

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tween thoſe two Preſſures. This Explication ſeems to be favoured by the following Experiments, which may all be accounted for in the ſame Manner, though I ſhall anon mention another Cauſe, which contributes to the Succeſs of the firſt and ſecond.

The firſt I ſhall mention is the famous Experiment of the Suſpenſion of Mercury purged of Air, to the Height of 70 or 75 Inches in the *Torricellian-Tube*, in the open Air. To which we may add the ſuſtaining of Mercury, likewiſe purged of Air, within the exhauſted Receiver, as related by the learned Monſ. *Papin* in his *Continuation du Digeſteur.* I forbear to mention the Suſpenſion of Water purged of Air in the *Vacuum*, which he deſcribes in the ſame Book; becauſe there is little Difference between that Experiment and our own above-mentioned; the very Top of the arched Part of his Tube, which Top we may ſuppoſe as ſmall as we pleaſe, ſupplying the Place of the fine Capillary at the Top of our Tube. But we muſt not omit the Experiments made by the famous Monſ. *Huygens* *, of the cohering of poliſhed Plates, with a conſiderable Force in the exhauſted Receiver; as likewiſe of the running of Water and Mercury, when purged of Air, thro' a Siphon of unequal Legs in the *Vacuum:* All which he accounts for from the ſame Principle, and much in the ſame Manner, as we have uſed for explaining the Experiment above.

* Vid. ſupra V. II. p. 24.

As to the Exiſtence of ſuch a Medium, I ſhall content myſelf to refer to what has been ſaid by Sir *Iſaac Newton* in the Queries at the latter End of the laſt Edition of his *Opticks:* And as I have lately produced ſome Experiments upon Quickſilver, which were exactly the Reverſe of thoſe made by Dr. *Taylor*, the late Mr. *Hawkſbee* and my ſelf upon Water; by which I am now enabled to throw this whole Affair into a little Syſtem by itſelf, I ſhall lay it down in the following Propoſitions, the Proof of which is contained in the Experiments annexed.

Prop. 1. *The Particles of Water attract one another.*

This, I think, is now univerſally acknowledged, and therefore needs no Demonſtration; the Sphericity of the Drops of Rain, and the running of two Drops of Water into one another upon their Contact, manifeſtly proving it.

Prop. 2. *The Particles of Quickſilver attract one another.*

This is likewiſe manifeſt from the Spherical Figure, into which a Drop of Mercury forms itſelf upon a Table; and from two of them immediately running together, as ſoon as they come to touch.

Prop. 3. *Water is attracted by Glaſs.*

This plainly appears from all the Experiments that we have ſhewn upon this Subject.

Prop. 4. *Quickſilver is attracted by Glaſs.*

Experiment 1. If a ſmall Globule of Quickſilver be laid upon a clean Paper, and be touched with a Piece of clean Glaſs; upon drawing the Glaſs gently away, the Quickſilver will adhere to it, and be drawn away with it. And if the Glaſs be lifted up from the Paper, the Quickſilver

ſilver will be taken up by it, in the ſame Manner as a Piece of Iron is drawn up by the Loadſtone, and will ſtick to the Glaſs by a plain Surface of a conſiderable Breadth, in Proportion to the Bulk of the Drop, as manifeſtly appears by an ordinary Microſcope. Then if the Glaſs be held a little obliquely, the Drop of Mercury will roll ſlowly upon its Axis along the under ſide of the Glaſs, till it comes to the End, where it will be ſuſpended as before.

Exp. 2. If a pretty large Drop of Mercury be laid upon a Paper, and two Pieces of Glaſs be made to touch it, one on each ſide; upon drawing the Glaſſes gently from each other, the Drop of Mercury will adhere to them both, and will be viſibly drawn out from a globular to an oval Shape; the longer Axis paſſing from the middle of thoſe Surfaces, in which the Drop touches the Glaſſes.

Prop. 5. *The Particles of Water are more ſtrongly attracted by Glaſs, than by one another.*

This manifeſtly appears from the riſing of Water in ſmall Tubes above the Level. For when the Water begins to riſe into a Capillary Tube, all the Particles of Water, which touch the ſmall *Annulus* at the Bottom of the Tube, muſt have quitted the Contact of the other Water, and have riſen contrary to their Gravity, to come into Contact with the Glaſs. After the ſame Manner the other Experiments of Dr. *Taylor*, Mr. *Hawkſbee* and myſelf, upon this Subject, are eaſily explicable. For upon a careful Examination, it will be found in them all, that ſome Parts of the Water quit the Contact of the other Water, and join themſelves to the Glaſs.

Prop. 6. *The Particles of Quickſilver are more ſtrongly attracted by one another, than by Glaſs.*

Fig. 197. *Exp.* 1. If a ſmall Tube as A B, open at both Ends, be dipt into a Glaſs Veſſel fill'd with Mercury, and be held cloſe to the ſide of the Veſſel, that the riſe of the Mercury within it may appear; the Mercury will partly enter into the Tube, but will ſtand within it at ſome Depth, as C E, below the Surface of the Quickſilver in the Veſſel, C D; and this Depth will always be reciprocally as the Diameter of the Tube.

In this Experiment a Column of Quickſilver of the height C E endeavours to force the Mercury higher into the Tube; and as Glaſs has been already prov'd to attract Quickſilver, the Attraction of the annular Surface on the Inſide of the Tube, which is contiguous to the upper Part of the Mercury, will likewiſe conſpire to farther its Aſcent. What oppoſes the Aſcent of the Quickſilver, is the Power by which that Part of it, which endeavours to riſe into the Glaſs, is drawn back by the Attraction of the other Mercury, with which it is in Contact laterally, and this does not only balance the Attraction of the Glaſs, but likewiſe the Weight of the Column of Mercury C E, and conſequently this Attraction is conſiderably ſtronger than the Attraction of the Glaſs.

The

The Cauſe therefore that ſuſpends the Weight of the Column of Mercury CE, being the Difference between the Attraction of the annular Surface of the Tube at E, and that of an equal Surface of the Quickſilver in the Ciſtern, from which the Mercury, that endeavours to riſe into the Tube, muſt recede, in order to unite itſelf to ſuch an Annulus of the Glaſs, will always be proportional to that annular Surface, or to the Diameter of the Tube. And ſince the Column ſuſtained muſt be proportional to the Cauſe that ſuſpends it, that Column muſt likewiſe be as the Diameter of the Tube. But the Column ſuſpended, is as the Square of the Diameter of the Tube, and the height CE conjointly; from which it follows, that the height CE muſt be as the Diameter of the Tube reciprocally, as it is found to be by Experiment.

The Experiment of the Aſcent of Water above the Level in a Capillary Tube, is juſt the Reverſe of this.

Exp. 2. Quickſilver being poured into the inverted Siphon ACB, one of whoſe Legs AC is narrower than the other CB; the height CE, at which the Mercury ſtands in the wider Leg CB, is greater than the height CD, at which it ſtands in the narrower Leg CA. Fig. 198.

On the contrary, Water ſtands higher in the narrower Leg, than in the wider.

Exp. 3. ABCD repreſents a rectangular Plane of Glaſs, which makes one ſide of a wooden Box. On the Inſide of this is another Glaſs Plane of the ſame Size, which at the End AC is preſs'd cloſe to the former, and opens to a ſmall Angle at the oppoſite End BD. When Mercury is poured into this Box to any height as CE, it inſinuates itſelf, between the two Glaſs Planes, and riſing to different heights between the Glaſſes, where the opening is greater or leſs, it forms the common Hyperbola CGF; one of whoſe Aſymptotes EF is the Line on which the Surface of the Mercury in the Box touches the inner Glaſs; the other is the Line AC, in which the Planes are joined. This Hyperbola being carefully examined by Mr. *Hawkſbee* and my ſelf, the Rectangle EHG, whereſoever taken, proved always equal to itſelf, to as great an Accuracy as could be expected, when the Planes were opened to any conſiderable Angle: But when the opening was very ſmall, the Inequalities of the Planes, though the beſt I could procure, bearing a greater Proportion than before to the Diſtance between them, occaſioned a ſenſible Variation. Which, by the Way, I take to be the Reaſon why the Ordinates found by the late Mr. *Hawkſbee*, in examining the Curve produced in a contrary Situation, upon dipping two Glaſs Planes ſo join'd into Spirit of Wine, do not anſwer to thoſe of the Hyperbola. Fig. 199.

Exp. 4. AB is a Perpendicular Section through two Glaſs Planes join'd at A, and open'd to a ſmall Angle at B. C repreſents a pretty large Drop of Mercury, the larger the better, which being made to deſcend as far as C, by holding the Planes in an erect Poſture, with the End A downwards, retires from the Contact of the Planes to D, upon inclining the Planes towards an horizontal Situation; and the Fig. 200.

Diſtance C D becomes greater or leſs, as the Planes are more or leſs inclin'd towards the Horizon.

A Drop of any Oily or Watery Liquor moves the contrary Way, as has been ſhewn by the late Mr. *Hawkſbee.*

Fig. 201. *Exp.* 5. A B is a Tube open at both Ends, and a Foot or two in Length, whoſe lower Part is drawn out into a fine Capillary at B. This Tube being filled with Mercury, the whole Column of Quickſilver will be ſuſtained in it, provided the Capillary Tube at B be ſufficiently ſmall. But if the Mercury in the End B be ſuffer'd to touch any other Mercury, it runs all out of the Tube. If, without letting it touch any other Mercury, a ſmall Part of the End B be broken off, the Mercury will run out, till it comes to ſome leſſer height as B C, at which it will again ſtop, the height B C being nearly in a reciprocal Proportion to the Diameter of the ſmall End of the Tube.

The Seventh Experiment in the former Paper is the Reverſe of this.

Fig. 202. *Exp.* 6. Is the ſame in Subſtance with the former, but made with a large Glaſs Funnel AB, inſtead of a Tube.

The Reverſe of this in Water is the thirteenth Experiment in the former Paper.

In all theſe Experiments it is eaſily ſeen, that the Effect is owing to the Difference between the two Attractions, by which Mercury tends to Glaſs and to its own Body; they being always oppoſed to one another, ſo that a particular Explication is no Way neceſſary. But perhaps it may ſave ſome little Trouble to the Reader, to remove the following Objection, which will readily occur to him.

In the Experiments brought to demonſtrate the fourth Propoſition, the Globule of Mercury adheres to the Glaſs in a Plane Surface, which cannot be done without increaſing the Surface of the Globule, and conſequently removing ſome of its Particles from the Contact of one another. If therefore they tend more ſtrongly to one another than to the Glaſs, why do they not recede from the Glaſs, and aſſume a Figure perfectly Spherical, that they may all have the greateſt poſſible Contact with each other?

To this we may anſwer, That the Power, by which Mercury is attracted either by Glaſs, or by other Mercury, is proportional to the attracting Surface; and therefore, though, *cæteris paribus*, the Tendency of Mercury to Glaſs, is not ſo ſtrong as its Tendency to other Mercury, yet in this Caſe a much greater Number of Mercurial Particles coming into Contact with the Glaſs, than what recede from the Contact of one another, it is no Wonder that the Attraction of the Glaſs prevails, and cauſes the Globule to adhere to it. For the Number of Mercurial Particles, which loſe their Contact with the other Mercury, is no more than what makes up the Difference of Surface, which ariſes from changing the Figure of the Drop: Whereas the Particles, which by this Means come to adhere to the Glaſs, are all thoſe that conſtitute the plane Surface, in which the Globule touches it.

Which

Which Conſideration ought likewiſe to be apply'd to the Suſpenſion of Quickſilver in Glaſs-Tubes, either at extraordinary heights in the open Air, or at leſſer heights in a *Vacuum*, as above-mentioned. For the Top of the Tube being Spherical, or nearly ſo, it will be found, that the Contact of the Mercury with the Extremity of the Tube, is to the Contact with other Mercury, which would be gained by its leaving the Top of the Tube, and deſcending a very ſmall Space, in a *Ratio* infinitely great; and conſequently that the Contact of the Mercury with the Top of the Tube is one Cauſe of its Suſpenſion.

Corol. 1ſt. From this Propoſition it appears, that in a Barometer made with a narrow Tube, the Quickſilver will never ſtand at ſo great a height as in a wider. Which accounts for the *Phænomenon* ſo often mentioned, in the yearly Hiſtory of the Royal Academy of Sciences at *Paris*, by Monſ. *De la Hire*; that in the Barometer, which he conſtantly made uſe of for his annual Obſervations, the Quickſilver did not riſe ſo high, as in another he kept by him, by about three Lines and a half, which is near a third of an Inch our Meaſure: For he tells us, that the Tube of his Barometer is very ſmall. So that there is no need to have recourſe to any Peculiarity, either in the Quickſilver or the Glaſs of which that Tube was made; or to an unperceived Remnant of Air left in the Tube, from ſome of which Cauſes that Effect, and ſome others of the ſame kind, were imagined to proceed.

Corol. 2d. In a Barometer made with a ſmall Tube, the Mercury will riſe and fall irregularly. For, as the height of the Mercury depends partly upon the Diameter of that Part of the Tube that touches the upper Surface of the Mercury; it is plain, that the unavoidable Inequalities in the Diameter of the Tube will be more conſiderable, in reſpect to the whole Diameter; and conſequently will affect the height of the Mercury, more in a ſmall Tube than in a wider. And this I take to be the Reaſon, why it is ſo very difficult, not ſo ſay impoſſible to make two Barometers which ſhall exactly agree in the height of the Quickſilver in all Conſtitutions of the Air, eſpecially if the Tubes be very narrow. This Irregularity is ſtill more conſiderable in the Pendent Barometer, in which the Quickſilver moves through a large Space in order to make a ſmall Alteration in the length of the Column ſuſpended. The ſame Conſideration is eaſily extended to thoſe Levels, that depend upon the riſing of Mercury to the ſame height, in the oppoſite Legs of a bent Tube, an Inſtrument of which kind has been lately offered. And as the Effect is juſt contrary in Levels made with Water or Spirit of Wine, due Regard ought to be had to this Property in the Conſtruction of thoſe Inſtruments, by making the Tubes ſufficiently wide, in order to diminiſh the Error as much as poſſible.

IV. We often ſee the Motion of Water, when it runs out at a Hole in the Bottom of a Veſſel, to be compared with other Powers, not only in Hydraulicks, but in the Application of its Principles to the Animal Oeconomy. *Of the* Motion *of* Running Waters, *by the* ſame, n. 355. p. 748.

Oeconomy. The Quantity of which Motion as no one that I know of has hitherto rightly determined, in its Place the Writers on Hydraulicks are used to have Recourse to the Weight of a Column of Water incumbing on the Hole. They that do this do not consider, that no Motion can be compared with a Weight at rest. Now the Motion of running Water may easily be determined after the following Manner.

Fig. 203. Let *SHAHS* be the indefinite Superficies of Water, *CC* a circular Hole made at the Bottom, *AB* a perpendicular right Line drawn through the Center of the Hole, *SGCCGS* a Column or Cataract of Water running through the Hole *CC*, *SGC* a Curve by the Rotation of which about the Axis *AB* a Solid is generated, or the Cataract *SGCCGS*. For when Water descends freely, and by an accelerated Motion like all heavy Bodies, it will be necessarily contracted into a lesser Space, as it acquires a greater Velocity by falling, and flows out of the Hole *CC* with such a Velocity as is acquired by falling from the Height *AB*.

But the Velocity of a heavy Body acquired by falling, as has been demonstrated by *Galilæus*, is in a subduplicate Ratio of the Altitude from from whence it falls. Wherefore if any Ordinate *DE* be drawn to the Curve *SGC*, and this be call'd y, and *AD* be made x; the Velocity of the Water in the Section *EE* will be expounded by $\sqrt{x}$, and the Product of that Velocity drawn into the Section it self, will be $yy\sqrt{x}$.

This Product is as the Quantity of Water passing through that Section in a given Space of Time; and as the same Quantity of Water in a given Time passes through all the Sections of the Cataract, that Product will always be the same, and will be $yy\sqrt{x}=1$, or $xy^4=1$.

This is the Equation of the Curve *SGC*, Part of which (comprehended within a given Vessel,) the great *Newton* has delineated, and has plainly indicated its Equation, *Prop.* 32. *L.* 2. of his *Principia*; and is the first who has explained to the learned World the true Velocity of running Water, derived from its genuine Principles.

The Curve itself is an Hyperboloid of the fourth Order, one of whose Asymptotes is the right Line *AS* parallel to the Horizon, the other is *AB* perpendicular to the same.

The Power of which is the Quadrato-Cube of the Ordinate *FG*, drawn at the Point *G*; where the right Line *AG*, bisecting the Angle contained by the Asymptotes, meets the Curve.

The Space *SADES*, included between the Curve *SGE*, the Ordinate *DE*, and the Asymptotes *AD*, *AS*, is equal to four Thirds of the Rectangle *HD*, contained by the Abscis *AD* and the Ordinate *DE*. And therefore the Space *SHE* is one third Part of the said Rectangle.

The Solid *SGEEGS*, generated by the Rotation of the Space *SADES* about the Axis *AD*, is double to the Cylinder incumbing on the Section *EE*. Whence the Concave Solid, which the Space *SHEGS* produces by its Conversion about the same Axis, is equal to the incumbent Cylinder. All which Things are easily found by the inverse Method of Fluxions.

Theorem

Theorem 1. If Water runs out of a Veſſel of an infinite Extent, through a circular Hole made at the Bottom, the Motion of the whole Cataract of Water towards the Horizon is equal to the Motion of a Cylinder of Water, under the Hole itſelf and the Altitude of the Water, whoſe Velocity is equal to the Velocity of the Water running through the Hole; or is equal to the Motion of a Quantity of Water which runs out in any given Time, of which the Velocity is the ſame as that, by which a Space equal to the Altitude of the Water may be deſcribed in the ſame given Time.

Demonſtration of the firſt Part. To the Curve *S G C* let another Ordinate *d c* be drawn as near the former *D E* as may be.

The Curve being converted about the Axis *A B*, the Ordinates *D E*, *d e*, will generate two Circles, between which the naſcent Solid *E E e e* will be intercepted. That Solid is equal to the Product of the Altitude *D d* drawn into the Section *E E*; and its Motion is equal to the Product of the Solid itſelf drawn into the Velocity of the ſame, or to the Product of the Altitude *D d*, the Section *E E*, and the Velocity of the Water in that Section. And ſince it is ſhewn above, that the Product of any Section of the Cataract and the Velocity of the Water in that Section, is a conſtant Quantity; the Motion therefore of the whole Cataract will be equal to the Product of that conſtant Quantity drawn into the Sum of all the Altitudes *D d*, or into *A B*, that is, to the Motion of the Cylinder under the Hole itſelf, and the Altitude of the Water, whoſe Velocity is equal to the Velocity of the Water flowing through the Hole. *Q. E. D.*

Corol. 1. The Altitude of the Water being given, the Motion of the Cataract will be in the Ratio of the Aperture.

2. The Aperture being given, the Motion of the Cataract will be in a ſeſcuplicate Ratio of the Altitude, or in a triplicate Ratio of the Velocity, with which the Water runs through the Hole.

3. The Motion of the Cataract being given, the Aperture will be reciprocally in a ſeſcuplicate Ratio of the Altitude, or in a triplicate Ratio of the Velocity reciprocally.

Demonſtration of the ſecond Part. The Quantity of Water running in a given Time is to the Cylinder under the Aperture, and Altitude of the Water, as the Space which Water running out with an equable Velocity will deſcribe in that given Time, is to the Altitude of the Water. And ſince the Velocity which is communicated to the Quantity of flowing Water is to the Velocity of the Cylinder in the ſame Ratio reciprocally, the Quantities of the Motions on each ſide will be equal. *Q. E. D.*

Corol. 1. The Altitude of the Water and the Quantity running out being given, the Motion of the Cataract is in a reciprocal Ratio of the Time, in which that Quantity runs out.

2. The Altitude and Time being given, the Motion of the Cataract will be as the Quantity of Water running out in that Time.

3. The

3. The Time and Quantity of the running Water being given, the Motion of the Cataract will be as the Altitude.

4. The Motion of the Cataract and Altitude being given, the Quantity of the Water is as the Time.

5. The Motion of the Cataract and Quantity of running Water being given, the Altitude is at the Time.

6. The Time and Motion of the Cataract being given, the Quantity of running Water will be reciprocally as the Altitude.

Fig. 204. *Theorem* 2. If BA be taken to BD, as DG^4 to $DG^4—BC^4$; and if the Water runs out of a given Cylindrical Veſſel $GGEE$, which is always full, through a circular Aperture CC made in the Middle of the Bottom; the Motion of the Cataract of Water towards the Horizon will be equal to the Motion of the Cylinder under the Aperture and Altitude AB, whoſe Velocity is equal to the Velocity of the Water going out at the Aperture. Or it will be equal to the Motion of the Quantity of Water which flows out in any given Time, of which the Velocity is ſuch by which a Space may be deſcribed in the ſame given Time equal to the Altitude AB.

Demonſtration of the firſt Part. Let AS be drawn parallel to DG, and with Aſymptotes AS, AB, through the Points G, C, let *Newton*'s Curve SGC be ſuppoſed to be deſcribed.

That the ſame Altitude of the Water may continue, the Place of that which runs out muſt be ſupply'd with the Cylinder of Water $ggGG$, which deſcends with that uniform Velocity which is acquired by falling from A to D; as the aforeſaid excellent Author teaches us in that Propoſition.

The Motion of the Cataract $SSGG$ is equal to the Motion of this Cylinder, by the foregoing Theorem. Therefore the Motion of the deſcending Water, being compounded of the Motion of the aqueous Cylinder $ggGG$, and of the Motion of the Cataract $GGCC$, will be equal to the Motion of the whole Cataract $SGCCGS$, that is, by the firſt Theorem, to the Motion of the aqueous Cylinder under the Aperture and Altitude AB, the Velocity of which is equal to the Velocity of the Water running out at the Aperture. *Q. E. D.*

The ſecond Part follows from the firſt.

Corol. 1. Hence ariſe all the Corollaries of the foregoing Propoſition, by ſubſtituting the Altitude AB, for the Height of the Water.

2. If the Veſſel was of a Figure different from a Cylinder, or the Figure of the Aperture inſtead of Circular was Square, Triangular, or any other, or the Aperture were not in the Middle, or were in the Side of the Veſſel, the Motion of the Cataract will be the ſame, that is, equal to the Motion of an aqueous Priſm under the Aperture and Altitude AB, whoſe Velocity is equal to the Velocity of Water running out. For the ſame Quantity of Water will paſs with the ſame Velocity as in the former Hypotheſis, both through the Aperture itſelf, as alſo through all the Sections of the Cataract.

3. If

3. If the Diameter of the Veſſel ſhould have a very large Ratio to the Diameter of the Aperture, the Altitude *A D* might be neglected, and the Altitude of the Veſſel itſelf might be uſed for the Altitude of the Cylinder, or of the aqueous Priſm.

Hitherto we have conſidered only that particular Caſe, in which the Water runs out of the Veſſel by the Force of its Gravity. This we did the more willingly, as well becauſe Mathematicians are commonly uſed to admit that only, when they treat of the Impetus of Fluids, as alſo becauſe we think that Property of the Hyperbolical Curve above explained, in which it forms a Cataract of deſcending Water, not to be unworthy of the Conſideration of Geometricians. Otherwiſe that Caſe might have been eaſily deduced from the general Theorem, which we ſhall next propoſe.

Theorem 3. If Water flows through any full Canal *A B C D*, according to the Line *E F*, to which both the Orifices of the Canal *A B* and *C D* are perpendicular; the Motion of the Water towards the Orifice *C D*, or the Motion of the Impediment, which being oppoſed in the Orifice itſelf, ſtops the Motion of all the Water, is equal to the Motion of an aqueous Priſm under any Section of the Canal *C H*, and the Line of Direction or the Length of the Canal *E F*, which is moved with the ſame Velocity with which the Water flows through that Section; or is equal to the Motion of a Quantity of Water which in any given Time flows out of the Canal, the Velocity of which is the ſame by which a Space equal to the Length of the Canal may be deſcribed in the ſame Time. Fig. 205.

Caſ. 1. Let the Line of Direction be any right Line *E F*.

The firſt Part is eaſily demonſtrated in the ſame manner as the firſt Theorem. For the Product of any Section of the Canal *C H*, and of the Velocity of the Water in that Section, is a conſtant Quantity.

The ſecond Part follows from the firſt.

Caſ. 2. If the Line of Direction *A B C D E* is compounded of ſeveral right Lines *A B*, *B C*, *C D*, *D E*, inclined to each other, the Motion of the Water will be the ſame. For the Motion of the Water in the whole compounded Canal *A B C D E* is made up of the Motions of the Water in the Parts of the Canal *A B*, *B C*, *C D*, *D E*, added together. Now it is determined, that Water running according to the right Line *A B*, if it changes that Direction into another, by which it proceeds according to the right Line *B C*, loſes none of its Motion. For Fluids do not obſerve the Laws which are obſerved in the Motion of ſolid Bodies, whenever their Direction is changed. Otherwiſe a Fluid would quite ſtop, when it changes its Direction into another Perpendicular to the former, which we do not find by Experiments. Wherefore Water running out of a Hole in a Veſſel, whether downwards, or horizontally, or if it is forced directly upwards, maintains the ſame Velocity. Now if at any time it ſhould be diſcover'd, either by Experiment or by ſome ſtricter way of reaſoning, that any Change of Motion ſhould Fig. 206.

follow

follow from a Change of Direction, then an Account must be taken of it.

Fig. 207. If the Line of Direction *A B* be a Curve, it must be refer'd to this Case, as it is to be conceived as composed of many little right Lines.

Fig. 208. *Cas.* 3. If the Canal *A B* is divided into several Branches *B C*, *B D*, *B E*, equal in Length, the Motion of the Water will be found after the same Manner, taking for the Line of Direction the Length *A B D*, compounded of the Length of the principal Canal *A B*, and the Length of each Branch *B D*. Now it is all one whether the Water flows from the principal Canal towards the Branches, or from the Branches towards the principal Canal. Now if the Branches are unequal, the Motion of the Water must be found in each Branch, taking for the Line of Direction a Length composed of the Length of each Branch, and the Length of the principal Canal.

This is easily deduced from the second Case.

Fig. 209. *Cas.* 4. If the unequal Branches into which the Canal *A B* is distributed, are again united into one *F G*, to find the Motion of the Water for the Line of Direction we must make use of the whole Length *A B D F G*, compos'd of the Length of the principal Canal *A B*, of each Branch *B D F*, and of the recompounded Canal *F G*. If the Branches are unequal, the Motion of the Water must be found in each, and the Sum of their Motions must be added to the Motion of the Water in the recompounded Canal. This follows from Cas. 2 and 3.

Corol. 1. The Length of the Canal being given, and any Section of the same, the Motion of the Water will be in the Ratio of the Velocity with which the Water flows through that Section.

2. Any Section being given, and the Velocity of the Water flowing through that Section, the Motion of the Water will be as the Length of the Canal.

3. The Length of the Canal being given, and the Velocity of the Water in any Section, the Motion of the Water will be in the Ratio of that Section.

4. The Motion of the Water being given, and also any Section, the Length of the Canal will be in the reciprocal Ratio of the Velocity.

5. The Motion of the Water being given, and the Length of the Canal, any Section will be reciprocally as the Velocity.

6. The Velocity being given in any Section, and the Motion of the Water, that Section will be reciprocally as the Length.

7. The Length of the Canal being given, and the Quantity of Water running out in any certain Time, the Motion of the Water will be reciprocally as that Time.

8. The Length of the Canal, and the Time being given, the Motion of the Water will be as the Quantity running out.

9. The Time being given, and the Quantity of Water running out, the Motion of the Water will be as the Length of the Canal.

10. The Motion of the Water, and the Length of the Canal being given, the Quantity flowing out will be as the Time.

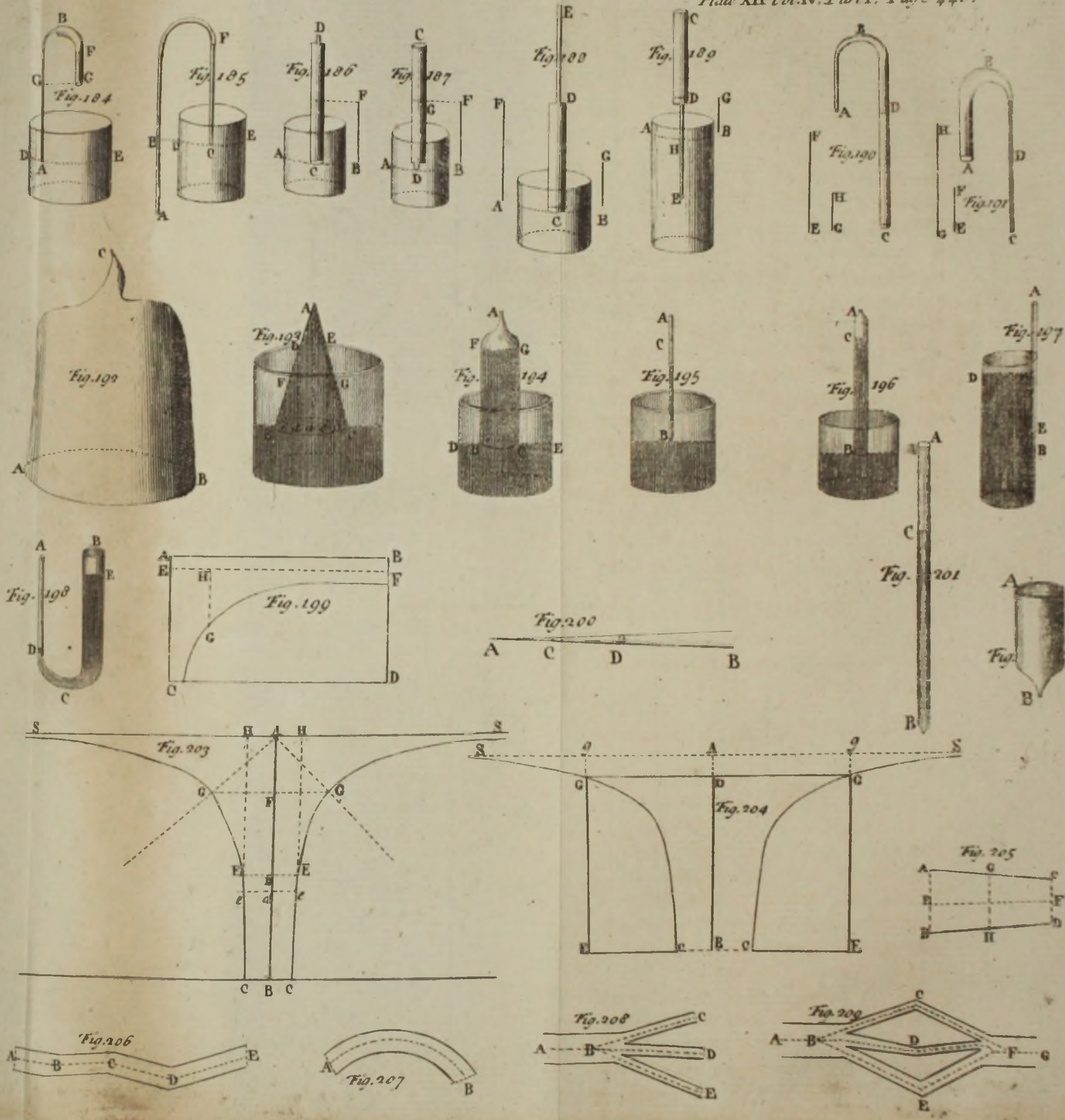
Fig. 184
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Fig. 196
Fig. 197
Fig. 198
Fig. 199
Fig. 200
Fig. 201
Fig. 202
Fig. 203
Fig. 204
Fig. 205
Fig. 206
Fig. 207
Fig. 208
Fig. 209

11. The Motion of the Water, and the Quantity running out being given, the Time will be as the Length of the Canal.

12. The Time being given, and the Motion of the Water, the Quantity running out will be reciprocally as the Length of the Canal.

13. If two Quantities of Water meet directly with a contrary Motion, and the Superficies with which they impinge are alike, as alſo the Velocities with which thoſe Superficies meet; and if one of the Quantities of Water is only equal to one little Drop, and the other Quantity is a whole Ocean, or an infinite Quantity of Water; it may be ſo order'd that the Drop ſhall ſuſtain the whole Ocean, or force it to move the contrary Way with the ſame Velocity as before, and it ſelf ſhall proceed the ſame Way after meeting. Which is a wonderful Paradox in Hydraulicks.

14. If a certain Quantity of Water flows through a Canal which is compoſed of two cylindrical Tubes of unequal Diameters, and runs from the larger Tube towards the narrower; and the Motion of the Water is neither leſſened nor increaſed as it flows; as ſoon as the firſt Part of the Water ſhall enter at the Beginning of the leſſer Tube, it will immediately begin to run ſlower, and by a continual Efflux out of the wider Tube into the narrower, the Water by Degrees will be more retarded in the narrower Tube, till the whole ſhall come into that Tube. The Matter will happen juſt on the contrary, when the Water flows out of the leſſer Tube towards the wider. This is another Paradox in Hydraulicks. But the Water is ſuppoſed every where to cohere with it ſelf.

Theſe two Corollaries ariſe from Caſe 1.

15. From Caſe 2 a Method is ſupply'd for eſtimating the Motion of the Blood in any of the Arteries. —*Of the* Motion *of the* Blood *in the* Arteries.

16. Any two Arteries being given that tranſmit an equal Quantity of Blood, the Impetus of the Blood is greater in that which is more remote from the Heart, than in the nearer. This is a remarkable Paradox in the Animal Oeconomy.

17. From the third Caſe ariſes another Paradox in the Animal Oeconomy, that the Motion or Impetus of the Blood is greater in all the capillary Arteries taken together, than in the Aorta it ſelf. Alſo that it is greater in the capillary Veins than in the Arteries.

18. From the fourth Caſe a Method is derived of determining the Motion of the Blood in any of the Veins.

19. From the ſame is derived a third Paradox in the Animal Oeconomy, that the Impetus of the Blood is greater in any Vein, than in the Artery correſponding to that Vein; and therefore that it is greater in the Vena Cava than in the Aorta.

Problem I. To find the Motion of the Air ruſhing out of the Lungs. —*Of the* Motion *of the* Air *flowing out of the* Lungs *in* Expiration.

Let l be the Length of the whole Aerial Duct, from the Mouth and Noſtrils to the furtheſt Branches of the Trachea.

$q =$ to the Quantity of Air emitted from the Lungs at a moderate Expiration.

$\mathcal{Q} =$ to the Quantity expell'd at a very moderate Expiration.

$t =$ to the Time of a moderate Expiration.

$T =$ to the Time of a very ſtrong Expiration.

Then by *Theor.* 3. *Caſ.* 3. the Motion of the Air ruſhing out of the Lungs at a moderate Expiration will be $= \frac{q\,l}{t}$.

At a very ſtrong one will be $\frac{Q\,l}{T}$.

That is, the Motion of the Air ruſhing out of the Lungs is equal to the Motion of the Quantity of Air which is emitted at one Expiration, of which the Velocity is the ſame, by which the Length of the whole Aerial Canal is deſcribed in the Time of an Expiration. *Q. E. I.*

The famous Philoſopher *Alphonſus Borellus* has determin'd by Experiment, that the Quantity of Air emitted by a moderate Expiration is about 18 or 20 cubical Inches. Now it is different not only in different Men, but in the ſame Man at different Times. I have made an Experiment after this manner.

I hung a Weight to the lower End of a wet Bladder, and fitting a Glaſs Tube to the upper Part of about an Inch Diameter, ſtopping my Noſe I breath'd Air gently into the Bladder, for the Space of three Seconds, the Weight in the mean time being at Reſt upon the Table. Afterwards I dipt the Bladder, with the Air included and the Weight hanging to it, into Water that was contain'd in a cylindrical Veſſel, carefully obſerving to what Height the Water was raiſed, When this was done the Quantity of Water was eaſily found, which being poured into the Veſſel aroſe to the Height before obſerved. This Experiment being repeated ten times, and the Quantities being added together which were found at each Time, their tenth Part, or the mean Quantity of Water contain'd in the Veſſel, was found to be equal to 35 cubical Inches. And this is the Quantity of Air contain'd in the Bladder; then adding about one twelfth Part, or three cubical Inches, becauſe of the Condenſation of the Air made by the Coldneſs of the Water, it being then Winter-Seaſon, it becomes 38 cubical Inches. Beſides a little muſt be added, both becauſe of the Preſſure of the Water in the Bladder, as becauſe of the Vapour which is ſent forth with the Breath into the Moiſture ſqueezed together; which muſt neceſſarily be from the Coldneſs of the Water, and the Contact of the wet Bladder. Therefore I eſtimated the Quantity of Air that was emitted by gentle Expiration in the Time of three Seconds, in a round Number of 40 cubical Inches.

By a very ſtrong Expiration I emitted 125 cubical Inches in the Time of one Second.

And by ſuch a very ſtrong Expiration, with a violent ſtraining of the Lungs, continued almoſt to choaking, I emitted from my Breaſt 220 cubical Inches. Whence it is plain, which I ſhall take notice of by the bye, that much more Air remains in the Lungs than is emitted at one moderate Expiration.

If

If therefore we ſuppoſe $l = 2$ Feet,

$q = 40$ cubical Inches,

$Q = 125$ cubical Inches,

$t = 3$ Seconds,

$T = 1$ Second,

The ſpecific Gravity of Air to the Gravity of Water, as 1 to 1000,

A cubical Foot of Water = 1000 Ounces Avoirdupois,

The moderate Motion of Air going out of the Lungs will be equal to the Motion of the Weight of four Scruples and nine Grains, which moves one Inch in a Second ; or to the Motion of a Weight of $1\frac{1}{3}$ Grain, which in the ſame Time deſcribes the Length of 5 Feet and 7 Inches. This is the Velocity of the Air ruſhing through the Larynx, ſuppoſing the Section of the Larynx equal to one fifth of a ſquare Inch.

The greateſt Motion of the Air expell'd out of the Breaſt is equal to the Motion of a Weight of about $1\frac{1}{4}$ of an Inch, deſcribing one Inch in a Second ; or to the Motion of a Weight of $1\frac{1}{3}$ of a Grain, deſcribing 52 Feet in the ſame Time. This is the Velocity of the Air ruſhing through the Larynx in the ſtrongeſt Expiration.

Corol. 1. The Quantity of Air being given, and the Length of the Aerial Canal, the Motion of the Air is in a reciprocal Ratio of the Time of Expiration.

2. The Quantity of Air and the Time being given, the Motion will be in a direct Ratio of the Length.

3. The Length and Time being given, the Motion is as the Quantity of Air.

4. The Motion and Quantity of Air being given, the Length will be in the direct Ratio of the Time.

5. The Motion and Length being given, the Quantity of Air will be directly as the Time.

6. The Motion and Time being given, the Quantity of Air will be reciprocally as the Length of the Aerial Canal.

7. The Motion of the Air is in a Ratio compounded of the quadruplicate Ratio of any homologous Diameter of the Animal, and the inverſe Ratio of the Time of Expiration. Or in a Ratio compounded of the Ratio of the whole Weight of the Animal, a ſubtriplicate Ratio of its Weight, and the reciprocal Ratio of the Time.

For the Weight of the Animal, the Cube of any homologous Diameter and the Quantity of Air expell'd are in the ſame Ratio. Now it is ſuppos'd, that the Bodies of Animals are Machines made after the ſame manner.

Scholium. You are to underſtand the Length here made uſe of to be either the Length of the Aerial Canal, if all the Branches of the Trachea are ſuppos'd equal in Length, or the mean one between the different Lengths, if the Branches are unequal.

Problem II. To determine the Impetus, or the Impreſſion, which the internal Surface of the Lungs receives by expiring the Air.

Since Action and Reaction are equal and contrary, it muſt neceſſarily follow, that by whatever Motion the Air to be expired is urged by the internal Superficies of the Lungs, by the ſame on the contrary the Superficies of the Lungs is repell'd by the Air.

Whence by the foregoing Problem the ſaid Impetus in a moderate Expiration will be equal to $\frac{q\,l}{t}$,

in a very ſtrong one it will be equal to $\frac{Q\,l}{T}$. *Q. E. I.*

Hence ſuppoſing the ſame Things as are ſuppos'd above, the moderate Impetus of the Air upon the Lungs is equal to the Motion of about $1\frac{1}{2}$ Drachm, which in the Space of a Second deſcribes one Inch. Or to the Motion of the Weight of 19 Pounds, moving $\frac{1}{1642}$ of an Inch in the ſame Time, which is the Velocity of the Air in Contact of the inward Superficies of the Lungs. But we ſuppoſe with the very learned Dr. *James Keil*, that the internal Superficies of the Lungs is equal to about 21900 ſquare Inches.

But the greateſt Impetus of the Air upon the Lungs is equal to the Motion of the Weight of about $1\frac{1}{4}$ Ounce, moving one Ounce in a Second; or to the Motion of the Weight of 19 Pounds, which deſcribes the $\frac{1}{175}$ Part of an Inch in the ſame Time. This is the Velocity of the Air at the Superficies of the Lungs in a violent Expiration.

Corol. 1. The Corollaries ſubjoin'd to the foregoing Propoſition follow from hence.

2. A moderate Impetus incumbing upon a Part of the Surface of the Lungs, which is equal to the Section of the Larynx, is the Motion of the Weight of $\frac{1}{12312}$ of a Grain deſcribing the Space of an Inch in a Second; or the Motion of the Weight of $1\frac{1}{3}$ of a Grain, which deſcribes the $\frac{1}{1642}$ Part of an Inch in the ſame Time. But the greateſt Impetus upon an equal Superficies is the Motion of the Weight of the $\frac{1}{131}$ Part of a Grain, which deſcribes one Ounce, or the Motion of the Weight of $1\frac{1}{3}$ of a Grain, which makes $\frac{1}{175}$ Part of an Inch in every Second of Time.

3. The Impetus of the Air in a moderate Expiration impreſs'd upon the Lungs, is equal to the Motion of a Column of Water that runs one Inch in a Second, the Baſe of which Column is the internal Surface of the Lungs, and its Height is $\frac{1}{85138}$ of an Inch. And in the moſt vehement Expiration of all, the Altitude of the Column is the $\frac{1}{7300}$ Part of an Inch.

4. The Impetus incumbent upon a Superficies equal to a great Circle of a Globule of Blood, in a gentle Expiration, is the $\frac{1}{14}$ Part of the Weight of a Globule of Blood, in a vehement Expiration it is $\frac{2}{3}$ of the ſame Weight moving one Inch in a Second. But by the way I think fit to explain after what manner I meaſur'd the Diameters of the Globules of Blood, ſince it may be of Uſe for determining the Magnitudes of other minute Objects. I took a fine Hair which was pretty long, and wound it ſeveral times about a fine Needle, ſo that all the Convolutions might

might exactly touch one another, as I could plainly perceive by the Help of a Microſcope. Then I took with my Compaſſes the Diſtance between the extream Circumvolutions on each Side, and apply'd it to a Diagonal Scale, and divided the Space found by the Scale by the Number of Circumvolutions. Whence was found the Breadth of one Circumvolution, or the Diameter of the Hair. Then I cut the ſame Hair into a great Number of very ſmall Parts, and ſcatter'd them on the Plain of my Microſcope, on which a little Blood had been ſmear'd ſo as that the Globules might be diſtinctly diſcern'd. When I look'd upon them with my Microſcope, in ſome Places I found the Bits of Hair ſo conveniently diſpoſed, that I could count how many Globules were oppoſed to the Diameter of a Segment or Bit of Hair. But the Segments were unequal in Diameter, becauſe the Hair was ſlenderer towards its Extremity than nearer the Root, ſo that ſometimes 7 or 8, ſometimes 12 or 13 Globules anſwer'd to a tranſverſe Section of the Hair. Now when both Experiments were often repeated, at laſt I eſtimated the mean Diameter of the Hair at the $\frac{1}{324}$ Part of an Inch, and the Diameter of a Globule of Blood at a tenth Part of the Diameter of the Hair, or at the $\frac{1}{3240}$ Part of an Inch.

5. The Impetus which is ſuffer'd by the internal Superficies of the Lungs by expiring the Air, is leſs than the Motion of the mildeſt Particle of Dew falling from the Heaven.

Scholium. In the Solution of the two foregoing Problems the Conſideration is neglected of that Impediment, which the Air ſuffers at its going out of the Lungs, by its Friction againſt the Sides of the Artery Trachea and its Branches; ſince it is but little, nor can it be eaſily eſtimated exactly by any Experiment. Nor have we been very ſolicitous about keeping nicely the Ratios of the Numbers, ſince the only thing we propoſed was, to explain the Method of eſtimating thoſe Forces, ſomething more certainly than has hitherto been done, by which in Expiration the Air acts upon the Blood-veſſels, that involve the internal Superficies of the Lungs. Whence it may be known, whether thoſe Forces are ſufficient to produce thoſe Effects, which are attributed to them by ſome very learned Writers on Medical Subjects.

Problem III. To determine the Impetus of the Blood, in the Vena Cava, near the right Auricle of the Heart; or the Motion of the Blood flowing through all the Arteries and the Veins, except the Veins of the Lungs. —*Of the* Motion *of the* Blood, &c.

Let q denote the Quantity of Blood projected into the Aorta, by one Syſtole of the Heart.

$l =$ to the mean Length of the intire Arterio-venous Duct, taking in both the longer and ſhorter Branches.

$t =$ to the Time between two Pulſes,

Thence by Theor. 3. Caſ. 4. the Impetus required $= \frac{q\,l}{t}$.

That is, the Impetus of the Blood in the Vena Cava is equal to the Motion of the Quantity of Blood which is projected into the Aorta by one

one Systole, of which the Velocity is such, that the whole Length of the Arteries and Veins may be described in the Space of Time intercepted between two Pulses. *Q. E. I.*

If in an human Body are supposed

$q = 2$ Ounces Avoirdupois,

$l = 6$ Feet,

$t = \frac{3}{4}$ Seconds,

The Impetus of the Blood in the Vena Cava will be equal to the Motion of the Weight of 12 Pounds, which describes the Length of one Inch in a Second. Or to the Motion of the Weight of Pounds, which in the same Time describes half a Foot. This is nearly the Velocity of the Blood flowing in the Cava. But we suppose, by the Mensuration of the learned Man above named, that a Section of the Cava is $\frac{3}{4}$ of a square Inch.

Corol. All the Corollaries of the first Problem, changing what is to be changed, result from this Problem.

Problem IV. To determine the absolute Motion of the Blood in the Vena Cava, or the Motion of the Blood flowing through all the Arteries and Veins, except those of the Lungs; abstracting from the Resistance of the Vessels.

Let the natural Velocity of the Blood be to that Velocity with which the Blood would flow, abstracting from all Resistance, as 1 to x. And whereas by the Corollary of the foregoing Problem, and Corol. 1. Prob. 1. the Motion of the Blood is in the Ratio of the Velocity, thence the Motion required is $= \frac{xql}{t}$. *Q. E. I.*

Now if the Proportion found by the Experiment made by the above-mention'd learned Man, be admitted as near the Truth, it will be $x=2, 5$.

Whence the same Things being suppos'd as above, the absolute Motion of the Blood in the Vena Cava is equal to the Motion of the Weight of 30 Pounds, which describes the Length of an Inch in a Second; or to the Motion of a Weight of 2 Pounds, describing $1\frac{1}{4}$ Foot in the same Time. With this Velocity nearly the Blood moves through the Cava, abstracting from all Resistance.

Problem V. To find the Motion of the Blood in the Pulmonic Vein, near the left Auricle of the Heart, or the Motion of the whole Blood flowing through the Lungs.

Besides the Characters used in Prob. 3. let λ be the mean Length of the Pulmonic Arterio-venous Canal.

Whence by Theor. 3. Cas. 4. the Motion required is found $= \frac{q\lambda}{t}$.

That is, the Motion of the Blood flowing through the Lungs is equal to the Motion of the Quantity of Blood, which is projected at one Systole into the Pulmonic Artery, having that Velocity with which the Length of the Pulmonic Arteries and Veins may be described in the Time contain'd between two Pulses. *Q. E. I.*

If

If in the human Body we ſuppoſe $\lambda = 1\frac{1}{2}$ Foot,

The Motion of the Blood in the Lungs will be equal to the Motion of the Weight of 3 Pounds deſcribing the Space of an Inch in a Second.

Problem VI. To determine the abſolute Moment of the Blood in the Pulmonic Vein.

By the ſame Argumentation as is uſed in Prob. 4. the Motion required will be found $= 2, 5 \times \frac{q\lambda}{t}$. *Q. E. I.*

The ſame Things being ſuppos'd as above, the abſolute Motion of the Blood flowing through the Lungs is equal to the Motion of the Weight of $7\frac{1}{2}$ Pounds, which every Second deſcribes one Inch.

Scholium. By the Experiment of Dr. *Keil* the Proportion is determin'd, which the natural Velocity of the Blood flowing through the Aorta and its Branches obtains, to the Velocity with which the Blood would flow through the ſame, abſtracting from the Reſiſtance of the Arteries and the preceding Blood. We have transfer'd the ſame Proportion to the Blood flowing through the Pulmonic Artery. Becauſe if we take away or diminiſh in any Ratio the Reſiſtance which the Blood ſuffers as it flows through each Artery, the Blood of neceſſity will be alike accelerated in each Artery. For unleſs it were ſo, the two Ventricles of the Heart would either not be contracted in the ſame Time, or would not eject the ſame Quantity of Blood. Either of which Things could not be done without the greateſt Perturbation and Danger of the whole Machine.

Corollary to the three foregoing Problems.

Hence follow the Corollaries ſubjoin'd to Prob. 5. *mutatis mutandis.*

Scholium to the four Problems above.

It is to be obſerved, that the Velocity of the Blood flowing as well through the Lungs as through the other Parts of the Body, though in reality it is not equable, yet here it is ſuppos'd to be ſo, that the mean Motion of the Blood may be found.

General Scholium. If any one ſhall think the Numbers not to be ſufficiently accurate, which are here interſperſed in ſpecious Characters, he may eaſily correct them by deriving other Numbers from Experiments that approach nearer to Truth, as alſo the aforeſaid Examples of Motions; or by the Aſſiſtance of the Corollaries of the Propoſitions themſelves.

V. I am buſy at preſent for a Coal-mine, which hath been left off becauſe of the Impurity of the Air; I have therefore improved the *Heſſian* Bellows: An Account of that Contrivance is printed *Lipſiæ* in *Actis Eruditorum anno* 1699. with this Title, *Rotatilis Suctor et Preſſor Heſſiacus:* And it may be applied for Wind as well as for Water. At that Time the Shape of the *Tympanum* was Cylindric, as may be ſeen *Fig.* 210. where *DAFC* is the Circumference: *CP*, *DP*, *AP*, are the *Radii* which bear the Wings *Cm*, *Dn*, *Ao*: *CE* is the Aperture through which the Wind muſt be driven in the Direction of the Tangent *CB*: And it may be obſerved, that when the Engine is working, every Wing from the End of the

The Heſſian *Bellows improv'd, by Mr.* D. Papin. n. 300. p. 1990.

Fig. 210.

the Aperture *E*, till it comes to the Beginning of the ſame Aperture *C*, drives always the ſame Air, with the ſame Swiftneſs, and at the ſame Diſtance from the Center: So that in peruſing all that Circumference, the Air doth find Reſiſtance by Friction, and gets nothing at all. I do therefore now make the Circumference of the *Tympanum* in the Spiral Shape,
Fig. 211. which is to be ſeen *Fig.* 211, where the Spiral Circumference is *AFGB*, the *Radii* are *AP*, *CP*, *DP*, *&c.* The Wings are *AM*, *CN*, *DO*, *&c.* The Aperture is *AB*. And it is to be obſerved, that every Wing in going round drives new Air, becauſe the Air which is firſt in Motion finds Place to recede from the Center towards the Spiral Circumference; and ſo it gives room to new Air to come to the Wing: And when the Wings come near to the Aperture, they drive their new Air into the Aperture without any Friction; and the Air which hath been firſt driven and removed from the Wing, cannot loſe its Swiftneſs, becauſe the Wings which continually follow do continually drive new Air, which keeps that which is before always in the ſame Swiftneſs. This new Shape of the *Heſſian* Bellows affords alſo another Advantage; becauſe the Air in going round follows the Spiral-line, which is nearer to the ſtrait Line than a circular Circumference; and when the Air comes to the Aperture, it gets into it without any Loſs of Subſtance; but in the Cylindrical Machine, the Air doth always go round in a circular Circumference, and when it comes to the Aperture, the Wind is driven directy in the Direction of the Tangent, but juſt in the Beginning at *C*; and afterwards the Impulſion is oblique: And this Obliquity is always increaſing until the Wing comes to the *punctum A*: Now it is known how much Diminution ſuch an Obliquity can make to the Strength. I believe therefore that this Spiral Figure is a good Improvement to this Engine. And indeed I have made ſuch Bellows, where the *Radius AP* is but $10\frac{1}{2}$ Inches, the Wing *Am* 2 Inches broad and 9 Inches high; becauſe the *Tympanum* is alſo ſo high, or little more; the Aperture *AB* is alſo 9 Inches, or a little more, ſo that it makes a ſquare Hole. When I work this Engine with my Foot, it makes ſuch a Wind, that it may raiſe up two Pounds Weight; and without doubt, a ſtronger Man could do much more: But this is more than ſufficient for our Purpoſe, ſince we muſt but drive Air enough for the Reſpiration of ſuch Men that can work in the Mine; and we may eaſily with Boards make wooden Pipes, to carry the Wind to the very Bottom: So that the Air within will be continually renewed as well as without.

As to the Engine to demonſtrate the Power of Water expanded by Fire, we have here made very good Experiments of that Matter before Winter. We have raiſed Water to the height of 70 Foot, by a very commodious Way, which may be yet very much improved. The *Heſſian* Bellows may be very uſeful to a Furnace, I have already made a little Trial of it, and I had a very ſtrong Fire in a Furnace, to melt Glaſs, Iron, or any other hard Metal; and yet I could open the Furnace above the Matter to be wrought upon, and yet no Flame would get out through the Aperture; nor cold Air from without get into the Furnace: So that it it is very like this will be a great Conveniency for ſeveral ſorts of Work, ſince Men may work the Matters when they are moſt ſoftened in the Fire; and they may be drawn up perpendicularly, that they may not be bent, as they are when we draw them horizontally. I believe that would be good, eſpecially to make eaſily Glaſs Pipes and Looking-Glaſſes of an extraordinary Bigneſs.

VI. *Account of a Book omitted.*

Joannis Poleni in Gymnaſio *Patavino* Phil. Ord. Prof. & Scient. Societatum Regalium, quæ *Londini* & *Berolini* ſunt, Sodalis, *De Motu Aquæ Mixto*, Libri Duo, *&c.* *4to.* *Patavii* 1717.

CHAP. VI.

Geography. Navigation.

I. SEveral Perſons have given us, as they have ſuppoſed, the juſt Number of Acres contained in *England*, or *South Britain*, or very near it. Sir *William Petty* reckons about 28 Millions; others, 29 Millions; others, a few more. But they have all been miſtaken in under-reckoning.

A Demonſtration of the Number *of* Acres *in* England, *by Dr.* N. Grew. n. 330. p. 266.

And the Reaſon of their Miſtakes ſeems to have been, their reckoning only by the Maps; that is, by computed, and not by meaſured Miles; by which only the Number of Acres can be known.

I have ſeen an Account of the Number of Acres in each County: Which Account, whether taken from Doomſday-Book, or from any other Regiſter, cannot be true. For tho' we have loſt ſome Land, yet there is a great deal more now gained, which in the Conqueror's Time lay under Sea. Within 120 Years, very much has been recovered out of the Seas, and maintained by Banks, in the Marſhes and Fenns of *Eſſex*, *Kent*, and the Iſle of *Ely*. And in ſome Parts of *Lincolnſhire*, the Land has gained of the Sea four Miles in a direct Line from Land to Sea, in the Memory of Men now living.

Nor is it the truer, for having been taken from any other Record: For if the Numbers of Acres, according to the ſaid Account, in each Shire, be put together, they exceed not 39 Millions and a Quarter: Which Number, though it comes much nearer to the Truth than any of the former, yet is a great deal ſhort of it.

For however, according to vulgar Computation, *England*, or *South Britain*, is reckoned in Length but 305 Miles; and in Breadth, about 290 Miles: Nevertheleſs, it appears by an exact Wheel-meaſure, That from *New-Haven* in the *South* of *England* to *London*, are 56 meaſured Miles; and that from thence by a ſtrait Line continued to *Berwick* in the *North*, are 339 of the ſame meaſured Miles; in all 395 meaſured Miles, the true Length of *England*. And again, that from the *South*

Foreland in the *Eaſt*, to the Land's End in *Cornwall*, are about 367 Miles of the ſame Wheel-meaſure, the true Breadth of *England*.

This being known, it is eaſy to know alſo, how many ſquare Miles and conſequently how many Acres are contained in *England*, or *South Britain*.

If a Line be drawn on a Chart of *England*, from the *South* Foreland in *Kent* to *Berwick*; and from the two Ends of this Line, two more Lines meeting at the Land's End in *Cornwall*, they will make the Tri-
Fig.212. angle ABC: Which Triangle, as it excludes as much more of the Land, as it includes of the Sea, as may anſwer the ſmall Number of Miles obtained by the Curvity of the Roads; it may therefore be allowed to be equal to the Area of *England*, or *South Britain*.

Next, if to the Triangle ABC, another ſimilar and equal Triangle BCD, be added; both together make the Rhomboid ABDC. Which being divided at EF, maketh the Rhomboids ACEF, and BDEF, equal to one another. One of which is therefore equal to the Triangle ABC. And the Rectangle AGHF, ſtanding upon the ſame Baſe, and between the ſame Parallel Lines with the Rhomboid ACEF, by the 35th of the 1ſt of *Euclid*, is equal to the ſaid ACEF; equal to the Triangle ABC; equal to the Area of *England*, or *South Britain*.

Now the Length between *Berwick* and the South Foreland in *Kent*, being about 5 Miles more than between *Berwick* and *New-Haven*, which is 395 Miles: Therefore the Line AB, may be taken for 400 Miles; and ſo the Line AF, for 200. And the Line AG being leſs by about 7 Miles, than between the South Foreland in *Kent*, and the Land's End in *Cornwall*, which is 367 Miles, the ſaid AG may be taken for 360 Miles. Therefore AG, 360, being multiply'd by AF, 200, produceth 72000 ſquare Miles: And 72000 being multiplied by 640, the Number of Acres contained in one ſquare Mile, produceth 46 Millions and 80000, the Number of Acres contained in *England*, or *South Britain*.

Whence it appears, Firſt, that if the Province of *Holland* contains, as is computed, but one Million of Acres, then *England* is more, by a Fraction of 80000 Acres, than 46 times as big as *Holland*.

Next, if in the Province of *Holland*, containing but one Million of Acres, are two Millions and 400 Thouſand Souls, or two Millions and four 10ths, as they are ſaid to be; then *England*, which contains 46 Millions of Acres, to be proportionably populous, ſhould have twice 46 Millions of People, and four 10ths of 46; that is, about 110 Millions.

But to allow room enough for Perſons of all Degrees, if *England* were half as populous as *Holland*, with only 55 Millions, it were a good Proportion, and would be near five times our preſent Number: And about 22 times as many as in the Province of *Holland*.

To people *England* in a competent Time with this Number, there are many Ways practicable: By which, I have computed, the preſent Number may be doubled in 24 or 25 Years. And probably quadrupled in about 36 Years.

One

One of these Ways, though not the speediest, would be the introducing of Strangers: Yet to make use of this, or of any other Way, to multiply the People, before we have provided the Means of employing them, would be preposterous.

But when we shall mind our true Interest, in employing and encouraging every where our own Hands, and the Hands of other Nations, as the *French* and *Dutch* do, in all the sorts of Husbandry, Manufactury, and Merchantry: When our Nobility and Gentry themselves, shall be Examples in some or other of these Particulars: When we shall hereby be universally engag'd to inclose, and to improve every Foot of our Land; to make the utmost Use of all our Home Growths, above and under Ground; and of all our Ports, (about 200 great and small,) more than in all the Kingdoms and States of *Europe* put together: And when *Scotland* and *Ireland* shall both of them afterwards be improved in like Manner; when all Mens Heads and Hands shall be thus employ'd about some one honest and profitable Business, it is easy to foresee how highly it will advance the *British* Monarchy and People, at Home, and all over the World, in Beauty, Strength and Glory.

The Difference of Longitude *between* London *and* Cambridge *in* NewEngland, *by Mr.* J. Hodgson. n. 292. p. 1637.

II. I conclude that the Eclipse of the Moon of *Dec.* 12*th* 1703. began at *London* at about 31 or 32 Minutes after 4 in the Morning.

At *Cambridge*, about 4 Miles from *Boston* in *New England*, Mr. *Brattle* found, † that at 44 Minutes after 11 at Night, part of the Moon's Disk looked somewhat duskish, and that at 52 Minutes, the Shadow was well enter'd: So that from hence, as well as from a Comparison of the Ingress and Egress of the principal Spots, it probably began there about 49 Minutes after 11; whence it follows, that *Cambridge* in *New England* lies $4^h\ 4'\ 2\frac{1}{2}$, or $70\ 37'$, to the Westward of the Meridian of *London*.

† *Vid.* supra p. 271.

The Longitude *of the* Cape Good Hope, *&c. by Dr.* E. Halley. n. 361. p. 992.

III. 'Tis now above thirty Years, since I had a Dispute with some of the *French Geographers* about the Longitude of the Cape of *Good Hope*, said to have been observ'd by the Religious Missionaries sent to *China* in the Year 1685. By an Emersion of the first Satellite of *Jupiter*, they determined that Cape to be $1^h\ 11'$, or $17\frac{3}{4}$ *gr.* more Easterly than *Paris*; that is, 20 *gr.* from *London*: Which, for the Reasons I then gave, ‖ I concluded could not be more than 17 *gr.* Very lately I have fallen upon an Observation which I believe will determine the Controversy in my Favour; for I had accidentally a Journal of an Officer of the Ship *Emperor*, put into my Hands, who in his Return from *India*, on the fifth of *March* 1718. observ'd the End of a Lunar Eclipse, when the visible Altitude of the Moon's Center was $13^\circ\ 25$, he being then in the Latitude of $34^\circ\ 23$ *South*, and as they found afterwards, just 180 Leagues to the Eastwards of Cape *Bonne Esperance*. By Calculation I find, that in that Latitude, the Moon had that height at $7^h\ 17'\frac{1}{2}$ *P. M.* and by comparing this Eclipse with that we observ'd with great Exactness on *Feb.* 11°, 1682. (which agrees perfectly well with our Numbers) I conclude the middle of this to have happen'd at *London* at 3^h

‖ *Vid.* Supra V. I. C. VII. S. XXVI.

48′ *P. M.* to which adding $1^h\ 46'$ for the Semiduration (this being very certain from the obferved Continuance of the Eclipfe of 1682) the End will be found to have been at *London* at $5^h\ 34'$. The Ship was therefore in a Meridian 26° to the Eaftwards of *London:* But fhe was at that Time 180 Leagues to the Eaftwards of the *Cape*, which Diftance in that Latitude, gives eleven Degrees of Longitude; this therefore being deducted from the Longitude of the Ship, leaves juft 15 *gr.* or one Hour, for the Difference of Meridians between *London* and the *Cape.* So that by this Account, the *Cape* is yet nearer our Meridian than I had formerly plac'd it, and near fix Degrees nearer than M. *De la Hire* places it in his Tables.

This Eclipfe was attended with all the Circumftances requifite to make the Conclufion as certain as the Nature of the Thing will admit of: For the Moon was nearly *in Perigæo*, and the Eclipfe almoft central; fo that fhe emerged out of the Shadow as fwiftly as poffible. The Sea was very fmooth, there having been little Wind for above 30 Hours before; and the Moon was not too high to be well obferved with a Foreftaff: Nor were they long at Sea, before they made the Land; for in lefs than five Days, on the tenth of *March*, at Noon, they had paft Cape *d'Agulhas*, the moft Southerly Promontory of *Africa*, which then bore from them *North Eaft*, about feven Leagues diftant. The End of this Eclipfe, though not vifible here, might have been feen in *Germany*, both at *Nurenburg*, *Leipfick* and *Berlin*; but we do not hear that it was any where obferved there: However, our Numbers in this Cafe may be fecurely relied on.

On this Occafion, I fhall infert an Obfervation or two I procured to be made at the *Cape*, by Mr. *Alexander Brown*, a *Scotch* Gentleman. He carried with him a very good *Brafs Quadrant* of above two Foot Radius, and at the *Dutch* Settlement at *Table Bay*, having rectify'd his *Pendulum Clock* by correfpondent Altitudes, on the *4th* of *Auguft* 1694. at $5^h\ 59'$ *Manè*, the Diftance of the bright Limb of the Moon from the Right Shoulder of *Orion*, was obferv'd to be 25° 3′. And the next Morning *Aug.* 5. at $5^h\ 21'\ 12''$, the fame Limb was diftant from *Procyon* 25° 57′, and at $5^h\ 36'\ 48''$ from the *Lucida Arietis* 58° 29′.

It were much to be wifh'd, that the Moon had, either of thefe Mornings, been obferv'd at *Greenwich* or *Paris*, or at fome Place in *Europe* whofe Longitude from them is well known: But that failing us, I had recourfe to the Period of the Lunar Motions, which is perform'd in 18 Years and ten or eleven Days; after which, the Errors of our Lunar Computations return very nearly the fame; and I found among my own old Obfervations, one that agreed well with that of the *4th* of *Auguft*, *viz.* Anno 1676. *July* 23° $13^h\ 11'\ 35''$ at *Oxford*, I obferv'd the Moon to apply to the Star *in medio Collo Tauri*, by *Bayer* mark'd *A.* The Star at that Time was diftant from the Southern and neareft Cufp of the Moon, by the Micrometer 20′ 32″, and at $13^h\ 17'\ 15''$, when it feem'd to immerge upon the bright Limb

Limb of the Moon, it was diſtant from the Northern Cuſp 23′ 20″; but this is leſs certain, by reaſon of the hazy Air. The Star at that time was in ♉ 28° 56′ with 1° 13′ 20″ *North* Lat. whereby I found, that our Lunar Tables, founded on Sir *Iſaac Newton*'s Theory of her Motion, gave her Place at that Time only two Minutes too ſlow; which Error being allowed on the *4th* of *Auguſt* 1694. the Reſult was, that 5^h 59′ at Cape *Bonne Eſperance*, was at *London* 4^h 53′; whence the Difference of Longitude 16 ½ Degrees, ſufficiently near what we had before determin'd.

Of the Variation *at* Paraiba, *&c. And the* Longitude *of the* Magellan Straights, *&c. by Dr.* E. Halley. n. 341. p. 165.

IV. The Gentlemen of the *Royal Academy of Sciences* in *France*, have, for ſome Years paſt, apply'd themſelves with much Candour and Diligence, to examine the Chart I publiſh'd in the Year 1701. for ſhewing at one View the Variations of the Magnetical Compaſs, in all thoſe Seas with which the *Engliſh* Navigators are acquainted; and I find, that what I did ſo long ago, has been ſince abundantly verified by the concurrent Reports of the *French* Pilots, who of late have had frequent Opportunities of inquiring into the Truth thereof. So that I am in Hopes, I have laid a ſure Foundation for the future Diſcovery of the Law or Rule by which the ſaid Variations change, in Appearance regularly, all the World over. Of this I have long ſince given my Thoughts, † and as yet I ſee no Cauſe to retract what I there offer for a Reaſon of this Change; but of this we might be more certain, had we a good Collection of Obſervations made in that Ocean, which divides *Aſia* and *America*, and occupies about two Fifths of the whole Circumference of the Globe. This, we hope, may be effectually ſupply'd by the *French*, who may return from *Peru* by the *Eaſt Indies*.

† *Vid.* Supra V. II. C. IV. S. VIII.

In the mean time I cannot omit to take Notice of two Particulars, ſeeming to call in Queſtion the Truth of my aforeſaid Map, in the *Memoirs* of the *Royal Academy of Sciences*.

Of the Variation *at* Paraiba *in* Brazile.

The one is in the *Memoirs* of the Year 1700. concerning the Variation obſerved at *Paraiba* in *Brazile*, about 25 Leagues to the Northwards of *Pernambouc*, by M. *Couplet le fils*, whoſe Words are theſe:

'*May* 20. 1698. Having before carefully drawn a Meridian Line, which I made uſe of for Aſtronomical Obſervations, I obſerved the Declination of the Needle touch'd by the Loadſtone to be 5° 35′. N. W.' And the ſame Obſerver tells us, that he found the Latitude of the Town of *Paraiba* 6° 38′ 18″. Now it happen'd, that I was in the River of *Paraiba*, in *March* 1699. and there fitted and clean'd my Ship; ſo that I had full Opportunity to obſerve the Variation both on Board and on Shore, and found it conſtantly to be above 4 *gr. North Eaſt*; ſo that I am willing to believe this to be an Error of the Preſs, putting N. W. for N. E. or rather of the Memory of M. *Couplet*, who, it ſeems, loſt all his Papers by Shipwreck in his Return. The ſame may be ſaid of the Latitude of *Paraiba*, which, though I did not obſerve myſelf, yet at the Fort of *Cabo Dello*, at the Mouth of the River, and which is about 3 Leagues more Northerly than the Town, I found the Latitude not leſs than 6° 55′ South, and by Conſequence that of the Town more than 7 Degrees.

The other is in a Diſcourſe of M. *de Liſle*, in the *Memoirs* of 1710; where he compares the Variations obſerved in ſome late Voyages, with my Map of the Variations. Among other Things, 'tis there ſaid, that on the Eaſt-ſide of the Iſland St. *Thomas*, under the Equinoctial Line, M. *Bigot de la Canté*, had, in the Beginning of the Year 1708, found the Variation 11½ *gr.* whereas my Chart makes it but 5½ *gr.* I never indeed obſerved myſelf in thoſe Parts; and 'tis from the Accounts of others, and the Analogy of the whole, that in ſuch Caſes I was forc'd to ſupply what was wanting; and 'tis poſſible, that there may be more Variation on that Coaſt than I have allowed. But conſulting my Chart, (which was fitted to the Year 1700,) I find I then make the Variation at the Iſle of St. *Thomas*, full 7½ *gr.* and not 5½ *gr.* the which, by the Year 1708, might well ariſe to near 9 *gr.* So that the Difference will become very tolerable; whereas an Error of 6 *gr.* ſuch as is here repreſented, would render the Credit of my Chart juſtly ſuſpected.

Of the Longitude *of the* Magellan *Straights.*

But a further Thing I might complain of, is, that in the ſame *Memoir* of M. *de Liſle*, the *Geography* of my Chart is called in Queſtion; and we are told, that I have placed the Entrance of the *Magellan* Straights at leaſt 10 Degrees more Weſterly than I ought to have done: For that the Ship St. *Louis*, in the Year 1708, ſailing from the Mouth of *Rio Gallega*, in about the Latitude of 52 *gr.* South, and not far from Cape *Virgin*, directly for Cape *Bonne Eſperance* (which Courſe perhaps was never run before) had found the Diſtance between the two Lands not more than 1350 Leagues, which, he concludes, is much leſs than my Chart of the Variation makes it. I know not from what Computation M. *de Liſle* has drawn this Conſequence; but I find by my Chart that I have made the Longitude of *Rio Gallega* 75 *gr.* Weſt from *London*, and that of Cape *Bonne Eſperance* 16½ Eaſt from it; that is in all 91½ *gr.* Difference of Longitude. This with the two Latitudes, gives the Diſtance, according to the Rhumb-line, 1364 Leagues; but according to the Arch of a great Circle, no more than 1287 Leagues. So that inſtead of invalidating what I have there laid down, it does abſolutely confirm it, as far as the Authority of one ſingle Ship's Journals can do it.

I do not pretend, that I have had Obſervations made with all the Preciſion requiſite, to lay down inconteſtably the *Magellan* Straights in their true Geographical Site; but it has not been without good Grounds, that I have placed them as I have done. For when Sir *John Narborough*, in the Year 1670, wintered in Port St. *Julian*, on the Coaſt of *Patagonia*, Capt. *John Wood*, then his Lieutenant, and an approved Artiſt in Sea-Affairs, did obſerve the Beginning of an Eclipſe of the Moon, *Sept.* 18. *Stil. vet.* at juſt 8 at Night: And the ſame Beginning was obſerv'd by M. *Hevelius* at *Dantzick*, at $14^h\ 22'$; whence

Port

Port St. *Julian* is more Westerly than *Dantzick* 6h 22′, or than *London* 5h 6, that is 76½ *gr.* Besides, I have had in my Custody a very curious Journal of Capt. *Strong*, who went into the *South Seas* in quest of a Plate-wreck, and who discover'd the two Islands he called *Falkland*'s *Isles*, lying about 120 Leagues to the Eastwards of the *Patagon* Coast, about the Lat. of 51½. This Capt. *Strong* had a quick Passage from the Island of *Trinidada* (in 20½ South) to the *Magellan* Straights; and in this Journal, which was very well kept, I found, that Cape *Virgin* was, by his Account, 45 Degrees of Longitude more Westerly than that Island, whose Longitude I know to be just 30 Degrees from *London*; that is in all, 75 *gr.*

From these concurrent Testimonies, I adventured to fix the Longitude of this Coast as I have done; and I can by no Means grant an Error of 10 Degrees to be possible in it, though perhaps it may need some smaller Correction. I will however readily grant, that those that go thither from *Europe*, shall find the Land more Easterly than is here express'd, by reason of a constant Current setting to the Westward near the Equator, where Ships are many times long detained by Calms, whilst the Stream carries them along with it; which Thing befals all Ships bound to any Part of the East Coast of the *South America*.

V. *The* Variation *of the* Compass *in the* Atlantic *and* Æthiopic *Oceans.* A. D. 1706. *by Mr.* J. Maxwell. n. 310. p. 2433.

Variation.			Latitude.			Longit. from *London.*		
8°	32′	West.	49°	18′	North.	07°	29′	West.
6	42		44	31		13	45	
5	30		41	06		15	08	
5	04		40	22		14	54	
4	22		39	11		15	35	
3	30		32	21		15	39	
3	35		32	42		15	38	
1	20		18	50		20	52	
1	14		09	26		17	59	
1	10		00	49		18	42	
1	00		01	09	South.	18	58	
0	16		02	32		19	48	
0	00		03	17		20	05	
0	40	East.	03	58		20	27	
1	02		05	09		21	39	
1	30		06	21		22	08	
1	50		08	03		23	15	
2	10		09	07		23	35	
3	32		12	03		25	03	
6	04		18	53		26	30	
6	19		19	51		27	02	

Variation

Variation.	Latitude.	Longit. from *London*.
6° 20′ Eaſt.	21° 26′ South.	28° 14′ Weſt.
6 30	21 48	28 10
7 00	21 58	28 23
6 45	24 45	27 56
6 36	27 11	27 17
5 04	33 53	16 58
0 00	34 21	01 29 30″
1 00 Weſt.	34 15	01 33 Eaſt.
4 16	33 41	06 23
8 46	34 39	13 02
11 56	14 30	16 15 *at the Cape of*
11 30	32 51	13 41 Good Hope.
10 00	30 21	11 46
09 44	29 51	11 44
09 34	29 28	11 31
09 22	28 56	11 05
09 04	27 38	10 01
08 30	26 55	08 45
08 02	25 41	07 22
07 32	24 32	05 43
01 52	16 00	06 30 Weſt *at the Iſle of St.* Helena.

A Mechanical Way to divide *the* Nautical Meridian Line *in* Mercator's *Projection. And the* Relation *of that* Line *to the* Curva Catenaria, *by Mr.* J. Perks. n. 345. p. 331.

VI. The moſt uſeful Projection of the Spheric Surface of Earth, and Sea for Navigation, is that commonly call'd *Mercator's*; tho' it's true Nature and Conſtruction is ſaid to be firſt demonſtrated by Mr. *Wright*, in his *Correction of the Errors in Navigation*. In this Projection the Meridians are all parallel Lines, not divided *equally*, as in the common plain Chart (which is therefore erroneous) but the Minutes and Degrees (or ſtrictly, the *Fluxions of the Meridian)* at every ſeveral Latitude are proportional to their reſpective *Secants*. Or a Degree in the projected Meridian at any Latitude, is to a Degree of Longitude in the Equator, as the *Secant* of the ſame Latitude is to *Radius*.

The Reaſon of which Enlargement of the Elements of Latitude is, to counterbalance the Enlargement of the Degrees of Longitude. For in this Projection, the Meridians being all parallel, a Degree of Longitude at (ſuppoſe) 60 Deg. Lat. is become equal to a Degree in the Equator, whereas it really is (on the Globe's Surface) but *half* as much, the Radius of the Parallel of 60 Deg. (that is its *Coſine)* being but *half* the Radius of the Equator. Therefore to proportion the Degrees of Latitude to thoſe of Longitude, a Degree (or Elemental Particle)

ticle) in the Meridian, is to be as much greater than a Degree (or like Particle) in the Equator, as the Radius of the Equator is greater than the Radius of the Parallel of Latitude, *viz.* its *Cosine.*

Let the Radius CD represent half of the Equator, DM an Arc of the Meridian; MS its Sine, CE its Secant; then is CS equal to its *Cosine:* and $CS : CM :: CD\ (=CM) : CE$, that is, as Cosine : to Radius : : so is Radius : to Secant. The Cosines being then, in this Projection, suppos'd all equal to Radius, or (which comes to the same) the Parallels of Latitude being all made equal to the Equator, the Radius of the Globe, at every Point of Latitude, (by the precedent Analogy) is supposed equal to the Secant of Latitude; and consequently the Elements (Minutes, *&c.*) of the Meridian must be proportional to their respective Secants. Fig. 214.

The Way Mr. *Wright* takes for making his Table of *Meridional Parts*, is by a continual Addition of Natural Secants, beginning at 1 Minute, and so proceeding to 89 Deg. Dr. *Wallis* (in *Phil. Transf.* N° 176.) finds the Meridional Part belonging to any Latitude by this *Series*, putting S for its Natural Sine, *viz.* $S + \frac{1}{3}S^3 + \frac{1}{5} + {}^5 + \frac{1}{7}S^7 + \frac{1}{9}S^9$ &c. which gives the *Merid. Part* required. How to find the same mechanically by Means of an easily constructed Curve Line, is what I shall now shew.

1. Prepare a Rular AB of a convenient Length, in which let Bo be equal to the Radius of the intended Projection. To the Point o as a Center (on the narrower Edge of the Rular) fasten a little Plate-Wheel wh tight to the Rular, and of a Diameter a little more than the Thickness of the Rular. Let KR (*Fig.* 214.) represent another long Rular, to which AR is a perpendicular Line. Place the Rular AB upon the Line AR, with the Center of the Wheel at A. Then with one Hand holding fast the Rular KR, with the other Hand slide the End B of the Rular AB by the Edge of KR; so will the little Wheel wh describe on the Paper a Curve Line ACB, to be continued, as far as is convenient. Fig. 213. Fig. 214.

2. Having drawn the Curve ACB, draw a straight Line KR by the Edge of the Rular KR: which Line is the Meridian to be divided, and also an Asymptote to the Curve ACB.

3. In this Meridian, (accounting R to be the Point of its Intersection with the Equator,) the Point answering to any Degree of Latitude is thus found. In the Perpendicular AR, make RG equal to the *Cosine* of Latitude (Radius being AR,) and from G draw GC parallel to KR, and intersecting the Curve in C. With Center C and Radius $CM = AR$, strike an Arc, cutting the Meridian at M; so is M the Point desir'd!

4. In the Curve AC, let c be a Point infinitely near to C, and cm, $(=CM,)$ a Tangent to the Curve at c, making the little Angle MCm, to which let the Angle RAr be equal: So is $Rr = Md$ (a Perpendicular from M to cm.) Draw CD equal and parallel to AR, intersecting

KR in S. With Center C and Radius CD draw the Arc DM, and its Tangent DE and Secant CE.

5. Becauſe of the like Triangles CDE, Mdm; $CD : CE :: MD : Mm$, that is, as Radius to Secant of the Arc DM, (whoſe Coſine is $CS = GR$,) :: ſo is Md ($= Rr$ a Degree or Particle of the Equator :) to Mm the Fluxion or correſpondent Particle of the Meridian Line RM. Whence, and from what is premiſed concerning the Nature of this Nautical Projection, 'tis evident, that RM is the *meridional Part*, anſwering to the Latitude whoſe Coſine is GR. Or thus; With Center R and Radius AR deſcribe the Quadrant $A\varkappa\alpha$, in which let the Arc $A\varkappa$ be equal to the given Lat. From $\varkappa$ draw $\varkappa C$ parallel to KR, and interſecting the Curve in C, ſo is $C\varkappa$ the Meridional Part deſir'd, being equal to RM, as is eaſy to ſhew.

6. As to the other Properties of this Curve, 'tis evident, from its Conſtruction, that its *Tangent* (as CM) is a *Conſtant Line* every where equal to AR; the Curve being generated by the Motion of the Wheel at the End of the Rular which is its Tangent. And from hence the Curve ACB may, for Diſtinction, be call'd the *Equitangential Curve.*

7. The Fluxion of the Area $ARMC$ is the little Sector or Triangle MCd, which ſame is alſo the Fluxion of the Sector CDM: whence the Areas $ARMC$, CDM are equal, and the whole Area ACB, &c. KMR being infinitely continued, is equal to the Quadrant $AR\alpha$.

8. To find the Radius of Curvature of any Particle, as Cc, from C draw an indefinite Line CT perpendicular to CM, (on the Concave ſide of the Curve) and from c another Line perpendicular to cm, which Lines, (becauſe of the Inclination of CM to cm) will ſomewhere meet as at T, making an Angle $CTc = MCm$. Theſe Angles being equal, their Radii are proportional to their Arcs: therefore, $Md : Cc :: MC : CT$. But $Cc = dm$ (becauſe of $CM = cm$) ſo that $Md : dm$ ($:: CD : DE$) $:: CM : CT$. But $CD = CM$, therefore $CT = DE =$ Tangent of the Arc DM.

9. So that ſuppoſing ATt a Curve Line, in which are all the Centers of Curvature of the Particles of ACB, any point as T being found as before, the Length AT (by the Nature of *Evolution of Curves*,) is every where equal to the *Tangent* of its correſpondent circular Arc DM. The Point T is alſo found by making MT perpendicular to RM, and equal to the Secant CE: for ſo is the Angle $CMT = MCD$; and the Triangle MCT equal to the Triangle CDE.

10. Let AHb be an Equilateral Hyperbola, whoſe Semiaxis is AR and Center R. In the Meridian let RP be equal to the Tangent DE. Join AP, and draw $PH = AP$ and parallel to AR. Compleat the Parallelogram $HNRP$, ſo will the Point H be in the Hyperbola, and its Ordinate HN ($= RP = DE = CT$) be equal to the Curve ATt. From whence, and from *Prop.* 3. *Coroll.* 2. of Dr. *Gregory*'s *Catenaria* (*Phil. Tranſ.* N° 231.†) it appears, that the Curve ATt is that called the

† *Vid.* Supra. V. I. C. I. S. XIII.

the *Catenaria* or *Funicularia*, *viz.* the Curve, into whose Figure a *slack Cord or Chain* naturally disposes its self by the Gravity of its Particles.

" 11. Hence we have another Property of the *Catenaria* not hitherto " taken Notice of (that I know of) *viz.* that supposing AR ($= a$, the " constant Line in Dr. *Gregory*) equal to the *Radius* of the Nautical " Projection, and RN the Secant of a given Latitude, then is NT the " *Catenaria*'s Ordinate at N, equal to RM the Meridional Part answer- " ing to the Latitude, whose Secant is RN.

12. That TA is the *Catenaria* is also demonstrable from Dr. *Gregory*'s first *Prop.* Let Tu be the Fluxion of the Ordinate NT: and tu ($= Nn$) the Fluxion of the Axe AN. Then because of like Triangles TCM, Tut, $CM : CT$ ($= TA$) $: : Tu : ut$, that is, as CM a constant Line is to TA the Curve : : so is the Fluxion of the Ordinate to that of the Axe ($\dot{y} : \dot{x}$) according to *Prop.* 1. *Catenariæ.*

13. From the Premisses the Construction and several Properties of the *Catenaria* are easible deducible; one or two of which I'll set down.

The Area $ATMR$ is equal to $AOPR$ a Rectangle contained by Radius AR, and RP the Tangent answering to Secant $HP = TM$. For because of the like Triangles CMm, CEe; $CM : CE : : Mm : Ee$, (that is, putting r, s, t, m for Radius, Secant, Tangent and Meridional Part RM) $r : s : : \dot{m} : \dot{t}$ whence $r\dot{t} = s\dot{m}$, and all the $r\dot{t}$ = all the $s\dot{m}$, that is $AOPR = ATMR$, which agrees with Dr. *Gregory*'s *Cor.* 5. of *Prop.* 7.

14. Supposing the former Construction, let be added the Line RH, including the *Hyperbolic Sector* ARH. I say the same Sector is equal to half the Rectangle $ARMQ$ contained by Radius AR and the Meridional Part RM, ($= \frac{1}{2} rm$) For the Sector ARH = Triangle RNH wanting the Semisegment ANH, The Fluxion of the Triangle RNH is $\frac{s\dot{t} + t\dot{s}}{2}$. The Fluxion of ANH is $t\dot{s}$. So the Fluxion of the Sector ARH is $\frac{s\dot{t} + t\dot{s}}{2} - t\dot{s} = \frac{s\dot{t} - t\dot{s}}{2}$. 'Tis found before (*Sect.* 13.) that $r : s$ ($s : \frac{ss}{r}$) $: : \dot{m} : \dot{t}$; whence $s\dot{t} = \frac{ss}{r}\dot{m}$. And because of the like Triangles CDE, Efe, $CD : DE : : Ef : fe$. But $Ef = Mm = \dot{m}$, because both Ef and Mm are to Md in the same Reason, *viz.* as s to r; therefore $r : t$ ($t : \frac{tt}{r}$) $: : \dot{m} : \dot{s}$: whence $t\dot{s} = \frac{tt}{r}\dot{m}$, and $\frac{s\dot{t} - t\dot{s}}{2} = \frac{ss - tt}{2r}\dot{m} = \frac{rr}{2r}\dot{m} = \frac{1}{2}r\dot{m}$, = the Fluxion of the hyperbolic

Sector ARH, whose flowing Quantity is therefore equal to $\frac{1}{2} rm = \frac{1}{2} ARMQ$. *Q. E. D.*

15. This shews another Property of the *Catenaria*, *viz.* that it squares the Hyperbola; for RM is equal to NT the Ordinate of the *Catenaria.*

Fig. 215. 16. Let AR be Radius, ACB the Equitangential Curve; MRN its Asymptote, in which let M, N, be any two Points equally distant from R. Upon M draw ML parallel to AR and equal to the *Difference* of the Secant and Tangent of that Latitude, whose Meridional Part is RM (by *Sect.* 3, 4.) Upon N draw NO parallel to AR, and equal to the *Sum* of the foresaid Secant and Tangent. Do thus from as many Points in the Asymptote, as is convenient, and a Curve drawn equably through the Points L—A—O, &c. will be a *Logarithmic Curve*, whose *Subtangent* (being constant) is equal to Radius AR.

17. Let no be an Ordinate infinitely near and parallel to NO. $Op = Nn$ the Fluxion of the Asymptote; OT the Tangent, and TN the Subtangent to the Logarith. Curve in O. Then $op : pO :: ON : NT$. But $ON = s + t$, therefore $op = \dot{s} + \dot{t}$. $pO = \dot{m}$ (the Fluxion of the Meridian or Asymptote.) So the Analogy is $\dot{s} + \dot{t} : \dot{m} :: s + t : NT$. By *Sect.* 13, 14. $\dot{s} : \dot{m} :: t : r$. also. $\dot{t} : \dot{m} :: s : r$. and thence $\dot{s} + \dot{t} : \dot{m} :: t + s : r$. wherefore is NT (the Subtangent to LAO) equal to Radius AR a constant Line, and consequently the Curve LAO is the Logarithmic Curve, and its Subtangent known.

18. The same Demonstration serves for LM (any Ordinate on the other Side of AR) only changing the Sine $+$ into $-$; and then it agrees with Mr. *James Gregory*'s *Prop.* 3. *pag.* 17. of his *Exercitations*, viz. *That the Nautical Meridian is a Scale of Logarithms of the Differences whereby the Secants of Latitude exceed their respective Tangents, Radius being Unity.* So here RM is the Logarithm of ML, the Difference of the Secant and Tangent of the Latitude, whose Meridional Part is RM.

19. Supposing the precedent Construction, if through any Point C of the Curve ACB be drawn a right Line GCW parallel to MR, terminated with the Logarithmic Curve in W and the Radius AR in G: I say, that the same right Line WG is equal to the intercepted Part of the Curve Line AC.

20. Let wg be a Line infinitely near and parallel to WG, and terminated by the same Lines; and CS, $W\sigma$, perpendicular to the Meridian; CS intersecting wg in z, and $W\sigma$ in y. Let CM be a Tangent to AC in C; $W\tau$ a Tangent to AW in W; so is $CM = \sigma\tau$. Because of like Triangles Czc, CSM; and Wyw, $W\sigma\tau$; $CS : CM :: Cz : Cc$: also $W\sigma : \sigma\tau :: Wy : yw$. But $W\sigma = CS$; $\sigma\tau = CM$; $Cz = Wy$; therefore is yw the Fluxion of GW, equal to Cc, the Fluxion of the Curve AC. Consequently $GW = AC$ *q. e. d.*

21. It may be noted, that this Equitangential Curve gives the Quadrature of a Figure of Tangents standing perpendicular on their Radius. In *Fig.* 214. let $A\gamma\Gamma$ be a Curve, whose Ordinates as $g\gamma$, $G\Gamma$, are equal to the Tangents of their respective intercept Arcs Ak, Ax. Let ΓG be produced to touch the Curve AC in C: then is the Area $A\Gamma G$ equal to the Rectangle contained by Radius AR and GC the produced Part of the Ordinate; or $A\Gamma G = AR \times GC$. The Demonstration of which, and of the following *Section*, I for Brevity omit.

22. If we suppose the Figure ACB, &c. KR (*Fig.* 214.) infinitely continued, to be turned about its Asymptote RK as an Axe, the Solid so generated will be equal to a rectangled Cone, whose Altitude is equal to AR; and its Curve Surface will be equal to half the Surface of a Globe whose Radius is AR. So that if the Curve be continued *both ways* infinitely (as its Nature requires) the whole Surface will be equal to that of a Globe of the same Radius AR.

The Description of the Rular and Wheel, *Fig.* 213. is sufficient for the Demonstration of the Properties of the Curve: But in order to an actual Construction for Use, I have added *Fig.* 216. where AB is a Brass Rular; wh the little Wheel, which must be made to move freely and tight upon its Axe (light a Watch-wheel) the Axe being exactly perpendicularly to the Edge of the Rular. s represents a little Screw-pin to set at several Distances for different Radii, and its other End is to slide by the Edge of the other fix'd Rular. p is a Stud for the convenient holding of the Rular in its Motion. Fig. 213. Fig. 216.

N. B. *Most of the Properties of this* Curve *by the Name of* la Tractrice, *are to be found in a* Memoire *of M.* Bomie *among those of the* Royal Academy *of* Sciences *for the Year* 1712, *but not published till* 1715: *Whereas this Paper of Mr.* Perks *was produced before the* Royal Society *in* May 1714, *as appears by their* Journal.

VII. 1.] I have lately thought of a new Instrument for drawing a Meridian Line; it is easy in its Use, and sufficiently exact.

A New *Way of drawing a* Meridian Line, *by Mr.* S. Gray. n. 268. p. 763.

Take the Gnomon of an horizontal Dial for the Latitude of the Place, and to the *Hypothenusa* fix two Sights, whose Centers may be parallel to the same: let the Eye-sight be a small Hole; but the other's Diameter must be equal to the Tangent of the double Distance of the *North-Star* from the Pole, (the Distance of the Sights being made *Radius.*) Let the Stile be riveted to the End of a straight Ruler: When you would make use of it, lay the Rular on an horizontal Plane, so that the End to which the Stile is fix'd may over-hang; then look through the Eye-sight, moving the Instrument, till you see the North-Star appear to touch the Circumference of the Hole in the other Sight, on the same Hand with the Girdle of *Cassiopeia*; or on the opposite Side to that, whereon the Star in the Great Bear's Rump is, at that Time:

Time: then draw a Line by the Edge of the Rular; and it will be a true Meridian Line, as it is eaſy to demonſtrate.

I do not hear that any of the Occultations of *Aldebaran* by the Moon were obſerv'd laſt Year: I expected ſeveral, but was always hinder'd by the Weather from obſerving any.

—*on the* ſame *by the* ſame. n. 270. p. 815.

2.] I have ſent ſome farther Thoughts upon the Inſtrument for drawing a Meridian Line, and have improved it ſo far, that no other *Star* will be made uſe of than the *Polar* one to obtain the Hour and Minute of the Day or Night.

Let there be taken a Teleſcope of about 16 Foot, or longer if you pleaſe; in the Plane of its Focus place a Ring of Braſs at right Angles to the Axis of the Glaſs, the Diameter of the inward Circle equal to the double Tangent of the Pole-Star's Diſtance from the Pole; the focal Length of the Object Glaſs being made Radius, as was ſaid in the Deſcription of the Meridian Inſtrument; let the Ring be divided into 24 Hours, with their Minutes number'd from the Right-hand towards the Left, as in our common Nocturnals; the Eye Glaſs muſt be equal in its Diameter to the Horary Ring: but this perhaps will be thought too chargeable, eſpecially for ſuch large Teleſcopes as I am ſpeaking of, which has made me think of this Contrivance: The Eye Glaſs muſt lie in a broad Index towards one End, this is to turn on a Center Pin, that lies in the Center of the Glaſs, and conſequently over the Center of the Horary Ring, from which it muſt be equal to the Diſtance of the Focus of the Eye Glaſs; then let the Tube be elevated to the Height of the Pole, and directed to the Pole-Star, till by turning the Index through the Eye Glaſs, you perceive the *Star* to touch the Horary Ring on that Side the Star in the *Great Bear's Rump* lies, or on the oppoſite to that in the Hip of *Caſſiopeia*; but on the contrary, had not the Glaſs inverted the Object, then bring one of the twelves to be in a Perpendicular to the other by a Plumb-line; ſo will the Star ſtand at its Horary Diſtance from the Meridian; or if the Latitude of the Place be unknown by the Right Aſcenſion of the Sun and Star, the Time of its coming to the Meridian will be eaſily obtained; and then the Hour of the Night found, will as eaſily give the Star's Horary Diſtance from the Meridian; then elevate the Tube towards the Star, bringing the Meridian, or 12 and 12 into the Plane of the Perpendicular; turn the Glaſs about, till you ſee the Pole-Star ſtand at its Horary Diſtance from the Meridian; ſo will the Inſtrument when fixed, ſhew the Horary Diſtance throughout the whole Day, or as long as it remains in this Poſition, by the apparent Motion of the Star in the Ring. The beſt Time to fix the Inſtrument will be, when this, or any of the other two Stars above-mentioned, are about 6 Hours from the Meridian. It is to be obſerv'd, that the Latitude of the Place is now given with the utmoſt Preciſeneſs: for the Axis of the Glaſs lies now in the

the Axis of the World; and if one of the Sides of the Tube be parallel thereto, as it ought to be at the upper End, hang a Line and Plummet from the Point of the Suſpenſion; find another Point equal in Diſtance to the Length of the Line, or a Knot towards the lower End, the Diſtance from this Knot to the former Point will be but the Chord of the Latitude; and if from the ſame Edge of the Index, another Line and Plummet be hung towards the lower End of the Tube, theſe two Lines, when at reſt, will be in the Plane of the Meridian.

This Inſtrument may be made to ſhew the Hour with as much Facility as a Clock or Sun-dial, if the Horary Ring be made to move within a larger fixed one; and the outward Circle of the former be divided into the Days of the Month, reſpect being had to the Right-Aſcenſion of the Sun and Star: Then by bringing the two oppoſite Points in the fixed Circle to the Perpendicular, which is done at the fixing the Inſtrument, move the Circle till the Day of the Month come to any of theſe, and the Ring is rectified for that Day; and if the Air be clear, you will ſee the Star ſtand at the true Time of the Day or Night.

It may be objected, that in a few Years, by the Annual Increaſe of its Declination, the Pole-Star will, by moving in a leſſer Circle, be brought too far from the Edge of the Ring, that the exact Hour and Minute cannot well be diſtinguiſh'd: but this Inconveniency, when it is one, may be eaſily remedied ſeveral Ways; either by making a leſſer Ring, or by extending a fine Thread of Silk croſs the Ring, till it cuts the Star, and at the ſame Time it gives the Hour; or, which will yet make this Inſtrument commodious for other Purpoſes, there may be made an Index to move on the Center of the Hour-wheel, which being brought to cut the Star with the Edge that proceeds from the Center, it will at the ſame Time cut the Hour: And now we need not be ſollicitous about the exact Diameter of the Ring, provided it do but a little exceed the Diſtance of the Pole-Star from the Pole, the focal Length of the Glaſs being made Radius.

Mr. *Flamſteed* has diſcovered, that there is a Parallaxis of the Earth's Annual Orbit at the Pole-Star of about 40 or 45 Seconds; whereby the Diameter of the Star's Parallel is greater in *June* than in *December*, by about 1 Min. 2 Seconds; which he has evinced from ſeven Years ſucceſſive Obſervations, whereby the Earth's Motion is indubitably demonſtrated, as appears from his Letter to Dr. *Wallis* on that Subject.

Now if on the Edge of this Index there be drawn a Scale of Degrees, Minutes and Seconds, to the Radius of the Glaſs, we ſhall not only have a very accurate Inſtrument for the Hour, but be furniſhed with one, whereby we ſhall ſee the Truth of the Earth's Motion confirmed by the Acceſs and Receſs of our Star towards and from the Pole, according to the Earth's Place in the Ecliptick, as that learned Perſon has diſcovered; and that not only when the Star tranſits the Meridian,

Meridian, but in clear Air at any Time of the Day; one shall likewise observe that Annual Increase of the Pole-Star's Declination, caused by the Precession of the Equinox.

My own Observations assure me, that the Pole-Star may be seen in the Day time with a Telescope of 16 Foot; for with one of this Length I saw that Star on the 26*th* of *April* 1701, from 4 o'Clock in the Morning till 7, and could have seen it longer, had not Clouds interposed; and again the first of *May*, I did not look for the Star, till the Sun had been up more than half an Hour, *viz.* at 5 in the Morning, yet I soon found it, and saw it afterwards as oft as I pleased, till half an Hour after 9 the same Morning; so that I doubt not, this Star may be seen in a clear Day throughout the whole Year.

The Declination of the Pole-Star for the Year 1700, is 87° 42′ 51″, as I find it by *Ricciolus*'s Catalogue of fixed Stars, in the Appendix to Sir *Edward Sherbourn*'s Sphere of *Manilius*, *&c.* Hence its Distance from the Pole at this Time may be assumed 2° 17′. the focal Length of my Object Glass is 15 Foot 6 Inches, so that the Diameter of the Ring will be 14 Inches, and 84 hundredth Parts of an Inch, which is the natural Tangent of the former Arch 2° 17′ doubled; a Circle large enough to be divided into Minutes and Halves, which will be so magnified by the Eye-Glass, that it will be easy to distinguish the Time to a few Seconds.

It is true, there is some Difficulty in fixing up this Instrument; and when it is so, to keep it from varying from its due Position; but yet it is not insuperable: But for small Instruments, of about 2 or 3 Foot long, there cannot be a more accurate, easy, and expeditious Way than this for drawing a Meridian Line. But whether the many Benefits that may accrue to Astronomy, do not make the larger one worth the Charge and Trouble of compleating it, I leave to the Consideration of the Learned.

An Instrument *for finding the* Meridian. *By Mr.* W. Derham. n. 291. p. 1578.

VIII. Among all the Ways contriv'd for finding the Meridian of any Place, the most commodious, I think, is an Instrument of Sir *Chr. Wren*'s, or two of Mr. *Gray*'s, or one publish'd in the Appendix of a Book call'd the *Artificial Clockmaker*.

Sir *C. Wren*'s Contrivance, I am informed, is thus: At one End of a Rular, erect a Sight, to see the Pole-Star, *&c.* through. At the other End set up two Circles of small Wire, one within the other; the Diameter of the innermost, equal to the doubled Tangent of the Distance of the Pole-star from the Pole, the Distance of the Sight being Radius; and the Diameter of the outermost Circle, equal to the double Tangent of the Distance of the next Star to the Pole-Star, from the Pole. Your Instrument thus prepared, if you look thro' the Sight, and bring the two Circles to the two Stars, whose Distances from the Pole they represent; a Line passing through a Sight and Center of the Circles, is the Elevation of the Pole: and two Plumb-lines hung up, one over the

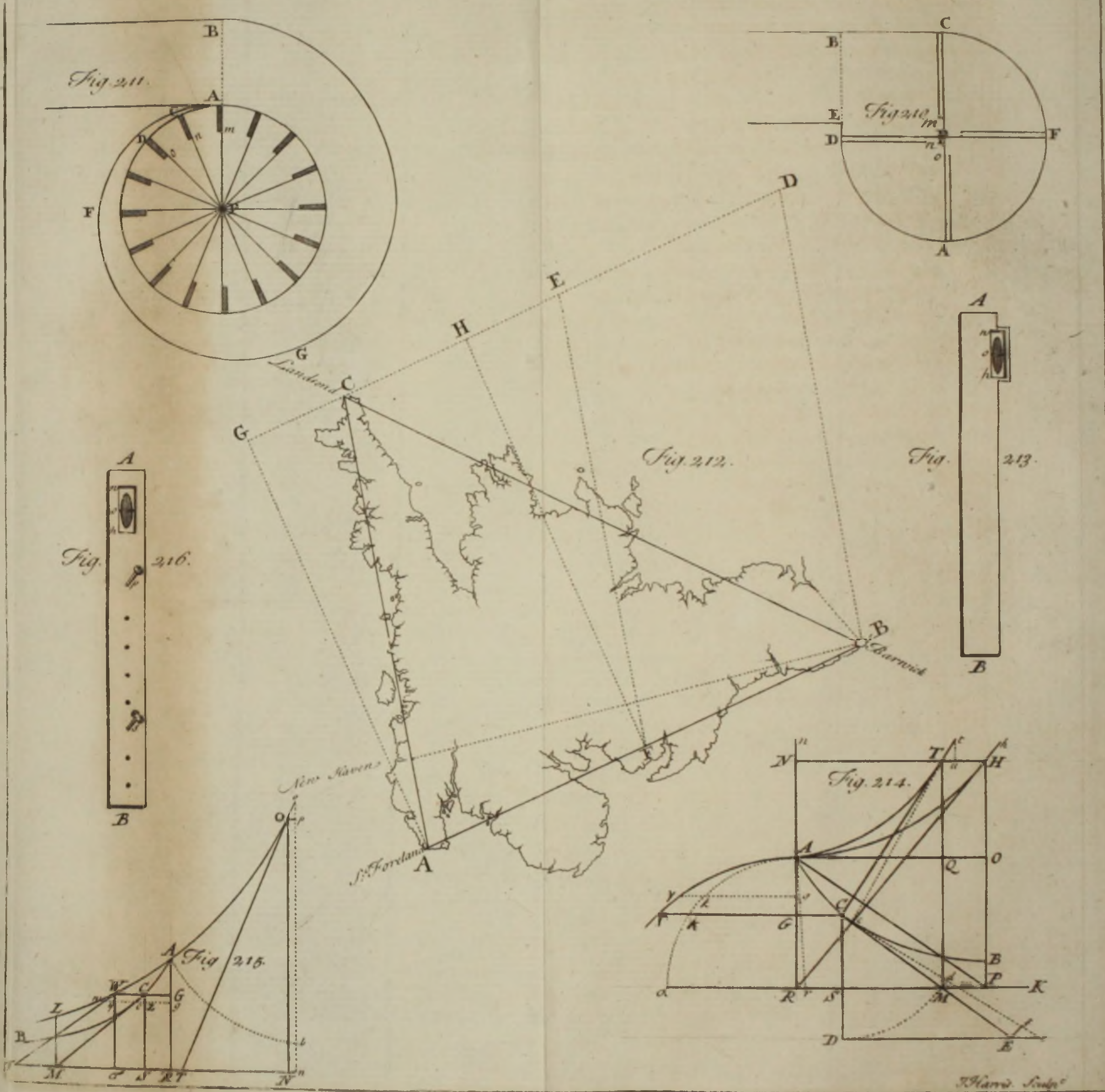


Fig. 211.
Fig. 210
Fig. 212.
Fig. 213.
Fig. 214.
Fig. 215.
Fig. 216.
Landsend
Berwick
New Haven
S. Foreland
J. Harris Sculp.

the Sight, the other over the Center of the two Circles, will exactly lie in the Meridian of the Place.

Mr. *Gray*'s Contrivances being printed, I need not give any Description of them.

The last Instrument is what I have made use of for several Years, and I would recommend it, upon my own Experience, for a very nice Way to find the Meridian of any Place, and to see the *Transits* of the Celestial Bodies over it, whether Northward or Southward.

The Instrument is thus made of Wood, or rather Iron, or Brass, to endure the Weather, without swelling or contracting, *viz.* Prepare a small flat Iron Bar, *C*, *C*, at each End of which rivet on two upright Sights, to turn stiffly, at the Joints *I*, *I*. Let one of the Sights *c*, *d*, have a Perforation big enough to see the *Pole-Star* through it; the other Sight *a*, *b*, a very small Perforation, to see the *Sun* through. Just behind the Joints fix two upright Arms *C*, *D*, and *C D*, but to bend off, so as to be out of the Way of the Sights, when you look through them. These Arms ought to be long enough for the Plumb-lines to reach the Polar-Star, on the one Side; and the Sun at his greatest Height, on the other Side, when you look through either of the Sights. The Plumb-lines therefore are Tangents to their opposite Sights, and their Lengths may be found by a Table of natural Tangents, and making the Distance of the two Sights Radius. Thus in the Latitude of *London*, if the Instrument be two Feet from Sight to Sight, the Southern Plumb-line hath need to be near four Feet, and the Northern Plumb-line near two Feet ten Inches. On the Tops of these two Arms, place two small cross Pieces *D E* and *D E*, to turn with a Point at *D*, which cross Pieces are to hold the Plumb-lines *E F* and *E F* and to turn off and on, so as to bring the Plumb-lines to the Sights exactly. Place this Instrument on a Pedestal *H* to turn round on it stiffly at the Pin *G*.

Your Instrument being thus prepared, the Way to *Set and Use it* is thus; plant it in a convenient Place, where the Polar-Star may be seen by Night, and the Sun by Day. When that Star is on the Meridian, is the Time to set this Instrument, which is thus to be done, *viz.* Through the Sight with the large Hole *c*, *d*, look at the Pole-Star, and turn the whole Instrument about, until you see the opposite Plumb-line nicely to intersect the Pole-star. Or when you have brought the Plumb-line near the Star, you may more easily bring the Plumb-line to intersect, by moving the Sight *c*, *d*, backward or forward, at the Joint I, instead of the moving the whole Instrument. And that you may more easily see the Pole-Star through the Sight, let the Plumb-line be a very fine Cats-gut String or Horse-hair, &c. And if it be white, or some such light Colour, it will be the better seen, with the Help of a Candle shining on it by Night, when it is necessary.

The Sight *c*, *d*, and opposite Plumb-line being thus set in a direct Line with the Polar-Star on the Meridian, it is manifest, that the Instrument lieth exactly in the Meridian, so as to see any Star on the Meridian to the North. And that you may see the same Southerly; the next Day, or when you please, you may hang up the Plumb-line E, F, upon the Southern Arm *C*, *D*, so as that the Plumb-line may exactly intersect the Perforation *c*, *d*. This may easily be done by moving the top Joint, with the Plumb-line on its Cross-piece backward and forward, till the Plumb-line hangeth to your Mind. If the Sight with the lesser Perforation *a*, *b*, be not exactly under the Northern Plumb-line, it must be brought to be so, by turning the Sight, by Help of its Joint at I; and then all the Instrument is set right, so as to see the Sun, Moon or Stars, come on the Meridian towards the South.

But to see the Sun transit the Meridian, it is necessary to guard the Eye, with a colour'd Glass, or a Glass darkened with the Smoak of a Lamp or Candle.

A Glass *to look through upon the* Sun.

Chuse two Pieces of Glass cut into the same Size and Figure; but take care they do not refract vitiously; which may be known by moving the Glass before the Eye. If the Objects you look on seem to dance about, the Glasses are false and refract; but true if all seems steady. Smoak one of these Glasses over the Flame of a Lamp or Candle, until it be obscured enough to take off the Sun-rays sufficiently, but not so as to darken it too much. This may be seen by looking upon the Sun with it, or upon the Candle. One of the Glasses being thus darkened, lodge them both together, and fasten them in a little Case fit for the Purpose, with the smoaked Side innermost, and an edging of Card between, to keep the Glasses asunder, so as that the Soot may not be rubb'd off, or disordered.

'Tis good to have two Glasses thus prepared, one for a strong Sun; the other less darkened, for the Sun behind a thin Cloud, Mist, or, *&c.*

With one of these Glasses held behind, or before the Sight *a*, *b*, you may plainly see the Sun pass.

— the same Instrument varied.

Instead of an intire Instrument, prepare only two Sights (as in *Fig.* 218.) with Perforations as before. Let these Sights be nailed or screwed down, upon the Tops of two Stakes at *I*, *I*, so as to turn stiffly upon them. The Plumb-lines (one at least) may be hung up at the End of an

Fig. 218.

House (as at *K*, *Fig.* 218.) or on the Bough of a Tree (if the Wind would not shake it) or any where you see fit: And the Sights must be stuck up, so as to bring the Pole-Star to intersect, and all be performed, as hath been before directed.

This, although in a Manner the same with the Instrument before, yet is more convenient in some Respects. Chiefly because the Plumb-lines may be made longer, and the Sights set farther asunder, than in the Instrument before can conveniently be done; which is some, altho' no great Advantage for seeing the Transits. Also, these Sights may be made so light, as to be easily carried about; or they may be easily made, or imitated in any Place where-ever you come.

To

To know when the *Polar-Star* comes on the *Meridian*, the Way is this; subtract the *Right Ascension* of the *Sun* from the *Right Ascension* of the *Pole-Star*, the *Remainder* giveth the Degrees, Minutes and Seconds when the Pole-Star transits the Meridian above the Pole. Divide these Degrees by 15, it gives the Hours; and every Degree under 15 multiplied by 4, gives the Minutes; and every Minute multiplied by 4, gives the Seconds, of apparent Time of the Pole-Star's Southing. I scarce need say, that it comes under the Pole at 12 Hours Distance, only making some small Allowance for the Alteration of the Sun's *Right Ascension* in that 12 Hours Time. *To know when the* Polar-Star *comes on the* Meridian.

But you may shorten your Labour, by using Tables of the Sun's R. Asc. in *Time*, instead of his R. Asc. in *Degrees*, &c.

If the Sun's R. Asc. exceedeth the Pole-Star's, add 360 Degrees, or 24 Hours, and then subtract.

The R. Asc. of the Pole-Star is determin'd by Mr. *Flamsteed* to be $0^h\ 33'\ 4''$ of *Time*, Anno 1690, and the Increase of its R. Asc. in 10 Years $1'\ 16''$ of *Time*. Therefore this present Year 1703, the R. Asc. of the Pole-Star is $0^h\ 35'\ 22''$ of *Time*.

Or you may see, when the Pole-Star cometh to the Meridian, by hanging up a Plumb-line, and observing when the *Thill Horse* in *Charles*'s *Wain* called *Alioth*, comes near the Line, together with the Pole-Star, on one side the Pole; or the bright Star of the Third Magnitude in *Cassiopeia*'s *Thigh* on the other side, as is represented in *Fig.* 219. Fig. 219.

The foregoing Instruments may be set by any other Star, as well as the Pole-Star. But the Pole-Star in our Northern Hemisphere, is most convenient, because it maketh but a small Circle round the Pole, and therefore moves slower, and consequently is longer in transiting the Meridian. And therefore a small Error in Calculation, or a little Expence of Time in setting the Instrument, may be admitted.

The Uses of these Instruments are, 1. You may see with all imaginable Exactness, when it is *Noon*, even to 1, 2, or at most 3 Seconds of Time. For you may see, when the very Limb of the Sun toucheth the Meridian, and whilst all his Disk is passing it. So that by much it exceeds all Sun-Dials: so far that if you once use this Instrument, you will be ready to lay aside all Sun-Dials; the best of which (unless we except Mr. *Molineux*'s) can never shew the Time to one or many Seconds. *The Uses of these Instruments.*

But besides all this, another vast Conveniency is, That it will *fit most Latitudes*. So that there is no Need of having a strict Regard to the Elevation of the Pole, nor any Danger of Error in making and setting, as is in most other Instruments, but all is with Ease and Certainty performed. Therefore,

2*dly*, Into whatsoever Place you come, you may easily see the Errors of the Sun-Dials there, and which go truest, and which false.

3*dly*, As the Sun, so also the fix'd Stars may be seen to transit the Meridian, whereby the *Hour of the Night* may as exactly be known, as of the Day by the Sun, knowing the R. Aſc. of the Star that tranſits. For (as before for the Pole-Star) ſubtract the R. Aſc. of the Sun from the R. Aſc. of the Star, the Remainder converted into Time, is the Time of that Star's Culmination or Southing. And if 12 Hours be added or ſubtracted (making due Allowance for the Alteration of the Sun's R. Aſc. in that Time) it ſheweth the exact Time of that Star's coming to the Meridian Northward.

4thly, The Hour of the Day and Night being thus to 1, 2 or 3 Seconds, diſcoverable by the aforeſaid Inſtruments, I doubt not, but that they may be uſeful in finding the exact *Differences of Meridians*, either by the *Eclipſes of Jupiter*'s *Satellites*, or the *Occultation* of the *Fix'd Stars* by the *Moon*.

I do not pretend, that theſe Inſtruments are any otherwiſe uſeful in finding the *Longitude*, than by ſhewing the exact Time of the Day or Night; which is one Thing abſolutely neceſſary in this Matter. Neither indeed will they ſerve without a well adjuſted Pendulum-Watch or Pocket-Watch, that will keep Time exactly from one Obſervation by the Meridian-Inſtrument to another. Nor indeed are they uſeful on Shipboard, but only on Land, where they remain fix'd. But on Head-Land, or any where on Shore, they may be uſeful to the Seaman: And indeed (until better Diſcoveries are made) theſe Meridian-Inſtruments may be Uſe, where-ever long Teleſcopes can be of Uſe, for ſeeing the *Appulſes* of the *Moon* to the *Fix'd Stars*, or the Eclipſes of *Jupiter*'s *Satellites*; which is only on Land: Unleſs (which I have thought feaſible) a convenient ſtanding for a Man, a Teleſcope might be hung penduloully in a Ship, which (eſpecially in a calm Sea) may be as little ſubject to Diſturbance, as the Pendulums of Watches are, which will retain their Motion at Sea.

5thly, You may with all Exactneſs continue a Meridian-Line for many Miles, by looking through either Sight, and ſeeing what Objects are interſected by the Plumb-Lines.

Of a Meridian Line *drawn thro'* France, *&c.* *Communicated by Mr.* Geoffroy, n. 278. p. 1097.

IX. Monſieur *Caſſini* open'd the *Aſſembly* (of the *Academy Royal of Sciences*, Nov. 12. 1701.) with a Diſcourſe containing the Obſervations he had made in his laſt Voyage, with a Deſign to determine the Paſſage of a *Meridian-Line* (taken from a Point in the *Obſervatory* at *Paris*) from one End of *France* to the other. In the firſt Part of this Diſcourſe, he went back to the moſt ancient Aſtronomers, and recounted their Opinions of the Spheric Figure of the Earth, and their Methods to know its Dimenſion; and then proceeded to thoſe of the Moderns. And in the laſt Place, he related the Method of the late Monſieur *Picard*, of the *Academy Royal*, as the moſt exact. Then he ſpoke of his own Obſervations on the ſame Subject, of the Uſe he had made of the *Satellites* of *Jupiter*, more fit for this than the Eclipſes of the Moon, in that they are more frequent; and ſaid that his Obſervations had been confirmed by the like made in *China*. He ſhewed the Method he took to determine

mine the Paſſage of the *Meridian* taken from a Point in the Obſervatory at *Paris*. By the Means of Triangles, which he made through the whole Courſe of his Journey, and very exact Calculations, he determined the Place of this Meridian, and marked all the conſiderable Places through which it paſſed, from *Paris* to the higheſt Mountains of the *Pyreneans*, which ſeparate *Rouſſillon* from *Catalonia*; among theſe Mountains he obſerved one of a prodigious Height, it being 1440 Toiſes high. But the moſt extraordinary Obſervation was that of the Inequality of the Degrees of the Meridian on the Earth; which is ſuch, that Monſieur *Caſſini* found that going Southward one Degree ſurpaſſed another an 800*th* Part, which may give great Reaſon to doubt of the exact Roundneſs of the Earth. Upon this Occaſion he reported two different Opinions, the one of Monſieur *Huygens* and *Newton*, the other of a Mathematician of *Straſburg* named *Eiſenſchmidius*. The two former hold, that the Earth is flatted towards the Poles, ſo that it is ſomething of the Shape of an *Holland* Cheeſe: Which they both conclude by Phyſical and Algebraical Deductions, from an Obſervation made at *Cape Verd*; that the Pendulums, though of the ſame Length, make their Vibrations there much ſlower than in the Northern Countries. The other Mathematician holds, that the Figure of the Earth is Elliptique, ſo that it is ſtretch'd out towards the Poles, and has the Form of an Egg. M. *Caſſini* left the Queſtion undecided. The Cities through which he obſerv'd the *Meridian* of *Paris* to paſs, are *Dunkirk*, *Amiens*, *Aubigny*, *Bourges*, *Aurillac*, *Rodez*, *Alby*, and *Carcaſſione*.

X. *A Paper Omitted.*

Guilielmi Muſgrave Regiæ Societatis Socii de *Britanniâ* quondam *Pene-Inſulâ*, Diſſertatio. n.352. p. 589.

Chap. VII.

MUSIC.

I. HAving made the Trial of a Muſical Experiment before the *Society*, I ſhall give a farther Account of it; that the *Theory* of *Muſic*, which is but little known in this Age, and the *Practice* of it which is arriv'd to a very great Excellency, may be fixed upon the ſure Foundations of *Mathematical* Certainty.

The Theory of Muſic *reduced to* Arithmetical *and* Geometrical Proportions, *by Mr.* T. Salmon. n. 302. p. 2072.

The Propoſitions, upon which the Experiment was admitted, were; That Muſic conſiſted in *Proportions*, and the more exact the Proportions, the better the Muſic: That the Proportions offer'd were the ſame that

that the Ancient *Grecians* us'd: That the Series of Notes and half Notes was the ſame our Modern Muſic aimed at: Which was there exhibited upon Finger-boards calculated in Mathematical Proportion. This was demonſtrated upon a Viol, becauſe the Strings were of the greateſt Length, and the Proportions more eaſily diſcern'd; but may be accommodated to any Inſtrument, by ſuch mechanical Contrivances as ſhall render thoſe Sounds, which the Muſic requires.

To prove the foregoing Propoſitions, two Viols were Mathematically ſet out, with a particular Fret for each String, that every Stop might be in a perfect Exactneſs: Upon theſe, a Sonata was perform'd by Mr. *Frederick* and Mr. *Chriſtian Stefkins*; whereby it appear'd, that the Theory was certain, ſince all the Stops were owned by them, to be perfect. And that they might be prov'd agreeable to what the beſt Ear, and the beſt Hand performs in modern Practice; the famous *Italian*, Signor *Gaſperini*, play'd another Sonata upon the Violin in Conſort with them, wherein the moſt compleat Harmony was heard.

The full Knowledge and Proof of this Experiment may be found in the two following Tables, wherein Muſic is ſet forth, firſt Arithmetically and then Geometrically: The Mathematician may, by caſting up the Proportions, be ſatisfied that the five ſorts of half Notes here ſet down, do exactly conſtitute all thoſe Intervals, of which our Muſic does conſiſt. And afterwards he may ſee them ſet forth upon a Monochord, where the Meaſure of all the Notes and half Notes comes exactly to the middle of the String. The Learned will find, that theſe are the very Proportions which the old *Greek* Authors have left us in their Writings, and the practical Muſician will teſtify, that theſe are the beſt Notes he ever heard.

The Explication of the Firſt Table.

Between the two loweſt Lines, you have the Series of all the 12 half Notes in an Octave, from A *re* to A *lamire*, which added together make an Octave or exact Duple Proportion: The ſeveral Parts alſo added together make all thoſe Intervals of which it is conſtituted. As for Example, the two half Notes from A to A ※ $\frac{17}{18}$, and from A ※ to B $\frac{16}{17}$ make a Major Tone $\frac{8}{9}$; to which if an Hemitone from B to C $\frac{15}{16}$ be added, you have a leſſer Third $\frac{5}{6}$.

In like Manner between the two next Lines, you have the Series of all the 12 half Notes, in an Octave from C *fa ut* to C *ſol fa ut*: The two firſt Tones added together make a greater Third: And ſo you may add a Tone or Hemitone, till you arrive at every Interval in the Octave, which is ſo call'd, becauſe eight Sounds are required for expreſſing thoſe ſeven gradual Steps, whereby we commonly aſcend to it.

It may be alſo obſerved, that the Proportions falling upon the ſame Notes in two Keys, one Finger-board will be ſufficient for both.

It is acknowledg'd by all that are acquainted either with Speculative or Practical Muſic, that every Interval is divided into two Parts, whereof one is greater than the other: An Eighth $\frac{1}{2}$ into a Fifth $\frac{2}{3}$ and a Fourth

Fourth $\frac{3}{4}$. Again, a Fifth $\frac{2}{3}$ into a greater Third $\frac{4}{5}$, and a leſſer Third $\frac{5}{6}$. Thus alſo a greater Third $\frac{4}{5}$ muſt be divided into a Tone Major $\frac{8}{9}$ and a Tone Minor $\frac{9}{10}$. The Leſſer Third (to comply with the Practice of Muſic) is rather compounded of, than divided into a Tone Major $\frac{8}{9}$ and an Hemitone, which is its Complement, $\frac{15}{16}$.

TABLE I.

TABLE the 1ſt. Containing the Proportions ſet out *Arithmetically.*							
							An Eighth $\frac{1}{2}$
						A Seventh $\frac{8}{15}$	
					A Sixth $\frac{3}{5}$		
				A Fifth $\frac{2}{3}$			
			A Fourth $\frac{3}{4}$				
		A Greater Third $\frac{4}{5}$					
	$\frac{8}{9}$	$\frac{9}{10}$	$\frac{15}{16}$	$\frac{8}{9}$	$\frac{9}{10}$	$\frac{8}{9}$	$\frac{15}{16}$
	Tone Major :	Tone Minor	Hemitone	Tone Major	Tone Minor	Tone Major	Hemitone

An Octave with a greater Third. C $\frac{17}{18}$ ※C $\frac{16}{17}$ D $\frac{19}{20}$ ※D $\frac{18}{19}$ E $\frac{15}{16}$ F $\frac{17}{18}$ ※F $\frac{16}{17}$ g $\frac{19}{20}$ ※g $\frac{18}{19}$ a $\frac{17}{18}$ ※a $\frac{16}{17}$ b $\frac{15}{16}$ c.

A $\frac{17}{18}$ ※A $\frac{16}{17}$ B $\frac{15}{16}$ C $\frac{17}{18}$ ※C $\frac{16}{17}$ D $\frac{19}{20}$ ※D $\frac{18}{19}$ E $\frac{15}{16}$ F $\frac{17}{18}$ ※F $\frac{16}{17}$ g $\frac{19}{20}$ ※g $\frac{18}{19}$ a An Octave with a leſſer Third $\frac{5}{6}$.

Three Tones Major, two Tones Minor, and two of the aforefaid Hemitones, placed in the order found in the Scheme, exactly conftitute the practical Octave; which is fo call'd, becaufe it confifts of eight Sounds, that contain the feven gradual Intervals. But it is alfo neceffary to fet down the Divifions of the whole Tones, which are the true Chromatic half Notes, becaufe there is great Ufe of them in Practical Mufic.

To make all our whole Notes, and all our half Notes of an equal Size, by falfifying the Proportions, and bearing with their Imperfections, as the common Practice is, may be allow'd by fuch Ears, as are vitiated by long Cuftom: But it certainly deprives us of that fatisfactory Pleafure, which arifes from the Exactnefs of fonorous Numbers; which we fhould enjoy, if all the Notes were truly given according to the Proportions here affign'd.

It is very eafie to fatisfie our felves in the Arithmetical Scheme, by thofe Operations, which *Gaffendus* has fet down in his *Manuduction* to the *Theory* of *Mufic*, Tom. V. pag. 635. As for Example, his Rule for Addition is, That two Proportions being given, if the greater Number of one be multiplied by the greater Number of the other, and the leffer by the leffer, the two Numbers produc'd exhibit the compounded Proportions. Thus take a Practical Fifth $\frac{3}{2}$ and a Practical Fourth $\frac{4}{3}$ for the two Proportions given, multiply 3 by 4 and you have 12: then multiply 2 by 3 and you have 6: which compounded Proportion of 12 to 6 makes the Practical Octave $\frac{2}{1}$.

Thus, according to his Arithmetical Operations of Addition, Subtraction, Multiplication, of Continuation and Divifion, is our whole Syftem proved, which for the more eafy Application to Practical Mufic, fhall be alfo fet forth Geometrically upon the fix Strings of a Viol.

The Explication of the Second Table.

Thefe fix Lines reprefent the fix Strings of the Viol in the common Tuning.

The founding Part of each String from the Nut to the Bridge is fuppos'd to be 30 Inches long; the two middle Strings C and E are drawn out to 15 Inches, the half of the whole.

'Tis eafie to meafure every Interval with a Pair of Compaffes. Suppofe you are to take the 20*th* Part of the String G; 'tis an Inch and a half for the firft half Note; if you take the whole Note from G to A, 'tis the tenth Part, and muft be three Inches.

After thefe are taken away, your String will be but 27 Inches long, fo that if you advance one Note, or a Major Tone further, you muft take a 9*th* Part of it, which will be three Inches more, whereby you arrive at a greater Third, being the fifth Part of the whole String. Thus the Series of all the Notes may be demonftrated.

All the Strings are Unifon at the Stops where the tuning requires: So that though the Proportions be carried on as far as the Frets allow, yet the String is open the fame with the Stop of that String to which it is

3 tuned;

tuned; and accordingly the Series of the Notes proceeds as if they were all upon a Monochord.

This Calculation ſerves but for two Keys *A* and *C*, which are called Natural, becauſe they have no eſſential Flats or Sharps.

But becauſe the Compoſer begins upon any Key, and the Series of Notes muſt take its *terminus à quo* from thence; the Inſtrument-maker can provide ſuch moveable Finger-boards as will ſerve exactly for every Key. They are taken out and put in upon the Neck of the Viol, with as much Eaſe as you pull out and thruſt in the Drawer of a Table. Three, or at moſt five of them, will be ſufficient to accommodate all the Keys that are made uſe of.

This Mathematical fixing of the Frets enables every Practitioner, who ſtops cloſe to them, to give the Proportions of the Notes in a greater Exactneſs, than can be done upon the Baſs-Violin, or Violin itſelf: Since they may be ſet forth more perfectly by a Pair of Compaſſes dividing a Line, than the niceſt Ear can direct.

Though the Frets for the ſeveral Strings do not ſtand in a ſtrait Line, and the Places are alſo ſhifted in different Keys, yet the Ear naturally directs the Fingers to them; inſomuch that thoſe Perſons, who have all their Lives Time been accuſtom'd to ſtop upon Frets, that go quite croſs the Finger-boards of their Inſtruments, do with very little Practice fall right upon theſe. Such is the Power of a muſical Genius, as may be undeniably proved by thoſe that play upon the Violin; who, when they change the Key, fall upon the right Stops, though they have no viſible Direction where to ſtop, nor Time to alter, by the Ear, the Note they firſt pitched upon.

By this Standard of Regular Proportions may the Voice be formed to ſing the pureſt Notes; they are all the ſame in Vocal and Inſtrumental Muſic; if then the Inſtrument which governs the Voice be perfect, the Ear will of Neceſſity bring it to Perfection. It is pity that a good natural Voice ſhould be taught to ſing out of Tune, as it muſt do, if it be guided by an imperfect Inſtrument; and this may be the Reaſon why ſo few attain to that Melody, which is ſo much valued; but ſince we now know wherein Perfection lies, a conſtant Practice will come to the Attainment of it. The dividing Wholes into Chromatic Hemitones is very neceſſary, but very difficult for the Voice to be broken to: If it learns from an Inſtrument whoſe whole Notes, and whoſe half Notes are ſuppoſed to be equal, the Sound muſt needs be very uncertain and unharmonical; whereas the Proportions truly fixed, would bring it to a Perfection in the niceſt and moſt charming Part of Muſic.

The Chromatic Hemitones are the ſmalleſt Intervals our modern Muſic aims at, though the Ancients had their Enharmonic quarter Notes, which they eſteem'd their greateſt Excellency: Theſe may alſo in Time be recover'd, ſince we know their Proportions; for as the Diatonic Tone is divided into Chromatic Hemitones, ſo after the ſame manner may the Chromatic Hemitones be divided into thoſe leaſt Enharmonic Intervals, which were ever made Uſe of. But if we go no further, yet this Experiment demonſtrates the true Theory of Muſic, and brings the Practice of it to the greateſt Perfection.

Of the Ancient Greek *and* Roman Lyre; *and a* Paſſage *in* Horace *explain'd by Dr.* T. Molyneux, n. 282. p. 1267.

II. Reading over lately the *Third* Ode of the *Fourth* Book of *Horace*, which *Scaliger*, *Dacier*, and the reſt of the Critics and Commentators ſo much admire, I hit upon a Paſſage, which I think none of them (and I have examin'd the Chief) have clearly explain'd. The *Ode* begins:

Quem tu Melpomene, &c.

The Paſſage I ſpeak of is this;

O Teſtudinis Aureæ
Dulcem quæ ſtrepitum, Pieri, temperas!
O Mutis quoque Piſcibus
Donatura Cygni, ſi libeat, ſonum!

At firſt it ſeem'd to me a wild Rant, or extravagant Whim for *Horace*, ſo great a Judge and Maſter in the Art of Poetry, ſo particularly remark'd for his Propriety of Thought, in ſo labour'd and exquiſite a Poem, to ſay that his Muſe could give even to *Mute Fiſhes* the melodious Voice of the Swan; I look'd upon the Fancy as perfectly forc'd and groundleſs, founded upon nothing that was real or true in Nature: But upon a ſecond Conſideration, I fancied this might be the Meaning of the Paſſage;

That, after he had in the Verſes going before, acknowledged how much he was indebted to the Bounty of his Muſe, he here makes a ſudden Exclamation to extol her great Art and Myſtery, who by mixing various Notes, could compoſe ſuch ſweet Harmony on the *Gilded Lyre* or *Teſtudo*, and by her ſurprizing Power, could give even to *Mute Fiſhes*, or the hollow Shells of the *Teſtudines Aquaticæ* or *Water Tortoiſes*, (a Sort of Fiſh of which I imagin'd they made their *Lyres* in old Times) the ſweet Melody of the Swan. As for the Compariſon to the Voice of a *dying Swan*, though that be a Fiction, yet a Vulgar Error univerſally embrac'd, was ever ſufficient Authority for a Poet or Orator to draw from it a Compariſon or a Simile.

This put me upon ſearching for Matter of Fact, whether or no the ancient Lyre was made of the Shell of a Tortoiſe; and looking into ancient Authors, I find that it was a current Piece of Hiſtory generally received among the Ancients, that *Mercury* was the firſt Inventor of the *Lyre* (whence *Horace* in his 10th Ode of the 1ſt Book ſtyles him *Curvæ Lyræ Parentem*) and that he made it of the Shell of a dead *Tortoiſe*, which he accidentally found on the Banks of the River *Nile*. Out of many, I will produce two Teſtimonies to this Purpoſe;

Nicander, who wrote above 100 Years before *Horace*, in his *Alexipharmaca*, ſpeaking of Antidotes proper againſt the Poiſon of the *Salamander*, recommends both the *Sea* and the *Mountain Tortoiſe* in theſe Words,

Ἀμμίγδην ἁλίοιο καθεψηθέντα χεμώνης
Πυιοις, ἢ ταχινῇσι διαπλώῃ πτερύγεσσιν,
Ἄλλοτε δ' οὐρείης κυτισηνόμου, ἥν τ' ἀκάκητα
Αὐδήεσσαν ἔθηκεν ἀναύδητον πέρ ἐοῦσαν
Ἑρμείης, ζαρκὺς γὰρ ἀπενόσφισε χέλειον
Αἰόλον, ἀγκῶνας δὲ δύω παρετείνατο πέζαις.

Thus

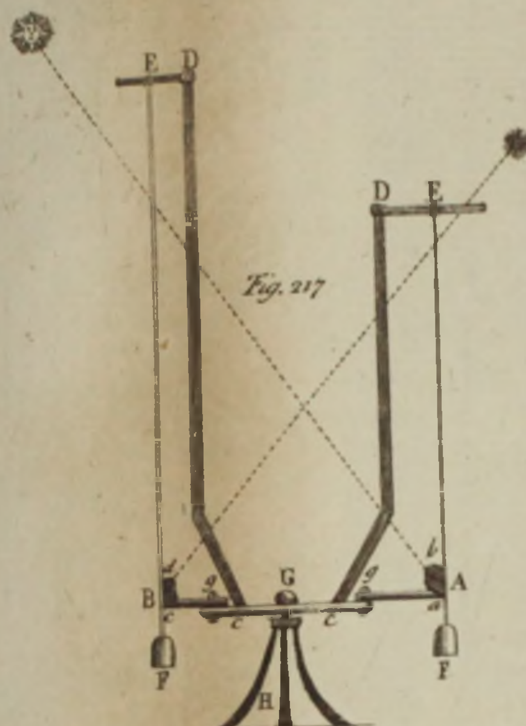


Fig. 217

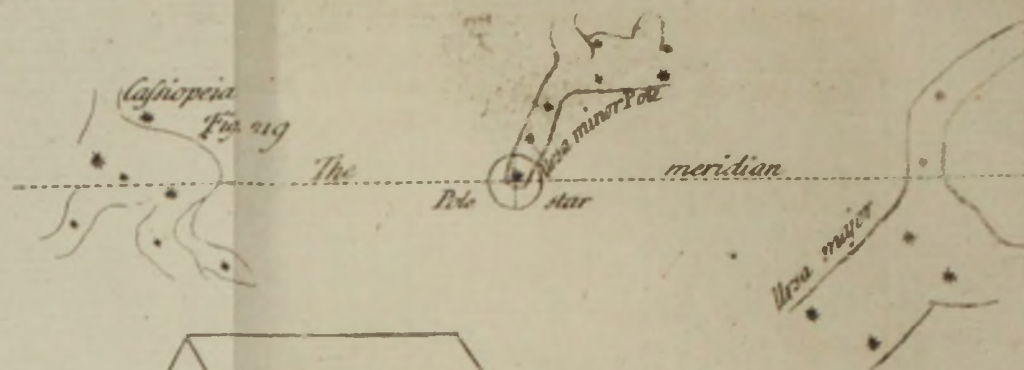


Fig. 219

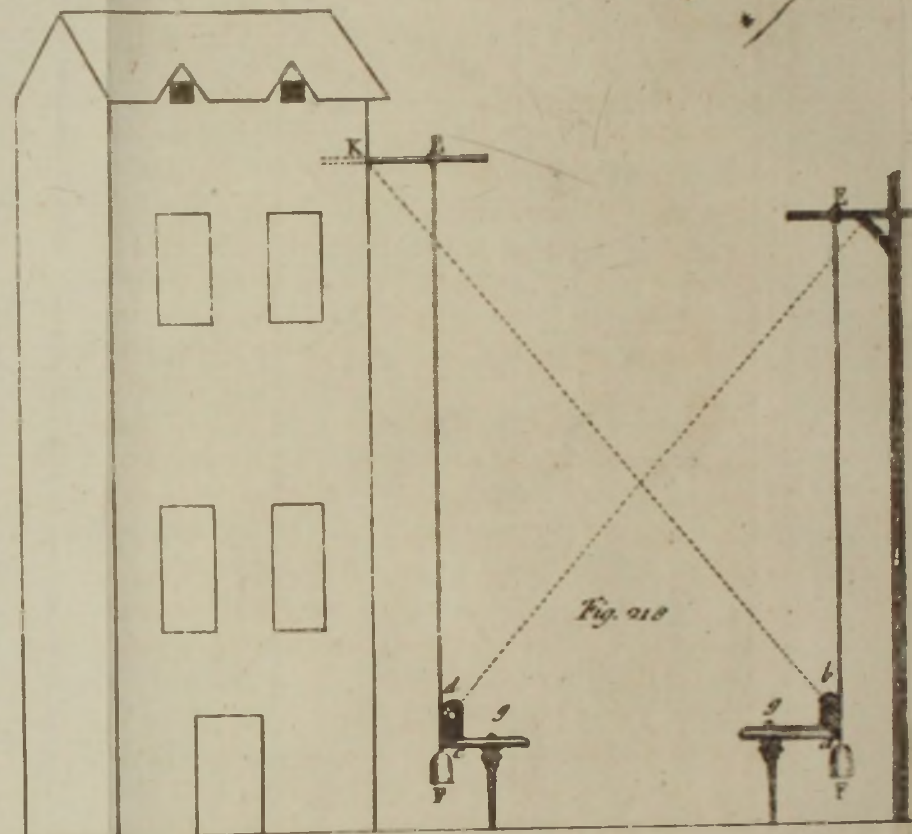


Fig. 218

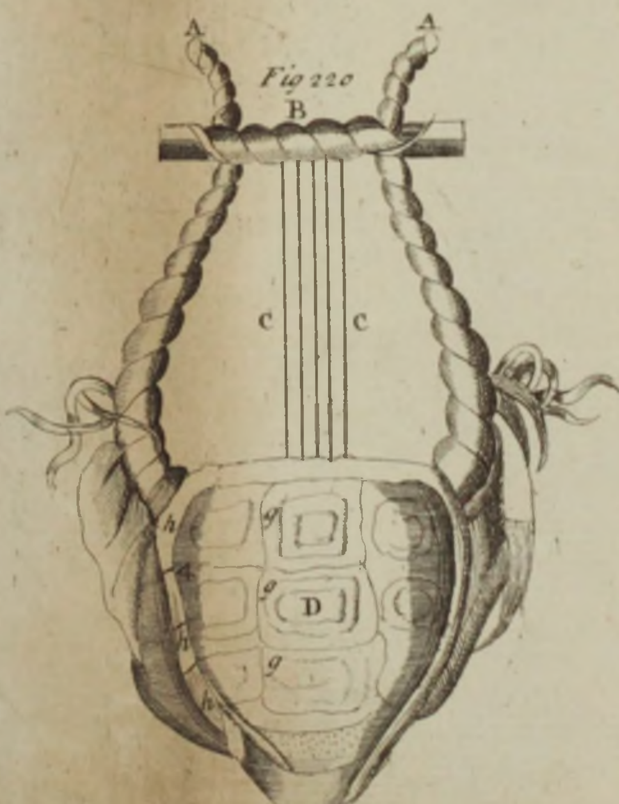


Fig. 220

Fig. 221

Thus turn'd by *Joannes Gorræus,*

Cum curvâ auxilio veniunt Testudine——
Quæ Pelagi fluctus velocibus innatat alis,
Aut montana etiam Cytiso quæ vescitur, & quam
Reddidit e muta modulanti voce canoram
Mercurius, picto insontis qui Cortice carnem
Exemit, geminumq; Ancona intendit in oris.

Grevinus in his Treatise *de Venenis* in the Chapter *de Salamandra*, pag. 119, gives us a Comment on these Verses, and relates at large the History of the first Lyre, but I cannot but take Notice that this Verse

Αὐδήεσσαν ἔθηκεν ἀναύδητόν περ ἐοῦσαν;

Reddidit e muta modulanti voce canoram——Is so home and apposite to our Purpose, and comes up so close to *Horace*'s Thought,

O mutis quoque Piscibus
Donatura Cygni si libeat sonum,

that it does not only explain the true Meaning of it, but makes me inclinable to believe, that *Horace* might have in his View this very Passage; which he seems also again to allude to (though not so fully and expresly) in his 11*th* Ode of the 3*d* Book, where he invokes his Lyre,

Tuque Testudo resonare septem
Callida nervis,
Nec Loquax olim neque grata——

The other Instance is from one of *Lucian*'s Dialogues, who writ above a hundred Years after *Horace*; whence 'tis plain the Mechanism of the ancient Lyre, and the Opinion concerning its first Invention, prevail'd since, as well as before, *Horace*'s Days. In this Dialogue he introduces *Apollo* and *Vulcan* talking after his jocose way of *Mercury* to this Purpose.

Απ. χελώνην πȣ νεκρὰν εὑρὼν, ὄργανον ἀπ' αὐτῆς συνεπήξατο, πήχεις γὰρ ἐναρμόσας, καὶ ζυγώσας, ἔπειτα καλάμους ἐμπήξας, καὶ μαγάδιον ὑποθεὶς, κατατεινάμενος ἑπτὰ χορδὰς, μελῳδεῖ πάνυ γλαφυρὸν, ὦ Ἥφαιστε, καὶ ἐναρμόνιον.

Ap. *Testudinem mortuam alicubi offendens Instrumentum ex ea concinnavit; Brachia enim adaptans Jugum opposuit, deinde Clavos infigens, & Hæmisphærium repandum infra subjiciens, septem Chordas extendebat, atq; modulabatur quiddam valde sonorum, O Vulcane, & ad Musicæ Melodiam compositum.*

Fig. 220. is taken from Father *Mersennus* (*Lib.* 1. *de* Instrumentis, *p.* 7.) which he tells us he copied from the Sculpture of an antique Gem, that belong'd to one *Jacobus Gaffarellus*. *A*, *A*, shew the πήχεις of *Lucian*, the Ἀγκῶνες or *Brachia* of *Nicander*, made of the Horns of some Beast. *B* the Ζυγὸς, or *Jugum*, in which were fastened the κάλαμοι *Clavi*, Pegs that raised or deprest *c*, *c*, the χορδαὶ, or Strings, which were fix'd at their t'other End to *D* the μαγάδιον *Hemisphærium* or Belly: This is very like *Figure* 221. which is an entire *Testudo Aquatica* or rather *Fluviatilis*, taken from *Johnstonus de Animalibus* as delineated in his *eightieth* Table *de Quadrupedibus*; making Allowances for their different Posture, one being represented full and flat, whilst only half of the other appears because 'tis shewn side-ways.

Fig. 220.

Fig. 221.

 The

Fig. 220. Fig. 221. The Belly of *Merſennus*'s Lyre mark'd *D*, agrees nicely in Figure and Shape with the Back or Shell of *Johnſtonus*'s *Teſtudo Aquatica*, mark'd *E*. They are both curiouſly teſſelated, checker'd into Areas or Scales *F*, *F*, *F*, *F*, *F*, *F*, of ſomewhat a Square Figure; and each of theſe Scales again in both is neatly wrought about their Edges with a Line running parallel to their Margins *g*, *g*, *g*, *g*, *g*, *g*; and the Shell of the Lyre, as that of the Tortoiſe, terminates in a narrow Limb or Verge, cut into ſmaller Scales *h*, *h*, *h*, *h*, *h*, *h* incompaſſing the whole: ſo that both theſe Figures, though drawn by different Artiſts, perhaps at two thouſand Years Diſtance, do manifeſtly own the Lineaments of the ſame natural Original.

Pauſanias too in his Deſcription of *Greece* (as I find it quoted by *Geſner*) mentions a Mountain in *Arcadia* called *Parthenius Mons*, *qui Teſtudines exhibet ad compingendas Lyras aptiſſimas*; and in another Place, *Arcadum Querceta ingenti magnitudine Teſtudines exhibent*, *ex quibus Lyras conficeres æquales illis quæ ex Indica Teſtudine componuntur*. From whence 'tis plain the Ancients made their *Lyres* of the Shells of *Tortoiſes*, perhaps not very curious in the Choice of their Materials, but might take promiſcuouſly the *Land* or *River Tortoiſe*, which occaſions *Pauſanias* and *Nicander* to mention the *Mountain*, whereas *Horace* ſpeaks of the *River Tortoiſe*.

And indeed moſt of the Inſtruments, *&c.* now in Uſe were at firſt rude, plain, and ſimple, tho' improv'd by Length of Time, and Fancy of Artificers: Thus the *Flute*, *Flagelet*, *Hautboy*, and *Organs*, are only Improvements of the *Tenues Avenæ* or *Oaten-Pipes* of the Field, or the *Calami impares Juncti* of the Ancients, *Reeds* of unequal Lengths rudely put together; thus their Trumpets were at firſt made only of rude *Horns* of Beaſts, and ſometimes of the common *Buccina Whelks* or large Sea-ſhells, hence *Virgil*,

Rauco ſtrepuerunt Cornua cantu. And *Perſius*,

Buccina jam priſcos cogebat ad arma Quirites.

And thus their Lyres were at firſt made of the Tortoiſe-Shell; tho' in After-Ages the Number of the Strings was encreaſed, and the Model alter'd; and the Inſtrument tho' improv'd, and very unlike its firſt Original, yet ſtill retain'd its Ancient Name.

This appears from thoſe other Schemes *Merſennus* gives in the ſame Table of ſeveral Sorts of the ancient Lyres (but theſe I take to be more Modern than that which is here expreſt) and from thoſe deſcrib'd by *Leonardo Agoſtini*, in the Second Part of his Collection of the *Gemme Antiche*, which ſhew us, that as the Fancy of the Workman, the Mode of the Times, real Convenience or an imaginary Beauty in the Inſtrument determin'd it, they were faſhion'd into various Shapes, and frequently like their Lamps of Old into capricious, fantaſtical odd Figures.

The End of the FIRST PART.

The Philosophical Transactions

ABRIDG'D.

PART II.

Containing the

PHYSIOLOGICAL PAPERS.

CHAP. I.

Physiology. Meteorology. Pneumatics.

0		I. THE Heat of Winter Air, when Water begins to freeze. This Heat is known by rightly placing the Thermometer in Snow pressed together, at what Time it begins to thaw.
0, 1, 2.		The Heat of Winter Air.
2, 3, 4.		The Heat of the Air in Spring and Autumn.
4, 5, 6.		The Heat of the Air in Summer.
6.		The Heat of the Air at Noon, about the Month of *July*.
12.	1	The greatest Heat that the Thermometer receives by the Contact of a Human Body. This Heat is much the same as that of a Bird sitting upon her Eggs.
$14\frac{3}{11}$	$1\frac{1}{4}$	The Heat of a Bath, which is almost the greatest that any one can endure long, with his Hand agitated and immersed in it. The same almost is the Heat of Blood just let out.
17	$1\frac{1}{2}$	The greatest Heat of a Bath that any one can endure long, his Hand being immersed and at rest in it.
$20\frac{2}{11}$	$1\frac{1}{4}$	The Heat of a Bath in which Wax swimming and melting, by moving about grows hard and loses its Transparency.
24	2	The Heat of a Bath in which Wax swimming grows liquid by the Heat, and is preserved in continual Flux without Ebullition.
$28\frac{6}{11}$	$2\frac{1}{4}$	The intermediate Heat between the Degrees in which the Wax melts and the Water boils.
34	$2\frac{1}{2}$	The Heat by which Water boils violently, and a Mixture of two Parts of Lead, of three Parts of Pewter, and

A Scale of the Degrees of Heat, by . . . n.270. p.824.

		and of five Parts of Bifmuth grows ftiff in cooling. Water begins to boil by a Heat of 33 Parts, and in boiling conceives a Heat of more than $34\frac{1}{2}$ Parts. But Iron with a Heat of 35 or 36 Parts ceafes to excite an Ebullition, when hot Water is dropt upon it; and of 37 Parts, when cold Water does the fame.
$40\frac{4}{11}$	$2\frac{1}{4}$	The leaft Heat by which a Mixture of one Part Lead, of four Parts Pewter, and of five Parts Bifmuth, grows hot and melts, and is preferved in a continual Flux.
48	3	The leaft Heat by which a Mixture of equal Parts of Pewter and Bifmuth melts. This Mixture cools and coagulates by a Heat of 47 Degrees.
57	$3\frac{1}{4}$	A Heat by which a Mixture of two Parts of Pewter, and one Part of Bifmuth is melted, as alfo a Mixture of three Parts of Pewter, and two Parts of Lead. But a Mixture of five Parts of Pewter, and of two Parts of Bifmuth, cools and grows ftiff with this Heat. And a Mixture of equal Parts of Lead and Bifmuth does the fame.
68	$3\frac{1}{2}$	The leaft Heat by which a Mixture of one Part of Bifmuth, and eight Parts of Pewter is melted. Pewter alone is melted with a Heat of 72 Parts, and cools and grows ftiff by a Heat of 70 Parts.
81	$3\frac{3}{4}$	The Heat by which Bifmuth is melted, as alfo a Mixture of four Parts of Lead, and one Part of Pewter. But a Mixture of five Parts of Lead, and one Part of Pewter, grows ftiff when melted, and cools in this Heat.
96	4	The leaft Heat by which Lead is melted. Lead grows hot and melts in a Heat of 96 or 97 Parts, and cools and grows ftiff in a Heat of 95 Parts.
114	$4\frac{1}{4}$	The Heat by which Bodies heated in the Fire by cooling quite leave off to fhine in the Darknefs of the Night, and again by growing hot begin to fhine in the fame Darknefs, but with a very faint Light which can hardly be perceived. In this Heat a Mixture of equal Parts of Pewter and Regulus Martis will melt; but a Mixture of feven Parts of Bifmuth, and four Parts of the fame Regulus Martis, will cool and grow ftiff.
136	$4\frac{1}{2}$	The Heat by which Bodies heated in the Fire grow red hot, but not fo in the Twilight. By this Heat a Mixture of two Parts of Regulus Martis, and of one Part of Bifmuth, as alfo a Mixture of five Parts of Regulus Martis, and one Part of Pewter, by cooling grows ftiff. The Regulus by itfelf grows ftiff with a Heat of 146 Degrees.

161	4¼	The Heat by which Bodies heated in the Fire plainly grow red hot in the Twilight, juſt before the Riſing or Setting of the Sun, but not ſo in open Day-light, or but very obſcurely.
192	5	The Heat of burning Coals in a ſmall Kitchen Fire, made of bituminous foſſile Coals, and without blowing with Bellows. The ſame is the Heat of Iron in ſuch a Fire, that grows red hot as much as it can. The Heat of a ſmall Culinary Fire made of Wood is ſomething greater, perhaps of 200 or 210 Degrees. But the Heat of a large Fire is ſomething greater ſtill, eſpecially if provoked by the Uſe of Bellows.

In the firſt Column of this Table we have the Degrees of Heat in Arithmetical Progreſſion, beginning the Computation from that Degree in which Water begins to freeze, as it were from the loweſt Degree of Heat, or the common Limit of Heat and Cold, and making the external Heat of a Human Body to be 12 Degrees. In the ſecond Column are had the Degrees of Heat in Geometrical Progreſſion, ſo that the ſecond Degree is as great again as the firſt, the third as great again as the ſecond, and ſo on; and the firſt is the external Heat of the Body of a Man adequate to Senſe. Now it appears from this Table, that the Heat of boiling Water is almoſt three Times greater than the Heat of the Human Body, and that the Heat of melted Pewter is ſix Times greater, and the Heat of melted Lead is eight Times greater, and the Heat of melted Regulus is twelve Times greater, and that the ordinary Heat of a Culinary Fire is 16 or 17 Times greater, than the ſame Heat of a Human Body.

This Table was conſtructed by the help of a Thermometer and a piece of red hot Iron. By the Thermometer I found the Meaſure of all the Degrees of Heat, till I came to the Heat with which Pewter is melted, and by the red hot Iron I found the Meaſure of the reſt. For the Heat which red hot Iron communicates to cold Bodies which are contiguous to it, in a given time, that is, the Heat which the Iron loſes in a given time, is as the whole Heat of the Iron. Therefore if the Times of cooling are taken equal, the Heats will be in a Geometrical Ratio, and therefore are eaſily found by a Table of Logarithms.

Therefore firſt I found, by a Thermometer conſtructed with Linſeed Oyl, that when the Thermometer was put into melting Snow, the Oyl took up a Space of 10000 Parts. The ſame Oyl rarified by a Heat of the firſt Degree, or by that of a human Body, took up the Space 10256; and by the Heat of Water juſt beginning to boil, it took up the Space 10705, and by the Heat of Water boiling vehemently it took up the Space 10725, and by the Heat of melted Pewter cooling, when it began to be ſtiff and put on the Conſiſtence of an Amalgama, it took up the Space 11516, and the Space 11496 when it was quite ſtiff. Therefore the rarified Oyl was to the dilated in the Ratio

Ratio of 40 to 39, by the Heat of the human Body; in the Ratio of 15 to 14 by the Heat of boiling Water; in the Ratio of 15 to 13 by the Heat of cooling Pewter, when it began to grow ſtiff and coagulate; and in the Ratio of 23 to 20 by the Heat by which cooling Pewter grows quite ſtiff. The Rarefaction of Air with equal Heat was ten times greater than the Rarefaction of Oyl, and the Rarefaction of Oyl was about 15 times greater than the Rarefaction of Spirit of Wine. And from what is here found, by ſuppoſing the Heat of the Oyl proportional to its Rarefaction, and for the Heat of the human Body writing 12 Degrees, the Heat of Water when it begins to boil will come out 33 Degrees, and when it boils vehemently 34 Degrees; and the Heat of Pewter either when it melts, or when it begins to cool and becomes of the Conſiſtency of an Amalgama, will be of 72 Degrees, and when it cools and grows hard, of 70 Degrees.

Theſe things being known, that I might find the reſt, I heated a thick piece of Iron till it was red hot, and taking it out of the Fire with a hot pair of Pincers, I immediately put it in a cold Place, where the Wind blew conſtantly; and putting upon it little Particles of different Kinds of Metals, and other Bodies that would melt, I obſerved the Times of Cooling, till all the Particles grow ſtiff and loſt their Fluidity, and the Heat of the Iron was equal to the Heat of the human Body. Then ſuppoſing that the Exceſſes of the Heat of the Iron and the rigid Particles above the Heat of the Atmoſphere found by the Thermometer, are in Geometrical Progreſſion when the Times are in Arithmetical Progreſſion, all the Degrees of Heat became known. I placed the Iron in a Wind blowing uniformly, and not in a quiet Air, that the Air heated by the Iron might always be carry'd away by the Wind, and the cool Air might ſucceed in its Place with an uniform Motion. For thus equal Parts of the Air would be made hot in equal Times, and would conceive a Heat proportional to the Heat of the Iron.

Now the Heats thus found will have the ſame Proportion to one another with the Heats found by the Thermometer, and therefore we have rightly aſſumed, that the Rarefactions of the Oyl are proportional to its Heat.

An Account of Dr. Hook's *Marine Barometer, by Mr.* E. Halley, n. 269. p. 791.

II. Dr. *Hook*, who has made many Attempts to improve the *Barometer*, and to render the minute Diviſions on the Scale thereof more ſenſible, judging that it might be of great Uſe at Sea, contrived ſeveral Ways to make it ſerviceable on Board a Ship; one of which he explained to the *Royal Society* at their weekly Meeting in *Greſham-College*, *January* 2. 166$\frac{7}{8}$. ſince which Time he hath further cultivated the Invention, and ſome Years ago produced before the ſaid *Society*, the Inſtrument I am now to deſcribe.

The Mercurial Barometer requiring a perpendicular Poſture, and the Quickſilver vibrating therein with great Violence upon any Agitation, is therefore uncapable of being uſed at Sea, (though it hath lately

lately been contrived to be made portable.) So it remained to find out ſome other Principle, wherein the Poſition of the Inſtrument was not ſo indiſpenſably neceſſary : For this, all thoſe that uſe the Sea are obliged to the great Facility Dr. *Hook* has always ſhewn, in applying philoſophical Experiments to their proper Uſes.

It is about 40 Years ſince, that the *Thermometers* of *Robt. de Fluctibus*, depending on the Dilatation and Contraction of included Air by Heat and Cold, have been diſuſed, upon Diſcovery that the Air's Preſſure is unequal ; that Inequality mixing itſelf with the Effects of the Warmth of the Air in that Inſtrument. And inſtead thereof was ſubſtituted the ſeal'd Thermometer, including Spirit of Wine (firſt brought into *England* out of *Italy* by Sir *Robert Southwell)* as a proper Standard of the Temper of the Air in relation to Heat and Cold ; that æthereal Spirit being of all the known Liquors the moſt ſuſceptible of Dilatation and Contraction, eſpecially with a moderate Degree of either Heat or Cold. Now this being allowed as a Standard, and the other Thermometer that includes Air being graduated with the ſame Diviſions, ſo as at the Time when the Air was included, to agree with the Spirit-Thermometer in all the Degrees of Heat and Cold, noting at the ſame Time the preciſe Height of the Mercury in the common Barometers : It will readily be underſtood, that whenſoever theſe two Thermometers ſhall agree, the Preſſure of the Air is the ſame it was, when the Air was included and the Inſtrument graduated : That if in the Air-Thermometer the Liquor ſtand higher than the Diviſion marked thereon, correſponding with that on the Spirit-Glaſs, it is an Indication that there is a greater Preſſure of the Air at that Time, than when the Inſtrument was graduated. And the contrary is to be concluded, when the Air-Glaſs ſtands lower than the Spirit, *viz.* that then the Air is ſo much lighter, and the Quickſilver in the ordinary Barometers lower than at the ſame Time of Graduation.

And the Spaces anſwering to an Inch of Mercury will be more or leſs, according to the Quantity of Air ſo included, and the Smallneſs of the Glaſs Cane, in which the Liquor riſes and falls, and may be augmented almoſt in any Proportion, under that of the ſpecific Gravity of the Liquor of the Thermometer to Mercury. So as to have a Foot or more for an Inch of Mercury, which is another great Convenience.

It has been obſerved by ſome, that in long keeping this Inſtrument, the Air included either finds a Means to eſcape, or depoſits ſome Vapours mix'd with it, or elſe for ſome other Cauſe becomes leſs elaſtic, whereby in Proceſs of Time it gives the Height of the Mercury ſomewhat greater than it ought ; but this, if it ſhould happen in ſome of them, hinders not the Uſefulneſs thereof, for that it may at any Time very eaſily be corrected by Experiment, and the riſing and falling thereof are the Things chiefly remarkable in it, the juſt Height being barely a Curioſity.

In theſe Parts of the World, long Experience has told us, that the riſing of the Mercury forbodes fair Weather after foul, and an Eaſterly or Northerly Wind, and that the falling thereof, on the contrary, ſignifies Southerly or Weſterly Winds, with Rain, or ſtormy Winds, or both; which latter is of much more Conſequence to provide againſt at Sea than at Land; and in a Storm, the Mercury beginning to riſe, is a ſure Sign that it begins to abate, as has been experienced in high Latitudes both to the Northwards and Southwards of the Equator.

Fig. 1. *A B* repreſents the Spirit-Thermometer, graduated from 0, or the freezing Point, through all the poſſible Degrees of the Heat or Cold of the Air, at leaſt in theſe Climates.

C D is the Air-Thermometer, graduated after the ſame Manner, with the like Degrees.

E F is a Plate applied to the Side of the Thermometer *C D*, graduated into Spaces anſwering to Inches and Parts of an Inch of Mercury, in the common Barometers.

G, a Hand ſtanding on the Plate at the Height of the Mercury thereon, as it was when the Inſtrument was graduated, as ſuppoſe here at $29\frac{1}{2}$ Inches.

L M a Wire on which the Plate *E F* ſlips up and down, parallel to the Cane of the Thermometer *C D*.

K, any Point at which the Spirit ſtands at the Time of Obſervation; ſuppoſe at 38 on the Spirit-Thermometer; ſlide the Plate *E F* till the Hand *G* ſtand at 38 on the Air-Thermometer, and if the Liquor therein ſtand at 38 likewiſe, then is the Preſſure of the Air the ſame as at the Time of Graduation; *viz.* 29, 5; but if it ſtand higher, as at 30 at I, then is the Preſſure of the Air greater; and the Diviſion on the ſliding Plate againſt the Liquor, ſhews the preſent Height of the Mercury to be 29 Inches 7 Tenths.

I had one of theſe Barometers with me in my late Southern Voyage, and it never failed to prognoſtick and give early Notice of all the bad Weather we had.

A New *Baroſcope, by Mr.* Caſwell. n. 290. p. 1597. Fig. 2.

III. I have made a new Sort of Baroſcope, 'tis cheap and very exact, I here ſend you its Calculation as it occurr'd to my Thoughts before I made it. Suppoſe *A B C D* is a Bucket of Water, in it the Baroſcope *x r e z y o s m*, which conſiſts of a Body *x r s m*, and a Tube *e z y o*, the Body and Tube are both concave Cylinders communicating with each other, and made of Tin (for Want of Glaſs:) The Bottom of the Tube *z y* has a Lead-weight to ſink it, ſo that the Top of the Body may juſt ſwim even with the Surface of the Water by the Addition of ſome Grain-Weights on the Top. The Water when the Inſtrument is forced with its Mouth downwards gets up into the Tube to the Height *y u*. There is added on the Top a ſmall concave Cylinder, which I call the Pipe, to diſtinguiſh it from the Bottom ſmall Cylinder, which I call the Tube: This Pipe is to ſuſtain the Inſtrument from ſinking to the Bottom, md is a Wire, mS, dE are two Threads oblique to the Surface of the Water, which Threads perform the Office of Diagonals: For that while the Inſtrument ſinks more or leſs by the Alteration of the Gravity of the Air, there, where the Surface of the Water cuts the Thread, is form'd a ſmall Bubble, which Bubble aſcends up the Thread, while the ☿ of the common Baroſcope aſcends.

The Circumference of the Body is 21 Inches, therefore its Area $= 35$: the Altitude $ms = 4$, therefore the Body's Solidity $= 140$, each Baſe xm, rs, has a Convexity whoſe Altitude is 6.5, therefore the Conoid on each Baſe is nearly $= 11\frac{1}{2}$, therefore d the whole Body is $= (140 + 11\frac{1}{2} + 11\frac{1}{2} =)$ 163, and b the entire Altitude of the Body $= (4 + .65 + .65 =)$ 5.3. The inner Circumference of the Tube is 5.014, therefore its Area $n = 2$ the Length of the Tube $= 45$, therefore the Tube's Capacity $= 9$, therefore C, the Content of the Body and Tube $= 163 + 9 = 172$ Cubic Inches, that is almoſt $2\frac{1}{2}$ Quarts.

Suppoſe the Air's Preſſure when greateſt $= 30.5$ Inches of ☿ $=$ $(30.5 \times 14 =)$ 427 of Water, and $f = 427$, therefore $fc = 73444$. Put a for the Depth ou, of the Air in the Tube when the Body is juſt all immerſed, the Air in the Inſtrument on Immerſion contracts ſomewhat by the Cold of the Water; this Contraction I find is nearly as much as would be produc'd by an Addition of 1 Inch to the Atmoſphere's Altitude 427; this in cold Weather, but in warm Weather 'tis probably twice as much: But we will now ſuppoſe it $= 1$, therefore the Depth of the Surface of the Water in the Tube below the Surface of the outer Water is $= b + a$, therefore the Preſſure on that inner Surface, is as the Altitude of the Atmoſphere above it $= f + b + 1 + a = F - a$ (putting $F = f + b + 1$.) Then for that the Spaces into which the Air is contracted, are reciprocal to their reſpective Preſſures, and for that while the Inſtrument is out of the Water, the Preſſure f anſwered to the Space C, therefore,

$F + a : f :: C : \dfrac{fc}{F + a} =$ Space which the Air takes up in the Inſtrument under Water; therefore, $\dfrac{fc}{F + a} - d =$ that Part of the Tube which is poſſeſſed by Air $= an$ (ſuppoſing the Tube's Area $2 = n$).

Therefore $fc - Fd - ad = Fan + aan$. Therefore $aa + F + \frac{d}{n}$

$\times a = \frac{fc - Fd}{n}$. Put $F + \frac{d}{n} = 2g$, therefore $aa + 2ga$, $= \frac{fc - Fd}{n}$ therefore $a = \sqrt{:\frac{fc - Fd}{n} + gg} - g$.

Then ſuppoſe the Atmoſphere's Gravity leſs, ſo much as to ſink the ☿ $\frac{1}{10}$ Inch = 1.4 of Water, and therefore putting $\phi = F - 1.4$, and in the laſt Equation α inſtead of a, and γ inſtead of g, you have $\alpha = \sqrt{\frac{fc - \phi d}{n} + \gamma\gamma} : - \gamma$. Thus I find $\left.\begin{array}{l} a = 2.72 \\ \alpha = 2.94 \end{array}\right\}$ and therefore $\alpha - a = .22$, which $.22 \times n$ gives .44 Cubic Inches, and (ſuppoſing a Cube-Inch = 253 Grains) $.44 \times 253 = 111$ Grains Weight of Water that was gotten up into the Tube in the firſt Caſe more than in the ſecond, and therefore the Baroſcope requires an Addition of 111 Grains on its Top to ſink it with the Level of the Water in the ſecond Caſe more than in the firſt, and this upon the ſinking of the ☿ in the common Baroſcope only $\frac{1}{10}$ Inch; now 1 Grain in this new Baroſcope is nearly as diſcernable as $\frac{1}{10}$ Inch in the common, and therefore this new Baroſcope is more exact than the common 111 Times.

Put $f = 247$.$c = 172$.$d = 163$.$n = 2$ as above, only change F, put F, put F 437.3, that is, ſuppoſe the Body ſunk in Water 4 Inches lower; in this Caſe $\alpha = 208$, therefore $a - \alpha = .64$, which multiplied into $\phi\, n = 1.28$ Cubic-Inches, which $\times$ 253 gives 324 Grains, and ſo much the Body's Top $x\,m$ being ſunk 4 Inches under Water, the Body becomes heavier, than while $x\,m$ was at the Surface of the Water. Therefore this 1.28 divided by the aforeſaid Depth 4 gives .32 the Area of the Top-Pipe, ſuch as would ballance or buoy up the Body at any Depth. Strictly ſpeaking, the Pipe ſhould be gradually bigger upward in order to ſuſtain the Inſtrument at any Depth, but as to Senſe it is cylindrical, and its Circumference = 2.005. But for that the leaſt Alteration of the Air would make the Body's Top $x\,m$ in that Caſe paſs through the 4 Inches (which 4 Inches I ſuppoſe, all the Variety of Depth that the Inſtrument has room given it in the Bucket to aſcend or deſcend) therefore the Pipe is made a ſmall Matter bigger, *(viz.)* its Circumference is 2.14; whereby the Pipe, according as the Body ſinks more, gives more Reſiſtance to the deſcending Body. The Pipe's Area is .3643; therefore the Capacity of the Pipe in 4 Inches Altitude is = 1.457. But as abovesaid to give juſtly no Reſiſtance, its Capacity ſhould be 1.28. Therefore this 1.28 taken from 1.457, leaves .177 the actual Reſiſtance in 4 Inches Depth, *viz.* $(.177 \times 253 =)$ 44 Grains.

But

But this Resistance will not be the same in all Weathers, in order therefore to calculate what it will be when the ☿ of the common Baroscope is very low: For Example, but 28 Inches high = 392 of Water; f must be supposed = 392, therefore $F = f + b + 1 = 398.3$, and the rest as before; *viz.* $d = 163$, $fc = 67424$. $Fd = 649229$. thence by the aforesaid Equation $\left.\begin{array}{l} a = 2.59 \\ \alpha = 2.84 \end{array}\right\}$ therefore $\alpha - a = .25$, which $\times\ n$ gives .50 Cubic Inches, which × 253 = 126 Grains. So that this Baroscope when the ☿ is lowest, is more exact than the common 126 Times, supposing the Body immersed afresh when the ☿ is so low.

Next while the ☿ is so very low, suppose the Top of the Body depress'd 4 Inches under Water; therefore $\varphi = F + 4 = 402.3$, the rest are as before, *viz.* $fc = 67424$, then α will be 19: but before, while the Top of the Body was at the Surface, α was 2.59; therefore the Difference 69 × Tube's Area 2, gives 1.38 Cube-Inches, which × 253 gives 349 Grains, and so much the Baroscope is heavier, when the Top $x\,m$ is 4 Inches under Water; or which comes to the same, supposing that ☿ at 28, and $x\,m$ at the Surface; this Baroscope by the ☿'s ascending $\frac{4}{14}$ Inch will become heavier 349 Grains. The Pipe's Capacity in 4 Inches Altitude was 1.457, from which take the abovesaid 1.38, the Residue = .077, which × 253 gives 19 Grains in 4 Inches; so that the Pipe will sustain the Baroscope, and also 44 when the ☿ is $30\frac{1}{2}$ high, and but 19 Grains when the ☿ is 28 high. The fewer Grains Difference there are in its sinking, through 4 Inches, the more nice the Baroscope will be.

There where the Thread cuts the Surface of the Water, is form'd a Bubble, therefore this Bubble while the Instrument sinks in Water 4 Inches, which is all the room that I give it, the Bubble moves on the 2 Diagonal Threads 20 Inches, it follows therefore that 120 Grains Difference would make the Bubble walk over 120 Inches, if the Threads were so long, but as it has been above calculated, about 120 Grains Difference of Weight of the Instrument is produc'd by so much of the Alteration of the Air, as would make the ☿ of the common Baroscope $\frac{1}{10}$ Inch; therefore when the ☿ ascends $\frac{1}{10}$ Inch, the Bubble of this new Baroscope ascends 120 Inches; therefore this new Baroscope is more exact than the common Baroscope by about 1200 Times.

1. While the ☿ of the common Baroscope is often known to be stationary 24 Hours together, the Bubble of the new Baroscope is rarely found to stand still one Minute. *Observations made with this Baroscope.*

2. Suppose the Air's Gravity encreasing, and accordingly the Bubble ascending, during the Time that it ascends 20 Inches, it will have many short Descents, of the Quantisy of $\frac{1}{2}$ Inch 1, 2, 3, or more Inches, each of which being over it will ascend again. These Retrocessions are frequent, and of all Varieties in Quantity and Duration, so that there is no judging of the general Course of the Bubble by bare Inspection, though you see it moving but by waiting a little Time.

3. A

3. A small Blast of Wind will make the Bubble descend; a Blast that cannot be heard in a Chamber of the Town, will sensibly force the Bubble downward. The Blasts of Wind sensible abroad cause many of the abovesaid Retrocessions, or Accelerations in the general Course; as I found by carrying my Baroscope to a Place where the Wind was perceptible.

4. Clouds make the Bubble descend. A small Cloud approaching to the Zenith, works more than a great Cloud near the Horizon. In cloudy Weather the Bubble descending, a Break of the Clouds (or clear Place) approaching to the Zenith, has made the Bubble to ascend; and after that Break had passed beyond the Zenith a considerable Space, the Bubble again descended.

5. All Clouds (except one) hitherto by me observed, have made the Bubble to descend. But the other Day the Wind being *North*, and the Course of the Bubble descending, I saw to the Windward a large thick Cloud near the Horizon, and the Bubble still descended, but as this Cloud drew near the Zenith, it turned the Way of the Bubble, making it to ascend, and the Bubble continued ascending till the Cloud was all passed, after which it resumed its former Descent. It was a Cloud that yielded a cold Shower of small Hail.

An Account of a New Thermometer, *by Mr.*Geoffroy. n. 274. p. 961, 962.

IV. This New Thermometer is compos'd of a Bowl or Bottle of Glass, *A*, which has no other opening, but by a little Tunnel at the End; and which descends almost to the Bottom of the Bowl. This Tunnel is open at both Ends *B C*. *B* dips into the Liquor *E*, which is at the Bottom of the Bowl.

The Space of the Bottle of Glass is fill'd with Air, which has no Communication with the exterior Air.

Fig. 3.

When the Air contain'd in this Space is rarified by the exterior Air which touches the Bottle, it presses at the same Time the Liquor *E*, and obliges it to rise by *B* in the Tunnel *B C*. On the contrary, when it condenses by the exterior Cold, by not pressing the Liquor *E*, it permits that which is in the Tunnel to fall.

The Readiness with which the Air condenses or rarifies by Cold and Heat, makes the Effects of this Thermometer much more sudden than those of any other sort: Besides, the Effects of this is much greater, the Air being capable of a greater Rarefaction, or of a greater Condensation, than any other Liquor.

The Cause *of the* Variation *of the* Barometer, *&c. by Mr.* J. T. Desaguliers. n. 351. p. 570.

V. *Remarks on the* Second *Paper in the* History *of the* Royal Academy *of* Sciences, *for the Year* 1711, *concerning the* Cause *of the* Variation *of the* Barometer: *Shewing that the Way of accounting for it in that Paper is insufficient, and that the Experiment made use of to prove what is there asserted, does no Way prove it.*

The Paper is as follows.] " It appears by the Barometer, that when it " rains, or a little before the Rain, the Air commonly becomes lighter.

" That

" That it must rain when the Air becomes lighter is easy to imagine; " for the imperceivable Particles of Water, that swim about in the " Air in prodigious Quantity, not being sufficiently sustain'd when the " Air has lost a certain Degree of its Weight, begin to fall, and several " of them joining together in the Fall make Drops of Rain. So when " about half of the Air is drawn out of the Recipient of the Air-Pump, " (and consequently the remaining Air is as weak again as at first) some- " thing like a small Rain falls. But why should the Air become lighter? " One might imagine that in the Place where it rains, it may have lost " some of its Weight and Bulk, by Means of the Winds carrying away " some Part of it: But Monsieur *Leibnitz*, in a Letter to the Abbot " *Bignon*, gives a more ingenious and more new Reason for it.

" He pretends that a Body, which is in a Liquid, weighs with that " Liquid, and makes up Part of its whole Weight, so long as it is " sustained in it; but if it ceases to be sustained, and consequently falls, " its Weight no longer makes a Part of the Weight of the Liquid, which " thereby comes to weigh less. This may naturally be applied to the " above-mentioned Particles of Water; they encrease the Weight of " the Air when it sustains them, which is diminished when it lets them " fall: And as it may often happen that the Particles of Water that " are highest, fall a considerable Time before they join with those that " are low, the Gravity of the Air diminishes before it rains, and the " Barometer shews it.

" This new Principle of Monsieur *Leibnitz* is surprizing. For must " not a strange Body, whether sustained in a Liquid or not, always " weigh? Can it gravitate upon any other Bottom than that which " sustains the whole Liquor? Does that Bottom cease to carry a " strange Body, because it falls? And is not that Body all the while it " is falling, part of the said Liquid as to the Weight? At that rate, whilst " a Chymical Precipitation is made, the whole Matter ought to weigh " less, which has never been observed, and scarce appears credible.

" Notwithstanding these Objections the Principle holds good, when " more closely examin'd. What sustains a heavy Body is press'd by it. " A Table, for example, which sustains a Pound Weight of Iron, is press'd " by it, and is so only because it sustains the whole Action and Effect of " the Cause of Gravity, (whatever it be) to push that Lump of Iron lower. " If the Table should yield to the Action of that Cause of the Weight " (or Gravity) it would not be press'd, and therefore would carry no- " thing. After the same Manner, the Bottom of a Vessel, which contains " a Liquid, opposes itself to all the Action of the Cause of Gravity against " the said Liquid: If a strange Body swims in it, the Bottom opposes itself " also to the said Action against that Body, which, being *in Æquilibrio* with " the Liquid, is in that respect really a Part of it. Thus the Bottom is " press'd both by the Liquid and the strange Body, and sustains them both. " But if the Body falls, it yields to the Action of Gravity, and conse- " quently the Bottom does no longer sustain it; neither will it sustain

" it,

" it, till the said Body is come down to the Bottom. Therefore during the whole Time of the Fall, the Bottom is eased of the Weight of that Body, which is no longer sustained by any Thing, but push'd down by the Cause of Gravity, to which nothing hinders it from yielding.

" Monsieur *Leibnitz* to confirm his Notion, proposed an Experiment. He says, that two Bodies must be tied to the two Ends of a Thread, the one heavier, and the other lighter than Water, yet such as both together may swim in Water: Put them into a Tube full of Water, the Tube being tied to one End of the Beam of a Ballance whose other End has a counterpoising Weight: Then if we cut the Thread which ties the Bodies together (that are of unequal Weight) so that the heaviest may presently descend, he says, that in such a Case the Tube would be no longer *in Æquilibrio*, but its counterpoising Weight would preponderate, because the Bottom of the Tube would be less press'd. It is plain, that the Tube must be sufficiently long, that the falling Body may not reach the Bottom before the Tube has Time to rise. In Chymical Precipitations, the Vessels are either too short, or what is precipitated falls sometimes too fast and sometimes too slow; for then the little Bodies are always (as to Sense) *in Æquilibrio* with the Liquor that contains them.

" Monsieur *Ramazzini*, the famous Professor at *Padua*, to whom Monsieur *Leibnitz* had proposed his Experiment, has made it with Success, after some fruitless Trials. Monsieur *Reaumur* (to whom the Academy had recommended it) has also made it with Success.

Fig. 4. *Remarks upon Monsieur* Leibnitz's *New Principle.*] Let *A B* be the Bottom of a Vessel full of any Fluid, whose Top is either wider than the Bottom as *G H*, narrower as *E F*, or equal to it as *C D*. The Pressure of the Fluid upon the Base *A B* will be equal to the Weight of *C B*, or of a Cylinder or Prism of the same Fluid, made up of the Area of the Base multiplied into the perpendicular Height above it.

If the Fluid be equally dense every way as Water, or of a Density uniformly diminish'd as you go upwards, this Proposition (called by Mr. *Boyle* the Hydrostatical Paradox) will hold good. This is demonstrated by all Hydrostatical Writers.

Fig. 5. Let *E F* represent Part of the Surface of the Earth, and *G E F H* a Pillar of the Atmosphere, whose Height is *G E* the whole Height of the Air. Let us imagine the Vapours rising out of the Earth to form themselves into two Clouds *A* and *B*, and to settle in that Place where the Air is of the same specific Gravity with themselves. It is evident that they will cause the Air to rise so much higher as their Bulk amounts to, and will therefore make the Surface which was at *G H* to rise up to *I K*, so that the Bottom *E F* which was press'd by a Pillar of Air as *G E F H*, is now press'd by an higher Pillar as *I E F K*. Now if the Clouds *A*, *B*, by any Cause soever, change their Place, so as to come downwards, (for *Example* to *C*, *D*) the

Height

Height of the Pillar $IEFK$ will remain the ſame as it was, and therefore the Bottom EF will be preſs'd as before by the foregoing Propoſition.

Corol. 1.] If the Clouds AB deſcend, and in their Deſcent keep the ſame Bulk as they had before, the Surface IK will remain the ſame, and therefore EF will be preſs'd as before.

Corol. 2.] Whether a Body be ſpecifically lighter or ſpecifically heavier than a Fluid; ſo long as it is detain'd in it, it will add to the Fluid as much Weight as the Weight of an equal Bulk of that Fluid: Wherefore a Body does not loſe all that Weight which it added to the whole Weight of the Fluid, when it ceaſes to be ſuſtain'd in the ſaid Fluid; contrary to Monſieur *Leibnitz*'s Principle.

Schol.] If a Cloud (by any Cauſe whatſoever) becomes ſpecifically heavier than that Part of the Air in which it ſwims, the Exceſs of its Gravity above an equal Bulk of Air will make it deſcend, and accelerate its Motion downwards; and then indeed it will loſe of its Weight by the Reſiſtance of the *Medium*, till it comes to an uniform (or ſenſibly uniform) Motion: but all the Weight that it will loſe will only be the Exceſs of its Gravity above that of the Air; for with the reſt of its Weight it will ſtill make up Part of the Weight of the Air.

Experiment 1.] Having with a Weight in the Scale C of the Balance AB counterpois'd the long Glaſs of Water EI, with a Horſe-Hair I let down the leaden Weight W into the Water, which from FG aroſe up to EH; and therefore the Water became heavier by the Weight of a Bulk of Water equal to the Lead. Having with another Weight in C made up the Counterpoiſe to the whole, with fine Sciſſars I cut the Thread of the Plummet; and all the while the Plummet was falling, the Water deſcended rather than roſe; and when the Lead was at the Bottom the Water overpois'd, becauſe it had then added to it all the Exceſs of Weight of the Lead above an equal Bulk of Water, which by Experiment is about $\frac{10}{11}$ of its Weight. Had Meſſieurs *Reaumur* and *Ramazzini* tried the Experiment thus, the Succeſs had been the ſame; but M. *Ramazzini* (as I underſtood from a Gentleman who was preſent) tried it in the following Manner, as I have ſince done. Fig. 6.

Experiment 2.] Making uſe of the abovementioned Machine, after I had balanc'd the Water and Lead in it, I fix'd to the End of the Beam B the Thread of the Plummet, which in the former Experiment I held in my Hand. This added to the Weight hanging at B, and oblig'd me to put into the other Scale a Weight equal to $\frac{10}{11}$ of the Lead, to recover the *Æquilibrium*. Then cutting the Thread or Hair, the Scale with the Weights overpois'd whilſt the Lead was falling; but the *Æquilibrium* was reſtor'd when it came to the Bottom. So that the Lead even then muſt have loſt only its Exceſs of Weight above Water. Fig. 7.

Experiment 3.] I tried the Way propoſed by Monſieur *Leibnitz* in the following Manner.

Fig. 8. I took a Cork *C* weighing an Ounce, and ſomething more than four times lighter than an equal Bulk of Water, and a Ball of Antimony *W* about four times ſpecifically heavier than Water, and of four Ounces Weight. The Cork laid upon the Water in the Veſſel *E A B D* rais'd the Water from *S S* to *G G*, and added an Ounce to the Weight of the whole Water: then ſuſpending the Ball of Antimony by a String, and letting it hang in the Water at *N*, it rais'd the Water from *G G* to *H H*, and ſo added another Ounce to the Weight of the Water. Then tying the Antimony to the Cork, the Cork had added to it three Fig. 9. Quarters of the Weight of the Antimony which the Hand before had ſuſtain'd, and made it ſink ſo as to be almoſt cover'd, and raiſed the Water to *i k*, adding three Ounces to its Weight. Hanging this Veſſel of Water upon the Balance, and a Counterpoiſe at the other End, upon cutting the String the Veſſel of Water was rais'd up, and the *Æquilibrium* was not reſtor'd till the Antimony came to the Bottom.

By obſerving that as the Cork (being freed from the Weight of the Antimony) aroſe, and that during the Fall of the Body, the Water ſunk to *h h*, it appears that this is, in Effect, the ſame Experiment as the former, and concludes no more. As to the real Cauſe of the Variation of the Barometer, namely, the Accumulation of the Air by Winds over the Place where the Barometer riſes; and Part of the Air being blown away where the Mercury in the Barometer ſinks, ſee Dr. *Halley*'s Account of it in the *Phil. Tranſact.* Num. 181.

Vid. Supra. V. II. p. 20.

Poſtſcript.] In making the firſt Experiment before the R. Society, of a Piece of Lead ſuſpended by a Thread, whilſt it was wholly cover'd with Water in the large Tube in which it hung (whoſe Length was four Feet) it was obſervable, not only that the End of the Balance (to which the Tube of Water with the Lead in it was fixed) did not riſe when the Thread was cut, (to let the Lead fall from the Top to the Bottom of the Tube) as it muſt have done according to M. *Leibnitz*'s Principle; but that the ſaid End of the Balance began to deſcend from the Time that the Lead began to fall. Therefore to be ſure that it was not the Plummet's rubbing againſt the Sides of the Tube in its Fall, which cauſed that *Phænomenon*, I hung to the Balance a long Glaſs of three Inches Diameter inſtead of the Tube, and making the Experiment as before, it ſucceeded in the ſame manner: The End of the Balance, which carried the Veſſel of Water, ſunk as ſoon as the Thread of the Plummet was cut; tho' this Glaſs was not above half ſo long as the Tube.

When by holding of the String I drew the Lead upwards and downwards in the Water, there was no ſenſible Alteration of the *Æquilibrium*. Neither was it alter'd by cutting the String of a Stone-Plummet, becauſe of the Shortneſs of the Glaſs, and the little Exceſs of ſpecific Gravity in the Stone: for the greater the Difference is betwixt

betwixt the Body made uſe of in this Experiment and Water, as well as the bigger the Body itſelf is, the better the Experiment will ſucceed.

Hence it appears, that when a Body, ſpecifically heavier than a Fluid, is (by what Cauſe ſoever) detain'd in any Place of the ſaid Fluid, it adds as much to the Weight of the whole Fluid as an equal Bulk of the ſaid Fluid amounts to: And when the ſaid Body, by the Action of its Exceſs of ſpecific Gravity above the Fluid, deſcends with an accelerated Motion; ſo long as that Motion is accelerated, the Reſiſtance of the Fluid (which is as the Square of the Velocity) takes off ſomething of the whole Weight of the Body; but as much as the Body loſes, ſo much the Water gains, over and above what was given it by its riſing on Account of the immers'd Body.

A Body therefore that falls in a Fluid is ſo far from making the Fluid lighter as it falls, that it makes it preſs more upon the Bottom that ſuſtains it, when it is falling, than when it was at reſt in the Fluid.

If the Veſſel of Water be long enough for the falling Body to come to an uniform Motion before it reaches the Bottom, the Force impreſs'd on the Water under the Body will make it preſs the Bottom, as much as if the Body were actually at Bottom, the Body in that Caſe loſing all its Exceſs of Gravity above that of the Water, and the Water gaining it.

Hence it follows, that a falling Cloud, when it comes to an uniform Motion, will not only add to the Weight of the Air as much as the Weight of an equal Bulk of Air; but even as much as its whole Weight amounts to, tho' it be ſpecifically heavier than the Air about it.

All the Diminution of Weight that can be allow'd in this Caſe is this. If we imagine the Air to have a ſmooth, regular Surface, as we have at firſt ſuppos'd, (or if that not allow'd, we may take any imaginary Surface of it above the Clouds) when a falling Cloud is diminiſh'd in Bulk, (as when it is chang'd into Rain) the Surface of the Air will ſubſide in Proportion to that Diminution, and therefore will weigh leſs, by ſo much as is the Weight of a Quantity of Air equal to the Bulk that Cloud has loſt: But when the Drops of Rain after Acceleration (occaſion'd by their Exceſs of Gravity above that of the Air) are come to an uniform Motion by the Reſiſtance of the Air, they reſtore to the Air the Weight that it had loſt. Now this uniform Motion being acquir'd in about two Seconds of Time, and the Diminution of Gravity in the Air being inſenſible, when compared to near three Inches of Mercury (for ſuch is the Variation of the Barometer with us) can no way be the Occaſion of thoſe ſo ſenſible Alterations in it, which happen ſome time before Rain or Fair Weather.

Add to this, that the whole Quantity of Rain that falls in England *and* France, *in the Space of one Year, ſcarce ever equals two Inches of* Mercury: *And in moſt Places between the* Tropicks, *the Rains fall, at certain Seaſons, in very great Quantities, and yet the* Barometer *ſhews there very little or no Alteration.*

Experiments made with the Barometer, *by Dr.* J. Scheuchzer. n. 344. p. 266.

VI. Theſe Barometrical Experiments, for inquiring into the different Elaſticity of different Kinds of Air, were made by help of a Tube 32 Digits long, *Paris* Meaſure, and 2 Lines Diameter, in different Parts of *Switzerland*, on occaſion of an Excurſion upon the *Alps*, undertaken in *September* 1714.

The firſt Column ſhews the Air left in the Tube. The ſecond the Height of the Mercury above the Superficies of the Quickſilver. The third the Spaces of the expanded Air. The fourth ſhews the Deſcent of the Mercury becauſe of the Air that is left.

Sept. 6. At *Zurich*, the Height of the whole Barometer at 8 a-Clock before Noon was 26 *Paris* Digits, 4 Lines. But at 9½ it was 26 Dig. 4½ Lines.

Column I.	Column II.		Column III.		Column IV.	
Digits.	Digits.	Lines.	Digits.	Lines.	Digits.	Lines.
3	19	9 Twice.	12	6½ Twice.	6	7½
6	16	8	15	7½	9	8½
	16	7½	15	8	9	9
9						
12	11	11 Twice.	20	3 Twice.	14	5½
15	9	9 Twice.	22	6 Twice.	16	7½
18	7	5½	24	8½	18	11
	7	6	24	8	18	10½
21	5	3	27	0 Twice.	21	1½
24	3	3	28	11 Twice.	23	1½
27	1	6	30	7½ Twice.	24	10½
30	0	4	31	10½ Twice.	26	0

Sept. 11. in a Plain of the *Alps* call'd **Ennenſewen gen Abeten**, near the Mountain *Liber*, under the Government of *Glarys*, at one in the Afternoon, the Sky being clear, the Altitude of the whole Barometer was 23. 10 Twice.

Column I.	Column II.		Column III.		Column IV.	
Digits.	Digits.	Lines.	Digits.	Lines.	Digits.	Lines.
3	18	7	13	6	5	3
6	15	7½	16	4	8	2½
9	13	3	18	7	10	7
12	11	1½	20	9	12	8½
15	9	0	22	9	14	10
18	6	11	25	0	16	11
21	4	11	26	10	18	11
24	3	0	28	10	20	10
27	1	4	30	5	22	6
30	0	2	31	8	23	8

Sept. 12. At 7 before Noon, the Sky clear, **auff Scherf** an Eminence of the Mountain *Liber*, the Altitude of the whole Barometer was 21. 8.

Column I. Digits.	Column II. Digits.	Lines.	Column III. Digits.	Lines.	Column IV. Digits.	Lines.
3	17	6	14	6	4	2
6	14	7	17	3	7	1
9	12	6	19	6	9	2
12	10	5	21	6	11	3
15	8	5	23	6	13	3
18	6	5	25	3	15	3
21	4	7	27	1	17	1
24	2	9½	29	0½	18	10½
27	1	4	30	6	20	4
30	0	2	31	8	21	6

Sept. 12. At 9 before Noon, the Sky clear, **auff dem Blattenstock**, an Eminence of the Mountain *Liber*, the Altitude of the whole Barometer was 21. 6.

3	17	2½	14	6	4	3½
6	14	5	17	5	7	1
9	12	4	19	6	9	2
12	10	4½	21	5	11	1½
15	8	7	23	4½	12	11
18	6	7	25	3	14	11
21	4	8	27	3	16	10
24	2	9	29	0	18	9
27	1	3	30	5	20	3
30	0	3	31	6	21	3

Sept. 14. At 12 within the Iron Mine at *Saruneta*, about 300 Paces from the Entrance, the Sky without being clear. The Height of the whole Barometer was 24. 4. and 24. 3.

3	18	9	13	1	5	7
6	15	9	16	1	8	7
9	13	5	18	5	10	11
12	11	3	20	7	13	1
15	9	1	22	9	15	3
18	7	0	24	10	17	4
21	4	11	27	0	19	5
24	3	0	28	10	21	4
27	1	4	30	6	23	0
30	0	3	31	6	24	1

Out of this Metallic Mine, in the open Air, I obſerved the ſame Altitude of the Mercury in the whole Barometer, alſo in 3 and 9 Digits of Air that was left in the Tube. But it muſt be obſerved, that in the inmoſt Parts of the Mine, where I made my Experiments, the Air was rarified by means of a Fire lighted the Day before, (with which the Miners mollify the Hardneſs of the Vein,) and the Place was moderately warm'd like a Stove.

N. B.

N. B: *It is found by many Experiments made before the Royal Society, that the Elastic Force of compress'd Air is as the compressing Weights directly. By these Observations of the learned* Scheuchzer *it appears, that the same Rule obtains very nearly in rarified Air. For though some Difference is found, yet it is not so great but that it may be owing to the unequal Diameters of the Tube. Now that these Experiments may be rightly made, it is necessary, that the Capacity of the Tube be divided into equal Parts, by putting the Mercury in by degrees, instead of taking Parts equal in Length.*

VII. *Observations on the* Weather *in a Voyage to* China, 1709, *by Mr.* J. Cunningham. n. 292. p. 1639.

Weather.	Winds.	Needles Inclin. or Depression of the N. or S. Point under the Horizon.	Needles Variation E. or W.	Longitude E. from St. *Jago.*	Latitude N. or S.	Philos. Barom Altit.	Ther. Altit. below Extream Heat.	Day of the Mon.
Rainy and Cloudy.	Variable Breezes, and 10 Hours calm.	dif. 8° 30 N		6° 16 E	1° 26 N		5 Div.	*Jan.* 31.
Rainy and cloudy with Lightning.	N S by W S W by S and 16 Hours calm.			6 11 E	1 18 N		4	*Febr.* 1
Fair.	Variable Breezes and 11 Hours calm.			6 13 E	1 14 N		5	2
Fair with Rain at noon	Variable.			5 57 E	1 07 N		5	3
Fair and cloudy, Rain in the Forenoon.	Variable.			5 49 E	0 27 N		2	4
Rainy and cloudy.	Variable with 9 hours calm.			5 44 E	0 00		4	5
Fair and cloudy with some Rain.	Variable with 4 hours calm.			5 35 E	0 30 S		4	6
Fair and cloudy.	Small Gales from E by S to S E by S.	3 30 S		5 08 E	1 46 S		5	7
Fair and pleasant.	Easy Gales from E by S to S E by E.			4 56 E	3 00 S		$4\frac{1}{2}$	8
Fair and pleasant.	Easy at E S E, and S E by E.			4 19 E	4 29 S		5	9
Fair and pleasant.	Easy at S E by E and S E.	10 00 S		3 54 E	5 57 S		5	10
Fair and pleasant.	Fine Gales S E by E.		20 17 E	3 05 E	7 10 S		6	11
Fair and pleasant.	E S E fine.		2 20 E	2 22 E	8 46 S		6	12
Fair and pleasant with one Shower.	E S.S and E by S.		2 50 E	1 37 E	10 29 S		8	13
Fair and cloudy with some Squalls.	E by S and E.		3 30 E	1 20 E	12 05 S		7	14
Fair, with one Squall.	E by N to E S E.		4 12 E	1 10 E	13 44 S		$7\frac{1}{2}$	15
Fair and pleasant.	E by S and E.	19 00 S	4 10 E	1 10 E	15 09 S		$7\frac{1}{2}$	16
Fair and pleasant.	E.		4 52 E	1 14 E	16 32 S		8	17
Fair and cloudy with one Shower.	E.		5 14 E	1 19 E	18 01 S		$8\frac{1}{2}$	18
Fair with one Shower	E by S to E by N.	30 00 S	5 42 E	1 29 E	19 14 S		$7\frac{1}{2}$	19
Fair and cloudy with several Squalls.	S E by E to E by N small.		5 18 E	1 15 E	20 05 S		8	20
Fair with 2 Showers.	NNE to E by N easy.			1 35 E	21 16 S		7	21

Day of the Mon.	Ther. Altit. below Extream Heat.	Philos. Barom Altit.	Latitude N. or S.	Longitude E. from St. *Jago.*	Needles Variation E. or W.	Needles Inclin. or Depression of the N. or S. Point under the Horizon		Winds.	Weather.
22	7		22 23 S	1 54 E	7 02 E		dif.	N N E to E by N.	Fair with a Shower.
23	5		23 47 S	2 39 E				N N E to N N W.	Fair and pleasant.
24	5		25 21 S	3 28 E				N N E to N E by E.	Fair and pleasant.
25	6½		26 38 S	4 09 E	8 00 E	39 00 S		N E by N to N N E.	Fair and pleasant.
26	6½		27 49 S	4 52 E				N E by E to N.	Fair and pleasant.
27	8		29 16 S	6 07 E				N to N W.	Fair sometimes cloudy
28	8		30 01 S	7 34 E				W N W to W S W.	Cloudy & squally with Rain and Lightning.
29	10½		31 26 S	10 20 E				W N W to N by E fresh.	Squally with much Rain & some Thunder
March 1.	12½	8	31 10 S	11 49 E				N W to W moderate.	Uncertain with Thunder and Rain.
2	13	10	31 16 S	13 57 E	4 50 E	42 00 S		W to N W moderate.	Fair and cloudy.
3	17½	14	31 13 S	16 04 E				W to S fresh.	Cloudy and squally.
4	23½	18	30 26 S	17 51 E				S by E to S E by E.	Squally and cloudy.
5	26	19½	30 58 S	17 17 E				E S E and E by S.	Fair and cloudy.
6	24	20	32 07 S	17 08 E	3 05 E			E by S to E by N moderate.	Fair and cloudy.
7	23	26	33 06 S	16 54 E				E by N to S E by E small.	Fair and cloudy.
8	32	32	33 14 S	18 26 E	2 05 E			S E to S S W moderate.	Fair and cloudy.
9	33	38½	32 45 S	19 59 E				S to S S W blowing hard.	Close and squally.
10	29	34	32 27 S	21 13 E				S by E & S moderate.	Gray and cloudy.
11	29	32½	32 01 S	22 32 E				S by E to S E by E.	Squally, close and cloudy.
12	28½	33½	32 36 S	12 23 E				S S E to E by S.	Dry, close & cloudy
13	27	34	33 42 S	22 41 E				E by S to E by N easy.	Fair and close.
14	26	31	34 08 S	23 23 E	1 15 E			Variable and small.	Fair and close.
15	26	30	34 19 S	25 26 E	0 57 E			N to N W by N easy.	Fair and serene.
16	24	30	34 40 S	27 45 E	0 20 E			N by W to N N E.	Fair and serene, sometimes close.
17	25	30	34 55 S	29 48 E	0 22 W			N by E to N N W a fine Gale.	Fair and serene.
18	25½	30	34 59 S	33 27 E	1 19 W	47 00 S		N to N N W.	Fair and serene, this Forenoon cloudy.
19	34	38	34 44 S	36 25 E				N N W to S W and S by W.	Thick and squally, this Forenoon fair.
20	26	31½	34 22 S	37 45 E	6 25 W			S W to N W easy.	Fair and cloudy.
21	28	33	34 29 S	39 04 E				NW by N to W easy.	Fair and cloudy.
22	37	42	34 08 S	41 31 E				W to S S E fine.	Fair and sometimes squally.
23	30	36	34 35 S	43 41 E	7 05 W			S to W S W fine.	Variable Weather.
24	26½	32	34 08 S	45 06 E	8 30 W			SSW to WSW small.	Fair and pleasant.
25	27	30½	33 58 S	47 02 E	8 52 W			S W by S to S E by S easy.	Variable.

Weather

Weather.	Winds.	Needles Inclin. or Depref. of the N. or S. Po. under the Horizon.	Needles Variation E. or W.	Longitude E. from St. *Jago.*	Latitude N. or S.	Philof. Barom. Altitude.	Ther. Altit. below Extream Heat	Day of the Mon.
Fair and cloudy.	S E by S to S by W moderate.		9 43 W	48 32 E	33 35 S	26½	25½	26
Variable.	S W by S to S moderate.		10 39 W	49 56 E	33 51 S	29	27	27
Fine moderate Weather.	S W by S to S by W fmall.			50 36 E	34 03 S	30½	35½	28
Fair and pleafant.	S E fmall, fometimes frefh.			50 36 E	34 03 S	34½	33	29
Fair and pleafant.	SW to SE moft Part calm.	48 00 S		50 36 E	34 03 S	31	30	30
Fair and pleafant.	Variable Breezes and calm.			50 36 E	34 03 S	25	26	31
The fame.	S to S E fmall.			At the	Cape			*Apr.* 1.
The fame.	S E fmall.			of				2
The fame.	S E and S E by S moderate.			*Good-*	*Hope.*			3
The fame.	S E and S S E fmall and calm.							4
The fame.	S E by S fmall, at Night frefh.							5
The fame.	S E fmall.							6
The fame.	Variable and calm.							7
The forenoon foggy, Afternoon clear'd up.	Calm, at Night blowing frefh.					34	30½	8
Fair and hazy.	S S E and S by E blowing hard.			Longit. from the				9
Fair and pleafant.	S S E fine Gales.			Cape of *Good Hope*				10
Fair and gray.	S E by S to N E moderate.			00 43 W	34 45 S	29	31	11
Fair and cloudy.	Variable and fmall.		11 29 W	00 25 W	35 01 S	28	26	12
Squally.	S S E to S E by E moderate.			00 08 W	35 30 S	42½	33	13
Variable.	S E to E by N.			00 36 W	36 52 S	44½	34½	14
Fair and cloudy.	E to E N E fmall.			00 13 W	37 51 S	29	25½	15
Variable.	S by E to S W by S fmall, fometimes calm.		12 40 W	1 03 E	37 57 S	31½	27	16
Variable.	S S W to E S E.			2 38 E	37 41 S			17
Fair and clofe.	E by S to E N E fmall.			3 29 E	38 45 S	38	31	18
Fair and cloudy.	E by S to N E by N moderate.			4 50 E	39 53 S	34	26	19
Cloudy and overcaft.	N E by E to E by S moderate.			5 58 E	40 42 S	39	31	20
Moderate Weather.	N E by E to N E by N eafy.			7 49 E	41 06 S	34	32½	21
Variable.	W S W to S E frefh.			10 06 E	39 52 S	45½	40	22
Fair and cloudy.	S E to S S E fmall.		19 06 W	11 34 E	38 51 S	44	38	23
Fair and pleafant.	S E to N N E fmall.			11 50 E	39 09 S	33½	32	24
Fair and cloudy.	Variable Gales.			14 37 E	39 14 S	41	30½	25

Day of the Mon.	Ther. Altit. below Extream Heat.	Philos. Baro. Altit.	Latitude N. or S.	Longitude E. from the Cape of *Good-Hope.*	Needles Variation E. or W.	Needles Inclin. or Depression of the N. or S. Point under the Horizon	dif.	Winds.	Weather.
26	46	55	38 00 S	17 06 E				W by N to S W blowing hard.	Variable with Rain and Hail.
27	36	45	37 41 S	19 10 E		60 00 S	8	S W by S to W N W fresh.	Close and cloudy.
28	31	37	37 45 S	21 35 E	21 23 W			W N W to N W by N moderate.	Fair somewhat cloudy
29	25	36	37 27 S	23 32 E				N W to S W easie.	Variable.
30	23	33	37 30 S	23 57 E	22 27 W			W S W to S E by S small.	Close Weather.
May 1	33	42	37 58 S	25 24 E				N N E to E small.	Close and cloudy.
2	32	45	38 16 S	27 00 E	24 30 W			N E by E to N by E easie.	Close.
3	32	40	38 47 S	29 40 E				N E by N to N N W fresh.	Close and squally.
4	30	31½	38 50 S	33 20 E				N N W to N W fresh.	Fair and cloudy.
5	37	37	38 43 S	36 20 E		60 00 S	7	N W to S W fresh.	Variable and squally.
6	38½	45½	38 35 S	39 54 E				N W to S W easie.	Variable and squally.
7	38½	45½	38 17 S	42 07 E		69 00 S	8	W N W to S W moderate.	Fair Weather.
8	46½	53	37 49 S	45 51 E	24 20 W			W N W to S W by W fresh.	Variable.
9	35	40	38 48 S	49 19 E				W S W and W N W fresh.	Variable.
10	32	39	38 30 S	52 41 E	24 00 W	72 00 S	8	W by N to N W by W fresh.	Variable.
11	35	16	38 09 S	56 08 E				W N W and W fresh.	Moderate with some Rain.
12	32	42	37 25 S	58 16 E				W to S W easie.	The same.
13	33½	47½	37 25 S	59 11 E	20 48 W			S W to N N E small	Cloudy Weather.
14	34	45½	37 10 S	60 54 E	20 00 W	75 00 S	8	W N W to N W by N small.	Fair and pleasant.
15	30	38½	37 13 S	63 29 E	19 00 W			W N W to S W fine Gales.	Variable.
16	34	41	36 53 S	65 04 E	18 43 W	70 00 S	6	S W to W N W easie.	Close and hazy with Rain.
17	31½	36½	36 48 S	68 14 E	17 40 W			W N W to N W fresh.	Variable.
18	35	1	36 56 S	71 12 E		68 00 S	8	W N W and W by N fine.	Fair and cloudy.
19	3[illegible]½	41	36 00 S	74 16 E				W N W and W fresh.	Moderate.
20	32	34	35 01 S	76 39 E				N W by W to W fine.	The same.
21	32½	37	34 07 S	78 32 E	14 28 W	68 00 S	8	W by S to N W by N	The same
22	27	25½	33 [illegible]7 S	81 05 E	14 00 W			N W by N and N W	The same.
23	28½	24½	32 53 S	83 38 E	3 3 W			N W by N to N E	Variable.
24	35	33	31 35 S	85 13 E				N by E to W by S	Thick and squally.
25	29½	30½	30 11 S	86 28 E		68 00 S	8	S W to W N W	Variable.
26	31	34½	29 2[illegible] S	87 06 E	10 04 W			S W by W & S W by S	Fair and pleasant.

Weather.	Winds.	Needles Inclin. or Depression of the N. or S Point under the Horizon		Needles Variation E. or W.	Longitude E. from the Cape of *Good-Hope.*	Latitude N. or S.	Philos. Baro. Altit.	Ther. Altit. below Extream Heat.	Day of the Mon.
		dif.							
The same.	S W to S E small.				87 59 E	28 01 S	35	31	27
Variable.	S E. and S E by E.			3 58 W	89 24 E	25 55 S	33	30	28
Squally & Rainy with Thunder & Lightning	S E by E to S S E.				90 29 E	23 59 S	20	27	29
Variable.	S E and S E by E.	8	62 00 S	[illegible] 12 W	91 32 E	22 40 S	24	25	30
Fair and serene.	S E to S S E.			7 00 W	92 42 E	21 23 S	22	22	31
Fair and pleasant.	S E and S E by E.			6 35 W	93 20 E	19 54 S	19	19	*June* 1
The same.	S E by E and E S E.				94 10 E	18 02 S	17	17	2
The same	S E by E and E S E.			4 40 W	94 55 E	16 30 S	13	14	3
The same.	E by S to E by N.	7	53 00 S	4 50 W	95 15 E	15 02 S	10	12	4
The same.	E by N to E by S.			5 12 W	95 34 E	13 40 S	7	7½	5
The same.	E to N E by E.	4	52 00 S	5 20 W	95 42 E	13 01 S	3	5	6
The same.	E by N to E S E.			5 15 W	95 56 E	12 59 S	3	5	7
The same.	E by N to S E.			4 42 W	96 17 E	12 6 S	3	5	8
The same.	E S E to E N E.			4 26 W	96 28 E	11 43 S	1	5½	9
The same.	E by N and E.			3 52 W	96 34 E	11 10 S	1	5	10
The same.	E by N to E S E.			3 32 W	96 54 E	10 18 S	0	4	11
The same.	E by S to E S E.	6	46 00 S	3 26 W	97 37 E	8 45 S	1	5	12
The same.	E by S to N N E.			3 12 W	97 21 E	7 52 S	1	5½	13
The same.	E S E to N N E.				95 56 E	7 23 S	0	4½	14
The same.	S S E to S E.				95 16 E	6 45 S	0	5	15
The same.	S S E, S S W and calm.				Betwixt		0	4	16
The same.	S S W, S E and calm.				*Java Head*		0	2½	17
The same.	S E and E. by S.				and		0	2	18
The same.	E N E and S E.				*Batavia*		0	2	19
The same.	N E by E to S S E.						0	1	20
The same.	N N E to S.	3	40 00 S		Longit. from *Batavia*		0	1	21
The same.	N to S S E.				00 05 W		0	1½	22
The same.	N W by W and S W.				00 47 E	5 27 S	0	4½	23
Cloudy with some Rain.	S by W and S E by E.				1 48 E	5 24 S	0	2	24
Squally with Thunder Lightning and Rain.	E S E to N by E.				1 55 E	5 51 S	0	3	25
Cloudy with Rain.	E S E to N E by E.	4	38 00 S		1 51 E	6 12 S	2½	6½	26
The same.	Calm.				1 59 E	6 16 S	0	5	27
Variable.	E by N to E S E small.				2 02 E	6 29 S	0	5	28
Fair.	E S E and S E small.				2 18 E	6 21 S	0	5	29
Uncertain.	Variable Breezes.				2 33 E	6 29 S	0	5	30
Fair with some Rain.	E by S to N E by E				2 43 E	6 44 S	0	4	*July* 1
Fair with some Rain.	N E by E to S E by E.				2 55 E	6 39 S	0	4½	2
Fair and serene.	N E by E to S E by E.				3 34 E	6 35 S	0	3½	3
The same.	N E to S S E.				3 49 E	6 [illegible] S	0	3½	4
[illegible]	N E by N to S E.	8	45 00 S		3 49 E	6 35 S	0	5	5

Day of the Mon.	Ther. Altit. below Extream Heat.	Philoſ. Barom Altit.	Latitude N. or S.	Longitude E. from *Batavia.*	Needles Variation E. or W.	Needles Inclin. or Depreſſion of the N. or S. Point under the Horizon	dif.	Winds.	Weather.
6	8	2	6 12 S	4 07 E				S E ſmall.	The ſame.
7	6	0	6 21 S	4 28 E				S E by S to E.	The ſame.
8	6	0	6 22 S	4 43 E				S E by E to E N E	The ſame.
9	6	1	6 28 S	5 13 E				E by S to E by N.	The ſame.
10	8½	3	6 50 S	5 37 E	1 25 W	41 10 S	2	S E by E to E by N.	The ſame.
11	6	0	6 44 S	5 47 E				E N E to S S E.	The ſame, with ſome Rain.
12	5	½	6 23 S	6 05 E				E by N to S E.	Fair and ſerene.
13	5	0	6 05 S	6 17 E				S E to E by S.	The ſame.
14	6	2	5 10 S	6 54 E				S E to E S E.	The ſame, with ſome Rain.
15	4	0	4 20 S	7 09 E				S E by S to E S E.	Fair and cloudy.
16	5	0	4 08 S	7 27 E				S E by E to N E by E.	Fair and ſerene.
17	5	0	4 00 S	7 30 E	At the *Banjar*	Bar of on *Borneo*		E to S E and calm.	Fair and cloudy.
18	8	4	4 00 S	7 30 E				E to S E by S ſmall.	Cloſe and cloudy with Rain.
19	7½	1	4 00 S	7 30 E				S E freſh.	Cloſe and clearing up.
20	6	0	4 15 S	7 06 E		40 00 S	0	S S E to S E	Fair and ſerene.
21	5	0	4 51 S	5 08 E				S E by S and S E.	Fair and cloudy.
22	2½	0	4 59 S	3 12 E				S E by E to E by S.	The ſame.
23	4	0	4 45 S	1 25 E				S by E to E by S.	Cloſe with ſome Rain.
24	7	4	4 09 S	0 17 E				Variable.	Squally with Thunder
25	[illegible]	0	4 01 S	0 13 E		35 00 S	6	S to E by S.	Fair and cloſe with Lightning.
26	1½	0	3 32 S					E by S to S E by S.	Fair and ſerene.
27	3	0	In the	Streights				E N E to S by E.	Variable.
28	5	0	of	*Banca.*				S E by S to S.	Fair and ſerene.
29	4½	0	1 59 S					S E by E to S.	The ſame.
30	4	0	0 31 S	Longit.	1 50 W			S E to N N E	The ſame.
31	4	0	1 25 N	from *Pulo*	*Condore.*	31 00 S	5	S S E to S E by S.	The ſame.
Aug. 1.	2½	0	3 19 N	1 36 W				S S E to S by W.	Fair and cloſe.
2	1	0	5 07 N	0 52 W				S by W to S E.	Variable with Thunder and Lightning.
3	1	0	6 04 N	0 25 W				Variable.	Fair and cloſe.
4	1	0	7 47 N	0 08 E				S S W to S W by S.	Fair and pleaſant.
5	3½	0	9 16 N	1 25 E				W to S W.	Fair, cloſe and hazy.
6	2	0	10 14 N	3 07 E				S W by W to W.	Fair and hazy.
7	1	0	12 04 N	4 33 E		5 30 S	12	S W by W to S W by S	Fair and ſerene.
8	1	0	24 05 N	5 58 E		4 00 S	13	S W and S W by W.	The ſame.
9	1½	0	16 15 N	7 02 E				W by S to S W by S.	Fair and cloudy.
10	2½	0	18 14 N	7 20 E				Variable.	Squally.
11	1½	0	19 49 N	7 12 E				W S W to W N W. blowing hard.	Squally and much overcaſt.
12	½	0	21 17 N	7 02 E		2 30 S	1	W S W to S S W. moderate.	Fair and cloſe with ſome Rain.

Weather.	Winds.	Needles Inclin. or Depref. of the N. or S. Po. under the Horizon.	Needles Variation E. or W.	Longitude from *Pulo Condore*	Latitude N. or S.	Philof Barom Altitude.	Ther. Altit. below Extream Heat.	Day of the Mon.
Fair and ferene.	S S W to S E by S fmall.	dif.		7 36 E	21 58 N	above Extr.	$1\frac{1}{2}$ Heat.	13
Fair and pleafant with one Shower.	S E to N N E.	2 6 00 N		7 47 E	22 15 N	0	1 below	14
Fair and pleafant.	E by N to N E by E.			8 09 E	22 18 N	0	1	15
The fame.	Variable and calm.		1 20 W	8 43 E	22 32 N	0	0	16
Fair, with fome drizling Rain.	W S W to S W eafy.			9 48 E	23 10 N	0	$\frac{1}{2}$	17
Fair and ferene.	Variable and fmall.			10 04 E	23 32 N	0	$1\frac{1}{2}$	18
The fame.	N to N E by N frefh.			10 18 E	23 56 N	0	$2\frac{1}{2}$	19
Fair and hazy.	N by E to N E by N			10 28 E	24 12 N	0	4	20
The fame.	N E by E to N by E.	6 12 00 N		10 43 E	24 22 N	0	1	21
The fame.	N E to N N E.			11 01 E	24 32 N	0	3	22
Fair and pleafant.	N E by E to N N E eafy.			11 15 E	24 47 N	0	$4\frac{1}{2}$	23
Variable.	E N E to N E by N frefh.			11 28 E	24 56 N	0	6	24
Fair and pleafant.	E N E to N by E moderate.			11 38 E	25 09 N	0	5	25
The fame.	N by E to N E by E.			12 08 E	25 09 N	0	6	26
The fame.	E N E to N N E.			12 20 E	25 07 N	0	5	27
Variable.	E by N to N N E.			11 50 E	25 29 N	0	5	28
Variable.	N E by E to N.			12 02 E	25 37 N	0	4	29
Clofe and fqually.	N by E to N E by E frefh.			11 47 E	25 49 N	0	$3\frac{1}{2}$	30
Variable.	N E by N to E by N.			11 53 E	26 02 N	0	2	31
Fair and pleafant.	N N E to N E by E.			11 56 E	26 05 N	0	$2\frac{1}{2}$	*Sep.* 1.
The fame, with fome Rain at Night.	N E fmall.			At *Crocodile*	Ifland.			2
Fair and pleafant.	N N E fmall.							3
The fame.	N by E to N E.							4
Fair and cloudy.	N E moderate.							5
The fame, at Night much Rain.	N E frefh, at Night S W blowing hard.							6
Cloudy with fome Rain.	N E frefh and moderate.							7
Fair and pleafant, at Times overcaft.	N E moderate.							8
Fair and hazy.	N E moderate.			12 38 E	25 44 N	0	3	9
The fame.	N E by N to N E by E.			12 13 E	26 11 N	0	$3\frac{1}{2}$	10
The fame.	N E to N N E.			12 47 E	26 11 N	0	5	11
Fair and pleafant.	N E by N to E by S fmall.			12 37 E	26 22 N	0	$4\frac{1}{2}$	12
Fair.	E N E to N E moderate.			12 40 E	26 26 N	0	$8\frac{1}{2}$	13
Fair and cloudy.	N N E to E N E.			12 59 E	26 37 N	0	$6\frac{1}{2}$	14

Day of the Mon.	Ther. Altit. below Extream Heat.	Philos. Barom Altit.	Latitude N. or S.	Longitude E. from *Pulo Condore*.	Needles Variation E. or W.	Needles Inclin. or Depression of the N. or S. Point under the Horizon	dif.	Winds.	Weather.
15	5	0	27 05 N	13 11 E				N E by E to E by N	Fair close and cloudy.
16	7	0	26 55 N	14 01 E				E N E to N by W.	Fair and cloudy.
17	8	0	27 07 N	14 13 E				N to N E by E.	The same.
18	10	$7\frac{1}{2}$	27 26 N	14 13 E				N E to N by E.	Variable.
19	9	$6\frac{1}{2}$	27 42 N	14 02 E				N by E to N E by E.	Fair.
20	$7\frac{1}{2}$	4	27 46 N	14 25 E				N E by E to N by E fresh.	Fair and cloudy.
21	12	10	28 03 N	14 25 E				N E to N N E moderate.	Fair and pleasant.
22	15	$12\frac{1}{2}$	27 49 N	15 31 E				N E by N to N by E.	Fair with some Rain.
23	17	14	27 51 N	15 01 E				N N E to N fresh.	Fair and cloudy.
24	4	13	27 53 N	16 26 E				N by W to N by E small.	Fair and much overcast.
25	14	12	28 10 N	15 36 E				N by E to N E by N.	Fair and close.
26	$13\frac{1}{2}$	13	28 47 N	15 04 E				N by E to E by N.	Variable.
27	14	13	29 06 N	14 49 E		17 30 N	5	N N E to N E by E.	Fair and pleasant.
28	$15\frac{1}{2}$	17	28 55 N	15 09 E	100 E			N N E to N E.	Fair and cloudy.
29	15	13	29 02 N	15 09 E				N E by N.	Fair sometimes close.
30	15	17	29 43 N	15 19 E				Variable from S S E to N W.	Fair pleasant and cloudy.
Oct. 1.			29 56 N	14 55 E				W N W to N E by E.	Fair and close.
2			29 56 N	14 55 E				N N W and N small.	Thick and foggy with Rain.
3								N N W to N W small.	Variable, Forenoon serene.
4								N to N E moderate.	Variable.
5								N N W to N W blowing fresh.	Fair and cloudy.
6								Forenoon calm, Afternoon S E fresh.	Fair and serene.
7								N N E to N E.	Cloudy with Rain.
8								N to N N W.	Cloudy and overcast
9								N by W to N E.	Cloudy hazy & Rain
10	20	20	29 56 N	14 55 E				N E to S E moderate	Cloudy and fair.
11	12	16	30 29 N	14 20 E				S E to S W small	Fair and serene.
12	17	15	30 25 N	14 20 E	At	*Chusan*.		S W small.	The same.
13	25	32						N N E to N E.	Fair and cloudy.
14	24	30	30 25 N	14 20 E	00 56 E	21 00 N		N E moderate.	Fair and cloudy.
15								N small.	Fair and cloudy
16	25	29						N E to E moderate.	Cloudy with some Rain
17	16	14						S E small.	Thick and hazy with Rain.
18	15	13						S E moderate.	Thick and hazy with much Rain.

Weather

Weather.	Winds.	Needles Inclin. or Depref. of the N. or S. Pole under the Horizon.	Needles Variation E. or W.	Longitude from *Pulo Condore*	Latitude N. or S.	Philof. Barom Altitude.	Ther. Altit. below Extream Heat	Day of the Mon.
Fair and cloudy.	N E fmall.	dif.				31	$23\frac{1}{2}$	19
The fame.	N N E to E N E.					30	24	20
Cloudy with fome Rain.	N N E to S E.					29	24	21
Fair and cloudy.	N to N W frefh.					30	25	12
Fair and pleafant.	N by W to W N W fmall.					30	26	23
The fame.	S and S S E fmall.					32	$27\frac{1}{4}$	24
The fame.	S E moderate.					28	24	25
The fame.	SE to S by E moderate					26	$22\frac{1}{2}$	26
Cloudy with fome Rain.	N W to N by W moderate					27	20	27
Thick and hazy with Rain.	N W fmall, fometimes frefh.					45	$36\frac{1}{2}$	28
The fame.	N W to N moderate.					58	45	29
Fair and cloudy.	N N W to N by E fmall.					59	46	30
The fame.	N W fmall.					65	47	31

Note 1. That the Altitude of the Spirits in the Thermometer and Philofophical Barometer was commonly taken at Noon.

Note 2. That the Account of the Winds and Weather at Sea, is from Noon to Noon.

Note 3. That the middle Inclination of the Dipping-Needle is fet down with the Difference alfo which was made, as either Side of the Compafs was turn'd Eaft or Weft: Which Difference at firft was not taken Notice of.

From whence this Difference fhould arife, I cannot determine, the Compafs feeming to be juftly pois'd and equally divided.

For the few Chafmata in the Columns of the Thermometer and Barometer, there needs no other Apology, than that I was not on Board to take an Account of them my felf.

A Regifter of the Weather, &c. at Chufan *in* China, 1700. *by Mr.* J. Cunningham. n. 292. p. 1648.

VIII. *Note* 1. The Ifland *Chufan* is in 30° 25′ N. Latitude upon the Coaft of *China*.

Note 2. That the following Obfervations were of a portable Barometer from *England*; which by a Barometer fet up here were always $\frac{6}{20}$ of an Inch lower.

Note 3. That the Barometer ftood about 18 Feet above the *Superficies* of the Sea at high Water.

1. Grey

November 1700.

1. Grey cloudy Weather, very cold and moderate Gales from N W to N.

2. Grey cloudy Weather with moderate Gales from N W to N, and very cold.

3. Grey cloudy Weather, very cold and small Gales from N by W and N N W. At Night little Wind and more serene.

4. Grey cloudy Weather, very cold and moderate Gales at N N W and N by W.

5. Fair and serene Weather with small Gales from N N W to N. The Air temperate.

6. Fair and pleasant Weather, with small Gales at N W and N N W.

7. Fair and pleasant Weather, somewhat hazy, and small Gales at N N W. At Night calm.

8. Fair and pleasant Weather, with small Gales at N. At Night little Wind.

9. Fair and serene Weather, with small Gales at N. At Night calm.

10. Very serene and warm Weather, with small Gales at N by W. In the Night calm.

11. Dry and serene Weather, with small Gales from S E. At Night calm.

12. Dry and serene Weather, with some small Northerly Breezes. At Night calm.

13. The Morning foggy, all Day serene with small Breezes at N N W.

14. The Morning grey and cloudy, toward Noon thick hazy Weather with drizling Rain till 8 at Night. All Day fresh Gales from W by S to N W. At Night less Wind and fair.

15. Grey cloudy Weather with moderate Gales from N W to N.

16. Fair and pleasant Weather with small Gales from N to N by E.

17. Dry and serene Weather with small Gales at N N W.

18. Grey cloudy Weather, with moderate Gales from N W. In the Night cold.

19. Dry Weather somewhat cloudy, with small Gales from N W to N. Cold at Night.

20. Dry and pleasant Weather, with small Gales at N N W.

21. Fair and pleasant Weather, with small Gales from W by S to N W.

22. Fair and pleasant Weather, somewhat hazy with Gales at S E.

23. Fair and pleasant Weather, the Afternoon overcast, with moderate Gales at N W to N.

24. Dry

24. Dry Weather, ſomewhat cloudy, with moderate Gales from W N W to N.

25. Fair and pleaſant Weather, with ſmall Gales from N by W. At Night calm and cold.

26. Fair and pleaſant Weather, with ſmall Breezes at N W, for the moſt part calm. ☿ Altitude $30\frac{8}{20}$.

37. Grey cloudy Morning, with ſmall Gales at W N W. ☿ Altitude $30\frac{6}{20}$. In the Afternoon fair and pleaſant with ſmall Southerly Breezes. At Night calm.

28. Fair and pleaſant Weather, with ſmall Gales at N W. ☿ $30\frac{6}{20}$. The Afternoon ſmall Breezes at S W. At Night calm, and the Breezes veering to N, the Air temperate. ☿ $30\frac{4}{20}$.

29. Fair and ſerene Weather, with Calms. ☿ $30\frac{5}{20}$. In the Afternoon ſmall Gales at S E. At Night grey cloudy Weather, the Gale veering to N. ☿ $30\frac{6}{20}$.

30. A grey cloudy Morning, with moderate Gales at N W and W N W. ☿ $30\frac{6}{20}$. All Day more ſerene. In the Evening overcaſt, and ſome Rain at 9 of the Night.

December 1700. 1. Grey cloudy Weather all Day and Night with freſh Gales from N to N E. ☿ $30\frac{9}{20}$ falling to $30\frac{8}{20}$.

3. In the Morning fair and ſerene. ☿ $30\frac{5}{20}$. Since overcaſt and grey cloudy Weather, with moderate Gales at N N E and N. In the Evening ſome Drops of Rain, and ſometimes blowing freſh in the Night, and very cold.

4. Grey cloudy Weather, with moderate Gales at N to N W. ☿ $30\frac{11}{20}$, at Night falling to $30\frac{10}{20}$, then almoſt calm.

5. A grey cloudy Morning, towards Noon more ſerene. ☿ $30\frac{11}{20}$. Freſh Gales at N E. In the Evening overcaſt, with ſome Drops of Rain. Little Wind all Night.

6. A grey Morning ſomewhat cloudy, and a ſmall Breeze at N N E. ☿ $30\frac{9}{20}$. In the Afternoon overcaſt, with the Wind at S, and ſmall Drops of Rain. The ☿ falling to $30\frac{8}{20}$. Some Rain in the Night.

7. Cloſe thick Weather with drizling Rains all Day and Night, and ſmall Winds at N N E and N. ☿ $30\frac{7}{20}$.

8. A grey cloudy Morning with ſome drizling Rain, and moderate Gales at N and N by W. ☿ riſing above $30\frac{7}{20}$. All Day and Night overcaſt; the ☿ as before.

9. A grey cloudy ſharp Morning, and ſmall Winds at N N W. ☿ $30\frac{6}{20}$. Cloudy all Day, and towards Evening cloſe Weather and calm. ☿ $30\frac{5}{20}$.

10. Dry Weather ſomewhat grey and cloudy, with ſmall Breezes at N N W. ☿ $30\frac{4}{20}$. All Day overcaſt with variable Breezes intermix'd with Calms. ☿ as before.

11. Fair and pleaſant in the Morning, ſince overcaſt with variable Breezes, the ☿ as before.

12. Grey cloudy Weather, in the Forenoon ſmall Gale at S E. ☿ 30 $\frac{4}{20}$. In the Afternoon the Gale freſhning at N W. ☿ 30 $\frac{5}{20}$. With ſome Rain all Night.

13. Grey cloudy Weather, with moderate Gales from N W to N. ☿ 30 $\frac{4}{20}$. Cold all Night.

14. A ſharp Morning and fair pleaſant Weather, with ſmall Gales at N N W and N. ☿ 30 $\frac{6}{20}$. At Night calm. ☿ 30 $\frac{5}{20}$.

15. Fair and pleaſant Weather, with moderate Gales at S E. ☿ 30 $\frac{4}{20}$ falling to 30 $\frac{4}{20}$.

16. The Morning ſomewhat cloudy, with ſmall Gales at S E. ☿ 30 $\frac{2}{20}$. At Noon veer'd to N W, and the Sky overcaſt; at Night ſome Rain, much Wind and Cold.

17. A ſharp Morning and grey cloudy Weather, with moderate Gales from N W to N N W. ☿ 30 $\frac{7}{20}$. All Day overcaſt, at Night little Wind and much Rain.

18. Thick cloſe rainy Weather, all Day and Night, with ſmall Gales at N W.

19. Grey cloudy Weather, with moderate Gales at N W to N. ☿ $\frac{6}{20}$. Some Rain at Night and very cold.

20. Grey cloudy Weather, with moderate Gales at N N W. ☿ 30 $\frac{7}{20}$. At Night little Wind.

21 Grey cloudy Weather and cold, with moderate Gales from N N W to N W. ☿ 30 $\frac{7}{20}$. At Night drizling Rains. ☿ 30 $\frac{6}{20}$.

22. In the Morning cloſe Weather, with drizling Rains and moderate Gales at N W and N N W. ☿ 30 $\frac{6}{20}$. The Afternoon dry, grey and cloudy.

23. A grey cloudy Morning and calm, ☿ 30 $\frac{5}{20}$, towards Noon more ſerene and a ſmall Breeze at E S E. In the Evening overcaſt with ſome Rain and freſh Gales, all Night at N N W.

24. Grey cloudy Weather, ſomewhat cloſe, with freſh Gales from N N W to N W by W.

25. Grey cloudy Weather, with moderate Gales at N W.

26. Cloſe Weather with drizling Rains, and ſmall Breezes at N E, for the moſt calm. In the Night the Gale freſhned at N. ☿ 30 $\frac{6}{20}$.

27. The Forenoon grey cloudy Weather, and ſmall Gales at N N E. ☿ 30 $\frac{7}{20}$. The Afternoon and all Night cloſe Weather with drizling Rains, the Breeze veering to E N E.

28. Grey cloudy Weather with ſome drizling Rains, and ſmall northerly Gales. ☿ 30 $\frac{7}{20}$.

29. In the Morning ſomewhat fair, then overcaſt with drizling Rains and cloſe Weather all Day and Night, ſmall Gales from N N E to N E. ☿ 30 $\frac{6}{20}$.

30. A grey cloudy Morning, with Gales at E N E. ☿ $30\frac{7}{20}$. All Day the foreſaid Weather.

31. In the Morning fair and pleaſant, with ſmall Gales at S E. ☿ $30\frac{5}{20}$. In the Forenoon overcaſt, ☿ falling to $30\frac{3}{20}$. The Afternoon and Night cloſe thick foggy Weather, with ſome drizling Rain and calm.

January, 1701.

1. Dry Weather, ſomewhat cloſe, with ſmall Gales at S E. ☿ $30\frac{1}{20}$. The Afternoon overcaſt and cloſe Weather, with ſmall Gales at E S E. ☿ falling to $29\frac{19}{20}$, and much Rain in the Night.

2. Cloſe Weather and drizling Rains, with moderate Gales at N N E. ☿ $30\frac{1}{20}$. At Night blowing freſh, ☿ riſing to $30\frac{3}{20}$.

3. Cloſe and cloudy Weather, with drizling Rain, and moderate Gales at N. ☿ $30\frac{5}{20}$.

4. A grey cloudy Morning, with moderate Gales at N E. ☿ $30\frac{1}{20}$. Afternoon cloſe and thick Weather, with drizling Rains. ☿ 30. Much Rain in the Night.

5. Thick cloſe rainy Weather, with moderate Gales at N E. ☿ $30\frac{1}{20}$. At Night fair and cold.

6. A grey cloudy Morning, with freſh Gales at N E. ☿ $30\frac{6}{20}$. At Noon $30\frac{7}{20}$. At Night $30\frac{6}{20}$, and ſome Rain.

7. Cloſe and cloudy Weather, in the Morning. ☿ $30\frac{5}{20}$. Towards Noon drizling Rains, and encreaſing in the Afternoon, with ſmall Winds at N E. ☿ $30\frac{4}{20}$.

8. Thick cloſe and cloudy Weather, with drizling Rains and ſmall Gales at N E. ☿ $30\frac{2}{20}$.

9. Cloſe and cloudy Weather, with drizling Rains and ſmall Gales at E S E. ☿ $30\frac{4}{20}$. At Night fair. ☿ $30\frac{6}{20}$.

10. A grey cloudy Morning with freſh Gales at N. ☿ $30\frac{7}{20}$. In the Evening and all Night rainy Weather.

11. A cloſe and cloudy Morning with drizling Rains and ſmall Gales at N. ☿ $30\frac{5}{20}$. All Day the aforeſaid Weather.

12. Thick cloſe rainy Weather all Day and Night, with moderate Gales at N N W. ☿ $30\frac{2}{20}$. Much Rain at Night.

13. Very thick cloſe rainy Weather, with freſh Gales at N W. ☿ 30. In the Night cold.

14. A ſharp cold Morning, with much Snow falling and cloſe Weather, with freſh Gales at N W. ☿ $30\frac{2}{20}$. Continu'd Snowing a little all Day and Night following. ☿ $30\frac{3}{20}$.

15. Fair Weather, freezing hard, with ſome Sun-ſhining and freſh Gales at N N W and N by W. ☿ $30\frac{5}{20}$.

16. Fair and ſerene Weather all Day and Night, freezing hard, with moderate Gales at N N W. ☿ $30\frac{7}{20}$. The Sun melting the Snow.

17. Fair and ſerene Weather, (the Sun diſſolving moſt of the Snow) with freſh Gales at N by W. ☿ $30\frac{9}{20}$. At Night ſomewhat cold, freezing hard.

18. Grey

18. Grey cloudy Weather, freezing hard, with moderate Gales at N by W. ☿ $30\frac{11}{20}$. At Night rising to $30\frac{11}{20}$.

19. Grey cloudy Weather, with little Frost, and moderate Gales at N N W. ☿ $30\frac{10}{20}$.

20. Grey cloudy Weather, somewhat close, with moderate Gales at N N W. ☿ $30\frac{9}{20}$, freezing in the Morning, and inclining to thaw at Night.

21. In the Morning somewhat serene, the rest of the Day overcast, with moderate Gales at N W, and some Thaw. ☿ $29\frac{9}{20}$.

22. Fair and serene Weather, with small Gales at W N W. ☿ $30\frac{7}{20}$. thawing all Day with the Heat of the Sun, at Night cold, but not freezing. ☿ as before.

23. Grey and cloudy Weather, with small Gales at S E, thawing a little. ☿ $30\frac{5}{20}$. At Night much Rain and calm. ☿ $30\frac{4}{20}$.

24. A fair and serene Morning with small Gales at N N W and N. ☿ $30\frac{4}{20}$. All Day overcast, and drizling Rains all Night.

25. Close hazy Weather, with drizling Rain and no Wind. ☿ $30\frac{4}{20}$, At Night much Rain.

26. Very close hazy Weather, with drizling Rains and small Breezes at S E. ☿ $30\frac{1}{20}$. In the Afternoon falling to $29\frac{18}{20}$, at Night much Rain, the Wind veering to N N W. blowing sometimes in Gusts.

27. Close Weather, with drizling Rains and moderate Gales at N W. ☿ $30\frac{2}{20}$. In the Afternoon fair, and at Night freezing.

28. Grey cloudy Weather, freezing hard all Day, with moderate Gales from N W to N. ☿ $30\frac{4}{20}$. *This Day being New Moon, begun the* Chinese *New Year.*

29. Fair and serene frosty Weather, with moderate Gales at N by W. ☿ $30\frac{6}{20}$.

30. The Morning fair and serene, all Day overcast with moderate Gales at N to N W. ☿ $30\frac{7}{20}$. Frosty Weather.

31. Fair and serene Weather, freezing hard, with moderate Gales at W N W. ☿ $30\frac{7}{20}$.

1. Fair and pleasant Weather, with small Gales from W, veering to S S E, and at Night to N N W, but no Frost. ☿ $30\frac{6}{20}$. *February.*

2. Fair and pleasant Weather, in the Morning, little Wind, in the Forenoon fine Gales at N W. ☿ $30\frac{8}{20}$. At Night little Wind.

3. Dry Weather, somewhat overcast, with small Gales at N W. ☿ $30\frac{6}{20}$. At Night little Wind.

4. Dry Weather, somewhat close, with small Gales at S E. ☿ $30\frac{6}{20}$.

5. Fair and cloudy Weather, with small Gales at N W. ☿ $30\frac{4}{20}$.

6. The Morning close and overcast, the Afternoon serene, with small Gales at N W. ☿ $30\frac{5}{20}$.

7. Fair and cloudy Weather, with ſmall Gales at N N W. ☿ $30\frac{6}{20}$.

8. Fair Weather, ſomewhat cloudy, with variable Breezes round the Compaſs. ☿ $30\frac{6}{20}$.

9. Fair and pleaſant Weather, with ſmall Breezes at S E. ☿ $30\frac{7}{20}$. At Night little Wind from N N W.

10. Cloudy Weather, with moderate Gales at N by W, in the Afternoon and all Night drizling Rains. ☿ $30\frac{4}{20}$.

11. Cloſe Weather, with drizling Rains and ſmall Gales at N by W. ☿ $30\frac{5}{20}$. All Night much Rain.

12. Cloſe Weather, with ſome drizling Rains and ſmall Gales at N by W, and N N W. ☿ $30\frac{3}{20}$.

13. Fair and pleaſant Weather, with ſmall Gales from N to S S E. ☿ $30\frac{5}{20}$.

14. Cloſe and cloudy Weather, with drizling Rains, and ſmall Gales at N N W. ☿ $30\frac{1}{20}$. Afternoon and Night fair, pleaſant and calm. ☿ as before.

15. Fair and ſerene Weather, and no Wind. ☿ $30\frac{3}{20}$. The Afternoon overcaſt, with cloſe Weather, and moderate Gales at S E, and ſome Rain. ☿ 30.

16. Cloudy Weather and ſomewhat cloſe, with ſmall Gales at N N W. ☿ $29\frac{19}{20}$. In the Night freſher Gales.

17. Grey cloudy Weather with moderate Gales at N N E. ☿ $30\frac{3}{20}$.

18. Grey cloudy Weather, with moderate Gales at N. ☿ $30\frac{4}{20}$.

19. Grey cloudy Weather, with moderate Gales at N N W. ☿ $30\frac{4}{20}$.

20. Grey cloudy Weather, with ſmall Gales at N by W. ☿ $30\frac{6}{20}$. Very cold with ſome Snow at Night.

21. Grey cloudy Weather, with ſmall Gales at N. ☿ $30\frac{9}{20}$. Some Snow this Morning, whitening the Tops of the Hills and lying all Day.

22. In the Morning ſome Sun-ſhining diſſolving the Snow; all Day grey cloudy Weather, and temperate with ſmall Gales at N N W. ☿ $30\frac{8}{20}$.

23. Dry Weather, ſomewhat cloudy, calm in the Morning, at Noon blowing freſh from N W till Night, then little Wind. ☿ $30\frac{6}{20}$.

24. Fair and pleaſant, with ſmall Gales at S E, the Afternoon calm, at Night moderate Gales from N N W. ☿ $30\frac{6}{20}$.

25. Fair and pleaſant, with moderate Gales at S S E and S E. ☿ $30\frac{6}{20}$.

26. Grey cloudy Weather, with drizling Rains all Day and Night, and moderate Gales at S E. ☿ $30\frac{6}{20}$.

27. Fair Weather, clearing up with ſmall Gales at S E.

28. Fair and pleaſant Weather, with ſmall Gales N W. In the Afternoon veering to W S W, and about to S E. ☿ 30 $\frac{6}{20}$.

1. Dry Weather, ſomewhat cloudy, with moderate Gales at S E. ☿ 30 $\frac{3}{20}$. *March.*

2. The Morning fair and very ſerene, the Afternoon overcaſt with ſmall Gales at S E. ☿ 30 $\frac{6}{20}$.

3. The Morning fair and ſerene, the Afternoon overcaſt with moderate Gales at E S E and S E. ☿ 30 $\frac{5}{20}$.

4. Grey cloudy Weather with moderate Gales at S E. At Night blowing freſh from N E. ☿ 30 $\frac{4}{20}$.

5. Grey cloudy Weather, with ſome Rain, and moderate Gales at S E. At Night ſome Thunder, Lightning and Rain. ☿ below 30.

6. The Morning ſerene and temperately warm, with ſmall Southwardly Breezes. The Forenoon and all Day overcaſt, in the Afternoon ſome Rain, cloſe hazy Weather with ſmall Gales at N by E and N. ☿ 30.

7. Grey cloudy Weather, with ſmall Gales at E S E. At Night much Rain. ☿ 30.

8. Grey cloudy Weather, ſomewhat hazy, with moderate Gales at N and N by E. ☿ 30 $\frac{4}{20}$.

9. Fair and pleaſant Weather in the Forenoon, with ſmall Gales at S E. ☿ 30 $\frac{6}{20}$. The Afternoon overcaſt, and little Wind all Night. ☿ 30 $\frac{5}{20}$.

10. Dry Weather, ſomewhat cloſe, with ſmall Gales at S E, in the Evening ſome Rain. ☿ 30 $\frac{4}{20}$. In the Night much Wind and Rain.

11. Cloſe and cloudy Weather, with ſmall Rains and moderate Gales at N N W. ☿ 30 $\frac{4}{20}$.

12. Cloſe and cloudy Weather, with moderate Gales from S E to N E. ☿ 30 $\frac{4}{20}$.

13. Grey cloudy and cloſe Weather, with ſome Rain and ſmall Gales at S E. ☿ 30.

14. Grey cloudy and cloſe Weather, with ſmall Gales variable from S E to N W. ☿ 30 $\frac{3}{20}$.

15. Very cloſe hazy Weather, calm all Day. ☿ 29 $\frac{17}{20}$. At Night ſmall Gales at W N W with ſome Rain.

16. Cloſe and cloudy Weather, with ſmall Gales from N W to N. ☿ 30.

17. Fair and ſerene Weather, with ſmall variable Gales. ☿ 30 $\frac{4}{20}$.

18. In the Morning overcaſt with moderate Gales at N W, preſently veering to S E, with drizling Rains all Day. ☿ 30 $\frac{3}{20}$, falling to 29 $\frac{17}{20}$.

19. Grey cloudy Weather, for the moſt part calm. ☿ 29 $\frac{17}{20}$.

20. Very

20. Very thick foggy Weather all Day, with ſmall variable Breezes, for the moſt part calm. ☿ 29 $\frac{17}{20}$.

21. In the Morning foggy, the reſt fair and ſerene Weather, with ſmall Southwardly Gales ☿ 29 $\frac{18}{20}$.

22. The Morning fair and ſerene, the Afternoon overcaſt with ſome Rain and variable Gales. ☿ 30.

23. Grey cloudy Weather, the Afternoon and all Night drizling Rains and moderate Gales at N N E. ☿ 30.

24. Cloſe Weather with continu'd drizling Rains, and at Night much Rain with moderate Gales from N N E to N, and very cold. ☿ 30 $\frac{1}{20}$.

25. The Morning ſomewhat cloudy, the Afternoon ſerene, with moderate Gales at N. ☿ 30 $\frac{1}{20}$.

26. Dry Weather, ſomewhat cloudy, with ſmall variable Gales. ☿ 30 $\frac{4}{20}$.

27. Grey cloudy Weather, with ſmall variable Gales, ſometimes calm. ☿ 30 $\frac{2}{20}$.

28. Cloſe hazy Weather, with ſome Rain and ſmall variable Gales, ſometimes calm. ☿ 29 $\frac{18}{20}$.

29. The Forenoon grey cloudy Weather, the Afternoon very cloſe and hazy with much Rain. ☿ 29 $\frac{17}{20}$. Small variable Breezes, for the moſt part, calm.

30. Cloſe and cloudy Weather, with moderate Gales at N N E. ☿ 29 $\frac{18}{20}$. At Night riſing to 30, and the Gale freſhning.

31. Fair Weather, ſomewhat cloudy, with ſmall variable Gales. ☿ 30.

April. 1. The Forenoon overcaſt, Afternoon more ſerene, with ſmall Southerly Gales. ☿ 30 $\frac{1}{20}$.

2. Fair and pleaſant Weather, with ſmall Gales at S W. ☿ 30 $\frac{4}{20}$.

3. Grey cloudy Weather, blowing freſh in the Forenoon at N E. Afternoon moderate. ☿ 30 $\frac{4}{20}$.

4. Fair and ſerene Weather, the Horizon ſomewhat cloſe, with eaſy Gales at S. ☿ 30 $\frac{4}{20}$. In the Afternoon the Gale veer'd to W, and the Sky ſomewhat hazy.

5. Cloſe and cloudy Weather, with ſome drizling Rain in the Forenoon, and ſmall Gales from S to E S E. The Night calm and rainy. ☿ 30.

6. Cloſe foggy Weather, with drizling Rains and calm. ☿ 29 $\frac{19}{20}$.

7. Fair and pleaſant Weather, ſomewhat cloudy and calm. In the Evening ſmall Gales at E to N E. All Night cloſe foggy Weather. ☿ 29 $\frac{15}{20}$.

8. Cloſe foggy Weather, with ſmall Northerly Breezes, for the moſt part calm. ☿ 29 $\frac{15}{20}$.

9. The Forenoon cloſe and cloudy, Afternoon fair and pleaſant, with ſmall Gales at N. In the Evening calm. ☿ 29 $\frac{16}{20}$.

10. Fair

10. Fair and pleaſant Weather, ſometimes overcaſt, with ſmall Gales from S W to S. ☿ $29\frac{18}{20}$.

11. Fair and pleaſant Weather, with ſmall Gales from S to S E. ☿ 30.

12. Fair Weather, ſometimes overcaſt, with moderate Gales at N. ☿ $30\frac{2}{20}$.

13. Dry Weather, ſomewhat hazy, with freſh ſharp Weſterly Gales. ☿ $30\frac{4}{20}$. In the Evening ſerene and little Wind. ☿ $30\frac{3}{20}$.

14. Fair and pleaſant Weather, with ſmall Gales at S E. ☿ $30\frac{3}{20}$.

15. Fair and pleaſant Weather, with ſmall Gales from S to S E. In the Evening calm. ☿ $30\frac{2}{20}$.

16. The Forenoon fair and pleaſant, with moderate Gales at S E. ☿ $30\frac{2}{20}$. In the Afternoon overcaſt, with ſome Rain and ſmall Gales at N W. ☿ 30.

17. The Forenoon fair and pleaſant, with moderate Gales at S E. ☿ $30\frac{2}{20}$. The Afternoon overcaſt, and the Gale freſhning at Night. ☿ 30.

18. Fair and pleaſant Weather, ſomewhat cloudy, with ſmall Gales at S E. ☿ 30.

19 Cloudy Weather, with a hazy Sky, and ſmall Gales at S E. ☿ $29\frac{18}{20}$.

20. Dry cloudy Weather, with fine Gales at S E. ☿ $29\frac{17}{20}$.

21. Grey cloudy Weather, with ſome Rain in the Forenoon, and ſmall Breezes at S E, for the moſt part calm ; the Afternoon fair. ☿ 29.

22. Cloſe and cloudy Weather, with hazy and calm ; in the Afternoon ſome Rain and ſmall Breezes at S E. ☿ $29\frac{16}{20}$.

23. Dry Weather, ſomewhaat foggy and cloudy, with ſmall Gales at S E. ☿ $29\frac{16}{20}$. In the Evening thick foggy Weather.

24. Grey cloudy Weather, ſomewhat foggy, with moderate Gales at S E. ☿ $29\frac{17}{20}$. At Night much Fog.

25. Cloudy and foggy Weather, with fine Gales at S E. ☿ $29\frac{15}{20}$. At Night much Rain, Thunder and Lightning, with little Wind. ☿ $29\frac{14}{20}$.

26. Cloſe and cloudy Weather, with ſmall Gales at N W. $29\frac{16}{20}$. In the Afternoon ſomewhat hazy, with ſmall drizling Rain.

27. The Weather clearing up with eaſy Gales at S E. ☿ $29\frac{17}{20}$. The Afternoon overcaſt, and n the Evening much Rain with ſome Thunder and Lightning, the Wind veering to N W, and back to S E. ☿ $29\frac{15}{20}$.

28 Grey cloudy Weather, with ſome Fog, and ſmall Gales from W to N W. ☿ $29\frac{17}{20}$.

29 Cloſe and cloudy Weather, with ſmall Gales from N W to N. ☿ $29\frac{17}{20}$. At Night ſome Rain.

30. Grey cloudy Weather, ſomewhat cloſe, with freſh Gales at N W. ☿ $29\frac{17}{20}$.

May. 1. A fair and ſerene Morning, with ſmall Breezes at W by S. ☿ $29\frac{18}{20}$. All Day and Night fair and pleaſant, with ſmall Breezes at N W. ☿ $29\frac{19}{20}$.

2. Fair and ſerene Weather, with ſmall Breezes at S E, and ſometimes calm. ☿ 30^{d}. In the Afternoon and all Night freſh Gales.

3. The Weather ſomewhat cloudy, with freſh Gales at S E. ☿ $29\frac{17}{20}$. In the Afternoon falling to $29\frac{14}{20}$. The Wind veering to W N W, had much Rain with Thunder and Lightning all Night.

4. The Morning cloſe foggy Weather, almoſt calm. ☿ $29\frac{17}{20}$. The Afternoon clear'd up, blowing freſh from N to N E. ☿ $30\frac{1}{20}$.

5. Fair and pleaſant Weather, ſomewhat cloudy, with moderate Gales at S E. ☿ 30^{d}.

6. Fair and pleaſant Weather, ſomewhat cloudy, with moderate Gales at S E. ☿ $29\frac{19}{20}$. Towards Noon little Wind. In the Afternoon a fine Gale at S E by E, and at Night thick foggy Weather. ☿ $29\frac{11}{20}$.

7. Grey cloudy Weather, ſomewhat foggy, with ſmall Gales at S E, ☿ below $29\frac{18}{20}$.

8. Fair and pleaſant Weather, ſomewhat foggy on the Hills, and ſmall variable Breezes from S W to N W. ☿ $29\frac{18}{20}$. At Night much Fog.

9. Fair and pleaſant Weather, with ſome Fog on the Hills, with ſmall variable Breezes from S W to N W. ☿ 30^{d}. The Afternoon ſerene.

10. The Forenoon fair and pleaſant, ☿ falling from $29\frac{18}{20}$ to $29\frac{17}{20}$. The Afternoon overcaſt with ſmall Gales from S to S E.

11 Grey cloudy Weather, with ſmall Gales N E. ☿ $29\frac{17}{20}$. In the Afternoon ſome Rain.

12. Fair and pleaſant Weather, ſomewhat cloudy, with ſmall Northerly Gales. ☿ $29\frac{18}{20}$.

13. Fair and ſerene Weather, with ſmall Gales at S W, ſometimes calm. ☿ above $29\frac{11}{20}$.

14. Fair and ſerene Weather, with ſmall Gales variable from W to N E. ☿ 30^{d}. The Afternoon ſomewhat overcaſt.

15. Fair and pleaſant Weather, with ſmall Gales at N E. ☿ $29\frac{18}{20}$. At Night calm.

16. Fair and ſerene Weather, with ſmall Gales from N E to S E. ☿ falling to $29\frac{17}{20}$. In the Evening overcaſt with ſome Fog.

17. The Morning ſome Fog on the Hills, all Day fair and ſerene, with ſmall Gales at S E. ☿ $29\frac{18}{20}$. At Night calm.

18. The Morning ſomewhat cloſe and foggy, all Day fair and pleaſant with fine Gales at S E. ☿ 30^{d}.

19. Fair and pleaſant Weather, with fine Gales at S E. ☿ 30^{d}.

20. Fair and pleaſant Weather, with freſh Gales at S E, the Sky ſomewhat hazy. ☿ $29\frac{18}{20}$.

21. Grey cloudy Weather, with freſh Gales at S E. ☿ falling below $29\frac{18}{20}$. At Noon ſome ſmall Rain.

22. Cloudy hazy Weather, with ſmall drizling Rains and moderate Gales at S E. ☿ below $29\frac{17}{20}$.

23. Cloſe and cloudy Weather, with moderate Gales at S E. ☿ $29\frac{16}{20}$.

24. Thick hazy Weather, with continual Rain, and freſh Gales from N N E to S E. ☿ falling below $29\frac{12}{20}$.

25. Cloſe and cloudy Weather, with moderate Gales from E to N E. ☿ $29\frac{11}{20}$.

26. The Forenoon thick hazy Weather, and the Afternoon grey cloudy Weather, with ſmall Gales at N E. ☿ riſing above $29\frac{14}{20}$.

27. Grey cloudy Weather, with moderate Gales at S E. ☿ above $29\frac{15}{20}$. Some Rain at Night.

28. Grey cloudy Weather, with the Wind from S to S S E, ſometimes ſmall Gales, and ſometimes blowing freſh. ☿ $29\frac{15}{20}$. Rain at Night.

29. Grey cloudy Weather, with fine Gales at S E, and drizling Rains. ☿ $29\frac{15}{20}$. At Night ſmall Gales at N E, ſometimes calm with thick Weather.

30. Thick foggy Weather, for the moſt Part calm, and ſmall drizling Rains. ☿ $29\frac{13}{20}$. At Night ſmall Gales at S E.

31. Grey cloudy Weather, ſomewhat foggy, with ſmall Gales at S E. ☿ $29\frac{18}{20}$. At Night ſome Rain.

1. Grey cloudy and foggy Weather, with ſome Rain in the Forenoon, and ſmall Gales at W. ☿ $29\frac{13}{20}$. In the Afternoon, riſing to $29\frac{15}{20}$. The Wind veering to N, and the Weather clearing up. *June.*

2. The Forenoon fair and pleaſant Weather and calm. ☿ $29\frac{17}{20}$. The Afternoon overcaſt, with ſome ſmall Rain, and ſmall Gales at S E. ☿ falling below $29\frac{16}{20}$. At Night ſome Rain.

3. All this Forenoon cloſe thick rainy Weather, with ſmall Gales at N E. ☿ falling below $29\frac{13}{20}$ The Afternoon dry, cloudy and calm.

4. Fair and pleaſant Weather, with fine freſh Gales from S E to S S E. ☿ above $29\frac{14}{20}$.

5. Fair and ſerene Weather, very hot, with ſmall Gales from W S W to N W. ☿ above $29\frac{15}{20}$.

6. Fair and ſerene Weather, with variable Gales round the Compaſs. ☿ $29\frac{17}{20}$. The Afternoon ſomewhat cloudy, and at Night calm.

7. The Forenoon overcaſt and foggy, with ſmall Gales at S E, and ſince Noon drizling Rains. ☿ falling to $29\frac{14}{20}$. Much Rain in the Night, blowing in Guſts.

8. Cloſe hazy Weather, with drizling Rains all Day, and ſmall variable Breezes, for the moſt Part calm. ☿ $29\frac{13}{20}$. At Night fair.

9. This Morning clearing up, with ſome Drops of Rain and calm. ☿ riſing to 29 $\frac{16}{20}$. The Afternoon overcaſt, and ſome Rain in the Evening.

10. Grey cloudy Weather, ſomewhat foggy, with ſmall Gales at S E, ſometimes calm. ☿ 29 $\frac{18}{20}$. The Afternoon and Night drizling Rains.

11. This Morning cloudy and foggy, with drizling Rains and calm. ☿ below 29 $\frac{18}{20}$. The Afternoon cloſe and foggy, with drizling Rains and ſmall Gales from E S E to S E. ☿ falling to 29 $\frac{14}{20}$.

12. Cloſe foggy Weather, with little Wind at S E, and ſometimes calm. ☿ 29 $\frac{13}{20}$. The Afternoon and all Night very much Rain.

13. Cloſe foggy Weather, with much Rain in the Forenoon, and drizling in the Afternoon, with ſmall variable Breezes, ſometimes calm. ☿ 29 $\frac{13}{20}$.

14. The Morning cloſe and foggy, the Forenoon clear'd up, with fair and pleaſant Weather, and ſmall Breezes at S W. ☿ 29 $\frac{14}{20}$. The Afternoon overcaſt, with ſmall Gales at S E. In the Evening ſome vehement Thunder, with Lightning and much Rain. ☿ below 29 $\frac{14}{20}$.

15. The Forenoon cloſe foggy Weather, the Afternoon grey and cloudy, with ſmall Gales from S E to N E, ſometimes calm. ☿ below 29 $\frac{14}{20}$.

16. The Forenoon cloſe and foggy, with ſome Rain; at Noon cleared up with ſmall Gales from S E. ☿ above 29 $\frac{11}{20}$. At Night cloudy with ſome Lightning.

17. The Morning very hazy and calm, the Forenoon clear'd up with ſmall Gales at S E. ☿ above 29 $\frac{14}{20}$. The Afternoon overcaſt, with the Gale veering to N by E, much Rain, Thunder and Lightning. ☿ as before.

18. Grey cloudy Weather in the Morning, and cleared up in the Afternoon, with ſmall Gales at S E. ☿ 29 $\frac{16}{20}$. The Afternoon overcaſt with freſh Gales continuing all Night. ☿ as before.

19. This Forenoon grey cloudy Weather, ſometimes clearing up, with freſh Gales at S E. ☿ 29 $\frac{16}{2}$.

20. Fair and cloudy Weather, with moderate Gales at S E. ☿ 29 $\frac{17}{20}$.

21. The Forenoon fair and ſomewhat cloudy, the Afternoon ſerene and pleaſant, with ſmall Gales at S E. ☿ below 29 $\frac{16}{20}$.

22. The Forenoon fair and pleaſant, with fine Gales at S E. ☿ below 29 $\frac{16}{20}$. The Afternoon grey and cloudy, with little Wind.

23. Fair and pleaſant Weather, with ſmall Breezes at S E, for the moſt Part calm, with ſome Lightning in the Night. ☿ 29 $\frac{16}{20}$.

24. Fair and pleaſant Weather, with ſmall Gales at S E, ſometimes calm. ☿ 29 $\frac{16}{20}$. At Night overcaſt and calm, with ſome Lightning.

25. Fair and ſerene Weather, with ſmall Gales at S E, in the Forenoon. ☿ 29 $\frac{16}{20}$. The Afternoon fine freſh Gales from S E to S by E, and blowing very freſh all Night.

26. Fair and pleaſant Weather, blowing very freſh from S by E to S by W, about Noon little Wind. ☿ below $29\frac{16}{20}$. The Afternoon moderate Gales at S E.

27. Fair and ſerene Weather, with moderate Gales at S E. ☿ below $29\frac{16}{20}$.

28. Fair and ſerene Weather, with moderate Gales at S E. ☿ as before.

29. Fair and ſerene Weather, with fine Gales at S E. ☿ $29\frac{14}{20}$. The Evening overcaſt and blowing freſh all Night.

30. Cloudy Weather, and ſomewhat hazy, with freſh Gales from E S E to N E. ☿ below $29\frac{14}{20}$. The Afternoon much Rain and blowing hard all Night at S E. ☿ $29\frac{12}{20}$.

July.

1. Cloudy and hazy Weather, with ſome Rain and hard Gales at S E. ☿ $29\frac{12}{20}$.

2. The Forenoon fair and ſomewhat cloudy, the Afternoon ſerene, with fine freſh Gales at S E. ☿ below $29\frac{11}{20}$. All Night cloudy.

3. Dry Weather, ſomewhat cloudy, with fine Gales at S E. ☿ $29\frac{14}{20}$.

4. Fair and ſerene Weather, with fine freſh Gales at S E. ☿ $29\frac{14}{20}$. The Evening overcaſt.

5. Fair and pleaſant Weather, ſomewhat cloudy, with a ſmall Shower in the Forenoon, ſome Thunder and eaſy Gales at S E. ☿ above $29\frac{15}{20}$. At Night little Wind.

6. Fair and ſerene Weather, with ſmall Gales at S E. ☿ $29\frac{16}{20}$. At Night little Wind.

7. Fair and pleaſant Weather, ſomewhat cloudy, with fine Gales at S E. ☿ $29\frac{14}{20}$.

8. The Forenoon overcaſt, with ſome Rain and Guſts of Wind at S E; the Afternoon fair and pleaſant, with eaſy Gales at S E. ☿ above $29\frac{11}{20}$. At Night ſome ſmall Rain.

9. Fair and pleaſant Weather, ſometimes cloudy, with fine Gales at S E. ☿ above $29\frac{11}{20}$.

10. The Morning ſomewhat hazy and cloudy, all Day fair and ſerene Weather, with eaſy Gales at S E. ☿ below $29\frac{11}{20}$.

11. Fair and pleaſant Weather, with ſmall Gales at S E. ☿ below $29\frac{12}{20}$. The Afternoon overcaſt and little Wind. ☿ falling to $29\frac{11}{20}$. Some Thunder and Lightning.

12. The Forenoon fair and pleaſant, with ſmall Gales at S E. ☿ below $29\frac{10}{20}$. The Afternoon overcaſt, with ſeveral ſmall Showers of Rain and little Wind, all Night calm.

13. Fair and pleaſant Weather, with ſome Clouds and calm. ☿ $29\frac{12}{20}$. Towards Noon overcaſt, and the Wind in ſmall Gales veering to S W. with cloſe rainy Weather all the Afternoon, in the Evening dry and cloudy, with ſmall Gales at W. ☿ $29\frac{14}{20}$.

 14. This

14. This Morning and Forenoon cloſe and cloudy Weather, with much Rain and ſmall Gales at W. ☿ 29 $\frac{14}{20}$. The Afternoon dry and cloudy, the Wind and ☿ as before.

15. Fair and pleaſant Weather, with ſmall Breezes from W to S W, ſometimes calm. ☿ 29 $\frac{16}{20}$.

16. Dry cloudy Weather, with ſmall Gales at S E. ☿ 29 $\frac{14}{20}$.

17. Fair and pleaſant Weather, ſometimes cloudy, with fine freſh Gales at S E by S. ☿ above 29 $\frac{14}{20}$.

18. Fair and pleaſant Weather, ſometimes overcaſt, with freſh Gales at S E. At Night blowing very hard. ☿ 29 $\frac{14}{20}$.

19. Fair and pleaſant Weather, with ſmall Gales at S E. ☿ 29 $\frac{16}{20}$.

20. Fair and ſerene Weather, with fine freſh Gales in the Forenoon, and fine freſh Gales in the Afternoon. ☿ 29 $\frac{15}{20}$. At Night calm.

21. Fair and ſerene Weather, with fine Gales at S E. ☿ 29 $\frac{15}{20}$.

22. Fair and ſerene Weather, with fine Gales at S E. ☿ 29 $\frac{15}{20}$. At Night ſometimes little Wind, at other Times blowing freſh.

24. Fair and ſerene Weather, with fine Gales at S E. ☿ 29 $\frac{16}{20}$. Afternoon blowing freſh.

25. Very fair and ſerene Weather, with ſmall Gales at S E. ☿ 29 $\frac{17}{20}$. At Night ſome Lightning.

26. Grey cloudy Weather, with ſmall Gales at S E, at Night calm and ſome Lightning.

27. Grey cloudy Weather, with ſmall Gales at N N W in the Forenoon, veering to N E in the Afternoon. ☿ 29 $\frac{16}{20}$. At Evening much overcaſt, with Thunder, Lightning and ſome Rain.

28. Fair and pleaſant Weather, ſomewhat cloudy, with ſmall variable Gales from N W to N E. ☿ 29 $\frac{15}{20}$.

29. Grey cloudy Weather, with ſmall variable Gales from N E to S W, ſometimes calm. ☿ 29 $\frac{15}{20}$. In the Evening a ſmall Shower of Rain, with ſome Thunder and Lightning.

30. Fair and ſerene Weather and calm. ☿ 29 $\frac{17}{20}$. In the Afternoon ſmall Gales at N N W, very hot and ſultry; at Night little Wind with ſome Thunder and Lightning.

31. Fair and pleaſant Weather, ſometimes cloudy, with fine Gales at S E. ☿ 29 $\frac{16}{20}$. Some Thunder and Lightning in the Afternoon.

Auguſt. 1. The Morning ſomewhat overcaſt, all Day fair and pleaſant, with ſmall Gales at S E. ☿ 29 $\frac{16}{20}$. The Afternoon fine Gales.

2. Very fair and ſerene Weather, with fine Gales at S E. ☿ 29 $\frac{17}{20}$.

3. Fair and ſerene Weather, with fine Gales at S E. ☿ 29 $\frac{18}{20}$.

4. Fair and ſerene Weather, with fine Gales at S E. ☿ 29 $\frac{18}{20}$.

5. The Forenoon grey cloudy Weather and calm, the Afternoon ſerene with ſmall Gales at S E by E. ☿ 29 $\frac{18}{20}$.

6. Fair and ſerene Weather, with ſmall Gales at N N W. ☿ 29 $\frac{16}{20}$. The Evening overcaſt, with ſome Lightning.

7. Fair Weather, ſometimes overcaſt, with freſh Gales from S to S S E. ☿ 29 $\frac{17}{20}$.

8. Fair

8. Fair and pleaſant Weather in the Forenoon, and cloudy in the Afternoon, with ſmall Gales at S S E. ☿ $29\frac{16}{20}$.

9. Fair and ſerene Weather, with ſmall Gales in the Forenoon, and freſhning in the Afternoon at S E. ☿ $29\frac{18}{20}$.

10. Fair and ſerene Weather, with ſmall Gales from S E to E by S. ☿ $29\frac{18}{20}$. At Night calm.

11. Fair and ſerene Weather, calm in the Forenoon, and ſmall variable Gales in the Afternoon, and Lightning at Night. ☿ $29\frac{18}{20}$.

12. Fair and ſerene Weather, with ſmall Gales at S S E, at Night calm and hazy. ☿ $29\frac{17}{20}$.

13 Fair and ſerene Weather, calm in the Forenoon, and ſmall variable Breezes in the Afternoon. ☿ $29\frac{18}{20}$

14. Fair Weather, ſometimes overcaſt, with ſmall Gales at N W by W. ☿ $29\frac{15}{20}$.

15. Fair Weather, ſomewhat cloſe, and ſome Rain in the Afternoon, with ſmall variable Gales. ☿ $29\frac{15}{20}$.

16. Fair and ſerene Weather, calm in the Forenoon, and ſmall variable Breezes in the Afternoon. ☿ $29\frac{15}{20}$.

17. The Weather cloudy and overcaſt, with ſome Rain in the Afternoon, and ſmall variable Gales round the Compaſs. ☿ $29\frac{16}{20}$.

18. Grey cloudy Weather, with ſome Rain, and eaſy Gales at N E. ☿ $29\frac{16}{20}$ In the Night very freſh Gales from N E to S E, and ſometimes at N W.

19. Grey cloudy Weather, with ſome Rain, and freſh Gales from S E to E S E. ☿ $29\frac{15}{20}$

20. Fair and pleaſant Weather, with fine Gales at S E. ☿ $29\frac{16}{20}$.

21. Fair and pleaſant Weather, with fine Gales at S E. ☿ $29\frac{16}{20}$.

22. Fair and ſerene Weather, with ſmall Gales at S E. ☿ $29\frac{18}{20}$.

23. Fair and pleaſant Weather, with ſmall Gales at S E. In the Afternoon ſomewhat cloudy, and the Gale freſhning. ☿ $29\frac{18}{20}$. In the Night blew very freſh.

24. Fair and ſerene Weather, with moderate Gales at S E. In the Evening much overcaſt, and at Night much Rain with ſome Thunder and Lightning. The Wind at N W. ☿ $29\frac{15}{20}$.

25. The Morning grey and cloudy, all Day fair and pleaſant, with ſmall Gales at N W. In the Afternoon veering to S E, and at Night calm. ☿ $29\frac{16}{20}$.

26. Fair and pleaſant Weather, with ſmall Gales at S E. ☿ $29\frac{16}{20}$.

27. Grey cloudy Weather, with drizling Rains this Morning and Forenoon; and fair in the Afternoon, with ſmall variable Gales. ☿ $29\frac{15}{20}$.

28. Fair and pleaſant Weather, with ſmall Gales from N to N E. ☿ $29\frac{17}{20}$. At Night calm.

29. Fair pleaſant Weather, with ſmall Gales at N E. ☿ $29\frac{17}{20}$. At Night ſome drizling Rain.

30. Fair

30. Fair and pleaſant Weather, ſomewhat cloudy, with eaſy Gales at N E. ☿ $29\frac{16}{20}$.

31. Grey cloudy Weather, with ſome Rain, and moderate Gales at N E. In the Night blowing in Guſts. ☿ $29\frac{14}{20}$.

September. 1. Fair Weather, ſometimes overcaſt, with freſh Gales from N to N N W. ☿ $29\frac{14}{20}$.

2. Cloudy Weather, with ſome Rain, and bluſtering Gales at N N W. ☿ $29\frac{15}{20}$.

3. Fair and pleaſant Weather, with moderate Gales at N N W, ſometimes overcaſt and blowing freſh; at Night little Wind. ☿ $29\frac{16}{20}$.

4. Fair and pleaſant Weather, with ſmall Gales from N W to N. At Night calm. ☿ $29\frac{18}{20}$.

5. Fair and pleaſant Weather, with ſmall Gales at N. ☿ $29\frac{18}{20}$.

6. Fair and pleaſant Weather, with ſmall Gales at N and N by W. ☿ $29\frac{18}{20}$.

7. Fair and pleaſant Weather, with ſmall Gales at N by W and N N W. ☿ below $29\frac{18}{20}$.

8. Fair and pleaſant Weather, with ſmall Gales at N N W. ☿ below $29\frac{18}{20}$.

9. Fair and ſerene Weather, with ſmall Gales at N. ☿ $29\frac{18}{20}$.

10. Fair and ſerene Weather, with ſmall Gales at N. ☿ $29\frac{18}{20}$.

11. Fair and ſerene Weather, with ſmall Gales at S E, and at Night veering to E N E. ☿ $29\frac{18}{20}$.

12. Fair and ſerene Weather, with ſmall Gales at S S E. ☿ $29\frac{18}{20}$.

13. The Forenoon fair and pleaſant, the Afternoon grey cloudy Weather, with ſmall Gales at E S E. ☿ 30^d.

14. Grey cloudy Weather, with ſmall Gales from E S E to E N E, ſometimes calm. ☿ 30^d. In the Night variable Gales with ſome Rain.

15. Grey cloudy Weather, with ſmall Gales from N E to S E by E. ☿ 30^d.

16. The Morning cloudy and overcaſt, all Day clear'd up with eaſy gentle Gales at S E. ☿ 30^d.

17. Fair and ſerene Weather, with moderate Gales at S E. ☿ 30^d.

18. Fair and ſerene Weather, with moderate Gales at S E. ☿ 30^d.

19. Fair and ſerene Weather, with ſmall Gales at S E. ☿ $29\frac{19}{20}$.

20. Cloſe hazy Weather, with ſmall Gales from N E. ☿ $30\frac{1}{20}$. In the Afternoon ſome Rain.

21. Grey cloudy Weather, with moderate Gales at N by E and N N E. ☿ $30\frac{3}{20}$. In the Evening little Wind.

22. Dry temperate Weather, ſomewhat grey and cloudy, with ſmall Gales at N by W. ☿ $30\frac{5}{20}$.

23. Fair and pleaſant Weather, ſomewhat cloudy, with ſmall Gales at N N W. ☿ $30\frac{4}{20}$.

24. Fair

24. Fair and pleaſant Weather, with ſmall Gales at N N W. ☿ 30 $\frac{2}{10}$.

25. Fair and pleaſant Weather, with ſmall Gales at N N W. ☿ 30 $\frac{1}{10}$.

26. Fair Weather, ſomewhat cloudy, in the Afternoon ſome Showers of Rain, and ſmall Gales from N N W to N W by W. ☿ 30 $\frac{2}{10}$, riſing at Night to 30 $\frac{4}{10}$.

27. Fair and cloudy Weather, with ſmall Gales from N N W to N. ☿ 30 $\frac{4}{10}$.

28. Fair and pleaſant Weather, with ſmall Gales from N by W to N by E.

29. Dry and cloudy Weather, with ſmall Gales at N N W.

30. Fair and pleaſant Weather in the Forenoon, with ſmall Gales at N N E, the Afternoon overcaſt with Guſts of Wind and ſome Showers of Rain.

1. The Morning overcaſt, all Day fair and pleaſant, with ſmall Gales from N N E to N. *October.*

2. The Morning grey and cloudy, all Day fair and pleaſant, with ſmall Gales at N by W.

3. Grey cloudy Weather, with moderate Gales at N and N by E, ſometimes blowing freſh; at Night thick and hazy with ſome Rain.

4. Thick hazy Weather, towards Noon clear'd up, Afternoon overcaſt with Rain and dark Weather. Small Gales at N N E.

5. Grey cloudy Weather, with ſmall Gales at N N E, inclining to Rain.

6. Thick hazy Weather, with much Rain and ſmall Gales at N N E. ☿ below 30^d^.

7. The Morning cloſe and cloudy Weather, with ſome Rain, and ſmall Gales at N and N by W. ☿ below 30^d^. All Day thick hazy Weather, with drizling Rains.

8. Hazy Weather, with drizling Rains and ſmall Gales at N N E and N E. ☿ 30 $\frac{1}{10}$.

9. Cloſe hazy Weather, with drizling Rains, and ſmall Gales at N E. ☿ 30 $\frac{4}{10}$. ☿ at Night 30 $\frac{5}{10}$.

10. Cloſe and cloudy Weather, with ſome Rain, and moderate Gales at N E. ☿ 30 $\frac{3}{10}$.

11. Grey cloudy Weather, with ſome Rain at Night and ſmall Gales at N N E. ☿ 30 $\frac{6}{10}$.

12. The Morning ſerene, with ſmall Gales at N E. ☿ 30 $\frac{7}{10}$. The Afternoon grey cloudy Weather. ☿ 30 $\frac{6}{10}$.

13. Dry cloudy Weather, with ſmall Gales at N by E. ☿ 30 $\frac{6}{10}$. The Afternoon more ſerene and almoſt calm.

14. Grey cloudy Weather, with moderate Gales at N E, ſometimes blowing freſh. ☿ 30 $\frac{3}{10}$.

15. Grey

15. Grey cloudy Weather inclining to Rain, with moderate Gales at N by W. ☿ $30\frac{2}{20}$. At Night much Rain.

16. Grey cloudy Weather, with ſome Rain at Night, and freſh Gales at N and N by W. ☿ $30\frac{4}{20}$.

17. Grey cloudy Weather, with moderate Gales at N by E, blowing freſh in the Night at N N W. ☿ $30\frac{4}{20}$.

18. Fair and pleaſant Weather, with moderate Gales at N W by N. ☿ $30\frac{5}{20}$.

19. Fair and ſerene Weather, with ſmall Gales at N W. ☿ $30\frac{6}{20}$.

20. Fair and ſerene Weather, with ſmall Gales at W N W and W by N. ☿ $30\frac{6}{20}$.

21. Fair and ſerene Weather, with ſmall Gales at N E and N E by E. ☿ $30\frac{6}{20}$.

22. Fair and ſerene Weather, with ſmall Gales at N and N by E. ☿ $30\frac{6}{20}$.

23. Fair and ſerene Weather, with ſmall Northerly Breezes, ſometimes calm. ☿ $30\frac{1}{20}$.

24. A grey cloudy Morning, blowing freſh at N N W. All Day fair and pleaſant, with moderate Gales at N. ☿ $30\frac{5}{20}$.

25. The Forenoon overcaſt, with fine freſh Gales at W N W, and ſharp Weather, the Afternoon more ſerene and ſmaller Gales. ☿ $30\frac{7}{20}$.

26. Fair ſerene Weather, with fine ſharp Gales at N N W. ☿ $30\frac{8}{20}$.

27. Fair and ſerene Weather, with ſmall Gales at E S E and S E, ſometimes calm. ☿ $30\frac{6}{20}$.

28. Grey cloudy Weather, with freſh Gales at N N W and N. ☿ $30\frac{8}{20}$.

29. Fair Weather, ſomewhat cloudy, with moderate Gales from N by W to N W. ☿ $30\frac{8}{20}$

30. Grey cloudy Weather, with moderate Gales at N W. ☿ $30\frac{8}{20}$.

31. Fair and ſerene Weather, with moderate Gales at N by W and N. ☿ $30\frac{10}{20}$. Very cold.

November.

1. Fair and ſerene Weather, with ſmall Gales at W N W. ☿ $30\frac{6}{20}$.

2. Grey cloudy Weather, with ſome Rain in the Evening, and moderate Gales at N N W. ☿ $30\frac{5}{20}$.

3. Fair and ſerene Weather, ſomewhat hazy, with very freſh Gales at N by W and N N W. ☿ below $30\frac{6}{20}$.

4. Fair and ſerene Weather, with ſmall Breezes at N by E. ☿ $30\frac{6}{20}$. At Night calm.

5. Fair and ſerene Weather, with ſmall Breezes at S E. Sometimes calm. ☿ $30\frac{6}{20}$.

6. Fair and ſerene Weather, with ſmall Breezes from W S W to W N W. ☿ $30\frac{5}{20}$.

7. Fair

7. Fair and ſerene Weather, with ſmall Gales at N W. ☿ 30 $\frac{6}{20}$.

8. This Morning foggy; all Day ſerene, with moderate Gales from S W to N W. ☿ 30 $\frac{6}{20}$.

9. Fair and pleaſant Weather, ſometimes overcaſt, with moderate Gales at S E and E S E. ☿ 30 $\frac{4}{20}$. At Night little Wind and calm.

10. Fair and ſerene Weather, with moderate Gales at S E. ☿ 30 $\frac{5}{20}$.

11. Grey cloudy Weather, with ſmall Gales at N and N by W. In the Evening calm. ☿ 30 $\frac{3}{20}$.

12. Fair and pleaſant Weather, with ſmall Gales at N. In the Evening calm. ☿ 30 $\frac{4}{20}$.

13. Fair and pleaſant Weather, with ſmall Gales from S by E to S E. ☿ 30 $\frac{4}{20}$. At Night calm.

14. Grey and cloudy Weather, ſomewhat hazy, with moderate Gales from W N W to N W. ☿ 30 $\frac{5}{20}$.

15. Grey cloudy Weather, with freſh Gales at N N E, and ſome ſmall Rain. ☿ 30 $\frac{5}{20}$.

16. Fair and pleaſant Weather, with moderate Gales from S to S E. ☿ 30 $\frac{6}{20}$.

17. Grey cloudy Weather, with moderate Gales from S by W to E S E. ☿ 30 $\frac{5}{20}$. At Night blowing hard, and veering to N W, with much Rain.

18. Grey cloudy Weather, with freſh Gales at N W. In the Afternoon blowing very hard. ☿ 30^d.

19. Grey cloudy Weather, with moderate Gales at N W. ☿ 30 $\frac{6}{20}$. At Night riſing to 30 $\frac{10}{20}$.

20. Fair and pleaſant Weather, with ſmall Gales from N to E, and about to S E. In the Evening little Wind. ☿ 30 $\frac{10}{20}$, falling to 30 $\frac{6}{20}$.

21. Grey cloudy Weather, for the moſt Part calm, with ſmall Northerly Breezes. ☿ 30 $\frac{6}{20}$.

22. Grey cloudy Weather, with moderate Gales at N W. ☿ 30 $\frac{6}{2}$.

23. Grey cloudy Weather, with fine Gales at N W. ☿ 30^{6}.

24. Grey cloudy Weather, with eaſy Gales from W N W to N W. ☿ 30 $\frac{6}{20}$.

25. Grey and cloudy Weather, with eaſy Gales at N W. ☿ 30 $\frac{6}{20}$.

26. Fair and ſerene Weather, with ſmall Gales at N W, ſometimes calm. ☿ 30 $\frac{6}{20}$.

27. Thick hazy Weather, with drizling Rains, and at Night much Rain, with ſmall Southerly Breezes, for the moſt Part calm. ☿ 30 $\frac{3}{20}$.

28. Hazy Weather, with drizling Rains all Day and Night, and ſmall Gales from N to N N E. ☿ 30 $\frac{1}{20}$, riſing to 30 $\frac{6}{20}$.

 29. Grey

29. Grey cloudy Weather, with ſmall Gales at N N E. ☿ $30\frac{5}{20}$.

30. Grey cloudy Weather, with fine Gales at N. ☿ $30\frac{8}{20}$.

December, 1701.

1. Grey cloudy Weather, and very cold with fine Gales at N. ☿ $30\frac{11}{20}$. Some Froſt at Night.

2. Dry Weather, ſomewhat cloudy, with moderate Gales at N N W. ☿ $30\frac{8}{20}$.

3. Fair and ſerene Weather, with ſmall Gales at N W. ☿ $30\frac{6}{20}$.

4. Fair and ſerene Weather, with ſmall Gales at N W and W N W. ☿ $30\frac{6}{20}$. At Night overcaſt.

5. Dry cloudy Weather, with moderate Gales at N by W. ☿ $30\frac{9}{20}$.

6. Fair and pleaſant Weather, with fine Gales from E to S E, and at Night calm. ☿ $30\frac{7}{20}$, falling to $30\frac{5}{20}$.

7. Grey cloudy Weather, with freſh Gales at N W and W N W. ☿ $30\frac{8}{20}$.

8. Fair Weather, ſomewhat cloudy, with freſh Gales at N W. ☿ $30\frac{11}{20}$.

9. Fair and pleaſant Weather, with fine Gales at N N W. ☿ $30\frac{11}{20}$.

10. Fair and ſerene Weather, with ſmall Gales at N W to N. ☿ $30\frac{11}{20}$. At Night calm.

11. Fair and ſerene Weather, with ſmall Gales at N W. ☿ $30\frac{1}{20}$.

12. Fair and ſerene Weather, ſometimes cloudy, with ſmall Gales at N W. ☿ $30\frac{9}{20}$.

13. Fair and ſerene Weather, with ſmall Gales at N W. ☿ $30\frac{7}{20}$. At Night calm.

14. Fair and ſerene Weather, with ſmall Gales at N W. ☿ $30\frac{6}{20}$. At Night calm.

15. Fair and ſerene Weather, with ſmall Gales at S S W. ☿ 30^{8}. At Night the Gale freſhned from S S E.

16. Grey cloudy Weather, with moderate Gales at S E. ☿ $30\frac{6}{20}$. At Night ſome Rain.

17 Grey cloudy Weather, with drizling Rains, and ſmall Gales at N W. ☿ $30\frac{5}{20}$. At Night calm.

18. Grey cloudy Weather, with moderate Gales at N W. ☿ $30\frac{6}{20}$. Some Sleet in the Night.

19. Grey cloudy Weather, with moderate Gales at N W. ☿ $30\frac{5}{20}$.

20. Fair and ſerene Weather, with ſmall Gales at N W. ☿ $30\frac{5}{20}$. At Night $30\frac{4}{20}$.

21. The Forenoon ſomewhat hazy and calm. ☿ $30\frac{5}{20}$. The Afternoon ſerene, with ſmall Gales at N W. ☿ $30\frac{6}{20}$.

22. Fair

22. Fair and ſerene Weather, freezing, with freſh Gales at N N W. ☿ $30\frac{11}{20}$.

23. Serene Weather, freezing hard, with freſh Gales at N N W. ☿ $30\frac{11}{20}$. At Night little Wind.

24. Serene Weather, freezing hard, with moderate Gales at W N W. ☿ $30\frac{11}{20}$.

25. Fair froſty Weather, with moderate Gales at N W. ☿ $30\frac{10}{20}$.

26. Fair froſty Weather, with ſmall Gales from W N W to N N W. ☿ $30\frac{10}{20}$. At Night Fog.

27. Fair and ſerene Weather, freezing with little Wind at N W, for the moſt part calm. ☿ $30\frac{9}{20}$.

28. Fair and ſerene Weather, freezing with ſmall Breezes at N, and ſometimes calm. ☿ $30\frac{8}{20}$.

29. Fair and ſerene Weather, freezing with ſmall Northerly Breezes, for the moſt part calm. ☿ $30\frac{6}{20}$. At Night ſome Wind and Rain.

30. The Morning foggy, all Day fair and pleaſant, with ſmall Gales from S E to E by S. ☿ $30\frac{4}{20}$.

31. Cloſe and cloudy Weather, with drizling Rains, for the moſt part calm. ☿ $30\frac{2}{20}$.

1. Thick hazy Weather, with drizling Rains, for the moſt part calm, with ſmall variable Breezes. ☿ $30\frac{3}{20}$. *January*, 1702.

2. Grey cloudy Weather, with ſome Rain, and ſmall Gales at N E. ☿ $30\frac{4}{20}$.

3. Grey cloudy Weather, with ſmall Gales at N and N N W. ☿ $30\frac{8}{20}$.

4. Grey cloudy Weather, with ſmall Northerly Gales. ☿ $30\frac{9}{20}$.

5. Grey cloudy Weather, with ſmall Northerly Gales. ☿ $30\frac{6}{20}$. Afternoon and Night ſerene.

6. Fair and pleaſant Weather, with moderate Gales at S E. In the Evening and all Night calm. ☿ $30\frac{3}{20}$.

7. Cloſe and hazy Weather, with ſmall Gales at E S E and S E, ſometimes calm. ☿ $30\frac{2}{20}$.

8. Cloſe and cloudy, in the Morning little Wind, towards Noon blowing ſomewhat freſh at N N W, and in the Evening moderate. ☿ $30\frac{2}{20}$, riſing to $30\frac{6}{20}$.

9. Grey cloudy Weather, with ſmall Northerly Gales. ☿ $30\frac{4}{20}$.

10. Grey cloudy Weather, with ſmall variable Gales, ſometimes calm. ☿ $30\frac{4}{20}$.

11. Fair and ſerene Weather, with ſmall Breezes at S E. ☿ $30\frac{5}{20}$.

12. The Weather overcaſt, with moderate Gales at W N W. ☿ $30\frac{6}{20}$.

13. Fair and pleaſant Weather, with moderate Gales at N W. ☿ $30\frac{10}{20}$.

14. Fair and ſerene Weather, with ſmall Gales at N. ☿ $30\frac{10}{20}$.

15. Fair and ſerene Weather, with ſmall Gales at S E. ☿ $30\frac{3}{20}$.

16. Grey cloudy Weather, with ſmall Gales from S S W to S E. The Afternoon cloſe and hazy, with ſome Rain. ☿ $30\frac{2}{20}$.

17. Cloſe hazy Weather, with moderate Gales from N W to N by W. ☿ $30\frac{2}{20}$. At Night little Wind. *The* Chineſe *New-Year began this Day.*

18. Cloſe and cloudy Weather and calm. ☿ $30\frac{2}{20}$. The Afternoon ſmall drizling Rains.

19. Grey cloudy Weather, with freſh Gales at N by W, and a little Snow. ☿ $30\frac{5}{20}$.

20. Grey cloudy Weather, with ſmall Gales at N W, the Afternoon ſerene, and freezing hard at Night. ☿ $30\frac{6}{20}$.

21. Fair and ſerene Weather, freezing, with ſmall Gales at N N W, for the moſt part calm. ☿ above $30\frac{6}{20}$.

22. Fair ſerene Weather, freezing hard, with ſmall Northerly Breezes, for the moſt part calm. ☿ $30\frac{8}{20}$.

23. Grey cloudy Weather and calm. ☿ $30\frac{6}{20}$.

24. Cloſe hazy Weather, with drizling Rains and calm. ☿ $30\frac{7}{20}$.

25. Grey cloudy Weather, with ſmall Gales at N E. ☿ $30\frac{6}{20}$. At Night little Wind and ſome Rain.

26. Cloſe and cloudy Weather, with ſmall Gales at N E. ☿ $30\frac{8}{20}$. At Night calm. $30\frac{2}{20}$.

27. Thick hazy Weather, with drizling Rains and ſmall Gales at N by E. ☿ $30\frac{1}{20}$, blowing freſh at Night.

28. Grey cloudy Weather, with freſh Gales at N N W. ☿ $30\frac{6}{20}$. At Night $30\frac{8}{20}$.

29. Grey cloudy Weather, with fine Gales at N W. ☿ $30\frac{8}{20}$.

30. Fair and pleaſant Weather, with moderate Gales at N W. ☿ $30\frac{7}{20}$.

31. Fair and ſerene Weather, with ſmall Gales at N W. ☿ $30\frac{6}{20}$.

A Regiſter of the Weather *for* 1692. *by Mr.* J. Locke. n. 298. p. 1917.

IX. I herewith ſend you a Copy of my Regiſter of the Weather at *Oates* in *Eſſex*, from the 9*th* of *December* 1691, to the End of the Year 1692. I ſhall firſt explain ſome Things in the Table.

The *Firſt* Column having D at the Top, contains the *Day* of the Month.

The next with H, the *Hour* of the Day, which beginning from 1 of the Clock in the Morning, I count round in one continued Series to 24, which is 12 of the Clock at Night.

The

The Column *Ther* is that of the Thermoſcope, which was a ſealed one, whereof you will find a larger Account hereafter.

The Column *Bar* marks the height of the *Mercury* in the Baroſcope. The firſt Number is the Inches of its height, the ſecond Number marks the 20th Parts of an Inch above that Inch mark'd by the firſt Number.

The Column *Hyg* is that which marks the Moiſture of the Air. The Inſtrument I us'd was the Beard of a wild Oat, of which each Turn was divided into ſixteen Degrees.

The Column of the Wind mark'd the Point the Wind was in, but not always exactly, becauſe the Weathercock viſible out of my Window, was ſtiff, and turn'd not eaſily; nor was the Houſe it ſtood on ſituate exactly Eaſt and Weſt; ſo that it was not eaſy by the ſtanding of the Weathercock, to know exactly the Point of the Wind: Wherefore I contented myſelf to ſet down barely one of the 4 Cardinal Points, when the Wind was pretty near it; and when it was more remote, the two Cardinal Points between which it was, putting the Letter of the Cardinal Point firſt to which it was neareſt; as when the Wind was between the South and the Weſt, if it were nearer the Weſt than the South, I writ W. S. and ſo of the reſt.

I mark'd beſides, the Force of the Wind, which I divided into four Degrees. 1. When it juſt moved the Leaves. 2. When it blew a pretty freſh Gale. 3. When it was hard and whiſtling Wind. 4. When it blew a Storm. Though theſe Diviſions were not made with that Exactneſs as they might have been, had one had an Inſtrument on purpoſe, yet they may give ſome Help to thoſe who would make Obſervations from ſuch Regiſters as theſe, (o) was when there was not Wind enough to move a Leaf as I could ſee.

As to the Weather, *Cloudy* ſignifies more of the Sky (viſible out of the Windows of my Study, which were Eaſt and South) cover'd with Clouds, than not. *Fair* the contrary. *Between* when it was uncertain whether more of the Sky was covered or clear. *Cover'd* when no part of the clear Sky appear'd. *Cloſe* when the Sky was cover'd with one uniform thick Cloud.

I have often thought that if ſuch a Regiſter as this were kept in every County in *England*, and ſo conſtantly publiſhed, many Things relating to the Air, Winds, Health, Fruitfulneſs, *&c.* might be collected from them, and ſeveral Rules and Obſervations concerning the Extent of Winds and Rains, *&c.* be in Time eſtabliſh'd, to the great Advantage of Mankind. From this ſolitary one there is little to be collected, beſides the ordinary Obſervation, which I ſet down commonly every Morning, there ſeldom happen'd any Rain, Snow, or other remarkable Change, which I did not ſet down.

N. B.

N. B. That the Thermofcope mark'd 4, which I made ufe of till *Mar.* 7. 1701. was one of thofe fold by Mr. *Tompion*, wherein o mark'd Temperate, and the Figures from thence increafing both upwards and downwards, fhew'd the Increafe of Heat and Cold from Temperate. *Sept.* 22. 1701. I began to ufe a new fealed Thermofcope, adjufted to a Scale made by Mr. *John Patrick*, who places o at the Top, fuppofing it to be the Heat under the Line, and fo the Figures increafe downwards, with the Increafe of Cold. Temperate being placed at 45. This Thermofcope is mark'd 5 in my Regifter.

December, 1691.

D.	H	Ther. 4	Bar.	Hyg.	Wind.		Weather.
9	16	1. 2	30. $\frac{4}{20}$		S	1	Fog.
10	17	2. 1	4		S E	1	Fair.
11	10	3. 1	4		S E	1	Fog.
12	9	3. 6	3		E	1	Froft, Clofe.
13	9	3. 4	30. 0		E	1	Clofe.
14	9	3. 7	29.17		E	2	Froft, Fair.
15	9	4. 2	30. 0		E	3	Froft, Fair.
16	9	4. 4	4		E	3	Froft, Fair.
17	9	4. 6	6		E	1	Froft, Fair.
18	9	4.	5		N E	1	Clofe.
19	9	3. 3	3		N E	1	Fog, Thaw.
	24		29.19			1	Cloudy, a little Rain next Morning.
20	10	3. 4	18		N	1	Clofe.
21	9	3. 6	19		N	1	Fog, Froft.
22	9	4.	18		N	1	Clofe.
24	10	2. 6	14		S W	2	Cloudy.
25	21	2. 1	15				Fair.
26	21	1. 7	13		W	3	Clofe.
	24		12				Rain before next Morning.
27	18	1. 6	6				
28	1	2. 6	9		S W	2	Fair.
	22		8				Rain.
	24		7		S	2	Rain hard.
29	9	3.	4				Clofe, and fome Time after Rain.
	19	2. 7	4				Snow beginning at 6.
30	8	3. 7	10		N W		Fair.
31	9	4. 7	12		N W	0	Fair, hard Froft.
	17	3. 3	10		S E	1	Cloudy, Thaw.

January 1692.

D.	H	Ther. 4	Bar.	Hyg.	Wind.		Weather.
1	9	3. 7	9. 11	47	S E	1	Cloſe, Snow at 10.
2	9	3. 7	14	43	W	1	Fair.
3	9	3. 5	8	41	S	2	Clouds.
4	8	3. 6	11	43	S	1	Cloudy.
5	9	3. 7	13	43	S	1	Cloudy.
6	8	4. 3	11	36	S	2	Cloſe.
	16	4.	8	36	E	3	Cloſe.
	20	4. 2	6	41			Snow.
7	10	4. 1	7	45	E	2	Cloſe.
	4	3. 7	17				Cloſe.
8	9	4. 1	30.	45	N E	1	Cloſe.
9	9	2. 6	29. 18	50	S	3	Cloſe.
	20		15				Rain.
10	9	2. 7	18	44	S W	1	Fair.
	12			26			In the Shade.
11	9	2. 6	30. 4	48	W N	1	Cloſe, with a little Fog.
	24	2. 5	30. 8	41			Very fair all the Afternoon, now cloſe.
12	9	2. 7	9	44	N	1	Miſty.
13	9	3. 4	9	46	S	1	Cloſe.
14	9	3. 7	7	48	E	1	Fog.
15	9	4. 4	6	31	E	1	Fog, Froſt.
16	9	4. 5	2	45	N	1	Fair, hard Froſt.
17	10	4. 1	29. 18	43	N E	2	Cloſe, hard Froſt.
18	8	4. 1	12	35	N W	2	Fair, Rain & Snow for an Hour about 16.
19	9	3. 7	9	44	N W	2	Snow till 13.
20	8	4. 5	5	43	N W	2	Fair. Some Clouds. Snow in the After.
21	9	5. 7	6		N	2	Fair. Snow in the Afternoon.
22	9	5. 6	9	18	N W	2	Very fair, hard Froſt.
	10	6. 2	11				In a Northern Cloſet without Fire at any Time.
	24	6. 5½					In the ſame Cloſet, very hard Froſt.
23	9	5. 7	11	31	N W	2	Fair, Snow a little at 14.
24	9		11	28	N W	1	Cloudy, hard Froſt.
25	9	5. 6	10	31	N W	1	Fair, hard Froſt.
26	9	6. 0	9	32	N W	1	Cloudy, hard Froſt.
27	7	5. 3	7	33	N	1	Cloudy, hard Froſt.
28	9	4. 6	4	33	N	1	Snow in the Night, and Snow ſtill.
29	9	5. 6	2	32	W	1	Cloſe, Snow all Afternoon.

N. B.

N. B. I ſuſpect that from the 23d to the 29th incluſively, the Hygroſcope has been counted 16 Degrees, *i. e.* one whole Turn too high, it being all that while very hard Froſt.

D.	H	Ther.		Bar.	Hyg.	Wind.		Weather.
			4					
30	9	4.	4	2		N W	2	Snow.
31	9	5.	3	8		N W	1	Cloſe.
February.								
1	8	5.	0	10	19	W		Fog.
2	9	5.	6	11	19	W	1	Fog.
3	8	5.	7	11	20	W	1	Fog.
4	9	5.	5	9		E	2	Cloſe.
5	9	5.	4	7	26	E	3	Cloſe, Snow in the Evening.
6	9	5.	1	29. 0	20	E	1	Snow.
7	9	4.	6	2		N	1	Cloſe Fog, little Thaw at 12.
8	9	4.	6	29. 0	40	N E	2	Fog, Snow in the Night.
9	8	5.	3	29. 0		N E	2	Snow.
10	9	5.	5	1		N E	2	Cloſe.
11	8	6.		8		W	1	Fair.
12	9	7.	0	11		N W		Cloſe, freezing exceſſively.
13	9	4.	2	5		S W	1	Cloſe Thaw.

It may be obſerv'd between the Thermoſcope to-day and yeſterday Morning, there is the Difference of two whole Degrees and $\frac{6}{8}$ a greater Riſe than one ſhall ordinarily find. The Thermoſcope was unmov'd in a Corner of a very large Room, out of all Reach of the Fire, whereby it might be alter'd.

D.	H	Ther.		Bar.	Hyg.	Wind.		Weather.
14				7				Cloſe.
16	12			10				Cloſe.
17	19			9				Fog.
18	12			5		W		Snow.
26	2			11		W	2	Cloſe.
27	9	2.	3	29. 8		W	2	Fair, Rain from 15 to 19.
28	9	3.	1	6		W	1	Cloſe, Snow laſt Night.
29	9	3.	6	6		S W	2	Cloudy, Snow laſt Night.
March.								
1	8	2.	7	28.17		S W	3	Fair.
2	8	3.	1	29. 5		S W	3	Fair.
3	7	1.	6	8		W S	2	Fair.
	16	1.	4	8		W S	1	Rain very gently till 20, hard till 23.
4	9	3.	1	14		N E	2	The Ground cover'd with Snow. Cloſe.

D.	H	Ther. 4.	Bar.	Hyg	Wind.	Weather.
5	9	3. 4	18		W 2	Cloudy.
6	9	2. 4	11		W 2	Rain till 16.
	16	2. 0	9		W 1	Rain.
	22	2. 2	10			Fair.
7	9	3. 5	10		W 2	A little dropping.
	12		9		W N 2	Snow for ¼ Hour.
8	9	3. 3	14		W 1	Fair.
9	8	3. 3	15	25	S W 2	Fair.
10	9	6	19	17	S 1	Fair.
	17	2. 2	18	19	W 2	A little Rain.
11	9	1	19	25	W N 1	Cloſe.
12	9	5	30. 1	23	N W 1	Cloſe.
	14	2. 0	30. 0	24	W N 2	Cloſe. Small Rain at 15 till 16.
	16	1. 7	29.19	40	W N 1	Very gentle Rain.
	23	7	16		N W 2	Hard Rain for 3 or 4 Hours paſt.
13	9	2. 3	13		W N	Very gentle ſmall Rain.
	12		11		N W 2	Fair.
14	9	2. 6	11	27	N W 2	Cloſe.
	16	5	13	25	N E 3	Fair.
15	8	4. 0	16	26	N E 2	Cloſe, Froſt laſt Night.
16	9	3. 4	14	25	N W 1	Cloſe.
	13	0	13	20	N W 1	Fair. Rain in the Night.
17	9	3. 0	8	25	N E 1	Cloſe.
	13		7		S E 1	Cloudy.
	18	2. 2	6		S E 2	Cloſe.
	19		5	29		Small Rain.
	23		4	35		Cloſe. Much Rain in the Night.
18	8	2. 3	1	41	W S 2	Cloſe.
19	9	6	3	34	S W 2	Cloudy.
20	9	2	2	31	S 2	Cloudy. Rain laſt Night, Houſes yet (dropping.
21	8	3. 3	3	29	0	Cloſe.
	14	1. 3	3	27	N E 2	A very little Shower.
22	8	3.	6	27	N E 1	Cloſe, a little, very little Snow in the Afternoon, ☿ riſing.
23	8	3. 5	10	26	1	Clouds.
	17	2. 7	10	20	S W 1	Cloſe.
24	8	3. 5	29.14	24	S W 1	Very fair, little Snow and Hail about 12, the ☿ falling a little.

D.	H	Ther. 4	Bar.	Hyg.	Wind.	Weather.
25	8	2. 7	10	27	S W 2	Fair.
26	8	1. 6	2	35	3	Rain very little at Night.
	12		5		S W 4	Cloſe.
	18	1. 0	7	29	S W 2	Cloſe.
	24		7	34	1	Hard Rain, the ☿ a little fallen.
27	8	1. 0	7	37	W S 2	Cloſe.
	17	0. 2	2	33	W 1	Small Rain.
28	7	0. 7	12	33	W 1	Cloſe.
	16	0. 5	12	30	W 1	Fair.
29	8	1. 6	10	32	W 1	Rain. Rain a great deal laſt Night.
	13		8	2	N 1	Cloſe, miſling Rain all this Morning till now.
	15		7		N 1	Cloſe.
	22	1. 6	5	31		Fair.
30	8	2.	1	32	W 2	Cloſe Rain laſt Night.
	15		2		N W 3	Rain as almoſt all this Morning.
	19		3		N W 3	Rain gently, as the great Part of this Afternoon.
	24		6			Fair.
31	9	3. 1	6	29	N W 1	Fair.
	24		3			Cloſe, Rain in the following Part of the Night.
April. 1	9	2. 6	2	52	S 1	Small Rain.
	24		1			The greateſt Part of the foregoing Day gentle Rain.
2	9	1. 7	28.16	39	S E 2	Cloſe, Rain a great deal laſt Night, ſo as to make a Flood.
	16	1	17		S W 2	Fair.
	24		19			Cloſe, Rain in the following Part of the Night.
3	9	2. 2	29. 0	23	N 2	Cloſe.
	14	1. 7	2	32	N W 3	Small Rain.
4	8	3. 4	10	30	N 2	Fair.
5	8	3. 2	19	26	N W 1	Cloudy.
	14	2. 1	19	26	N W 1	Rain, the ☿ a little riſen, the Rain laſted about an Hour.
	14	2. 1	19	26	S E 1	Rain.

D.	H	Ther. 4	Bar.	Hyg.	Wind.		Weather.
	17	2. 1	19	26	S E	1	Rain.
6	8	3. 2	30. 3	27	N E	1	Cloudy.
7	8	3. 0	4	25	N	1	Fair.
	3		2	25			Cloſe.
8	8	2. 4	2	23	N E	1	Cloſe.
	24	1. 7	1	22			Cloſe.
9	8	2. 0	29. 19		N E	1	A little Fog.
10	9	3	30. 0	21	N E	2	Fair, $\frac{15}{16}$ | 23 $\frac{16}{16}$.
11	8	4	2	17	N E	1	Not a Cloud 16 | $\frac{15}{16}$.
12	8	2	1	12	N E	1	Scarce a Cloud 15 | $\frac{9}{16}$.
13	8	1. 4	34. 0	7	N E	1	Not a Cloud.
	13	0.—2	29. 19	3	E	1	Not a Cloud, but a thick Air called a red Wind.
14	8	1.—0	16	10	N E	1	High Clouds.
	23	0.—3	15	8			Clouds.
15	7	0.—3	15	9	N E	1	Very fair.
16	8	0	18	8	W	2	Very fair.
17	9	0. 3	17	6	S W	2	Very fair.
	23	0.—6	16	7			Small Rain for a little while.
18	19	0. 1	15	18	S W	2	Cloudy.

In the Cloſet on the North Side of the Houſe.

D.	H	Ther.	Bar.	Hyg.	Wind.		Weather.
	15	0. 3	14	16	S W	2	Cloſe.
	23	0. 2	14	18			Rain.
19	9	1. 0	15	18			Fair.
	16	0. 5	15	16	S W	3	Cloſe.
20	9	0.—2	15	16	W	1	Fair.
	9	0.——	6	18	S W	2	Cloſe, a little Rain this Morning.
21	23	0. 4	1	27		2	Rain, as it had been moſt part of the Day.
22	8	1. 1	29. 0	17			
30	24	2. 3	4	21	N E	3	Rain, and all the Day before Rain.
1	10	2. 2	8	29	N E	2	Cloudy. May.

In the Chamber on the South Side of the Houſe.

D.	H	Ther.	Bar.	Hyg.	Wind.		Weather.
2	8	2. 1	12	29	N W	2	Cloudy, Rain laſt Night, ☿ was at 23 laſt Night as now.
	18	3	11	27		1	Fair, two or three Showers ſince 8.
3	9	2	14	29	N E	1	Fair.

D.	H	Ther. 4	Bar.	Hyg.	Wind.		Weather.
4	11	1.	14	26	N W	1	Fair.
5	8	2. 6	16	23	N W	2	Close.
6	8	6	16	24	N W	1	Fog.
7	8	0. —2	14	21	W	1	Fair.
	19		13				Rain.
8	9	0 ‾‾‾ 3	17	16	N E	1	Cloudy.
9	8	0 ___	19	13	N E	1	Fair.
		2 1					
10	7	2. 2	19	12	N E	2	Fair.
13	17	0 ___ 2	19	5	N E	2	Fair.
14	10	1. 0	19	12	N E	2	Fair.
15	9	0 — 2	19	12	N E	1	Rain a little.
	16	0 ___ 4	19	13	N E	1	A few Drops, the ☿ sunk a very little.
16	7	0	17	13	E	1	Fair.
17	8	0 ‾‾‾ 4	29.14	8	N E	1	Not a Cloud.
18	7	0 ___ 6	29.12	5	S	1	Clouds, a little Shower about 10.
19	9	0 ___ 5	12	6	W	2	Cloudy, *i. e.* more Clouds than clear Sky.
20	9	3	10	5	S E	1	Cloudy.

N. B. Cloudy signifies more of the Sky cover'd than clear. Fair signifies more open Sky than cover'd with Clouds.

D.	H	Ther.	Bar.	Hyg.	Wind.		Weather.
21	9	0	9	6	S W	1	Cloudy.
	18	0 — 2	9	5	S W	1	Rain for about an Hour.
22	8	0 — 1	12	6	N E	1	Close, *i. e.* the Sky no where to be seen for Clouds; a Shower about 17.
23	6	0 ___ 2	18	7	W	1	Fair, *i. e.* more Sky than Clouds.
	9	2	18	8	S W	1	Very fair.
	15	1. — 1	18	7	W N	2	Hard Rain about ¼ of an Hour.
	17	1. 1	18	7	S W	2	Hard Rain.
24	7	0 ___ 4	17	8	N E	1	
25	8	5	14	8	S E	1	
26	8	1. 1	10	6	E N	1	
	18	3	10½	8	N E	2	Close, a Thunder Shower at 19 the ☿ being risen to 11.

D.	H	Ther. 4	Bar.	Hyg.	Wind.	Weather.
27	7	7	12	7	N E 2	Cloſe.
28	7	0 — 1	14	7	N E 2	Cloudy.
	12	0 — 7	14	9	N E 2	Cloudy.
29	8	1. 1	13	10	W 2	Cloudy.
	13	1. 3	13	10	W 2	Rain.
30	6	0 — 2	15	8	W S 1	Cloſe.
	11	0 —	13	8	S W 3	Rain till Night.
	14	0 — 1	11	9	S W 3	Rain.
	21	3	3	14	3	Rain hard.
	24	3	29.	17	4	Cloſe.
31	7	0 — 3	1	14	W 4	Cloudy.
	24		2		W 4	Cloudy.
	24	1	4	12	2	Fair.
1	7	0 — 4	6	12	W 3	Fair. June.
2	7	0 —	4	13	W 2	Cloudy.
	10	0 — 1	4	13	W 4	A Shower for about half an Hour,

and then fair and calm again, at 11 Rain again, the ☿ a little raiſed. Several Showers in this Day.

D.	H	Ther.	Bar.	Hyg.	Wind.	Weather.
3	8	0 — $\frac{1}{2}$	13	14	W 1	Cloſe.
	9	0 —	14	14	W 1	Rain a little.
4	6	0 — 7	14	13	S 1	Fair.
	23	0 — 5	7	13	1	Cloſe.
5	9	0 —	$4\frac{1}{2}$	13	S W 3	Cloudy.
	12	0 — 6	4	14	S W 2	Hard Rain $\frac{1}{4}$ Hour, ſeveral ſuch Showers this Afternoon.
6	7	0 — 5	29. 5	14	S W 1	Cloudy.
	10	0 — 3	5	14	S W 2	Hard Rain $\frac{1}{4}$ Hour.
7	7	0 — 3	6	13	S E 2	Betwixt Cloudy and Fair.
	11	0 — 1	6	13	S E 2	Hard Rain $\frac{1}{4}$ Hour, and a very great Shower 1 Hour, ☿ ſtanding as it was.
8	8	0 — 4	7	13	o	Fair. (all this Morn.
9	9	0 — 1	$6\frac{1}{2}$	15	W S 1	Rain a good part of laſt Night, and
	24	0 — 6	11	18	2	Rain all the Afternoon till 20 or 21, the ☿ all the while riſing, now cloſe.
10	9	0 — 4	12	18	0	Cloſe, Rain the paſt morning.

13

D.	H	Ther. 4	Bar.	Hyg.	Wind.	Weather.
11	9	0 ———	13	19	S W 2	Cloudy.
	11	0 ——— 3	14	19	S W 2	Rain ¼ Hour.
12	9	0 ——— 1	17	17	S W 2	Very Cloudy.
13	8	0 ——— 7	17	19	W 1	Cloudy.
14	18	1 1	16		W 1	Cloudy.
15	7	1. 0	15	18	N 1	Very fair.

July. In my Abſence, the Thermoſcope being obſerv'd, it was found from the 15th of *June* to the 11th of *Auguſt*, never to get ſo high as 3, and was very often below Temperate; ſo cold was this Summer.

Auguſt.

D.	H	Ther.	Bar.	Hyg.	Wind.	Weather.
13	19	2 6	29.10			Cloudy, Rain about 21.
14	9	1 5	29.10		S W 2	Cloudy.
	11	1 6	29.10	17	S W 2	Rain.
	23	1 0	12	17		Fair.
15	9	1 2	8	25	E 2	Cloudy, Rain laſt Night.
	12	1 5	7	26	S E 2	Rain.
16	8	0 ——— 5	12	25	W 2	Fair.
17	8	0 ——— 2	15	21	S E 1	Fair.
18	7	1 1	14	22	S W 1	Cloudy.
	24	2 1	13	22		Small miſty Rain.
19	9	1 4	15	22	W 2	Cloſe.
20	8	1 1	18	20	S W 1	Very Cloudy.
21	8	1 0	16	19	S 1	Fair.
	10	1 3	16	19	W 1	Very ſmall Rain.
	12	1 5	16		W 1	Very Cloudy.
22	8	1 6	18	18	N W 2	Fair.
23	9	0 ——— 5	30. 0	16	S W 1	Not a Cloud.
24	8	0 ——— 3	30. 0	15	0	Not a Cloud.
25	8	1 1	29.19	15	S W 1	Not a Cloud. (ſince Yeſterday Morn.
26	8	1 2	16		S E 1	Not a Cloud, ☿ gently ſinking ever
	17	3 5	16		1	Fair.
27	7	1 5	14	12		Thick Fog.
28	8	1 1	29.16	12	N W 2	Very fair.
29	9	0 ——— 7	16	10	N W 1 rac. S E	Very Cloudy.

D.	H	Ther. 4.	Bar.	Hyg.	Wind.		Weather.	
	17	1. 3	15	12		0	Close, some small Drops.	
30	8	1 5	13	12	N W	2	Close.	
	20	0. 0	12	13	NE	1	A little Rain.	
31	8	0 2	13	12	NW	1	Very fair.	
1	9	1. 0	11	23	S W	3	Cloudy.	*September.*
	12	1. 3	11	13	S W	3	A little Rain.	
	14	1. 4	11	13	S W	2	Hard Rain for 1 Hour, and soft till 17.	
2	3	0 2	14	11	W	2	Fair.	
3	9	0 6	14	10	W	2	Cloudy.	
4	8	2. 0	15	8				
5	8	1. 6	16				Very fair.	
18	22	0 6	29.11		W	2		*October.*
19	9	0	4		S	3	Rain.	
20	9	1 2	5		S W	2	Very fair.	
	18	0 3	2	16	S W	4	Rain.	
21	9	1. 4	10	17	W N	2	Not a Cloud.	
	4	0 5	15	16			Rain.	
22		0	16	37	W S	3	Cloudy.	
	16	0 3	17	33	W S	2	Cloudy.	
23	9	0 3	17	33	S W	1	Cloudy.	
	15	0 3	16	24	S W	1	Fair.	
24	9	0 5	15	27	S W	2	Fair.	
	14	0 1	16	30	S W	1	Small Mist.	
25	8	0 5	30.	30	W	2	Close.	
	15	0 3	29.19	29	W	2	Scarce a Cloud.	
26	9	0 5	16	28	W	1	Cloudy.	
27	9	1. 1	13	27	W	1	Close.	
	23	0 7	13	27		1	Close, a little misling Rain good part of the Afternoon.	
28	9	1. 0	15	28	N W	1	Cloudy.	
29	9	1. 1	17	28	E	1	Close, a little Fog.	
		0 7	18	29	E	1	Close, a little gentle Rain about 15.	
30	9	2. 1	30. 1	26	E	2	Cloudy.	
31	8	3. 0	29.19	18	N E	1	Fair.	
	11	2. 6	18	19	N E	1	Hard Snow, which lasted till past Midnight.	

	D.	H	Ther. 4	Bar.	Hyg.	Wind.		Weather.
November	1	9	3. 5	29. 9	19	N E	1	Fair, scarce a Cloud.
	2	11	4. 0	7	20	S W	1	Frost.
		15	3. 6	7	20	S W	1	Snow, which began at 13, and lasted to 20.
	3	9	3. 5	7	23	W	1	Close.
		12	3. 0	5	25	W S	1	Rain, which began at 11, and lasted till 17.
		24	2. 0	1	39		3	Close, Rain before 1, and so the greatest Part of the Night.
	4	9	1. 5	3	40	S	2	Close.
	5	9	1. 4	13	36	N E	1	Close, Rain last Night.
	6	9	2. 0	15	33	N E	1	Close.
	7	8	2. 2	16	32	N E	0	Close.
	8	9	2. 1	14	32	E N	1	Close.
		16	2. 0	14	33	N E	1	Rain, which began at 14, and continued till 24.
	9	9	2. 2	13	33	N	2	Small Rain but for a very little while.
	10	9	2. 4	17	32	N W	1	Not a Cloud.
	11	9	2. 4	17	30	W	1	A little Fog.
		16	1. 6	14	32			Hard Rain.
	12	9	1. 5	10	33	W	1	Rain.
	13	9	3. 1	4	30	W	0	Fair, Frost.
	14	9	4. 2	12	28	N W	2	Very fair, hard Frost.
	15	9	4. 6	16	26	W	0	Very fair, hard Frost.
	16	10	4. 1	8	31	S	3	Close, Snow last Night from 11 the greatest Part of this Day.
	17	9	3. 7	8	34	S	1	Very Cloudy.
		1	3. 0	6	35	S W	2	Rain.
	18	8	4. 0	10	37		0	Fog, in the Closet on the North side of the House.
	26	2[illegible]	4. 2	18			3	Close, in the Closet on the North side of the House.
	27	10	4. 4	16	34	E	2	Cloudy.
	28	9	4. 6	18	32	E	3	Fair, Frost.
	29	9	4. 6	16	31	E	3	Close, Frost.
	30	9	4. 5	18	33	E	3	Cloudy, Frost.

December.

D.	H	Ther. 4	Bar.	Hyg.	Wind.		Weather.
1	9	4. 5	30. 0	33	E	2	Rain mix'd with Hail, the Rain as it fell, froze, and continued misling till 14 or 15.
2	9	4. 1	30. 0	38	S E	2	Rain, which began last Night, and lasted all this Day.
	17	3. 4	29.17	39			Fog.
3	9	3. 6	19	45	S W	0	Thick Fog, Frost.
4	9	2. 6	5	48	S	2	Rain all the Morning.
	12	2. 3	3	49	S W	2	Cloudy.
5	10	2. 4	5	54	S W	2	Cloudy.
6	11	2	7	53	W	2	Cloudy, some Rain this Morning.
7	9	3	3	54	W	2	Close.
8	9	1. 4	18.18	56	S W	4	Rain and Stormy Wind all this Day.
	23	0	15	58	S W	4	Hard Rain.
9	9	4	16	55	S W	3	Cloudy.
	17	5	15	53		3	Hard Rain.
	24	2. 1	15	52		3	Fair.
10	10	2. 4	29. 1	52	S W	1	Very Cloudy.
	24	3. 1	11	52		0	Very Fair.
11	9	3. 4	13	53	W	0	Very Fair.
	18	2	13	52	E	1	Very Fair.
12	10	3. 0	10	54	S	3	Close.
	15	2. 6	6	53	S E	3	Rain a little and short.
13	9	3	4	54	S E		Close.
14	9	1. 6	5	56	S	2	Rain all Day.
	18	1. 4	3	57		2	Rain yet.
15	9	2. 0	11	56	W	1	Very Fair.
	15	1	13	56	W	1	Thick Fog.
16	9	4	11	54	S	2	Close.
	15	2. 0	11	57	S W	2	Rain from 12.
17	9	1. 6	14	56	S W	1	Cloudy.

In my Chamber on the South Side of the House.

D.	H	Ther.	Bar.	Hyg.	Wind.		Weather.
	11	1	15	54	W N	1	Fair.
	17	4	17	49	W N	1	Fair.
18	10	2. 7	30. 3	43	E	0	Very Fair, Hoar Frost.
19	9	3. 2	0	43	S	0	Mist. A Mist is when the Drops manifestly fall.

D.	H	Ther. 4	Bar.	Hyg.	Wind.	Weather.
20	8	2. 6	19.17	45	N E 0	Miſt, as it was all Day yeſterday.
21	9	4	12	46	S W 2	Cloſe.
	19	1	4	48	S W 3	Rain.
22	9	1	6	48	S 1	Not a Cloud.
	24	5	0	42	3	Rain.
23	9	3	28.10	51	W 4	Rain, which had been all Night.
24	10	3. 2	19	45	W 1	Cloſe.
25	11	3. 3	29.12	40	W 1	Fair, Froſt.
26	9	2. 0	3	57	S W 3	Rain all laſt Night.
27	9	6	3	51	S 3	Cloſe.
	11	2	0	51	S W 4	Cloſe.
	12	1	28.19	51	S W 4	Rain all Day.
28	11	3. 2	18	48	S W 2	Rain.
29	9	6	29. 6	48	S 1	Fair, Froſt.
	24	2	28.19	47	S 4	Rain.
30	9	2. 3	11	57	W 4	Rain very hard all laſt Night till now.
	24	3	17	48	4	Fair.
31	9	6	15	46	W 4	Not a Cloud.
	20	5	29. 8	39	2	Rain.
	24	4	10	40	1	Cloudy.

Obſervations on the Weather, Rain, Barometer *for* 1699, 1700, 1701, 1702, *by Mr.* W. Derham. n. 286. p. 1443.

X. 1.] Mr. *Townley* having communicated to me an Account of the Weather, *&c.* I ſhall compare my Obſervations on that Subject, made at *Upminſter* in *Eſſex*, with thoſe which he made at *Townley* in *Lancaſhire*.

As to the moſt remarkable Weather, eſpecially Rain of 1702, and the Effects thereof: Mr. *Townley* tells me, that it is a general Complaint in the North of *England*, that there were but ſmall Crops of Hay, which Calamity befel the Southern Parts alſo; the Cauſe whereof may be perceiv'd by the following firſt Table of Monthly Rain; in which the growing Months of *March* and *April* appear to have been dry Months in *Lancaſhire*, and *May* no wet Month, conſidering the Quantities of the other Months, and of other Years. Here at *Upminſter*, *April* was fortunately a wet Month, till the 23d, or elſe, no doubt, we ſhould have ſuffer'd more than we did in the Want of Hay; for the growing Month of *March* was a dry

dry Month, by the following Table; and *May* (which by the same Table seemeth to have had near a due Quantity of Rain) was a very dry Month: For very little Rain fell from *April* 23 till *May* 29, and then fell in great Showers, the greatest Quantity of that Month's Rain. Mr. *Townley* doth not tell me Particulars, but I guess it to have been after this manner with them in the North of *England*; for besides that *March* and *April* were dry Months with them, and *May* somewhat more wet; yet probably the Wet of *May* did not fall early in *May*; for it appears by the following Table the third, that the *Mercury* was high, and on somewhat a fix'd Station on *May* 13.

Thus much for the Weather in the Spring-Months of the Year 1702, and the Effects it had on Hay, which Effects I have some reason to think extended to many Parts of this Kingdom.

As to the other Months, there is little remarkable, besides the vast Disproportions of Rain between *Lancashire* and *Essex*, which I should scarce take notice of, if it was not what happeneth almost every Year, as will appear by the following Table the first: The Cause of this I cannot judge of, unless it be that *Lancashire* is a more hilly Country than *Essex*, which sort of Lands, as they more need Wet than Vales and low plain Countries do, so have greater Shares of it than these have; and perhaps something may be attributed to the Western Situation of *Lancashire* near the Sea; from which Quarter the Winds in *England* blow more than from the Eastward.

At the Foot of the Table of Rain, besides the Quantity which fell in each Year, I have added the Depth thereof in Inches; or what Depth it would have been of, if the Earth had not imbib'd it, but it had stagnated on the Surface thereof.

I have added two Tables more, of the Stations of the *Mercury* in the *Barometer* at *Townley* in *Lancashire*, and at *Upminster* in *Essex*, with the Differences thereof; and this observ'd at three times of the Day, *viz.* in the Morning, and about three in the Afternoon at *Townley*, but at Noon at *Upminster*, and at nine a-Clock at Night. One Table to the first Day of every Month; the other the most remarkably low, high and more settled Stations of the ☿ the last Year 1702.

The Mercury *rises and falls in all Places at the same* Time, *but not in the same* Proportion.

By these *Barometrical* Tables it may be seen how far true that Opinion is of some learned Men, *viz. That the ☿ ascendeth and descendeth in all Places at the same Time, and in the same Proportion.* It is manifest, that the ☿ doth commonly rise and fall in one distant Place, when it doth so in another, but not alike: Also when any considerable Variation is in one Place it is so in another; when remarkably high, remarkably high; when low, low; when a great Ascent or Descent, generally the same elsewhere; but only the Differences of all these are not in

 equal

equal Proportion in all Places; all which ſeemeth reaſonable to be expected, by reaſon of the Difference of Weather in different Places, eſpecially as to wet and dry.

A remarkable Deſcent *of the* Mercury.

There is one thing more in the following third Table, which I think deſerves Remark, becauſe I believe it to be the moſt conſiderable Alteration of the Mercury, that hath ever happen'd ſince the Invention of the *Baroſcope*, and that was the Deſcent on *Febr.* 3*d* and 4*th* 1702; concerning which, Mr. *Townley* in a former Letter gives me this Account, " That on *Febr.* 3*d* the ☿ was at three in the Morning at 29. 15. at " 3h. 28. 50. and at 10 at Night at 27. 5. The next Day it fell yet " lower, and about 12 was at the loweſt, *viz.* 27. 39; but for an Hour " before, and as much after, it varied only ſo much as to make it ſen- " ſible that it was fallen, and began to riſe again; the loweſt he had " ever ſeen it before was on *Nov.* 18, 1674, when it fell to 27. 63. " That Mr. *Flamſteed* at the Obſervatory obſerv'd as remarkable a De- " ſcent of his ☿; and that it happen'd about the ſame Time of the " Day, *viz.* two of the Clock in the Afternoon at both Places.

And *laſtly*, he tells me, " That the Deſcent in *Febr.* laſt was the " greateſt that has been ſince the filling his Tube, which was in *March* " 1665." The Particulars which I obſerved here at *Upminſter* about that Deſcent were, That on *Febr.* 2 the ☿ was high, *viz.* 29. 80. the next Morning 29. 50. at Noon 29. 16. at Night 28. 43. the next Morning, (*viz. Febr.* 4.) at ſeven of the Clock, it was fallen to 28. 5. and was globoſe, as if it had riſen, or was inclin'd to riſe: But it continu'd in the ſame Station till Afternoon, and then began to riſe about two of the Clock, and roſe haſtily. The Weather accompanying was fair on *Febr.* 3*d* in the Morning, hazy at Noon, and Rain at Night, and a violent Tempeſt in the Night, and all the next Morning, of *Febr.* 4*th.*

TABLE

TABLE I.

A Table, shewing how many Pounds, and Centesimals of a Pound *Troy* of Rain, fell at *Townley* in *Lancashire*, and at *Upminster* in *Essex*, in each Month of the Years 1699, *&c.* with the Quantity and Depth every Year.

	1699		1700		1701		1702	
	Townl.	*Upmr.*	*Townl.*	*Upmr.*	*Townl.*	*Upmr.*	*Townl.*	*Upmr.*
Januar.	17 91	8 91	20 84	3 91	22 41	14 96	21 10	9 81
Febru.	32 70	60 5	19 12	7 64	16 78	8 78	21 27	7 30
March	17 92	5 63	7 58	1 55	7 10	3 91	2 48	2 37
April	10 47	3 44	18 65	7 60	6 11	1 43	5 34	10 90
May	4 00	2 67	17 92	6 91	19 67	9 11	8 81	6 49
June	10 37	40	13 15	7 60	11 34	5 79	23 00	13 46
July	16 51	6 36	15 26	4 24	17 58	9 49	25 31	43 09
August	19 77	8 57	12 05	8 14	23 66	6 57	20 12	6 88
Sept.	16 53	8 06	23 52	14 85	21 30	5 63	23 01	8 05
Octob.	18 90	13 49	26 44	17 15	24 59	10 21	28 57	7 92
Nov.	14 65	1 91	13 69	5 24	25 60	8 22	37 11	14 05
Dec.	16 78	5 77	26 88	10 30	10 19	9 35	41 63	10 27
Total	196 51	75 55	215 30	95 13	206 33	93 45	257 75	101 89
Depth	39 302	15 110	43 060	19 026	41 266	18 690	51 55	20 378

TABLE II.

A Table shewing the Height of the ☿ at *Townley* and *Upminster*, on the first Day of every Month in the Year 1702, three times a Day, *viz.* about 7 in the Morning, and 9 at Night; and about 3 Afternoon at *Townley*, with the Difference of the ☿'s Variation, and its Difference between both Places.

First Day of the Month.	☿ Height at Town.	☿ Height at Upmr.	Daily Differ. at Town.	Daily Differ. at Upmr.	☿ lower at Town.
Jan.	29 06	29 28			22
	28 90	21	16	07	31
	58	10	32	11	52
Feb.	29 58	96			31
	40	91	18	05	51
	30	80	10	11	50
Mar.	36	68			32
	36	66	00	02	30
	40	58	04	08	18
April	70	79			09
	68	73	02	06	05
	69	79	01	06	10
May	20	51			31
	49	49		02	
	09	44	11	05	35
June	56	78			22
	61	82	05	04	21
July	84	98			14
	90	30 01	06	03	11
	92	11	07	01	19
Aug.	62	29 80			18
	49	74	13	06	25
	47	67	02	07	20
Sept.	92	30 09			17
	95	12	03	03	17
	95	11	00	01	16
Octob.	56	29 74			18
		76		02	
	54	75	02	01	21
Nov.	46	71			25
	50	75	04	0	25
	58	76	08	01	18
Dec.	3	50			15
		34		16	
	10	09	25	25	01

TABLE III.

A Table shewing the *Lowest Stations* of the ☿ in the Year 1702, at *Townley* and *Upminster*; with the Difference of the ☿ at both Places

Day of the Month.	☿ at Town.	☿ at Upmr.	Differ.
Feb. 3	29 15	29 50	35
	28 50	16	66
	27 50	28 43	93
		05	
4	39	05	66
		62	
Dec. 23	77	18	31
	71	12	41
	90	10	35
		25	
High Stations of ☿ An. 1702.			
	Upmr.	Town.	
Jan. 30	30 25	29 95	30
	25	95	30
	19	83	36
Mar. 12	33	30 02	31
	35	05	30
	32	07	25
Oct. 21	07	29 94	13
	18	99	19
	22	30 00	27
More settled Stat. An. 1702.			
	Town.	Upmr.	
Apr. 27	30 04	30 13	09
	00	15	15
	02	16	14
	03	16	13
28	01		
	29 99	15	16
	98	15	17
29	93	13	20
	86	06	20
May 13	78	29 89	11
	72	87	15
	74	87	13
June 10	30	68	03
	40	71	31
	44	70	26
	50	70	20
11	51	72	21
	59	70	11
Aug. 30	45	86	41
	57	86	29
	59	88	29
Sept. 27	73	91	18
	72	91	19
	7	91	18
	74	91	17
28	78	[illegible]	[illegible]

2.] *A Prospect of the* Weather, Winds *and* Height *of the* Mercury *in the* Barometer, *on the* first Day *of the* Month; *and of the whole* Rain *in every* Month *in the Year* 1703, *and the Beginning of* 1704. *Observ'd at* Townley *in* Lancashire, *by* R. Townley, *Esq*; *and at* Upminster *in* Essex, *by Mr.* W. Derham.

Observations on the Weather, &c. 1703, 1704, by Mr. W. Derham. n. 297. p. 1877.

The Mon.	*Weather at* Town.	*Weather at* Upmin.	*Winds at* Town.	*Winds at* Upminst.	*Barom. at* Town.	*Barom. at* Upm.	*Rain at* Town.	*Rain at* Upm.
Jan.	*Overcast*	*Overcast*	S E 5	S E 1	29 04 28 91 80	29 39 35 22	15 17	8 89
Feb.	*Overcast*	*Frost and fair*	S S E 3	E. 1 *Clouds.* S	29 29 37 47	62 68	15 88	6 41
Mar.	*Chequer'd*	*Frost and fair. Snow*	W S W N E	N W 1	64 67 73	82 83 92	20 02	4 75
Apr.	*Chequer'd and Cloudy*	*Fair* *Cloudy* *Fair*	W	W S W 1 W 4	59 59 55	93 91 91	17 63	12 49
May	*Cloudy*	*Cloudy Thunder with Hail and Rain*	N N E	N W *by* N 3 N *by* W 3	49 00 60	66 70 76	17 64	20 77
June	*Cloudy*	*Cloudy* *Clear*	S	S 0 S 1	38 49 63	61 75 88	24 06	14 55
July	*Cloudy*	*Fair*	SSE. SE E 1	E 1 N *by* E 2	84 80 77	99 96 90	3 65	14 90
Aug.	*Cloudy*	*Thunder and Rain* *Fairer*	S E 1 4 4	N 2 N *by* W 2 *Clouds.* SE	57 55 58	72 72 69	14 21	3 36
Sept.	*Clear* *Chequer'd*	*Cloudy* *Fairer*	E	W N W 1 N W 1	80 00 18	30 40 40	32 40	14 86
Oct.	*Chequer'd*	*Fair*	N	N N W 4	28 76 83 86	87 30 08	7 04	9 55
Nov.	*Overcast*	*Cloudy*	E	N *by* E 1	29 57 51 52	29 72 69	28 56	7 27
Dec.	*Overcast* *Cloudy*	*Overcast* *Rain*	W 3		36 45 48	81	10 24	2 14
							196 60	119 94

The Mon.	*Weather at* Town.	*Weather at* Upmin.	*Winds at* Town.	*Winds at* Upminſt.	*Barom. at* Town.	*Barom. at* Upm.	*Rain at* Town.	*Rain at* Upm.
$170\frac{3}{4}$ Jan.	*Overcaſt*	*Overcaſt*	S S E S E 2	E 0 S E *by* E 1	80 82 85	30 07 10 10	31 39	4 06
Feb.	*Overcaſt*	*Miſling Cloudy*	W	N W *by* N 1	90 02 02	23 26 26	5 93	2 19
Mar.	*Overcaſt*	*Overcaſt Fairer*	S	S E *by* E 0 S E 1	29 11	29 58 45 40	20 78	16 04
Apr.	*Cloudy and Chequer'd*	*Rain with hail Fair*	W 4 N W 7	S *by* W 4 *Clouds* S W *by* W	28 72 94 07	17 18 38		

From theſe Tables it is to be obſerv'd,

1. That much more Rain falleth at *Townley* than *Upminſter*.

I have an Extract of the Rain at *Paris* and *Liſle*, as far as the *French* have publiſh'd their Obſervations. And by comparing the Rain of one Place with that of another, I find that there is about twice as much Rain falleth at *Townley*, as doth either at *Upminſter*, *Paris*, or *Liſle*. Mr. *Townley* hath formerly obſerv'd, that as much more Rain falleth at *Townley* as *Paris*. And M. *de la Hire* obſerves, that more Rain falleth at *Liſle* than *Paris*. But *Townley* doth far exceed.

At *Liſle*, one Year with another, the Depth of the Rains amounts to 22 Inches 3 Lines, *Paris* Meaſure, or 23 in 3 *l.* which make about $23\frac{1}{2}$ Inches *Engliſh*, or $24\frac{1}{2}$. At *Paris*, one Year with another, they amount to 20 Inches $3\frac{1}{2}$ Lines *Paris* Meaſure, which is near 22 Inches *Engliſh*. But at *Townley*, one Year with another, according to Mr. *Townley*'s Computation formerly, the Rains amount to above 41 Inches Depth. And by taking eight other Years, in which the Rain was obſerved both at *Townley* and *Upminſter* (*viz.* from 1696 to 1704) I found that all the eight Years Rain at *Townley* amounteth to above 1700 *l. Troy*, at *Upminſter* 823 *l* only. Which ſaid Sums being divided by 8, give 212 *l.* $\frac{1}{4}$ one Year with another, at *Townley*, and 103 *l.* at *Upminſter*. Each of which Sums being doubled, and making a Decimal Fraction of the laſt Figure, doth nearly give the Number of Inches, which all the Rain would have riſen to, if the Earth had not ſwallow'd it up, *viz.* $42\frac{1}{2}$ Inches at *Townley*, and about $20\frac{1}{2}$ Inches at *Upminſter*. Wherefore the Rain at *Upminſter* is leſs than at *Paris*, at *Paris* than at *Liſle*, and at every one of the Places leſs than at *Townley* by much. The Reaſon of which vaſt Surmount at *Townley*, is doubtleſs from the Height of the Hills thereabouts, which retard or ſtop the Weſterly Clouds: From which

which Point the Winds blow more than any other here in *England.* But,

2. Notwithstanding the great Disproportion of Rain between one Place and another, yet there is a great Agreement between our Barometers; one rising or falling when the other doth; and that much, or little, as the other doth; altho' not always so exactly in the same Proportion.

And this is what I find Monsieur *Maraldi* hath observ'd, by comparing his Observations at *Paris* with mine at *Upminster*, in the Years 1697 and 1698. Only at *Paris* the ☿ is commonly three or four Lines lower than at *Upminster*. And so it appears to be at *Townley*, from this and some other Tables, *viz.* three or four Tenths of an Inch lower at *Townley* than an *Upminster*. Which is an Argument that *Townley* and *Paris* are situated higher above the Surface of the Sea than *Upminster* (which is nearly in the same Level with *London)* is.

3. There is some Agreement between the Winds at *Townley* and *Upminster*. Which altho' not always exactly in the same Point, yet do often tend the same way, blowing within a Point or two perhaps of the same Course; especially when the Wind is somewhat strong. Or if the Winds have differ'd, yet the Scudd (as the Seamen call the Current of the Clouds) hath commonly shown the Motion of the upper Air to agree thereto.

This doth often happen, tho' not always. And this Monsieur *Maraldi* hath observ'd at *Paris* in the aforesaid Years, *viz.* " That there are " a great many Days, during the different Seasons of the Year, where " the Winds are the same in both Places, [*i. e. Paris* and *Upminster.*] " When the Wind was the same, both in one part and the other, it " was ordinarily pretty strong, and of long Continuance. And also " he observed that the Winds had changed alike in both Places. *Vid. Hist. de l'Acad. Roy. des Sciences, An.* 1699.

Tables *of the* Weather, *&c. for* 1705, *by the* same. n. 309. p. 2378.

JANUARY.				7 . . . 12 . . . 9				
	Weather	Wind	Course of the Clouds	Barom.		Th.	Rain	
	Frost	N b W o		30	14	79		
5	Fair	1	N E		17	90		
	Cloudy				16	87		
	Frost with thick	W o			21	82		
10	Air	E b S o			22	93		
					25	87		
	Hard Frost	W b N 1			00	74		
15	and cloudy	S W o		29	97	85		
	Thaw				93	95		
	Frost and fine	E 1		30	03	73		
20	Weather	E b S 1			06	96		
					13	82		
	Mist	E b S 1		29	79	92	0	36
25	Brighter	E S E 1			80	103		
	Cloudy				81	92		
	Cloudy	N 1	N b E	30	03	87		
30	Snow	N b E 1			02	97		l.
	Cloudy			22	99	90	1	11

FEBRUARY.				7 . . . 12 . . . 9				
	Weather	Wind	Course of the Clouds	Barom.		Th.	Rain	
		S 2		28	89	98		
5	Rain	S E b S 2			77	102		
					78	93	2	68
	Fair	S S E 1		29	85	97		
10	Cloudy	S 5	S W b S		81	115		
	Rain				66	107		
	Frost	N b W o		30	39	84		
15	Sun-shine with							
	thin Mist				31	88		
	Misty	E b N 2			03	92		
20				29	97	100		
	Cloudy				86	9[illegible]		
	Frost	S b E 1	S S W		28	81		
25	Fair	S 2			27	115		
	Rainy				19	95		
30				The Sum of the Rains				l.
							5	53

MARCH.				6...12...9		
	Weather	Wind	Course of the Clouds	Barom.	Th.	Rain
5	Cloudy	N N E 3	N E	29 61	94	
	Fair			65	90	
10	Frost	S 0	W b N	41	86	
	Sun-shine	E N E 1		40	112	
	Cloudy			40	90	
15		E 1	S	24	94	
	Rain					
				17	105	
20	Dark with Storms of Snow	N b E 2		65	88	
25	Brighter with small Snow	N b W 2		21	83	
				33	104	
				48	81	
30		S b W 3		35	116	
	Turbid and rainy	S W 8		30	127	l.
				22	108	5 55

APRIL.				5...12...9		
	Weather	Wind	Course of the Clouds	Barom.	Th.	Rain
5	Cloudy	W S W 1		29 72	102	
	Dirty	S W 5		70	122	
	Drops			66	120	
10		E b N 1	S b W	34	98	
	Rainy			32	117	
				25	106	
15	Frost with	N 0		82	79	
	Sun-shine			90	116	
	Cloudy			96	98	
20	Cloudy	W b S 1		91	101	
		W 2		92	134	
	Milder			85	120	
25	Hoar-frost	W 0	N b W	70	86	
	Thunder with	1	N b W	80	125	
	much Rain			87	117	
30		W b N 1		68	100	
	Fair and warm					l.
				73	109	5 15

MAY.						5 . . . 12 . . . 9	
	Weather	Wind	Courſe of the Clouds	Barom.		Th.	Rain
		S W b S 1		29	61	115	
5	A cloudy Day	S b W 2			56	150	
					55	120	
	Cloudy	S S W 1	N W	30	07	104	
10		W b N 2			05	142	
	Fair			29	98	124	
	Showers of Hail	N W b N 2			80	92	
15	and Rain						
					95	97	0 22
	Cloudy	N 1			78	106	
20		N b E 2			78	124	
	Fair				80	113	
	Cloudy and	N b E 2	N N E		85	93	
25	Cold	N 2			91	120	
	Clearer				96	101	
		W b N 1			71	98	
30	Cloudy and cold						I.
					85	99	2 03

JUNE.						5 . . 12 . . . 9	
	Weather	Wind	Courſe of the Clouds	Barom.		Th.	Rain
	Cloudy and	S W 1	W	29	62	119	
5	Warm	N N W 1	W N W		65	143	
	Dropping				64	126	
	Sun-ſhine with	E S E 2		30	03	110	
10	Heat	E b S 2	S E b S		01	153	
	Cloudy, Rainy			29	94	126	0 30
	Miſt	W b S 1			92	127	
15	Hot	2			92	152	
	Sun-ſhine				93	132	
	Cloudy	E 1			95	125	
20	Hot						
	Not ſo hot				94	125	
	Cloudy	S W b W 2			62	137	
25	Drops	W 3			59	151	
	Showers				50	131	0 08
	Miſt	N 0			84	106	
30	Sun-ſhine	W 1			62	156	I.
	Hot				04	134	3 29

JULY.

JULY.				5...12...9		
	Weather	Wind	Courſe of the Clouds	Barom.	Th.	Rain
	Sun-ſhine	W 0	N W	30 08	122	
5	Rain	N W 2		08	139	
				10	116	0 82
	Cloudy	N b W 3		29 79	112	
10	Showers and not	5		88	120	
	ſo hot			30 80	118	
	Sun-ſhine	E b S 0		29 96	112	
15	Very hot	S 2	N	93	163	
				90	145	
	Sun-ſhine with	S W b W 0		30 03	115	
20	Heat	N N W 2		03	164	
	Cloudy			03	136	
	Sometimes Sun-	W b S 0		29 72	110	
25	ſhine, ſometimes	S b W 3	W	69	170	
	cloudy			66	148	
	Sun-ſhine	W b S 1		79	116	
30	Cloudy	S W 2		82	151	1.
	Leſs cloudy			82	135	5 56

AUGUST.				5...12..9		
	Weather	Wind	Courſe of the Clouds	Barom.	Th.	Rain
	Cloudy	W S W 1	W	29 80	133	
5	Leſs cloudy	W b N 1		86	158	
				86	148	
	Sun-ſhine	S S W 3	S W	48	131	
10	Windy	S W 6		56	158	
	Miſty			44	138	1 00
	Thick	S W 0		97	109	
15	Cloudy	S S W 1		94	153	
				82	132	
	Sun-ſhine	S 1	S W	61	120	
20	Drops					
	Leſs cloudy			65	129	
	Sun-ſhine	W 1	N W	34	105	
25	Drops	W N W 6		42	128	
	Leſs cloudy			49	117	0 07
		S 1	S S W	68	127	
30	Much Rain	S b W 4		71	142	1.
				74	124	10 81

SEPTEM.

SEPTEMBER. 6 . . . 12 . . . 9

	Weather	Wind	Course of the Clouds	Barom.	Th.	Rain
5	Sometimes Sun- shine, sometimes cloudy	S W b W o	W S W	29 88 88 84	107 144 118	
10	Cloudy Showers	S S W o	S W above N W below	77 86	102 108	
15	Cloudy Misty Cloudy	W N W o W 1	N b W N W	30 13 15 15	119 145 129	
20	Cloudy	E 1 E b N 4		25 25 22	124 138 128	
25	Small Rain	S W b S 1 S S W o		29 36 38 40	110 122 113	
30	Hoar-frost and Sun-shine	N W o W b N 1		48 50 42	93 112 92	1. 0 04

OCTOBER. 7 . . . 12 . . . 9

	Weather	Wind	Course of the Clouds	Barom.	Th.	Rain
5	Sometimes showers sometimes Sun- shine	S 1 S b W 2		28 93 99 29 10	111 127 111	0 56
10	Thick Mist Sun-shine	S E o E S E o		93 98 30 05	101 128 108	
15	Cloudy Sun-shine	W S W 2	W	29 75 78	98 108	
20	Sun-shine Cloudy Thunder, &c.	S E 1 E b S 2 E N E 2	S E b E	2 18 16 22	86 112 108	5 69
25	Sun-shine and cold	N 2 3		86 92 30 04	89 103 87	
30	Frost with Sun- shine	N b E 2		29 94 30 00	87 89	1. 16 01

NOVEM-

NOVEMBER.				8 . . . 12 . . . 9		
	Weather	Wind	Course of the Clouds	Barom.	Th.	Rain
	Frost and	E N E 1	E	29 68	83	
5	Sun-shine	E b N 1		72	103	
	Cloudy			72	101	
	Cloudy	S 2		80	103	
10						
	Rainy	6		44	120	0 27
	Frosty					
15	and					
	Sun-shine			30 05	80	
	Thick Mist	N 2		29 57	89	
20	Rain and	W 1		57	89	
	Turbid	9		35	98	0 91
	Frost and	W 0		65	79	
25	thin Mist			67	85	
				71	76	
	Thin Mist	S b W 2		10	102	
30	Cloudy	4	S S W	10	112	1.
	Rain			28 84	111	5 84

DECEMBER.				8 . . . 12 . . . 9		
	Weather	Wind	Course of the Clouds	Barom.	Th.	Rain
	Sometimes cloudy,	W 2		29 10	92	
5	sometimes fair	W b N 3	N W	25	102	
				46	92	
		N N W 1		45	88	
10	Cloudy					
				61	89	
	Small Rain	S W b W		34	100	
15		W b S 3		36	101	
	Rain			31	95	1 21
	Rain	S 1		28 73	94	
20	Clear			73	100	
	Frost			76	84	1 83
	Cloudy	N b E 1		29 72	94	
25	Rain		N N E	76	96	
	Snowy			90	88	0 48
		S E b S 1		92	87	
30	Dark	S E 1		90	88	1.
				86	85	21 70

In

Explication of the Tables.

In these Tables I have selected only every fifth Day of every Month, which may give a commodious View of the whole Year.

These Tables cannot want much Explication beyond their Titles, except in the Columns of Winds and Clouds, in which I have made use of the Marks in *English.* For their Variety was not known to the *Romans.*

The four principal Quarters are denoted by these Letters: N the North, S the South, E the East, and W the West.

The intermediate Points are denoted by a Conjunction of these Letters; N W denotes the Wind which *Seneca* calls *Corus*, we the North-West, which he says blows from the Western Solstice; S W *Africus*, or the South-West; S b W, South and by West, denotes that Point which lies next to the South; S S W, South-South-West, the next to this; S W b S, South-West and by South, the next to this, or that which lies between this and the South-West: And so of the rest.

The numeral Figures annexed to the Winds shew the Strength of the Winds. The Cypher 0 denotes the Tranquillity of the Air, or that no Wind blows: 1 denotes so mild a Wind, that it would hardly extinguish a lighted Candle: 2 is a stronger Wind: 7, 8, *&c.* as far as 15 or 20, denote more violent and raging Winds.

As to the Column of the Thermometer, it is to be observed, that the Degree of Freezing is about 85. But Hoar-frost will happen about 90, or something higher.

In the Column of Rain, sometimes I have put down the Weight of the Rain that has fallen on the rainy Days mark'd in the Table. And at the End of every Month is set down the Quantity of all the Rain fallen in that Month. In this whole Year there fell 84,62 Pounds of Rain, of those Pounds which we in *England* call *Troy* Weight.

The Funnel which receives the Rain is circular, the Diameter of which is equal to 12 *English* Inches.

Lastly, These Observations were made three times in a Day, except I was absent or otherwise employ'd. The Hours of observing, both before and after Noon, are mark'd at the Top of each Month.

Remarks *on the Weather,* &c.

About the End of *February*, and the greatest part of *March*, our People were generally seized with a Difficulty of Breathing, and a Cough. Let the Physicians judge whether this might not proceed from the Eastern Wind, together with a cold and moist Temper of the Heavens, which the Tables shew were then cold but not frosty.

Apr. 1. in the Morning *Parelia* were seen by some, but I did not happen to see them.

The Month of *June* was so hot and dry, that the Springs fail'd, the Fruits languish'd, and the Grass was dry'd up. Honey-dews were also frequent. Likewise the Corn was blasted.

Aug. 11. the Wind was so fierce, that it did great Injury to the Fruits of the Trees.

And though Plenty of Rain succeeded, yet the Ponds were dry in *September*, and there was a great Scarcity of Hay.

Dec.

December 19. This Morning (I think) was the greatest Descent of the *Mercury* in the Barometer, in the following Manner.

$8^h\frac{1}{4}$	In the Morning	28,28 Inches.
10		28,06
11		27,94
$11\frac{1}{2}$		27,94
$12\frac{1}{2}$		28,03
1	Afternoon	28,13
1	10′	18
1	20′ Wind West 7	20
2	Wind West 9	34

The Changes of the Heavens and of the Weather were not so remarkable as those of the *Mercury*. Only in the Afternoon the Wind was violent, and much Rain fell in the Night. But we heard that Day there was a dreadful Tempest at *Corbeil*.

This greatest Descent of the *Mercury* was observed by others. In the Observatory at *Greenwich* the *Mercury* descended to 27,80 Inches; in the City of *Canterbury* to 27,90.

In the Reckoning of the Rain I had almost forgot to mention the Depth of the Rain. If the Earth had not absorb'd it, it would have arose to 16,924 *English* Inches: Therefore this Year is to be accounted a dry Year: For the mean Proportion of Rain every Year is about $20\frac{1}{2}$ Inches at *Upminster*; and $42\frac{1}{2}$ at *Townley* in *Lancashire*; also 22 at *Paris* in *France*, and 24 Inches at *Lisle* a City of *Flanders*, as I observed above.

Tables *of the* Barometrical *Altitudes for* 1708, *at* Zurich *in* Switzerland; *and of the* Rain *at* Pisa *in* Italy, *and* Zurich, *and* Upminster, *for* 1707, 1708, *by Mr.* W. Derham. n. 341. p. 342. *With* Remarks *on the Wind, Heat, and Cold,* &c.

XI. The Society having put into my Hands Dr. *Scheuchzer*'s Observations of the Weather, *&c.* made at *Zurich* in the Year 1708, and having received from Dr *Mich. Angelo Tilli* the Quantity of Rain which he observed to fall at *Pisa*; I have compar'd these Observations with mine made at the same time at *Upminster*; and have put what I could of them into the annexed Tables. In the former of which, I have represented Dr. *Scheuchzer*'s and my Barometrical Observations: In the latter, his Rain Observations, those of Dr. *M. A. Tilli*, and mine own; all reduced to our *English* Measure. But because I am not as yet certain of the true Proportion between the *Tuscan* and *English* Weight, I have therefore given Dr. *M. A. Tilli*'s Rain, both in the *Tuscan* Pounds and Ounces, as he sent it me; as also reduced to our *English Troy* Pound and Centesimals of that Pound, according to Mr. *Greaves*'s Proportion, which is different from that assigned by Sir *Jonas Moor*.

Observations *with the* Thermometer.

As for the *Thermometer*, it would have been in vain to have compared Dr. *Scheuchzer*'s Observations with mine, by Reason we have not

not yet a Standard for Thermometers, as we have for the Barometers: they being every where in all, or moſt Reſpects different; ſome with ſmall Bottles of Spirits; ſome accordingly with longer, ſome with ſhorter; ſome with wider, ſome with narrower Canes or Shanks; ſome fill'd with more highly rectify'd, and conſequently more expanſive Spirits; ſome with more phlegmatic and duller Spirits.

The Difference particularly between Dr *Scheuchzer*'s and my Thermometer is, his is about one Foot long; that I obſerved with all along ('till it was broken this Year) about two Feet and a half; and that I now obſerve with three Feet and a Quarter; the Bore of the Stalk is ſmall, and the Ball is large, and conſequently the Rang great, anſwering every the leaſt Alteration of Heat and Cold.

But yet, thus much I have been able to obſerve by comparing Dr. *Scheuchzer*'s and my Thermometrical Obſervations, *viz.* That notwithſtanding the *Alpine Snows* have mighty Effects on the Weather in *Switzerland*, and other conterminous Places, yet there is much more Agreement between the Heats and Cold at *Zurich* and *Upminſter*, than before comparing them I imagin'd. (I ſpeak with relation to the laſt Year only, having no other Obſervations.) For in Winter, altho' I imagine we have more warm Days than they; and in Summer, that they have greater Heats than we; yet I obſerve that the Colds and Heats in both Places, begin and end nearly about the ſame Time: And that oftentimes any remarkable Weather (eſpecially if of ſomewhat long Continuance) affecteth one as well as the other Place. Thus, for Inſtance, *June*, which was (ſome part of it at leaſt, particularly the very Day after the Solſtitial-Day, *June* 12.) remarkably cold in *England*, ſeems to have been not very different at *Zurich*; Dr. *Scheuchzer*'s Thermometer divers Times that Month (tho' not on the very ſame Days perhaps) deſcending as low, or rather lower than in the Month before, yea, as many Days in the Winter Months. I obſerved too, that all this Month their cold Weather conſtantly preceded ours here about five or more Days. An Indication that the Weather in both Places was influenced by the ſame Cauſes, whether from the *Alpine* Hills and Cold, or the Influx of the Moon and other heavenly Bodies, or any other Cauſe.

And as in *June* there was a great Agreement in the unuſual Cold, ſo in *Auguſt* there was not much leſs Agreement in Heat; the Heats in both Places being great, and beginning to abate about the ſame Time, only a little ſooner here than there.

In Winter alſo, although I imagine we have a greater Number of warmer Days than they, yet I find that a warm Winter Month there is ſo here; and a cold one there, is a cold one here likewiſe. Thus in *February* and *March*, *October* and *November*, a great Agreement ſeems to have been between the Heats and Colds of both Places, ſome Days excepted. But *January* was at the Beginning not ſo conſtantly cold, for

for the Season at *Upminster*. as it seems to have been at *Zurich*. And *December* last, which from the eighth Day to *Christmas*-Day, was here moderate and open Weather, and after that more intensely cold than even in the *Long-frost*, *Anno* 1683, by the fewer Thermometrical Observations which Dr. *Scheuchzer* made then, than in other Months, the greatest part of that Month seems to have been intensely cold at *Zurich*, as the latter part thereof was with us remarkably in *England*.

As to the *Winds*, which I did not enter into Tables, because it may be sufficient to observe in general, that although many Days they agree in both Places, yet there are many more in which they differ. When they do agree, I find it is chiefly when the Winds are strong, and of long continuance; and more, I think, when Northerly and Easterly, than in the other Points. Also, I have observ'd, that a strong Wind in one Place hath been a weak one in the other. *—of the* Winds.

As to the *Barometrical Observations*, I have thought it worth while to specify them. Mine own Observations I selected, which were made at Noon; and Dr. *Scheuchzer*'s as near Noon as might be. For which reason, I commonly took his Morning Observations, because made, for the most part, about ten or eleven a-Clock. Also I took those made with his bent Barometer; because they seem'd to me (especially at the beginning of the Year) to be the most accurate. *—of the* Barometrical *Observations.*

The Altitudes of this *Mercury* he measureth by the *Paris* Foot, which I have reduced to our *English* Measure; for which reason I have also all along noted their Differences.

It is manifest from the Tables, that throughout the whole Year, the *Mercury* was lower at *Zurich* than at *Upminster*, by sometimes one, sometimes above two Inches *English*. The most remarkable Difference was at the latter End of *September* and Beginning of *October*, when the Difference was for a good while above two Inches *English*. The reason of which, I guess, was because at *Zurich*, I imagin'd the Air was more inclin'd to wet, at that Time, than at *Upminster*; as also, because the Winds were then at Northerly and Easterly with us, which, 'tis well known, do make our Barometers rise, even in our wet Weather. But the mean Difference between Dr. *Scheuchzer*'s and my Barometers, I take to be about half an Inch *English*. From whence I conclude, that the Situation of *Zurich* is near a Quarter of an *English* Mile higher than that of *Upminster* above the Surface of the Sea; or else that that Part of the terraqueous Globe, lying nearer the Line, is (according to the receiv'd Opinion) higher, or farther distant from the Center, than ours is, lying nearer the Pole.

It may be observed from the Barometrical Tables, that (as near the Equinoctial the Barometer is observed to stand nearly at a Stay, but the more Northerly the Latitude, the greater the Rang of the *Mercury*, so) at *Zurich*, the Difference last Year was not so great between the highest and lowest Stations of the ☿, as it was either at *Paris* or *Upminster*.

 For

For at *Zurich*, the Difference was only one Inch *Paris* Meaſure; at *Paris*, Dr. *Scheuchzer* ſaith, 'twas one Inch two Lines and an half; but at *Upminſter* it was eighteen Inches, (and ſome Years it is more) which is greater than either of them.

I obſerve, although there be ſome, and that a pretty deal of Agreement between the riſing and falling of our Barometers, one being very often high or low, when the other is ſo; and one oftentimes riſing or falling when the other doth ſo; and one riſing much or little, or falling much or little when the other doth; yet it is not ſo certainly ſo, as it is nearer home. I have before given a Table of ſome Heights of the *Mercury* obſerved at *Upminſter*, and at 200 Miles diſtance in *Lancaſhire* at the ſame time. And in the *Hiſt. de l'Acad. Roy. des Scien. Anno* 1699. Monſieur *Maraldi*, by comparing his Obſervations at the *Paris Obſervatory* with mine at *Upminſter*, takes notice, "That there "is a great Agreement between the Variation of the Heights of the "Barometers in both Places; that he finds almoſt always that when "one riſeth or falleth, the other doth ſo too, although not always "alike: That the Days in each Month whereon the *Mercury* hath been "higheſt or loweſt, it hath been the ſame at *Paris* as at *Upminſter*, but "ordinarily ſomewhat more than three or four Lines lower at *Paris* "than *Upminſter*." But the Agreement between the Variations of Dr. *Scheuchzer*'s Barometers and mine, although often great, yet is not ſo conſtantly, nor ſo certainly great as nearer home, *viz.* at *London*, *Lancaſhire*, *Paris*, and other Places, with which I have made the Compariſon.

—*on the* Rain. The Rain was obſerved at *Piſa* in *Italy*, by Dr. *Mich. Angelo Tilli*; and at *Zurich* in *Switzerland*, by Dr. *J. J. Scheuchzer*, and by myſelf at *Upminſter* in *Eſſex*.

The Rains for the moſt part are more frequent at *Upminſter*, that either at *Zurich* or *Piſa*; I mean, we have more Rainy Days than they. But yet the Rains in both theſe Places are much greater in Quantity, in the whole Year, and in ſome Months, eſpecially the Autumnal and Winter Months, than our Rains are at *Upminſter*. *May*, *June*, and *July*, and a great part of *Auguſt*, in 1707, ſeem to have been very dry, and I ſuppoſe ſearching Months at *Piſa*, as in ſome meaſure ſome of them were here: And in that Time leſs Rain fell there than here. But the following Autumnal Months made at *Piſa* ſufficient Amends, either by the great Quantity that fell at a Time, I ſuppoſe in Thunder, or ſuch like haſty large Showers; or elſe by the Quantity and Frequency of both. What a prodigious Quantity was that, for inſtance, of above 32 Pounds on *Auguſt* 19? (If it all fell on that, and not ſome on the preceding Days). But we find very large Quantities at a time to have fallen on divers Days, where it is manifeſt the Rain was weighed every Day, *viz.* ten Pound, nine Pound, and other large Quantities for ſeveral Days together, in the cooler

cooler Autumnal Months. But as the Weather groweth warmer, I imagine their Rains at *Piſa* are fewer; and what falleth, falleth in large Quantities. For which reaſon the Quantity of Rain in the Spring Months of *March*, *April*, and *May*, 1708, (oftentimes dripping Months in *England*) is nearly the ſame both at *Piſa* and *Upminſter*.

As to the Rain at *Zurich*, I obſerve, that altho' their Rains are leſs frequent than ours in *Eſſex*, yet they ſeem to be more frequent than theirs at *Piſa*: But the Quantity at *Zurich* is greater than at *Upminſter*, and leſs than at *Piſa*.

'Tis Dr. *Scheuchzer*'s Opinion, "That more Rain falleth in *Swit*-"*zerland* than in *France*; at *Zurich* than at *Paris*." To confirm which, he giveth us this Table of eight Years Rain at *Paris*, to which I ſhall add mine for *Upminſter*.

The Rain at Paris *in eight Years.*						*At* Upm.	
The Year.	*Depth in Lines of* Paris *Meaſure.*	*Depth in Inches of* Paris *Meaſure.*		*Depth in* Engliſh *Inches & Centeſim.*		*Depth in* Engliſh *Inches & Centeſim.*	
		Inch	*Lin.*				
1699	224 ¼	18	8 ¼	19	93	15	11
1700	240 ½	20	½	21	37	19	03
1701	256 ¼	21	4 ¼	22	77	18	69
1702	196 ¼	16	4 ¼	17	45	20	38
1703	208 ¼	17	4 ¼	18	51	23	99
1704	238 ½	19	10 ½	21	20	15	80
1705	266 ¾	13	10 ¾	14	82	16	93
1706	183 ½	15	3 ½	16	31	24	29
Total	*Depth*	142	10 ¼	152	36	154	22

It is manifeſt from this Table, that the *Zurich* Rain laſt Year (altho' it amounted not to the Quantity which fell at *Piſa* in a whole Year, yet) exceeded both the *Paris* and *Upminſter* annual Rains of eight Years before. But whether it conſtantly doth ſo or not, will appear from future Obſervations.

I take notice too, that there is a greater Difference between theſe laſt eight Years Rain at *Paris* and *Upminſter*, than I found in the eight Years, in which I formerly compared the Rain of *Townley*, *Paris*, *Liſle*

Lisle and *Upminster* together; for by that Comparison it appeared, that less Rain fell at *Upminster* than at either of the other three Places. But according to these later eight Years in the Table, a small Matter more falleth at *Upminster* than at *Paris.* For the mean Proportion for *Paris* (which according to former Years was above 20 Inches *Paris* Measure, or 22 Inches *English*) is according to these last eight Years, no more than 17 Inches, 9 Lines *Paris* Measure, or 19 Inches *English:* And *Upminster* Rain, which I formerly computed at, Year by Year, about 20 Inches and an half *English*, is for these eight Years much the same, or a little more than that at *Paris.*

The Proportions therefore which I shall now lay down for the yearly Rain of all Places, whose Rain I have had Information of, are these; for *Zurich* ('till farther Observations are made) $32\frac{1}{4}$ Inches; for *Pisa* ('till farther Observations) $43\frac{1}{4}$ Inches; for *Paris*, 19 Inches; for *Lisle*, 4 Inches; for *Townley* in *Lancashire*, $42\frac{1}{2}$ Inches; for *Upminster*, $19\frac{1}{4}$ Inches; all the same, that is *English* Measure.

The last Observation I make is, the great Use of Cold to the making of Rain. That Exhalations and Vapours are the Matter of Rain, is not to be doubted; how they are raised, I shall not enquire; it is sufficient for my present Purpose to say, that when those Vapours are raised, they are constipated and condensed into Clouds and Rain, chiefly by the Cold of the Air to which they are elevated. And the greater the Quantity of Vapours raised is, and withal the more intense the Cold of those airy Regions, the greater is the Quantity of Rain. Now this is manifest from the annexed Tables, compared with Dr. *Scheuchzer*'s and my Weather, *&c.* Observations. Thus for instance *January*, which Dr. *Scheuchzer* frequently observed was sometimes warm, sometimes cold, and appeareth farther to have been so by his Thermometrical Column, and which was the same with us in *South Britain*, that Month, I say, had Plenty of Rain at *Zurich*, *Upminster*, and *Pisa* too. The same might be said of *February* for *Zurich*, and probably *Pisa* too. So also for *December* in 1707, at *Pisa* and *Upminster*; and *December* last at *Zurich* and *Upminster.* But with us *February* was for the most part a cold Month, and the Rain the less, by reason the Vapours either could not be raised in Plenty enough, or not be carried high enough, or suspended long enough to be united, but soon were precipitated back again to the Earth.

From these Causes assign'd, the Plenty of Exhalations and Cold of the airy Regions, I conceiv'd it is, that at *Upminster*, about the Equinoxes, we have often more Rain than at other Seasons. But I cannot say this is certain and constant. Thus it was at the autumnal Equinox in 1707, not only at *Upminster*, but at *Pisa* too: So at *Zurich*, *Pisa* and *Upminster*, about the Vernal in 1708; and at *Zurich* and *Upminster* the last autumnal Equinox: And this 28*th* of *March* 1709, is a pregnant Proof of this. For, not only unusual Cold of the Winter hath been

been ſucceeded by as unuſual Quantities of Rain all this Month, but at this very Time the Weather is open; but withal cool. Particularly *March* 26, many Vapours aroſe, ſo as to fill the Air with a warm ſtinking Fog. The Night following a ſmart Shower of Hail fell, a manifeſt Indication of the Cold of the Middle, or Top of the lower Region of the Air. And the Day after, *viz. March* 27, proved ſo wet a Day, that almoſt five Pound of Rain fell through my Tunnel, a large Quantity for the Compaſs of twelve Inches Diameter in fourteen or fifteen Hours Time. The Winds and Clouds were all the while calm and ſtill, and frequently changing from Point to Point, near round the whole Compaſs; and the Rain that fell, fell thick in ſmall Drops. Which makes me think, that the warm foggy Vapours, raiſed in great Plenty the Day or two before, as ſoon as they were mounted aloft, met with ſudden extreme Cold of the middle Region, and were thereby haſtily condenſed, and the Air being at the ſame Time very light (the Barometer being then very low) they ſpeedily tumbled down in ſmall and thick Drops of Rain.

And this I take to be the very Caſe of the Vernal and Autumnal Rains, that in Spring, when the Earth and Waters are looſed from the Brumal Conſtipations, the Vapours ariſe in great Plenty. So alſo in Autumn, when the Heats that diſſipated them in Summer, and alſo warmed the ſuperior Regions, are abated, the Vapours raiſed then in great Plenty are ſoon condenſed by the Cold of the ſuperior Regions, and ſo are forced down in more plentiful Rains, than at other Seaſons, when either the Vapours are fewer, or Cold of the ſuperior Regions leſs.

For a farther Illuſtration of this, let us caſt an Eye upon *June* laſt, a Month as unſeaſonably Wet, as 'twas unuſually Cold. The Cold thereof I have already taken notice of; and the wet Weather accompanying it was ſo unſeaſonable to us in *South Britain*, that altho' we had great and welcome Crops of Hay after a great Scarcity the preceding Year, yet we had ſcarcely any good Weather to make it in. So Dr. *Scheuchzer* ſaith it was with them in *Switzerlaud*, in his Remarks on that Month: *This Month, as appears from meaſuring the Rain, was more than ordinary wet, to the Injury both of Men and Vegetables. Much Hay was rotted, the Corn alſo that was not yet cut grew too luxuriant. The Vines and their Bloſſoms received much Damage from the continual Rains; their tender Buds fell off, their Leaves were canker'd, and there appear'd but ſmall Hopes from the approaching Autumn,* &c.

Having conſider'd the Uſe of Cold to the Production of Rain, I ſhall remark one Thing concerning the *Alps*; and that is, I cannot but think that thoſe, and all ſuch like high Mountains, and the Snows they are cover'd with, are of great Uſe to the neighbouring, and more diſtant Countries, in generating their Rain, and performing other great Offices

Offices of Nature. From ſome Obſervations I have made in comparing Dr. *Scheuchzer*'s and my own larger Tables, having ſo frequently obſerved the Riſing and Fallings of the Barometer, ſome of the moſt conſiderable Variations of the Wind, the moſt remarkable Alterations of Heat and Cold, and of Wet and Dry; I have ſo often obſerved many of theſe to precede in one Place what hath followed in another, that I am apt to think that even *England* may ſometimes partake of the Effects of the *Alpine Mountains* upon the Air and Vapours. It is certain that their very cold Weather in *December* laſt, and the Relaxation thereof preceded ours: Which makes me inclin'd to think it might probably be derived from them to us. All the former part of that Month, eſpecially from about the *8th* Day till the *24th*, was here mild and open. But on *Chriſtmas* Day it began to be colder, and the following Days to freeze harder and harder; inſomuch that on *December* 30, my Thermometer was a great deal lower than ever I had ſeen it before. And two Perſons in *London* told me, that the Spirits in their Thermometers fell ſeveral Degrees lower this laſt Winter, than they had done in the ſelf-ſame Thermometers during all the long and remarkable Froſt in the Year 1683. Whether at *Zurich* the Cold was more exceſſive than it uſed to be in other Years, Dr. *Scheuchzer* doth not ſay; but he noteth the Air to have been exceſſively cold, and his Thermometrical Obſervations ſhew it to have been ſo ſome time before, in, and after *Chriſtmas*. And Dr. *Newton*, in a Letter to me from *Florence*, ſays, "The Cold was there ſo great, that for twenty "Years paſt they had not been ſenſible of greater; it wanting on "*Twelfth-day* but half a Degree of the Extremity". Their *Twelfth-day*,, I reckon, fell on *December* 26, *O. S.* and conſequently their ſo eminenly Freezing Day preceded ours about four Days.

And as their Cold, ſo by Dr. *Scheuchzer*'s Obſervations, I find the Relaxation thereof preceded ours a ſhort Time. For about the latter End of *December*, the Weather appears to have been milder, at leaſt leſs intenſely cold with them. And ſo was ours at the Beginning of *January*, about as many Days after theirs, as their Cold preceded ours.

Thus I have given one eminent Inſtance of what I found leſſer Examples frequently, as I run over Dr. *Scheuchzer*'s laſt Year's Obſervations. But whether there may be any farther Reaſons for any ſuch Concluſions about the Influences of the *Alpine* Eminences and Colds upon far diſtant Places, future Obſervations will determine. But as to their Influences nearer home, Dr. *Scheuchzer* ſaith, *The* Alps *are not only the fruitful Mother of Rivers and Clouds, but alſo of Snow and Rain. It is very credible, that ſuch Places as are near the Sea and the* Alps, *abound more with Rain than Places which are more remote.*

To

To these Remarks I might add Dr. *Scheuchzer*'s Observations of the Occurrences in each Month, of what was curious as to Meteors, the State of Health and Diseases, *&c.* Also the Increase and Decrease of their *Zurich* River, the *Limat*, which (like other Rivers that have their Source in the *Alps*) he puts beyond all Doubt (in my Opinion) to receive greater Increments from the melting of the *Alpine* Snows, than from all the Wet proceeding from their Rains. But for them, I refer to his Observations at large.

From Dr. *Tilli*'s Table of Rain, compar'd with the other Tables, it appears, that although, in the Year before, *June* and other Summer Months were dry, yet last *June* was a wet Month at *Pisa*, as well as *Zurich* and *Upminster*; and so likewise was it about the Autumnal Equinox: And for the same Reasons I imagine, which I have already mentioned.

As to the Excess of the *Pisa* Rain above that of other Places, he attributeth it to the same Cause, that I did that of *Lancashire*, namely, *the Height of the Hills, and the Blowing of the Winds for a long Time from some one Quarter.* His Observation is this, *I easily allow, that our Rain always or for the most part exceeds yours, for the Reason you have observed. And especially if the rough* Corsican *Mountains, at the Time of Autumn, are covered early with Snow. Then the Southerly Winds and Showers prevail for a long Time. But it plainly appears that the North Winds blow more frequently about the Hills of* Florence *than the City of* Pisa. *For this City is surrounded with Hills on the North-side, and is distant about five Miles from the Sea at an equal Interval.*

The same Account of the Situation of *Pisa*, and the great Quantity of Rain falling there, I remember I had some Time since from a very ingenious Member of this Society, Mr. *Aston*, who hath been there.

A Table shewing the Heights of the Mercury in the Barometer *in* English *Inches and Centesimals of an Inch, both at* Zurich *in* Switzerland, *and at* Upminster *in* South Britain, *together with the Differences of those Heights, for the Year* 1708.

	January.			February.			March.		
D. of M.	Zurich *in* Engl. *Inches.*	Upm. *in* Engl. *Inches.*	*Dif. in* Engl. *Inches.*	Zurich *in* Engl. *Inches.*	Upm. *in* Engl. *Inches.*	*Dif. in* Engl. *Inches.*	Zurich *in* Engl. *Inches.*	Upm. *in* Engl. *Inches.*	*Dif. in* Engl. *Inches.*
1	28 1	29 4	1 3	28	29 59	1 59	27 90	29 22	1 32
2				8	52	1 44	28	23	1 23
3	17	14	97	17	53	1 36	27 85	42	1 57
4	26	43	1 17	17	39	1 22	82	64	1 83
5	17	42	1 25	75	37	1 12	28	73	1 73
6	1	28	1 27	17	44	1 27	12	44	1 32
7	17	43	1 26	12	35	1 23	6	73	1 67
8	27 99	5	1 6	8	40	1 32	27 81	30 12	2 31
9	64	28 78	1 14	40	67	1 27	81	29 46	1 65
10	46	50	1 4	15	87	1 72	85	18	1 33
11	46	97	1 51	14	30 2	1 88	75	30	1 55
12	99	29 38	1 39	27 90	2	2 12	81	50	1 69
13	73	23	1 50	83	29 67	1 84	85	59	1 74
14	73	11	1 38	28	49	1 49	85	45	1 60
15	27 99	28 99	1	27 82	45	1 63	28 6	37	1 31
16				84	47	1 63	13	47	1 34
17	28 17	29 19	1 2	81	32	1 51	8	52	1 44
18	8	28 89	81	27 95	33	1 38	27 90	33	1 43
19	27 90	29 3	1 13	81	36	1 55	28 6	21	1 15
20	90	15	1 25	28	25	1 25			
21	81	28 80	99	17	8	91	27 85	34	1 49
22	73	95	1 22	38	41	1 3	72	8	1 36
23	99	29 12	1 13	35	48	1 13	72	13	1 41
24	90	20	1 30	17	47	1 30	85	6	1 21
25							81	34	1 53
26				20	25	1 5	75	29	1 54
27	81	57	1 66	15	22	1 7	85	16	1 31
28	81	95	2 14	27 95	28 99	1 4	28 4	38	1 34
29				85	94	1 9	27 90	37	1 47
30							95	6	1 11
31	28 08	76	1 68				28	14	1 14

D. of M.	April. Zurich in Engl. Inches.	April. Upm. in Engl. Inches.	April. Dif. in Engl. Inches.	May. Zurich in Engl. Inches.	May. Upm. in Engl. Inches.	May. Dif. in Engl. Inches.	June. Zurich in Engl. Inches.	June. Upm. in Engl. Inches.	June. Dif. in Engl. Inches.
1	28	28 97	0 97	28 0	29 53	1 53	27 95	29 65	1 70
2	27 95	94	0 99	6	69	1 63	28 6	55	1 49
3	90	29 28	1 38				27 81	86	2 5
4	81	53	1 72	8	50	1 42	83	30 7	2 24
5	90	45	1 55	0	44	1 44	35	29 96	2 61
6	85	65	1 80	8	41	1 33	90	69	1 79
7	90	65	1 75	6	36	1 30	28 8	52	1 44
8	28	50	1 50				27 90	50	1 60
9	00	60	1 60	26	62	1 36	90	56	1 66
10	27 64	62	1 98	12	63	1 51	28 0	56	1 56
11	75	77	2 2	0	46	1 46	27 90	58	1 68
12	72	62	1 90	27 90	66	1 76	28 6	59	1 53
13	90	80	1 90	81	79	1 98	27 81	36	1 55
14	28	91	1 91	75	83	2 8	85	49	1 64
15	6	89	1 83	72	68	1 96	90	60	1 70
16	8	88	1 80	54	66	2 12	85	47	1 62
17	27 95	93	1 98				28 3	44	1 41
18	95	85	1 90	64	44	1 80	27 81	40	1 59
19	28	89	1 89	72	58	1 86	81	47	1 66
20	6	88	1 82	90	74	1 84	85	73	1 88
21	6			28 0	81	1 81			
22	8	30	1 92	0	59	1 59	90	70	1 80
23	00	29 80	1 80	27 95	54	1 59	97	70	1 73
24				28 8	67	1 59	85	43	1 58
25	8	80	1 72	17	80	1 63	96	45	1 49
26	12	85	1 73	15	86	1 71			
27				27 85	84	1 99	72	81	2 9
28	6	76	1 70	28 8	81	1 73	73	99	2 26
29				0	87	1 87	72	98	2 26
30	00	37	1 37	0	84	1 84	75	80	2 5
31				27 90	78	1 88			

D. of M.	July. Zurich in Engl. Inches	July. Upm. in Engl. Inches.	July. Dif. in Engl. Inches.	August. Zurich in Engl. Inches.	August. Upm. in Engl. Inches.	August. Dif. in Engl. Inches.	September. Zurich in Engl. Inches.	September. Upm. in Engl. Inches.	September. Dif. in Engl. Inches.
1	28	29 72	1 72	27 85	30 2	2 17	28 16	29 58	1 52
2	27 95	92	1 97	81	29 84	2 3	00	54	1 54
3	97	89	1 92	85	70	1 85	17	50	1 33
4	95	80	1 85	28 6	72	1 56	8	48	1 40
5	28	67	1 67	10	67	1 57	27 72	61	1 89
6	27 75	69	1 94	8	56	1 48			
7	81	72	1 91	0	57	1 67	72	45	1 73
8	28	69	1 69	27 81	93	2 12	72	49	1 77
9	8	68	1 60	85	98	2 13			
10	12	80	1 68	90	62	1 72	81	45	1 64
11	0	84	1 84	95	66	1 71			
12	27 81	90	2 9	28 8	89	1 81	64	67	2 3
13	93	83	1 90	0	93	1 93	46	48	2 2
14	28 6	68	1 62	27 85	93	2 8	72	43	1 71
15				75	84	2 9	85	28 65	0 80
16				72	80	2 8			
17	27 95	63	1 68	73	55	1 82	72	29 30	1 58
18	90	77	1 87	72	19	1 47	69	67	1 98
19	28	76	1 76	90	61	1 71			
20	27 72	84	2 12	95	81	1 86	72	88	2 16
21	28	66	1 66	72	93	2 21	75	86	2 11
22	6	50	1 44	81	77	1 96	75	85	2 10
23	17	56	1 39	90	93	2 3	75	96	2 21
24	8	84	1 76	95	53	1 58	81	30 20	2 39
25	6	73	1 67	28 00	51	1 51	56	17	2 61
26				00	51	1 51	50	12	2 62
27	27 95	82	1 87	27 95	52	1 57	64	89	2 25
28	85	54	1 69	85	63	1 78	81	94	2 13
29	95	66	1 71	28	51	1 51			
30	85	61	1 76	27 85	62	1 77	72	73	2 1
31	81	96	2 15	28 6	56	1 50			

October.

D. of M.	October. Zurich in Engl. Inches.	October. Upm. in Engl. Inches.	October. Dif. in Engl. Inches.	November. Zurich in Engl. Inches.	November. Upm. in Engl. Inches.	November. Dif. in Engl. Inches.	December. Zurich in Engl. Inches.	December. Upm. in Engl. Inches.	December. Dif. in Engl. Inches.
1	27 72	29 92	2 20	28 22	30 21	1 99	28 26	29 36	1 10
2	69	93	2 24	24	18	1 94	30	45	1 15
3							22	5	0 83
4							27 99	28 96	0 97
5	72	73	2 1	15	6	1 91	73	92	1 19
6	85	30 5	2 20	08	12	2 4	77	29 11	1 34
7				17	29 86	1 69	85	11	1 26
8	81	29 94	2 13				99	15	1 16
9				22	30 8	1 86	28 15	24	1 9
10	72	86	2 14	17	10	1 93	15	24	1 9
11	64	86	2 22	22	29 78	1 56	08	28	1 20
12	72	30 6	2 34				27 99	40	1 41
13				22	30 10	1 88	90	59	1 69
14	90	29 41	1 51	17	00	1 83	73	83	2 10
15									
16				22	15	1 93			
17	87	79	1 92	26			75	74	1 99
11	28 6	59	1 53				90	77	1 77
19	00	49	1 49	22	29 88	1 66	95	52	1 57
20	27 72	70	1 98				99	60	1 61
21	72	30 00	2 28	22	50	1 28	28 6	61	1 55
22	90	2	2 12				6	50	1 44
23	95	29 76	1 81	27 77	27	1 50	27 99	77	1 78
24	95	74	1 79	90	60	1 70	28 17	28	1 11
25	64	89	2 25	28 17	84	1 67	26	36	1 10
26	85	80	1 95				17	30 14	1 97
27	81	67	1 80				22	28	2 6
28	81	63	1 74						
29	28 00	64	1 64				17	29 83	1 66
30	27 90	80	1 90	26	45	1 19	15	80	1 65
31	28 00	73	1 73				17	49	1 3

A Table

A Table of the Rain at Pisa *in* Italy, *both in* Tuscan *and* English *Troy Weight, which fell through a Tunnel of half a Brace square, from* May *till the End of* December 1707: *As also the Quantity of Rain at* Upminster *in* Essex *at the same Time, which fell through a round Tunnel of* 12 *Inches Diameter, in Pounds Troy, and Centesimals of a Pound.*

	May.			June.			July.		
D. of M.	Pisa *Rain in* Tuscan *Weight.*	Pisa *Rain reduced to* Eng. *W.*	*Rain at* Upminster.	*Rain at* Pisa *in* Tuscan *Weight.*	Pisa *Rain reduced to* Eng. *W*	*Rain at* Upminster.	Pisa *Rain in* Tuscan *Weight.*	Pisa *Rain reduced to* Eng. *W.*	*Rain at* Upminster.
	l. oz	*l. dec.*	*l. dec.*	*l. oz.*	*l. dec.*	*l. dec.*	*l. oz.*	*l. dec.*	*l. dec.*
1						0 17			0 1
2						2 90			
3						0 05			0 92
4									0 40
5									0 25
6						0 55	2 0	1 84	
7									
8				5 7	5 12				
9									
10						0 06			
11						0 2			0 60
12									0 12
13									0 3
14									
15						0 29			
16									0 17
17									0 16
18							0 3	0 23	
19									
20									
21									
22	0 9	0 69	2 70						0 94
23			1 26						0 65
24			0 17			0 85			
25						0 42			
26									
27						0 33			0 92
28						0 23			0 20
29			0 56						
30			0 56			0 81			
31 Tot.wt		0 69	5 25		5 12	6 68		2 7	6 37
Dep.in Inch		0 12	1 05		0 88	1 34		0 25	1 27

D. of M.	August.			September.			October.		
	Pisa *Rain in* Tuscan *Weight*	Pisa *Rain reduced to* Eng. *W.*	*Rain at* Upminster.	*Rain at* Pisa *in* Tuscan *Weight.*	Pisa *Rain reduced to* Eng. *W.*	*Rain at* Upminster.	*Rain at* Pisa *in* Tuscan *Weight.*	Pisa *Rain reduced to* Eng. *W.*	*Rain at* Upminster.
	l. oz.	*l. dec.*	*l. dec.*	*l. oz.*	*l. dec.*	*l. dec.*	*l. oz.*	*l. dec.*	*l. dec.*
1									
2	1 1	1 0	1 90						2 54
3							4 8	3 28	
4			0 76						
5			0 09						0 69
6			3 34			0 51			
7			0 16				7 7	6 96	
8			0 45			0 02			
9			0 81			0 04			
10						1 40	1 9	1 61	
11				0 50	0 38	0 70			
12							2 3	2 06	
13			0 10	0 4	0 31	0 51			
14			0 07	2 11	2 68	0 06			
15							3 10	3 52	
16							1 8	1 63	
17			1 68						
18						0 10			
19	32 5	29 75	0 10			0 87			
20	1 8	1 53	1 02	3 8	3 36				
21				7 9	7 11	1 65			1 62
22				10 0	9 18				
23				3 2	2 91				0 80
24	1 4	1 23		6 10	6 27	2 73			0 26
25						0 85			
26						0 46			
27						2 76			
28				5 10	5 35				0 71
29			0 31			1 84			
30			0 09						
31	*Total.*	33 51	88		37 55	14 50		19 66	6 62
Dep. in Inch		5 76	2 176		6 45	2 90		3 43	1 324

D. of M.	November. Rain at Pisa in Tuscan Weight.	Pisa Rain reduced to Eng. W.	Rain at Upminster.	December. Pisa Rain in Tuscan Weight.	Pisa Rain reduced to Eng. W.	Rain at Upminster.
	l. *oz.*	*l.* *dec.*	*l.* *dec.*	*l.* *oz.*	*l.* *dec.*	*l.* *dec.*
1						
2				2 8	2 45	0 44
3			0 33			0 65
4						
5						
6		1 84				0 63
7						0 25
8				5 4	4 89	
9	9 4	8 57				0 84
10				2 10	2 60	
11						
12				2 8	2 45	1 67
13						
14			0 53			
15						1 24
16				3 10	3 52	0 96
17			0 32			
18						
19	3 9	3 44				
20				5 6	5 05	0 82
21			0 86			1 38
22						0 22
23	9 5	8 64				0 25
24	0 6	0 46	3 08			
25	2 4	2 14				0 16
26			0 28	3 2	2 91	
27	1 5	1 30		7 3	6 65	0 62
28						1 98
29			0 50	7 5	6 80	
30						0 03
31	*Tot.wt*	24 55	5 90		37 32	12 14
Dep.inInch		4 22	1 18		6 39	2 428

A Table of the Rain at Zurich *in* Switzerland, *at* Pisa, *and* Upminster, *in the Year* 1708. *All reduced to the Depth in* English *Inches, and Centesimals of an Inch.*

D. of M.	January.								February.							
	Rain at Zurich.		*Rain at* Pisa.		Pisa *Ra. reduced.*		*Rain at* Upmin.		*Rain at* Zurich.		*Rain at* Pisa.		Pisa *Ra. reduced.*		*Rain at* Upmin.	
	Inches.		*l.*	*oz*	*l.*	*dec.*	*l.*	*dec.*	*Inches.*		*l.*	*oz.*	*l.*	*dec.*	*l.*	*dec.*
1			5	8	5	20										
2	1								2	1/2	3	3	2	98		
3																
4			7	2	6	58	0	97	1	1/4						
5	1						0	29		1/4	0	10	0	76		
6							0	7	2						0	2
7																
8									2							
9			6	3	5	74	0	53			2	4	2	14		
10							2	48			1	8	1	53		
11							1	26								
12									2	1/2	0	9	0	69		
13							1	91								
14			3	4	3	6	1	88								
15	4						0	4		1/2					0	18
16							0	92								
17			5	6	5	5			1	3/4						
18			2	9	2	52	1	26								
19									4	3/4						
20	1	1/2								1/6	2	1	1	90	0	54
21	1														0	51
22	4						0	91								
23	2										7	2	6	58	0	64
24	2		3	6	3	21										
25	1	1/2					1	7							0	19
26							0	80								
27			4	1	3	75										
28												9	2	52		
29		1/2								1/2					0	22
30																
31			2	5	2	22										
Tot	18	1/2			37	33	14	39	18	2/3			19	10	2	30
Dep		6			6	41	2	88	1	65			3	28	0	46

D. of M.	March. Rain at Zurich. Inches.		Rain at Pisa. l.	oz.	Pisa Ra. reduced. l.	dec.	Rain at Upmin. l.	dec.	April. Rain at Zurich. Lines.		Rain at Pisa. l.	oz.	Pisa Ra. reduced. l.	dec.	Rain at Upmin. l.	dec.
1	2	¼							9	¼					1	16
2											2	10	1	68	0	16
3									7	¼	2	4	2	14		
4											0	9	0	69		
5															1	54
6							1	27								
7									1							
8									3							
9															0	16
10							0	38	1	¼					0	24
11							0	37		¼						
12	3	½													0	85
13			8	1	7	40										
14							0	37		⅓						
15		½														
16									4	¾						
17									5	½	0	10	0	76		
18			6	3	5	74			3	¼						
19	2	¼					0	25			2	2	1	99		
20							0	8	5							
21							0	20								
22							1	37								
23	2	⅙					1	06	6							
24							1	37		½						
25							1	43	3	¼						
26		¼								⅓						
27							1	29								
28							0	15								
29							0	54							0	65
30	1	½													0	01
31	3	½	2	6	2	29										
Tot	17	⅙			15	43	10	13	52	¼			7	26	4	77
Dep	1	51			2	65	2	3	4	69			1	25	0	96

D. of M.	May: Rain at Zurich.		Rain at Pisa.		Pisa Ra. reduced.		Rain at Upmin.		June: Rain at Zurich.		Rain at Pisa.		Pisa Ra. reduced.		Rain at Upmin.	
	Lines.		*l.*	*oz.*	*l.*	*dec.*	*l.*	*dec.*	*Lines.*		*l.*	*oz.*	*l.*	*dec.*	*l.*	*dec.*
1									1	½						
2											4	1	3	75	0	66
3							1	09	21	¼	2	2	1	99		
4			5	1	4	66										
5							0	27	2	½	3	4	3	06		
6			4	4	3	98					2	8	2	45		
7			3	10	3	52	3	28							0	60
8							0	82	1	¼	3	3	2	98	0	20
9							0	11							0	90
10											0	9	0	69	0	11
11																
12	1	¼													1	66
13							0	32		½					0	15
14		⅓									2	1	2	68	0	76
15	2	⅓					1	11	3	¼	0	10	0	76		
16							0	82								
17		⅓					0	01							0	01
18							0	96			11	1	10	17	0	93
19	5	¼													0	04
20	3	¼							19							
21							0	53							2	66
22							0	79	1							
23															1	23
24									6	¼					0	80
25			3	4	3	06									0	44
26									1	¼					0	38
27	2	¼														
28										½						
29	1															
30			2	10	2	06			6	¼					0	08
31	4	½	1	8	1	53										
Tot	21	½			19	35	10	11	66	½			28	53	11	61
Dep	1	91			3	33	2	02	5	91			4	90	2	32

	July.				August.			
D. of M.	*Rain at* Zurich.			*Rain at* Upmin	*Rain at* Zurich.	*Rain at* Pisa.	Pisa *Ra. reduced.*	*Rain at* Upmin.
	Lines.			*l. dec.*	*Lines.*	*l. oz.*	*l. dec.*	*l. dec.*
1	1			0 06				
2								
3	2 ½							
4				0 03				
5								
6	25 ½							
7								3 38
8				0 93				
9				0 49				
10				0 59				
11				0 47				0 07
12				0 01				
13				0 11				
14						10 1	9 25	
15				1 00		1 8	1 53	
16				0 16				
17						2 8	2 45	1 05
18	3				3 ¼			0 56
19								
20	2 ⅖							
21				0 01				
22								
23					27 ¼			
24				0 01				0 15
25				0 41	1 ¼			0 76
26	¼							
27								0 31
28	¼			1 20				
29	⅙				2 ½			1 32
30	3			0 04	¼			
31								7 10
Tot	39 ⅓			5 52	35 ½		13 23	14 70
Dep	3 50			1 11	3 15		2 27	2 94

	September.								October.							
D. of M.	*Rain at* Zurich.		*Rain at* Pisa.		Pisa *Ra. reduced.*		*Rain at* Upmin.		*Rain at* Zurich.		*Rain at* Pisa.		Pisa *Ra. reduced.*		*Rain at* Upmin.	
	Lines.		*l.*	*oz.*	*l.*	*dec.*	*l.*	*dec.*	*Lines.*		*l.*	*oz.*	*l.*	*dec.*	*l.*	*dec.*
1			1	2	1	07					5	1	4	67		
2															0	03
3			10	0	91	8	0	74								
4							0	06								
5															0	08
6							0	64								
7	2		12	2	11	17										
8	7	½					0	40								
9	4	¼									4	4	3	98		
10							0	05								
11							0	01	4	¼	14	5	13	13		
12							1	41							0	06
13															0	37
14	5	¾					1	24							0	16
15	2	¼	2	8	2	45	1	10			2	3	2	06		
16																
17																
18	7	¼					0	29			2	6	2	29	0	31
19											1	9	1	61		
20																
21			10	4	9	48			13	½						
22																
23							0	27							0	02
24			9	4	8	57										
25																
26									6	¼	3	7	3	29		
27	3	¼														
28															0	06
29							0	99								
30							0	08								
31									2	¼					0	05
Tot	34				41	92	7	28	27	½			31	03	1	14
Dep	3	02			7	21	1	46	2	44			5	33	0	23

	November.								December.				
D. of M.	*Rain at* Zurich.		*Rain at* Pisa.		Pisa *Ra. reduced.*		*Rain at* Upmin.		*Rain at* Zurich.			*Rain at* Upmin.	
	Lines.		*l.*	*oz.*	*l.*	*dec.*	*l.*	*dec.*	*Lines.*			*l.*	*dec*
1			3	3	2	98						0	10
2													
3												0	61
4							0	92				0	08
5									17 ½			0	27
6													
7													
8												0	30
9			2	0	1	84						0	66
10													
11							2	22					
12		¼	3	6	3	21	2	26					
13									8				
14													
15													
16													
17												0	54
18													
19												1	84
20													
21												1	22
22													
23	6	¼										4	00
24												0	22
25													
26													
27													
28													
29													
30							0	90	½				
31									3 ½				
Tot	7				8	03	4	30	29 ½			9	84
Dep	0	62			0	13	0	86	2 62			1	97

A Prospect of all the Rain *in the foregoing Tables, in every Month, Half Year, and the whole Year, from* June 1. *N. S. or* May 21. *O. S.* 1707. *to the End of the Year* 1708.

	Depth of the Pisa *Rain.* Engl. *Inches.*		*Depth of the* Upm. *Rain.* Engl. *Inches.*		*Depth of the* Zurich *Rain.* Engl. *Inches.*	
May.	0	12	1	05		
June.	0	88	1	34		
July.	0	36	1	27		
August.	5	76	2	18		
September.	6	45	2	90		
October.	3	43	1	33		
November.	4	22	1	18		
The Half Year's Rain.	21	22	11	25		
December.	6	39	2	43		
Anno 1708.						
January.	6	41	2	88	1	64
February.	3	28	0	46	1	65
March.	2	65	2	03	1	51
April.	1	25	0	96	4	69
May.	3	33	2	02	1	91
The Half Year's Rain.	23	31	10	78		
Depth of the whole Year's Rain.	44	53	22	03		
June.	4	90	2	32	5	91
The Half Year's Rain.			10	67	17	31
July.			1	11	3	50
August.	2	27	2	94	3	15
September.	7	21	1	46	3	02
October.	5	33	0	23	2	44
November.	0	13	0	86	0	62
December.			1	97	2	62
The Half Year's Rain.	19	84	8	57	15	35
The whole Year's Rain.			19	24	32	66

An Account of the Rain at Upminster *for* 18 *Years, by* Mr. W. Derham. n. 341. p. 130.

XI. The last Year (1714) having been so remarkably dry, that the Ponds hereabouts are for the most part dry, and the Springs generally either very low or quite failing, I made an Extract (out of my Registers of the Weather, &c.) of the *Quantity of Rain* which fell at *Upminster* the last 18 Years. The Particulars of which, every Year, may be seen in the Table. In one Column of which, the Weight of the Rain in Pounds *Troy*, and Centesimals of Pounds, may be seen; in the other, the Depth of it in Inches and Centesimals of inches, or what Height it would have been, had it not been imbib'd by the Earth, or lessen'd by Exhalations, but been suffer'd to have stagnated on the Ground.

A Table of Rain which fell at *Upminster*, from the Year 1697, to the Year 1714.

Year.	Weight		Depth.	
	l.	Cent.	Inch.	Cent.
1697	77	60	15	52
1698	122	32	24	46
1699	75	54	15	11
1700	95	13	19	03
1701	93	45	18	69
1702	101	89	20	38
1703	119	94	23	99
1704	79	02	15	81
1705	84	62	16	93
1706	121	43	24	29
1707	81	55	16	31
1708	96	09	19	22
1709	132	82	26	56
1710	91	84	18	37
1711	118	02	23	60
1712	118	78	23	76
1713	115	80	23	16
1714	55	95	11	19

Remarks on the Dry Year 1704.

Among the *Dry Years*, 1704 was complain'd of for one; which I remember the News-Papers reported to have been so considerable at *Venice*, that they were forc'd to fetch their Water in Barks five Leagues off, as far as the *Brenta*; so that publick Prayers were put up for Rain. Yet we may observe, that several other Years were drier that that with us at *Upminster*. But among them all, none comparable to the last Year 1714. In which the whole Quantity of Rain was no more than 55 *l.* 95 Hundredths, or 11 Inches 19 Hundredths; whereas the least Quantity of any of the preceding 18 Years, exceeded 15 Inches in Depth.

What Effects this Drought hath had in the Bodies of Animals, I leave others to judge. It is well known how contagious and fatal a Distemper hath raged among, not only our own black Cattle, but in many other Parts of *Europe*. And I observed the *Itch* was Epidemical among the poorer Sort, at the Beginning of the Year; that the *Measles* were very common some Parts of the Year; and that *Pleurisies* and *Malignant Fevers* infested a great many, especially in the Summer Months. But how far these Distempers might be owing to the dry Season, I leave to the Physicians.

To

To compare with these, we have collected out of the *Memoirs* of the *Royal Academy* of *Sciences*, the Quantity of *Rain* and dissolved *Snow* which has fallen at the *Observatory* at *Paris* for 23 Years together; according to the accurate Observation of M. *De la Hire*, and have reduced the *French* Measure to our own. But it is to be observed, that the Diversity of *Stile* makes the Years not exactly the same, though, as to this Matter, the Difference may seem very inconsiderable.

The Rain at Paris *for* 23 *Years.*

	French		*English.*	
Anno	Inch	Lin.	Inch	Cent.
1689	18	$11\frac{1}{2}$	20	23
1690	23	$3\frac{1}{4}$	24	87
1691	14	$5\frac{1}{4}$	15	40
1692	22	$7\frac{1}{2}$	24	14
1693	22	8	24	18
1694	19	9	21	07
1695	19	$7\frac{1}{4}$	20	96
1696	19	$5\frac{1}{2}$	20	76
1697	20	3	21	68
1698	21	9	23	20
1699	18	$8\frac{1}{4}$	19	93
1700	20	$0\frac{1}{3}$	21	38
1701	21	$4\frac{1}{4}$	22	78
1702	16	4	17	42
1703	17	$4\frac{1}{4}$	18	51
1704	19	$10\frac{1}{3}$	21	20
1705	13	$10\frac{1}{4}$	14	82
1706	15	$3\frac{1}{6}$	16	32
1707	17	11	19	11
1708	18	$3\frac{1}{2}$	19	51
1709	21	$9\frac{1}{6}$	23	21
1710	15	$8\frac{1}{4}$	17	10
1711	25	2	26	84

A Storm of Rain at Denbigh *in* Wales, *by*——n. 308. p. 2348.

XIII. Upon *Tuesday*, the 16*th* Day of *July* 1706, about Eight of the Clock in the Morning, it began to rain in and about *Denbigh*, which continued incessantly for 30 Hours, but not very violently, till about Three or Four a-Clock in the Morning upon *Wednesday*, when it rain'd somewhat faster, attended with a terrible Noise (like Thunder) with some Flashes of Lightning, and a boisterous Wind. About Break of Day the Rain and Wind began to abate of their Violence, which lessen'd gradually every Hour, till about One or Two a-Clock in the Afternoon, and then it perfectly ceased, and the Air became clear and somewhat calm.

Upon *Tuesday* the Wind blew South West; but on *Wednesday* it was come to the North West.

The Effects of this great Storm were dismal, for it occasion'd the overflowing of all the Rivers in *Denbighshire*, *Flintshire*, and *Merionethshire*, &c. which spoil'd a great deal of Corn, and took off all the Hay that was mowed, near the Banks of the Rivers, which was carried

carried by the Stream in ſuch vaſt Quantities down to the Bridges, that it choak'd the Arches and Inlets, inſomuch that it broke down above a Dozen great Bridges. Great Oaks and other large Trees were unrooted and ſwept away, with ſeveral Quickſet Hedges; and ſome Quillets by the Side of the River *Elwy*, ſo covered with Stones and Gravel, that the Owners can't well tell whereabouts their Hedges and Landmarks ſtood; and the ſame River has alter'd its Courſe in ſome Places, ſo as to rob the Landlords on one Side of ſome Acres, and beſtowed as much on the oppoſite Side. Two or three Rivulets, that convey'd Water to ſome Mills, have been ſo choak'd up with Stones and Gravel, that the Owners don't think the Profit will countervail the great Charge of clearing them.

It is affirmed by many, that the great Floods were not ſo much the Effects of the Rain, as the breaking out of an infinite Number of Springs, in ſuch Places, as they were never known to flow from before. In the Town of *Denbigh* a great many broke out in the Houſes and Stables, eſpecially in that Part which lies next the Caſtle on the North Side of it; ſome of which broke out with a great deal of Violence, and in ſuch a Quantity, that it is affirmed by ſeveral Men of the Town, that three of theſe new Springs, which flowed out of the Stables of three Inns, were ſufficient to turn any Corn Mill.

At a ſmall Diſtance, Northward of *Denbigh*, lies *Park-Snodiog*, a Rocky Hill, out of which broke out a great many Springs, which flowed ſo plentifully for nine or ten Days, that the Cattle water'd in them for that Time; whereas before and after, the People were forc'd to water them all Summer long at a Well in the Highway, at ſome Diſtance from this *Park-Snodiog*. There are ſeveral deep Holes and Trenches cut in the Highways adjoining to the River *Elwy*, &c. ſome ſo very large, as to hide three or four Horſes, which is not attributed ſo much to the overflowing of the River, as to the breaking out of Springs in thoſe very Places.

In *Comb* Mountain there is a Pit of a circular Form, which in the Summer time uſed to have little or no Water in it, and in Winter, as much Water as would ſwell the Surface to about fourteen or ſixteen Yards croſs over: But now in the midſt of Summer it roſe up at leaſt a Yard and a half higher than it was ever known to do in the wetteſt Winters; and overflowing its Banks, it fell down the Hill with ſuch Violence, as to penetrate into the very Body of a Rocky Road, and dug Pits in it, that will bury the biggeſt Horſes; and the Road, which was a common Highway, is now become irreparable.

XIV. 1.] In the Month of *March* 1701, in the Forenoon, between the Hours of 10 and 11, I obferved a remarkable *Water-Spout* in the *Downs*. It bore N by E off our Ship, about two Leagues diftance by Eftimation; the Wind at E N E. a Top-fail Gale, and very cold. The Horizon was entirely open and ferene, except the Northern Parts thereof from N N W to N E by E. or thereabouts. The higheft Part of the Cloud appear'd to make an Angle of 45 Deg. of Elevation. About one half of the Cloud, (*viz.* the upper) was very white, and the other extreamly black. The Spout itfelf, (which hung from the lower Part of the whitifh Cloud) hovered up and down for about 20 Minutes, and during two or three Minutes of the Time, that Part of the Sea exactly under the Spout, did fparkle up Water to a confiderable Height. The fparkling run along to the Leeward, (the Cone of the Spout moving that Way, and making, it feems, a Difcharge, tho' not vifible to us in its Fall) and continued running along for fix Ships length. Afterwards the Body of the Spout did quickly contract itfelf, and then difappear'd. About two Hours afterwards the Heavens were intirely overcaft, and during that Afternoon there fell abundance of Hail, and both Wind and Cold increafed. I have feen feveral Water-fpouts in the *Mediterranean* fome Years ago, and thofe ufually during the Time of a ftark Calm and hot Summer Weather; but to fee one in our Northern Climate at this Time of the Year, and during Weather both Cold and Windy, is, I think, a little unufual.

A Water-Spout *obferv'd in the* Downs, *by Mr.*P.Gordon, n. 270. p. 805.

2.] The 27th of *Auguft* 1701. being upon the Coaft of *Barbary*, to the Northward of the Town of *Bona*, upwards of 10 Leagues diftance at Sea, about 7 a-Clock at Night, foon after Sun-fetting, there appeared in the N E. (which was directly up the Gulf of *Lyons* from us) great and continued Flafhes of Lightning one after another, with fcarce any Intermiffion, and this without Thunder continued till the next Morning; the Flafhes of Lightning fometimes reprefenting the fudden Appearance of a Star, and at other times of a Flaming Sword, and again of a Silver Cord ftretched along the Clouds, or as the irregular Crack of a Wall from Top to Bottom.

Spouts *in the* Mediterranean, *by Dr.* A. Stuart. n. 277. p. 1077.

About eight next Morning we had Thundring, with a Continuation of Lightning of the Kind and Appearance above-mentioned, all from the N E. or thereabouts.

About nine the fame Morning, fell down from the Clouds (which look'd difmally black, lowring, and, as it were, heavy with Rain) in the faid N E. quarter, three Water-fpouts, that in the middle being the greateft, feem'd as big as the Maft of a Ship, and I judg'd it to be at leaft a League and a half diftant from us; fo that in itfelf, no doubt, it was bigger than three Mafts. The other two were not by by half ſo big. All of them black, as the Cloud from whence they fell; all of them ſmooth, without any Knot or Irregularity; only at firſt falling, ſome fell perpendicularly down, and ſome obliquely, and all of them ſmaller at the lower end than above, giving the Repreſentation of a Sword; ſometimes alſo one of 'em would bow itſelf, and again become ſtraight, and alſo ſometimes became ſmaller, and again increaſed its Bulk; ſometimes it would diſappear, and immediately fall down again; ſometimes it became extenuated to the Smallneſs of a Rope, and again became groſs as before.

There was always a great boiling and flying up of the Water of the Sea, as in a *Jette d'eau*, or Water-work; or this riſing of the Water had the Appearance of a ſmoaking Chimney in a calm Day. Some Yards above the Surface of the Sea the Water ſtood as a Column or Pillar, and then ſpread itſelf, and was diſſipated as Smoak: And the Sword-like Spout from the Clouds either came down to the very middle of this Pillar, and as it had been joined with it, as the greateſt, which fell perpendicularly down, ſtill did from beginning to end: Or elſe it pointed to this Column of Water, at ſome Diſtance, either in a perpendicular or oblique Line, as the other two leſſer.

There were three or four Spouts more, which appear'd at the ſame time in the ſame Quarter of the Heavens, but neither for Bulk or Duration like theſe three: Thoſe appear'd or diſappear'd ſeveral times, during the Continuance of theſe three.

It was hardly diſtinguiſhable whether the Sword-like Spout fell firſt down from the Cloud, or the Pillar of Water roſe firſt from the Sea, both appearing oppoſite to one another all of the ſudden, as in the twinkling of an Eye. Only I obſerved of one, that the Water boiled up from the Sea to a great Height, without the leaſt Appearance of a Spout pointing to it either perpendicularly or obliquely, and here the Water of the Sea never came together in the Form of a Pillar or a Column, but did fly up ſcatteredly, the Sea being in a boiling Rage round the Place. The Wind being then N E. the ſaid boiling advanced towards the S W. as a flitting or moving Buſh upon the Surface of the Sea, and at laſt ceaſed. This proves that the boiling or flying up of the Water of the Sea may begin before the Spout from the Cloud appears to us: And indeed if there be any ſmall Matter of Priority betwixt theſe two Appearances, the boiling or throwing up of the Sea-water has it: Which begins firſt to boil, and then frames it ſelf into a Pillar of Water, eſpecially on the lower Part thereof.

It was obſervable of all of them, but more perceptible of the great one; that towards the end it began to appear like a hollow Canal, only black in the Borders, but white in the Middle; and though at firſt it was altogether black and opaque, yet now one could very diſtinctly

ſtinctly perceive the Sea-Water to fly up along the Middle of this Canal, as Smoak up a Chimney, and that with great Swiftneſs, and very perceptible Motion. And then ſoon after the Spout or Canal brake in the Middle, and diſappear'd by little and little: The boiling up, and the Pillar-like Form of the Sea Water continuing ſtill the laſt, even for ſome conſiderable Time after the Spout diſappear'd, and perhaps till the Spout appear'd again, which it commonly did in the ſame Place as before, breaking and forming itſelf again ſeveral times in a quarter of an Hour, or half an Hour's Time.

The Middle one of the three, exceeded all the reſt in Bigneſs, Perpendicularity, Conſtancy of Form and Situation, as well as Duration; but at laſt vaniſhed.

I know not, if any has accounted for this *Phænomenon*, but I imagine it may be ſolv'd by Suction (improperly ſo call'd) or rather Pulſion, as in the Application of a Cupping-glaſs to the Fleſh, the Air being firſt voided by the kindled Flax.

It was farther obſervable, that the oblique Spouts pointed always from the Wind; that is, that the Wind being at N. E. the oblique Spouts always pointed to the S. W. tho' at the ſame Time and Moment there were others perpendicular, which remained ſtill ſo, notwithſtanding the Wind.

Alſo that ſuch as were curved had ſtill the Convex ſide from the Wind, and the Concave towards it; that is, the Wind being at N. E. the Concave was towards the N. E. and the Convex towards the S. W.

It rained a great deal during the Continuance of theſe Spouts, and after their total Diſappearance we had half an Hour's violent Gale of Wind from the N. E. with very little Rain, the Weather afterwards clear'd up.

The Explication of the Tables.

A The Spout of a black Colour, falling out of a black Cloud perpendicularly. Fig. 10.

B The Water of the Sea, riſing in the Form of a Pillar or Column in the Middle, and ſcatter'd round about the ſaid middle Column, in form of Smoak, or rather like the falling of a *Jette d'eau*. Theſe two meet one another directly, and the Column of Water from the Sea is commonly groſſer than the Spout from the Clouds.

A A curved Spout, joining with the riſing Water of the Sea at B. Fig. 11.

A Repreſents a black Spout, falling obliquely from the Clouds of the ſame Colour. Fig. 12.

B Repreſents the aſcending Column of the Sea-Water as in *Fig.* 10. with this Difference, that here the Spout and Column of Water meet not.

E and W in this Fig. ſignify Eaſt and Weſt. Fig. 13.

Fig. 13. 1 2 3 Represent the successive Progression of the boiling of the Sea from East to West, or from N. E. to S. W. and that without any Appearance of a Spout from the Clouds, pointing to either of these Places.

Fig. 14. A Represents the big perpendicular Spout a little before its breaking, white in the Middle.

B The Column of Sea Water joining therewith.

2 2 2 2 The Water of the Sea, ascending in the Form of Smoak up a Chimney, all along the Column at B to the Clouds.

Fig. 15. A The breaking of a perpendicular Spout, commonly beginning in the Middle at A.

B The Rise of the Sea Water, which begins to fail, and the middle Column to disappear.

Fig. 16. A An oblique Spout, which after reaching to the Sea in a curved Line, or obtuse Angle, does soon after break at A, and disappear.

B the Rising of the Sea Water also beginning to cease.

Fig. 17. A A perpendicular Spout beginning to fall.

B The beginning Ascent of the Water of the Sea under it.

Fig. 18. A An oblique Spout beginning, or darting itself out of the Clouds.

B The rising or boiling of the Water, answering to it in an oblique Line.

These sometimes reach down to the Sea or rising Water, and sometimes they do not reach thither, but continue a while, as here represented.

A Water-Spout at Hatfield *in* Yorkshire, *by Mr.* Abr. de la Pryme.n.281. p. 1248.

3.] On the 15th of *Aug.* 1687. about two in the Afternoon, appeared a *Spout* in the Air at *Hatfield* in *Yorkshire*; it was about a Mile off coming directly to the Place where I was; I took my Prospective Glasses to observe it as well as I could.

The Season was very dry, the Weather extreme hot, and the Air very cloudy, the Wind aloft, and pretty strong, and (which is remarkable) blowing out of several Quarters at the same Time, and filling the Air hereabouts with mighty thick and black Clouds, layer upon layer; the Wind thus blowing soon created a great *Vortex*, Gyration, and Whirling amongst the Clouds; the Center of which every now and then dropt down in the Shape of a thick long black Pipe, commonly call'd a Spout; in which I could distinctly view a Motion like that of a Screw, continually drawing upwards, and screwing up (as it were) whatever it touch'd. In its Progress it mov'd slowly over a Hedge-Row and Grove of young Trees, which it made to bend like Hazle Wands, in a circular Motion; then going forward to a great Barn, it twitch'd off in a Minute all the Thatch, and fill'd the whole Air therewith. Coming to to a very great Oak Tree, it made it bend like the foregoing Trees, and broke one of the greatest and strongest Branches, that would not yield to its Fury, and twisting it about, flung it a very considerable Distance off; then coming to the Place where I stood, within 300 Yards of me, I beheld this odd *Phænomenon*, and found that it proceeded from nothing but a Gyration of the Clouds by contrary Winds meeting in a Point or Center; and, where the greatest Condensation and Gravitation was, falling down into a Pipe or great Tube (something like the *Cochlea Archimedis*) and that in its working or whirling Motion, either sucks up Water, or destroys Ships, &c. having travell'd about a Quarter of a Mile farther, it dissolv'd by the Prevalency of the Wind that came out of the East.

4. I have seen another Spout in the same Place, which very much confirms me in my Notion of the Origin and Nature of them. The Weather here in this Part of the Country, hath been exceeding wet and cool, insomuch that it seem'd rather to be Spring than Midsummer; yet the 21st of *June* 1702 was pretty warm; on the Afternoon of which Day, about two of the Clock, no Wind stirring below, tho' it was somewhat great in the Air, the Clouds begun to be mightily agitated and driven together; whereupon they became very black, and were (most visibly) hurried round, from whence proceeded a most audible whirling Noise, like that commonly heard in a Mill. After a while, a long Tube or Spout came down from the Center of the congregated Clouds, in which was a swift spiral Motion like that of a Screw, or the *Cochlea Archimedis*, when it is in Motion, by which spiral Nature and swift turning, Water ascends up into the one as well as into the other. It travell'd slowly from West to North East, broke down a great Oak-Tree or two, frighted some out of the Fields, and made others lie down flat upon their Bellies, to save being whirl'd about and kill'd by it, as they saw many Jackdaws to be that were suddenly catch'd up, carried out of Sight, and then cast a great way amongst the Corn; at last it pass'd over the Town of *Hatfield*, to the great Terror of the Inhabitants, filling the whole Air with the Thatch that it pluck'd off from some of the Houses; then touching upon a Corner of the Church, it tore up several Sheets of Lead, and roll'd them strangely together; soon after which, it dissolved and vanished without doing any further Mischief.

—— another, *by the* same, n. 28½. p. 1331.

By all the Observations that I could make of this, and the former, I found that had they been at Sea, and join'd to the Surface thereof, they would have carried a vast Quantity of Water up into the Clouds, and the Tubes would then have become much more strong and opake than they were, and have continued much longer.

It

It is commonly ſaid that at Sea the Water collects and bubbles up a Foot or two high under theſe Spouts before that they be joined: But the Miſtake lies in the Pellucidity and Fineneſs of thoſe Pipes, which do moſt certainly touch the Surface of the Sea before that any conſiderable Motion be made in it, and that when the Pipe begins to fill with Water, it then becomes opaque and viſible.

As for the Reaſon of their diſſolving of themſelves after that they have drunk up a great Quantity of Water, I take it to be through the great Quantity of the Water that they have carried up, which muſt needs thicken the Clouds, and impede their Motion, and by that Means diſſolve the Pipes.

A Fall *of Water from a* Spout *in* Lancaſhire, *by Dr.* R. Richardſon. n. 363. p. 1097.

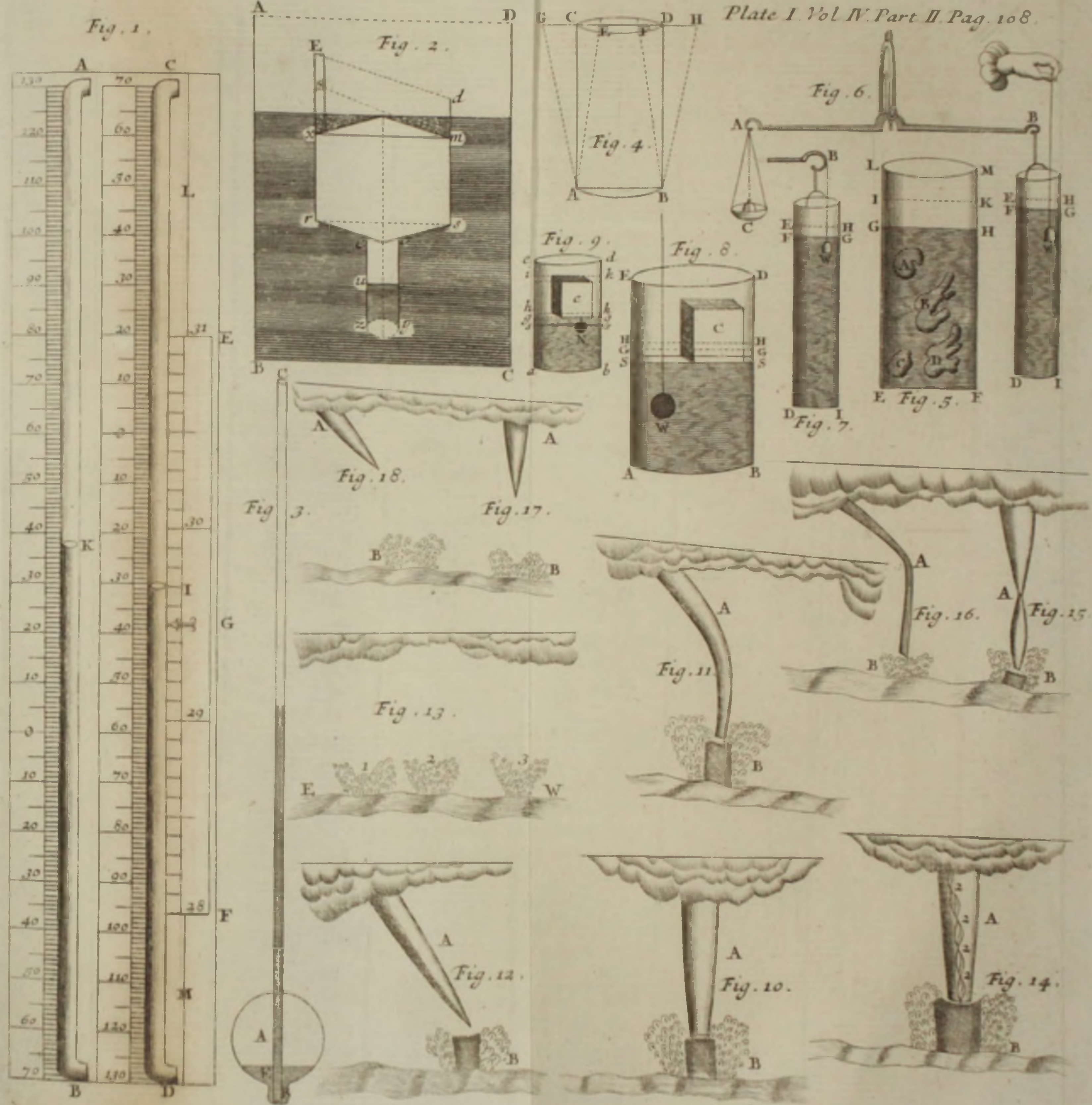
5.] We have frequent Accounts of Damage done at Sea by Spouts of Water, yet ſuch rarely happening at Land, induc'd me to take the following Relation of a remarkable one, which fell on *Emott-more*, nigh *Coln* in *Lancaſhire*, on *Tueſday* the 3d of *June* 1718, about Ten in the Morning; when ſeveral Perſons, who were employ'd in digging *Peat* nigh the Place where this Accident happen'd, upon a ſudden were ſo terrified with an unuſual Voice in the Air, that they left their Work and ran Home, which was about a Mile from the Place: But to their great Surprize, they were intercepted by Water; for a ſmall Brook in the Way was riſen above ſix Feet perpendicular in a few Minutes Time, and had overflown the Bridge.

It is to be obſerv'd, that there was no Rain at that Time on *Emott-more*, only a Miſt, which is very frequent upon thoſe high Mountains in Summer-time. There was a great Darkneſs in the Place where the Water fell, without either Thunder or Lightning, (as I had my Information from an Eye-witneſs.) The Meadows at *Wicolae* were ſo much floated, that the like had not been ſeen in ſeveral Years before, tho' there it was a very bright Day.

I went to view the Place where the Water fell; tho' I believ'd this Inundation might proceed from an Eruption of Water out of the Side of the Mountain; ſuch being not unfrequent, where Lead or Coal have been dug, but neither have ever been ſought for here. The Ground was torn up to the very Rock, where the Water fell, which was above ſeven Feet deep, and a deep Gulf made for above half a Mile, and vaſt Heaps of Earth caſt up on each Side of it, ſome Pieces remaining yet above twenty Feet over, and ſix or ſeven Feet thick. About ten Acres of Ground were deſtroyed by this Flood. The firſt Breach where the Water fell is about ſixty Feet over, and no Appearance of any Eruption, the Ground being firm about it, and no Cavity appearing. The Ground too on each Side the Gulf was ſo ſhaken, that large *Chaſms* appear'd at above thirty Feet Diſtance.

On

Plate I. Vol IV. Part II. Pag. 108



XV. On the *7th* of *June* 1711, there happened a very great Storm of Hail, accompanied with terrible Thunder and Lightning. It begun about *Rotherham* (a little beyond which was something of a Hurricane) where it burnt a noted Tree. About One of the Clock it reached *Wentworth-Woodhouse*. The Hail-stones were from 3 to 5 Inches in Circumference, and some say larger, which killed several Pigeons; but the chief Damage done here was in the Glass-Windows. In *Wash* Field, about two Miles from thence, it did vast Damage. Some Part of the Field escaped, and the *Barley* received no Damage; but the Generality of the *Wheat* was cut off, about half a Yard from the Ground, and the *Rye* about two Feet. The Stubble, tho' green at first, turn'd white, that it look'd like a Field newly shorn. The *Rye* was afterwards mown, instead of shearing, and yielded not above a Bushel of Corn in a Wain-load. Some of the *Wheat* took Root, and grew up. The Breadth of this Storm was about half a Mile, as appear'd by the Effects. In Places adjoining there was no Hail, but large Drops of Rain. A Joyner measured one of the Hail-stones with his Compasses, and it was an Inch and half in Length; but these were not Globular, but mostly Oblong. The Generality of them at *Bolton-upon-Dearne* were of the Bigness of Cherries; tho' one was taken up that was an Inch and half in Diameter, and round, not long, and somewhat flat, as the others were. Great Quantities of Twigs and small Boughs were beaten off the Trees.

A Storm of Hail in Yorkshire, by Mr. R. Thoresby. n.335. p.514.

XVI. 1.] Having receiv'd from Mr. *Townley* an Account of the State of the Atmosphere in *Lancashire*, during the late great Storm, I shall compare his Observations with mine at *Upminster*: I shall not give a long History of the Devastations, *&c.* but shall make a few Remarks of a more Philosophical Consideration.

Observations on the Great Storm, Nov. 26. 1703. by Mr. W. Derham. n. 289. p. 1530.

To look back then to the preceding Seasons of the Year, *April*, *May*, *June* and *July* were wet Months in our Southern Parts. In *April* there fell 12, 49 *l.* of Rain thro' my Tunnel. (And about 6, 7, 8, or 9 *l.* I esteem a moderate Quantity for *Upminster.*) *In* May *there fell more than in any Month of any Year since the Year* 1696, viz. 20, 77 l. *June* likewise was a dripping Month, in which fell 14, 55 *l.* And *July*, altho' it had considerable Intermissions, yet had 14, 19 *l.* Above 11 *l.* of which fell on *July* 28*th* and 29*th* in violent Showers; and I remember the News Papers gave Account of great Rains that Month from divers Places of *Europe*; but the *North of England* (which also escaped the Violence of the late Storm) was not so remarkably wet in any of those Months; at least not in that great Proportion more than we, as usually they are; as I guess from the Tables of Rain, which I had from Mr. *Townley*: Particularly *July* was a dry Month with them, there being no more than 3,65*l.* of Rain fell through Mr. *Townley*'s Tunnel of the ſame Diameter with mine.

September was a wet Month, eſpecially the latter Part of it; there fell of Rain in that Month 14,86 *l.*

October and *November*, tho' not remarkably wet, yet have been open warm Months for the moſt Part. My Thermometer (whoſe freezing Point is about 84) hath been very ſeldom below 100 all this Winter, and eſpecially in *November*.

I have given this Account of the preceding Diſpoſition of the Year, particularly as to Wet and Warmth, becauſe I am of Opinion, that theſe had a great Influence in the late Storm; not only in cauſing a Repletion of Vapours in the Atmoſphere, but alſo in raiſing ſuch Nitro-ſulphureous or other heterogeneous Matter, which, when mix'd together might make a ſort of Exploſion (like fir'd Gunpowder) in the Atmoſphere. And from this Exploſion, I judge thoſe Coruſcations or Flaſhes in the Storm to have proceeded, which moſt People, as well as myſelf, obſerved, and which ſome took for Lightning.

On *Thurſday*, *Nov.* 25. in the Morning was a little Rain, the Winds high in the Afternoon S. by E. and S. In the Evening there was Lightning, and between Nine and Ten of the Clock at Night a violent but ſhort Storm of Wind and much Rain at *Upminſter*, and of Hail in ſome other Places. There fell in that Storm 1,65 *l* of Rain. The next Morning, *Friday Nov.* 26. the Wind was S. S. W. and high all Day, and ſo continued. About 12 at Night the Storm awaken'd me, which gradually increas'd till near Three that Morning. And from thence till Seven it continued in the greateſt Exceſs; and then began ſlowly to abate, and the *Mercury* to riſe ſwiftly. The Barometer I found at 12 h. $\frac{1}{2}$ P. M. at 28,72, where it continued till about 6 the next Morning, or 6$\frac{1}{2}$, and then haſtily roſe; ſo that it was got to 82 about 8 a-Clock, as in the Table.

How the Wind ſate during the late Storm, I cannot poſitively ſay, it being exceſſively dark all the while, and my Vane was blown down. But by Information from ſome that were forc'd to venture Abroad, and by my own Gueſs, I imagine it to have blown about S. W. by S. or nearer to the S. in the Beginning, and to veer about towards the Weſt towards the End of the Storm, as far as W. S. W.

The Degrees of the Wind's Strength being not meaſurable, but by gueſs, I thus determine, with Reſpect to other Storms. On *Feb.* 7. 169$\frac{8}{9}$ was a terrible Storm that did much Damage. This I number 10 Degrees; the Wind then W. N. W. Another remarkable Storm was *Feb.* 3. 170$\frac{1}{2}$, at which Time was the greateſt Deſcent of the ☿ ever known. This I number nine Degrees. But this laſt of *November*, I number at leaſt 15 Degrees.

 As

As to *November 17th* (whereon Mr. *Townley* mentions a violent Storm in *Oxfordſhire*) it was a ſtormy Afternoon here at *Upminſter*, accompany'd with Rain, but not violent, nor ☿ very low. *November* the 11*th* and 12*th* had both higher Winds and more Rain; and the ☿ was thoſe Days lower than even in the laſt Storm of *November* 26*th*.

I have had Accounts of the Violence of the Storm at *Norwich*, *Beccles*, *Sudbury*, *Colcheſter*, *Rochford*, and ſeveral other intermediate Places.

A Table, ſhewing the Height of the *Mercury* in the Barometer, at *Townley* and *Upminſter*, before, in, and after the Storm.

Townley.				*Upminſter.*			
Day.	Hour.	Height of ☿		Day.	Hour.	Height of ☿	
Novr.	7	28	98	Novr.	8	29	50
25	3		64	25	12		39
	9½		61		9		14
	7		80		8		33
26	3		70	26	12		28
	9½		47		9		10
					12½	28	72
	7		50		7½		82
27	3		81	27	12	29	31
	9		95		9		42
	7	29	34		8		65
28	3		62	28	12		83
	9		84		9	20	07
29	7		88	29	8		25

I have receiv'd an Account from a Clergyman at *Lewes* in *Suſſex*, not only that the Storm made great Deſolations thereabouts, but "That a Phyſician travelling ſoon after the Storm to *Tiſehyrſt*, about 20 Miles from *Lewes*, and as far from the Sea, as he rode pluck'd ſome Tops of Hedges, and chewing them, found them ſalt. Some at *Lewes* hearing this, taſted ſome Grapes that were ſtill on the Vines, and they alſo had the ſame Reliſh. The Graſs on the *Downs* in his Pariſh was ſo ſalt, that the Sheep in the Morning would not feed till Hunger compell'd them, and afterwards drank like Fiſhes, as the Shepherds report. This he attributeth to Saline Particles driven from the Sea.———He

" heareth alſo, that People about *Portſmouth* were much annoyed
" with ſulphureous Fumes, complaining they were almoſt ſuffocated
" therewith.

—*on the* ſame in Suſſex, *by* J. Fuller, *Eſq*; ibid.

2.] We live ten Miles off the Sea in a direct Line, and yet can ſcarce perſuade the Country People, but that the Sea Water was blown thus far, or that during the Tempeſt the Rain was ſalt, for all the Twigs of the Trees the Day after were white, and taſted very ſalt; as I am inform'd almoſt by every Body, though I did not taſte them time enough myſelf, nor obſerve it, and that not only upon this Hill where we live facing the Sea, but in all other Places within 14 or 15 Miles of the Sea, as well in the Vallies, between which and the Sea are ſeveral high Hills, as on the Hills themſelves.

—*on the* ſame *by* Mr. Lewenhoek, *ibid.*

3.] On the 8*th* of *December* 1703, we had a dreadful Storm from the South-Weſt, inſomuch that the Water mingled with ſmall Particles of Chalk and Stone, was ſo daſh'd againſt the Glaſs-windows, that many of them were darken'd therewith.

The lower Windows of my Houſe, notwithſtanding that they look to the North Eaſt, and ſo ſtood from the Wind, were ſo cover'd with the Particles of the Water, which the Whirlwind caſt againſt them, that in leſs than half an Hour's time they loſt moſt of their Tranſparency. Suppoſing this might be Sea-water, I view'd theſe Particles with my Microſcope, and found they had the Figure of our common Salt: As to the upper Windows, where the Rain had beat againſt them, there was little or no Salt to be found ſticking upon them.

During the ſaid Storm, and about eight of the Clock in the Morning, I caſt my Eye on my Barometer, and obſerv'd that I had never ſeen the *Mercury* ſo low; but half an Hour after the *Mercury* began to riſe, though the Storm was not at all abated, at leaſt to any Appearance; from whence I concluded the Storm would not laſt long, which accordingly ſo happen'd.

Some think that the ſcattering of this Salt Water will do a great deal of harm to the Fruits of the Earth; but I am of Opinion that a little Salt ſpread over the Surface of the Earth, eſpecially where it is heavy Clay-ground, does render it fruitful; and ſo it would be if the Sand out of the Sea were made uſe of to the ſame Purpoſe.

——*on the* ſame, *by the* ſame, n. 295. p. 1793.

4.] From the Waves of the Sea, which the Winds had carried over our Meadows and Orchards, I preſaged a fruitful Year. Since that ſeveral Perſons concern'd in Tillage and Grazing have aſſured me, that they never knew ſuch a plentiful Year for Graſs, as alſo for Peaſe and Beans, as the laſt Summer was.

XVII. The

XVII. The Society having put into my Hands ſome Papers relating to the late *great Froſt* and having myſelf receiv'd Accounts thereof from my Friends at home and abroad, I ſhall endeavour to give an Account of the *Degree* and *Effects* of this remarkable *Froſt*. *The* Hiſtory *of the* great Froſt, 1708, *by Mr* W. Derham. n. 324. p. 454.

As to the *Degree* of this Froſt, I believe it was greater (if not more univerſal alſo) than any within the Memory of Man. The greateſt that hath happen'd within our Memory, was the *Long Froſt* in 1683; but the late Froſt, although of ſhorter Continuance, was more intenſe than that. My Thermometer was lower on *December* 30. (†) than it had ever been ſince 1697, when I firſt began my Thermometrical Obſervations: The ſelf-ſame Thermometer in our Repoſitory in *Greſham-College* was lower than ever it was before: [The Particulars of its greateſt Deſcents are theſe; *January* 26. 1696. 41 *Gr.* *January* 5. 1683. 40 *Gr.* and *January* 3. 1708-9. 43 *Gr.*] And laſtly, that in another ſelf-ſame Glaſs in *London*, [Mr. *J. Patrick*'s] the Spirits were four or five Degrees lower than in 1683. *The Degree of the Froſt in* England.

(†) *Vid.* Supra S. XI.

In *London* the greateſt Contraction of the Spirits was on *January* 3. which was an exceſſive cold Day at *Upminſter* alſo: But the far greateſt Contraction with us was on *December* 30. before. The Reaſon of the Difference is, becauſe my Thermometer is always abroad in the open Air, where no Sun-ſhine toucheth; but thoſe two *London-Glaſſes* are within Doors, in Rooms where no Fires are made. And it is eaſy to obſerve, that the Froſt doth not preſently exert its greateſt Force, nor ſo ſoon abate its Force within Doors, as without.

Of this Intenſeneſs of the Cold with us, I have receiv'd Confirmations from other Places in the Southern Parts of our Iſland.

The Deſcent of the Spirits in my Thermometer on *December* 30. was within One tenth of an Inch as great as the Deſcent effected at another Time (and that in a *Cold Day* too) with artificial Freezings performed both with Snow and Salt, and alſo Snow and Spirits. Both which Mixtures I have ſeveral Times made uſe of, and find them nearly of equal Power: If any Difference be, I have ſometimes thought the Preference due to the Mixture of Spirit of Wine with the Snow.

Although the Froſt was ſo rigorous in the Southern Parts of our Iſle, yet the Northern felt little thereof; as I have been certified by Perſons that have come from thence, as well as by ſeveral Letters. Dr. *Sloane* writes to me in general, that he hath received many Informations from thoſe Parts, which do all agree that the Winter was no way extremely cold there, but as other Winters. The Lord *Biſhop* of Degree *of the Froſt in* Scotland.

of *Carlisle* in a Letter to Dr. *Woodward*, says, " In *January* last, I had " a sufficient Occasion to take Notice of the Frost and Cold being " more intense in the Southern Parts than here, and the Snow much " thicker. I began my *London* Journey on the 26th of that Month, " three Days before the Thaw, and assure you that for several Miles " (near the Banks of the River *Eden*, in both the Counties of *Cumberland* and *Westmoreland*) my Horses hardly ever trod upon Snow. " When we came to *Stanemoor*, on the Confines of *Yorkshire*, we found " the Ground covered pretty thick, and the deeper still the farther " we came to the South. None of our Rivers or Lakes were frozen " over; and the extraordinary Flocks of Swans that resorted hither " (nothing of the like having been seen by the eldest Man living) " was a sure Argument that the Temperature of Climates was strange- " ly inverted."

From *Edinburgh*, Sir *Robert Sibbald* says, " I can learn no extraor- " dinary Effects of the cold Season here. It was a long Winter: The " Cold came early in *Octob.* and continued till near *May*. There was " much Snow, which lay long upon our South Hills near this Place. " We had not much Snow to speak of, and it lasted not long."

In Ireland.

In *Ireland*, I am inform'd, the Frost was very favourable, by a Letter from Mr. *S. Molyneux*, who says, " They had there an harder Winter " than usual, but judgeth they suffered not so much as their Neigh- " bours: They had two or three pretty hard Frosts, and some Snow, " but not of any remarkable Continuance, as he remembers."

Degree *of the Frost in other* Parts *of* Europe.

In Switzerland.

(*) *Vid.* Supra. S. XI.

In the Comparison made before between Dr. *Scheuchzer*'s Observations at *Zurich*, and mine here, I said (*) That he noted the Cold to have been excessive there; but whether more than usual he saith not. But by a Letter I have lately seen from his Brother, it appears to have been in as great and unusual Excess there, as here it was with us.

In Italy.

In that Paper too I observed, to what Excess the Frost arriv'd in *Italy*, *viz.* " That the Cold there was so great, that for 20 Years past, " they had not been sensible of a greater; and on *Twelfth Day* it want- " ed but half a Degree of the Extremity."

In Denmark.

As to the *Northern Parts*, Dr. *Woodward* tells me, that in a Letter he received from Mr. *Otho Sperling*, from *Copenhagen*, he calls it *Hyems Atrocissima*. And I find it noted in the Minutes of the *Royal Society* of *May* 4. 1709. " That Dr. *Judichar* said the Ice was frozen " in the Harbour of *Copenhagen* 27 Inches; and that *April* 9. " *N. S.* People had gone over between *Schonen* and *Denmark* on the " Ice." Which Accounts give me a better Opinion of some Papers I have by me, which were shew'd to the Society, concerning the Frost at *Copenhagen*, said to be taken from the Observations of Mr. *Romer*. 'Tis said there, " That such a Frost hath

" not

"not been known in the Memory of Man in these Countries, and "that the Frost on *January* 7. and *February* 23. 170⅝. did very nearly "approach the Point of Artificial Freezing."

In the Northern Parts of *Germany* it was much the same; of which I have a printed Account. The Title of the Book is, *Consideratio Physico Mathematica Hyemis proximè præterlapsæ, &c.* being an Academical Exercise, perform'd in the University of *Hall*, *June* 13. 1709. by *G. Remus*, a *Dantzicker*, and printed at the same Place [*Halæ Magdeburgicæ.*] This Dissertation being, I suppose, in but few Hands, I shall give a short Account of it. —*in* Germany.

The Author having complain'd of the Defects of Meteorology and Meteorological Instruments; and given some Directions concerning observing the Winds, *&c.* tells us, he had the Help of the Observations of three eminent Persons in his Dissertation about the Winter, of Dr. *Wolfius* of *Hall*; Dr. *Hamberger* of the University of *Jena*; and Reverend Mr. *Teuber* of *Ciza*. The Winter he distributes into five Periods. The first of which he begins at *October* 19. 1708. at which Time he says the cold Weather began with them, the Northerly Winds then blowing, and frosty Weather accompanying it. But with us at *Upminster*, it began sooner: For all the latter End of *September*, the Winds were Northerly, and an Hoar-Frost on *Michaelmas*, and the following Days. After which, a great Part of *October* to the 23*d* Day, my Register shews the Weather to have been, for the most part Hoar-frosty, or Frosty, very agreeably to Mr. *Remus*'s Observations. The End of this first Period he placeth on *November* 5. the same with our *October* 23. *O. S.* their Stile, I perceive by divers Comparisons, and Hints in his Paper, being the New Stile.

As to his next Period, which with its Interval takes in *November* and *December*, I find a pretty deal of Agreement between his Observations and mine, the Weather often being warm, or cold here, as it was there, and the Winds also not very different. Only I observe the Cold in one Place commonly to precede the other. Also the furious Wind, that he saith blew the Night before *December* 13. was not perceivable here 'till the second Day after. *viz. December* {14 *N. S.* / 3 *O. S.*} about Noon: At which Time it had much spent itself, and was only a brisk Easterly Wind, but no Storm.

The third Period he begins on *January* 5. Of which he saith, *The Scene was presently changed, and to the Astonishment of all* Europe *a Period began, which was very remarkable for its unusual Cold.* The very same {*Jan.* 5. / *Dec.* 25.} the Wind and Weather began here to change, as there he saith it did, and the Cold also to increase. The most remarkable Depressions of the Spirits there, may be seen with mine in this Table.

Day

Day of the Month. O. S.	Degree of the Thermometer at Hall, at $10^{h.}$ p. M.	Degree of the Thermometer at Upminster, at $9^{h.}$ p. M.
Dec. 27		65
28	$84\frac{1}{2}$	75
29	$84\frac{1}{2}$	58
30	$92\frac{1}{2}$	45
31	100	52
Jan. 1	Totus intra	63
2	Sphæram.	54

It is to be obſerved that the Scale of their *Thermometer* runs downwards from ſome Point above, down towards the *Ball.* But the *Ball*, or the Bottom of the Stalk, being a certain Place that all Thermometers agree in, and every one is acquainted with, I therefore make the Degrees of the Scale of my Thermometers to begin at the Top of the Ball, or (which is all one) at the Bottom of the little Tube, or Stalk, and ſo reckon upwards; every Degree being one Tenth of an *Engliſh* Inch; the *Freezing-Point* in my old Thermometer (here noted) at 82 *gr.* equal to 8 Inches two Tenths from the Ball; and the moſt *intenſe Cold* at 44 *gr.* But in my later Thermometers (which I now uſe, and are much nicer than my old one) the *Freezing-Point* is at 100 *gr.* ten *Engliſh* Inches from the Ball, and the moſt *intenſe Froſt* near to, or juſt in the Ball.

It may from the foregoing Table be perceived, that the Froſt kept a pretty equal Pace in both Places at its Beginning. And my Notes give me Reaſon to think it did the ſame the greateſt Part of its Duration: But I cannot be very ſure thereof, my old Thermometer (the only one I then had) happening to be broke on *January* 11. For which Reaſon I am unable to give ſuch another Thermometrical Table of his next Period, as I have done in this.

This third Period he makes to end *January* $\left\{\begin{matrix} 25\ N.\ S. \\ 14\ O.\ S. \end{matrix}\right\}$ with a Weſterly Wind, and a Thaw, which held for a few Days. With us the Wind was Southerly at the ſame Time, and a Thaw accompanying it for a few Days likewiſe.

The fourth Period he begins *January* $\left\{\begin{matrix} 31\ N.\ S. \\ 20\ O.\ S. \end{matrix}\right\}$ in which I obſerve there is a great Agreement between our Obſervations as to the Cold; and thoſe Days on which he noteth the Weſterly Winds

to

to have been ſtrong, it was the ſame here. And ſome Agreement alſo, but leſs, is in the coaſting and ſhifting of the Winds throughout this Period.

The fifth and laſt Period he placeth between *February* $\left\{\begin{matrix}17\\6\end{matrix}\right.$ and *March* $\left\{\begin{matrix}17\ N.\ S.\\6\ O.\ S.\end{matrix}\right\}$ In this, he ſays the cold Weather returned, and continued long: And the ſame it did with us. But as to the End of this Period, I find ſome Difference, and ſome Agreement between our Obſervations. The Snow was more with them than us; the Winds changed with us from the Eaſterly Points, to the Weſterly and Southerly, a Day or two ſooner than with them; then agreed with them; and ſoon after veer'd about to the Eaſterly and Northerly, as it did with them. And I obſerve, that when the Winds agreed in both Places, my Notes ſhew the Wind to have been of ſome Force here.

As to the Warmth of the Weather all this time, I find a pretty deal of Agreement; only as the Wind changed two Days ſooner here, ſo we had the mild Weather, he mentions, two Days ſooner: Then it grew colder here, as he ſaith it did with them. And whereas he noteth *April* $\left\{\begin{matrix}13\ N.\ S.\\2\ O.\ S.\end{matrix}\right\}$ to have been the firſt Day on which the Spirits roſe to the Point of Warmth, I found by my Thermometer the Day before to have been as warm as that, as alſo were the following Days; and each of them warmer than had been all the preceding Winter; but yet that we had divers warm Days before that Time, particularly *March* 12, 13, 14, 18, 19, 28. *O. S.* were warm Days, but the reſt in that Month for the moſt part cold.

The Waters were the firſt Thing that felt the dire Effects of this Froſt. And theſe were in many Places frozen to an extraordinary Depth; although I hardly believe to that Depth, as in the Long Froſt in 1683. Of which Froſt we have a ſufficient Inſtance in our River of *Thames*; whoſe Waters were ſo frozen, that above Bridge, 'tis well known, many Booths were erected, Fires made, and Meat dreſs'd; and on *January* 10. 168¾, I ſaw a Coach and two Horſes drive over the River into *Southwark*, and back again, a great Number of People accompanying it. But this laſt Winter the Caſe was greatly different, according to the Account I receiv'd from Mr. *Lowthorp*, who ſays, "He "ſaw ſeveral People croſs the *Thames* at ſome Diſtance above the "Bridge: But that was only towards Low-water, when the great "Flakes of Ice that came down, ſtopp'd one another at the Bridge, "till they made one continued Bed of Ice from thence almoſt to the "*Temple*. But when the Flood came, the Ice broke, and was all car- The Effects of the Froſt on Fluids.

" ried with the Current up the River. I was told the like happened
" between *Westminster* and *Lambeth*, a little above *Whitehall*.

As for other Waters, they also had their Share; especially where they lay exposed to the Northerly and North Easterly Winds. The Sea-waters were cover'd with Ice in many Places near the Shore, in Harbours, and where they lay calm and still. Of this I have already given a pregnant Instance in the Harbour of *Copenhagen*, and the Sea between *Denmark* and *Schonen*. And in a Letter from Dr. *Newton*, he tells me, " The Sea was frozen both on the Coast of *Genoa* and *Leg-*
" *horn*.

As for the Northern Parts of *Germany*, the last cited Dissertation gives this Account of its Effects on Fluids: *Water was frozen into Ice beyond the usual Depth, and other Liquids appear'd to be congealed, which in Midst of Winter are thought to be out of Danger of freezing. A Fountain in a certain Village of* Silesia, *which tho' at other times was found to be cool in Summer and warm in Winter, yet this Winter it was cover'd with pretty thick Ice, to the great Wonder of every one. The publick News have sometimes told us of hot Baths converted into Ice. Tho' this cannot happen to those that are very hot- - - at* Hall *we saw Iceicles adhere to the Salt Fountains, which is reported not to have happen'd for an Age.* D. Breynius *assures me by Letter, that the Sea itself was cover'd with Ice on the 8th of* April, *as far as his Eye could reach. He exposed a Lixivium to the Air, which had been plentifully impregnated with Pot-ash, which by those who had used it for many Years was affirm'd never to freeze; yet in a short time it was found converted into Ice. He adds, that a certain Friend of his had observed, that a dephlegmated Spirit of Tartar had froze. The Observations of* Hall *relate, that Spittle hardly dismiss'd from a Man's Mouth became Ice. The Rivers were froze thrice, even those which for their Rapidity were always Proof against freezing.* Thus far D. *Remus*.

These Effects, I am apt to think, the Waters felt not only in *England*, *Denmark*, *Germany*, *France* and *Italy*; but in all the *Northern World* also, excepting *Scotland*, *Ireland*, and probably some other Islands, or Places near the Sea; altho' even some of these appear from the foregoing Account to have been great Sufferers too. This Universality of the Frost, I suspect from the Multitudes of divers kinds of Birds (utter Strangers to these Parts, and many of them Inhabitants of the Northern colder Countries) which were seen and kill'd in many Parts of *England*. In our *Essex-Marshes*, we had many wild Swans, Brent-Geese, many of the rarer Gull-kind, and divers other sorts of Birds, utter Strangers to these Parts. Mr. *Bellers* gave Dr. *Woodward* this following Catalogue of Birds kill'd within four or five Miles of *Coln St. Aldwins*, or *Edwins*, in *Gloucestershire*, between the Beginning of *November*, and the latter End of *March* 1708, which he saith are never found there in moderate Winters.

1. *Lanius*

1. *Lanius cinereus major*, the greatest Butcher-Bird, or Mattagess; sometimes seen in *Derbyshire*, but commonly in *Germany*, as Mr. *Willoughby* saith. 2. *Fringilla montana*, the Brambling. 3. *Numenius, sive Arquata*, the Curlew. These Birds, though Strangers to the inland Parts, I have seen common enough on the Sea-coasts of *Essex:* And Dr. *Woodward* says, he saw them several Times this last Winter at the *Poulterers* in *London*. 4. *Gallinula Erythropus major*, the Red-shank, or Pool-Snipe. 5. *Gallinula Hypoleucos Gesneri*, the Sand-piper. 6. *Schæniclos*, the Stint. 7. *Corvus aquaticus minor, sive Graculus Palmipes*, the Shag. 8. *Merganser*, the Goosander. 9. *Mergus cirratus longiroster*, the Dun-diver. 10. *Mergus major cirratus*, the Smew, or white Nun. 11. *Colymbus major*, the Greater Loon. 12. *Larus major*, the Greater Gull. 13. *Cygnus ferus*, the Elk, or Hooper, or wild Swan. 14. *Brenta*, the Brent-Goose. 15. *Anas niger Aldrovandi*; seldom seen in *England*, but frequent in *Norway*. 16. *Tadorna*, the Shell-Drake, or Burrough-Duck. 17. *Anas Fuligula prima Gesneri*, the Tufted-Duck. 18. *Anas fera fusca Gesneri*, *Penelops Veterum*, the Poker. 19. *Anas Platyrhynchos mas Aldrov.* the Golden-Eye. 20. *Anas Platyrhynchos rostro nigro & plano*, the Cadwall.

Effects of the Frost on Animals.

In the Dissertation before-cited, we are told, how Animals suffered both with them, and in other Places; " That the Fresh-water *Fish* were " every where kill'd in their Parts, and that a vast Destruction befel " their *small Birds*. Both which, he is inform'd, happen'd in his own " Country also at *Dantzick*. Nay, some did not stick to affirm, that " they saw Birds, as they flew along, drop down out of the Air, their " Strength failing: That the *Lusatia Letters* said, many Cows were " frozen to Death in their Stalls. And many Travellers on the Road " were some quite frozen to Death, others lost their Hands, Feet, " Noses or Ears; and others fainted, and were in great Danger of Life " or Limb, when brought too soon near the Fire. Of these Particu- " lars, he gives divers Instances from their News Papers; of two " Gentlemen, and a Smith in *England*, and above 60 Men, and many " Cattle near *Paris*; and the like at *Venice*, and 80 *French* Soldiers " near *Namur*, all kill'd on the Road, with the Cold." Whether any such Persons perished on our Roads in *England*, I have not heard: But we were told of some that did; particularly some Post-Boys, and, I think, some Drovers. Our Fresh-water Fish were, many of them, destroyed, in Ponds that were shallow, and especially if long frozen over; some for want of Air, where the Ponds were not kept open; and some with the cold Air at the Holes in the Ice, where in great Numbers they came to get Breath. On the *Italian* Coast some of our " Mariners on Board our Men of War, died of the Cold; and several lost " Parts of their Fingers and Toes: As Dr. *Newton* informs me.

 But

But the greatest Sufferers in the Animal-Kingdom, were *Birds* and *Insects*. *Robin Redbreasts*, which before the Frost were numerous, are since that very scarce about us; and notwithstanding their Recruits in the following Summer, yet even still, in this succeeding Winter, their Scarcity remains. *Larks*, both *Wood* and *Sky-Larks*, became, in a Manner, Rarities in our Country the following Spring and Summer; nor are they as yet become so numerous as heretofore. But whether this was an universal Calamity that befel that Family of small Birds, or whether it only happened to our *Essex-Larks*; or whether they were not driven from these Parts by the Frost, I cannot say; because I have been told, that in some other Counties of *England*, which abound in large common plough'd Fields, and where Larks are commonly more numerous than about us, they have had large Flights of Larks this present Winter 170⅞. But I have enquired of the *London* Poulterers, and they tell me, they have Larks from almost all Parts of *England*, and have not this following Year receiv'd a Quarter, scarce a tenth Part, of the Larks they used to have; by reason the Frost kill'd them, as the Bird-Catchers say.

In the *Insect-Tribe*, I have particularly observ'd the *Death-Watch* to be great Sufferers; notwithstanding that Insect's great Precaution, and Art, to secure itself against the hard Weather, in dry Places within Doors, under downy light Dust, *&c*. Few of them appear'd the following Summer; and in Places where they used in *July* to be very sonorous with their ticking Noise, only now and then one was heard; a manifest Sign of their being either kill'd, or render'd less fertile and venereous.

Effects of the Frost on Vegetables.

But among all the Sufferers by the Frost, the *Vegetables* were the most universal; few of the tender Sorts escaping. About us, *Bays*, *Rosemary*, *Cypresses*, *Myrtles*, most of the *Phillyrea*'s, even *Junipers*, among Shrubs; and *Artichokes*, *Colly-Flowers*, and a great many other Olitory Plants, suffer'd greatly. By Enquiries made on Purpose among the *London Gardeners*, I have been inform'd, some of them have lost to the Value of 80*l.* 100*l.* 200*l.*

But the most exact Account I have met with, is from Mr. *Ja. Bobart* of the *Oxford* Physick-Garden. He takes Notice, that the Damages of this Frost do not come up to those in 1683; which Frost being of longer Continuance, cleft the *Oaks*, and Bodies of the *Vines*, &c. But in the last Frost there were Intervals of Relaxation, besides several considerable Snows, which prov'd a good Guard to many Plants. But the Snow melting, and the Cold withal continuing, prov'd of evil Consequence to many bulbous and tuberous Roots, and abundance of other Things. " But (he says) the sharp, dry, and cutting Winds " from the North, and North-East, were destructive to many of the

" Orna-

" Ornaments of our Gardens, which before seem'd to be almost
" naturaliz'd to our Clime; as *Cypress*, *Bays*, *Rosemary*, *Alaterni*, *Phil-*
" *lyrea*'s, *Arbuti*, *Laurustines*, &c as also to most of our frutescent
" Herbs, such as *Lavenders*, *Abrotonums*, *Rue*, *Tyme*, and divers others
" of such Race, especially such as had their Heads above the kind
" Covering of the Snow. And not such Exoticks only, but some
" of our own Natives, as is visible in most of our *Furze-fields*,
" and divers *Hollies*, especially of the finer strip'd Race, have felt the
" Smart of the Rigour of the Season, by the Loss of their Leaves,
" and sometimes their Lives.

" And what hath been more observable this Year, than in others,
" is the Sap of our finer mural Fruit-Trees, as of *Peaches*, *Necta-*
" *rines*, *Apricocks*, &c. was so congeal'd and disorder'd, that it prov'd
" stagnated in the Limbs and Branches, and equal to Chill-blains in
" Human Bodies; which, in too many Parts of the Tree, turn'd to
" so frequent Mortifications, that it is very much to be doubted, whe-
" ther sufficient Vigour is ever to be expected from them, to be worth
" their standing, notwithstanding their weak Endeavours of shoot-
" ing.

" And it is no less observable, that the very Buds in these finer
" Trees, as well Leaf-Buds, Blossom-Buds, (which are but the Ova-
" ries of the succeeding Fruits) were quite kill'd, and dry'd into a fa-
" rinaceous Matter, by the too great Sharpness of the Cold, before
" they grew out, though Life remain'd in the Branch.

" The *Plumbs*, being more hardy, produc'd their Blossoms well
" enough; but through the chilling Wets, which happen'd too plen-
" tiful about that Time, and the great Defect of nutritive Warmth,
" they grew weak; with their little Stalks, or Pedicles, languish-
" ing, and turning yellow, generally dropt off, and came to no-
" thing.

" It might reasonably have been suppos'd, that such conjoin'd Cold,
" with repeated Wets, should have destroyed the injurious *Insects*,
" which usually infest the first Product; but even in this Year, they
" have prov'd vivid, in too great Plenty, among the *Apples* and *Pears*
" (especially the former) whose Blossoms, as well as Leaves, have been
" a Pabulum for these voracious *Erucas*, whose Eggs lay dormant all
" the Winter, so dry in their Bags, that there were so many escap'd
" from being frozen, that in many Places they prov'd enough to de-
" stroy the whole Verdure.

" *Fig-Trees*, whose softer Texture was more easily penetrated, have
" suffered much, most of them being cut down.

" Many *Exotick Greens*, and rare Plants coming from *Africa* and
" other warm Regions, have mightily suffered, especially in such Stoves
" and Conservatories as had not Fire enough.

What he obſerveth concerning the Deſtruction of *Wheat*, was, I believe, a general Calamity, as alſo the Particulars he takes Notice of much the ſame in other Places too, *viz.* " Where the Land was poor, " and coldly expoſed, there the *Wheat* was kill'd ; that many Lands " of Wheat eſcaped tolerably well on the warm Side, when the other " Side was quite kill'd with the Extremity of Cold.

By the *warm and cold Sides*, I ſuppoſe our Obſerver means the ſunny and ſhady Sides. But with us the Wheat ſuffer'd rather more on the Southern, ſunny Side, than the Northern ; I ſuppoſe by reaſon the Ground was ſomewhat open'd by the Sunſhine, and the covering of Snow melted, and Way thereby made to the Severity of the nocturnal Froſt. Upon which Account I have heard it ſaid by ſome Obſervers, *that Vegetables ſuffered more the laſt Winter from the Sun, than the Froſt.*

In *Eſſex* alſo, I obſerved many ſmall Fields of three or four Acres of Wheat, to eſcape pretty well, where fenced with thick high Hedges againſt the cold Winds, eſpecially where they were cover'd long with Snow ; at leaſt they came off better than other Parcels of Land expoſed to the Winds, that diſlodg'd the Snow, and aggravated the Cold alſo. So at *Upminſter*, the beſt Pieces of Wheat were ſuch, I obſerv'd, as lay on gentle Deſcents facing the Weſt or S. W. eſpecially when guarded on the Eaſtern Side with a Hill, or a Wood ; which fenced off the cold piercing Eaſterly and North Eaſterly Winds.

Not only *Shrubs* and *Plants*, but the *larger Trees*, have in ſome Places had their Share of Suffering too. But it was obſerv'd by ſome Perſons of the *Society*, that the Calamities which befel Trees, aroſe not purely from their being frozen, but principally from the Winds ſhaking and rocking them at the ſame Time, which rent and parted their Fibres.

Theſe have been ſome of the moſt remarkable Effects of the Froſt on the *Vegetables* of the more *Southerly* Parts of our Iſland, the *Northerly* eſcaping better. From *Edinburgh*, Sir *Rob. Sibbald* ſays, " The " Corn did not riſe, and ripen, ſo ſoon as wont ; but there hath been " a plentiful Harveſt, well brought into the Barns and Yards. And " the Price of Victuals (which was high) falls lower daily. There " was no greater Number of thoſe who died, than was uſual during " the Winter formerly.

As to other Places, I find the Effects were, in the more Southerly Parts of *Europe*, much the ſame on their Vegetables as on ours. In *Italy* Dr. *Newton* ſaith, " Almoſt all the *Lemon* and *Orange-Trees*, " with thoſe of the like Kind, are deſtroyed in this Country by the " Froſt, and a great many *Olive-Trees*. The Leaves of the *Bay-Trees* " have the ſame Colour now, as all others have when they are falling " in *October*. Beſides which, there are two other Accidents he tells me of, owing probably to the Froſt. One happen'd at *Florence*, where,

where, " on the side of a Hill were formerly many Buildings, " which twice falling down, by the Earth giving way, a Wall was " erected in the Time of this Great Duke's Grandfather, with an In- " scription on the Wall, which separates the Ground from the next " Street, that for the future no Person should build there. After the " Great Frost, this Wall hath fallen down too. The Hill is full of " Stones, and they will have it, that as those increase, the Ground is " push'd forward, and thereby thrown down." But I am apt to think, the Frost might have a great Concern herein.

The other Accident was at *Pisa*, " where, upon the melting of the " Snows, and the great Rains which fell after the Frost, altho' the *Arno* " did not swell over the Banks at *Pisa*, yet the Water at some Distance " from the River, in a middle Row of Houses betwixt the River and " the great Street on the Northside, with great Violence broke out, and " if it had not been immediately perceiv'd, and the Breach stopp'd by " the throwing in of a great Quantity of Bricks and Timber, that Part " of the Town might have been in Danger of being drowned, where " the *Palace*, and the *Publick-Schools*, or (as they call it) the *Sapienza* " stand.

Dr. *Mich. Angelo Tilly* tells me from *Pisa*, " That the Frost hath de- " stroy'd a World of Trees both in City and Country about them.

In *Switzerland*, among the high *Alpine* Ridges, they felt dire Effects of the Frost; yet some Places escap'd. Of which Dr. *Woodward* imparted to me the following Account from Mr. *John Scheuchzer*, " Some " Places that were defended towards the North by very high Moun- " tains, did not feel those dreadful Effects, which our Trees suffer'd last " Winter, especially our Walnut-trees and Vines. At *Vesena* near the " Lake *Rivarius*, the Trees and Vines suffer'd no Damage, so that the " Vintage is good there, but we have none. The Walnut-trees " were loaded with Fruit, and other Trees likewise, as if they had grown " in a Climate different from their Neighbours. The Village *Vettis* is " situate at the Bottom of *Galanda*, a very high Mountain on the Con- " fines of the *Grisons*. The Inhabitants of this assure us, that they hard- " ly ever knew a milder Winter; when on the contrary the Inhabitants " of the next Village *Valentia*, lying near the *Fabarian* Baths, were much " afraid lest all the *Vettians* should perish with Cold, all Intercourse be- " tween them being intercepted by the hard Weather. Also the Woods " which were expos'd to the North Wind, which were planted with " hardy Trees, such as Firs, Yew-trees, and Larch-trees, became burnt " up, rusty, and stript of their Leaves.

As to the Northerly Parts of *Germany*, the Case was there after the manner it was with us; as Mr. *Remus* informs us. " The cold Wea- " ther destroy'd the Trees and Shrubs in great Numbers, especially such " as appear'd above the Surface of the Snow. The Cherry-trees, Apple- " trees, and Plumb-trees despised the Severity of the Winter. Our

" President

"President (Dr. *Wolfius*) apply'd many Particles of the Boughs to his "Microscope in the Month of *March*, but could not perceive that any "thing was wanting to the Intireness and Turgidness of the Fibres. "There were Plenty of Blossoms on the Cherry-trees, few on the Apple-"trees, *&c.* The Almonds, Peaches, and Apricocks, of whatever sort "they were, very rarely escaped. The Pears suffer'd much. Such Vines "were preserved, as were cover'd by the Earth, and thereby secured "from the Cold; but such as were neglected and not sufficiently de-"fended were all lost. This we saw, and were inform'd of it by the "News. But we shall relate what the President took notice of. When "we could visit our Gardens soon after the Equinox, the Snow being "melted and the Ice thawed; the Bark, the Wood, and the Pith of "such Trees as had been spoiled by the Frost, especially the Pears and "the Apricocks, were grown black, so that many pull'd them up. "When we apply'd to our Microscopes some Pieces of such Boughs "as had grown the last Summer, we found the little Fibrils torn as if "the Wood had been rotten. But in other Parts of the Branches no "such Disruption was observed, only there was no Greenness or Sap. "For as about the middle of *April* the Trees were cherish'd by the "Heat of the Sun, in the Apricock Trees many new Buds put forth "from the old Wood, and in some from the younger Wood, where "the Blossoms ought to grow; in some there was no Succor. In the "Pears all the Buds put out, and Blossoms grew; yet not so vigorous "as usual, and leaving no Rudiments of Fruit. At that Time the "Bark obtain'd its full Greenness, blacker proceeded from the Center "of the Pith towards the outside, the Substance of the Wood recover'd "its Whiteness. The Fibrils of the new Year were black still, yet "when seen through a Microscope they seem'd to abound with Sap, "not otherwise than the same Fibrils of the Cherry-tree or Apple-tree, "which the Frost had left untouch'd. The Pith under the Buds was "tinged with an unusual Blackness, yet the Root of the Bud, when "push'd on into a Succor, appear'd through the Microscope to be very "green and turgid. Now it is very remarkable, that as the Frost had "spared the Plumbs, so it also spared the Buds of the Apricocks, that "were grafted into the Bark of the Succors of the Plumbs, which now "grew up into tall Leaves according to Trees of its Kind, in which the "Frost had not spared so much as one Bud.

As to the *Causes* of this Great Frost, they are, I confess, to me so very much hidden, that I intended wholly to have pass'd over that Matter. But Mr. *Remus* having ingeniously enquir'd into them, I shall briefly give his Opinion.

The Causes of the Frost.

The Fountain of Heat enjoy'd by the Earth, being the Sun, and that Heat being not always the same, he enquires into the Reason why it is not so. The Variation of the mutual Distance between the Earth and Sun at the Apogee and Perigee; the Mutuation of the

the Earth's Place in Respect of the Heavens, or its being justled at a greater Distance from the Sun, and the Obstruction of the Solar Rays by the Spots on the Sun, he rejects. And as to the true Causes, having assign'd good Philosophical Reasons for the perpendicular warming more than the oblique Rays, for the Wind cooling the Air, and the North and East more than other Winds, *&c.* he then enumerates his Causes in these Words: *On the Sun's Part is required a very great Distance from the Zenith, and a small Continuance above the Horizon. On the Earth's Part is required an Atmosphere full of Exhalations, and abounding with Clouds: Also Easterly and Northerly Winds, and especially violent ones. But what is most of all necessary, that the Action of the Sun should be hinder'd for a long Time, chiefly then when the Causes of the Frost concur.*

Having assigned his Causes, he applies them to his five Periods, and the more remarkable Accidents that happen'd in them.

But after all, there are some other more hidden extraordinary Causes, that he hath not reach'd. For we have all his Causes very commonly concurring in other Winters, without the same Effects as in the last. This present, next succeeding Winter 17$\frac{09}{10}$, we have had (besides what is common to all Winters, the Obliquity of the Sun's Rays, *&c.*) the Winds as much Northerly and Easterly, and as strong; and as much dark Weather; and all concurring too together, as happen'd during the Great Frost: And yet no more than ordinary severe Weather.

But as to misty, cloudy, dark Weather, which he reckons among his principal Causes, I am so far from thinking it a Cause, that I rather take it to be the Reason we have not more frequent severe Frosts, at least in our Island Places, surrounded by the warm Vapours of the Sea. Clouds and Vapours do indeed intercept, and keep off the Sunbeams; and probably imbibe and retain a great deal of Warmth themselves; nay, perhaps they may (as he saith) reflect back some of the Sun-rays: But we constantly in Winter find, that the fewer the Exhalations are, and the clearer the Air, and after the Warmth of the Sun by Day, the sharper the Frost is at Night.

I do not pretend to assign Causes; yet thus much seems to me reasonable: That the great Mint of Meteors being the superior Regions of the Air, and the Source of Exhalations being the Terraqueous-Globe, in those two Places we are to seek for the farther, and more grand Causes of the late Frost. And in the fourteen and more Years Observations I have made of the Weather, *&c.* I have found a great deal to be attributed to the Increases and Decreases of the Cold of the upper Regions, as also to the inner Dispositions of our Globe, at least to the greater or less Plenty of Vapours and Exhalations. But not as yet having Observations enough to clear and demonstrate my Hypothesis, I shall defer what I might have said.

Effects of Thunder and Lightning *in* Ireland, *communicated by* S. Molyneux, *Esq;* n. 313. p. 36.

XVIII. 1.) Strange were the Effects of the Thunder and Lightning which happen'd at Mrs. *Close*'s House at *New-Forge* in the County of *Down* in *Ireland*, on the 9*th* of *Aug.* 1707. I waited on her about a Fortnight after to inform myself of the Particulars. She told, that the whole Day was close, hot, and sultry, little or no Wind stirring till towards the Evening; that there was a small Breeze with some misling Rain, which lasted about an Hour; that as the Air darken'd after Sun-set, she saw several faint Flashes of Lightning, and heard some Thunder-claps, as at a Distance; that between 10 and 11 of the Clock, both were very violent and terrible, and so increased and came on more frequent till a little before 12 of the Clock; that one Flash of Lightning, and one Clap of Thunder came both at the same Time louder and more dreadful than all the rest, which, as she thought, shook and inflam'd the whole House; and being sensible at that Instant of a violent strong sulphureous Smell in her Chamber, which she did not perceive before, and feeling a thick gross Dust falling on her Hands and Face as she lay in Bed, she concluded that Part of her House was thrown down by the Thunder, or set on Fire by the Lightning; that arising, and calling for Candles, she found her Bed-Chamber full of Smoak and Dust, as also the Kitchen that was beneath it: The rest of the House being safe, she only observ'd the Looking-glass, that hung in her Chamber to be broken.

The next Day she found, upon further Search and Enquiry, that Part of the Top or Cornish of the Chimney, which stood without that Gable-end of the House where her Chamber was, was struck off; that Part of the Copeing of the Splay of the Gable-end itself was broken down, and the Shingles on the Roof adjoining thereto (to the Number of 12 or 16) were raised or ruffled, but none shatter'd or carry'd away; that Part of the Ceiling in her Chamber beneath those Shingles was forc'd down, and Part of the Plaister and Pinning-stones of the adjoining Wall, was also broken off and loosen'd, (the whole Breach 16 or 20 Inches abroad.) That at this Place there was left on the Wall a smutted Scar or Trace, as if made black by the Smoak of a Candle, which was directed downwards towards another Place on the same Wall whereon a Breach was also made as the former, and of the same Dimensions, Part of which was behind the Place where the Looking-glass did hang; that the Boards on the Back of a large Hair-Trunk full of Table and other Linnen, standing beneath the Looking-glass, were forc'd in, and splinter'd as if by the Blow of a Smith's Sledge; that two Parts of three of the Linnen within this Trunk were pierced or cut through, the Cut appearing of a Quadrangular Figure, and between two or three Inches over; that the End of the Trunk was likewise forc'd out, as the Back was drove in; that at about two Feet distance from the End of this Trunk (where

(where the Floor and the Side-Wall of the Houſe joined) there was a ſmall Breach made in the Plaiſter, where a ſmall Chink or Crevice was to be ſeen between the ſide Board of the Floor and the Wall, ſo wide as that a Man could thruſt his Fingers down; and that juſt beneath this again, in the Kitchen, the Cieling was forced down, and ſome of the Lime or Plaiſter of the Wall broke off; that exactly under this again, ſtood a large Tub or Veſſel of Wood incloſed with a Crib made of Brick and Lime, which was broke and ſplinter'd all to Pieces, and moſt of the Brick and Lime-Work about it forc'd and ſcatter'd about the Kitchen.

I went from Place to Place, viewing each Particular; and as I found all was done on or near the Gable-end of the Houſe, I have endeavoured to explain it by a Draught, wherein the ſeveral Breaches are Fig. 21. diſtinguiſh'd: And as I conceiv'd all to be effected by ſome irreſiſtible Body, I have alſo by two Parallel Lines traced out its irregular Motion.

The Looking-glaſs was broke with that Violence, that there was not a Piece of it to be found of the Largeneſs of Half-a-Crown: Several Pieces of it were ſticking like Hail-ſhot in the Chamber-Door (being of Oak) and on the other Side of the Room; ſeveral of the Edges and Corners of ſome of the Pieces of the broken Glaſs were tinged of a light Flame Colour, as if heated in the Fire; the Curtains of the Bed were cut in ſeveral Pieces, thought to be done by the Pieces of the Glaſs; ſeveral Pieces of Muſlin and wearing Linnen, left on a Trunk, were thrown and ſcatter'd about the Room, no way ſinged or ſcorched; and yet the Hair on the Back of the Trunk, where the Breach was made, was ſinged; the uppermoſt Part of the Linnen within the Trunk was ſafe and well, and the lowermoſt Parcel, conſiſting of 350 odd Ply of Linnen, pierced through, of which, none was any way ſmutted, but the uppermoſt Ply of a Table-cloth that lay above all the reſt. She told me, there was a yellow Singe or Stain perceivable on ſome Part of the other Linnen ſo damag'd the next Day; and the whole Linnen ſmell'd ſtrong of Sulphur; but neither this yellow Stain or Smell was perceivable when I was there: That the Glaſs of two Windows in the Bed-Chamber above, and two Windows in the Kitchen beneath, was ſo ſhatter'd, that there was ſcarce one whole Pane left in any of them; that the Pewter, Braſs, and Iron Furniture in the Kitchen were thrown down, and ſcatter'd about the Kitchen, particularly a large Girdle about 20 Pounds Weight, that hung upon an Iron Hook near the Cieling, was found lying on the Floor: That a Cat was found dead the next Morning in the Kitchen, with its Legs extended as in a going Poſture, in the Middle of the Floor, with no other Sign of being hurt, than that the Furr was ſinged a little, about the ſetting on of the Tail.

She told me too, that about ſome few Days before this Accident happen'd, ſhe remov'd a Table Preſs-Bed from the Place where the Hair-Trunk ſtood, wherein two little Girls (her Daughters) uſed to lie; which ſhe look'd upon as a particular Piece of Providence.

The Wall both above and below a little Window in the ſame Gable-end, was ſo ſhatter'd at the ſame Time, that the Light could be ſeen through the Crevices in the Wall; and that upon a large Stone on the outſide of the Wall beneath this Window, was to be ſeen a Mark, as if made by the Stroke of a Smith's Sledge, or large Iron Crow, with which a Splinter or Piece of the Stone was broken off of ſome Pounds Weight. I was further informed, that from the Time of that great Thunder-clap, both the Thunder and Lightning diminiſh'd gradually, ſo that in an Hour's Time all was ſtill and quiet again.

—*at* Ipſwich *by Mr.* O. Bridgman, n. 316. p. 317.

2.] There happen'd at *Ipſwich* on the 16*th* of *July* 1708, a moſt violent Storm of Thunder and Lightning; it began about Six to be perceiv'd at ſome Diſtance, and aroſe in the South-Weſt. I was then on the higheſt Eminence about this Town, whence I could plainly diſtinguiſh the working of the Storm: The Inſtant I perceiv'd the Flaſh (which I judg'd to be about four Miles diſtant) it ſeem'd to extend itſelf like a Bow, and caſt its Light a conſiderable Way round it, and the Shaft of Lightning (if I may ſo call it) did not run in a waving angular Figure, as uſual, but in a ſtraight Shaft of Fire, like the Fuze of a Bomb, directly from the Cloud to the Ground; upon which, and finding the Storm approach, I haſten'd Home; and ſoon after, we had two or three prodigious Flaſhes of Lightning, and the Noiſe of the Thunder that ſucceeded them was ſo great, and cauſed ſuch an Emotion in the Air, that it made the Rooms ſhake, and the Windows rattle, as in a great Storm of Wind. Dr. *Dade* aſſur'd me, that at that Time the Lightning ſeem'd to dwell ſome conſiderable Space on the Ground, and that he could very plainly feel the Heat of it in his Face. The Paſſage-Boat was at that Time coming from *Harwich*, and juſt got to the Town, when a terrible Flaſh came, which kill'd the Maſter, and three more Perſons that were on Board. I ſaw one of them the next Day; he had a Wound in his Thigh, his Breaſt was lacerated, as if he had been whipp'd with Wires, and his Face and Body as black, as if he had been blown up with Gunpowder, and Thouſands of ſmall black Spots about him. The Maſter of the Veſſel was not at all disfigur'd, had only one Wound on his Side, like a freſh Burn, no other Mark about him; the Chain of his Watch was melted, yet no Burn could be perceiv'd on his Breeches or Cloaths. The third Perſon was very much torn and ſhatter'd about the Head, the Crown of his Hat was taken clear out, as if it had been cut out, and ſeveral Parcels of his Hair drove into the Subſtance of the Hat. The fourth was very little disfigur'd, only a black Spot on his Side, and a ſmall Wound, as if made with a cauterizing Iron. There were ſeveral others a-board wounded and ſtunn'd. One *Artis* had his Hair burnt cloſe to his Head behind, and his Peruke untouch'd: He had a Scratch on his Arm about four Inches long, and a ſmall Hurt below the Elbow; he fell that Night into a violent Fever, grew delirious, and is pronounc'd irrecoverable. Whether he receiv'd any Hurt on his Brain, or the Violence of the Fever cauſes the Delirium, remains undetermin'd. There was no Mark to be ſeen on his Coat, Waſtecoat, or Shirt, where he had his Hurt on his Arm. Two of the Perſons kill'd, were on the outſide, and the other two under the Tilt of the Boat; and what is pretty remarkable, the two that were within the Tilt, ſate on each Hand of a Woman, that receiv'd no Damage. One Perſon had the Soal of his Shoe unripp'd from the Leather, and no other Damage. I wonder the Blaſt lighting ſo directly on the Boat, did not ſhatter it all to Pieces: There was another Boat that follow'd them, and receiv'd no Damage, and took out the reſt of the poor frighted Wretches; the Maſter of which does affirm, he ſaw the Fire light on the Bow-ſprit of the former Boat, where meeting a ſmall Reſiſtance, it flew into ſmall Streams like a Rocket, part into the Boat, part into the Water; which, if true, no doubt, was the Cauſe of the Miſchief being done in ſo many different Parts of the Boat; and does in ſome Meaſure ſolve the ſeeming Difficulty of the Woman's being unhurt between the two Perſons that were kill'd.

—*at* Colcheſter *on the* ſame Day, *by Mr.* J. Nelſon, *ibid.*

3.] At *Colcheſter*, on *July* 16, 1708. about Eight of the Clock at Night, (the greater Part of the Afternoon being cloudy, but more thick toward Night, with Thunder at a Diſtance for above an Hour before, and much Lightning) I hard a Thunder-Crack ſo loud, as if it were cloſe to me, (the like I never heard before;) at which Time the Thunder and Lightning broke into Mr. *King*'s Houſe, beginning at the South-ſide thereof, at the Gable-end, breaking ſeveral Roof-Tiles, and near 20 other, as at *c* in the *Figure*, continuing its Courſe perpendicular, and in a ſtraight Line (the only Motion that ſeems conſiſtent with ſuch Violence, which, it ſeems, was otherwiſe in the Gentlewoman's Houſe in *Ireland*) it went into a Lean-to, and lighting on a bunching out of the Wall at *d*, it enter'd into the Strong-Beer Buttery through the Laths, and forc'd a Cork out of the lower Tap-hole of a But: In its Way at *a*, it ſhiver'd a Stud about three Inches ſquare, ſo that one Side remain'd nail'd to Laths, yet not much thicker than a Lath, and alſo brake it in two, as if it were a Tobacco-pipe. Below the Beam at *b*, it Fig. 20.

clave

clave or ſplit a Stud, about four Inches ſquare, ſeveral Feet down, which is there ſtanding; this was from its violent razing on the outſide. At the Time of this Blow, Mr. *King* was in the Lean-to, but receiv'd no Hurt; he ſmell'd a ſtrong ſulphureous Scent. It caſt the broken Wall divers Rood with the Violence. There was ſome little Damage done to *Alhallows* Church about the ſame Time in the ſaid Town.

Divers Boats were carrying Perſons from *Harwich* to *Ipſwich* on the *Orwell*; the Violence of the Thunder and Lightning kill'd four dead immediately, made a Lad run mad, and wounded the reſt that were in that Boat, which were twelve Perſons, and melted a Watch and the Chain all of a Lump which was in a dead Man's Pocket; this was about the ſame Time of the aforemention'd. Mr. *Thomas Holborow* of *Colcheſter* was Eye-witneſs to this: Being in one of the Boats, he ſmell'd ſuch a Scent of Sulphur, as he could not bear. This was about 18 Miles N. E. of *Colcheſter*, and one Mile S. E. it was no more violent than an ordinary Storm.

——*in* York-ſhire, *with violent* Rain; *by Mr.* R. Thoreſby. n. 319. p. 289.

4.] On the 5*th* Day of *July* 1708, we had *a Storm of Thunder, Lightning, and violent Rain*; I was then at the *Spaw* at *Harrow-gate*, near *Knaresborough*; where having a ſpacious View upon the open Foreſt, I obſerv'd the Motion of the Clouds and Storm, which began in the Weſt, and wheel'd about by the North and Eaſt to the South. When the Night drew on, the Lightning appear'd more dreadful. The Intermiſſion between the Flaſhes was very ſmall; the Claps of Thunder were very loud, and the Flaſhes of Lightning were continu'd. The Reverend Mr. *Furnis* of *Bewerly* writes, that *Thomas Horner*, with others, flying from the Violence of the Rain, which ſeem'd rather to fall in *Spouts* than Drops, took ſhelter in a neighbouring Barn, whence, after ſeveral frightful Thunder-claps, they were expell'd by the *Bolt*, as they term'd it, but really the Lightning, which ſinged the Hair of the ſaid *Horner*, blew another Man backward, who was climbing up the Hay-Mow, left a ſulphureous Stench behind it, and burnt the Barn and Hay. The *Inundation* of Rain was ſurprizing; it tore up much of the Road and Street, from the Church to the Bridge, and made Pits in ſome Places, ſeveral Yards deep, threw down Part of a Barn and a Stable, both of them lately built; it puſh'd into moſt of the Houſes in the Town; the Water, in ſome, was as high as the Soals of the Windows, and block'd up the Door of one Houſe with Gravel, almoſt to the very Top. Several Perſons were in great Danger, but only one Woman drown'd: She was hurry'd away with the Violence of the Stream, and not found till the fourth Day after. It removed the Bole of a large Oak ſeveral Yards; bore down the moſt Part of four Wood Bridges, and has left at the End of the great Stone Bridge, or within about 100 Yards of it, as much Gravel, *&c.* as is computed

at

at above a thouſand Cart Loads. For all this Deluge, the River *Nidd* kept within Bounds.

5.] We had much Thunder and Lightning in *Yorkſhire*, on the 12*th* of *December* 1710, in the Morning. The Lightning was ſo ſevere, that one *Sainor* a Gardener, as he was riding over *Bramham-moor*, thought his Hair had been burnt, and his Face ſcorch'd at one Flaſh, which being more ſevere than the reſt, did actually ſet on Fire the Stick he had in his Hand, as he was ready to depoſe upon Oath before the Mayor of *Leeds*, who preſented me with the ſaid Hazle Rod, which the Gardener had given him. It yet retains Part of the Blackneſs, tho' the Man (little minding it as a Curioſity) had beat off much of the End of the Rod in forcing his Horſe forward.

—*in* Yorkſhire, *by the* ſame. n. 331. p. 321.

6.] At *Sampford Courtney* in *Devonſhire*, on the 7*th* of *October* 1711, in the Afternoon, when the Miniſter was officiating, (from whom I have this Account) there was ſo great a Darkneſs, that he could hardly ſee with Spectacles: As ſoon as Prayers were over, ſome Men went to ringing, and ſeveral others were talking in the Church-Porch; a great Fire-Ball, on a ſudden, fell in between them, and threw ſome one way, ſome another, but no one received any Hurt: The Ringers ſaid they never knew the Bells go ſo heavy, and were forc'd to leave off: And being very weary, and looking out of the Belfrey into the Church, ſaw four Fire-Balls a little bigger than a Man's Fiſt, which of a ſudden broke to Pieces, ſo that the Church was full of Fire and Smoak.

—*in* Devonſhire, *communicated by Mr.* J. Chamberlayne. n. 336. p. 528.

John Goodman's Man receiv'd a full Blow in the Neck, which made him bleed both at the Noſe and Mouth; but he is very well now. He ſays, that the Fire and Smoak went up into the Tower, which broke a great Beam on which one of the Bells hung, which fell down on the Floor. It likewiſe carried away one of the Pinnacles of the Tower next the Town, and threw ſome of the Stones near a Barn Door at a pretty Diſtance from the Church, and has done ſome Damage to the Barn at one End. The Chimney of the Houſe was remov'd in ſuch a manner, by the Thunder and Lightning, that all People admir'd that it ſtood, and did not fall upon the Houſe. Tho' the People ran about in great Conſternation, no one was hurt.

XIX. I have collected what I can remember, relating to a *Meteor* I ſaw in *Jamaica* about the Year 1700. As I was riding one Morning from my Habitation, ſituated about three Miles North-Weſt from *St. Jago de la Vega*, I ſaw a Ball of Fire, appearing to me of the Bigneſs of a Bomb, ſwiftly falling down with a great

A Fiery Meteor, *&c. in* Jamaica, *by Mr.* H. Barham n. 357. p. 837.

Blaze. As I thought it fell into the Town; but when I came within a Quarter of a Mile of the Town, I ſaw many People gather'd together a little to the Southern in the *Savanna*, to whom I rode up, where they were admiring at the Ground's being ſtrangely broke and plough'd in by a Ball of Fire, which, as they ſaid, fell down there. I obſerv'd there were many Holes in the Ground, one in the middle of the Bigneſs of a Man's Skull, and five or ſix ſmaller Holes round about it, of the Bigneſs of a Man's Fiſt, and ſo deep (eſpecially the biggeſt) as not to be fathom'd by what long Switches or Sticks they had at Hand. I did not hear that any was ſo curious as to make any farther Search: It was obſerv'd, that the green Graſs was perfectly burnt near the Holes, and a ſtrong Smell of Sulphur remain'd thereabouts for a good while after.

Note, that we had a terrible rainy Night before, with much Lightning and great Thunder-Claps, which we have very frequently in *Jamaica*, often killing Cattle in the Fields. Mr. *Henry Lord*, who lives at *Dry-River*, had two Sons ſtruck dead with Lightning, in 1716, without any Wounds or Appearance of Hurt found about them. And as theſe Claps are much louder and ſtronger than any I ever heard in *Europe*, ſo are our Showers of Rain, pouring down in a moſt violent manner. We have Lightning all the Year round, but our great Rains are in the Months of *May*, *Auguſt*, and *October*. I knew *May* for two or three Years without Rain, which was look'd upon as a great Wonder: And we paid dear for it in our *Indigo*: For a Caterpillar appear'd and wove a fine Silk about the *Indigo-Plant*, and deſtroy'd it all, hurting nothing elſe. *May*-Rains uſed to deſtroy theſe Worms. *Auguſt* and *October* never go out without a Flood, we having then univerſal Rains all over the Iſland, coming from the Sea: For we have often Rains in the Mountains from the Clouds lodging there, when we have none in the Low-lands.

Our Iſland is full of Mines, and, if ſearch'd into, I queſtion not but very rich. We are very ſubject to Earthquakes, ſeveral happening every Year, eſpecially after great Rains, which fill up all our great Cracks in the Surface of the Earth: For in a very dry Time, we have them ſo very large, deep, and gaping ſo open and wide, that it is dangerous to ride over ſome Parts of the *Savannaes*, for fear a Horſe ſhould get his Legs into them. Our Earthquakes make a Noiſe or Rumbling in the Earth, before we feel the Shake; and ſeem to run ſwiftly to the Weſtward.

A Lunar Rain-Bow *ſeen in* Derbyſhire, *by* —, *communicated by Mr.* R. Thoreſby. n. 331 p. 320.

XX. A Gentleman of great Veracity told me, he had ſeen a *Lunar Rain-Bow* in *Derbyſhire* on *Chriſtmas* 1710. That walking towards

wards *Patterton Green*, about eight in the Evening, he obſerved with great Satisfaction the Bow, which the Moon had fix'd in the Clouds: She had then paſs'd her Full about 24 Hours, the Evening had been rainy, but the Clouds were diſpers'd, and the Moon ſhin'd pretty clear. This *Iris* was more remarkable than that which Dr. *Plot* obſerved at *Oxford*, the 23*d* of *November* 1675, that being only of a white Colour, but this had all the Colours of the *Iris Solaris*, exceeding pleaſant, diſtinct, and grateful to look upon; only faint, comparatively to thoſe we ſee in the Day; as muſt neceſſarily follow, both from the different Beams that cauſe it, and the Diſpoſition of the Medium. What puzzled him the moſt was the Largeneſs of the Arc, which was not ſo much leſs than that of the Sun, as the different Dimenſions of their Bodies, and their reſpective Diſtances from the Earth, ſeem to require: But as to its Entireneſs, and Beauty of its Colours, it was admirable and ſurprizing. It continu'd about ten Minutes, before the Interpoſition of a Cloud hinder'd his further Obſervation.

A Glade *of* Light *ſeen in the* Heavens, *by Mr.* W. Derham, n. 305. p. 2220. *Fig.* 21.

XXI. As I was obſerving the Immerſions of the third and fourth Satellite of *Saturn* on the 20*th* of *March* 170$\frac{8}{9}$, in the Evening; I eſpy'd a very odd ſort of Light in the Conſtellation of *Taurus*, the lower End of which was below the Bull's Eye, and the other a good Way above it, and that Star about the middle of the lower End thereof (as in *Fig.* 21.) which repreſents its Appearance to me. This Glade of Light had the ſame Motion that the Heavens had, and was much like the Tail of a Comet, but pointed at the upper End. This Light, I doubt not, is ſuch as Dr. *Childrey* firſt obſerved in *England*; and *Caſſini*, and others afterwards in *France*.

A Pyramidal Appearance *in the* Heavens *ſeen in* Eſſex, *by Mr.* W. Derham. n. 310. p. 2411.

XXII. On *Thurſday*, *April* 3, 1707, I perceived in the Weſtern Part of the Heavens, about a Quarter of an Hour after Sun-ſet, a long ſlender *Pyramidal Appearance*, perpendicular to the Horizon. The *Baſe* of this *Pyramid*, I judged to be the Sun (then below the Horizon.) Its *Apex* reach'd 15 or 20 Degrees above the Horizon. It was throughout of a ruſty red Colour; and when I firſt ſaw it, pretty vivid and ſtrong; but the Top-part fainter much than the Bottom, nearer the Horizon. At what time this Appearance began, whether at, or how ſoon after Sun-ſet, I cannot ſay. But about a Quarter of an Hour after Sun-ſet I perceived it, and had, for ſome time, a fair Proſpect of it, the Horizon being pretty free and open where I then was. But after a while, it grew by Degrees weaker and weaker, ſo that in about a Quarter of an Hour after I firſt ſaw it, the Top-part (*a. b. d.*) was ſcarce viſible. But the Lower-part remain'd vivid much longer; but yet grew by De-

Fig. 22.

grees ſhorter and ſhorter. I ſaw the Remains of the lower half (*b. d. e. f.*) a full Hour after Sun-ſet; and ſhould perhaps have ſeen it longer, had the Horizon been open. But it was often in my Walk pent up with Trees.

The whole Atmoſphere ſeem'd hazy, and full of Vapours, eſpecially towards the Sun-ſet. The Moon and Stars were that Evening bearded at that Time, and ſucceeded with an *Halo* about the Moon afterwards. Which Diſpoſition of the Air was probably the Cauſe of the Phænomenon. But the Pyramid was undoubtedly imprinted upon the far diſtant Vapours of the Atmoſphere; it being manifeſtly farther off, or lying beyond ſome ſmall thin Clouds (*C. L. c. l.*) that intercepted it, and in thoſe Parts cover'd and hid it.

I do not remember I ever ſaw any Thing like it, except the white Pyramidal Glade, which is now entituled by the Name of the *Aurora Borealis*; that being (except in Colour and Length) like it.

I have ſearched every Night ſince for this *Pyramis Veſpertina*, but have not ſeen any ſuch Appearance, altho' the next Evening was hazy and likely. I alſo look'd out to ſee whether the *Aurora Borealis* would ſucceed in the Room thereof, but diſcover'd no ſuch Thing.

A Meteor *in* Yorkſhire, *by Mr.* R. Thoreſby. n. 331. p. 322.

XXIII. A ſtrange Meteor was ſeen at *Leeds* 1710, on *Holy Thurſday*; the common People call'd it a Flaming Sword. It was ſeen not only in the Neighbouring Towns, but a great Way North, as alſo above fifty Miles South of *Leeds*. It appear'd here at a Quarter paſt ten at Night, and took its Courſe from South to North: It was broad at one End, and ſmall at the other; and was by ſome thought to reſemble a Trumpet, and mov'd with the broad End foremoſt: the Light was ſo bright, that People ſaw their own Shadows. I was reading (the Curtain of the Window being drawn) ſo ſaw nothing, except a ſudden Flaſh of Light. It is remarkable, that all Perſons (tho' at many Miles Diſtance from each other, when they ſaw it) thought it fell within three or four Furlongs of them, and that it went out with bright Sparklings at the ſmall End. An ingenious Clergyman told me, that it was the ſtrangeſt *Deceptio viſus* he was ever ſenſible of, if it was not abſolutely extinguiſh'd within a few Paces of him; and yet others ſaw it many Miles off, further North in a few Moments.

It has been ſeen in the Counties of *Nottingham* and *Derby*, as well as *York* and *Lancaſter*.

A ſtrange Meteor, *or* Aurora Borealis *ſeen in* Ireland, *by Mr.* Neve. n. 320. p. 310.

XXIV. 1.] On *Sunday*, *November* 16, 1707, after a froſty Morning, and fair ſtill Day, Wind North-Weſterly, about half an

Hour after eight in the Evening, there appear'd a very ſtrange Light in the North. The Evening was clear and Star-light, only the Horizon was darken'd with condenſed Vapours in the North, reaching, as I gueſs, ten or fifteen Degrees above the Horizon. Out of this Cloud proceeded ſeveral *Streams* or *Rays* of *Light*, like the Tails of ſome Comets, broad below, and ending in Points above. Some of them extended almoſt to the Tail of *Urſa Minor*, and all were nearly perpendicular to the Horizon, and it was as bright, as if the Full Moon had been riſing in the Cloud. But what I wonder'd at moſt, was, the Motion of the dark and lighter Parts running ſtrangely through one another in a Moment; ſometimes to the Eaſt, and ſometimes to the Weſt. It continued, after I firſt ſaw it, about a Quarter of an Hour, often changing its Face and Appearance, as to Form and Light; ſometimes broken, ſometimes entire and long Rays of Light in the clear Sky, quite ſeparate from and above the Cloud, and none below in the Cloud.

2.] Much ſuch another Appearance Mr. *Barret* was credibly inform'd was ſeen in his Neighbourhood in *Eſſex*, in *September* or *October* 1706. — *by Mr.* W. Derham. *ib.*

XXV. The *Theory* of the Air ſeems to be perfectly well underſtood, and the differing Denſities thereof at all Altitudes, both by Reaſon and Experiment, are ſufficiently defined: For ſuppoſing the ſame Air to occupy Spaces reciprocally proportional to the Quantity of the ſuperior or incumbent Air, I have (*) elſewhere prov'd, that at 40 Miles high, the Air is rarer than at the Surface of the Earth, about 3000 Times; and that the utmoſt Height of the Atmoſphere, which reflects Light in the *Crepuſculum*, is not fully 45 Miles: Notwithſtanding which, it is manifeſt that ſome Sort of Vapours, and thoſe in no ſmall Quantity, ariſe nearly to that Height. An Inſtance of this may be given in the great Light, (†) *September* 1676, mention'd by Dr. *Wallis*, which was ſeen in very diſtant Counties, almoſt over all the South Part of *England*. Of which, though the Doctor could not get ſo particular a Relation, as was requiſite to determine the Height thereof, yet from the diſtant Places it was ſeen in, it could not but be very many Miles high.

An Account *of ſeveral extraordinary* Meteors; *by Dr.* E. Halley. n. 341. p. 159.

(*) *Vid.* Supra, V. 11. C. 1. S. IX.

(†) *Vid.* Supra, V. 11. C. 1. S. LXXI.

So likewiſe that Meteor which was ſeen in 1708, on the 31*ſt* of *July*, between nine and ten a-Clock at Night, was evidently between 40 and 50 Miles perpendicularly high, and as near as I can gather, over *Sheerneſs* and the *Buoy on the Nore*. For it was ſeen at *London* moving horizontally from E. by N. to E. by S. at leaſt 50 Degrees high; and at *Redgrave* in *Suffolk*, on the *Yarmouth* Road, about 20 Miles from the Eaſt Coaſt of *England*, and at leaſt 40 Miles to the Eaſtward of *London*, it appear'd a little to the Weſtward of the

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South, ſuppoſe S. by W. and was ſeen about 30 Degrees high, ſliding obliquely downwards. We may conclude, that it was not many Miles more Weſterly than *Redgrave*, which is above 40 Miles more Eaſterly than *London*. Suppoſe it, therefore, where perpendicular, to have been 35 Miles Eaſt from *London*, and by the Altitude it appear'd at in *London*, *viz.* 50 Degrees, its Tangent will be 42 Miles; for the Height of the Meteor above the Surface of the Earth, which alſo is rather of the leaſt, becauſe the Altitude of the Place ſhewn me, is rather more than leſs than 50 Degrees: And the ſame may be concluded from the Altitude it appear'd in at *Redgrave*, near 70 Miles diſtant. Though at this great Diſtance it appear'd to move with an incredible Velocity, darting, in a very few Seconds of Time, for about 12 Degrees of a great Circle from North to South, being very bright at its firſt Appearance; and it died away at the End of its Courſe, leaving for ſome Time a pale Whiteneſs in the Place, with ſome Remains of it in the Track where it had gone; but no hiſſing Sound as it paſs'd, or Bounce of an Exploſion, were heard.

It may deſerve Enquiry, how ſo great a Quantity of Vapour ſhould be raiſed to the very Top of the Atmoſphere, and there collected, ſo as upon its Accenſion, or otherwiſe Illumination, to give a Light to a Circle of above 100 Miles Diameter, not much inferior to the Light of the Moon. 'Tis hard to conceive what ſort of Exhalations ſhould riſe from the Earth, either by the Action of the Sun, or ſubterranean Heat, ſo as to ſurmount the extreme Cold and Rareneſs of the Air in thoſe upper Regions.

Like to this, but much more conſiderable, was that famous Meteor which was ſeen to paſs over *Italy* on the 21*ſt* of *March O. S. Anno* 1676, about an Hour and three Quarters after Sun-ſet, which happen'd to be obſerv'd by the famous Profeſſor of Mathematicks in *Bononia Geminian Montanari*, as may be ſeen in his *Italian* Treatiſe. He obſerves that at *Bononia*, its greateſt Altitude in the S. S. E. was 38 Degrees, and at *Sienna* 58 to the N. N. W. that its Courſe by the Concurrence of all the Obſervers, was from E. N. E. to W. S. W. that it came over the *Adriatick* Sea, as from *Dalmatia*: That it croſs'd over all *Italy*, being nearly vertical to *Rimini* and *Savigniano* on the one Side, and to *Leghorn* on the other: That its perpendialar Altitude was at leaſt 38 Miles: That in all Places near this Courſe, it was heard to make a hiſſing Noiſe, *To make a Noiſe like a Sky-rocket, to hiſs through the Air like a Train of Gun-powder:* That having paſs'd over *Leghorn*, it went off to Sea towards *Corſica:* And laſtly, that at *Leghorn*, it was heard to give a very great Blow, *It thunder'd with a greater Report than that of a large Cannon:* Immediately after which, another ſort of Sound was heard, like the rattling of a great Cart running over Stones, which continued about the Time of a *Credo*.

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He concludes, from the apparent Velocity it went on with at *Bononia*, at above 50 Miles Diſtance, that it could not be leſs ſwift than 160 Miles in a Minute of Time. To this he adds the Magnitude thereof, which appear'd at *Bononia* bigger than the Moon in one Diameter, and above half as big again in the other; which with the given Diſtance of the Eye, makes its real leſſer Diameter above half a Mile, and the other in Proportion. This ſuppoſed, it cannot be wonder'd that ſo great a Body moving with ſuch an incredible Velocity through the Air, though ſo much rarified as it is in its upper Regions, ſhould occaſion ſo great a hiſſing Noiſe, as to be heard at ſuch a Diſtance as this was. But 'twill be much harder to conceive, how ſuch an *Impetus* could be impreſs'd on the Body thereof, which by many Degrees exceeds that of any Cannon Ball; and how this *Impetus* ſhould be determined in a Direction ſo nearly parallel to the Horizon, and what ſort of Subſtance it muſt be that could be ſo impell'd and ignited at the ſame Time: There being no *Vulcano*, or other *Spiraculum* of ſubterraneous Fire in the N. E. Parts of the World, that we ever yet heard of, from whence it might be projected.

I have conſider'd this Appearance, and think it one of the hardeſt Things to account for, that I have met with in the *Phænomena* of *Meteors*, and am induced to think, that it muſt be ſome Collection of Matter form'd in the *Æther*, as it were by ſome fortuitous Concourſe of Atoms, and that the Earth met with it as it paſs'd along in its Orb, then but newly form'd, and before it had conceived any great *Impetus* of Deſcent towards the Sun. For the Direction of it was exactly contrary to that of the Earth, which made an Angle with the Meridian at that Time (the Sun being in about 11 Degrees of *Aries*) of 67 Gr. that is, its Courſe was from W. S. W. to E. N. E. wherefore the *Meteor* ſeem'd to be moved the contrary Way: And beſides falling into the Power of the Earth's Gravity, and loſing its Motion from the Oppoſition of the *Medium*, it ſeems that it deſcended towards the Earth, and was extinguiſh'd in the *Tyrrhene Sea*, to the W. S. W. of *Legborn*. The great Blow being heard upon its firſt Immerſion into the Water, and the rattling, like the driving a Cart over Stones, being what ſucceeded upon its quenching; ſomething like which, is always obſerved upon quenching a very hot Iron in Water.

There has fallen into my Hands an Account of much ſuch another Appearance, ſeen in *Germany*, in the Year 1686, at *Leipſick*, by the late Mr. *Gottfreid Kirch*, who, in his Appendix to his *Ephemerides* for the Year 1688, gives us this remarkable Relation in the following Words.

On the ninth Day of July, *O. S. at Half an Hour paſt One in the Morning, there appeared a Ball of Fire with a long Tail, in* $8\frac{1}{2}$ *Degrees of* Aquary, *and* 4 *Gr. to the North, which continued immoveable for Half a Quarter of an Hour. Its Diameter was nearly equal to the Semidiameter of the*

the Moon. At first its Light was so great, that by it one might read without a Candle. It afterwards vanish'd in its Place by degrees. The same Phænomenon was also seen at the same Time by others in other Places, particularly at Schlaize, *a Town* 11 German *Miles distant from hence* (Leipsick) *to the South, at the Altitude of about* 60 *Degrees from the Southern Horizon.*

At the Time of this Appearance the Sun was in 26½ Gr. of *Cancer*, and by the given Place of the Meteor, 'tis plain, it was seen about ¼ of an Hour past the Meridian, or in S. *by* W. and by its Declination it could not be above 24 Degrees high at *Leipsick*, though the same, at *Schlaize*, was about 60 Gr. high: The Angle therefore at the Meteor, was about 36 Gr. Whence, by an easy *Calculus*, it will be found, that the same was not less than 16 *German* Miles distant in a right Line from *Leipsick*, and above 6½ such Miles perpendicular above the Horizon, that is, at least 30 *English* Miles high in the Air. And though he says of it, *that it continued immoveable for half a Quarter of an Hour*; 'tis not to be understood that it kept its Place like a fix'd Star, all the Time of its Appearance; but that it had no very remarkable progressive Motion. For himself has at the End of the said *Ephemerides* given a Figure of it, whereby it appears, that it darted downwards obliquely to the Right-hand, and where it ended, left two Globules or Nodes, not visible but by an Optick Tube.

The same Mr. *Kirch*, in the Beginning of a *German* Treatise of his, concerning the great Comet which appear'd in the Year 1680, intituled, **Newe himmels zeitung**, printed at *Nurenburg*, *Anno* 1681, gives us a Relation of such another luminous Meteor seen likewise at *Leipsick*, on the 22*d* of *May*, 1680, *st. vet.* about Three in the Morning: Which, though himself saw not, was observed by divers Persons, who made various Reports of it; but the more intelligent agreed, that it was seen descending in the North, and left behind it a long white Streak where it had pass'd. At the same Time at *Haarburgh*, the like Appearance was seen in N. E. or rather N. N. E. as also at *Hamburg*, *Lubeck* and *Stralsund*, all which are about 40 *German* Miles from *Leipsick:* But in all these Places, by Persons unacquainted with the Manner of properly describing Things of this Kind. So that all we can conclude from it is, that this Meteor was exceeding high above the Earth, as well as the former.

All the Circumstances of these *Phænomena* agree with what was seen in *England* in 1708.

An Account *of* Lights *seen in the Air*, March 6, 1715-16, *&c. by Dr.* E. Halley. n. 347. p. 406.

XXVI. 1.] The *Society* having received Accounts from many Parts of *Great Britain*, of the unusual *Lights*, which have appeared in the *Heavens*, desired me to draw up a general Relation of the *Fact*, and to explain more at large some Conceptions of mine, which I had proposed pofed to them, relating to the Caufe. The Account of this Appearance take as follows.

On *Tuefday* the fixth of *March*, *ft. vet.* in the current Year 1716, (the Afternooon having been very ferene and calm, and fomewhat warmer than ordinary) about the Time it began to grow dark, (much about feven of the Clock) not only in *London*, but in all Parts of *England*, where the Beginning of this wonderful Sight was feen; out of what feem'd a dufky Cloud, in the N. E. Parts of the Heaven, and fcarce ten Degrees high, the Edges whereof were tinged with a reddifh yellow, like as if the Moon had been hid behind it, there arofe very long luminous Rays or Streaks perpendicular to the Horizon, fome of which feem'd nearly to afcend to the Zenith. Prefently after, that reddifh Cloud was fwiftly propagated along the Northern Horizon into the N. W. and ftill farther Wefterly; and immediately fent forth its Rays from all Parts, now here, now there, they obferving no Rule or Order in their rifing. Many of thefe Rays feeming to concur near the Zenith, formed there a *Corona*, or Image, which drew the Attention of all Spectators. Some liken'd it to that Reprefentation of *Glory* wherewith our Painters in Churches furround the Holy *Name of God*. Others to thofe radiating *Stars*, wherewith the Breafts of the *Knights* of the Order of the *Garter* are adorn'd. Many compar'd it to the *Concave* of the great *Cupola* of St. *Paul*'s Church, diftinguifh'd with Streaks alternately light and obfcure, and having in the middle a Space lefs bright than the reft, refembling the Lantern. Whilft others, to exprefs as well the Motion as Figure thereof, would have it to be like the Flame in an Oven, reverberated and rolling againft the arched Roof thereof: Some thought it liker to that tremulous Light which is caft againft a Cieling by the Beams of the Sun, reflected from the Surface of Water in a Bafon that's a little fhaken; whofe reciprocal vibrating Motion it very much imitated. But all agree, that this *Spectrum* lafted only a few Minutes, and exhibited itfelf varioufly tinged with Colours, yellow, red, and a dufky green: Nor did it keep in the fame Place; for when firft it began, it appear'd a little to the Northwards of the Zenith, but by degrees declining towards the South, the long *Striæ* of Light, which arofe from all Parts of the Northern Semicircle of the Horizon, feem'd to meet together, not much above the Head of *Caftor*, or the Northern *Twin*, and there foon difappear'd.

After the firft *Impetus* of this afcending Vapour was over, the *Corona* appear'd no more; but ftill, without any Order as to Time, or Place, or Size, luminous *Radii*, like the former, continued to arife perpendicularly, now oftener, and again feldomer; now here, now there; now longer, now fhorter. Nor did they proceed as at firft out of a Cloud, but oftener would emerge at once out of the pure

pure Sky, which was more than ordinary ſerene and ſtill. Nor were they all of the ſame Form. Moſt of them ſeem'd to end in a Point upwards, like erect Cones; others like truncate Cones or Cylinders, ſo much reſembled the long Tails of Comets, that at firſt ſight they might well be taken for ſuch. Some of theſe Rays would continue viſible for ſeveral Minutes; when others, and thoſe the much greater Part, juſt ſhew'd themſelves, and died away. Some ſeem'd to have little Motion, and to ſtand, as it were fix'd, among the Stars, whilſt others, with a very perceptible Tranſlation, mov'd from Eaſt to Weſt under the Pole, contrary to the Motion of the Heavens; by which Means they would ſometimes ſeem to run together, and at other times to fly one another.

After this Sight had continued about an Hour and a half, thoſe Beams began to riſe much fewer in Number, and not near ſo high, and by Degrees that diffuſed Light, which had illuſtrated the Northern Parts of the Hemiſphere, ſeem'd to ſubſide, and ſettling on the Horizon formed the Reſemblance of a very bright *Crepuſculum:* That this was the State of this *Phænomenon*, in the firſt Hours, is abundantly confirm'd by the unanimous Conſent of ſeveral. For, by the Letters we have receiv'd from almoſt all the extreme Parts of the Kingdom, there is found very little Difference in the Deſcription from what appear'd at *London* and *Oxford*; unleſs that in the North of *England* and in *Scotland*, the Light ſeem'd ſomewhat ſtronger and brighter.

Hitherto I have related the Obſervations of others: As to myſelf, I had no Notice of this Matter, till between nine and ten of the Clock; upon the firſt Information of the Thing, I immediately ran to the Windows, which happen'd to regard the South and South-Weſt Quarter; and ſoon perceiv'd, that though the Sky was very clear, yet it was tinged with a ſtrange ſort of Light; ſo that the ſmaller Stars were ſcarce to be ſeen, and much as it is when the Moon of four Days old appears after Twilight. I perceiv'd at the ſame Time a very thin Vapour to paſs before us, which aroſe from the preciſe Eaſt Part of the Horizon, aſcending obliquely, ſo as to leave the Zenith about fifteen or twenty Degrees to the Northward. But the Swiftneſs wherewith it proceeded was ſcarce to be believed, ſeeming not inferior to that of Lightning; and exhibiting, as it paſs'd on, a ſort of momentaneous *Nubecula*, which diſcover'd itſelf by a very diluted and faint Whiteneſs; and was no ſooner formed, but before the Eye could well take it, it was gone, and left no Signs behind it. Nor was this a ſingle Appearance; but for ſeveral Minutes, about ſix or ſeven Times in a Minute, the ſame was again and again repeated; theſe Waves of Vapour regularly ſucceeding one another, and at Intervals very nearly equal; all of them in their Aſcent producing a like tranſient *Nubecula.*

By

By this Particular we were first assured; that the Vapour we saw, became conspicuous by its own proper Light, without Help of the Sun's Beams: for these *Nubeculæ* did not discover themselves in any other Part of their Passage, but only between the *South-East* and *South*, where being opposite to the Sun, they were deepest immers'd in the Cone of the Earth's Shadow; nor were they visible before or after. Whereas the contrary must have happen'd, had they borrow'd their Light from the Sun.

I then made all the Haste I could to a Place where there was a free Prospect of the Northern Horizon. Being come there, not much past ten of the Clock, I found, on the Western Side, *viz.* between W. and N. W. the Representation of a very bright *Twilight*, contiguous to the Horizon; out of which arose very long Beams of Light, not exactly erect toward the *Vertex*, but something declining to the South; which ascending by a quick and undulating Motion to a considerable Height, vanish'd in a little Time, whilst others, tho' at uncertain Intervals, supply'd their Place. But at the same Time, through all the rest of the Northern Horizon, *viz.* from the North-West to the true East, there did not appear any Sign of Light to arise from, or join to, the Horizon; but what appear'd to be an exceeding black and dismal Cloud seem'd to hang over all that Part of it. Yet was it no Cloud, but only the serene Sky more than ordinary pure and limpid, so that the bright Stars shone clearly in it, and particularly *Cauda Cygni*, then very low in the *North*; the great Blackness manifestly proceeding from the Neighbourhood of the Light which was collected above it. For the Light had now put on a Form quite different from all that we have been describing, and had fashion'd itself into the Shape of two *Laminæ* or Streaks, lying in a Position parallel to the Horizon, whose Edges were but ill terminated. They extended themselves from the *N.* by *E.* to the *North-East*, and were each about a Degree broad; the undermost about eight or nine Degrees high, and the other about four or five Degrees over it; these kept their Places for a long Time, and made the Sky so light, that I believe a Man might easily have read an ordinary Print by the Help thereof.

Whilst I was viewing this surprizing Sight, and expecting what was further to come, the Northern End of the upper *Lamina* by Degrees bent downwards, and at length closed with the End of the other that was under it, so as to shut up on the Northside an intermediate Space, which still continued open to the East. Not long after this, in the said included Space, I saw a great Number of small Columns or whitish Streaks to appear suddenly, erect to the Horizon, and reaching from the one *Lamina* to the other; which instantly disappearing, were too quick for the Eye, so that I could not judge whether they arose from the under, or fell from the upper, but by their ſudden Alterations, they made ſuch an Appearance, as might well enough be taken to reſemble the Conflicts of Men in Battle.

And much about the ſame Time, there began on a ſudden to appear, low under the Pole, and very near due North, three or four lucid *Areas*, like Clouds, diſcovering themſelves, in the pure but very black Sky, by their yellowiſh Light. Theſe, as they broke out at once, ſo after they had continued a few Minutes, diſappear'd as quick as if a Curtain had been drawn over them: Nor were they of any determined Figure, but both in Shape and Size might properly be compar'd to ſmall Clouds illuminated by the full Moon, but brighter.

Not long after this, from above the aforeſaid two *Laminæ*, there aroſe a very great *Pyramidal* Figure, like a *Spear*, ſharp at the Top, whoſe Sides were inclin'd to each other with an Angle of about four or five Degrees, and which ſeem'd to reach up to the *Zenith*, or beyond it. This was carried with an equable, and not very ſlow Motion, from the N. E. where it aroſe, into the N. W. where it diſappear'd, ſtill keeping in a perpendicular Situation, or very near it; and paſſing ſucceſſively over all the Stars of the *Little Bear*, did not efface the ſmaller ones in the Tail, which are but of the fifth Magnitude; ſuch was the extreme Rarity and Perſpicuity of the Matter whereof it conſiſted.

This ſingle Beam was very remarkable for its Height above all thoſe that for a great while before had preceded it, or that follow'd it.

It being now paſt eleven of the Clock, and nothing new offering itſelf to our View, but repeated *Phaſes* of the ſame Spectacle; being returned to my Houſe, I went to my upper Windows, which conveniently enough regarded the N. E. Part of the Heavens, and ſoon found that the two *Laminæ* or Streaks parallel to the Horizon, had now wholly diſappear'd; and the whole Spectacle reduced itſelf to the Reſemblance of a very bright *Crepuſculum* ſettling on the Northern Horizon, ſo as to be brighteſt and higheſt under the Pole itſelf; from whence it ſpread both Ways into the N. E. and N. W. Under this, in the middle thereof, there appear'd a very black Space, as it were the Segment of a leſſer Circle of the Sphere cut off by the Horizon. It ſeem'd to the Eye like a dark Cloud, but was not ſo; for by the Teleſcope the ſmall Stars appear'd through it more clearly than uſual, conſidering how low they were: And upon this as a *Baſis*, our *Lumen Auroriforme* reſted, which was no other than a Segment of a Ring or Zone of the Sphere, intercepted between two Parallel leſſer Circles, cut off likewiſe by the Horizon; or the Segment of a very broad *Iris*, but of one uniform Colour; *viz.* a Flame-Colour inclining to yellow, the Center thereof being about forty Degrees below

low the Horizon. And above this, there were ſeen ſome Rudiments of a much larger Segment, with an Interval of dark Sky between, but this was ſo exceeding faint and uncertain, that I could make no proper Eſtimate thereof.

I attended this Phænomenon till near three in the Morning, and the riſing of the Moon: But for above two Hours together, it had no manner of Change in its Appearance, nor Diminution, nor Increaſe of Light; only ſometimes, for very ſhort Intervals, as if new Fuel had been caſt on a Fire, the Light ſeem'd to undulate and ſparkle, not unlike the riſing of a vaporous Smoke out of a great Blaze, when agitated. But one Thing I aſſured my ſelf of, that this *Iris*-like Figure did by no Means owe its Origin to the Sun's Beams: For that about three in the Morning, the Sun being in the Middle between the North and Eaſt, our *Aurora* had not follow'd him, but ended in that very Point where he then was: Whereas in the true North, which the Sun had long paſs'd, the Light remain'd unchanged, and in its full Luſtre.

Thus I have endeavour'd by Words to repreſent what I ſaw; I have annexed a *Figure* exhibiting that particular Appearance of the two Fig. 23. *Laminæ*, which I ſaw at *London* between the Hours of ten and eleven: Becauſe I do not find, among the many Relations I have ſeen, any one that has taken Notice of it. In this Figure *A B* is the under *Lamina*, ſomewhat broader and brighter than the upper *C D*: It had near its under Edge the *Lucida Lyræ*, and below its Northern Extremity, on the Left-hand, *Cauda Cygni*: And as well above and below theſe, as in the intermediate Space between them, and indeed all round about that Part of the Heavens, the Sky was ſo unuſually dark and black, as if all that *Exotic Light* that had ſhew'd itſelf before, had been then collected into thoſe two Streaks. Only at *Q*, between the Weſt and North-Weſt, and no where elſe, out of a Brightneſs adjoining to the Horizon, there aroſe conical Beams, as *M*, *L*, *N*, after the ſame Manner as at firſt.

Whilſt we ſtood looking on, the Streak *C D*, at its Northern End, bent downward, and joined with the under *A B* at *E*, and included the Space *D C E A B*, which ſtill kept open at the other End towards the Eaſt; and in the mean Time, out of the very clear Sky, ſome luminous Spots, ſituated and figured as in the Scheme at *G*, *G*, *G*, *G*, preſented themſelves to the Eye, in Colour much like the *Laminæ*. Theſe did not ſhew themſelves all together, but came ſucceſſively, yet ſo as two or three of them were ſeen at a Time; and as their coming was inſtantaneous, ſo they went away in a Moment. At the ſame Time likewiſe, the ſeveral little white Columns mark'd *F*, *F*, *F*, *F*, occupied that Part of the Space between the two Streaks next to *E*, and by their ſudden and very irregular Motion, and the vaniſhing of ſome, whilſt others, at the ſame Time, emerged, gave occasion to the Conception of those that fancied Battles fought in the Air. Lastly, from about the Middle of *C D*, there arose suddenly a Cone or Obelisk of a pale whitish Light, greater than any we had yet seen, as *H*; which moving from East to West, with a Motion sufficiently regular, was translated to *K*, in the North West, and there disappear'd.

That we might by the same Scheme shew the Appearance of the last Hours, after Midnight; we have made the Light at *Q*, much bigger than what appear'd in the West about ten of the Clock; so as to represent truly that other. In this Case, the Point *Q* must, by the Imagination, be supposed transferr'd to the Intersection of the Horizon and Meridian under the Pole. The Scheme indeed could by no Means be contriv'd to answer the wonderful Variety this *Phænomenon* afforded; since even the Eye of no one single Observer was sufficient to follow it in the Suddenness and Frequency of its Alterations.

Thus I have attempted to describe what was seen, and am sorry I did not see the first and most surprizing Part thereof my self: The like is not recorded in the *English Annals* since 1574, that is, above a hundred and forty Years ago, in the Reign of Queen *Elizabeth*. Then, as we are told by the Historians of those Times, *Cambden* and *Stow*, for two Nights successively, *viz.* on the 14*th* and 15*th* of *November* that Year, much the same wonderful *Phænomena* were seen, with almost all the same Circumstances as now.

Nor, indeed, was this then so rare a Sight as it has been since: For we find, in a Book entituled, *A Description of Meteors*, reprinted at *London* in the Year 1654, whose Author writes himself *W. F.* D. D. that the same Thing, which he there calls *Burning Spears*, was seen at *London* on *January* 30, 1560; and again by the Testimony of *Stow*, on the 7*th* of *October* 1564. And from foreign Authors we learn, that in the Year 1575, the same was twice repeated in *Brabant*, *viz.* on the 13*th* of *February*, and 28*th* of *September*; and seen and described by *Cornelius Gemma*: Who in a Discourse he wrote of the Prodigies of those Times, after several ill-boding Prognosticks, thus very properly describes the *Cupola* and *Corona*, that he saw in the *Phasma* (as he calls it) of *February*. *A little while after new Flames rising like Spears, the Heaven seem'd to be on Fire on the Northern Side quite up to the Zenith. And lastly, that nothing might seem represented before which hitherto had happen'd, the Appearance of the Heavens was changed for the Space of an Hour, into the strange Likeness of a Box with which they play at Dice, blue and white continually changing, not with less Uncertainty and Swiftness than the Rays of the Sun, when they are reflected back by an interposed Speculum.* Here it is not a little remarkable, that all these four already mentioned, fell exactly upon the same Age of the Moon, *viz.* about two Days after the Change.

As

As to the other of *September* in the same Year 1575, these are the Words of *Gemma*. *It was not indeed so terrible, yet with greater Variety that other Phænomenon appear'd, which we saw in* October *following, just after the Sun was set. In this were many shining Bows, from which issued Spears, Cities with Turrets, and Armies of Soldiers. Hence the Rays proceeded every Way, as also the Floating of Clouds and Images of Battles. They fled from and pursued one another, with a wonderful Alternation.* From hence 'tis manifest, that this *Phænomenon* appear'd in our Neighbourhood three several Times, and that with considerable Intervals, within the Compass of one Year; though our *English* Historians have not recorded the two latter; nor did *Gemma* see that of *November* 1574, as 'tis most likely, by reason of Clouds. After this, in the Year 1580, we have the Authority of *Michael* * *Mastlin*; that at *Baknang* in the Country of *Wirtemburg* in *Germany*, these *Phasmata*, as he likewise stiles them, were seen by himself no less than seven Times within the Space of twelve Months. The first of these, and most considerable, fell out on the very same Day of the Month with ours, *viz.* on *Sunday* the sixth of *March*, and was attended with much the same Circumstances. And again the same Things were seen in a very extraordinary Manner on the *9th* of *April* and *10th* of *September*, following: But in a less Degree, on the *6th* of *April*, *21st* of *September*, *26th* of *December*, and *16th* of *February*, 1581. The last of which, and that of the *21st* of *September*, must needs have been more considerable than they then appear'd, because the Moon being near the Full, necessarily effaced all the fainter Lights. Of all these, however, no one is mentioned in our Annals to have been seen in *England*, nor in any other Place that I can find.

(*) M. Mastlin *Lib. de Cometa*, 1580.

The next that we hear of, was that of the Year 1621, on *September 2d, st. vet.* seen all over *France*, and well describ'd by *Gassendus* in his *Physicks*, who gives it the Name of *Aurora Borealis*. This, tho' little inferior to what we lately saw, and appearing to the Northwards both of *Rouen* and *Paris*, is no where said to have been observ'd in *England*, over which the Light seem'd to lie.

Another was seen all over *Germany*, in the Year 1623, thus describ'd by *Kepler*. *On the 17 Day of* November, *Anno* 1623, *a fiery Meteor was seen, or a burning Ball, flying over all* Germany *from West to East. In* Austria *they said it gave a Sound like a Clap of Thunder, which I cannot think is true; for the Descriptions that are extant do not confirm this.*

And since then, for above 80 Years, we have no Account of any such Sight, either from home or abroad. The first we find on our Books, was one of small Continuance seen in *Ireland* by Mr. *Neve*, on the *16th* of *November* 1707. And in the *Miscellanea Berolinensia*, published in 1710, we learn, that in the same Year 1707, both on the *24th* of *January*, and *18th* of *February*, *st. vet.* something of this

Vid. Supra, p. 134.

kind

kind was ſeen by Mr. *Olaus Romer* at *Copenhagen*, and again on the 23*d* of *February*, the ſame *Aſtronomer* obſerv'd there ſuch another Appearance, but much more conſiderable; of which yet he only ſaw the Beginning, Clouds interpoſing. But the ſame was ſeen that Night by Mr. *Gotfried Kirch*, at *Berlin*, above 200 Miles from *Copenhagen*, and laſted there till paſt ten at Night. To theſe add another ſmall one of ſmall Duration, ſeen near *London*, a little before Midnight between the ninth and tenth of *Auguſt* 1708, by the Lord Biſhop of *Hereford*; ſo that, it ſeems, in little more than eighteen Months, this ſort of Light has been ſeen in the Sky, no leſs than five Times in the Years 1707 and 1708.

Hence we may reaſonably conclude, that the Air, or Earth, or both, are ſometimes, though but ſeldom, and with great Intervals, diſpoſed to produce this *Phænomenon*: For though it be probable that many Times, when it happens, it may not be obſerv'd, as falling out in the Day-time, or in cloudy Weather, or bright Moon-ſhine: Yet, that it ſhould be ſo very often ſeen at ſome Times, and ſo ſeldom at others, is what cannot well be that Way accounted for. Wherefore conſidering what might be moſt probably the *Material Cauſe* of theſe Appearances; what firſt occurr'd was the Vapour of Water rarified exceedingly by ſubterraneous Fire, and tinged with ſulphureous Steams; which Vapour is now generally taken by our Naturaliſts to be the Cauſe of *Earthquakes*. And as Earthquakes happen with great Uncertainty, and have been ſometimes frequent in Places, where, for many Years before and after, they have not been felt; ſo theſe, which we might be allow'd to ſuppoſe produc'd by the Eruption of the pent-up Vapour through the Pores of the Earth, when it is not in ſufficient Quantity, nor ſudden enough to ſhake its Surface, or to open it ſelf a Paſſage by rending it. And as theſe Vapours are ſuddenly produc'd by the Fall of Water upon the Nitro-ſulphureous Fires under Ground, they might well be thought to get from thence a Tincture which might diſpoſe them to ſhine in the Night, and a Tendency contrary to that of Gravity; as we find the Vapours of *Gunpowder*, when heated in *Vacuo*, to ſhine in the Dark, and aſcend to the Top of the Receiver, though exhauſted: The Experiment of which, I ſaw very neatly performed by Mr. *J. Whiteſide*.

Nor ſhould I ſeek for any other Cauſe than this, if in ſome of thoſe Inſtances, particularly this whereof we treat, the Appearance had not been ſeen over a much greater Part of the Earth's Surface than can be thus accounted for. It having in this laſt been viſible from the Weſt Side of *Ireland*, to the Confines of *Ruſſia* and *Poland* on the Eaſt (nor do we yet know its Limits on that Side) extending over at leaſt thirty Degrees of Longitude; and in Latitude, from about fifty Degrees over almoſt all the North of *Europe*; and in all

Places

Places exhibiting at the ſame Time the ſame wondrous Circumſtances as we are informed by the Publick News. Now this is a Space much too wide to be ſhaken at any one Time by the greateſt of Earthquakes, or to be affected by the Perſpiration of that Vapour, which being included, and wanting Vent, might have occaſion'd the Earth to tremble. Nor can we this Way account for that remarkable Particular attending theſe Lights, of being always ſeen on the Northſide of the Horizon, and never to the South.

Wherefore laying aſide the explaining theſe Things by the ordinary Vapours or Exhalations of the Earth or Waters, we are forced to have Recourſe to other ſorts of *Effluvia* of a much more ſubtile Nature, and which perhaps may ſeem more adapted to bring about thoſe wonderful and ſurprizingly quick Motions. Such are the *Magnetical Effluvia*, whoſe Atoms freely permeate the Pores of the moſt ſolid Bodies, meeting with no Obſtacles from the Interpoſition of Glaſs or Marble or even Gold itſelf. Theſe by a perpetual Efflux do, ſome of them, ariſe from the Parts near the Poles of the Magnet, whilſt others of the like Kind of Atoms, but with a contrary Tendency, enter in at the ſame Parts of the Stone, through which they freely paſs; and by a kind of Circulation ſurround it on all Sides, as with an Atmoſphere, to the Diſtance of ſome Diameters of the Body.

That the Fact may be the better comprehended, I ſhall endeavour to exhibit the Manner of the Circulation of the Atoms concern'd therein, as they are expoſed to View, by placing the Poles of a *Terella*, or *Spherical Magnet*, on a Plane, as the Globe on the Horizon of a right Sphere: Then ſtrewing fine Steel Duſt or Filings very thin on the Plane all round it, the Particles of Steel, upon a continued gentle knocking on the underſide of the Plane, will by Degrees conform themſelves to the Figures in which the Circulation is perform'd. Thus, let *A B C D* be a *Terella*, and its Poles *A* the South, and *B* the North; and by doing as preſcrib'd, it will be found that the Filings will lie in a Right Line perpendicular to the Surface of the Ball, when in the Line of the Magnetical Axis continued. But for about forty-five Degrees on either Side, from *B* to *G* or *I*, and from *A* to *H* or *K*, they will form themſelves into Curves, more and more crooked as they are remoter from the Poles; and withal more and more oblique to the Surface of the Stone. Hence it may appear how this exceeding ſubtile Matter revolves; and particularly how it permeates the Magnet with more Force, and in greater Quantity in the circumpolar Parts, entering into it on the one Side, and emerging from it on the other, under the ſame oblique Angles: Whilſt in the middle Zone about *C* and *D*, near the Magnet's Equator (if I may uſe the Word) very few, if any of theſe Particles do impinge, and thoſe very obliquely. *Fig.* 24.

Now

Now by many and very evident Arguments it appears, that our *Globe of Earth* is no other than one great Magnet, or (if I may be allow'd to alledge an Invention of my own) rather two; the one including the other, as the *Shell* includes the *Kernel*; for fo and not otherwife, we may explain the Changes of the Variation of the Magnetical Needle. It fuffices that we may fuppofe the fame fort of Circulation of fuch an exceeding fine Matter to be perpetually performed in the *Earth*, as we obferve in the *Terella*; which fubtile Matter freely pervading the Pores of the Earth, and entering into it near its Southern Pole, may pafs out again into the *Æther*, at the fame Diftance from the Northern, and with a like Force; its Direction being ftill more and more oblique, as the Diftance from the Poles is greater. To this we beg leave to fuppofe, that this fubtile Matter, no otherways difcovering itfelf, but by its Effects on the Magnetic Needle, wholly imperceptible, and at other Times invifible, may now and then, by the Concourfe of feveral Caufes very rarely coincident, and to us as yet unknown, be capable of producing a fmall Degree of Light; perhaps from the greater Denfity of the Matter, or the greater Velocity of its Motion: After the fame Manner as we fee the *Effluvia* of *Electric* Bodies by a ftrong and quick Friction emit Light in the Dark: To which fort of Light this feems to have a great Affinity.

This being allow'd me, I think we may affign a Caufe for many of thefe ftrange Appearances, and for fome of the moft difficult to account for otherwife; as why thefe Lights are rarely feen any where elfe but in the North, and never, that we hear of, near the Equator: As alfo why they are more frequently feen in *Iceland* and *Greenland*, than in *Norway*, though nearer the *Pole* of the *World*. For the Magnetical Poles, in this Age, are to the Weftward of our Meridian, and more fo of that of *Norway*, and not far from *Greenland*; as appears by the Variation of the Needle this Year obferv'd, full twelve Degrees at *London* to the Weft.

The erect Pofition of the luminous Beams or *Striæ* fo often repeated that Night, was occafioned by the rifing of the Vapour or lucid Matter nearly perpendicular to the Earth's Surface. For that any Line erected perpendicularly upon the Surface of the Globe, will appear erect to the Horizon of an Eye placed any where in the fame fpherical Superficies; as *Euclid* demonftrates in a Plane, that any Line erected at right Angles to it, will appear to be perpendicular to that Plane from any Point thereof. That it fhould be fo in the Sphere, is a pretty Propofition, not very obvious, but demonftrated from *Prop.* 5. *Lib.* 1. *Theodofii Sphæric.* For by it all Lines erect on the Surface pafs through the Center, where meeting with thofe from the Eye, they form the Planes of Vertical Circles thereto. And by the Converfe hereof, it is evident, that this luminous

Matter

Matter aroſe nearly perpendicular to the Earth's Surface, becauſe it appear'd in this erect Poſition. And whereas in this Appearance, thoſe Beams which aroſe near the Eaſt and Weſt, as *L*, *M*, *N*, were furtheſt from the Perpendicular, on both Sides inclining towards the South, whilſt thoſe in the North were directly upright: The Cauſe thereof may well be explain'd by the Obliquity of the Magnetical Curves, making ſtill obtuſer Angles with the Meridians of the *Terrella*, as they are further from the Poles.

Hence alſo it is manifeſt, how that wonderful *Corona* that was ſeen to the Southwards of the *Vertex*, in the Beginning of the Night, and ſo very remarkable for its tremulous and vibrating Light, was produced; to wit, by the Concourſe of many of thoſe Beams ariſing very high out of the circumjacent Regions, and meeting near the Zenith: The *Effluvia* whereof they conſiſted mixing and interfering one with another, and thereby occaſioning a much ſtronger, but uncertain wavering Light. And ſince it is agreed by all Accounts that this *Corona* was tinged with various Colours, 'tis more than probable, that theſe Vapours were carried up to ſuch a Height, as to emerge out of the Shadow of the Earth, and to be illuſtrated by the direct Beams of the Sun: Whence it might come to paſs that this firſt *Corona* was ſeen colour'd and much brighter than what appear'd afterwards in ſome Places, where the Sight thereof was gone down much lower under the Horizon. Hence too it will be eaſily underſtood that this *Corona* was not one and the ſame in all Places, but was different in every differing Horizon; exactly after the ſame Manner as the Rainbow ſeen in the ſame Cloud is not the ſame Bow, but different to every ſeveral Eye.

Nor is it to be doubted, but the Pyramidical Figure of theſe aſcending Beams is Optical: Since, according to all likelihood, they are parallel-ſided, or rather tapering the other Way. But by the Rules of Perſpective, their Sides ought to converge to a Point, as we ſee in Paintings the Parallel Borders of ſtraight Walks, and all other Lines parallel to the Axis of Viſion, meet as in a Center. Wherefore thoſe Rays which aroſe higheſt above the Earth, and were neareſt the Eye, ſeem'd to terminate in Cuſps ſufficiently acute, and have been for that Reaſon ſuppos'd to repreſent *Spears*. Others ſeen from afar, and perhaps not riſing ſo high as the former, would terminate, as if cut off with Planes parallel to the Horizon, like truncate Cones or Cylinders: Theſe have been taken to look like the Battlements and Towers on the Walls of Cities fortified after the ancient Manner. Whilſt others yet further off, by Reaſon of their great Diſtance, good Part of them being intercepted by the Interpoſition of the Convexity of the Earth, would only ſhew their pointed Tops, and becauſe of their Shortneſs have gotten the Name of *Swords*.

Next the Motion of theſe Beams furniſhes us with a new and moſt evident Argument to prove the diurnal Rotation of the Earth. For thoſe Beams which roſe up to a Point, and did not preſently diſappear, but continued for ſome Time, had moſt of them a ſenſible Motion from Eaſt to Weſt, contrary to that of the Heavens; the biggeſt and talleſt of them, as being neareſt, ſwifteſt; and the more remote and ſhorter, ſlower. By which Means, the one overtaking the other, they would ſometimes ſeem to meet and joſtle; and at other Times to ſeparate, and fly one another. But this Motion was only Optical, and occaſion'd by the Eye of the Spectator being carried away with Earth into the Eaſt; whilſt the exceeding rare Vapour of which thoſe Beams did conſiſt, being raiſed far above the Atmoſphere, was either wholly left behind, or elſe follow'd but with Part of its Velocity, and therefore could not but ſeem to recede and move the contrary Way. And after the ſame Manner as the Stars that go near the Zenith, paſs over thoſe Vertical Circles which border on the Meridian, much ſwifter than thoſe Stars which are more diſtant therefrom; ſo theſe luminous Rays would ſeem to recede faſter from Eaſt to Weſt, as their Baſes were nearer the Eye of the Spectator; and *è contra*, ſlower as they were further off.

Nor are we to think it ſtrange, if after ſo great a Quantity of luminous Vapour had been carried up into the *Æther* out of the Pores of the Earth, the Cauſe of its Efferveſcence at length abating, or perhaps the Matter thereof conſumed; theſe *Effluvia* ſhould at length ſubſide, and form thoſe two bright *Laminæ* which we have deſcrib'd, and whoſe Edges being turn'd to us, were capable to emit ſo much Light. I choſe to call them *Laminæ*, becauſe, without Doubt, tho' they were but thin, they ſpread Horizontally over a large Tract of the Earth's Surface. And whilſt this luminous Matter dropp'd down from the upper Plate to the under, the many little white Columns were formed between them by its Deſcent, only viſible for the Moment of their Fall. Theſe by the Swiftneſs with which they vaniſh'd, and their great Number, ſhewing themſelves, and diſappearing without any Order, exhibited a very odd Appearance; thoſe on the Right ſeeming ſometimes to drive and puſh thoſe on the left, and *vice verſa*.

I have omitted ſeveral Particulars of leſs Moment: But theſe are the principal *Phænomena*; of whoſe Cauſes I ſhould have with more Certainty given my Thoughts, if I had ſeen the whole from Beginning to End; and could have added my own Remarks to the Relations of others; and eſpecially, if we could by any Means have come at the Diſtances thereof. If it ſhall by any be thought a hard Suppoſition that I aſſume the *Effluvia* of the Magnetical Matter for this Purpoſe, which in certain Caſes may themſelves become luminous,

luminous, or rather may ſometimes carry with them out of the Bowels of the Earth, a ſort of Atoms proper to produce Light in the *Æther*. I anſwer, that we are not as yet acquainted with any other Kinds of *Effluvia* of terreſtrial Matter, which may ſerve for our Purpoſe, than thoſe we have here conſider'd, *viz.* the magnetical Atoms, and thoſe of Water highly rarified into Vapour. Nor do we find any Thing like it in what we ſee of the celeſtial Bodies, unleſs it be the *Effluvia* projected out of the Bodies of Comets to a vaſt Height, and which ſeem by a *Vis centrifuga* to fly with an incredible Swiftneſs the Centers both of the Sun and Comet, and to go off into Tails of a ſcarce conceivable Length. What may be the Conſtitution of theſe Cometical Vapours, we Inhabitants of the *Earth* can know but little, and only that they are evidently excited by the Heat of the Sun; whereas this Meteor ſeldom is ſeen but in the Polar Regions of the World, and that moſt commonly in the Winter Months.

I beg Leave on this Occaſion, to mention what, near 25 Years ſince, I publiſh'd in N° 195 † of theſe *Tranſactions*, *viz.* That ſuppoſing the *Earth* to be concave, with a leſſer Globe included, in order to make that inner Globe capable of being inhabited, there might not improbably be contain'd ſome luminous *Medium* between the Balls, ſo as to make a perpetual Day below. That very great Tracts of the Ætherial Space are occupied by ſuch a ſhining *Medium*, is evident from former Inſtances *. And if ſuch a *Medium* ſhould be thus incloſed within us, what ſhould hinder but that we may ſuppoſe that ſome Parts of this lucid Subſtance may, on very rare and extraordinary Occaſions, tranſude through and penetrate the *Cortex* of our *Earth*, and being got looſe, may afford the Matter whereof this our *Meteor* conſiſts. This ſeems favour'd by one conſiderable Circumſtance, *viz.* that the *Earth*, becauſe of its diurnal Rotation, being neceſſarily of the Figure of a flat *Spheroid*, the Thickneſs of the *Cortex*, in the *Polar* Parts of the Globe, is conſiderably leſs than towards the *Equator*; and therefore more likely to give Paſſage to theſe Vapours: Whence a Reaſon may be given why theſe Lights are always ſeen in the North.

† *Vid.* Supra, V. II. C. IV. p. 615.

* *Vid. Supra* C. III. S. V.

2.] At *Paris*, the Light was ſo inconſiderable, that it was not regarded: But a Letter to Mr. *Alexander Geekie*, Surgeon, dated on Board a Ship in *Nevis* Road in *America*, *April* 19, 1716, informs us, " That on the ſixth of *March*, at Nine a Clock in the Evening, " we being then in the Latitude of 45° 36′ (off of the N. W. " Coaſt of *Spain*); A clear Cloud appear'd Eaſt to us, not far " diſtant from our Zenith, which afterwards darted itſelf forth in- " to a Number of Rays of Light, every way like the Tail of a " Comet, of ſuch a great Length, that it reach'd within a ſhort " Way

A Deſcription of the ſame Phænomenon *ſeen on the* Ocean. n. 348. p. 130.

" Way of the Horizon. There likewiſe appear'd a Body of Light, " N. N. E. of us, and continued as light almoſt as Day, till after " 12 a Clock. It appear'd at a good Diſtance from us, and darken'd " on a ſudden.

Hence it ſhould ſeem, that the Vapour which cauſed this Appearance, aroſe indifferently out of the deep Ocean Sea, as well as from the Land; by which we may conclude the great Subtilty of the Matter thereof, ſince it could permeate ſo great a Quantity of Water, and yet retain its Velocity.

A return of the ſame, ibid. 3.] Since this, moſt of the ſame *Phænomena* have been repeated three ſeveral Nights ſucceſſively, *viz.* on the laſt of *March*, and firſt and ſecond of *April*. The beſt and fulleſt Deſcription of the two firſt, is, from a Letter of Dr. *B. Taylor*, dated *April* 2, from *Cott rſtock*, near *Oundle* in *Northamptonſhire*, who thus deſcribes them. " On *Saturday* " Night laſt, and laſt Night, I ſaw Appearances of the ſame Kind, " with thoſe of *March* 6, but not to compare for Extent and Strength. " They both began ſoon after Sun-ſet, and continu'd till after 12, but " how much longer, I cannot tell. They were both about 10 or 15 " Degrees to the Weſtward of the North, and took up about 80 De- " grees of the Horizon; and the *Aurora* roſe about 30 gr. high, with " a dark Bottom, like what was ſeen in the firſt; and from whence " there ſprung out ſeveral Bodies of Light, which immediately ran in- " to Streams, aſcending about 30, or at moſt 40 gr. high. There " was no flaſhing nor waving Light, but, in all other Reſpects, theſe " Lights were of the ſame Kind with what we ſaw at *London*. Indeed " in that laſt Night, there was one *Phænomenon* like the flaſhing Light; " for a Body of Light about 15 or 20 Degrees long, parallel to the " Horizon, roſe till it came about 6 Degrees above the black " Baſis, and then ſent up two ſtrong Streams of Light about " 40 gr. high, which at Top daſh'd againſt one another, and diſap- " pear'd.

At *London*, the firſt Night, *March* 31. It did not begin to radiate, till towards Midnight, and was ſeen but by few, the Beams not riſing very high, and ſcarce appearing over the Houſes; but by the Relation of thoſe that ſaw it, it was much more conſiderable than the next Night following *Eaſter-day*; for it then ſent out but few, and very ſhort Beams, moſtly terminating in a ſharp Point, and preſently diſappearing: Only it beginning to ſtream as ſoon as it became duſky, it was very obſervable, that thoſe Rays which aroſe out of the Weſt-end of the luminous Arch, next the Sun, were enlighten'd by its Beams, and ſhew'd themſelves much brighter than thoſe which aroſe under the Pole, or to the Eaſtward thereof. And after nine, till Midnight, no more Beams aroſe; and the luminous Arch with its black Baſis, ſettled down very low in the Northern Horizon.

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The ſame two Nights, by the Obſervation of Mr. *William Lingen*, the like Appearance was ſeen at *Dublin*, about the Hours of nine or ten; at which Time, in the former Night, it was near as light as in a Moon-light Night. And from *France*, we have an Account, that both thoſe Nights, the ſame was ſeen at *Paris*, with much the ſame Circumſtances as at *Dublin*. So that, it ſeems, this Meteor, though no Ways comparable to that of the 6th of *March*, was ſeen not leſs than 150 Leagues, and probably much farther.

On *April* 2. when it began to be dark, a luminous Arch appear'd in the North, with a very narrow black Bottom under it, very low, and depreſs'd to the Horizon; nor was it ſeen at, or about *London*, to project any pointed Rays as the former.

But what was moſt remarkable that Evening, was, what was ſeen at *London*, by *Martin Folkes*, Eſq; about nine that Night. He being then in the open Air, ſaw in an Inſtant, a bright Ray of very white Light, appear in the Eaſt, out of the pure Sky, then very ſerene and ſtill; it very much reſembled the Tail of a Comet, and was about 20 gr. inclin'd from the Perpendicular to the Right, beginning about γ of *Bayer* in the *Corona Borea*, and terminating about the *Informis*, by ſome call'd *Cor Caroli*. This having appear'd but a very little Time, diſappear'd at once, as in a Moment: When, on a ſudden, ſuch another Beam was inſtantly produced, not exactly in the ſame Place, but in the ſame Situation. Its lower being about 20 gr. high, was terminated exactly between κ and γ, in the Right Hand and Arm of *Hercules*, and the Middle of it paſs'd over σ and ρ in the Girdle of *Bootes*, and thence proceeded Weſtwards, leaving *Cor Caroli* four or five Degrees to the Northwards. After it had continu'd in this Poſition near ten Minutes immoveable among the Stars, it began to move ſlowly towards the North: And the lower End paſſing over the Northern Edge of the *Crown*, and the Ray itſelf over *Cor Caroli*, it grew fainter, and vaniſh'd, having continued in all about 20 Minutes. This latter, with ſome Interruptions, was extended between *Caſtor* and *Pollux*, very far into the Weſt: And about that Time, the ſame, or ſuch another Beam, was ſeen at St. *Aſaph*, by Dr. *Stanley*.

Two Northern Auroras *ſeen in* Kent, *by the Reverend Mr.* E. Barrel. n. 351. p. 584.

XXVII. 1.] On *February* 5, 1716-17, at eight at Night, at *Sutton at Hone* in *Kent*, an *Aurora Borealis* appear'd. It occupied at leaſt $\frac{1}{5}$ or near $\frac{1}{4}$ of the Horizon; it was low, and ſhot out bright Rays, and, I believe, would have appear'd very light, had it not been that the Moon ſhone at the ſame Time, being about five Days old, and that the *Aurora* diſappear'd before the Moon ſet.

Again, on the 30th of *March* following, there was another *Aurora Borealis*. I ſaw it not till paſt nine: 'Twas dim then, and its higheſt Part cover'd the loweſt Star in *Caſſiopea's Chair*. It did not ſeem due North,

North, but one Point to the Weſt. About ten it ſhot out very bright Rays, high, and tending ſomewhat towards one another. Near eleven a Clock, there was (beſides the Northern Brightneſs) a long Streak, not very broad, extended Eaſt and Weſt; which beginning in the *Serpent's Head*, near *Hercules's Club*, and covering *Arcturus*, proceeded near *Berenice's Hair*, and ſo went over *Cor Leonis*, and thence to the *Canicula*, and ended a little beyond that Star. It ſhone very bright at firſt, but faded away in about eight or nine Minutes. If it had Motion (which I am not ſure of) it was ſouthward. I waited for the next Fit of Brightneſs of the *Aurora*; and in about ſeven Minutes, the eaſtern Part of the Streak, *viz.* from the *Serpent's Head* to near *Berenice's Hair*, became viſible again, though dim, and was quite effaced in four or five Minutes more: And I did not yet perceive any Change of its Place.

—one *of them ſeen at* London, *by* M. Folkes, *Eſq*; n.352. p.586.

2.] Being in the Street, between eight and nine a Clock, on *March* 8, 1717, I perceiv'd a Light over the Houſes to the Northwards, little inferior to that the Full Moon gives when ſhe firſt riſes. Upon this, I made all the Haſte I could into the Fields, where I was for ſome Time entertain'd with the Sight of an *Aurora Borealis*, attended with moſt of the *Phænomena* of that very remarkable one of the 6th of *March* 1715-16.

The whole Northern Part of the Horizon was in the ſame Manner cover'd with ſomewhat reſembling a very black Cloud, from behind which, there iſſued a conſiderable Light, whoſe lower Part was pretty well defin'd by the common Edge of the Cloud, but the upper died away more gradually. This upper Limb of the Light reſembling the Arch of a Circle, whoſe higheſt Point between nine and ten of the Clock (when the Meteor was moſt conſiderable) was elevated about 12 Degrees, and bore, as I imagin'd, about 20 Degrees weſtward of the due North. It touch'd the Horizon in the Weſt, at the Diſtance of about 65 or 70 Degrees from the North, whence the whole intercepted Arch of the Horizon would have been of near 100 Degrees, had not ſome few Degrees in the Eaſt been hid by Clouds, which lay between us and the Meteor.

The ſeeming black Cloud, when I firſt ſaw it, ran nearly parallel to the Horizon, and at the Diſtance of 6 or 7 Degrees, but in about Half an Hour, it changed its Figure very much, ſinking down in the North to about half its Height, and riſing in the Weſt near as much. What I principally took Notice of this for, was, that the Light iſſuing from behind it, did not change with it, but remain'd of the ſame Figure, however the Cloud approached, or receded from differing Parts of its Limb.

There aroſe at firſt, ſome Streams in the N. N. W. but of no conſiderable Length, few of them paſſing 5 Degrees above the Arch; but beginning

beginning from behind the seeming Cloud, so as to be about 12 Degrees high in all. They were pointed at the Ends, and nearly Vertical to the Horizon. Between Times there was nothing but the Arch to be seen, and that only resembling a common *Aurora*; and again in an Instant, by a sort of a tremulous Motion, several Parts of it would appear converted into a vast Number of parallel Streams, for the most Part very little higher than the Arch itself. About 20 Minutes before Ten, a small Part of the Arch, almost due North, grew remarkably lighter than the rest, and continued to increase for about half a Minute; when there suddenly broke out some very tall Streams of at least 60 Degrees high, as I found by one in particular, which arose full North, and passing over the Pole Star itself, reach'd some Degrees beyond it. This was the most remarkable Time of the Appearance; some such Lances, though not so high, immediately shooting out of the Place that first of all radiated, as did some more a good way to the East. They were all nearly perpendicular to the Horizon, and most of them did arise quite from the black Substance at Bottom, tho' I saw some few that did not reach so low, appearing as if their lower Parts had been broken off. Some of them were full as bright as any I saw the last Year, the Axes (if I may so call them) of some of the tallest Streams coming up very near to the Colour of that pale Fire we see in some Sorts of Lightning.

About this Time the Ground Westward was all cover'd with an odd sort of Mist, the same from which I remember last Year, a great many People said there came an ill Smell, which I did not at all perceive.

About 10 the *Phænomenon* very much decreas'd, and so continu'd till after 11, only sending up now and then two or three Streams; at half an Hour after 11 it was again pretty much increas'd, and I saw it again send out some Streams almost as considerable as I had before seen this Evening; the Arch yet continu'd, but not so entire; and from what I could judge, its middle was some Degrees nearer the North, than when I first took Notice of it. Till a Quarter of an Hour before 12 the Light continually abated, and then I left it; but I was inform'd that it continu'd till towards Day-break, but never stream'd remarkably after I went away.

Though I could not this Time see any Stars through the black Matter at Bottom, I am sensible it was not a Cloud, though it bore the Resemblance of one: For when a real Cloud (as several small ones did) came over any Part of it, their Difference was very conspicuous.

I have since receiv'd two Letters, one from *Wisbich* in the Isle of *Ely*, the other from within 14 Miles of the *Bath*, both which take Notice of it, though with no further Particulars, than that they had seen

ſeen the ſame Light, tho' not conſiderable, as in the Beginning of *March* the laſt Year.

An Account of an extraordinary Meteor, *&c. by Dr.* E. Halley. n. 360. p. 978.

XXVIII. On the 19*th* of *March* 1718-19, a wonderful Luminous Meteor was ſeen in the Heavens all over *England*. Some of its *Phænomena* are very hard to account for, according to the Notions hitherto receiv'd by our Naturaliſts; ſuch is the very great Height thereof above the Earth; the vaſt Quantity of the Matter; the extravagant Velocity wherewith it mov'd; and the prodigious Exploſions heard at ſo great a Diſtance, whoſe Sound, attended with a very ſenſible Tremor of the ſubject Air, was certainly propagated through a *Medium* incredibly rare, and next to a *Vacuum*.

† *Vid.* Supra. p. 135.

I have formerly † collected what I could find of ſuch Meteors, but none ſeem to come up in any Circumſtance to this late Appearance; of which I ſhall give an Account from the many Relations thereof communicated to the *Royal Society*; tho' it was not my good Fortune to ſee it myſelf.

Sir *Hans Sloan* being Abroad at that Time, happen'd to have his Eyes turn'd towards it, in its very firſt Eruption; and gave me an Account of it in the following Terms: " That paſſing along Eaſtward by the N. E. Corner of *Southampton-ſtreet* in *Bloomſbury-Square, London*, at about a quarter after Eight at Night, I was ſurpriz'd to ſee a ſudden great Light, much beyond that of the Moon, which ſhone then very bright. I turn'd to the Weſtward where the Light was; which I apprehended at firſt to be artificial Fireworks or Rockets. The firſt Place I obſerv'd it in, was about the *Pleiades* Northerly, whence it mov'd after the Manner of, but more ſlowly than a falling Star, in a ſeeming direct Line, deſcending a little beyond, and withal below, the Stars in *Orion's Belt*, then in the S. W. The long Stream appear'd to me to be branch'd about the Middle, and the *Meteor* in its Way turn'd Pear-faſhion'd, or tapering upwards. At the lower End it came at laſt to be bigger and ſpherical, tho' it was not ſo big as the Full Moon. The Colour of it was whitiſh, with an Eye of Blue, of a moſt vivid dazling Luſtre, which ſeem'd in Brightneſs very nearly to reſemble, if not ſurpaſs that of the Body of the Sun in a clear Day, beheld by the naked Eye. This Brightneſs oblig'd me to turn my Eyes (which had their Pupils adapted to the Light of the Moon) from it ſeveral Times, as well when it was a Stream, as when it was Pear-faſhion'd and a Globe; tho' I had a great Curioſity to obſerve it with Attention. It ſeem'd to move in about half a Minute or leſs, about the Length of 20°, and to go out, as I gueſs'd, about as much above the Horizon. There was left behind it, where it had

" paſs'd

" paſs'd, a Track of a cloudy or faint reddiſh yellow Colour, ſuch
" as red-hot Iron or glowing Coals have, which remain'd more than
" a Minute, ſeem'd to ſparkle, and kept its Place without falling.
" This Track was interrupted, or had a Chaſm towards its upper
" End, and about two Thirds of its Length. I did not hear any
" Noiſe it made, but the Place where the Globe of Light had been,
" remain'd after it was extinct, of the ſame reddiſh yellow Colour
" with the Stream for ſome Time, and at firſt ſome Sparks ſeem'd
" to iſſue from it, ſuch as come from red-hot Iron beaten on an
" Anvil.

All the Relations agree in this, that the Splendor was little inferior to that of the Sun; that within Doors the *Candles* gave no Manner of Light; and in the Streets, not only all the *Stars* diſappear'd, but the *Moon* then nine Days old, and high towards the Meridian, the Sky being very clear, was ſo far effaced as to be ſcarce ſeen, at leaſt not to caſt a Shade, even where the Beams of the Meteor were intercepted by the Houſes: So that for ſome few Seconds of Time, in all reſpects it reſembled perfect Day.

The Time when this happened was generally reckon'd at a Quarter paſt Eight; but by the accurate Account of the Reverend Mr. *Pound* (who only ſaw the Light) agreeing with what has been ſent us from the *Pariſian* Obſervatory, it appears to have been at $8^h\ 8'$ apparent Time at *London*. And the Sun being then in $9\frac{3}{4}$ *gr.* of *Aries*, the right Aſcenſion of the Mid-Heaven was 130 *gr.* 45′, whereby the Poſition of the Sphere of fix'd Stars is given. Hence the *Lucida Pleiadum* will be found at that Time to have been $25\frac{1}{4}$ *gr.* high, in an *Azimuth* 6 *gr.* to the Northward of the Weſt, and conſequently the Arch the Meteor mov'd in, was inclin'd to the Horizon with an Angle of about 27 *gr.* having its Node or Interſection therewith, nearly *South South Weſt*; as will more plainly appear from what follows:

At *Oxford*, five Minutes earlier, Mr. *John Whiteſide*, Keeper of the *Aſhmole Muſæum*, immediately after the Extinction of the Meteor, made Haſte out to ſee what it might be, and well conſider'd the Situation of the Track it had left in the Sky: He found it to have paſs'd about $1\frac{1}{2}$ Degree above the preceding Shoulder of *Orion*, and about $3\frac{1}{2}$ *gr.* above the middle of his *Belt*, where there appear'd a luminous *Nubecula* of a reddiſh Light, being a Dilatation of the Track, ſeeming to have been occaſion'd by ſome Exploſion there; and by what he could learn from thoſe that ſaw it, it was thereabout that it broke out, and firſt began to efface the Stars. Hence it proceeded as to Senſe in an Arch of a great Circle, and paſſing in the middle between the Tail of *Lepus* (θ *Bayero*) and β in the Fore-foot of *Canis major*, it terminated about ξ in the Breaſt of the ſame, nearly in 95 *gr.* of Right-Aſcenſion, with 23 *gr.* South Declination:

clination: and at the Place of its Extinction there remain'd a large whitish *Nebula*, much broader and of a stronger Light than the rest of the Track, which he took for a certain Sign of a very great Explosion made there. By Computation it will be found that the Angle this Track made with the Horizon of *Oxford* was nearest 40 *gr.* and its Intersection due S. S. W; and that the Place of its Extinction was about 9 *gr.* above the Horizon, in the Azimuth of 32 *gr.* to the West.

At *Worcester*, Mr. *Nicholas Fatio* saw this Meteor descend obliquely towards the South, making an Angle with the Horizon of about 65°, and intersecting it about S. S. W. ½ *S.* as may be collected from a Scheme thereof sent up by him to the *Royal Society*. By this the Track left all *Orion* and *Canis major* to the Westward, and divided the Distance between *Sirius* and *Procyon*, so as to be almost twice as far from *Procyon* as *Syrius*. The Time here was one Minute before Eight, this City being about 9′ of Time to the West of *London*, and consequently the Right-Ascension of the Mid-Heaven 128 ½ *gr.*

Now the Situation of the three Cities, *London*, *Oxford*, and *Worcester* being nearly on the same W. N. W. Point, whereon the Track of the Meteor had its greatest Altitude above the Horizon, equal to the Angle of its visible Way; if we suppose it at *London* to have been 27 *gr.* high, and at the same Time at *Worcester* to be 65 *gr.* high, in the Plane of the Vertical Circle passing through *London* and *Worcester*; supposing likewise the Distance between them, to be 90 Geographical Miles, or one Degree and half of an Arch of a great Circle of the Earth, we may by a Trigonometrical *Calculus* find the perpendicular Height to have been 64 such Miles; and the Point over which it was then perpendicular to have been 30 such Miles W. N. W. from *Worcester*. And the Geographical Mile to the *English* Statute Mile being as 23 to 20, this Height will be no less than 73½ *English* Miles. The Place also directly under it, will be found to be about *Prestain* on the Confines of *Hereford* and *Radnor*-Shires. The *Oxford* Observation too concurs nearly in the same Conclusion.

This Altitude being added to the Semidiameter of the Earth as *Radius*, becomes the *Secant* of eleven Degrees, so that the Meteor might be seen above the Horizon in all Places not more than 220 Leagues distant from it. Whence it will not be strange that it should be seen over all Parts of the Islands of *Great Britain* and *Ireland*, over all *Holland*, and the hither Parts of *Germany*, *France* and *Spain*, at one and the same Instant of Time.

Having thus fix'd one Point in the Line of its Motion, let us see what Course the Meteor took from thence; and first at the Town of *Kirkby-Stephens*, on the Borders of *Yorkshire* and *Westmorland*, in in a Meridian very little to the Weſtward of *Worceſter*, but about $2\frac{1}{2}$ *gr.* more to the North, it was obſerv'd to break out as from a duſky Cloud, directly under the Moon, and from thence to deſcend, nearly in a Perpendicular, almoſt to the Horizon. Now the Moon being at that Time in the third Degree of *Leo*, was about half an Hour paſt the Meridian, and conſequently much about a Point to the Weſt, or S. *by* W. and the Situation of *Preſtain* from *Kirkby-Stephens*, being ſufficiently near upon the ſame Point, it follows, that the Direction of the Track of the Meteor was according to the great Circle paſſing over thoſe two Places.

And this is further confirm'd by the Obſervation of *Sam. Cruwys*, Eſq; who at *Tiverton*, about twelve Geographical Miles, nearly due North from *Exeter*, obſerv'd the firſt Exploſion of this Meteor exactly in his Zenith, as he was aſſur'd by applying his Eye to the Side of his Door, which he took to be perpendicular, and looking upwards: And from thence he ſaw it deſcend to the Southward directly in the ſame Azimuth, without declining either to the Right or Left: Hence it it plain, that the Track likewiſe paſs'd over this Place, which by our Maps is found to lie in a Line with *Preſtain* and *Kirkby-Stephens*.

On this Suppoſition, that the firſt Exploſion, attended with the reddiſh *Nubecula*, was directly over *Tiverton*, let us compare the *Oxford* Obſervation with it, in order to determine more nicely the perpendicular Altitude there. At *Oxford* this *Nubecula* was found to be $3\frac{1}{2}$ *gr.* above the middle Star of *Orion's Girdle*, at $8^h\ 3'$, and was therefore $26\frac{1}{2}$ *gr.* above the Horizon; and the Diſtance between *Oxford* and *Tiverton*, being $1^\circ\ 55'$, or 115 Geographical Miles, it will be as the Sine of $61^\circ\ 35'$, to the Sine of $63^\circ\ 30'$. So the Semidiameter of the Earth being $3437\frac{1}{4}$ ſuch Miles, to 3498 Miles, the Diſtance of the Meteor from the Center of the Earth; from which deducting the Semidiameter, there remain $60\frac{1}{4}$ Geographical Miles for the Height of the Meteor above *Tiverton*: This is confirmed by the Obſervation of the Reverend Mr. *Derham*, who at *Windſor* ſaw the aforeſaid *Nubecula* about two Degrees above the moſt Southerly of the Seven Stars in the Shield of *Orion*; that is (the Time being $8^h\ 6'$) in the Altitude of $23\frac{1}{2}$ *gr.* Whence the Diſtance between *Tiverton* and *Windſor*, being 150 meaſur'd Miles, or 130 Geographical, by a like Proportion, we ſhall find the ſame Height of the Meteor 60 ſuch Miles, wanting only one Quarter. So that in a Round Number we ma conclude it to have been juſt 60 Geographical, or 69 Statute Miles, above the Earth's Surface. Nor is it poſſible to come at a preciſe Determination of this Matter, by reaſon of the Inaccuracy of our *Data*, which were only the Notes of Perſons under the Surprize of the Suddenneſs of the Light, and no ways pretending to Exactneſs: However, ſuch as they are, they abundantly evince the Height thereof to have exceeded 60 *English* Miles, not to say 38 or 40, as some would have it.

I was unwilling to leave off, till I had pitch'd upon some Hypothesis that might subject the Motion of this Meteor to a *Calculus*, that the Curious might be able to compute the visible Way thereof, either in respect of the Horizon, or among the Fix'd Stars. This I found might be perform'd with tolerable Exactness, supposing that it mov'd in the Arch of a Circle concentric with the Earth, but 60 Geographical Miles without it; and that the Point of the first Explosion was over the Latitude of 50° 40′, and 3° 40′ to the West of *London*; and that of the last Extinction over Lat. 47° 40′, with 4° 50′ West Longitude: The Time being fix'd to 8 Minutes past Eight at *London*. Hence it is easy, by a Trigonometrical Process, to obtain the visible Altitude and Azimuth of the Meteor at either of its Explosions, as seen from any Place whose Longitude and Latitude is known; and from the Time given, the Points in the Sphere of Stars answering to those Azimuths and Altitudes are readily deduced. Let those that contend for a much less Height of this Meteor, try if they can, on such their Supposition, reconcile the several *Phænomena* before recited with one another, and with the Observation of the Reverend Mr. *William Ella*, between *Gainsborough* and *Redford*. Here at $8^h\ 5'$, the Meteor was seen to pass precisely in the Middle between *Sirius* and the Fore-foot of *Canis major*, moving obliquely to the Southward, in a Line whose Direction seem'd to be from the Middle between the two Shoulders of *Orion*. The Latitude of the Place being nearly 53° 20′, and Longitude West from *London* 0° 45′. Let them try how they can account for its being seen five Degrees high at *Aberdeen* in *Scotland*, and near as much at *Peterhead*, half a Degree more northerly: And then let them judge whether it did not exceed the reputed Limits of our Atmosphere. Lastly, if the apparent Altitude of the Meteor at *Paris* was not $5\frac{1}{2}$, but 11 *gr.* on the W. *by* N. Point, when it must have been in its greatest Lustre, there will be no Pretence to bring it lower than I have made it, especially if it be allow'd to have follow'd the Track I have assign'd it, over *Prestain*, *Cardiff*, *Minehead*, *Tiverton*, and *Brest* in *Britany*.

Allowing this to have been the Path it mov'd in, the real Magnitude and Velocity of this Meteor might be assign'd, if the several Accounts of its apparent Diameter, and of the Time of its Passage from one of its Explosions to the other, were consistent. But some of them making its visible Appearance nearly equal to the Sun's, which, in the Opinion of many, it far exceeded, we may suppose with the least, that, at the Time when it first broke out over *Tiverton*, its Diameter was half a Degree. And its horizontal Distance being 150 Geographical Miles from *London*, and its Altitude tupe 60, the Hypothenusal or real Distance from the Eye, will be more than 160 such Miles; to which Radius the Subtense of half a Degree will be above an *English* Mile and half, being about 2800 Yards *quamproxime*. After the same Manner it is difficult to assign its Velocity, whilst some make it half, others less than a Quarter, of a Minute, in passing from its first Explosion to its last Extinction: But the Distance it mov'd in that Time being about 3 *gr.* or 180 Geographical Miles, we may modestly compute it to have run above 300 such Miles in a Minute; which is a Swiftness wholly incredible, and such, that if a heavy Body were projected horizontally with the same, it would not descend by its Gravity to the Earth, but would rather fly off, and move round its Center in a perpetual Orb, resembling that of the *Moon*.

Of several Accidents that were reported to have attended its Passage, many were the Effect of Fancy; such as the hearing it hiss as it went along, as if it had been very near at Hand: Others imagin'd they felt the Warmth of its Beams; and some there were that thought, at least wrote, that they were scalded by it. But what is certain, is, the wonderful Noise that follow'd its Explosion. All Accounts from *Devon* and *Cornwal*, and the neighbouring Counties, are unanimous, that there was heard there, as it were, the Report of a very great Cannon, or rather of a Broadside, at some Distance, which was soon follow'd by a rattling Noise, as if many small Arms had been promiscuously discharg'd. What was peculiar to this Sound, was, that it was attended with an uncommon Tremor of the Air, and every where in those Counties, very sensibly shook the Glass-windows and Doors in the Houses, and according to some, even the Houses themselves, beyond the usual Effect of Cannon, though near; and Mr. *Cruwys* at *Tiverton*, lost a Looking-Glass, that being loose in its Frame, fell out on the Shock, and was broken. Nor do we yet known the Extent of this prodigious Sound, which was heard, against the then Easterly Wind, in the Neighbourhood of *London*, as I am inform'd; and by the learned Dr. *Tabor*, who distinctly heard it beyond *Lewes* in *Sussex*; but whether the Report heard near *Lewes* were of that Explosion right over *Devonshire*, or rather of that latter, and much greater at the Extinction over *Britany*, I shall not undertake to determine, till we have some further Accounts from *France*, whence, hitherto, we have only had, that at *Paris*, the Time of the Appearance was at 17 Minutes past Eight.

It remains to attempt something towards a Solution of the uncommon *Phænomena* of this Meteor; and by comparing them with Things more familiar to us, to shew at least how they might possibly be effected. And first, the unusual and continued Heats of the last Summer in these Parts of the World, may be suppos'd to have excited

excited an extraordinary Quantity of Vapour of all Sorts; of which the aqueous, and moſt others, ſoon condens'd by Cold, and wanting a certain Degree of ſpecific Gravity in the Air to buoy them up, aſcend but to a ſmall Height, and are quickly return'd in Rain, Dews, *&c.* whereas the inflammable ſulphureous Vapours, by an innate Levity, have a ſort of *Vis centrifuga*, and not only have no need of the Air to ſupport them, but being agitated by Heat, will aſcend in *Vacuo Boileano*, and ſublime to the Top of the Receiver, when moſt other Fumes fall inſtantly down, and lie like Water at the Bottom; the Experiment whereof was firſt ſhewn me by the Reverend Mr. *Whiteſide*. By this we may comprehend how the Matter of the Meteor might have been raiſed from a large Tract of the Earth's Surface, and aſcend far above the reputed Limits of the Atmoſphere; where, being diſingag'd from all other Particles, by that Principle of Nature that congregates *Homogenea* viſible in ſo many Inſtances, its Atoms might in Length of Time coaleſce and run together, as we ſee Salts ſhoot in Water; and gradually contracting themſelves into a narrower Compaſs, might lie like a Train of *Gunpowder* in the *Æther*, till catching Fire by ſome internal Ferment, as we find the Damps in Mines frequently do, the Flame would be communicated to its continued Parts, and ſo run on like a Train fir'd.

This may explain how it came to move with ſo unconceivable a Velocity; for if a continued Train of Powder were no bigger than a Barrel, it is not eaſy to ſay how very faſt the Fire would fly along it; much leſs can we imagine the Rapidity of the Accenſion of theſe more inflammable Vapours, lying in a Train of ſo vaſt a Thickneſs. If this were the Caſe, it was not a Globe of Fire that ran along, but a ſucceſſive kindling of new Matter: And as ſome Parts of the Earth might emit theſe Vapours in greater Plenty than others, this Train might in ſome Parts thereof be much denſer and bigger than in others, which might occaſion ſeveral ſmaller Exploſions, as the Fire ran along it, beſides the great ones, which were like the blowing up of Magazines. Thus we may account for the rattling Noiſe like ſmall Arms, heard after the great Bounce on the Exploſion over *Tiverton*: The Continuance of which for ſome Time, argues, that the Sound thereof came from Diſtances that increaſed.

What may be ſaid to the Propagation of the Sound through a *Medium*, according to the receiv'd Theory of the Air above 300000 Times rarer than what we breathe, and next to a *Vacuum*, I confeſs I know not. Hitherto we have concluded the Air to be the Vehicle of Sound: And in our artificial *Vacuum*, we find it greatly diminiſh'd: But we have this only Inſtance of the Effect of an Exploſion

plosion of a Mile or two Diameter, the Immensity of which may perhaps compensate the extreme Fineness of the *Medium*.

XXIX. 1.] On *November* 10, 1719, in the Morning, about five of the Clock (as I was observing *Jupiter*) I found certain white Streaks in the Sky, seeming nearly perpendicular; which, whilst I consider'd them, seem'd instantly to vanish, and soon after others came as instantaneously in their Room: Looking up towards the *Zenith*, I perceiv'd an entire *Canopy* of such kind of white *Striæ*, seeming to descend from a white Circle of faint Clouds, about 7 or 8 Degrees in Diameter, which Circle sometimes would vanish on a sudden, and as suddenly be renew'd. I observ'd that the Center of this Place of Concourse was not exactly in the *Zenith*, but rather 14 Degrees to the Southwards thereof; which I estimated by a Star, which on each Return thereof shew'd its self about the Center of the Circle. This Star is the 33*d* Star of the *Great Bear* in *Tycho*'s Catalogue, whose Distance from the Pole at this Time is $52\frac{1}{2}$ Degr. and which about half an Hour past Five that Morning pass'd the Meridian; so that those Rays center'd very nearly on the Meridian itself. It was a very entertaining Sight, till the Day-break began to obscure these Lights, which were but faint, though sufficiently distinguishable. They came none of them lower than to about 30 or 40 Degr. of Altitude, and seem'd not to have ascended from the Horizon. The Sky was perfectly serene and calm, which seems to be one of the concomitant Circumstances attending the *Aurora Borealis*, of which this was certainly a Species. For the Night following, a Neighbour gave me notice of a strange streaming of Lights seen in the Air, which thereupon I attended from the Hours of $9\frac{1}{2}$ to 11, when a Fog came so thick as to put an End to my Prospect. But during that whole Time there ascended out of the E. N. E. and N. E. a continu'd Succession of whitish *Striæ*, arising from below; and after changing, as 'twere, into a sort of luminous Smoak, pass'd over Head with an incredible Swiftness, not inferior to that of Lightning; and as it pass'd, in some Part of its Passage, seem'd, as 'twere, gilded, or rather, as if the Smoak had been strongly illuminated by a Blaze of Fire below. Some of the *Striæ* would begin high in the Air, and a whole Set of them subordinate to one another, like Organ Pipes, would present themselves with more Rapidity than if a Curtain had been drawn from before them; some of which would die away where they first appear'd, and others change into a luminous Smoak, and pass on to the Westwards with an immense Swiftness. And I am of Opinion, that had it not been for the Moon, then ten Days old and very bright, this for the Time would have been reckon'd as considerable an Appearance as that of the *6th* of *March*, 1716.

An Account of an Extraordinary Aurora Borealis *at* London, *Nov.* 10. 1719. *by* *Dr.* E. Halley. n. 363. p. 1099.

2.] On

— the ſame in Devonſhire, *by Mr.* W. Maunder, *ib.* p. 1101.

2.] On the 26*th* of *October*, between Seven and Eight in the Evening, I ſaw ſome ſmall Appearance of an *Aurora Borealis*, viz. three or four large Coruſcations in form of Pyramids, of reddiſh Colour inclining to yellow, which roſe about 50 Degrees above the Horizon, and continued but few Minutes. But the North Part of the Hemiſphere was very bright and red all the Evening both before and after, till ten, if not longer.

November 10. Theſe Lights were ſeen again about four in the Morning, of which ſome ſay, that the Element open'd ſometime at one Place, then at another; from whence came great ſhining Lights that continu'd a while, and then went away by Degrees, and the Holes cloſed up again. This continu'd till Day-break.

The Evening following, coming from *Tiverton* about half an Hour after Eight, I ſaw the North Part of the Horizon very light and reddiſh (notwithſtanding the Moon being about ten Days old, was then in or paſt the Meridian, and ſhone very bright.) In a ſhort Time the ſtreaming luminous Rays began to appear very plain; ſome in one Shape, ſome another; many of them like Cones or Pyramids, but moſt of them badly terminated; ſome of which mounted very high, almoſt to the Zenith, to which Place, or near, they all or moſt ſeem'd to point. Shortly after there appear'd a long Streak of about 30 Degrees parallel to the Horizon and about 15 or 20 diſtant from it, and about two or three broad, but badly terminated, and of a fiery red Colour: Which ſent out ſome of the ſame ſtreaming Beams towards the Zenith. About ſix or ſeven Minutes after, there appear'd (ſomewhat ſudden) a circular Figure like an *Iris*, but twice as broad, of a pale Colour. The Eaſt Part was terminated by the Horizon at full Eaſt, if not ſomething to the South, and the Weſt End about North Weſt; the upper Part of its Arch being 50 or 60 Degrees high, great Numbers of luminous Rays darted from it upwards and downwards, (or elſe paſſing croſs it from the Horizon) at oblique Angles pointing to the Zenith, eſpecially from the North Eaſt Part. This continu'd, as near as I can gueſs, about eight or nine Minutes, when it divided and diſappear'd. After an Interval of three or four Minutes, another *Iris*-like Figure appear'd, (of a Colour, as it ſeem'd, paler than any of the ſtreaming Lights had been) whoſe Diameter was leſs than that of the former, and ſhew'd more than its Semicircle above the Horizon, the upper Part of its Arch approaching near the Zenith. I could not obſerve any Rays to paſs from, (or a-croſs) this as from the other. The Center of this laſt was much more to the Weſt than that of the firſt. After the Continuance of a Minute or two, it began to break in the upper Part of its Arch, and ſhining Particles being ſent out from both its broken Ends towards the Zenith, (to which they were near before) or rather a little beyond it

to

to the South or South-Weſt, they there formed a ſort of *Corona*, curving and bending ſomewhat like Flames reverberated on the Arch of an Oven: Tho' this expreſſeth it but badly, yet I know not how to deſcribe it better. It ſeem'd to me and others to be finely tinged with various Colours, Red, Yellow and Blueiſh, *&c.* and ſent out every way from it (except South and South-Weſt) long Flame-colour'd Rays. After this had continu'd about two Minutes, its ſhining Light abated, and it left behind it for ſome Minutes, ſomething like a whitiſh Cloud (like in Colour to what the Light on the 19th of *March* laſt left behind it, after the fiery Particles were extinguiſh'd, but thinner).

All this while the Moon ſhone very bright, from which this *Corona* was not very far diſtant, perhaps not twenty Degrees, to the North-Eaſt. After this there continu'd to be ſent up many fiery-colour'd or yellowiſh ſtreaming Lights, ſometimes more, ſometimes leſs; now here, now there, all along the North Part of the Hemiſphere, but moſtly from the North-North-Eaſt. All this while ſomething like ſmall whitiſh Clouds (which, to me, ſeem'd to move towards the Zenith, or to point a little more Southward, but diſappear'd as they approach'd the Moon) were carry'd very ſwiftly, and at very ſhort Intervals, moſtly coming from the Eaſt and North-Eaſt, but many alſo from North and North-Weſt. We took but little Notice of this at firſt, ſuppoſing it had been nothing but the Reflection of the other Lights, or the Shadows of the Clouds (whereof the North Parts were pretty full) as the Streams of Light paſs'd behind them: But at laſt, we obſerv'd, that when the Lights at any Time abated, theſe kinds of Clouds continu'd to fly as ſwift and frequent as ever. This I ſaw till Twelve or One next Morning: Many others ſaw it next Morning till almoſt Break of Day, when it appear'd much more red and fiery than it was in the Evening; the Moon perhaps being then ſet. Some People obſerv'd tall Cones to ariſe in the Eaſt, and to be carry'd to the Weſt pretty ſwiftly in an erect Poſition, but I ſaw them not. It has been repreſented here in all ſorts of Appearances, Armies, Battles, *&c.*

3.] On the 10*th* of *November*, the Afternoon having been very calm and ſerene, about Six in the Evening the Sky was tinged with a ſtrange kind of Light, and ſome Streams began to project from the North and N. E. One of them aroſe about N. by E. and was nearly a Subtenſe of an Arc between that and S. W. by Weſt; it was a little curvated toward the Sun, and what I ſaw of it (for the North Part of the Horizon was conceal'd by Houſes) very much reſembled the Tail of a Comet: About the ſame Time there was one or two which aroſe in the Eaſt, aſcending obliquely ſo as to leave the Zenith ſeveral Degrees to the Northward. *—the ſame at* Dublin, *by*— ibid. p. 1104.

Theſe *Striæ* continu'd to appear and diſappear alternately, till toward Eight in the Evening; they were *Pyramidal*, and their *Vertices* frequently projected ſeveral Degrees to the South of our Zenith.

Between Nine and Ten, I was agreeably ſurpriz'd with a kind of Coruſcation, or Flaſhing, that ſhew'd itſelf between twenty and ſixty Degrees from the Zenith, in the South or South by Weſt; and which from four or five, ſometimes from more Places at once, darting with a Velocity not much inferior to that of Lightning; and by interfering with each other produced a beautiful Tremor or Undulation in that ſubtile Vapour, which I cannot better illuſtrate, than by comparing it to the Beams of the Sun, reflected on a Cieling from the Surfaces of two or three Baſons of Water: Theſe *Waves* of Light were only viſible at the Inſtant of Coruſcation, and were of a pale whitiſh Colour, ſomewhat reſembling the Flaſhes produced by the violent Agitation of Quickſilver in an exhauſted Receiver; but ſo ſtrong, that a Gentleman, who was in a Room by himſelf without a Candle, aſſur'd me, he took it for common Lightning: Thus it continu'd inceſſantly for more than an Hour, during which Time ſeveral lucid *Areas*, like little Clouds, diſcover'd themſelves in the pure Sky, and after they had continu'd about five or ſix Minutes, as near as I could gueſs, would inſtantaneouſly diſappear; moſt of them pretty much reſembled a very thin white Smoak or Vapour illuminated by the Full Moon.

About three quarters paſt Ten, this Vapour was almoſt ſpent, or by a briſk Gale at South by Weſt diſpers'd and driven to the Northward; at which Time, between the Weſt and North, a vaſt Body of it, like a very bright Flame-colour'd *Crepuſculum*, ſeem'd to be fix'd: From this Baſis ſeveral Beams or *Striæ* of ſhining Matter were at uncertain Intervals, emitted; and though it was not ſo ſenſible to the Eaſtward of the North, yet ſeveral mighty Pillars were alſo ejected from thence; one, which, if I miſtake not, aroſe directly under the Pole, was above all others that had preceded it, both as to its Magnitude and Denſity ſo ſurprizing, that I am perſuaded the ſmalleſt Print might have been read by the Light thereof, had not that of the Moon, which ſhone very bright, pretty much effac'd it: 'Twas tinged with a kind of Yellow and Violet Colour. In about two or three Minutes it died away, and was ſucceeded by others of an inferior Order: It was now about a quarter paſt eleven of the Clock, and nothing but repeated *Phaſes* of the ſame Spectacle offering themſelves to view; the vibrating Motion had ceas'd; the Vapour ſhew'd itſelf no longer in lucid *Areas*, the Streams of Light were not ſo frequent, and thoſe more languid than before; and the bright *Aurora* having ſettled nearer the Horizon, I concluded the Scene was at an End, and accordingly gave over the queſt of new *Phænomena*, with

with only obſerving, that about N. E. there appear'd ſome Clouds that reflected an unuſual kind of reddiſh Light. Others, who ſat up longer than I did, repreſent the End with very ſurprizing Circumſtances.

On *Tueſday* the 24*th* of *November*, we had the ſame *Phænomena* repeated, though not with the ſame Variety: About a Quarter paſt Ten at Night, a vaſt Body of ſhining Matter was collected between N. W. *by* W. and N. *by* E. in the Form of the Segment of a Circle, whoſe Center was about 25 or 30 Degrees below the Horizon; from its *Periphery* a few ſhort *Pyramidal* Streams, of the ſame luminous Vapour, aſcended by a ſlow and nearly uniform Motion, and were exceeding rare, ſo as not to efface the ſmalleſt of the Fix'd Stars; and in a Minute or two vaniſh'd: The Light which that Collection of Vapour emitted, was ſo great, that in the otherwiſe very dark Night, I could thereby (at three Quarters paſt Ten) read the Title of the laſt *Philoſ. Tranſact.* which then happen'd to lie on my Deſk; and at four or five Yards diſtance, ſee the ſmalleſt Books in my Study.

— a return of the ſame.

XXX. At *Streatham* in *Surrey*, on *December* the 11*th*, about one a Clock at Night (or rather in the Morning of *Dec.* the 12*th*) I was call'd to obſerve Coruſcations which appear'd of a much different Colour, and in a very different Manner, from any I had before ſeen.

An Aurora Borealis *in* Surrey, *by Mr.* T. Hearne, n. 363. p. 1107.

The Streams of Light that darted upwards from the Horizon, ſeem'd to be at conſiderably a greater Diſtance, but not at all in leſs Quantity than thoſe of *Nov.* 10. But their meeting in a Point near the Zenith, and there forming a kind of Canopy, was what was particularly remarkable in theſe Coruſcations.

The Streams of Light roſe from the Horizon only towards the North, and on each Hand towards North-Eaſt and North-Weſt: But near the Zenith a Canopy was form'd of Streams of Light meeting in a Point, not only from thoſe Quarters, but alſo from the South, &c. Only to thoſe Points they extended downwards from the Zenith but a little Way, and were neither in ſo great Quantity, nor quite ſo bright as thoſe Northwards. At firſt I thought the Point in which the Streams met, was exactly the Zenith, but upon obſerving it ſomething longer, I found it was not ſo, but a few Degrees to the South of the Zenith. The Streams of Light near the Zenith, which form'd this Canopy, were of a pretty bright Colour, and in great Quantity, and darted very ſwiftly.

On each Side of the North, towards E. and W. but not exactly in the N. it ſelf (at leaſt when I ſaw it) from about 10 or 15 *gr.* to 40 or 50 *gr.* above the Horizon, the Streams were of a glowing red Colour, whereas all that I had ever ſeen before, were very pale. The Redneſs was like that of a burnt Brick, and neareſt of any Thing I have

have ſeen to the Colour, which remain'd for a few Minutes, like that Tract through which the Meteor paſs'd in the Spring.

The Streams appear'd of this fierce Colour when I firſt ſaw the Coruſcations, and continu'd ſo for ſome Time, till the Redneſs by degrees wearing off, in about $\frac{3}{4}$ of an Hour, they appeared of the uſual Paleneſs, when I left them ſtill forming a Canopy near the *Zenith*.

The Air was very calm and ſerene, not a Breath of Wind ſtirring; as I remember it was alſo, *Nov.* 10.

The Moon was now a Day or two older than it was on *Nov.* 10. and a good deal farther to the Weſt, than when I ſaw the Coruſcations that Night being full South. She had now round her what is commonly call'd a Burr, larger than ordinary, and ſeveral very lucid Clouds at a little Diſtance.

Northern Aurora's *ſeen abroad*, &c. *by* Dr. T. Robinſon, n. 349. p. 483.

XXXI. I am of Dr. *Halley*'s Opinion, that thoſe Phoſphorous or Luminous Appearances in the Firmament, proceed from the various *Effluvia* perſpir'd out of our Globe, or paſſing through it; for I have ſeen thoſe Lights over *Veſuvius*, the *Strombulo* Iſlands, and towards *Ætna*, in dark Nights, when thoſe *Vulcano's* were not flaming nor burning, their Sides and Tops being paſſable to Travellers at that Time, and all their outward Parts quiet. We are certain that *Iceland* and *Greenland* abound with *Vulcano's*; ſo may North-Eaſt *Lapland*, North *Ruſſia*, and *Tartary*, where vaſt Chains of Mountains are ſaid to run. The *Jeſuits*, and other Travellers, relate many prodigious Eruptions of Fires, and Earthquakes, towards the North of *China*; but nearer the Pole, the Earth muſt be clos'd and pent up many Months, by the long ſevere Freezings and continual Snow and Ice, which relaxing towards Spring, may give vent to that vaſt Maſs or Magazine of perſpirable Matter, that had been kept ſo long in hot Subterraneous Priſons. This may be one Reaſon why Animal Bodies themſelves are often ſenſible of Changes at that Seaſon in our Climate, when Perſpiration is upon ſuch an Increaſe.

Experiments on the Motion *of* Pendulums *in* Vacuo, *by* Mr. W. Derham. n. 294. p. 1785.

XXXII. Deſiring to know what Difference there might be between the *Vibration of Pendulums in Vacuo*, and in common *Air*, I recommended the Experiment to Mr. *Hawkſbee*, who having provided himſelf with a proper Receiver, and all other Things neceſſary, with a Friend of mine in *London*, made the Experiment. The Movements he tried with were an Eight-Day Clock vibrating Seconds, and an half Seconds Movement of mine. The Iſſue of their Experiment was, my Pendulum vibrated two Tenths of an Inch on each Side farther in *Vacuo*, than it did in the free Air, and went ſeven Seconds ſlower in twenty Minutes, than the other Movement.

But

But in the open Air, my Pendulum in twenty Minutes, went only $3\frac{1}{2}$ Seconds ſlower than the other Pendulum.

This Experiment I try'd over myſelf; the Inſtruments I made uſe of, were, firſt, an *Air-Pump* of Mr. *Hawkſbee*'s.

The next was a *Small Movement*, with a Pendulum of about ten Inches, that vibrates *Half-ſeconds*, and is driven by the Power of a Spring. This Inſtrument I thought commodious, not only for being eaſily fitted with a Receiver, but alſo for vibrating Half-ſeconds very nicely, and alſo becauſe its Vibrations are equal, not ſome large, ſome ſhorter.

The laſt Inſtrument was a very well regulated *Month-Piece*, that vibrates *Seconds* all the Year, with as much Exactneſs as moſt do.

Being thus furniſh'd, the Reſult of many repeated Experiments, Day after Day, was, That (as before) in *Vacuo* the Vibrations were always larger than in the Receiver unexhauſted. At the firſt, when my little Movement was newly clean'd, the Vibrations were above $\frac{1}{15}$ of an Inch larger than in the free Air. But afterwards (I ſuppoſe, from ſome of the foul'd Oil of the Pump ſpirtled on the Wheels, in letting in the Air, whereby the Force of the Spring on the Pallets was blunted, from hence, I ſay) as the Vibrations in the unexhauſted Receiver were a little contracted, ſo in the Receiver exhauſted, they were more contracted, and only about 0, 25 of an Inch larger than in the free Air.

The *Alteration* in *Time*, which this Difference of the Vibrations produc'd, was conſtantly only about two Seconds in an Hour ſlower, in the Receiver exhauſted, than in it unexhauſted. For if in four, five, or more Hours going, the two Pendulums did not vary a Quarter of a Second in the open Air, or when the Receiver was put over the little Movement, (but unexhauſted); yet when the Receiver was exhauſted, the Half-ſeconds Movement would loſe, at the Rate of two Seconds in every Hour, in every Experiment, in many Hours going.

And becauſe I had a Mind to ſee what Alterations would ariſe from varying the Vibrations, therefore by opening and ſhutting the Pallets, I cauſed the Vibrations in ſome Experiments to be as large as the Receiver would bear; in others, to be as ſhort as poſſible; always adjuſting the Pendulum to vibrate Half-ſeconds nicely in the Air. But ſtill the Succeſs was much the ſame, or the Difference ſcarce perceptible. But only I imagin'd when the Pendulum vibrated but a little Way from the Perpendicular, that the Vibrations in *Vacuo* were not ſo much enlarged, as when it vibrated in a larger Arch.

In all theſe Experiments (which were repeated divers Times with the ſame Succeſs) I had no other Reaſon to move me to think, but that the Vibrations were enlarg'd in *Vacuo* by the vaſt Rarefaction of of the *Medium*, but this, That perhaps the different State of the Air might alter the Force of the Spring, which drove the Movement. For the Trial of this, I put a well-adjusted *Pocket-Watch* (with *Hook*'s Regulator, *i. e.* the common small spiral Spring to the Balance) into the *Vacuum*; and after several Trials, at the same Pitch of the Spring, I found not the least Alteration in the Watch's going, in many Hours; neither the Springs, nor any other Part of the Watch, seeming to be in the least affected by the *Vacuum*: But the Balance circumvolving, or keeping the same Turns, as in the open Air.

I then try'd what the Success would be, by putting the Half-seconds Pendulum again into the Receiver, and only pumping out a Part of the Air. And accordingly I left no more Air in, than what kept the included Mercurial Gage at about six Inches Height; the Event of which was, that the Vibrations were then not above $\frac{1}{10}$ of an Inch larger on each Side, than in the Receiver unexhausted: And the Time lost but about Half a Second in an Hour, or $\frac{3}{4}$ at most. And so, according as the Mercurial Gage was more or less high, I always found the Vibrations greater or less; they gradually decreasing, according to the Quantity of Air re-admitted. From these Experiments we may remark,

1. What Mr. *Boyle* long since observ'd (from a cock'd Pistol going down as fiercely in his *Vacuum*, as in the Air) may be hereby farther confirm'd, *viz.* That the Air is not the Cause of the Motion or Restitution in Solid Bodies, as Springs. For if it was, it would certainly have been discover'd in so tender an Instrument as a well-adjusted Pocket-Watch, lying under the perpetual Influence of two Springs.

2. As in *Vacuo* (where the Pressure of the Atmosphere is taken off) heavy Bodies descend quicker than they do in the open Air; so it may be observ'd, that Pendulums move swifter in the Receiver exhausted, than in it unexhausted.

That heavy Bodies descend quicker in *Vacuo*, is evident, from the swift Descent of the less heavy Bodies, as *Cork*, the *Down of Sowthistles*, the *lightest Feather*, &c. which do all precipitantly descend, like a Stone, in a tall exhausted Receiver.

And that the Pendulum, in our Experiment, mov'd faster in *Vacuo*, is manifest, from its vibrating but two Seconds in an Hour slower, when the Vibrations were $\frac{2}{5}$ of an Inch on a Side, enlarg'd by the highest Rarefaction of the Air. Whereas I find by Experiment, that near the same Increase of the Vibrations, doth, in the open Air, make the Pendulum go six or more Seconds slower in an Hour. I say, *near the same Increase*, because it is scarce possible to manage the Pallets so, as nicely to make the same Vibrations as were in *Vacuo*.

3. The

3. The laſt Thing I ſhall deduce ſhall be by way of Query, *viz.* Whether the Variations of Pendulums obſerv'd under the Equinoctial, and between the Tropics, do not ariſe as much or more from the Rarity of the *Medium*, and the Encreaſe of the Vibrations conſequent thereupon? It is ſcarce, I think, to be doubted, but that the Air is much thinner and finer near the Line, than it is without the Tropics. And it is evident from the Barometer, that on the Tops of high Mountains the Atmoſphere gravitates leſs than nearer the Center. And therefore (although I like the Notion of the Decreaſe of Gravity from the Encreaſe of the Diſtance from the Earth's Center too well to diſcard it, yet) I am apt to think that this is not the only Reaſon of the *Phænomenon*.

I wiſh that Capt. *Halley*, when he obſerv'd at St. *Helena* his Clocks to go ſlower than in *England*, had at the ſame time obſerv'd whether the Vibrations were not enlarg'd. It might be worth the while for ſuch as have Opportunity, to take Notice, whether their Pendulums between the Tropics do not make larger Arches than higher Latitudes? Alſo in what Latitude they begin to alter? Whether the Vibration be greater near the Line, than in any other Parts between the Tropics? Or, whether the greateſt Encreaſe be not always in thoſe Places where the Sun paſſeth their Zenith? If the Vibrations be found larger under the Line, or in any other Part of the Torrid Zone, then it may be obſerv'd, how much larger they are, and in what Proportion they encreaſe, or decreaſe, by approaching nearer unto, or receding from the Place of their greateſt Encreaſe?

Alſo it may be worth obſerving, Whether Pendulums do not vary on the Tops of high Places, or in different States of the Atmoſphere, according as the *Mercury* is high or low in the Barometer? But then in this, and indeed in the former Caſes, it is neceſſary, or at leaſt very expedient, that the Movement be ſo exactly well made, that the Power, whether Weight or Spring, do at all Times exert the very ſame Force upon the Pads or Pallets. For moſt Clocks are apt to vibrate ſometimes larger, ſometimes leſſer Arches in the 24 Hours, according as the Weight or Spring doth more or leſs exert its *Forces* on the Work.

An Experiment of firing Gun-powder *in* Vacuo, *by Mr.* F. Hawksbee. n. 295. p. 1086.

XXXIII. 1.] A Candent Iron being included in a Recipient proper for that Purpoſe, and the Air withdrawn (which was in about two Minutes of Time) the *Mercury* then in the Gage ſtanding at 29 Inches $\frac{1}{4}$, a Quantity of Gun-powder was immediately made to deſcend upon the red hot Iron, which continu'd upon the Surface of it ſome ſmall Time before it went off, and then was obſerv'd not to fire all at once, and the laſt of the Quantity that did ſo ſeem'd to give the greateſt Flaſh; upon which the Mercurial Gage

Gage was taken Notice of to deſcend ſomething more than an Inch, it riſing again $\frac{1}{15}$ of the ſame: And upon ſeveral Repetitions of the like Quantity of Powder (the factitious Air being always withdrawn) the Appearances were very reſembling. Again upon purging the Recipient of the factitious Air, and the *Mercury* elevated in the Gage, as at firſt, three Quantities were cauſed to deſcend upon the Iron, whoſe Exploſion as well as the Air produc'd from them, ſeem'd in Proportion to the Quantity of Powder; the *Mercury* then in the Gage ſubſiding to 26. But upon dropping ſix Quantities (the Recipient being firſt purg'd as before) which Quantities not deſcending all at once, but ſucceſſively as faſt as might be, the Quantities that firſt reach'd the (ſtill Ignited) Iron taking Fire, by their Flame making an Exploſion of the whole, at once blowing up the Recipient, although the Weight of the Air incumbent on it was equal to 144 *l.* $\frac{1}{2}$, accounting the Receiver at 3 Inches $\frac{1}{2}$ Diameter, but was ſomething more, which does ſufficiently allow for the want of Height of *Mercury*. The Gage then ſtanding at 29 $\frac{1}{2}$, inſtead of 30, from which the Calculation is made. The Gun-powder us'd was the common glaz'd ſort; and the Weight of the ſix Quantities, which remov'd the Recipient, with ſo great a Preſſure incumbent on it, was but ſeven Grains, each Quantity weighing ſomething more than one. I did not obſerve the Recipient to be broke before it reach'd the Floor. It was thick lin'd with Sulphureous and Nitrous Steams, ſo that the Flaſhes of Fire through the Cloudineſs of the Glaſs ſeem'd very much to reſemble faint Lightnings. The Content of the Receiver was equal to about 25 Ounces $\frac{1}{2}$ of Water, allowing for the Bulk of Iron and Pedeſtal.

The Quality *of the* Air *pro-duc'd by* Gun-powder, *by the* ſame, *ibid.* p. 1807.

2.] Upon making the late Experiment before the *Society*, of firing Gun-powder *in Vacuo*, it was hinted as well worthy of Trial, Whether the factitious Air of fir'd Gun-powder was endu'd with any Quality differing from common Air? In order to the Satisfaction of the Query, I included a Candent Iron *in Vacuo*, the *Mercury* then in the Gage ſtanding at 29 Inches $\frac{1}{2}$: Upon dropping the firſt Quantity of Powder, (by a Quantity is to be underſtood ſomething more than a Grain weight) its Exploſion made a Deſcent of the *Mercury* in the Gage about an Inch, undulating very little. The ſecond Quantity being let fall, the *Mercury* ſubſided about $\frac{3}{4}$ of an Inch; and ſo for ſeveral Quantities following it deſcended by pretty equal Stages, till it had fallen about ſix or ſeven Inches; and it was obſerv'd, upon every Quantity fir'd, the Undulations of the *Mercury* increas'd. But after it had ſubſided ſix or ſeven Inches from 29 $\frac{1}{2}$, the ſeveral Deſcents of it became leſs, very little or nothing exceeding $\frac{1}{2}$ an Inch, although the Quantities firſt were equal; but

3 ſtill

ſtill the Undulations encreas'd, and the Exploſions manifeſtly did ſo too: 'Till at laſt the Receiver ſeem'd to be in great Danger of being blown up by a ſingle Quantity, the Undulations of the *Mercury* being then augmented to ſix or ſeven Inches. Now 26 Quantities or 32 Grains having been fir'd upon the Iron, and the *Mercury* in the Gage having fallen to 12$\frac{1}{4}$, I diligently attended to obſerve the Gage, which in ſeven Minutes had aſcended 2 Inches $\frac{1}{4}$, the next five Minutes it aroſe but 1 Inch $\frac{1}{4}$, and ſo leſs ſucceſſively every five Minutes, that in an Hour and 17 Minutes, it had attain'd but to 21 Inches, the Iron not being quite cold. At Nine the ſame Night I obſerv'd the Gage, and found the *Mercury* elevated to 22 Inches $\frac{3}{4}$ preciſely: Next Morning at Nine it had attained to 22$\frac{1}{2}$, and ſo continu'd all that Day, the Iron being then reduc'd to the Temperature of the outward Air. So that from 12$\frac{1}{4}$ to 22$\frac{1}{2}$, ſeems to be the Weight or Spring of Heat equal to about $\frac{1}{3}$ of an Atmoſphere of Air, which would preſs the *Mercury* upon the upper Part of the Gage, but equal to ſuch a Degree of Heat as was then contain'd in the Receiver, when the Gage was fallen to 12$\frac{1}{4}$: The remaining Space from 22$\frac{1}{2}$ to 29$\frac{1}{2}$ is ſuppos'd to be ſupply'd with factitious Air, and anſwers to about $\frac{1}{4}$ Part of the Recipient's whole Content, which was equal to 25 Ounces $\frac{1}{2}$ of common Water, allowing for the Iron and Pedeſtal. This Air produc'd from Gun-powder, I find to be actuated by Heat and Cold as common Air: For, holding my warm Hands upon the Receiver, the *Mercury* in the Gage would immediately deſcend, and riſe again when reduc'd to the Temperature of the outward Air. This I repeated ſeveral Times with the like Succeſs. What more occurs in this Experiment is, Why the Exploſions of the like Quantities of Gun-powder ſhould be greater when reſiſted by Air, than in *Vacuo*, where nothing ſeems to hinder the Extenſion of their Flame.

Deſcent of Malt Duſt *in* Vacuo, *by Mr.* F. Hawkſbee. n. 298. p. 1948.

XXXIV. I took ſome Malt Duſt, and having well dry'd the ſame, put a Quantity of it into a fine Muſlin Bag, where being looſely incloſ'd, it would, upon ſhaking, diſcover itſelf plentifully in the open Air, undulating and floating a conſiderable Time before it would deſcend; but being included within a Receiver, from which the Air was well exhauſted, and then ſhaken, the Duſt deſcended as a ponderous Body, precipitating in ſtraight Lines from the Top to the Bottom of a tall Receiver.

An Experiment *to prove an* Interſpers'd Vacuum, *by Dr.* J. T. Deſaguliers. n. 354. p. 717.

XXXV. Having had the Honour to make ſome Experiments laſt Year before *His Majeſty* and their *Royal Highneſſes the Prince and Princeſs of* Wales; among others, I ſhew'd that of a Guinea and a Piece of fine Paper; then of a Guinea and a Feather dropp'd together from the Top of an exhauſted Glaſs Receiver about 20 Inches

Inches high; both which fell to the Bottom at the fame Inftant of Time: Now fince the chief Refiftance of a *Medium* (and indeed almoft all of it) depends upon the (†) Quantity of its Matter; therefore this Diminution of Refiftance, whereby the Feather fell as foon as the Guinea, fhew'd a Diminution of the Quantity of Matter, and confequently prov'd an *interfpers'd Vacuum.* Some Time after this, I was inform'd, that fome *Plenifts* objected againft the Shortnefs of the Glafs-Receiver; as if the Difference of Time in the Fall of the two Bodies, which they affirm'd to be real, could not be perceiv'd in fuch a Glafs; and that fome Philofophers from abroad affirm'd, that in a Glafs-Receiver feven or eight Feet long, there would be fuch a manifeft Difference in the Time of the faid Bodies, as to fhew this Experiment no Proof of a *Vacuum:* To obviate this, I contrived a Machine for the Purpofe, which confifted of a ftrong Wooden Frame 15 Feet high, that held the Air-Pump and four Cylindric Glafs-Receivers of about two Feet long each, and fix Inches Diameter: Of thefe, having fet the firft upon the Air-Pump-Plate, I laid on the Top of it a Brafs-Plate of feven Inches Diameter, that had an oil'd Leather fix'd to it above and below, with an Hole through the Middle, of between four and five Inches Diameter; then on that Plate I fet the next Receiver, with a like Plate at Top; and after the fame Manner fix'd the other two with Plates between them: The upper Receiver being a little narrower at the Neck, went into the Hole of a Board, whereby it was fcrew'd down pretty hard on the other Glaffes, and fix'd to the whole Machine. On the Top of this upper Receiver, I had the Brafs Plate, wet Leather, and Brafs Springs which contain'd the Bodies to be dropp'd.

(†) *See Sir* If. Newton's *Principia*, Book II. Prop. 40.

Having acquainted his Majefty with what I had prepar'd, he order'd me to fhew him the Experiment with this long compounded Receiver, at *Hampton-Court*; and when I made it before him and her Royal Highnefs, he was pleas'd (by pulling down a String fix'd to a Leaver at the Top of the Machine) to let loofe the Bodies himfelf.

When the Receiver was full of common Air before Pumping, the Guinea came to the Bottom, juft as the Paper was about the Middle of the fecond Glafs; but when the Receiver was exhaufted, the Guinea and Paper came to the Bottom precifely in the fame Inftant of Time.

Upon my giving an Account of the Succefs of this Experiment to the *Royal Society*, they order'd me to repeat it before them on the *5th* of *December* 1717.

I made the Experiment firft with two of the Receivers; then with all the four; dropping a Guinea and a fmall Piece of Paper together; and the Succefs anfwer'd Expectation: But not being willing

willing to try with a Down Feather, becauſe I fear'd the Air might inſinuate between ſome of the Glaſſes, by reaſon the Number of Perſons preſent ſhak'd the Room, the Society order'd me to make the Experiment at Home, before one or more of their Members.

Martin Folkes, Eſq; was preſent when I made the Experiment at my Houſe, where we made four Trials in the following Manner:

The whole Machine being fix'd, as above mention'd, we firſt let fall a Guinea and two Papers, the one plac'd over, and the other under it, (before any Air was pump'd out) and the Guinea came to the Bottom when the Papers were only in the middle of the ſecond Glaſs from the Top. Then having laid a Feather on the Braſs-Springs cloſe by the Guinea, we let them looſe both together; and the Feather was fallen only down to the fourth Part of the Length of the firſt Glaſs, or one ſixteenth of the whole Diſtance, when the Guinea was got down to the Bottom of the Receiver. We then laid two Papers and two Feathers, one of each under, and the other over the Guinea between the Springs; and having drawn out ſo much of the Air as to bring up the *Mercury* in the Gage-Tube within a Quarter of an Inch of the greateſt Height to which it could be then rais'd by the Preſſure of the external Air, we caus'd the Bodies to fall all at once: And though the Papers came down to the Bottom at the ſame Time as the Guinea, yet the Feathers, being much lighter, wanted about three Inches. But at laſt, having laid the Papers, Feathers, and Guinea, as before, we pump'd out all the Air, and then the Feathers, as well as the Papers, came to the Bottom of the Receiver at the ſame Inſtant of Time as the Guinea.

Experiments *to find how much the* Reſiſtance *of the* Air *retards* Falling Bodies, *by Dr.* J. T. Deſaguliers. n. 362. p. 1071.

XXXVI. 1.] I took twelve Balls, (ſix of which were ſolid Leaden Globes, of about two Inches Diameter; three hollow Glaſs Balls of about five Inches Diameter; and three light Paſteboard hollow Globes of about the ſame Diameter) and having carried them to the upper Gallery in the Lanthorn, on the Dome of St. *Paul*'s Church, I caus'd them to fall down by two at a Time, in the following Manner;

Firſt, a Leaden Ball and a Glaſs Ball.

Secondly, a Leaden Ball and a Glaſs Ball.

Thirdly, a Leaden Ball and a Glaſs Ball.

Then I let fall, in the ſame Manner, the three other Leaden Balls, each with a Paſteboard Ball.

After that, having the Leaden and Paſteboard Balls brought up again, I repeated the Experiment twice more with a Leaden and Paſteboard Ball: Then I made the Experiment twice more with a Paſteboard Ball alone, to ſee how long it would be in falling.

Upon the whole, it appear'd, that the Leaden Balls were a very little longer than $4\frac{1}{2}$ Seconds in falling; the two largest of the Glass Balls 6 Seconds, and the Pasteboard Balls $6\frac{1}{2}$ Seconds.

The Height of the Gallery, from whence the Bodies fell, was 272 Foot above the Pavement of the Church (then cover'd with Boards) upon which they fell.

The Times of the Falls were taken two Ways above, *viz.* with a Wheel-Chronometer, which measures a small Part of Time accurately, nearer than to a Quarter of a Second, (made and contriv'd by Mr. *Graham)* and with an $\frac{1}{2}$ Second Pendulum: And the Differences of Time between the Fall of the Leaden Balls, and the other Balls, were taken below, by Sir *Isaac Newton*, *Martin Folkes*, Esq; and another Person, who all agreed in their Observations of the Time, which they made each with an half Second Pendulum.

The following Table gives the Marks, Weights, and Diameters, of the several Balls, in three Columns.

Leaden Balls.	*Troy Weight.* *l. oz d.*	*Diameters in Inches and Decimals.*
1 *c*	2 : 1 : $0\frac{1}{2}$	2, 1
2 *c*	1 : 11 : 4	1, 99
3 *c*	1 : 11 : 12	2, 0
4 *c*	1 : 11 : 12	2, 0
5 *c*	1 : 11 : 12	2, 0
6 *c*	1 : 10 : 00	1, 98
Pasteboard Balls.		
A	0 : 3 : 6	5, 5
B	0 : 1 : 14	5, 1
C	0 : 1 : 17	5, 1
Glass Balls.		
D	0 : 3 : $13\frac{1}{2}$	3, 9
E	0 : 5 : $3\frac{1}{2}$	5, 42
F	0 : 6 : $0\frac{1}{2}$	5, 55

N. B. The Polar and Equatorial Diameters of the Glass Balls being different, I have set down a mean Diameter for each of 'em; the true Diameters are thus, of D 4 and 3,8. of E 5,6 and 5,25. of F 5,7 and 5,4 Inches.

The particular Experiments are as follows.

Exp. I. Fall of 1 *c* and D. *c* fell by the Pendulum in $4\frac{1}{2}''$. The Fall of D was so near it, that the Difference was not taken either above or below.

Exp. II. Fall of 2 *c* and E.

2 *c* fell by the Chronometer in 5″, by the Pendulum in 4½″. Time of the Fall of E not taken above. The Difference taken below 1¾″.

Exp. III. Fall of 3 *c* and F.

3 *c* fell by Chronometer in 4½″, by the Pendulum in 4½″. F fell in ſix Seconds. The Difference taken below, was 1½″.

Exp. IV. Fall of 4 *c* and A.

4 *c* fell by Chronometer in 4¼, by the Pendulum in 4½. A fell in 6½ Seconds. Difference taken below = 2″.

Exp. V. Fall of 5 *c* and B.

We made no Obſervation above nor below.

Exp. VI. Fall of 6 *c* and C.

6 *c* fell by Chronometer in 4¾″, by the Pendulum in 4¾″. C not taken above. Difference below = 2¼″.

Exp. VII. Fall of 1 *c* and B.

1 *c* fell by Chronometer in 4¾″, by the Pendulum in 4¾′. B not taken above. Difference taken below 2¾″,

Exp. VIII. Fall of 5 *c* and A.

5 *c* fell by the Pendulum in 4¾′. A fell foul, and ſo was not obſerv'd at all. Difference taken below 2″.

Exp. IX. Fall of B alone.

By the Chronometer in 6½″, by the Pendulum in 6½″.

Exp. X. Fall of C alone.

By the Chronometer in 6½, by the Pendulum in 6½″.

By *Galileo*'s Theory, the Lead, which was 4½″ in falling, muſt fall four Foot the firſt ½″; or ſixteen Feet the firſt Second, which amounts to 324 Foot in 4½″. But as the Sound of the Ball (as it ſtruck the Bottom) by which we reckon'd our Time, had 272 Feet to move, we muſt abate a ¼ of a Second nearly, (ſuppoſing Sound to move one Mile in 4½″) which will take away 35 Feet, that the Body muſt have fallen in the laſt ¼ of a Second, and reduce the Number of Feet to 289: So that the Lead will have only fallen 17 Feet ſhort of the Theory, which muſt be attributed to the Reſiſtance of the Air.

The large Glaſs Ball in the 6 Seconds of its Fall, would in a *Vacuum* go through 576 Feet: But taking away the laſt ¼ of a Second 47 Feet, for Motion of Sound, it muſt only fall 529 Feet in *Vacuo*. Now ſince it fell but 272, there have been 257 Feet taken off from the Fall by the Air's Reſiſtance.

Likewiſe the Paſteboard Ball in 6½ Seconds muſt have fallen 676 Feet: But deducting the laſt Quarter of a Second, or 51 Feet, for the Motion of the Sound, there remains only 625 Feet for its Fall in *Vacuo*. But as it fell only 272 Feet, we muſt allow a Retardment of 353 Feet for the Reſiſtance of the Air.

At

At a Mean we may call the Weight of the Glafs Ball five *Oz. Troy*, and its Diameter 5 Inches and $\frac{1}{2}$; and the Weight of the Pafteboard Ball two Ounces *Troy*, and a little more than five Inches Diameter.

The Lead Balls all fell within near a Foot of one another, and made an Impreffion in the Boards of about $\frac{1}{3}$ of their Depth.

The Barometer ftood at 30,1 Inches, and the *Mercury* was very convex, and therefore inclin'd to rife ftill.

Some further Experiments *on the* fame, *by the* fame, *ibid.* p. 1075.

2.] Having found by our former Experiments, that thin Glafs Balls, and even Balls of pafted Paper, were too heavy to make fo confiderable a Difference between the Time of their Fall and the Fall of Leaden Balls, that it might be eafily obferv'd; I contriv'd a Way to make dry'd Hogs Bladders perfectly round, by blowing them (when moift) within a ftrong fpherical Box of *Lignum Vitæ*, and letting them dry in the faid Box before I took them out: Which I did by opening the Box that fcrew'd in the Middle, and had a Hole in the Pole of one of its Hemifpheres to let the Bladder pafs through, in order to tie it after blowing; and fome few fmall Holes all over the Box, that, in blowing, no Air might be confin'd between the infide of the Box and the Bladder, fo as to hinder it from putting on a fpherical Figure. Befides, I took off the Ends of the *Ureters*, the Fat, and a great deal of the upper Coats of the Bladders, before I blow'd them in the Box, to render them ftill lighter.

The Bladders I us'd, were fome of the thinneft I could find ready blown at a *Druggift*'s, which I moiften'd in Water, taking Care to leave none in the infide.

Having prepar'd five Bladders in the Manner aforefaid, I took them up to the upper Gallery in the Lanthorn on the Top of the *Cupola* in St. *Paul*'s Church; and there, by a Contrivance which I fhall defcribe, I let them fall by one at a Time, together with a Leaden Ball of about two Inches Diameter, and weighing 2*l. Troy*: And I took Notice of the Time of the Fall of each Bladder, knowing by former Experiments that the Balls are about $4\frac{1}{4}$ Seconds, or a little longer Time, in falling the fame Height, which is 272 Feet.

The following Table, confifting of five Columns, gives in the firft, the Marks of the Bladders; in the next their Diameters; in the third their Weights, in Grains *Troy*; in the fourth the Times of their Fall in Second Minutes of Time; and in the fifth, the Difference of Time between the Falls of the Leads and of each Bladder; taken below by Sir *If. Newton*, Dr. *Halley*, Dr. *Jurin*, *Martin Folkes*, Efq; and Mr. *Graham* the Clock-maker. The Time was taken above with Mr. *Graham*'s Chronometer; and below with the fame Inftrument,

ment, and three half Second Pendulums, all which agreed very well together.

The Experiments having been made twice over, the Table is twice ſet down; and thoſe Experiments in which the Bladders fell ſtraight down, and the moſt regularly, have this Mark before them (*).

Marks.	*Diameters in Inches.*	*Weight in Grains Troy*	*Time of the whole Fall.*	*Diff. between the Lead and Bladder.*
A	5, 3	128	$19\frac{1}{8}''$	$14\frac{2}{8}$ Seconds.
* B	5, 193	156	$17\frac{1}{4}$	$12\frac{1}{4}$
C	5, 33	$137\frac{1}{2}$	$18\frac{1}{4}$	$14\frac{5}{8}$
D	5, 26	$97\frac{1}{2}$	$22\frac{1}{8}$	$17\frac{6}{8}$
* E	5, 02	$99\frac{1}{8}$	$21\frac{5}{8}$	17
* A			$19''$	$14\frac{1}{2}$
B			$18\frac{5}{8}$	$14\frac{1}{4}$
* C			$18\frac{1}{8}$	14
D			24	$19\frac{1}{8}$
E			$21\frac{1}{4}$	$16\frac{6}{8}$

The Diameters and Weights may be rely'd upon, being taken the Day that the Experiments were made, and the Day after; but the Diameters and Weights taken ten Days before, not agreeing with theſe, I have left them out. For the Bladders by drying had loſt their Weight, and alter'd their Diameters.

As the Necks of the Bladders in drying ſhrink, ſo as to open a little, they muſt be blown before each Experiment. And for the Manner of letting them fall exactly in the ſame Inſtant of Time, it is deſcrib'd by *Figure* 25, in which

A, A, A, A, is the Hole through which the Bodies fell: 1, 2, is a Board laid over the Hole. G, D, D is another Board fix'd to the firſt Board by the two Wood-Screws D, D with a Pulley G at the other End of it, over the Hole. W, is a two Pound Ball of Lead faſten'd to a ſtrong Thread, which going over the Pulley, is ſtretch'd horizontally from G to the Nails F; to which it is faſten'd, ſo as to be about a quarter of an Inch above the Board. Fig. 25.

B is one of the Bladders, hanging with the Neck or heavieſt Part downwards, by Means of a Loop of fine Thread as E H, which goes over the Horizontal Thread G E F. Now when with a Pair of Sciſſars the Thread of the Lead (which in all is but one Foot long) is cut juſt at E, before the Loop of the Bladder, the Lead pulling away the String, the Loop of the Bladder ſlips off the remaining Thread F E, and begins to fall exactly in the ſame Inſtant as the Lead: But if the Thread ſhould be cut between E and

F, as

F, as the Lead falls, its Thread might give the Bladder an oblique Direction.

He that observes the Time either with a Pendulum or Chronometer may take it very exactly, by seeing the Motion of the Sciſsars as they cut the Thread.

N. B. As the Diameters of the Bladders were taken by wrapping a Thread twice round them, and ſomething muſt be allow'd for the Thickneſs of the Thread; I have here under ſet down the Diameters of the Bladders, as corrected by that Allowance. *Viz.* A 5, 28 Inches; B 5, 19; C 5, 30; D $5\frac{1}{4}$; and E juſt 5 Inches in Diameter.

The Bladder E was rough, with ſeveral Wrinkles and Inequalities, which made it be longer in falling than it ought to have been, according to its Diameter and Weight.

A Pail of Water thrown down met with ſuch a Reſiſtance in falling 272 Feet through the Air, that it was all turn'd into Drops like Rain.

XXXVII. *Papers*, &c. *of Mr.* Hawkſbee's *Omitted.*

n. 303. p. 2129. 1. An Account of ſeveral *Experiments* on the *Mercurial Phoſphorus*, made before the *Royal Society*.

n. 304. p. 2165. 2. An Account of ſeveral *Experiments* made before the *Royal Society*, concerning the *Attrition* of *Bodies* in various *Mediums*. —— Of *Amber* on *Woollen* in *Vacuo*. — Of *Flint* on *Steel* in *Vacuo*. — Of *Glaſs*, and various other Bodies in *Vacuo*.— Of *Glaſs* on *Woollen* — of *Glaſs* on *Oyſter-Shells* — of *Oyſter-Shells* on *Woollen* — of *Woollen* on *Woollen* —of *Glaſs* on *Glaſs*—of *Glaſs* on *Glaſs* under *Water*.

n. 307. p. 2277. 3. An *Experiment* concerning the Production of *Light* on a ſlight *Attrition* of the *Hands* on a *Glaſs Globe* exhauſted of its *Air*, &c.

n. 308. p. 2327. 4. An *Experiment* concerning the *Electricity* of *Glaſs*, produc'd by a ſmart *Attrition* of it.

Ibid. p. 2332. 5. A Continuation of the *Experiments* of the *Attrition* on *Glaſs*.

n. 309. p. 2372. 6. Some further *Experiments* relating to the *Electricity* of *Glaſs*, and of the *Effects* of the *Effluvia*, &c.

n. 310. p. 2313. 7. An *Experiment* confirming a former one, concerning the Production of *Light* by the *Effluvia* of one *Glaſs* falling on *another* in Motion.

Ibid. p. 2415. 8. An *Experiment* ſhewing the Difficulty of *Separating* two *Hemiſpheres*, upon the injecting of an Atmoſphere of *Air* on their *Outward Surfaces*, without *exhauſting* the included *Air*.

n. 305. p. 2221. 9. An *Experiment* concerning the *Proportion* of the Weight of *Air*, to the Weight of an equal Bulk of *Water*, without *knowing* the abſolute *Quantity* of either.

10. An

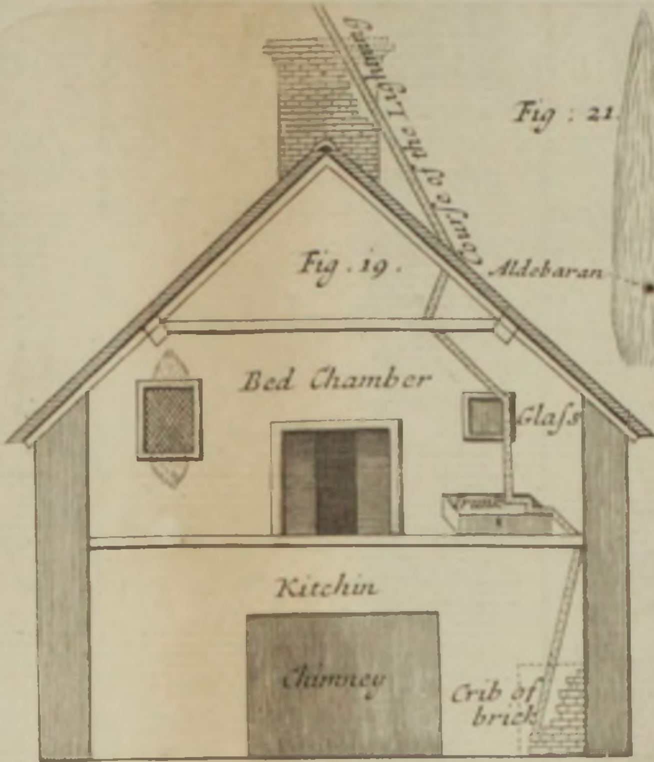


Fig. 19.

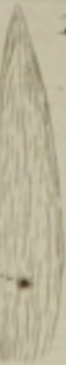


Fig: 21.

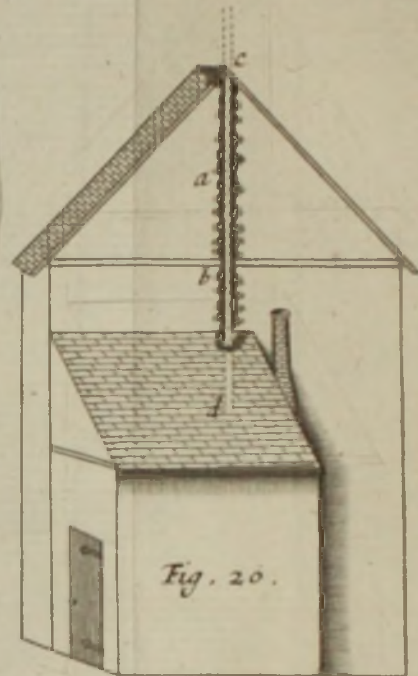


Fig. 20.

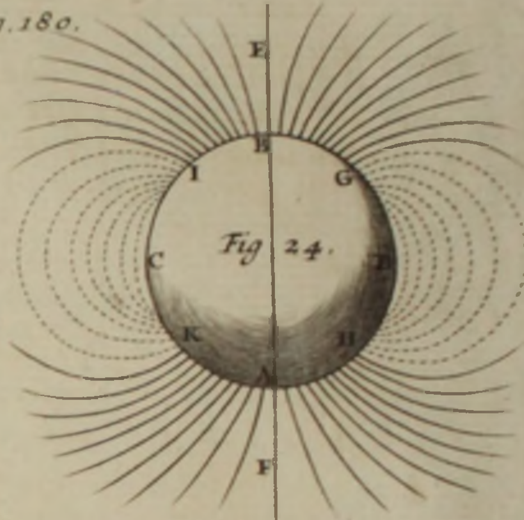


Fig 24.

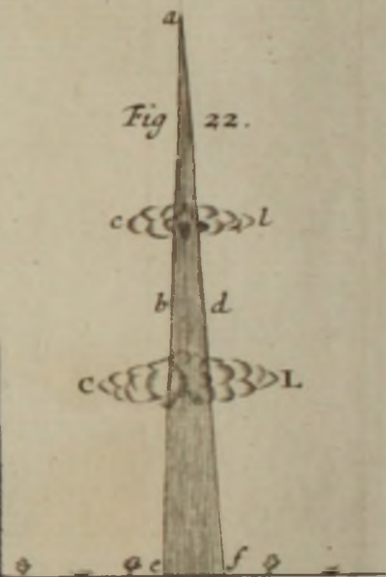
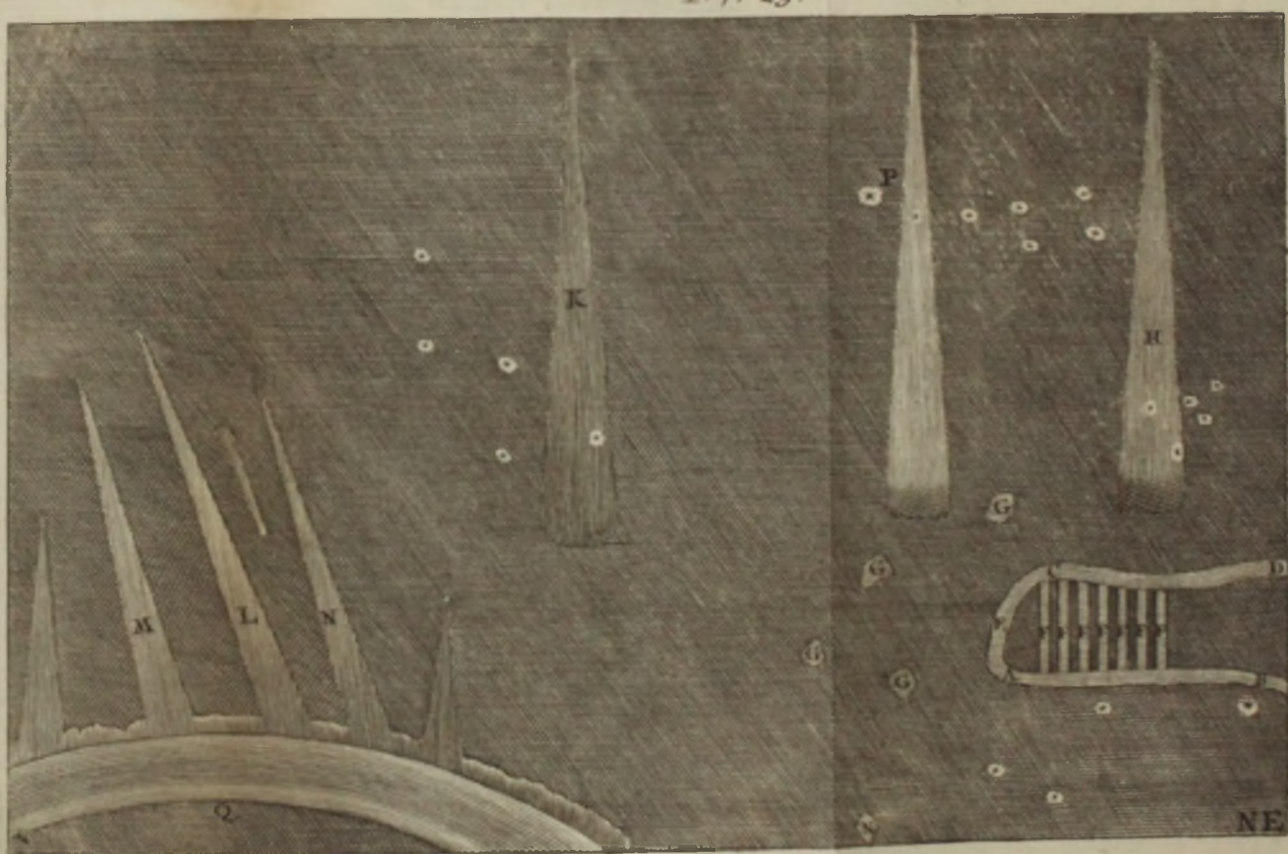


Fig 22.

Fig: 23.



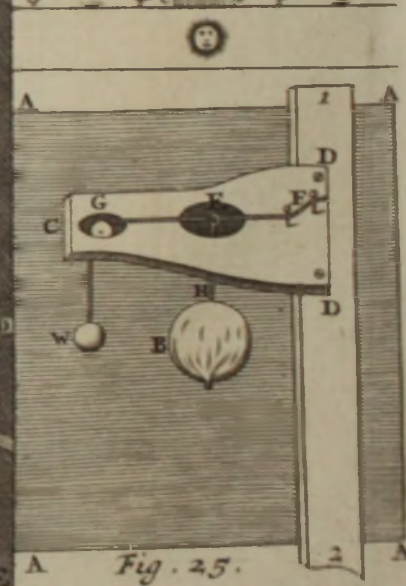


Fig. 25.

10. An *Experiment* shewing, that the *Ascent* of *Liquids* in *Small Tubes* open at both Ends, is the same in *Vacuo* as in the open *Air.* *Ibid.* p. 2223.

11. An *Experiment* concerning the *Quantity* of *Air* produc'd from a certain *Quantity* of *Gun-powder* fir'd in common Air. n. 311. p. 2409.

12. An *Experiment* shewing, that the *Springs*, or constituent Parts of *Air* are capable of suffering such *Disorder* by a violent *Impulse*, as to require Time to recover their Natural State. *Ibid.* p. 241·

13. An *Experiment*, shewing the *Cause* of the *Descent* of the *Mercury* in the *Barometer* in a *Storm.* n. 292. p 1629.

14. *Experiments* on the Production and Propagation of *Light* from the *Phosphorus* in *Vacuo.* n. 296. p. 1865.

15. *Experiments* concerning the Propagation of *Sounds* in *Condensed* and *Rarefied* Air. n. 297. p. 1902.

16. *Experiments* concerning the *Resilition* or Rebounding of *Bodies* in *Various Mediums.* n. 298. p. 1946.

17. Some farther *Experiments* concerning the *Electricity* and *Light* produc'd from various *Bodies* by *Attrition* — of *Glass* — of *Sealing-Wax* — of *Sulphur* and *Rosin.* n. 315. p. 82.

18. An Account of the Success of an *Attempt* to continue *several Atmospheres* of *Air* condens'd in the *Space* of *One*, for a considerable *Time.* n. 318. p. 217.

19. An *Experiment* concerning the Production of *Light* in an exhausted Glass, *lin'd* on the *Inside* with *Sealing-Wax*, upon an *Attrition* made on its *Outside.* n. 318. p. 219.

20. *Experiments* concerning the *Ascent* of *Liquids* between the *nearly contiguous Surfaces* of Bodies — between two *Glass Planes* in the *Open Air* — and in *Vacuo* — between *Marble* and *Brass Planes* in the *Open Air.* n. 319. p. 258.

21. The *Ascent* of *Liquors* between two *Round Glass Planes* in the *Open Air.* *Ibid.* p. 265.

22. The *Ascent* of *Water* through a *Tube* fill'd with *Ashes* in the *open Air* — and in *Vacuo.* *Ibid.* p. 262.

23. The *Ascent* of *Liquors*, in *Small Tubes* of *unequal Thickness*, but *equal Bores* or *Cavities.* *Ibid.* p. 260.

24. The *Ascent* of various *Liquors* between two *Square Glass Planes.* *Ibid.* p. 266.

25. An Account of an *Experiment* concerning the *different Densities* of the *Air*, from the greatest Degree of *Heat* to that of *Cold* in our Climate. n. 315. p. 93.

26. An Account of an *Experiment* concerning the *different Weights* of the *same sorts of Bodies*, but of very *unequal Surfaces* in *Water*, which were of *equal Weight* in *common Air.* n. 320. p. 306.

27. An *Experiment* concerning the *different Densities* of *common Water* from the greatest Degree of *Heat* in our Climate to the *Freezing* Point observ'd by a *Thermometer.* n. 319. p. 267.

28. *Experiments* in relation to the *Weight* of *common Water* under *different* Circumstances. *Ibid.* p. 269.

n. 320. p. 302. 29. An *Experiment* concerning the *Freezing* of *common Water*, and *Water purg'd of Air*.

Ibid. p. 304. 30. An *Experiment* concerning the *Freezing* of common *Water*, ting'd with a Liquid, said to be extracted from *Shell-Lac*.

n. 321. p. 367. 31. An *Experiment*, shewing that *actual Sound* is *not* to be *transmitted* through a *Vacuum*.

Ibid. p. 369. 32. An *Experiment* concerning the *Propagation of Sound* passing from the sonorous Body *into the common Air*, in *one Direction* only.

Ibid. p. 371. 33. An *Experiment* concerning the *Propagation of Sound* through *Water*.

n. 322. p. 391. 34. An *Experiment*, shewing that an *Object* may become *Visible* through such an *Opake Body* as *Pitch* in the *Dark*, while it is under the Circumstances of *Attrition* and a *Vacuum*.

n. 323. p. 439. 35. An *Attempt* to produce *Light* on the Inside of a *Globe Glass lin'd* with melted *Flowers* of *Sulphur*.

Ibid. p. 440. 36. A *Repetition* of the foregoing *Experiments*, &c.

n. 328. p. 196. 37. *Experiments* concerning the *Time* requir'd in the *Descent of Bodies* of *different* Magnitudes and Weights, *&c.*

Ibid. p. 199. 38. *Experiments* concerning the *Effects* of *Air* pass'd *through red hot Metals*, &c.

n. 328. p. 204. 39. A *Description* of the *Apparatus* for making *Experiments* on the *Refractions of Fluids*; With a Table of the *Specific Gravities*, *Angles* of *Observations*, and *Ratio* of *Refractions* of several Fluids.

n. 331. p. 325. 40. A *Repetition* of an *Experiment* of Dr. *Hook*'s, concerning *two Liquors*, which when *mix'd together possess less Space*, than when *Separate*; with another *Experiment* confirming the same.

Ibid. p. 328. 41. An Account of an *Experiment*, concerning an Endeavour to produce *Light* through a *Metallic Body*, under the Circumstances of *Attrition* and a *Vacuum*.

n. 332. p. 395. 42. An Account of an *Experiment* concerning the *Direction* of a *Drop* of the *Oil* of *Oranges* between two *Glass Planes*, towards any side of them that is nearest press'd together.

n. 333. p. 431. 43. *Experiments* on keeping *Fishes* in *Water* under *different Circumstances*.

n. 334. p. 473. 44. Of the *Angle* requir'd to suspend a *Drop* of the *Oil* of *Oranges*, at certain Stations, between two *Glass Planes*, plac'd in the Form of a *Wedge*.

n. 335. p. 511. 45. The *Specific Gravities* of several *Metallic Cubes* in Comparison with their *like Bulks* of *Water*.

n. 336. p. 539. 46. An *Experiment* concerning the *Ascent* of *Water* between two *Glass Planes* in an *Hyperbolic* Figure.

Ibid. p. 541. 47. A *Description* of several *Strata* of *Earth*, *Stone*, *Coal*, &c. found in a *Coal-Pit* at the West End of *Dudley* in *Staffordshire*, by Mr. *Fettiplace Bellers*, F. R. S. To which is added a Table of the *Specific Gravity* of *each Stratum*, by Mr. *F. Hawksbee*.

n. 335. p. 505. 48. *Experiments* concerning the *Proportions* of the *Power* of the *Load-stone* at *different Distances*; with a Description of the Load-stone made use of.

49. An

49. An *Experiment* concerning the *Proportions* of the *Ascent* of *Spirit* of *Wine* between two *Glass Planes*, whose Surfaces were *plac'd* at *different Distances* from each other. n. 337. p. 151.

50. *Experiments* concerning the *Ascent* of *Water* between two *Glass Planes* in an *Hyperbolic* Curve. Ibid. p. 153.

51. A farther Account of the *Ascending* of *Drops* of *Spirit* of *Wine* between two *Glass Planes twenty Inches and a half long*; with a *Table* of the *Distances* from the *Touching Ends* and the *Angles* of *Elevation*. Ibid. p. 155.

CHAP. II.

Hydrology.

An Account of the Lake Vetter, *in* Sweden; *by Dr.* Urban Hearne. n. 298. p. 1938.

I. BOTH *Olaus Magnus* and other Writers having related some Things surprising and unusual concerning the Lake *Vetter*, I thought it worth while to enquire more narrowly into the Nature of that *Lake*, and the Veracity of those Authors; and where I had not an Opportunity of making Observations myself, to ask the Testimonies of Persons of a good Character living in the Neighbourhood of it, who could solve all my Questions with Matters of Fact.

The Lake *Vetter* running from North to South, from *Askersund* in *Nericia* to *Jonekoping* in *Smoland*, measures fourteen *Swedish* Miles in Length, one of which is equal to five or six Miles *English*, and ten of them make almost a Degree: But in Breadth it is only three Miles, and in some Places hardly more than two. This Lake upon Account of the high Mountains about it, which in some Places begin upon the Borders of it, and in other Places are at a little Distance from it, appear depressed towards the Shore to People standing near it. It is remarkably deep, but its Depth is so unequal, that in some Places you find the Bottom at eighty Fathoms, but in several Parts on the Borders of *Ostrogothland*, and in a few of *Westrogothland*, at three hundred Fathoms you find no Bottom. An Inhabitant of *Vadsten*, *Benedictus Amberni*, in order to sound the Depth of the Lake upon the Borders of the State of *Grennen*, let down some Fathoms of Rope with a Hatchet at the End of it, but not finding any Bottom, when he gathered up the Rope, instead of the Hatchet he found a Horse's Skull very neatly fastened to it. An Abyss like to this at the Precipices of the Mountain *Obmen*, which are called the *Western Wall*, has always deceived those who have attempted to sound it; so that few dare venture upon it for fear of the West Wind, which growing stormy of a sudden easily loses the Vessels down the Sides of the Mountains in spite of all the Anchors they throw out on every Side. In the same manner formerly at a certain Province of *Westrogothland*, the Governor Count *Johannes Oxenstiern* after throwing out three hundred Fathoms could find no Bottom, which is attested by some Fishermen still alive, who were employed in that Affair. This Water is is no lefs clear than it is deep, fo that you can difcern a very fmall Piece of Money at a confiderable Depth. *Ericus Simonius* Paftor and Overfeer of *Vadften* fays, that in a clear Day he has been able to perceive a Silver Penny in the Water fixty Fathoms deep. But at fome Diftance from the Surface it appears tinged fomewhat green: And no wonder if the Filth coming from fo many fmaller Lakes, Mountains and Woods, fhould taint this limpid Water confiderably.

Although this *Lake of Vetter* is larger than moft others, yet the moft Part of it is free of Rocks, and there are very few Iflands in it. The chief of thefe is called *Vifingfoe*, formerly the Seat of the Family of the *Brahde*, and lies in the Middle of the *Lake* between *Grenna* of *Smoland* and *Weftrogothland*, and North from that is the Ifland of *Roknen* over againft the Baths of *Medevien*. There are befides thefe fome fmall Iflands near the Shore, but they are very fmall and few in Number. But being freely expofed to Winds and furrounded with very high Mountains, no wonder if it is but feldom calm, and the Boats are toffed upon it in frequent Storms; which come often fo unexpected that there begins a Motion within it while the Waters are as fmooth as a Mirror, before the leaft Breath of Wind is felt. Which feems to be owing to the Tempeft heaving up the Waters elfewhere, and gradually protruding them before the Winds have reached that Part. For it is no unfrequent Thing in the *Vetter* for the Veffels to be toffed by the Winds in one Part, while hard by it is fo calm that others are obliged to make ufe of Oars. This feems to indicate that Commotions may be raifed in the Waters by fubterraneous Winds, and *Varenius* attempts to explain the like Effects in his *Univerfal Geography*. A great many Phænomena confirm this Conjecture. For when the Tempeft and Clouds are threatening, you may perceive a kind of howling and thundering Noife in the Waters while the Air continues ferene, which I had occafion myfelf to hear feveral times while at the *Baths* of *Medevien* when the Air was quite calm, and was always followed with a violent Storm. But the Inhabitants of *Vifingfoe* are moft fenfible of this, having their Ears confounded with a Noife like that of Guns, from that Part of the Ifland whence they expect the Storm to come next Day. And when thefe Explofions are heard towards the Eaft, they have a Storm from that Quarter for the moft part attended with Hail and Rain. The various puffing up of the Waters, the fudden rifing of Vapours, and hafty flying out of Blafts, which fome People have obferved in this Lake, are worthy to be taken Notice of. Something of this Kind was obferved not without Surprize by *Abraham Winandz* the Architect, who happening once to pafs by thofe Coafts with fome Friends, while the Waters were quite calm, faw little Clouds darting up from the Bottom here and there, and joining themfelves with the Air in Form of Smoak, fell down in gentle Showers upon them every now and then all that Day; all which argue ftrongly that there are fuch Things as fubterraneous Winds.

Doubtlefs the fame Wind, with the Storm coming from above, is the Caufe why in the Spring, the Ice which juft now is ftrong and thick enough

enough to bear Horſes and Carriages, is ſo broken and diſperſed, that you may preſently ſail very ſafely upon it. But before this dreadful breaking of the Ice, the horrible Noiſe of the Waters which precedes it, warns the frightened Travellers that are upon it to get off as quick as poſſible; but if they happen to be far from the Shore, they are either immediately drowned, or drove up and down for ſome Time upon Fragments of Ice, and ſometimes the Ice preſently ſinks when the Air is very little moved.

Whether or not there are metallic Vapours ſerving to produce theſe ſubterraneous Winds, I ſhall not pretend to determine. But that theſe are not wanting there is plain, from ſeveral Mountains of *Nericia* and *Weſtrogothland*, on the North Side of the *Vetter*, with Veins of Iron Ore, and perhaps more noble Metals more lately diſcovered, and other different Kinds of Minerals, *viz.* Antimony, Loadſtone, Chalk, Mica Sterilis, ſeveral Sorts of Silver and Lead Ores, and Marchaſite, whence Sulphur, Vitriol, Allum, and other foſſile Subſtances uſe to be extracted. And in the Waters themſelves there is found a great Quantity both of Marchaſite, and a ruſty Kind of Oker. The *Ignis fatuus* muſt likewiſe be referred to this Place, being not only obſerved frequently upon the Borders of the Lake, but in the Night Time it is ſeen flying in the Middle of the Waters, and confounds the Fiſhermen, and a great many are perſuaded that this is owing to metallic and ſulphureous Exhalations. Nor is the Granate, Porphyry, Jaſper, Chryſtal, and other precious Stones produced in this Lake, formerly collected by Count *Peter Brabens*, and ſo finely poliſhed, as to be uſed amongſt the bridal Ornaments of the Ladies of *Viſingſburg*, generated without the Aſſiſtance of mineral Vapours. All theſe acknowledge a mineral Origin, not to mention the Baths of *Medevien*, in the Borders of the *Vetter*.

Amongſt the other Properties of this Lake, we muſt not paſs over the remarkable Whirl-pools and violent Torrents here, which though they have only one Vent, yet being directly oppoſed to the Winds and Waves, are very troubleſome to the Fiſhermen. Hence it has been ſuſpected on Account of its vaſt Depth, and the private Channels, and ſubterraneous Winds, that there muſt be a Communication under Ground of the *Vetter* with another Lake, ten *Swediſh* Miles to the Weſt of it, called the *Venner*. And this is the more probable, from the different Quick-ſands that are betwixt theſe two Lakes two of which in the Pariſh of *Fegren*, called the black and white Quick-ſands, *Haddorphius* had meaſured. He found them to be of a prodigious Depth, and obſerved in them an inteſtine Motion, as if they were turgid with a Kind of Ferment. The ſame Opinion is likewiſe ſtrengthened from this, that ſome Years the *Vetter* is conſiderably ſwelled without any manifeſt Cauſe, and falls again afterwards the ſame Way. Mr. *Daniel Ridley* Miniſter at *Motala*, has obſerved concerning this *Lake* theſe ſeven Years by-paſt, that in ſome certain Places it gradually decreaſed, ſo

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as you could walk dry-footed ſome Fathoms, where Boats uſed formerly to go, the Seaſons in the mean Time, *viz.* in the Years 1680, 1682, 1684, and 1685, being every where ſufficiently rainy. But in the Year 1686, towards Autumn, the Waters began again gradually to increaſe till the preſent Year 1688. But whether the *Vetter* keeps its ſtated Times, the ſame as they ſay of the *Venner*, increaſing ſeven Years, and then diminiſhing other ſeven, I am not able to determine. It is likewiſe ſurpriſing, that in a ſerene Air they can hear the Cannon at *Stockholm* and other Places thirty Miles diſtant. So that in the Year 1685, when the Princes of *Stockholm* were buried, they heard the Report of the Cannon exactly at ſix o'Clock. And with the ſame Eaſe in the Year 1676 they heard the Exploſion of the Guns in a Sea-Fight diſtinctly at about thirty Miles Diſtance.

As to what *Olaus Magnus*, *Meſſenius*, and other Hiſtorians relate concerning *Gilbert*'s *Cave* in the Iſland of *Viſingſoe*, I leave to the Credit of theſe Authors. This however is certain, that there ſtill remains a Cavern full of a ſulphureous and very nauſeous Stench, which I imagine, according to the Opinion of the People in that Place, to be owing to the Naſtineſs gathered in Length of Time in the Cave ſituated near the Waters, exhaling ſulphureous and moiſt Vapours. And that there appear upon the Borders of it different Spectres and Phantoms, for the moſt part reſembling Women, ſometimes Horſes and other Animals, friſking about, no Body who is intent upon thoſe Curioſities will deny. There are ſeveral Stories both of former Ages and of the preſent, which I could bring in to confirm this, if I was not reſolved to paſs it over ſo ſlightly.

But I muſt not neglect to mention the famous River *Motala*, which, as I ſaid, is the only Mouth of the *Vetter*, and at certain Times uſes to lay aſide its Fluidity, and ſtop its Courſe in ſuch a Manner, as you could go freely into it, and ſometimes take up Fiſhes that were left in the Bottom of it without any Impediment, as it happened in the Years 1682 and 1685, about *Chriſtmas.* The common People in that Country are unanimouſly perſuaded, that the Courſe of the River is never ſtopt without ſomething bad following it, and that it always preſages either a great Dearth or a War, or ſome public Diſaſter, in the ſame manner as a Whale's coming into the *Thames* is ſaid by the *Engliſh* to portend ſomething fatal. But whatever Stories they have about, if they are not conformable to the Laws of Nature, give no Satisfaction to a phyſical Perſon, enquiring into the natural Cauſes of ſuch ſurpriſing Effects. I was therefore ſollicitous in enquiring into thoſe Things which ſeemed to give Light to this Phænomenon, although I never had the Opportunity of ſeeing the Courſe of the River ſtopt. But however variouſly the Inhabitants near this River attempt to explain this Affair, imagining that at that Time the Waters retiring from the Shores ſink into the Channel, yet I have always ſuſpected that this Part of the River above where the Current was ſtopt muſt be obſtructed with Ice

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or Snow, ſo as to dam it up, while the Waters below run on towards the Sea. And what gives a Foundation for this Opinion is, that this ſtopping of the River never happens in the Spring, Summer, or Autumn, but always about *Chriſtmas*, or the Beginning of the new Year; and that it is always near the Bridge, where the Water being at leaſt above three Yards deep, the Stone Pillars upon which the Bridge is built retard a good deal the Courſe of the River there. That this Conjecture is agreeable to Truth, the above-mentioned Miniſter of the Church at *Motala* aſſerts, both from his own Experience and that of other Perſons. His Houſe ſtands juſt upon the Borders of the Bridge, and ſays that there are long Herbs, ſuch as Water-Graſs, Knot-Graſs, &c. which ſhooting out from the Bridge into the neighbouring Current germinate in the Water, and the Ice faſtens to them like Knobs, and congealed Snow, which being puſhed away by the River, and ſtopt by the Pillars of the Bridge, at laſt may be heaped up ſo as to ſtop the Current. There are likewiſe a great many Builders there, who ſay, that before they expect the Current to be ſtopt, there are large Heaps of Snow ſent out from the Lake, which ſticking like Glue to whatever Body they meet with, ſink gradually to the Bottom. Nor is it an uncommon Thing for the Waters in the Lake one Day to be very quiet, and the next Day all in Commotion ſtopt near the Bridge. But whatever is the Cauſe of it, it is ſtill ſurpriſing, that this does not happen when the Cold is very intenſe, but when the Air is more mild, generally about *Chriſtmas*, or the Beginning of *January*. Perhaps the Cold continues ſtill violent under the Waters, though the Air is become milder, or the Ice become ſofter ſticks to the Herbs or other Obſtacles which it meets with in its Way, and produces theſe Obſtructions.

Some Particulars of a Fountain *near the Lake* Vetter.

But I muſt not paſs over in Silence ſome remarkable Particulars of a *Fountain* not far from the *Vetter* in the Pariſh of *Nyen* (where are the Baths of *Medevien* too) near the Church and the Miniſter's Houſe, which I had from the Miniſter there, whoſe Name is *Jonah Frodelius*, a very worthy Man, and alſo from others. They call it the hungry Fountain, or the Foreteller of the Crop, becauſe it is never quite filled with Water but there follows a Dearth next Year. It is ſurrounded on all Sides with ſoft ſandy Hills, in the Middle of which is a flat Valley, but not at all marſhy. Out of this riſes the Fountain by ſecret Veins, which has this Particular, that in a rainy Summer for the moſt Part it becomes dry, and on the contrary, in the drieſt Summers, immediately before a Dearth, and according to others (whom I am not at leiſure to meddle with) likewiſe before a War, it lays the high Road between *Motala* and *Vadſten* under Water, as is certified by a great many People who live there. In the Year 1685, which was very rainy, this Fountain was almoſt dry, and not above half a Foot deep; and in the Summer following, the Water began to increaſe. And the Truth of this is confirmed this preſent very dry Summer, this Spring having greatly abounded with Water, whilſt all the reſt in the Neighbourhood

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were quite dried up. But the chief Things to be obſerved about it are theſe.

1. That it preſages a bad Crop or Dearth to *Oſtrogothland* only of all the Places that are nigh it.
2. This whole Country, eſpecially in the Neighbourhood of the Fountain, is of a very ſandy Soil, in ſome Parts mixed with a very hard Clay, and therefore requires a great deal of Water to make it fertile. Beſides too,
3. The Crops are only bad there in dry Seaſons, whilſt the contrary happens in *Iemptia*, and other more Northern Provinces.
4. That the Plenty of Meteors, and of the Seaſon, ſometimes depends upon the Diſpoſition of the Earth, and what lies hid under its Surface.
5. That the Waters of this Fountain, ſtrained as it were through the Sand near the Sand Hills, are collected together by inſenſible Veins.
6. That the Waters may be increaſed or ſwelled up before a dry Seaſon, and fall away in a rainy one, for phyſical Cauſes, to treat of which more largely I ſhall take another Opportunity.

A New Contrivance *for* Diving, *&c. by Dr.* E. Halley. 349. p. 492.

II. Many Methods have been propoſed, and many Engines contriv'd, for enabling *Men* to abide a competent while under *Water*: And the reſpiring freſh *Air* being abſolutely neceſſary to maintain Life in all that breathe, ſeveral Ways have been thought of for carrying this down to the *Diver*, who muſt, without being ſupply'd therewith, return very ſoon, or periſh.

We have heard of *Divers* for Spunges in the *Archipelago*, helping themſelves by carrying down *Spunges* dipp'd in Oil in their Mouths; but conſidering how ſmall a Quantity of *Air* can be ſuppos'd to be contain'd in the Interſtices of a *Spunge*, and how much that little will be contracted by the Preſſure of the *incumbent Water*, it cannot be believ'd that a Supply, by this Means obtain'd, can long ſubſiſt a Diver. Since by Experiment it is found that a Gallon of Air included in a Bladder, and by a Pipe reciprocally inſpir'd and expir'd by the Lungs of a Man, will become unfit for any further Reſpiration, in little more than one Minute of Time; and though its Elaſticity be not much alter'd, yet in paſſing the Lungs, it loſes its vivifying Spirit, and is render'd effete, not unlike the Medium found in Damps, which is preſent Death to thoſe that breathe it; and which, in an Inſtant, extinguiſhes the brighteſt Flame, or the ſhining of glowing Coals, or red hot Iron, if put into it. I ſhall not go about to ſhew what it is the Air loſes by being taken into the Lungs, or what it communicates to the Blood by the extreme Ramifications of the *Aſpera Arteria*, ſo intimately interwoven with the Capillary Blood-Veſſels; much leſs to explain how it is perform'd, ſince no Diſcovery has been made to prove that the ultimate Branches of the Veins and Arteries there, have any *Anaſtomoſes* with thoſe of the *Trachæa*; as by the Microſcope they are found ſound to have with one another. But I leave this to the Anatomiſts; and only conclude from the aforeſaid Experiment, that a naked Diver, without a Spunge, cannot be above a Couple of Minutes inclos'd in Water, (as I once ſaw a *Florida-Indian* at *Bermudas*) nor much longer with a Spunge, without ſuffocating; and not near ſo long without great Uſe and Practice: Ordinary Perſons generally beginning to ſtifle in about half a Minute of Time. Beſides, if the Depth be conſiderable, the Preſſure of the Water on the Veſſels is found by Experience to make the Eyes Blood-ſhot, and frequently to occaſion ſpitting of Blood.

When therefore there has been Occaſion to continue long under Water, ſome have contriv'd double flexible Pipes to circulate Air down into a Cavity encloſing the Diver, as with Armour, to bear off this Preſſure of the Water, and to give leave to his Breaſt to dilate upon Inſpiration: The freſh Air being forc'd down by one of the Pipes with Bellows, or otherwiſe, and returning by the other of them, not unlike to an Artery and Vein. This has indeed been found ſufficient for ſmall Depths, not exceeding twelve or fifteen Foot: But when the Depth ſurpaſſes three Fathoms, Experience teaches us, that this Method becomes impracticable: For though the Pipes and the reſt of the *Apparatus* may be contriv'd to perform their Office duly, yet the Water (its Weight being now become conſiderable) does ſo cloſely embrace the Limbs that are bare, or cover'd with a flexible Covering, that it obſtructs the Circulation of the Blood in them; and preſſes with ſo much Force on all the Junctures, where the Armour is made tight with Leather, Skins, or ſuch like, that if there be the leaſt Defect in any of them, the whole Engine will inſtantly fill with Water, which will ruſh in with ſo much Violence, as to endanger the Life of the Man below, who may be drown'd before he can be drawn up. Upon both theſe Accounts, the Danger encreaſes with the Depth. Beſides, a Man thus ſhut up in a weighty Caſe, as this muſt needs be, cannot but be very unwieldy and unactive, and therefore unfit to execute what he is deſign'd to do at the Bottom.

To remedy theſe Inconveniences, the *Diving-Bell* was next thought of; wherein the *Diver* is ſafely convey'd into any reaſonable Depth, and may ſtay more or leſs Time under Water, according as the *Bell* is of greater or leſſer Capacity. This is moſt conveniently made in the Form of a Truncate Cone, the ſmaller Baſis being cloſed, and the larger open; and ought to be ſo poiz'd with Lead, and ſo ſuſpended, that the Veſſel may ſink full of Air, with its greater or open Baſis downwards, and as near as may be in a Situation parallel to the Horizon, ſo as to cloſe with the Surface of the Water all at once. Under this Couvercle the Diver ſetting, ſinks down together with the included Air; and if the Cavity of the Veſſel may contain a Tun of Water, a ſingle Man may remain therein at leaſt an Hour, without much Inconvenience, at five or ſix Fathoms deep. But this included Air, as it deſcends lower, does contract itſelf according to the Weight of the Water that compreſſes it; ſo that at thirty three Feet deep or thereabouts, the Bell will be half full of Water, the Preſſure of it being then equal to that of the whole Atmoſphere. And at all other Depths, the Space occupied by the compreſs'd Air in the upper Part of the Bell, will be to the under Part of its Capacity fill'd with Water, as thirty three Feet to the Depth of the Surface of the Water in the Bell below the common Surface thereof. And this condens'd Air, being taken in with the Breath, ſoon inſinuates itſelf into all the Cavities of the Body, and has no ſenſible Effect, if the Bell be permitted to deſcend ſo ſlow as to allow Time for that Purpoſe. The only Inconvenience is found in the Ears, within which there are Cavities opening only outwards, and that by Pores ſo ſmall as not to give Admiſſion even to the Air itſelf, unleſs they be dilated and diſtended by a conſiderable Force. Hence on the firſt Deſcent of the Bell, a Preſſure begins to be felt on each Ear, which by Degrees grows painful, like as if a Quill were forcibly thruſt into the Hole of the Ear; till at length the Force overcoming the Obſtacle, that which conſtringes theſe Pores yields to the Preſſure, and letting ſome condens'd Air ſlip in, preſent Eaſe enſues. But the Bell deſcending ſtill lower, the Pain is renew'd, and again eas'd after the ſame Manner. But when the Engine is drawn up again, the condens'd Air finds a much eaſier Paſſage out of thoſe Cavities, and even without Pain. This Force on the auditory Paſſages might be ſuſpected to be prejudicial to the Organs of Hearing, but that Experience teaches otherwiſe. But what is more inconvenient in this Engine, is, the Water entering into it, ſo as to contract the Bulk of Air (according to the aforeſaid Rule) into ſo ſmall a Space, as that it ſoon heats and becomes unfit for Reſpiration, for which Reaſon it muſt be often drawn up to recruit it: And beſides the Diver being almoſt cover'd with the Water thus entering into his Receptacle, will not be long able to endure the Cold thereof.

To obviate theſe Difficulties which attend the Uſe of the common Diving-Bell, I have thought of Means to convey Air down to it, whilſt below; whereby not only the Air included therein, would be refreſh'd and recruited, but alſo the Water wholly driven out, in whatever Depth it is; and will furniſh Air at the Bottom of the Sea in any Quantity deſir'd. The Deſcription of my *Apparatus* take as follows:

The Bell I made uſe of was of Wood, containing about ſixty Cubick Feet in its Concavity, and was of the Form of a Truncate Cone, whoſe Diameter at Top was three Feet, and at Bottom five. This I coated with Lead ſo heavy, that it would ſink empty, and

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I diſtributed the Weight ſo about its Bottom, that it would go down in a perpendicular Situation, and no other. In the Top, I fix'd a ſtrong, but clear Glaſs, to let in the Light from above; and likewiſe a Cock to let out the hot Air that had been breath'd; and below, about a Yard under the Bell, I plac'd a *Stage* which hung by three Ropes, each of which was charg'd with about one hundred Weight, to keep it ſteady. This Machine, I ſuſpended from the Maſt of a Ship, by a *Spritt*, which was ſufficiently ſecur'd by *Stays* to the Maſt-head, and was directed by *Braces* to carry it over board clear of the Ship-ſide, and to bring it again within board.

To ſupply Air to this Bell when under Water, I cauſed a couple of Barrels, of about 36 Gallons each, to be cas'd with Lead, ſo as to ſink empty, each having a Bung-hole in its loweſt Part, to let in the Water, as the Air in them condens'd on their Deſcent; and to let it out again, when they were drawn up full from below. And to a Hole in the uppermoſt Part of theſe Barrels, I fix'd a Leathern Trunk or Hoſe, well liquor'd with Bees-Wax and Oil, and long enough to fall below the Bung-hole, being kept down by a Weight appended; ſo that the Air in the upper Part of the Barrels could not eſcape, unleſs the lower Ends of theſe Hoſe were firſt lifted up.

I fitted theſe Air-Barrels with Tackle proper to make them riſe and fall alternately, after the Manner of two Buckets in a Well; which was done with ſo much Eaſe, that two Men, with leſs than half their Strength, could perform all the Labour; and in their Deſcent they were directed by Lines faſten'd to the under Edge of the Bell, which paſs'd through Rings plac'd on both Sides the Leathern Hoſe in each Barrel; ſo that ſliding down by thoſe Lines, they came readily to the Hand of a Man, who ſtood on the Stage on purpoſe to receive them, and to take up the Ends of the Hoſe into the Bell. Through theſe Hoſe, as ſoon as theſe Ends came above the Surface of the Water in the Barrels, all the Air that was included in the upper Parts of them, was blown with great Force into the Bell, whilſt the Water enter'd at the Bung-holes below, and fill'd them: And as ſoon as the Air of one Barrel had been thus receiv'd, upon a Signal given, that was drawn up, and at the ſame Time the other deſcended; and by an alternate Succeſſion furniſh'd Air ſo quick, and in ſuch Plenty, that I my ſelf have been one of five, who have been together at the Bottom, in nine or ten Fathoms Water, for about an Hour and half at a Time, without any Sort of ill Conſequence: And I might have continu'd there as long as I pleas'd, for any Thing that appear'd to the contrary. Beſides the whole Cavity of the Bell was kept entirely free from Water, ſo that I ſat on a Bench, which was diametrically plac'd near the Bottom, with all my Cloaths on. I only obſerv'd, that it was neceſſary

ceſſary to be let down gradually at firſt, as about 12 Feet at a Time; and then to ſtop and drive out the Water that enter'd, by receiving three or four Barrels of freſh Air, before I deſcended farther. But being arriv'd at the Depth deſign'd, I then let out as much of the hot Air that had been breath'd, as each Barrel would repleniſh with cool, by means of the Cock at the Top of the Bell; through whoſe Aperture, though very ſmall, the Air would ruſh with ſo great Violence, as to make the Surface of the Sea boil, and to cover it with a white Foam, notwithſtanding the great Weight of Water over us.

Thus I found I could do any Thing that was requir'd to be done juſt under us; and that, by taking off the Stage, I could, for a Space as wide as the Circuit of the Bell, lay the Bottom of the Sea ſo far dry, as not to be over Shoes thereon. And by the Glaſs Window ſo much Light was tranſmitted, that, when the Sea was clear, and eſpecially when the Sun ſhone, I could ſee perfectly well to write or read, much more to take up any Thing that was under us: And by the Return of the Air-Barrels, I often ſent up Orders, written with an Iron Pen on ſmall Plates of Lead, directing how to move us from Place to Place. At other Times, when the Water was troubled and thick, it would be as dark as Night below; but in ſuch a Caſe, I have been able to keep a Candle burning in the Bell as long as I pleas'd, notwithſtanding the great Expence of Air requiſite to maintain Flame.

I take this Invention to be applicable to various Uſes; ſuch as *Fiſhing* for *Pearl*, *Diving* for *Coral*, *Spunges*, and the like, in far greater Depths than has hitherto been thought poſſible: Alſo for the fitting and plaining of the Foundations of Moles, Bridges, *&c.* upon rocky Bottoms; and for the cleaning and ſcrubbing of Ships Bottoms when foul, in calm Weather at Sea.

By an additional Contrivance, I have found it not impracticable for a Diver to go out of this Engine to a good Diſtance from it, the Air being convey'd to him with a continu'd Stream, by ſmall flexible Pipes; which Pipes may ſerve as a Clue to direct him back again, when he would return to the Bell.

Of an Eruption *of* Waters *in* Yorkſhire, *by Mr.* R. Thoreſby. n. 306. p. 2236.

III. I have lately enquir'd of a Neighbour concerning the extraordinary Eruption of Waters at *Craven*, which the Vicar of *Kildnick* ſent (†) an Account of to the *Royal Society*: I am not only fully ſatisfied of the Truth of what Mr. *Pollard* ſays, but alſo, that, as he conjectures, a great Part of the Land is not to this Day recover'd from the Sand and Stones, though a great Number of People were employ'd about it. Upon the *opening of the Rock*, at the Foot of which the Town of *Starbotham* ſtands, the Water guſh'd out in ſo vaſt a Quantity, as if it would have ſwept away the whole Town. The

(†) *Vid.* Supra V. II. C. II. S. XXXIV.

Waters

Waters came rolling impetuoufly down, almoft at once; feveral Houfes were utterly ruin'd, and others wreck'd up to the Chamber Windows; one, particularly, was fo cover'd, that a great Piece of the Rock was left upon the Top of the Chimney. My Neighbour was an Eye-witnefs of this fad Accident, and has fpent much Time in clearing fome Part of his Land.

Of Inundations *in* Ireland, *by Mr.* Neve. n. 320. p. 309.

IV. On *October* the *7th*, 1706, after a very rainy Day, and foutherly Wind, there happen'd a prodigious Flood in the North of *Ireland* (the like not in the Memory of Man) which broke down feveral Bridges, and the Sides of fome of the Mountains. It came running down in vaft Torrents from the Mountains, and drown'd abundance of black Cattle and Sheep, fpoil'd a great deal of Corn and Hay in the Stacks; it laid abundance of Houfes two or three Feet deep in Water, and broke down feveral of the Forges and Mill-Dams.

On *July 3d*, 1707, there happen'd another *Flood*, which came fo fuddenly from the Mountains, as if there had been fome fudden Eruption of the Waters. And on the *26th* of the fame Month, in the County of *Antrim*, there happen'd a very fudden and furprizing *Flood*, which rais'd the *Six-Mile-River* at that Rate, that it broke down two ftrong Stone Bridges, and three Houfes, and carried away 600 Pieces of Linnen Cloath that lay bleaching, fill'd feveral Houfes feveral Feet deep with Water, tore down fome large Rocks in its Paffage, and left feveral Meadows cover'd a Foot or two deep with Sand. In the South-Eaft Part of the County of *Derry*, they had that Day but little Rain, with fome Thunder; but beyond the Mountains, in the North-Weft Part of the County, the River *Roe* had a great Flood.

Obfervations *on the Lake* Lough-Neagh *in* Ireland, *by Mr.* F. Neville. n. 337. p. 260.

V. There is no petrifying Quality in the Water of the Lake *Lough-Neath*, as fome believe; I have liv'd 14 Years at *Dungannon*, within five Miles of it, and have been often there about the Skirts for many Miles, and in a Boat upon it feveral Times; I have view'd it often when the Waters have been very low, and a large Strand left in feveral Places: And many Trees lay in the Verge of the *Lough* (of which, I believe, fome might have lain there fome hundreds of Years) which had been over-turn'd by the *Lough*'s encroaching on the Land, where great Woods had grown; and many Roots of great Trees were ftanding in their proper Places, where the Water had prevail'd on the Land; and I perceiv'd no Alteration in the Wood at all, but it was firm, found Wood, without any Petrifaction.

Mr. *Brownlow* told me, that he had drove feveral Holly Stakes into the

the Ground within the Verge of the *Lough*, and that ſome of them continu'd there many Years, but that he found no Alteration.

There has indeed been great Quantities of ſuch ſort of Stone, like to Wood, found upon the Strand after great Floods and Storms of Wind, which have put the *Lough* into a Ferment; the Waves breaking down the Banks, encroaching on the Land, and tumbling over Trees, by which Encroachment this Sort of Stones are diſcover'd: And if ever they were Wood, they were petrify'd by the Earth, and not by the Water; of which kind I have ſeen ſeveral Pieces big and little, ſome like Oak, ſome Aſh, and ſome like Holly, with Bark, Grain, and Knots, like Wood; ſo that any by the Eye would judge it Wood, till they come to try it. I had a Piece about ſixteen Inches long, that look'd as if it had been a great Chip cut out of the Side of an Oak Block, with the Bark on it; and in cutting ſuch large Chips, there happens generally ſuch Shakes and Flaws, ſo that there will be a Separation of Parts at one End, and they remain firm at the other, as it was in this. I could have rais'd ſeveral of ſuch Splinters, of this large Chip, ſome bigger, and ſome leſs; and when ſo rais'd, they would have ſlapp'd down as though they were a Spring. Some of thoſe Stones would appear at one End as if rotten, and decay'd Wood; but trying it, it was as much Stone, as any other Part.

The Lake is reputed to be twenty four Miles long, and twelve Miles broad, and Navigable from *Charlemount* to *Portlenone*, which is about thirty five Miles. It does not abound with many Sorts of Fiſh; but, thoſe that are, are very good; ſuch as Salmon, Trout, Pike, Breame, Roach, Eels, and *Pollans*, with which laſt, it does abound. The *Engliſh* call them *Freſh-water Herrings*. They catch them in the Summer with Sieves, as they do Herrings. They are much in Shape and Bigneſs like to the largeſt Smelts, full of very large bright Scales, and pleaſant Meat, being eat freſh. Theſe were ſuppos'd to be a Fiſh peculiar to that Lake; but ſince I came here, I find *Lough Earne* has the ſame Sort, but not in ſo great a Plenty. They are generally caught here in their Eel-Nets, running to the Sea; ſo that I am of Opinion, that they are that Sort of Fiſh that is caught in the Sea, or between the freſh and ſalt Water, call'd *Shads*; and that the large ones come from the Sea, as the Salmon doth, and leave their Spawn in the *Lough*; which, when they grow to be big, go to the Sea, and there come to their full Growth: And that which confirms me in my Opinion, is, that at the Salmon Fiſhing at *Coleraine*, they catch many of the large ones going up to the *Lough*. There is one ſort of Trout in *Lough-Neagh* very large: I have ſeen one weigh 30 Pound weight; and the largeſt Salmon that I ever ſaw, weigh'd not more than 35. This ſort of Trout the *Iriſh* call a *Budagh*.

F. ſh Pollans.

That

That there is ſome healing Quality in the Water of this *Lough* is certain; but whether diffus'd through all Parts thereof is not known, nor pretended. The *Fiſhing Bay*, which is about half a Mile broad, hath a fine ſandy Bottom, where any one may walk with Safety and Eaſe from the Depth of his Ankle to his Chin, upon an eaſy Declivity, at leaſt three hundred Yards before a Man ſhall come to that Depth. Others, as well as myſelf, have obſerv'd that the Bottom has chang'd from Cold to Warm, and from Warm to Cold, and this in different Spots through the Bay.

The firſt Occaſion of taking Notice of this Bay for Cure, was in the Reign of King *Charles* the Second: There was one Mr. *Cunningham*, who had an only Son grown to Man's Eſtate. This young Man had the Evil to that Degree, that it run upon him in eight or ten Places: He had been touch'd by the King, and all Means imaginable had been us'd for his Recovery; but all did no good, and his Body was ſo waſted, that he could not walk. When all Hopes of his Recovery were paſs'd, he was carried to the *Lough*, where he was waſh'd and bath'd; and in eight Days Time, bathing each Day, all the Sores were dry'd up, and he was cur'd, and grew very healthy, married, begot Children, and liv'd nine or ten Years after. This Account I had from Capt. *Morris*, and his Brother, who were Eye-Witneſſes. After ſo remarkable a Cure, many came there, who had running Sores upon them, and were cur'd after a little Time. The Natives thought it could not do well, but upon ſome particular Time appropriated for that Service; and now great Crowds come there on *Midſummer-Eve*, of all ſorts of Sick; and ſick Cattle are brought there likewiſe, and driven into the Water for their Cure; and People do believe they receive Benefit. I know it dries up running Sores, and cures the Rheumatiſm, but not with once bathing, as People now uſe it; and the drinking the Water, I am told, will ſtop the Flux. I look upon it to be one of the pleaſanteſt Bathing Places I ever ſaw.

Of the Eruption of a Burning Spring in Shropſhire, by Mr. R. Hopton. n. 334. p. 475.

VI. About *June* 1711, at *Broſely* near *Wenlock* in *Shropſhire* (about two Nights after a remarkable Day of Thunder) there was heard a terrible Noiſe in the Night, which awaken'd ſeveral People; who riſing to ſee what it was, came at laſt to a Boggy Place under a little Hill, about 200 Yards off the River *Severn*; where they perceiv'd a mighty Rumbling and Shaking in the Earth, and a little boiling up of Water through the Graſs: They took a Spade, and digging up ſome Part of the Earth, immediately the Water flew up to a great Height, and a Candle, that they had, ſet it on Fire.

To prevent the Spring being deſtroy'd, there is an Iron Ciſtern plac'd about it, with a Cover upon it to be lock'd, and a Hole in the Middle thereof; that any who come may ſee the Water thro'.

If you put a lighted Candle, or any Thing of Fire to this Hole, the Water immediately takes Fire, and burns like Spirit of Wine, and continues to do ſo as long as you can keep the Air from it; but by taking up the Cover of the Ciſtern it quickly goes out: The Heat of this Fire much exceeds the Heat of any Fire I ever ſaw, and ſeems to have more than ordinary Fierceneſs with it.

Some People out of Curioſity, after they have ſet the Water on Fire, have put a Kettle of Water over the Ciſtern, and in it Green Peaſe, or a Joint of Meat, and boil'd it much ſooner than over any Artificial Fire that can be made. If any green Boughs, or any Thing elſe that will burn, is put upon it, it preſently conſumes them to Aſhes.

The Water of itſelf is as cold as any Water I ever felt; and what is remarkable, as ſoon as ever the Fire is out, if you put your Hands into it, it feels as cold as if there had been no ſuch Thing as Fire near it. It ſtill [in *September*] continues boiling up with a conſiderable Noiſe.

Of a Mineral Water *at* Canterbury, *by Dr.* Moulins. n. 312. p. 2462.

VII. About twelve Years ago a *Mineral Water* was accidentally diſcovered in *Canterbury*. In digging the Ground, they firſt met with a black fat Mold, extending itſelf three Feet deep, and gradually changing into another Sort of Earth, very fat, and like Butter. This ſecond Lay was two Feet thick; the Colour of it Yellow, ſomething mix'd; its Odour ſtrong and Mineral; and a Piece of it being for ſome Time expos'd to the Sun, ſmell'd much like burning Sulphur. After this they found a Quickſand of a darker Colour than the firſt Earth, mix'd with ſeveral little Stones, and the Smell ſtronger than before. Two Feet further, under the Quickſand, a hard Rock appear'd, out of which Water guſh'd with ſome Violence. They dug two Wells at about ſeven Feet diſtance from each other; one about eight or nine Feet from its Surface, and twelve from the Surface of the Ground about it, and reacheth the Rock: The other is not ſo deep by two Feet, and only toucheth the Sand. This laſt is ſomething ſtronger of the Sulphur, but the other is ſtronger of the Mineral Spirit and ferruginous Parts.

Two Drams of the ſecond Lay of Earth, found in digging, being put into four Ounces of Spirit of Vinegar, there preſently aroſe a conſiderable Ebullition; and ſoon after the Spirit was ting'd with a yellow browniſh Colour, which ſuffer'd no Alteration with the Infuſion of Logwood, nor with Galls, but with Oil of *Tart. p. deliq.* turn'd greeniſh, and with the Infuſion of *Lig. neph.* of a pale Red.

The Water taken up at the Spring is extraordinary limpid, but grows ſomething whitiſh in a quarter of an Hour, and in half an Hour the Spirit is loſt, and the Mineral hangs firſt on the Sides of the

the Glaſs, and then falls gradually to the Bottom. It won't keep quite ſo well as the *Spaw* or *Tunbridge* Water. Its Taſte is maſculine and auſtere; the Smell ferruginous and ſtrong, ſomething upon the Sulphur: People ſay it ſmells like Gun-powder. It will make the Root of the Tongue of the Drinkers look blackiſh. Linnen waſh'd in it turns yellow. It will not lather with Soap. The Glaſſes the Water is dipp'd with grow yellow, which no Scowering can take off, and are apt to fly. In froſty and cold Weather it is ſo warm as to melt Ice and Snow; in other Seaſons 'tis cold; tho' not ſo cold as ſome Spring Waters are.

The Weight of this Water varies much according to the Seaſons of the Weather. In *May* 1704, it weigh'd three Grains lighter than common Water in the Quantity of a Pound. In the Spring of 1705, it was equal in Weight to common Water; and is now ſtill heavier in *Auguſt* following, becauſe of the exceeding dry Weather of that Summer. But in general about *Midſummer*, if the Weather is no ways extraordinary, 'tis pretty equal to common Water in Weight.

A ſingle Grain Weight of good Gall will turn a Pint and a half of this Water of a very noble deep Red, and in an Inſtant. Syrup of Violets turns it of a Graſs green. With the Infuſion of *Braſile*, it giveth a deep lively Blue: With that of *Lign. nephr.* firſt a light Green, then a light Yellow, with a blue Crown: With the Infuſion of Logwood, a blue Black: With that of Fuſtick Wood, a duſky Yellow: With the Flowers of Pomgranates, a fair Violet: With the Leaves of Tea, a fine purpliſh Blue: With good *Nantz* Brandy, an elegant Sky-colour. It turns a Solution of the *Sacch. Saturni* milky in an Inſtant; and the Solution of *Sublimate* in ſome Time longer. *Ol. Tart. per deliq. Sp. Sal. Armon. Sp. Vitr. &c.* make no ſenſible Alteration.

In calm Weather, in Winter eſpecially, a thick oily Film covers the Surface of theſe Waters, of as great a Variety of Colours as a Rain-bow; a Spoonful of it drunk, hath the Effect of, and compoſes as much to Sleep as, a moderate Doſe of Opium. Some of this Scum, being dry'd by Evaporation, taſted very fat, and felt ſo between the Fingers. Some of this Powder being caſt upon a red hot Iron, moſt of it immediately burn'd away with ſparkling; and what remain'd was of the Colour of Ruſt of Iron, and taſted partly Stiptic and Earthy, and partly Saltiſh.

The Water itſelf, being gently evaporated, yields a yellowiſh Sediment, more or leſs, according to the Seaſons. Laſt Spring a Quart yielded ſix Grains of it; but in *September* following, the ſame Quantity afforded me nine Grains; whereas a Pound of *Tunbridge* Water gave but one ſingle Grain of Sediment to Mr. *Boyle*, as appears by his *Memoirs of Mineral Waters*. This Sediment, being boil'd in common Water, made a ſtrong *Lixivium*, with which Acids caus'd no ſenſible Fermentation; but Syrup of Violets turn'd it Green. This *Lixivium* being evaporated, yielded a fat Sulphurous Salt, that would not coagulate into Cryſtals. I can get but three or four Grains of it out of ten Grains of Sediment; but from the Colour and Taſte of the *Lixivium*, I have Reaſon to ſuſpect, that there is a larger Proportion of Saline Particles, which, as I conceive, being Volatile, evaporate away with the Water.

As for its Medicinal Virtues, from the many and truly wonderful Cures, I believe it to be one of the moſt excellent Waters of this Kind, as yet found out in *England*. The little Well is very uſeful in Diſeaſes of the Breaſt, as in Aſthmas, Coughs, Rheums and Catarrhs. It hath cur'd ſeveral given over of Conſumptions of the Lungs. Moſt Diſorders of the Stomach are cur'd by this Water. It ſeldom fails in the Cure of Rheumatick Gouty Pains of the Limbs, or other Parts of the Body, in the Scurvy and Melancholy Diſtempers, Jaundice, Vapours, all ſorts of Stoppages, Scabs, Itch, *&c.* But in Gravel, Cholick, and Green-ſickneſs, 'tis a true Specifick; as alſo in inward Ulcers, if not too far gone. A Porter of *Bolton*, who had been with many Doctors, and was laſt Spring diſcharg'd out of St. *Thomas*'s Hoſpital, as an incurable Perſon, hath been cur'd of his Ulcer in the Bladder this Summer, with drinking of this Water for three Months together.

In Agues it is beyond the Bark: I have ſeen ſome Rebellious ones, that could not be remov'd by the Bark, perfectly cur'd by this Water, and ſome Conſtitutions quite worn out by the frequent Relapſes of this Diſtemper, reſtor'd again. This is alſo remarkable, that it agrees beſt with old, decay'd, and weak Conſtitutions. The Water ſits pleaſantly upon the Stomach, works off by Urine very briſkly, cauſeth a good Appetite, clears the Spirits, and procures Sleep. It is not binding, as ſome other Chalybeats are, but keeps the Body open to moſt People, and upon ſome it brings now and then a gentle Looſeneſs, which carries off the Diſtemper. For theſe four Years I have preſcrib'd them to many Scores of People every Seaſon, and I could never obſerve any Inconveniency, or ill Symptom ariſe from the Drinking of them.

An Examen *of the* Spaw Waters, *proving them to be* Alcalies, *by* Dr. F. Slare n. 337. p. 247.

VIII. I have long ſince been of Opinion, that People have been miſtaken in their Notions about the Nature and Properties of thoſe Mineral Waters, which are of the *Chalybeat* or Iron Species. *Germany* abounds much with theſe Waters, and they beſtow one general Name upon them, and call them *Sour Brunns*, that is, *Sour* Wells, or Springs of Water. *Henricus ab Heers* agrees with *Vitruvius*, *Fallopius* and *Helmont*, in juſtifying the Acidity of ſeveral ſorts of *Spaw* and *Chalybeat* Waters; but, not being ſatisfied with their Reaſons, aſſigns

aſſigns others; and after a tedious Harangue, concludes, that they owe their Birth and great Virtues to Vitriol and Sulphur. He obſerves, that Vitriol and Sulphur are found in the Glebe or Earth from whence theſe Waters ſpring; but yet does not give us one Proof or Experiment of his having found any real Vitriol, or true Sulphur, or any Acidity in theſe Waters.

Dr. *Jordis*, who practiſed Phyſick at *Francford*, and often at *Swalbac* in Summer Time, at my Deſire, examin'd thoſe Waters. He gave me an Account of ſome *Ocres*, or Ferruginous Parts, which he calcin'd and tortur'd in the Fire, to make them confeſs their Sulphur Original; but in all his Experiments did not ſatisfy me, that the Water held one Drop of an Acid by Diſtillation, *&c.*

That which gave me the firſt Suſpicion, that the *Chalybeat* Waters did not contain any rough, or vitriolic, or acid Salts in them, proceeded from an accidental Uſe of a ſtrong Iron Water, in which I diſſolv'd Soap, and found it lather and waſh my Hands well, and then I us'd a Waſhball and ſhav'd with it, and try'd ſeveral other Waters of this ſort, which did the ſame, and much better than ſome Pump-Waters.

I conſulted my Palate, and try'd whether I could diſcover any Sharpneſs or Acidity in our *Engliſh* Steel-Waters at *Tunbridge*, at *Black-Boy* in the Pariſh of *Franfield* in *Suſſex*, *Hampſted*, *Sunning-Hill* in *Berkſhire*, &c. but I was ſo far from diſcovering any ſuch Thing, that theſe Waters ſeem'd rather to leave a ſweetiſh Flavour behind: Thus many Alkali Salts, if nicely examin'd (of the fix'd kind) have affected my Taſte.

I made Experiments with ſeveral Sorts of ſuch Spirits as are apt to ferment with Acids; ſuch as Spirit of *Hartſhorn*, of *Sal Armoniac*, &c. but theſe made no Ferment, nor any Motion or Change in theſe Waters.

I conſider'd the Diſeaſes in Human Bodies, which theſe Waters were preſcrib'd, by Phyſicians, to cure; that they were often ſuch as proceeded from ſharp, acid, or acrimonious Cauſes, as *Cardilage*, or Heart-burnings, ſour Vomitings, corroſive *Diarrhæas*, Cholicks, from Scurvies and Stranguries; and that for theſe Diſtempers ſweetning and alkaliſate Remedies are made uſe of.

I conſider theſe Waters as containing in them the Properties of Iron; and I find by Experience, that it is moſt oppoſite to Acids, being one of their great Correctors, and therefore rather to be eſteem'd an Alkali.

Take ſome Filings of Iron, perhaps a Dram, and pour on them about an Ounce of the milder Acids, ſuch as Vinegar, Verjuice, or the Juice of Lemons, and it will deſtroy the Sharpneſs of theſe Juices: Or if you pour on theſe Filings Mineral Acids, as the very corroſive Spirit of Nitre, or of Salt, or what is call'd Oil of Vitriol, they will immediately loſe their Acidity, be diſarm'd of their ſharp Points, and by Evaporation give a Salt that will taſte ſweetiſh, and is by Chymiſts call'd *Saccharum Martis*, if duly prepar'd; which is ſafely given inwardly, and is eſteem'd a good altering Medicine.

Steel beaten to a fine Powder, is, without any farther Preparation, given inwardly with great Succeſs for Stomachic Diſeaſes, as in the Green-Sickneſs, Hypochondriac, and various other Acid and Acrimonious Diſaffections.

I conſider'd Milk to be a very proper and obvious Subject to bring this Controverſy to a plain and unqueſtionable Deciſion. I made this Experiment with all poſſible Exactneſs: I firſt prov'd the *Chalybeat* Waters, more particularly the *Spaw* Waters, by trying whether they tinged with Galls. Theſe being very good, I put part of the Waters to cold Milk; ſome I only made lukewarm, and ſome I boil'd together, in equal Proportions: But they were ſo far from affording any Curd or Coagulation, that they continu'd ſeveral Days without being ſour.

The *German* Phyſicians (on the miſtaken Notion of their being Acid) ſtrictly prohibit their Patients the Uſe of all manner of *Lacticinia*, whilſt they are in a Courſe of theſe Waters.

This Prejudice too, has prevail'd much amongſt moſt of our Water Drinkers in *England*; but I do atteſt, that I have frequently advis'd, in ſome Caſes, Milk to be given daily in the Evening, through a whole Courſe of Steel Waters, with good Effect: Nay, I do affirm, that ſome others could not bear the Waters without having a third Part of Milk, or more, mix'd with them, and have continu'd them ſo for many Weeks, with good Advantage: Nor do I find the leaſt Reaſon to prohibit the Uſe of Milk in a Courſe of *Bath* Waters; having been there above a Year and half, making the beſt Scrutiny I can into the Properties, Virtues, and Vices (if they have any) of thoſe Waters.

Since our Experiments diſcover that thoſe Things which are of a ſweetning *Alkaliſate* Nature, do ſo very well agree with theſe Mineral Waters, it will appear by the following Experiment, that Acids do very much diſagree.

I put but one Drop of Oil of Vitriol to a large Glaſs full of ſtrong *Spaw* Waters, which before the Addition of this *Acid* did give a deep Purple to the Solution of Galls; but now would not give the leaſt Tincture, though I put in four Times as much of the Galls. From hence I conclude, that the Virtues of the *Chalybeat* Ingredients, which I take to be the Life and Soul of theſe Waters, were ſo far bound up or deſtroy'd, as to have loſt their Cordial or corroborating Faculty; and that the Bile or Gall in the Human Bowels, could not be able to ſeparate the *Chalybeat* (which are the only Medical) Particles, and mix them with the Chyle, in order to anſwer any End in Phyſic.

Let

Let this be a Caution to thoſe that deſign to make theſe Waters paſs better by Urine, that they do not make uſe of any *Acids*; it being a common Practice to uſe Spirit of Vitriol, *Spiritus Nitri dulcis*, &c. as a Diuretick: Unleſs it ſhould ſo happen that they have a Deſign to take off, and diveſt them of their warm Cordial or altering Power, and ſo to bring them near to common Water; which, I muſt confeſs, we are forc'd to do, eſpecially in the Uſe of *Bath* Waters, in ſome hot inflammatory Caſes.

I ſhall conclude with one ſhort Experiment in Favour of our *Alkalies*; that if you put any *Alkali* Salt, volatile or fix'd, ſuch as Volatile Salt of Hartſhorn, or of *Sal Armoniac*, or fix'd Salt of *Tartar*, or Wormwood, or any other true *Alkali*, you will then deſtroy the above-nam'd Acid Spirit, recover the Virtue of the Waters, and diſpoſe them to give their Tincture as they uſed to do in their natural State.

IX. Having procur'd about a Dozen Quarts of *Pyrmont* Waters this laſt Summer, I made ſome Trials with them. I found by the Taſte, that they contain'd a rich *Chalybeat* Virtue, and alſo made a very briſk and lively Impreſſion on the Palate, more grateful and ſpirituous, than the beſt *Spaw* Waters I ever taſted. The *Spaw* Waters are look'd upon as moſt excellent, if they ſparkle a little in a Glaſs; but theſe, in Summer-time, when pour'd into the Glaſs, nay, ſometimes even in the Bottle, as ſoon as the Cork was open'd, and the Air was admitted, would make a notable Ebullition, ſomewhat like bottled Cyder, though this was ſoon over; but they did yet continue their ſmart and briſk Taſte, and highly *Chalybeat* Reliſh, to the laſt Drop, though we were ſome Hours in drinking them off. In the Winter-time, theſe Waters do not ſparkle, nor ferment, at leaſt mine did not; but they were not carefully preſerv'd, being expos'd in cold Cellars; and yet, notwithſtanding, they loſt not the *Chalybeat* Taſte, and alſo retain'd a very pleaſant briſk Guſt. Theſe Waters have been reckon'd in the Number of the *German Acidulæ*, and ſome of my Friends, to whom I gave a Glaſs of the Water, have aſcrib'd to it a ſharp Taſte, and have been ready to run away with a poſſeſs'd Opinion of its being ſour: But, when I have deſir'd them to conſider, they have own'd that the ſmart and briſk Taſte miſled them to call it Acid or truly ſour: Thus Cyder and ſoft Ale, when bottled, will give ſuch an acute Affection to the Palate, when it is far from being ſour: And even *Volatile Alkalies* of *Sal Armoniac*, or of Hartſhorn, may be made to give the like Pungency to the Tongue.

Of the Nature *and* Virtues *of the* Pyrmont *Waters, by Dr.* F. Slare. n. 351. p. 564.

In order to a more nice Enquiry, whether any Acidity were diſcoverable in theſe *Pyrmont* Waters, we dropp'd in conſiderable Quantities both of Spirit of Hartſhorn, and of Spirit of *Sal Armoniac*, both

both juftly prepar'd; but could not difcover the leaft Luctation or Motion to appear upon this Conjunction, as it ufually does with an Acid.

I made a yet more nice and certain Examen of thefe Waters, by mixing Milk with them, fometimes in equal, fometimes in double Proportion; and in various Degrees of Warmth, both in lukewarm Degrees, and alfo with a boiling Heat, but I could not perceive any Curdling. But rather, on the contrary, the Water preferv'd the Milk from Coagulation, for four or five Days, even in *September*, it being hot Weather.

Take a very little Gall in Powder, about half a Grain to a Glafs of a Quarter of a Pint; this does in a Moment render it turbid, and make a dark Purple, efpecially if you ftir it: But if you drop the Powder on the Surface of the fame Water, it then caufes a fine blue Tincture. If you will make a very fine Tincture pleafant to the Eye, take five Leaves of ftrong Green Tea, put them into the Bottom of a Glafs holding a Quarter of a Pint, and you will fee thofe Leaves unfold themfelves, and in a Quarter of an Hour, tinge the Water with fuch a ceruleous Azure Blue, that few Vegetables do afford the like. We obferve, that the longer thefe Leaves, or any other Stiptics, (which are the Precipitators) do ftay together, the more they degenerate into a deep Purple, or even to an Atramentarious Colour.

In reference to the internal Ufe of thefe Waters, I drank about a Quart at a Time, after this Manner: I firft began with the *Spaw* Waters, which I procur'd very good, and drank them for a Week, and they agreed very well. I then drank the *Pyrmont* Waters for three or four Days, and continu'd the Ufe of thefe Waters alternately, until I had drank about twenty Days. By the Refult of my Experiment, it feem'd to me very plain, that the *Pyrmont* Water was more agreeable, gave more Strength and Spirit, and was as much or more preferable for its internal Virtue, as for its excelling the other in a brifker and more fprightly Tafte.

There is another Excellency in thefe Waters, which will make them more ufeful to us, than any foreign *Chalybeat* Waters we yet know, becaufe thefe will keep better; they are not fo foon fpoil'd by any accidental Infinuations of Air, as the *Spaw* are fubject to be. The *Chalybeat* Mineral is here throughly diffolv'd and well united, and mix'd in this Water, fo that it does not eafily precipitate: For which Reafon, it may alfo the better pafs the *vafa lactea*, and even enter into the Mafs of Blood it felf, and work the more confiderable Effects. That this is not a bare Hypothefis, may be prov'd by this Experiment.

Having fuffer'd the *Spaw* Water to be expofed in a Bottle which was half full, and unftopp'd twelve Hours, I examin'd it, and found

found it taste just like common Water; but the *Pyrmont* Waters that were open'd to the Air after the same Manner, tasted strong of the Mineral, and gave their Tincture as at first; nay, they continu'd thus for full two Days, and perhaps might have done so longer, but I thought that Time suffic'd.

Having had lately some Discourse about a *Purging* Quality contain'd in these Waters, I am now inquiring whether they in Reality do contain any purging Ingredients or Properties. *An Additional Account.* Ibid.

I evaporated about a Quart of this Water *ad siccitatem*; I then poured on the *Reliquiæ* some Rain-Water, enough to dissolve and take up the Salts, and exhal'd that Water, and had a Grain or two of the Salts, that tasted *Muriatic*, such as most River and Pump Waters give. It is well known that the Purging Waters have a very bitter Taste, and by Dr. *Grew*, that Salt was call'd *Sal Catharticum amarum*, which distinguish'd it from all other Species of natural Salts: That of the *Pyrmont* Water above-mentioned has no Relation to this, but to the Sea Salt, not being in the least bitter.

It is also well known, that unless our Waters be impregnated with a considerable Quantity of this bitter Salt, it will not purge at all: Two or three Grains signify nothing, nor have the least Cathartic Power. For Example, put two Drachms of the purging Salts to a Quart of common Water; and this Quantity will give but a Stool or two to one who is naturally very easy to work upon. I have examin'd several other *Chalybeat* Waters, and found much the like Ingredients, and never any that I could suspect to carry any purging Properties.

I think we can much better demonstrate that the *Chalybeat* Waters do contain Stiptic and Restringent Virtues, because they owe their Birth to the *Iron Mineral*, and more particularly to the *Pyrites*, which Dr. *Lister* suggests, (not without some Reason) to be the Parent even of all *Iron Ores*, as it is doubtless the Cause of all *Chalybeat* Waters: Thus I have often examin'd the Solution of the *Pyrites* by the Rain-Water at *Deptford*, and at other Places, where *Copperas* is made, and found it a very strong *Chalybeat* Water. It is from this *Mineral* we have our strong Stiptic and constringent Medicines, for external and internal Use; we have our Powders and Salts of *Steel*, or *Vitriol* of *Mars*, from hence; nay, even obstinate and inveterate *Diarrhæas* have, by a judicious Use of *Tunbridge* and other Iron Waters, receiv'd a Cure.

But it is asserted that the Waters really do purge at *Pyrmont*, where they are drank.

This we do allow to be true, that *Tunbridge* Waters do not only purge, but sometimes vomit, when drank hastily and in great Quantity; but our Physicians have corrected this Irregularity, and we hear of no such Complaints, where they observe a just *Regimen:*

And we do all agree, that thoſe Waters are, in their own Nature, binding, and do oft require ſome opening Medicine. The Quantities of Water drank at *Pyrmont* are very large, often two or three *Engliſh* Quarts. It is no Wonder that their Weight forces them through the Bowels; for any common Water, drank haſtily, and in ſuch Quantity, will do the ſame. Whereas, if you take this Method, and will drink *Pyrmont*, or any other *Chalybeat* Waters leiſurely, *viz.* a Pint-Glaſs in an Hour, or rather two Half-Pint Glaſſes, you may drink three Pints in ſo many Hours without Danger of loſing them by Dejection. But if any one will be careful, and take this Caution with him, he will ſcarce fail of Succeſs; that is, let him be very quiet and ſtill, both in Body and Mind; the leſs he ſtirs or walks, the better he will paſs off his Waters by Urine.

I ſhall mention only one Obſervation more, which is, that none of our *Engliſh* Steel Waters do ſtrike ſuch a Purple as the foreign celebrated *Chalybeat* Waters do; for ours do give a more turbid and dark Colour, and the worſe the Waters are, the blacker Sediment they make: Thoſe of *Iſlington* abound with a coarſe *Oker*, the Mineral is not well diſſolv'd, but gives an *atramentarious* Colour; but the *Pyrmont* Waters excel all I have happen'd to examine, in its bright *Cæruleous* Luſtre.

N. B. *Moſt of the Experiments alledg'd by Dr.* Slare, *in the foregoing Diſcourſe, were likewiſe by him ſhewn before the* Royal Society, Feb. 28. 1717, *and it was found that the* Pyrmont *Waters gave a much brighter Tincture with Galls and Tea, and had a much more exalted* Chalybeat *Taſte than the* Spaw; *and a ſmall Quantity of each being kept for ſome Time in Bottles, to compare them, the* Pyrmont *was found to have retain'd its Virtues much better than the* Spaw. *The Preſident, and ſeveral of the Members preſent, having drunk a Glaſs of it, found it of a very agreeable Reliſh, and to ſit eaſy on the Stomach.*

X. *Accounts of Books Omitted.*

n. 276. p. 1038. 1. *Aloyſii Ferdinandi* Comit. *Marſigli Danubialis Operis Prodromus*, Ad Regiam Societatem *Anglicanam*. Folio 1700.

n. 308. p. 2346. 2. Dr. *Ehm*'s Treatiſe of St. *George*'s Bath by *Landeck*, in the Lordſhip of *Glats* near *Sileſia*.

CHAP.

CHAP. III.

Mineralogy.

1. THE Marble for this Purpoſe ought to be very ſmooth without any Spot, and hard, that it may better bear the Force of the Fire, and therefore Alabaſter is by no means proper for theſe Uſes. *The Way of* Colouring Marble, *by* —— n. 268. p. 735.

2. Fire is requiſite to open the Pores, but in ſuch a Degree as that it ſhall not be ſcorched, for then the Colours will be deſtroyed; neither muſt it be too cool, for then, though it receives the Colours, yet they will be leſs fixed. For Marble even when it is cold, will imbibe ſome Colours, as Saffron, and Stone-blue for a blue Colour; but theſe Colours are eaſily diſſipated by the leaſt Heat of the Fire: And therefore the Degree of Heat ought to be ſufficient, gently to boil the Liquor that is poured upon the Marble.

3. The Menſtrua are different, according to the Diverſity of the Stuff to be diſſolved, a Lixive of Horſe's Urine made with four Parts of Pot-aſh, and one Part of Quick-lime, (*N. B.* Dog's Urine is better than Horſe's) Alſo Spirits of Wine, common Lixive, Wine and ſome oleagenous Bodies mixed.

4. The Colours laid on with a Vehicle are theſe. 1. *Stone-blue*, diſſolved in Spirit of Wine, or a Lixive of Quick-lime. 2. Lackmus in common Lixive. 3. Saffron or *Sapgreen* diſſolved in a Lixive of Urine and Quick-lime, or in Spirit of Wine. 4. Vermilion or Cochineal diſſolved as above. 5. Dragon's Blood diſſolved in Spirit of Wine according to Art. 6. Braſil Wood diſſolved in Spirit of Wine. 7. Alkanet Root extracted with the Oil of Turpentine; for it cannot be diſſolved in any other Menſtruum, neither in Spirit of Wine, nor a Lixive. 8. *Sapgreen the leſs*, mixed and diſſolved in Spirit of Wine, or a Lixive of Quick-lime as before. There is another kind of Dragon's Blood, called the Tears of it, which being mixed with Urine, produces a beautiful enough Colour, but it is hard to be got. Thoſe Colours that are mixed with Urine anſwer the beſt.

5. The Colours which are rubbed on without any Vehicle are theſe. 1. Dragon's Blood very well purified, for a red Colour. 2 Gum Gutta, for a yellow Colour. 3. Green Wax, for a green Colour. 4. Sulphur, Pitch, and Turpentine, for a brown Colour; it is only required that the Marble be ſufficiently hot, and ſo the earthy Colours are communicated to it by rubbing, which you will find by Experience.

Theſe Colours are either eaſily or difficultly waſhed out The red Colour may be extracted in ſix and twenty Hours, with Oil of Tartar by Deliquium, without hurting the Poliſh of the Marble in the leaſt; and the brown with Aqua fortis in a Quarter of an Hour, but the Poliſh will be hurt.

For a Golden Colour, take Sal Armoniac, white Vitriol, and Verdigreaſe, and reduce them to a very fine Powder.

Of a Quarry of Marble *in* Ireland, *by Mr.* F. Nevile. n. 337. p. 278.

II. I was lately with Mr. *Cole* in the Mountains in the County of *Fermanagh* in *Ireland*, where I had diſcover'd a Marble Quarry. The Country wherein it lies, is ſo ſtrange for the natural Wonders in it, that it would make a little Hiſtory to deſcribe all that is to be ſeen: It lies on the North-ſide of *Calcagh*, in the Pariſh of *Kilaſher*, in the County of *Fermanagh*. There are Marble-Rocks, whoſe perpendicular Height is 50 or 60 Feet, diſcover'd by ſubterraneous Rivers, which by Degrees have waſh'd away the Earth and looſe Stones, and diſcover'd theſe mighty Rocks. There are many great Pits fallen in on the Sides of the great Mountain; ſeveral of them in a ſmall Compaſs of Ground, ſo that it is dangerous travelling near them. There are many Caves form'd, ſome very large, the Sides and Arches of Marble, ſome of a Liver-Colour, varied in white with many little Figures; ſome of a light blue varied with white; but I could find no entire black and white amongſt them.

An Account of a Colliery blown up; *by* —— *Communicated by Dr.* A. Charlett. n. 318. p. 215.

III. On the 18*th* of *Auguſt* 1708, at *Fatfield*, in the Pariſh of *Cheſter Le Street*, about Three of the Clock in the Morning, by the ſudden Eruption of a violent Fire, which diſcharged itſelf at the Mouths of three Pits, with as great a Noiſe as the firing of Cannon, or the loudeſt Claps of Thunder, threeſcore and nine Perſons were deſtroyed in one Inſtant. Three of them, *viz.* two Men and a Woman, were blown quite up from the Bottom of the Shaft, fifty-ſeven Fathom deep, into the Air, at a conſiderable Diſtance from the Mouth of the Pit: One of the Men with his Head almoſt off, and the Woman with her Bowels hanging about her Heels.

The Engine, by which the Coals were drawn up, and is of a great Weight, was removed and caſt aſide by the Force of the Blaſt; and what is more wonderful, the Fiſh which were in the Rivulet, that runs twenty Yards under the Level, and at as great a Diſtance from the Mouth of one of the Pits, were in great Numbers taken up dead, floating upon the Water: Whether this happen'd by the violent Concuſſion of the Air, or whether they were choaked with the Sulphur (that to be ſure in Abundance diſperſed itſelf abroad) I leave to others to determine; only this I obſerve, that for ſeveral Days a very ſtrong and noiſome Smell continued to come out of the Pits.

As to the Cauſe of it, it is to be premiſed, that Coal Mines are in general ſubject to *Stith* or *Sulphur*.

Stith, as vulgarly ſo called by the Pitmen, I think corruptly, from Stench or Stink, is a want of Air, or rather ſuch a Foulneſs in the Air, that overcomes the Spirits of the Men, and ſo ſuffocates them, as well as extinguiſhes the Candles.

Sulphur differs in this, that as the other ſuffers not the Candles to burn, this makes them burn too faſt; and the Flame by the impulſive Quality of the Air, or attracted by the Sulphur, extends itſelf upwards into

into a prodigious Length, and as a Match lighted for the Diſcharge of a Cannon, as ſpeedily ſets on Fire that Vapour, equally deſtructive.

Now to prevent both theſe Inconveniencies, the Viewer of the Works takes the beſt Care he can to preſerve a free Communication of Air through all the Works; that as the Air goes down one Pit, it ſhould aſcend another; but it happen'd in this Colliery, that there was a Pit which ſtood in an Eddy, where the Air had not always a free Paſſage, and which in hot and ſultry Weather was very much ſubject to Sulphur: And it being then the Middle of *Auguſt*, and ſome Danger apprehended from the Cloſeneſs and Heat of the Seaſon, the Men were withdrawn from their Work in that Pit, and turned into another; but an Overman, ſome Days after this Change, and upon ſome Notion of his own, being induced, as is ſuppoſed, by a freſh, cool, froſty Breeze of Wind, which blew that Morning, and which always clears the Works of all Sulphur, had gone too near this Pit, and had met the Sulphur juſt as it was purging and diſperſing it ſelf; upon which the Sulphur immediately took Fire by his Candle, and ſo he proved the Occaſion of the Loſs of himſelf and ſo many Men, and of the greateſt Fire that ever was known in theſe Parts.

An Account of the Eruption *of Mount* Veſuvius *in* 1707. *by the Honourable* J. Valetta. n. 337. p. 22.

IV. 1.] The Eruptions of Mount *Veſuvius* happen ſo frequently, that that they are almoſt innumerable, and there is not a Month paſſes, far leſs a Year, when there is not ſome deſtructive Commotion in it, ſometimes greater, and ſometimes leſs. But in 1707, there happened a very great Eruption of it, in which there were many Things obſerved which have not been taken Notice of in any other neither before nor ſince.

In the Year 1707, when the Weather was very hot, in the latter End of *July*, Mount *Veſuvius*, which had remained quiet for a good while, began to give ſome Signs of an approaching Commotion, for firſt there were internal Roarings heard in the Middle of the Mountain; but as yet there was no Appearance neither of Smoak nor Flame. After theſe Sounds it began gradually to ſend out Smoak and Flame, which in the Night time eſpecially ſhone over all *Campania*. In the mean time at different Intervals, it ſent out ſuch dreadful Exploſions as are hardly to be imitated by the largeſt Artillery. After this it continued to throw up Aſhes, as of ſome Stuff that had been powdered, toſſing them up into the Air for ſeveral Days and Nights, and diſperſing them over the neighbouring Country, according as the Wind blew, ſometimes into the Sea, ſometimes on the *Stabian* Coaſt, ſometimes towards *Nola*, and ſometimes towards *Acerræ*. Nor muſt I neglect to mention the great Showers of Stones, which deſtroyed every Thing where they fell, even the very Cattle. Next there ruſhed out from its Mouth, as at other Times, a Torrent of Bitumen, which they call melted Gravel, which at firſt had the Appearance of a gentle Stream of Fire, moving downward only with ſuch a Celerity, as you may obſerve in melted Pitch, or other ſuch like Subſtances. This Matter, which I would compare to Glaſs made of Sand melted in the hotteſt Furnaces, in the ſame manner

ner as Glass, after it had cooled by going on, acquired a stony Hardness. But it is worth while to observe, that the upper Surface of this Matter when it was cold, was formed into small spongy Stones, while the lower resembled a solid, broad, and very hard Flint, which has been used of a long time in paving the Highways; as if that which was next to the Air had imbibed some of its Particles, while the under Part, having no Air mixed with it, formed a very compact Mass. But amongst a great many Phænomena of this boiling Mountain, there were two which had not been seen or known for many Ages: For the third or fourth Day, it began to send forth Flashes of Lightning from its Orifice, having the same Appearance as those which you see dart from the Heavens, but tortuous and slow, and at the same Time were heard Explosions like Claps of Thunder, so that at first we were afraid that it really thundered. The Flashes at first were so thick and frequent, that we expected it would rain, till we understood that they came from the Mountain, and that the dark Clouds were not owing to the Vapours, but to the Ashes flying so thick about. On the second of *August*, at four in the Afternoon, the Air at *Naples* was so full of Ashes, that the Sun's Rays being excluded, there was universal Darkness, and to such a Degree, that we could not know our Friends and Acquaintances in the Streets. No Night was ever darker than that Day; for if any one went abroad with a Torch, he was obliged to return again, which happened only in the Time of *Titus* as *Xiphilinus* informs us. The Magistrates of the City and the Priests, ordered Supplications to be put up by the People, and that the Relicks of St. *Januarius* the tutelar Saint of *Naples*, should be carried in Procession with the usual Ceremony to the *Capuan* Gate, which is towards the Mountain. After they had got there, in the Midst of the thickest Darkness, at length about the first or second Hour of the Night, one or two Stars began to shine towards the North, where perhaps the Ashes did not fly so thick, and the blue Sky to appear, and after that the Darkness which had robbed us of the Day gradually diminished. Then the Ashes were driven off from us towards the Sea. The next Day however was not quite clear, but the Air was still somewhat obscured with the Ashes, and retained-----*dubiæ discrimina Lucis.*

Thus *Vesuvius* laying waste the Country with Ashes, exhausted with throwing up melted Gravel for several Days, so that the black Torrent issuing out from it reached almost to the Sea; at last after fifteen Days almost became settled, and the People about *Naples* who had fled, returned Home. The Inhabitants too of the Town being at last freed from Fear, and desirous to perpetuate the Memory of St. *Januarius*, who assists them always in Straits, ordered a Gold and Silver Medal to be struck, on one Side of which was the Head of St. *Januarius*, with this Inscription, DIVO JANUARIO LIBERATORI URBIS FUNDATORI QUIETIS; and on the Reverse was *Vesuvius* quieted, and the following Inscription, POSTQUAM COLLAPSI CINERES ET FLAMMA QUIEVIT, CIVES NEAP. INCOLUMES. MDCCVII.

2.] *April* 17, 1717, with much Difficulty I reach'd the Top of Mount *Veſuvius*, in which I ſaw a vaſt Aperture full of Smoak, which hinder'd me from ſeeing its Depth and Figure. I heard within that horrid Gulph, certain odd Sounds, which ſeem'd to proceed from the Belly of the Mountain; a ſort of Murmuring, Sighing, Throbbing, Churning, Daſhing (as it were) of Waves, and between whiles a Noiſe like that of Thunder or Cannon, which was conſtantly attended with a Clattering, like that of Tiles falling from the Tops of Houſes in the Streets. Sometimes, as the Wind chang'd, the Smoak grew thinner, diſcovering a very ruddy Flame, and the Jaws of the Pan or *Crater*, ſtreak'd with Red, and ſeveral Shades of Yellow. After an Hour's Stay, the Smoak being mov'd by the Wind, gave us ſhort and partial Proſpects of the great Hollow, in the flat Bottom of which, I could diſcern two Furnaces almoſt contiguous; that on the Left ſeeming about three Yards in Diameter, glow'd with red Flame, and threw up red-hot Stones with a hideous Noiſe, which, as they fell back, cauſed the before-mention'd Clattering. *May* 8, in the Morning, I aſcended to the Top of *Veſuvius* a ſecond Time, and found a different Face of Things. The Smoak aſcending upright, gave a full Proſpect of the *Crater*, which, as I could judge, is about a Mile in Circumference, and an hundred Yards deep. A conical Mount had been formed ſince my laſt Viſit, in the Middle of the Bottom. This Mount, I could ſee, was made of the Stones thrown up and fallen back again into the *Crater*. In this new Hill remained the two Mouths or Furnaces already mention'd, that on our Left-hand was in the *Vertex* of the Hill which it had form'd round it, and raged more violently than before, throwing up every three or four Minutes, with a dreadful Bellowing, a vaſt Number of red-hot Stones, ſometimes in Appearance above a Thouſand, and at leaſt three hundred Feet higher than my Head, as I ſtood upon the Brink. But there being little or no Wind, they fell back perpendicularly into the *Crater*, increaſing the conical Hill. The other Mouth to the Right, was lower in the Side of the ſame new-form'd Hill. I could diſcern it to be fill'd with red-hot liquid Matter, like that in the Furnace of a Glaſs-Houſe, which raged and wrought as the Waves of the Sea, cauſing a ſhort abrupt Noiſe, like what may be imagin'd to proceed from a Sea of Quickſilver claſhing among uneven Rocks. This Stuff would ſometimes ſpew over, and run down the convex Side of the conical Hill, and appearing at firſt red-hot, it changed Colour, and harden'd as it cool'd, ſhewing the firſt Rudiments of an Eruption, or, if I may ſo ſay, an Eruption in Miniature. Had the Wind driven in our Faces, we had been in no ſmall Danger of ſtifling by the ſulphureous Smoak, or being knock'd on the Head by Lumps of melted Minerals, which we ſaw had ſometimes fallen on the Brink of the *Crater*, upon thoſe Shots from the Gulph at Bottom. But as the Wind was favourable, I had an Opportunity to ſurvey this odd Scene

—of Mount Veſuvius, *and its* Eruptions *in* 1717. *by Mr.* E. Berkeley. n. 354. p. 708.

for

for above an Hour and a half together; during which, it was very obſervable, that all the Volleys of Smoak, Flame, and burning Stones, came only out of the Hole to our Left, while the liquid Stuff in the other Mouth wrought and overflow'd, as hath been already deſcribed. *June* 5, after a horrid Noiſe, the Mountain was ſeen at *Naples* to ſpew a little out of the *Crater*. The ſame continued the 6*th*. The 7*th*, nothing was obſerv'd till within two Hours of Night; when it began a hideous bellowing, which continued all that Night, and the next Day till Noon, cauſing the Windows, and, as ſome affirm, the very Houſes in *Naples* to ſhake. From that Time it ſpew'd vaſt Quantities of molten Stuff to the South, which ſtream'd down the Side of the Mountain, like a great Pot boiling over. This Evening I returned from a Voyage through *Apulia*, and was ſurprized, paſſing by the North-ſide of the Mountain, to ſee a large Quantity of ruddy Smoak lie along a huge Tract of Sky over the River, of molten Stuff, which was itſelf out of Sight. The 9*th*, *Veſuvius* raged leſs violently; that Night we ſaw from *Naples* a Column of Fire ſhoot between whiles out of its Summit. The 10*th*, when we thought all would have been over, the Mountain grew very outrageous again, roaring and groaning moſt dreadfully. One cannot form a juſter Idea of this Noiſe, in the moſt violent Fits of it, than by imagining a mix'd Sound made up of the raging of a Tempeſt, the Murmur of a troubled Sea, and the Roaring of Thunder and Artillery, confuſed all together. It was very terrible as we heard it in the further End of *Naples*, at the Diſtance of above twelve Miles. This moved my Curioſity to approach the Mountain. Three or four of us got into a Boat, and were ſet aſhore at *Torre del Greco*, a Town ſituate at the Foot of *Veſuvius* to the South-weſt, whence we rode four or five Miles before we came to the burning River, which was about Midnight. The Roaring of the *Volcano* grew exceeding loud and horrible as we approach'd. I obſerved a Mixture of Colours in the Cloud over the *Crater*, green, yellow, red and blue; there was likewiſe a ruddy diſmal Light in the Air over that Tract of Land where the burning River flow'd; Aſhes continually ſhower'd upon us all the Way from the Sea-Coaſt. All which Circumſtances, ſet off and augmented by the Horror and Silence of the Night, made a Scene the moſt uncommon and aſtoniſhing I ever ſaw; which grew ſtill more extraordinary, as we came nearer the Stream. Imagine a vaſt Torrent of liquid Fire rolling from the Top down the Side of the Mountain, and with irreſiſtible Fury bearing down and conſuming Vines, Olives, Fig-trees, Houſes; in a Word, every Thing that ſtood in its Way. This mighty Flood divided into different Channels, according to the Inequalities of the Mountain. The largeſt Stream ſeemed half a Mile broad, at leaſt, and five Miles long. I walked ſo far before my Companions, up the Mountain, along the Side of the River of Fire, that I was obliged to retire in great Haſte, the ſulphureous

Steam

Steam having ſurprized me, and almoſt taken away my Breath. During our Return, which was about three a-Clock in the Morning, we conſtantly heard the Murmur and Groaning of the Mountain, which between whiles would burſt out into louder Peals, throwing up huge Spouts of Fire and burning Stones, which falling down again, reſembled the Stars in our Rockets. Sometimes I obſerv'd two, at others three diſtinct Columns of Flame, and ſometimes one vaſt one, that ſeemed to fill the whole *Crater*. Theſe burning Columns, and the fiery Stones ſeemed to be ſhot 1000 Feet perpendicular above the Summit of the *Volcano*. The 11*th* at Night I obſerved it from a Terraſs in *Naples*, to throw up inceſſantly a vaſt Body of Fire, and great Stones to a ſurprizing Height. The 12*th* in the Morning, it darken'd the Sun with Aſhes and Smoak, cauſing a ſort of Eclipſe. Horrid Bellowings this and the foregoing Day were heard at *Naples*, whither Part of the Aſhes alſo reached. At Night I obſerved it throw up Flame, as on the 11*th*. On the 13*th*, the Wind changing, we ſaw a Pillar of black Smoak ſhot upright to a prodigious Height. At Night I obſerved the Mount to caſt up Fire as before, tho' not ſo diſtinctly becauſe of the Smoak. The 14*th*, a thick black Cloud hid the Mountain from *Naples*. The 15*th* in the Morning, the Court and Walls of our Houſe in *Naples* were cover'd with Aſhes. In the Evening, Flame appear'd on the Mountain through the Cloud. The 16*th*, the Smoak was driven by a Weſterly Wind from the Town to the oppoſite Side of the Mountain. The 17*th*, the Smoak appear'd much diminiſh'd, fat and greaſy. The 18*th*, the whole Appearance ended, the Mountain remaining perfectly quiet without any viſible Smoak or Flame. A Gentleman, whoſe Window look'd toward *Veſuvius*, aſſur'd me, that he obſerv'd this Night ſeveral Flaſhes, as it were of Lightning, iſſue out of the Mouth of the *Volcano*. I ſhall not mention the Conjectures I have formed concerning the Cauſe of theſe *Phænomena*, from what I obſerved in the *Lacus Anſancti*, the *Solſatara*, &c. as well as in Mount *Veſuvius*. But this I may ſay, that I ſaw the fluid Matter riſe out of the Center of the Bottom of the *Crater*, out of the very Middle of the Mountain, contrary to what *Borelli* imagines, whoſe Method of Explaining the Eruption of a *Volcano* by an inflexed Siphon, and the Rules of *Hydroſtatics*, is likewiſe inconſiſtent with the Torrents flowing down from the very *Vertex* of the Mountain.

An Account of an Earthquake *in the* North *of* England, 1703. *by Mr.* R. Thoreſby. n. 289. p. 1555.

V. On the 28*th* of *December* 1703, there happen'd an Earthquake in theſe Parts. From *Hull* I am inform'd, that it was felt about three or four Minutes after five in the Evening; that it heav'd up Chairs and Tables, made Pewter-Diſhes and the Windows rattle, ſhook whole Houſes, and threw down Part of a Chimney. The Shock came and went ſuddenly, and was attended with a Noiſe like the Wind, though there was then a perfect Calm.

It

It was felt in much the fame Manner at *Beverly* and other Places, and particularly at *South-Dalton.* It was more violent near *Lincoln*, where it heav'd up the Chairs; and, as a certain Clergyman informed me, shook every Limb of him. At *Selby* it was felt pretty much; as also near *Navenby*, where the Noife which preceded it feemed to fome like the Rumbling of two or three Coaches driven furioufly, and immediately the Chairs they fat on were fhook violently, and the very Stones were feen to move.

Of Trees *found under* Ground *in* Hatfield-Chace, *by Mr.* Abr. de la Pryme. n. 275. p. 980.

VI. 1.) The Levels of *Hatfield-Chace* in *Yorkfhire*, were the greateft Chace of red Deer that King *Charles* the Firft had in all *England*; containing in all Limits above 180000 Acres, about half of which was yearly drowned and furrounded with an Ocean of Waters. This he bargains with, and fells to one Sir *Cornelius Vermuiden*, a *Dutchman*, to difchace, drain, and reduce to Arable and Pafture Land; which to the Surprize of all, and to the great Advantage of the Country round it, he at length effectually perform'd at the Expence of above 400000 Pounds.

In the Soil of all, or moft of thefe 180000 Acres of Land (of which 90000 were drained) even in the Bottom of the River of *Oufe*, in the Bottom of the adventitious Soil of all *Marfhland*, and round about by the Skirts of the *Lincolnfhire* Woolds unto *Gainfburg*, *Bautry*, *Doncafter*, *Baln*, *Snaith*, and *Holden*, are found infinite Millions of the Roots and Bodies of Trees, great and little, of moft of the Sorts, that this Ifland either formerly did, or at prefent does produce, as Firs, Oaks, Birch, Beech, Yew, Wirethorn, Willow, Afh, *&c.* the Roots of all, or moft of which ftand in the Soil in their natural Poftures, as thick as ever they could grow, as the Bodies of moft of them lie by their proper Roots. Moft of the great Trees, by all their Length about a Yard from their great Roots (unto which they did moft evidently belong, both by their Situation and the Samenefs of the Wood) with their Tops commonly North-Eaft, though the fmaller Trees lie almoft every Way crofs thofe, fome above, fome under, a third Part of all which are Firs, fome of which have been found of thirty Yards length and above, and have been fold to make Mafts and Keels for Ships. Oaks have been found of 20, 30 and 35 Yards long, yet wanting many Yards at the fmall End. Some of which have been fold for 4, 8, 10 and 15*l.* a-piece; which are as black as Ebony, and very lafting and durable. The Afhes are as foft as Earth, and are commonly cut in Pieces by the Workmens Spades, which as foon as flung up into the open Air, fall away into Duft; but all the reft, even the Willows themfelves, which are fofter than Afhes, preferve their Subftance and Texture to this Day. I have feen fome Fir-Trees, that as they have laid all along, after that they were fallen, have ftruck up great Branches from their Sides, which have grown into the Thicknefs and Height of confiderable Trees.

It

It is very obſervable, and manifeſtly evident, that many of thoſe Trees of all ſorts have been burnt, but eſpecially the Fir-Trees, ſome quite through, and ſome all on a Side, ſome have been found chopp'd and ſquared, ſome bored through, others half riven with great wooden Wedges, and Stones in them, and broken Ax-heads, ſomewhat like Sacrificing Axes in Shape; and all this in ſuch Places, and at ſuch Depths, as could never be opened from the Deſtruction of this Foreſt until the Time of the Drainage. Near a great Root in the Pariſh of *Hatfield*, were found 8 or 9 Coins of ſome of the *Roman* Emperors, but exceedingly conſumed and defaced with Time; and it is obſervable, that upon the Confines of this low Country, between *Burningham* and *Brumby* in *Lincolnſhire*, are ſeveral great Hills of looſe Sand, under which (as they are yearly worn and blown away with the Sand) are diſcovered many Roots of great Firs, with the Impreſſes of the Ax as freſh upon them, as if they had but been cut down a few Weeks; which I have ſeveral Times taken Notice of.

Hazle Nuts and Acorns have frequently been found at the Bottom of the Soil of thoſe Levels and Moors, and Fir-Tree Apples, or Cones, in great Quantities, by whole Buſhels together. And at the very Bottom of a new River or Drain, that the Drainers cut, (almoſt 100 Yards wide, and 4 or 5 Miles long, at the Charge of above 30000 *l.* beſides the great Sluice at the End thereof, which coſt near 30000 *l.* more) were found old Trees ſquared and cut, Rails, Stoups, Bars, old Links of Chains, Horſe-heads; an old Ax ſomewhat like a Battle-Ax, two or three Coins of the Emperor *Veſpaſian*; one of which I have ſeen, with the Emperor's Head on the one Side, and a Spread Eagle on the other; but that which is more obſervable, is, that the very Ground at the Bottom of the River was found in ſome Places to lye in Rigg and Fur, manifeſting thereby that it had been plow'd and tilled in former Days.

Mr. *Edw. Canby* told me, that about 50 Years ago, under a great Tree in this Pariſh, was found an old-ſhaped Knife, with a Haft of a very hard black ſort of Wood, which had a Cap of Copper or Braſs on the one End, and a Hoop of the ſame Metal on the other End, where the Blade went into it; which Blade ſoon mouldering away, he got a new Blade put therein, with this Diſtich upon it,

Ever ſince No's *Flood was I left,*
My old Blade's conſum'd, but this is the Haft.

The ſame Gentleman alſo found an Oak Tree within his Moors 40 Yards long, 4 Yards in Diameter at the great End, 3 Yards and a Foot in the Middle, and two Yards over at the ſmall End; ſo that the Tree ſeems to have been as long again; for which he was proffer'd 20 *l.* At another Time he found a Fir-Tree 36 Yards long, beſides the computed Length thereof, which might well be 15 Yards more. About 50 Years ago, at the Bottom of a Turf-pit, was found a Man lying at his length, with his Head upon his Arm, as in a common Poſture of Sleep, whoſe

Skin being as it were tann'd by the Moor-Water, preferved his Shape intire, but within, his Flefh and moft of his Bones were confumed and gone; an Arm of whom is now in the Poffeffion of Dr. *Nat. Johnfon.*

Though thefe Things may feem ftrange, yet many Authors have related the fame.

Cambden and others have told us, and it is a Thing well known, that moft of the great Moraffes, Moffes, Fens and Bogs in *Somerfetfhire, Chefhire, Lancafhire, Weftmoreland, Yorkfhire, Staffordfhire, Lincolnfhire,* and other Counties in *England,* are full of the Roots and Bodies of great Trees, moft of which are Fir; and that they have the fame Pofitions and Impreffions of the Fire and Ax upon them, that thofe have.

Giraldus Cambrenfis tells us, that in King *Henry* the Second's Days, by the Force of extraordinary Storms, the Sands were driven fo much off from the Sea-fhores, in *Pembrokefhire,* that under them were difcovered great Numbers of the Roots and Bodies of Trees in their natural Poftures, with the Stroaks of the Ax as frefh upon them, as if they had but been cut down yefterday, with a very black Earth, and fome Blocks like unto Ebony; the fame were difcover'd again at *Neugall,* in the fame County in 1590, and in *Cardiganfhire,* and other Places fince.

Dr. *Plot* mentions the like Roots and Trees to be found in *Shebben* Pool, the old *Pewit* Pool, and at *Layton* and other Places in *Staffordfhire*; and from their natural Situations and Poftures concludes, that they did certainly grow there.

Dr. *Leigh,* in his Hiftory of *Chefhire,* obferves, that in the draining of *Martin Meer* (which was perform'd but a few Years ago) were found Multitudes of the Roots and Bodies of great Firs, in their natural Poftures, with great Quantities of their Cones, 8 Canoes, fuch as the old *Britons* fail'd in, and in another Moor was found a Brafs Kettle, Beads of Amber, a fmall Millftone, the whole Head of an *Hippopotamus,* and Human Bodies intire and uncorrupted. I fuppofe he means, as to outward Appearance.

Many Places of the Soil of *Anglefea* and *Man,* as alfo of the Bogs of *Ireland,* are likewife full of Roots and Trees; but of what Sort I have not yet learn'd.

Verftegan tells us, that in many Places of the Moors and Moraffes of the *Netherlands,* great Fir-Trees are commonly found with their Tops lying to the North-Eaft, juft as they do in thefe Levels, and *Helmont* mentions the *Peel* there, a Turf Moor of 9 Miles broad.

I have likewife read in fome of the *French* Naturalifts, I think in Monfieur *de la Ferr,* that Trees and Roots are alfo frequently found in the Low Grounds, Levels and Moraffes of *France, Switzerland* and *Savoy.*

Rammazzini affures us, that in the Territories of *Modena* (which are feveral Miles long and broad, and at prefent a moft fruitful dry Country, tho' in the Time of the *Cæfars* it was nothing but a great Lake) are found at 30, 40, and 50 Feet deep, the Soil of a low marfhy Country,

Country, full of Sedge, Reeds, Shrubs, Roots, Trees, Nuts, Ears of Corn, Leaves of Trees, Branches and Boughs of Oaks, Elms, Wall-nuts, Aſhes, Willows, and the very Trees themſelves, ſome broke, ſome whole, ſome ſtanding upright, ſome lying at their Length, *&c.* with old Coins of the *Roman* Emperors, old Marbles and Stones ſquared, cut, carved, and wrought, with the Hands of Men, *&c.*

Moſt Men refer all this to *Noah*'s Flood; but if ſo, how comes it that the Trees and their Roots lie ſo near to one another, and why lengthways, from South-Weſt to North-Eaſt? Why ſome of them burnt, ſome chopt, ſome riven, ſome ſquared, ſome bored through? Why the Soil at the very Bottom of a great River lying in Rigg and Fur? And why the Coins of *Roman* Emperors found in thoſe Places, *&c.*? But I am of Opinion, that all thoſe Trees grew in the very Places where we now find them, both in this Country, and all others where they are found; to which I have heard but two Objections: The firſt, That *Cæſar* expreſly ſays, that no Fir-trees in his Time grew in *Britain:* But that *Cæſar* may have been miſtaken in this Point, may appear from what he mentions of the next Tree, the *Beech*, which he excludes alſo; and which is ſo common in every Part of this Nation: And in an old Deed relating to this very Chace, Fir-trees or Buſhes are mention'd as growing here and there one, about 300 Years ago; and it is very well known, that there was a Tree of the very ſame Wood growing upon *Hatfield* Moor Side within theſe 30 Years, which a while after was cut down, it being the very laſt of that Kind that was ſeen flouriſhing here.

The ſecond Objection is, That thoſe ſorts of Trees grow always on high Mountains and Rocks, and never thrive, nor naturally grow upon ſuch Low Grounds and Moraſſes, as theſe are, where we now find them; but though they do indeed in all cold Countries of the North, thrive beſt there upon the hardeſt Rocks and Mountains, yet are they ſometimes ſeen even there plentiful and great, in the Low Moraſſes of *Liefland*, *Courland*, *Pomerania*, and other Countries thereabouts; and in the Low Foreſts and Woods Weſt of *New England*, as I have heard Travellers affirm, what theſe Trees require, is a ſandy Soil; and if it lie never ſo high, or never ſo low, there they will grow, and there it is natural to them And as the Reverend Mr. *Earat* of *Hatfield* lately obſerv'd in the digging of a Pit of a great Decoy in theſe Levels, the Roots of the Firrs always ſtood in the Sand, and the Oaks in the Clay; and I have obſerved the ſame in Multitudes of Places of theſe Commons.

The Reaſon why all theſe Woods were deſtroy'd, we may learn from the *Roman* Hiſtorians; who frequently tell us, that when their Armies and Generals purſued the wild *Britons*, that they always fled into the Faſtneſſes of miry Woods and low watery Foreſts. *Cæſar* himſelf confeſſes the ſame; and ſays, that *Caſſibelan* and his *Britons*, after their Defeat, paſſed the *Thames*, and fled into ſuch low Moraſſes and Woods, that

that there was no Poſſibility of following them. We find alſo, that the ſtout Nation of the *Silures* did the ſame when they were ſet upon by *Oſtorius* and *Agricola*. The like did *Venutius* King of the *Brigantes*, who fled into the great woody Moraſſes of this Country, and perhaps into thoſe very ſame that formerly overſpread theſe Levels. And *Herodian* tells us, that it was the Cuſtom of the wild *Britons* to keep in the fenny Bogs and thick marſhy Woods, and when Opportunity offer'd, to iſſue out, and fall upon the *Romans*, who were at length ſo plagued with them, that they were forced to iſſue out Orders for the deſtroying and cutting down of all the Woods and Foreſts in *Britain*, eſpecially of all thoſe that grew upon low Ground and Moraſſes. This Order, I think, is mention'd in *Vopiſcus*; and that they were accordingly thereupon cut down, is evident in many Writers, who tell us, that when *Suetonius Paulinus* conquer'd *Angleſea*, he cut down all the Woods there. *Galen* tells us, that the *Romans* kept their Soldiers continually employed in cutting down of Woods, draining of Marſhes and Fens, and in paving of Bogs. It is manifeſt alſo, that they did not only do this themſelves, but alſo impoſed the ſame Taſk upon the *Britons*; for *Galgacus* in his Speech to his Soldiers, tells them, that the *Romans* made Slaves of them, and wore out their Bodies in cutting down of Woods and in cleanſing of Bogs, amidſt a thouſand Stripes and Indignities; and *Dion Caſſius* tells us, that the Emperor *Severus* loſt 50000 of his Men in a few Years time, in cutting down of the Woods, and cleanſing of the Fens and Moraſſes of the Nation.

As I have ſhew'd in general, that the *Romans* were the Deſtroyers of all thoſe great Woods and Foreſts, ſo now I ſhall ſhew in particular, that they actually were in this Part of the Country, and deſtroy'd this great and beautiful Foreſt of *Hatfield-Chace*.

The common Road of the *Romans* out of the South into the North, was formerly from *Lindum* (*Lincoln*) to *Segelocum* (*Little burrow* upon *Trent*) and from thence to *Danum* (*Doncaſter*, where they kept a ſtanding Garriſon of *Criſpinian* Horſe) a little off on the Eaſt and North-Eaſt of their Road between the two laſt named Towns, lay the Borders of the great Foreſt, which ſwarm'd with wild *Britons*, who were continually making their Sallies out of the ſame, and their Retreats into it again, intercepting their Proviſions, taking and deſtroying their Carriages, killing their Allies and Paſſengers, and diſturbing their Garriſons; which at length ſo enraged the *Romans*, that they were reſolved to deſtroy it; and that they might do the ſame more effectually, they marched with a great Army againſt the ſame, and encamped upon a great Heath or Moor, not far from *Finningly*, (as by their Fortifications there yet to be ſeen, is apparent) where it is probable, that a great Battle enſued; for hard by, is a little Town, called *Oſterfield*. Now as the latter Part of the Word is never uſed to be added to any other, but where there hath been a Battle; ſo the former ſeems to tell us what *Roman* General it was that fought, to wit, the famous *Oſtorius*, whom all the *Roman*

Hiſtorians

Historians assure us, was in those Parts. But who got the Victory, is not so easy to be judged of, though, no Doubt, it was the valiant *Romans*, who besides the Multitudes of the *Britons* that they slew, drove the rest back into the great Forest and Wood, that cover'd all this low Country: Whereupon the *Romans*, that they might both destroy it and the Enemy the easier, took the Opportunity of a strong South-West Wind, and set great Fires therein, which taking hold of the Fir-trees, burnt like Pitch, and consumed infinite Numbers of them; then when the Fir had done what Mischief and Execution it could, the *Romans* brought their Army nearer, and with whole Legions of captive *Britons* chopp'd and cut down most of the Trees, that were yet left standing, leaving only here and there some great ones untouched, as Monuments of their Fury, and unneedful of their Labour; which being destitute of the Support of the Underwood, and of their neighbouring Trees, were easily overthrown by the strong Wind; all which Trees falling cross the Rivers that formerly ran through this Low Country, soon damm'd up the same, turned it into a great Lake, and gave Origin to the great Turf Moors that are here, by the Gyrations and Workings of the Waters, the Precipitation there-from of terrestrial Matter, the Consumption and Putrefaction of rotten Boughs and Branches, and the vast Increase of thick Water Moss, which wonderfully flourishes, and grows upon such rotten Grounds; which, even now, since the Drainage, and since that the Country is laid dry for many Miles round about, yet for all that, are so turgid with Water, and so soft and rotten, that they will scarce bear Men to walk upon them.

Hence it is, that old *Roman* Coins, old *Roman* Ax-heads, *&c.* have been found by those Roots and Trees that lie at the Bottom of these Moors and Levels. Hence it is, that in all these Grounds are found great Numbers of Trees, that are burnt, some in two, and some lengthways, others hewn and chopp'd. Hence it is, that they lie by their own proper Roots with their Tops North-East: Hence it is, that some of the greatest Trees are found with their Roots on, and others, as they have laid all along, have had Branches growing out of the Sides, unto the Thickness and Height of considerable Trees. Hence it is, that both the Clay and Moor Soil of the Country, is in some Places two or three Yards higher than it was formerly, by the growing up of the same, and the daily Warp that the Rivers continually cast thereon, *&c.*

As the *Romans* were the Destroyers of this great Forest, so were they likewise of all those others that formerly grew upon the Low Countries of *Cheshire*, *Lancashire*, *Yorkshire*, *Lincolnshire*, *Staffordshire*, *Somersetshire*, &c. and of the very Countries before-mention'd beyond Sea, where such Trees are found. But as the *Romans* were not much in *Wales*, the *Isle of Man*, nor *Ireland*, so it cannot be supposed that it should be them that cut down their Woods; but though they did not,

yet

yet others did, for *Hollinſhead* and others of our Hiſtorians tell us, that *Edward* I. being not able to get near the *Welſh* to fight them, by their Continuance and Skulking in boggy Woods, commanded them all to be deſtroyed by Fire and Ax: And I doubt not at all, but that the Roots and Trees, before mention'd by *Cambrenſis* in *Pembrokeſhire*, were the Relicts of ſome of thoſe, that were then deſtroy'd: And as for thoſe in *Man*, and other Iſlands, they have all been cut down in the Time of War, and have laid till they were grown over with the Soil of the neighbouring Grounds: And as for thoſe that are found in the Bogs of *Ireland*, many of our Hiſtorians expreſly ſay, that *Henry* the Second, when he conquer'd it, cut down all the Woods that grew upon the Low Countries thereof, the better to ſecure his Conqueſt and Poſſeſſion of the ſame, to keep the Country in a ſettled Peace, and to diſarm the Enemy, who commonly truſting to ſuch Advantages, are apt to rebel.

I may alſo add, that it is a very common thing for Generals, even to this very Day, to deſtroy all the Woods that grow upon advantageous Places and Faſtneſſes in an Enemy's Country, if they intend to keep it; and that they always do it with Fire and Ax.

—on the ſame, *by the* ſame. n. 277. p. 1073.

2.] I have received ſome farther Informations about the *Fir-Trees* of *Hatfield-Chace.* I have been told by ſeveral Gentlemen, that about 20 Years ago, one *Sanderſon*, of *Hatfield*, died, aged near 80 Years, whoſe Father, much of the ſame Age, did frequently aſſure him, and other Gentlemen that were curious in the Matter, that he could very well remember many Hundreds of great Fir-trees, ſtanding one here and another there, in a languiſhing decaying Condition, half as high as Houſes, and ſome higher, whoſe Tops were all dead, yet their Boughs and Branches always green and flouriſhing, growing all of them in theſe Levels: And *John Hatfield* of *Hatfield*, Eſq; who is not above 40 Years of Age, has by him a large Twig that his Father pluck'd off from the Sprout of a green and flouriſhing Shrub of Fir, that grew from the great Root of one of the ſame kind in theſe Commons. And an old Man of *Croul* tells me, that he has heard his Father ſay, that he could remember Multitudes of Shrubs and ſmall Fir-trees growing here, while this Country was a Chace, and while the Vert was preſerved, before the Drainage. And in many old Charters, that I have ſeen, of *Roger de Mowbray*, Lord of *Axholm*, who lived in the Year 1100, relating to *Hurſt*, *Bell-wood*, *Roſs*, *Santoft*, &c. it appears, that then all theſe Places were cover'd with a great old decaying Foreſt or Wood; and not them only, but alſo all that low Common between *Croul* Cauſey and *Authrop upon Trent*; and though there be not one Stick of any ſuch Thing now to be ſeen, yet it is not only plainly manifeſt, that the ſame was true, from the Roots there found, but alſo from the ſaid Roots, that moſt of the Trees that then grew there, were Firs. All which were but the After-growth, and Relicts of the great Foreſt, that was deſtroyed by the *Romans*.

VII.

VII. There happen'd an Inundation at *Dagenham* and *Havering* in *Eſſex*, about four or five Years ago, by a Breach in the *Thames* Wall at an extraordinary high Tide; and by Means of the great Violence of the Water, a large Channel was torn up, or Paſſage for the Water of 100 Yards wide, and 20 Feet deep in ſome Places; and in ſome more, ſome leſs. By which means a great Number of Trees were laid bare, that had been there interred many Ages before.

Of Subterraneous Trees at Dagenham *in* Eſſex, *by Mr.* W. Derham. n. 335. p. 478.

The Trees were all of one Sort, except only one, which was manifeſtly a large Oak, with the greateſt Part of its Bark on, and ſome of its Head and Roots. The reſt of the Trees were taken to be *Yew*, from the Hardneſs, Roughneſs, and Weight of the Wood, notwithſtanding we have no *Yew* growing any where thereabouts; and it ſeem'd ſtrange to me, that *Yew* ſhould grow in ſuch vaſt Quantities, in ſuch a Soil, and ſo near the brackiſh Waters. Some took it to be *Horn-beam*, which grows plentifully alſo with us in the higher Lands (but I do not remember to have ſeen it in watery Places near us) but I rather incline to the Opinion of its being *Alder*, (which grows plentifully by our Freſh-Water Brooks) the Grain of the Wood, and Manner in which the Boughs grow, &c. much more reſembling that of *Alder*, than *Horn-beam.*

By lying ſo long under Ground, the Trees are become black and hard, and their Fibres are ſo tough, that one may as eaſily break a Wire of the ſame Size, as any of thoſe Fibres. This Toughneſs they maintain, if the Wood be kept dry; but by drying, thoſe Trees become cracked, and very flawy within, but look ſound outwardly, and with Difficulty yield to Wedges. But the Trees lying in the Marſhes, which are covered by every Flood, and laid bare by every Ebb, in a ſhort Time become very rotten.

There is no doubt, but thoſe Trees grew in the Place where they now lie, and that in vaſt Multitudes; they lying ſo thick upon, or near one another, that in many Places I could ſtep from one to another. And there is great Reaſon to think, that not only the Marſhes, which are now overflow'd (which are about 1000 Acres) are covered underneath with thoſe ſubterraneous Trees, but alſo all the Marſhes along by the River Side, for ſeveral Miles: For we diſcover theſe Trees all along the *Thames* Side over againſt *Rainham*, *Wennington*, *Purfleet*, and other Places: And in the Breach that happened at *Weſt Thorrock* about 21 Years ago, they were waſhed out in as great Numbers (as I have been inform'd) and of the ſame kind of Wood, as thoſe found lately in *Dagenham* and *Havering* Levels.

Theſe Trees are of different Sizes; ſome above a Foot Diameter, ſome leſs. I met with two of the leſſer Sort, ſtanding upright, in the ſame Poſture in which they grew; their Tops juſt above Low-Water, and their Bottoms (at leaſt the Bottom of the Channel) at 16 Feet Depth. We endeavoured to draw them out, but could not do it with

all

all our Strength. They feemed to be about two Inches Diameter in their Trunk, had fome of their Boughs on, were dead, and in all Likelihood, being young and light, efcaped the Force of what threw the other more large and unwieldy ones down.

Moft of the Trees had their *Roots* on, and many of them their *Boughs*, and fome a Part of their *Bark*. There was only one that I perceived had any Signs of the *Ax*, and its Head had been lopped off.

I could fee all along the Shores vaft Numbers of the *Stumps* of thofe fubterraneous Trees, remaining in the very fame Pofture in which they grew, with their Roots running fome down, fome branching and fpreading about in the Earth, as Trees growing in the Earth commonly are feen to do. Some of thofe Stumps I thought had Signs of the Ax, and moft of them were flat at top, as if cut off at the Surface of the Earth; but being rotten and battered, I could not fully fatisfy myfelf, whether the Trees had been cut or broken off.

The *Soil*, in which all thofe Trees grew, was a black oozy Earth, full of the Roots of Reed; on the Surface of which oozy Earth the Trees lay proftrate, and over them a Covering of grey Mould, of the felf fame Colour and Confiftence with the dry Sediment, or Mud, which the Water leaves behind it at this Day. This Covering of grey Earth is about 7 or 8 Feet thick, in fome Places 12 Feet or more, in fome lefs; at which Depths the Trees generally lie.

Another Thing I took notice of, was the *Pofture* in which the Trees lay, which was indeed in no kind of Order, but fome this way, fome that, and many of them a-crofs: Only in one or two Places I obferved they lay more orderly, with their Heads for the moft part towards the North, as if they had been blown down by a Southerly Wind, which exerts a pretty ftrong Force upon that Shore.

As to the *Age* in which thofe Trees were interred, it is hard to determine. Many think they have lain in that fubterraneous State ever fince *Noah*'s *Flood*. But although I have not the leaft Doubt, but that at this Day we have many Remains of the Spoils of that Deluge, even in the higheft Mountains; yet I rather think thefe Trees to be the Ruins of fome later Age, occafioned by fome extraordinary Inundations of the River of *Thames*, or by fome Storms, which blow fharply upon this Shore: Either of which Acts of Violence might be able to root up, and tumble down Trees growing in fo lax a Soil, as thefe manifeftly grew in at that Time. And as for extraordinary *Inundations* of the *Thames*, there is at this Day a Mark, which, if occafioned by an Inundation, was the Mark of an Inundation very prodigious, beyond all ever known to have been in that River; and that is a *Bed of Shells*, if not a kind of Marble too, lying crofs the Highway on the Defcent near *Stifford-bridge*, going from *S. Okendon.*

Below this Bed of Shells, at above 50 or 60 Yards Diftance in the Bottom of the Valley, runs a Brook that empties itfelf into the *Thames* at *Purfleet*, about three Miles from thence; which Brook ebbs and flows

flows as the *Thames* does, but not at any certain Height, by Reaſon of Mills ſtanding thereon; but above a pretty High-water in the Brook, the Surface of the Bed of Shells I find to lie above 20 Feet perpendicular. Conſequently if this Bed of Shells was repoſited in that Place by an Inundation of the *Thames*, that Inundation muſt be ſuch, as would have drowned a vaſt deal of the adjacent Country, and have over-topped the Trees by the River, in *Weſt-Thorrock*, *Dagenham*, and the other Marſhes, and probably by that Means overturned them.

Now had theſe Trees been left there by the *Univerſal Deluge*, we ſhould not find the Bed of Earth, in which they grew, ſo entire and undiſturbed, as it manifeſtly is at this Day, a ſpongy, light, oozy Soil, full of Reed-root; and I aſſure myſelf (although I never try'd it) of much leſs ſpecific Gravity than the *Stratum* above it is. Whereas I can from Experiments affirm, that in the three Places where I have tried it, the *Strata* are in a ſurprizing Manner gradually ſpecifically heavier and heavier, the lower and lower they lie.

As for the *Manner how theſe Trees came to be interred*, this I take to be from the gradual Increaſe of the Mud, or Sediment, which every Tide of the *Thames* leaves behind it. I preſume, thoſe Trees might be thrown down before the Walls or Banks were made, that keep the *Thames* out of the Marſhes; and then thoſe Trees were over-flown every Tide. And by Reaſon they lay thick, would ſoon gather a great deal of the Sediment. And after the *Thames* Walls were made, every Breach in them, and Inundation, would leave great Quantities of Sediment behind it; as I found in going over ſome of the Marſhes, ſoon after the late Breach, where I found the Mud generally above my Shoes, and in many Places above my Knees. And it is a Practice among us (of which we have divers Inſtances) that where a Breach would coſt more to ſtop, than the Lands over-flown will countervail, there to leave the Lands to the Mercy of the *Thames*; which by gradually growing higher and higher, by the Additions of Sediment, will in Time ſhut out the Water of the River, all except the higheſt Tides. And theſe Lands they call *Saltings*, when covered with Graſs; or elſe they become *Reed-ground*, &c.

That it was the Sediment of the *Thames* that buried theſe Trees, is farther manifeſt from what I ſaid before, of the Likeneſs of the Earth above them, in all Reſpects, to the Sediment the River now lets fall, when dry, which may be obſerved to conſiſt of many diſtinct Layers; ſome $\frac{1}{10}$ of an Inch thick, ſome leſs, and ſome ſcarce $\frac{1}{30}$ of an Inch. All which ſeveral Layers are, no doubt, the ſeveral Quantities, which every Tide left behind it. This Sediment, when dry'd by the Sun and Wind, becomes tough and hard, and looketh like a grey *Lapis Sciſſilis* or *Slate*, diviſible into many Plates or Layers. And what if we ſhould aſcribe the Conformation of *Slate*, *Muſcovia-glaſs*, and other the like laminated Concretions, to a like Work of Nature, by adding new Layers of ſuch Petrifactions, and Particles, as the Foſſil is made of?

I presume there will be no doubt but that the subterraneous Wood receives its Blackness from Vitriolic Juices in the Earth. I have try'd the Experiment, and find that *Alder-Wood*, whether green or old, becomes blackish, much of the same Colour as the Wood mentioned in this Paper, in a Solution of Copperas Which is not only an Argument, that the Blackness of the Wood is owing to Vitriol, but also that the Wood is *Alder*, or some such like Wood, that will become black with Vitriol; for I am informed that all subterraneous Wood is not black, particularly *Fir*. I have tried *Horn-beam* since, after the same Manner, and find that also becomes black, as the *Alder* doth.

Of strange Bones dug up near Canterbury. *And of the* Isthmus *between* Dover *and* Calais, *&c. by Mr.* W. Somner. n. 272. p. 882. Fig. 26, 27.

VIII. 1.] Mr. *John Somner*, in the Month of *September* 1688, sinking a Well at a new House of his in *Chartham*, a Village about three Miles from *Canterbury*, towards *Ashford*, on a shelving Ground or Bank-side, within twelve Rods of the River, running from thence to *Canterbury*, and so to *Sandwich* Haven; and digging for that purpose above seventeen Feet deep, through gravelly and chalky Ground, and two Feet into the Springs, there met with, and turned up a Parcel of strange and monstrous Bones, some whole, some broken, together with four Teeth, perfect and sound, but in a manner petrified, and turned into Stone; weighing (each Tooth) something above half a Pound, almost as big (some of them) as a Man's Fist: Cheek-Teeth, or Grinders, as to the Form, they are all, not much unlike, (but for the Bigness) the Grinders of a Man. I remember to have read in *Ludovicus Vives*, of such a Tooth, but a little bigger *(dens molaris pugno major)* which was shewed to him for one of St. *Christopher*'s Teeth, and was kept in a Church that bore his Name. Just such another Tooth, *of the Bigness*, he saith, *of an ordinary Fist*, was seen by *Acosta* in the *Indies*, digged out of the Ground, in one of their Houses there, with many other Bones; which put together represented a Man of a formidable, or as he speaks, *deformed Bigness*; as he judged of it. And so must we have judged of these Teeth, and of the Body to which they belonged, had not other Bones have been found with them, which could not be Man's Bones. Some that have seen them, by the Teeth and some other Circumstances, are of Opinion, that they are the Bones of an *Hippopotamus*, or *Equus Fluvialis*; that is, a *River Horse*; for a *Sea Horse*, as commonly understood and exhibited, is a fictitious Thing. Yet *Pliny* makes *Hippopotamum (mari, terræ, amni communem)* to belong to *Sea*, *Land*, and *Rivers*. But what the Differences and Properties of each Kind are, I leave to others to enquire. The Earth or Mould about them, and in which they all lay, being like a Sea-Earth, or Fulling-Earth, not a Stone in it, unless you dig three Feet deeper, and then it rises a perfect Gravel.

It is not easy to define or determine what the Creature was; and doubtless dubious enough it is, whether of the two, the Sea or the Land, may more rightly lay Claim to it. But I am of Opinion, that it is some Marine,

Marine, or Sea-bred Creature, to which the Land can of Right lay no Claim. But supposing it a Sea-bred Creature, how then (will some say) should it possibly come there? and at such a Depth under Ground? I answer, first, with as little Wonder as a Land-creature should, which who with Reason can imagine to have ever had at first so deep a Burial? Next, I say, the Mould, Soil, or Earth, wherein it lay, was altogether miry, like to that *cœnum* (*oose*, some call it) on many Parts of the Sea-coast, both in *England* and abroad. But how possibly (will it be said) a Sea-creature, when found at so remote a Distance from the Sea? For Solution (it may be) of this, and the like incidental Doubts, and removing all Rubs out of the Way of this Conjecture; I shall consider the four following Queries:

1. Whether the Situation and Condition, Face and Figure, of the Place, may possibly admit of the Sea's once insinuating itself thither?

2. Whether (that Possibility being granted, or evinced) the Sea did ever actually insinuate itself so far as to this Place, and when?

3. How in Probability, and when, this Valley or Level, being once Sea-land, should come to be so quite deserted and forsaken of the Sea, as it is at this Day; the Sea not approaching by so many, a dozen Miles, or more?

4. By what Means the Sea, once having its Play there, this Creature comes to lodge, and be found so deep in the Ground, and under such a shelving Bank?

1. As for the first (the Place's Capacity and Aptitude for the Sea's Influx or Insinuation) such as know the Situation, withal, cannot but know, and must agree it to be so. As for Strangers, and such as are unacquainted with the Place, they are to be informed, that the Place (the *locus loci*) we are upon, is a Part of that wide, fair, and fruitful Level, or Valley, extending itself not less than twenty Miles in Length, between a continued Series and Range of Hills, Downs, or high Grounds, lying at a pretty Distance each from other all the Way; beginning at the *East Kentish* Shore, and stretching itself Westward by *Sandwich*, *Fordwich*, *Canterbury*, *Chartham*, *Chilham*, *Godmersham*, *Wye*, *Ashford*, sometimes in a direct, sometimes in a winding Course, as far at length, as to that famous spacious Level of *Romney-marsh*; and it is wash'd and water'd all along, at least from about *Ashford*, by a sweet and pleasant River running through the midst of it, as far as to *Sandwich*, and there by the Creek, or Haven, emptying itself into the Sea: Nothing at all of Obstruction, by the Interposition of Hills, or high Grounds, hindering or controlling the Sea's free Play and Passage for so many Miles together. The Place then, with the Parts, the Tract above and below it, from the Condition or Constitution of it, is plainly not unapt or uncapable of the Sea's Insinuation and Influence.

If any ſhall object, *Canterbury*'s being in the Way, as an Obſtruction or Bar; they are eaſily enough anſwer'd. For although that City ſeems (and indeed is at this Day) for the moſt part ſomewhat elevated above the Pitch of the reſt of the Valley or Level we are upon; yet not ſo much as to defend itſelf many times from Floods and Overflowings, in the lower, and moſt depreſſed Parts of it, even by the Springs it ſtands upon; towards the helping whereof, by the Care and Providence of former Ages, it is very certain, and by digging Wells, Vaults, Cellars, and the like, daily experimented, that the moſt Part of the City, not excepting the very Heart and Center of it, is made and raiſed Ground: The Tokens of Foundations upon Foundations, to a very conſiderable Depth, daily appearing, and the Ground (as at *Amſterdam*, *Venice*, and elſewhere) for ſupporting Superſtructures, in ſeveral Places often ſtuck and ſtuffed with Piles of Wood, or long Poles and Stakes, forced into the Ground, as Wells and Cellar-diggers have informed me. And, as if, where now the Bull-ſtake Market-place is kept, the River had ſometime had its Courſe or Current, Pits, and other like Tanner's Utenſils, have, not many Years ſince, been met withal in digging for Cellars thereabouts. To this let me add, that my next Neighbour in *Caſtle-ſtreet*, within theſe thirty Years ſinking a Cellar, did a good Depth *(five or ſix Feet deep)* light upon, and was put to ſome Stop and Stand in his Work by, a ſtrong and well-couched arched Piece of *Roman* Tile or Brick, which he was fain to take, or break aſunder, and remove, before he could proceed. However, then *Canterbury* may now ſeem to ſtand in the *Æſtuary*'s Way, yet Time was, when in Probability it did not; when, I mean, the Place, the Soil, which now the City occupies, as the reſt of the whole Valley both above and below it, was of too low a Pitch to be an Obſtacle to it.

As to the ſecond Enquiry, (whether probably the Sea did ever actually inſinuate itſelf ſo far as to this Place, and when) the Anſwer is not ſo eaſy: Record of it, we have none. The beſt and eldeſt Account we have now of the Condition, Site, and Conſtitution, of theſe our Eaſtern Parts and Tract, we owe to *Julius Cæſar*, and the *Romans* after him; from whom we have not the leaſt Hint of any ſuch Thing, but rather the contrary; both the Sea-coaſt and Inland Parts, by his and their Relation, bearing in a manner one and the ſame Face and Figure then, as now. However, that the Level we are upon, was ſometime an *Æſtuary*, or Arm of the Sea, ſeveral *Criteria*, or Tokens are not wanting: For Example, beſides what may be argued and inferr'd from this Parcel of ſtrange Teeth and Bones now under Conſideration; much (as I conceive) there is of Probability for it, reſulting from our River's Name of *Stoure* more anciently, not ſeldom both called and written *Æſtur*, *Æſture*, &c. which I doubt not to proceed and come from the Latin *Æſtuarium*, and in proceſs of Time to have been corrupted and contracted into *Sture* and *Stoure*; giving Name in part to *Stourmouth*, a Place

about

about six Miles Eastward from *Canterbury*; so called from the River's disemboguing there into the Sea or Salt-water flowing up thither, as also giving Name to the Manor of the Archbishop's, at this Day, and for some Ages past called *Westgate-Court*, at *Canterbury*; but more anciently, as in the Conqueror's Time, (witness *Domesday* Book) called the Manor of *Esture* and *Esturesate*, from its Situation by the *Sture* or *Stoure*. From which Occasion, doubtless, the late Lord *Finch*'s Seat in ——— about five or six Miles nearer to the Spring-Head, at this Day vulgarly miscalled *East-Steward*, is of old sometimes called *Esture*, sometime *Est sture*. From *Saxon* Monuments and Records I could easily trace the Name up to a very high Date, by many Examples.

But to proceed to other *Criteria*; as by the Teeth and Bones now under Consideration, we have an Instance on that Side of the Valley for the Probability of the Sea's *quondam* Occupation of it; so I shall give here another no less remarkable from the other, or opposite side of it. By credible Relation, then, at a Place called *Webstere*, an obscure Village about three Miles from *Canterbury*, Eastward, lying under the Brow of the Hill stretching out by *Upstreete*, as far as to the West-end of *Sarrwall*, by which you make your Entrance into *Thanet*, upon the like Occasion to that here at *Chartham*, (the digging or sinking of a Well) at a very great Depth, store of Oysters and other-like Shells, together with an Iron Anchor, firm and unimpaired, were found and turned up in our Time. The like I have been told of an Anchor in our Days, digged up at *Broomedowne*, on the same side of the Level, somewhat above *Canterbury*, Westward.

As to the third Query, how in Probability, and when this Valley or Level, being once Sea-land, should come to be so quite deserted, and forsaken of the Sea, as it is at this Day, the Sea not approaching it by so many, a dozen Miles, or more? In Answer whereof, I must needs say and grant, that in case this Level were once Sea, an *Æstuary* I mean, or Arm of it; so very long it was ago, as we may not reasonably think, that *Canterbury* (whether as a City, or never so mean a *Pagus*, or Village) was then in *rerum natura*, or a Place inhabited; which haply it may have been, if not as long as *Julius Cæsar*'s Days, yet undoubtedly not long after. For an Account we have of it (as of some other Places in *Kent*) in the *Romans* Time, from *Ptolomy*, *Antoninus*'s *Itinerary*, and elsewhere. Now elder Records, either of *Kent*, or of *Britain*, that we may confide in as authentic, we have none, that I know of, before the *Romans* Time. We must either, therefore, throw off all further Inquiry, or else cast about for Information as we can. Such as are for this latter, will tell you, that the World is very aged, many thousand Years old, and that many and manifold are the Alterations which Time hath made in several Parts and Quarters of the World; to the Notice and Discovery whereof no written Record, or unwritten Tradition at this Day, can reach or direct us: Tradition itself (longer liv'd many times than any written Evidence) failing us for Age. Of such a Nature they

conceive

conceive may this of the *Æſtuary* be, ſo very ancient, as Time hath quite worn out the Memory of it; and that the Reaſon of the Sea's Receſs here, with an abſolute Valediction to the Place of its wonted Reſort, was its breaking, burſting, and cleaving aſunder, that *Iſthmus*, or Neck of Land, between *Gaul* and *Britain*, rendering the latter of the ſame Continent with the former: Such Things ('tis certain) have happen'd elſewhere. Thus (ſays *Seneca)* hath the Sea rent *Spain* from the Continent of *Africk.* Thus (as he adds) by *Deucalion*'s Flood was *Sicily* cut from *Italy.* More Inſtances of this Kind may be found in Mr. *Cambden*'s *Cantium*, and elſewhere. And although there be no certain Evidence of ſuch an Accident here, from ancient Hiſtorians or Geographers, yet is the Thing ſo ſtrongly and rationally argued, by him eſpecially, as by *Verſtegan* alſo, *Twine*, and others before him; and the Conjecture back'd with ſuch Plenty of probable *Criteria*, by the former, that I cannot but be of that Opinion; eſpecially, when to the Plenty of Arguments muſter'd up by Mr. *Cambden*, I ſhall have contributed this one, by him and the reſt omitted; which is, that by a received conſtant Tradition, *Romney-Marſh*, that large and ſpacious Level, containing (ſaith Mr. *Cambden*) fourteen Miles in Length, and eight in Breadth, was ſometime Sea-Land, lying wholly under Salt-water. And if I may gueſs at the Time and Occaſion of both that, and our *Canterbury* Level's Recovery from the Sea, I ſhall be apt to pitch upon that of the Sea's breaking through, and in Time working and waſhing away that *Iſthmus* between us and *France.* And then, whereas beforetime *Romney* Level (which had and hath its *Stoures* too, or *Æſtuaria* as well as ours) and this other not improbably (no high Lands, as we ſee, interpoſing or impeding their Conjunction) were but one and the ſame Level, and lay under the Sea; now both the one and the other (the Sea having ſo much more Play and Elbow-room, than formerly, by cleaving aſunder the *Iſthmus)* were reſcued from it, and of an *Æſtuary* became ſuch a rich and noble Valley or Level, as is ſecond to none in *England.*

And if from hence any one ſhall take an Hint, to conſider of the *Netherlands* or *Low Countries*, and enquire whether thoſe in whole, or in part, may not have ariſen out of, and conjecturally aſſigned for our *Kentiſh* Lowlands, I ſhould not at all wonder at it.

As to the fourth Query, by what Means the Sea once having its Play there (at *Chartham)* this Creature comes to lie and be found ſo deep in the Ground, and under ſuch a ſhelving Bank? My Anſwer is, That ſuppoſing this with the reſt of the Level or Valley once occupied by the Sea, or Salt-water; that being a Creature which by Fluxes and Refluxes always is in Motion, and thereby in Time beating upon, and working itſelf into the Bank, or riſing Ground there, might at length ſo far undermine, eat into, and looſen it, as to fetch down ſo much Mould or Earth upon, or over the Place, as to lodge the Creature at ſo great a Depth. Or elſe perhaps, the continual Agitation of the Water might in Time force, drive up, and caſt over it that great Quantity

tity of Ooſe, Earth, and other Matter, under which it lay. By the way, it is obſerved, that the Nature of the Soil here and there is ſuch, ſo looſe, ſupple, rotten and ſandy, that meerly of itſelf, it is apt to ſink, and fall in; as was lately experienced by a Saw-pit digg'd hard by, which after a little Time, by the Earth's giving way on each Side of it, fell in, and fill'd up itſelf.

Fig. 26, 27. *Repreſent two of the* Teeth *above mention'd.* Fig. 26, 27.

2.] Mr. *Somner* is of Opinion, (with Mr. *Cambden*, and other Antiquaries) That is highly probable (if not abſolutely certain) that *France* and *England* (or *Gaul* and *Britain*) were anciently joined by an *Iſthmus*, or Neck of Land, where now is the narrow Paſſage between *Dover* and *Calais:* Which, many Ages ſince (beyond the reach of any Hiſtory now extant) was (by the Seas violently beating upon it on both Sides) worn away, or broken through. Whereby, what was once an *Iſthmus*, is now become a *Fretum* or narrow Sea. *—on the ſame, by Dr.* J. Wallis. n. 275. p. 967.

Mr. *Cambden* is his *Britannia* (in that Chapter where he treats of *Kent*, or *Cantium*) gives us many cogent Arguments, which, if taken all together, ſeem to me a convincing Evidence, that there had once been ſuch a Conjunction; but not for many Ages now paſt.

To which I may add one more (of which Mr. *Cambden* takes no notice in this Place) from the *Unity of Language* between the ancient *Gauls* and *Britons*; and from the great Intercourſe between thoſe in *Gaul*, and the *Druides* in *Britain* (of which ancient Writers take notice:) which is not likely to have been, if there had not been an eaſy Communication between the one and the other. Which, though it be not a *Phyſical* Argument (as are thoſe of Mr. *Cambden*) is a good *Moral* Inducement, in Confirmation of them.

To thoſe Arguments of Mr. *Cambden*, Mr. *Somner* adds another, *a Parcel of ſtrange and monſtrous Teeth and Bones*, which (in the Month of *September* 1688.) upon digging a Well in the Pariſh of *Chartham* (about three Miles Southward from *Canterbury*) in the Land of Mr. *John Somner*, were found at the Depth of *ſeventeen Feet under Ground.* Which *Bones* and *Teeth* (from the Figure and Greatneſs of them, and from the Condition of the Earth wherein they were found) he judgeth to have been the Remainders of ſome *Hippopotamus*, or other large *Marine* Animal, which (many Ages ſince) had periſhed there; which hath ſince been covered with this Depth of Earth.

This *Chartham* he obſerves to lie about the middle of a large rich Valley, for about twenty Miles or more in Length, and of a conſiderable Breadth; having on each ide of it, at a conſiderable Diſtance, a long Tract of Hills, or high Grounds. Through which Valley, there now runs the River *Sture*, *Stoure*, or *Eſture*, for twenty Miles or more, by *Aſhford*, *Wye*, *Godmerſham*, *Chilham*, *Chartham*, *Canterbury*, *Fordwich*, and ſo to *Sandwich*, where it diſcharges itſelf into the Sea.

This

This long and large Vale, (from the Situation of the Place, the Nature of the Soil, and the Remains of this Marine Animal, lodged here at so great a Depth under Ground) he judgeth to have been (in former Ages) an *Æstuarium*, or Arm of the Sea, into which the Sea, (being stopped by the *Isthmus*, which then joined *France* and *England*, from the Course which now it takes) did discharge itself; which, in Process of Time, being filled up, (partly by the Earth, Sand, Oose or other Matter brought in by the Sea, and lodged there, partly by the Earth washed down, or falling upon it from the Hills on both Sides) is reduced to the State we now see.

Let us then consider, what must have been, if this Hypothesis be true; and how it agrees with what we see.

First, if such an *Isthmus* had once been, where now is the Pass between *Dover* and *Calais*, the great Seas on both Sides must continually beat upon it with a fierce impetuous Tide, twice in four and twenty Hours. The Northern Sea between us and *Holland* (called *Oceanus Germanicus*) on the Eastern Side; and the Western Sea between Us and *France* (called *Oceanus Britannicus*) on the Western Side. Which (in Process of Time) may well be supposed likely enough to wear away, or break through a narrow *Isthmus*.

The Western Tide coming in fiercely between Us and *France*, fretting on the Coast on both Sides, must needs be supposed to bring with it a great deal of Earth, Sand or Mud. But, being stopped in its Current by this *Isthmus*, did not deposit it (as might be thought) on the Side of it, (which might strengthen it) but found an Opportunity of discharging itself on the spacious Level of *Romney-Marsh*; (which, as *Cambden* tells us, is *fourteen Miles in length, and eight in breadth*) fretting that *Isthmus* as it comes along; and then (at standing Water, about the Tide's Recess) letting it fall on that Level, and lodging it there: But then again, fretting that *Isthmus*, and the Coast all along, as the Tide returns, with a like Force as it came in. Which gives us a fair Account, both how that *Isthmus* might be washed away, aud how that Level might be raised to that Height it now is. For no Man can doubt (who doth well know the Situation of the Place, and the Nature of the Soil) but that all that Level had heretofore been Sea. And, even at this Day, it lies so much lower than the Surface of the Sea at High-Water, that it would (much of it) be overflowed every Tide, if not defended (at a vast Charge) by *Dim-church* Wall, for many Miles together.

Whether it had a like Opportunity of such an Indraught (and in what Proportion) on the *French* Coast, I cannot tell. But, that this is the Condition of *Romney-Marsh*, no Man doubts.

The *Northern* Sea (between us and *Holland*) must, in like Manner, have beat on the East Side of that *Isthmus* with a like impetuous Tide, twice in four and twenty Hours. But, being there stopped in its Course, would have the like Opportunity of discharging itself on the Coast of

Holland, (as the Weſtern Sea on *Romney-Marſh*.) Whence it is, that *Holland* and *Zealand*, which (by the Conſent of all) is judged to have been once Sea, is now raiſed thirty or forty Feet higher than it had once been.

And the ſame *Northern* Sea, which (on this Account) hath ſo large an Inlet (Eaſtward) on the Coaſt of *Holland*; would (Weſtward) inſinuate itſelf likewiſe on the *Engliſh* Coaſt, where-ever it might find low Grounds. Which is the Caſe of this Valley, where now runs the River *Sture*, *Stoure* or *Eſture* (which Name it is ſuppoſed to have taken from the Corruption of *Æſtuarium*) for more than twenty Miles; (and nothing appears why we ſhould not think it had ſo done;) entering at the low Grounds near *Sandwich* (cloſe by that *Iſthmus*) and running up that Level (by *Canterbury*, *Chartham*, *Chilham*, and ſo forth) as far as *Aſhford*, or farther, which Valley had once been much deeper than now it is. For, it ſeems, that even at *Chartham* (which is now twelve Miles from the Sea) the Ground is raiſed at leaſt ſeventeen Feet; and the Soil, at that Depth, found to be of a like Condition, as where the Sea is known to have been; and nearer to the Sea, it may well be preſumed to have been yet deeper. Which is confirmed by the Reliques of this Marine Animal there found; and by Anchors, and Shells of Fiſhes, found elſewhere in the Borders of this Valley, at a great Depth under Ground.

Now, that the Sea may thus raiſe the Ground on ſuch In-draughts, by Sand, Earth and Mud, brought in and lodged there at every Tide, is not at all unlikely: For we ſee the ſame at this Day. Particularly, in the Iſle of *Oxney* (near adjoining to *Romney-Marſh*) there was a low Level, oft in Danger of being overflowed by the River *Rother*: But, ſomewhat more than threeſcore Years ago, the Sea being let in, hath raiſed that Level very conſiderably; by bringing in, and lodging there a conſiderable deal of Earth and Mud every Tide: But withal, it hath ſo fretted the Channel, by which it enters and goes out again, that the Channel by *Rye*, which (within my Memory) was ſo ſhallow near what was call'd *Kent*-Bridge, that Men were wont to ride through it; but now (by the Tide's entering and returning) that Bridge is long ſince ſwallowed up, and the Channel become ſo broad and deep, that a Veſſel of good Burden might ride there at Anchor. A fit Reſemblance of the Sea's fretting this *Iſthmus*, and filling up the *Æſtuaries* on both Sides of it.

The like, in good Meaſure, is to be ſeen at (what they call) the *Dogger Sands*, which is a Bank of Sands lying (obliquely) from about the Coaſt of *Norfolk* toward the Coaſt of *Zealand*, or North-part of *Holland*, which is the Place where the Northern and Weſtern Tides (ſince the Rupture of the *Iſthmus*) do now meet; and do there (at ſtill Water, for about half an Hour or more, or at the turning of the Tide) depoſit the Mud and Sand, which (by their rapid Motion) is both Ways brought thither. Which is ſuppoſed to be the true Cauſe of that

that ſandy Bank. Whether this in Tract of Time may there form a new *Iſthmus*, (if the World laſt long enough) I cannot ſay, but I am apt to think that the former *Iſthmus*, if the Tides had ſtopped there, and had not found thoſe In-draughts, on which to lodge what it waſhed from thence, might have continued, and been more ſtrengthened, by what (upon the Return of the Tide) would daily be lodged there.

And upon this Account (I think) it is, that the *Iſthmus* at *Corinth*, though beat upon by two Seas (which gave it the Name of *Bimaris Corinthus)* is not thereby deſtroyed: becauſe there are not ſuch Tides to waſh it away, nor ſuch In-draughts, on which to lodge what ſhould be waſhed from thence.

But the Caſe is much otherwiſe with this *Iſthmus* of ours; where are all Things to countenance this Hypotheſis. The ſteep Cliffs at *Dover*, and thoſe at *Calais*, anſwering directly the one to the other; and appearing to View, as if that between them had been violently torn away. And the Sea between them (even at this Day) being much ſhallower at that Place than on either Side of it (as *Cambden* doth well obſerve) which are ſtrong Preſumptions, that there had been formerly ſuch a Conjunction.

The greateſt Doubt in this Caſe is, that there is no Hiſtory extant, which takes notice of ſuch an *Iſthmus*, or ſuch a *Rupture*, in this Place, which being a Thing remarkable, might have been thought worthy to be reported.

Which yet need not be thought very ſtrange, conſidering that we have no particular Account of the *Britiſh* Coaſt (which might determine this Queſtion) older than the *Romans* Acceſs hither with *Julius Cæſar:* Whereas this might happen'd many Hundred of Years before that Time, when though the Iſland might be known, yet not the particular Coaſtings of it to the *Greeks* or *Latins*.

But I have this farther to ſay; *Plato* tells us a Story (as of a Thing which happened ſome Ages before his Time, and which at that Time was in a manner generally forgotten) of an Iſland ſomewhere in the *Atlantick* Ocean, which by a Deluge and Earthquake (in the Space of a Night and a Day) was deſtroyed and ſwallowed up by the Sea; whereby that Sea (formerly navigable) was for ſome Time become unnavigable or unſafe, by Reaſon of the Mud and Reliques of that abſorbed Iſland. The Words of *Plato* (as tranſlated in *Henry Stephens*'s Edition, p. 25.) runs thus; *Poſt autem, quum diluviorum & terræ motuum intemperies extitiſſet, unius noctis & diei ſpatio, omne illud bellicoſorum hominum genus in terram abſorptum fuit, illaque etiam Atlantica Inſula* (Ἀτλαντὶς νῆσος) *maris fluctibus plane obvoluta diſparuit, unde & illud mare trajectu difficile eſt, quum lutum adhuc copioſum Inſulæ iſtius remanſerit.*

Which ſeems to me very applicable to the Rupture of this *Iſthmus:* Whereby this Iſland was not indeed wholly deſtroyed, but was broken off from the Continent, to which it was before united. And, upon ſuch

such an Accident, the Sea must needs be disturbed, and put out of its Course, and render'd unsafe for Passage, before it came again to be settled. For though the first Breach might be made in the Space of one Night and Day, we cannot suppose the whole Bulk of it, when once broken, was presently carried smooth away; but first the Top or upper Part of it (in a Day and Night's Time) and afterwards the lower Parts of it by Degrees. Which would render that Sea, if not quite unpassable, at least troublesome and unsafe.

And if in some Circumstance this Narration chance to differ from the Matter of Fact, as calling the *Rupture* of this *Isthmus* the *Subversion* of an *Island*, this must be allowed in the Narrative of an old Tradition from Hand to Hand: For as such it is there brought in.

For *Plato* doth there introduce *Critias* (then an ancient Man) telling a Story, which (when a Boy of ten Years old) he had heard from his Grandfather (who was ninety Years of Age) of what *Solon* (long since dead) had told him; namely, that an *Ægyptian* Priest had (long before) told *Solon*, that it did appear from some old *Ægyptian* Records (of which the *Greeks* had no Knowledge) that such a Thing had happen'd in an Age so long before, as in comparison of which *the Greeks were but as Children*. And all this Tradition (through so many Hands, and at such great Intervals of Time) is, at every Step, reported from the Relator's present Memory. And 'tis very possible, that some one or other of these Relators might so far mistake, or misremember, as to call that a *Dissolution* or *Disappearance* of an Island (*ἠφανίσθη*) which was but a *Tearing it from the Continent*.

It serves, however, to the present Purpose, if at least so much of the Story be true, *That long before* Plato*'s Time, there had been some such Dissolution or Rupture of an Isle or Isthmus, somewhere in the* Atlantick *Ocean*, (that is, in the Northern Sea) *of which there were some Symptoms yet remaining in* Plato*'s Time*. For, this being admitted, it is as applicable to the present Case (as to any we know) of which there are so many Symptoms yet remaining to this Day.

I know that *Rudbeck* in his *Atlantica*, Cap. 7. Sect. 8. pag. 293. doth endeavour (in Favour of his *Sueonia*) to put an allegorical Sense upon this whole Passage.

But I see not why it may not be understood in a plain literal Sense, as a true Matter of Fact, (though perhaps a little disguised, as was wont to be the Fashion in that Age in relating old Stories) and is very consistent with all that *Rudbeck* cites out of *Plato*, in that whole seventh Chapter of his *Atlantica*.

For the Name of the *Atlantick Sea* (wherein this Island is said to be) was not then (nor is now) confined to the Coast of *Sweden*, but extended as far as the *British* Island, and much farther. And when *Rudbeck* tells us out of *Plato*, that the whole *Atlantica* was as big as *Libya* and *Asia* (which whether meant of *either of them* singly, as *Rudbeck* understands it, or of *both together*, as the Words seem to import, ἅμα Λιβύας καὶ Ἀσίας μείζων, I will not contend; we cannot ſuppoſe it to be *Plato*'s Meaning, that this *whole Region* was ſwallowed up; but rather ſome ſmall Part of it, from whence perhaps the whole might take its Denomination. And though he tells us from another Writer, that it was *five Days Voyage* from the *Britiſh* Iſland, to (that part of) his *Atlantica*, where for *thirty Days together the Sun doth not ſet*; this hinders not, but that the *Britiſh* Iſlands may be part of the *Atlantick Region*, though ſo far diſtant from the utmoſt Northern Cape of it.

I know not whether I may yet venture farther: This Author tells us, that this *Æſtuary* (from *Sandwich* to *Aſhford*) might perhaps flow ſo much farther, as to meet with that *Æſtuary* on *Romney-marſh*, and (both being conjoin'd) become *one Level*.

There is, I think, about three or four Miles diſtance between *Aſhford* and the neareſt part of *Romney-marſh*: How the intermediate Lands be qualified, I do not well remember.

But if this be admitted, that the two *Æſtuaries* (that of *Stoure*, and that of *Romney-marſh*) in former Times may thus have met; this opens a new Scheme, of which before we were not aware. For then we muſt ſay, that the two Tides (that from the North and that from the Weſt) which now meet at the *Dogger Sands*, did then meet at the Confluence of theſe two *Æſtuaries*: And then (as was ſaid of the *Dogger Sands*) bringing, on both Sides, Earth, Mud, and Sand, to this Place, and lodging it there, might firſt form an *Iſthmus* there, and, by Degrees, fill up thoſe *Æſtuaries* on both Sides: In the mean while, waſhing away that *Iſthmus* between *Dover* and *Calais*, and opening a new Paſſage as now it is.

There be many other *Æſtuaries* in *England*, where the Sea now enters a great Way into the Land; and how far it might have entered farther in former Times, who can tell? As that Sea by *Briſtol* between *Wales* and *Cornwall*: That of the *Humber* between *Yorkſhire* and *Lincolnſhire*: And we may reaſonably think, that the *Waſhes* and the *Fens* in *Lincolnſhire*, may have heretofore been Sea, or overflowed by the Sea at high Tides: And that of the *Thames* (between *Kent* and *Eſſex*) which now flows (above *London* and *Brentford*) within a Mile of *Kingſton* (at Spring-Tides); it may perhaps ſeem too daring, to think it may formerly have flowed as far as *Oxford* (between *Shot-over* Hill and *Foxcomb* Hill) and ſo onward toward *Wallingford* (in the *Romans* Time called *Galena*); but there is this to countenance it, that (if I be not much miſinformed) there be frequently found (in our Stone-Quarries and Gravel-pits) about *Oxford*, Fiſh-ſhells, and even the Bodies of Fiſh petrified, at great Depths under Ground. And there have been (no doubt) and now are (in *England*) many other *Æſtuaries*, Creeks, or Arms of the Sea (entering a great Way within Land) ſome whereof may be in a Manner filled up, and become firm Land; others much narrower, ſhallower and ſhorter, than in former Times they have been. For it is the Nature of *Æſtuaries*, where the Tides flow in, to leave

behind

behind them, at their Return, much of Mud, Ooſe, or Sleech, as they all it, which doth in Time come to be firm Land.

3.] At *Hythe* in *Kent* (which is one of the *Cinque Ports*) there was (in our Fathers Time) a convenient Harbour for ſmall Veſſels, which is now ſwarved up; ſeveral Attempts have been made to recover the Harbour, but with ſmall Succeſs. For when (with great Labour and Charge) they have (in ſome Meaſure) opened it, it hath ſoon been filled up again, by what the Sea caſts up. And whoever conſiders the vaſt Quantity of (what they call) *Beach*; that is, a vaſt Multitude of ſmall looſe Stones and Fiſh-ſhells, caſt up by the Sea at *Hythe*, *Lyd*, and elſewhere, on the Coaſt of *Romney-marſh*, (for divers Miles in Length and Breadth, and to a great Depth) will not think it ſtrange, that a *Creek* or *Æſtuary* ſhould come in Time to be filled up, and become firm Land. And in many Places of this *Beachy* Ground, where (within the Memory of Perſons now living) nothing was to be ſeen, but ſuch looſe Stones and Shells (to a great Depth) it comes (by Degrees) to be covered with Earth, and becomes Paſture Ground. *—on the* ſame, *by the* ſame, *ibid.* p. 978.

On the contrary, that what was formerly Firm-land, may be ſo deſtroyed or waſhed away, as to become Sea, is evident from (what they call) the *Goodwin-Sands*, on the Coaſt of *Kent*, which is ſaid to have been the Lands of Earl *Goodwin*, but loſt by an Inundation about the Time that *Tenterden* Steeple was built, (which gave Occaſion to that Ironical Proverb of Things Cotemporary, that *Tenterden Steeple was the Cauſe of* Goodwin *Sands*.) The Occaſion of ſuch different Effects, depending on the different Situation of the Shores, and the ſetting of the Tides; ſo as to waſh off from one Place what it lodgeth on another.

And many ſuch Alterations (no doubt) have been of the Face of the Earth, all the World over, of which we have no particular Hiſtories. For the World was of great Age, before the writing of any Hiſtories (except the Bible) now extant.

And who knows, but that in former Ages, even amidſt the *Alps*, there may have been large *Lakes*, which, in proceſs of Time, (by Earthquakes or other Accidents) may have been drain'd of their Water, and become fruitful Valleys: Of which it is ſaid, divers Symptoms have been diſcover'd, even amidſt the *Alps*, in later Ages.

And ſomething of the like Nature hath happen'd within ſome few Years laſt paſt, in *Jamaica*, in *Sicily*, and other Places.

4.] Since I wrote laſt on the Subject of the *Iſthmus*, &c. I find Mr. *Luffkin* † has informed us of divers Bones of an extraordinary Bigneſs, found lately in a Gravel Pit, not far from *Harwich* in *Eſſex* (much like thoſe found at *Chartham* in *Kent*,) at a great Depth under Ground, which Bones he thinks rather to have been the Bones of an Elephant, than of an *Hippopotamos*, or other Marine Animal. *——on the* ſame, *by the* ſame. n. 276. p. 1030. † *Vid.* infra, S. X.

But

But which ever it is, it will equally prove thoſe Valleys to have been much deeper in former Times, than now.

I obſerve, that the River in *Eſſex*, and that in *Kent*, near which the Bones were found, are both of them named the *Stowr*; which, whether it be a Corruption of the *Latin Æſtuarium* (as Mr. *Somner* conjectures) or of the *Britiſh ys-dwr*, (that is, *the Water*) I will not diſpute.

And that the Bones were found (in both Places) much at the *ſame Depth*, (about 16 or 17 Feet under the Surface of the Earth) which therefore may (probably) have been lodged (in both Places) much about the ſame Time: And perhaps, when the Emperor *Claudius* brought his *Elephants* into *Kent* and *Eſſex*; as Mr. *Luffkin* intimates out of *Dion Caſſius*.

I obſerve alſo, that thoſe petrified Bones, in both Places, were found in *gravelly Grounds*, (as are thoſe petrified Shells, and Bodies of Fiſhes, in Gravel Pits and Stone-Quarries near *Oxford*). How far the Steams, Fumes, or Fluors of the Earth, which contribute to the Formation of Stone or Gravel, may conduce to the petrifying of theſe Bones, Shells, or other Bodies; I leave to the Conſideration of the Naturaliſts.

And whether the Impregnation of ſuch *Steams*, may not *Swell* ſuch petrified Bodies, to a larger Proportion than before they had. Like as we obſerve Wood (and other like Materials) in a moiſt Air to Swell; by the Diſtention of their Pores, upon the Intromiſſion of moiſt Particles. For I take all Petrifications to be made, either by *Incruſtation* or *Intromiſſion* of ſtony Particles.

And I well remember, that (many Years ago) at *Moldaſh* in *Kent*, (not far from *Feverſham*) on ſome high Grounds, and very ſtony, (which uſed to be ſometimes Paſture, and ſometimes plow'd) I have obſerved divers *Oyſter-ſhells* (petrified, or partly ſo) much larger and thicker than the ordinary Proportion of Oyſters in thoſe Parts, and very weighty; which Oyſter-Shells might have been purpoſely thrown there long before, as being reputed a good Manure for Land; and might have been there impregnated with like *Halitus*, *Effluvia*, as are the numerous Stones on thoſe Lands.

I have known the Inhabitants, heretofore, have uſed to cauſe the Stones, in thoſe Lands (becauſe they are very numerous) to be gather'd up, and carried off the Lands by Cart-loads, to make more Room for the Graſs to grow. But of later Years, they forbear (I have been told) ſo to do, as thinking the Warmth (or ſomewhat equivalent) of thoſe Stones, is rather an Help, than Hindrance of the Earth's Fertility.

But (to return to what I was ſpeaking of) I ſee not why we may not think the *Stowr* in *Eſſex*, and the *Stowr* in *Kent*, to have been (both of them) *Æſtuaries* of the *Northern Tyde*; before the Rupture of that *Iſthmus* between *Dover* and *Calais*: (And the like of the River

near

near *Malden*, and other ſmall Creeks on the Coaſt.) Though not ſo great as thoſe of *Humber* and the *Thames*: which were then *Æſtuaries* of the ſame Sea; as are many others on the Coaſt of *Scotland*.

I ſay, *before that Rupture*; For, ſince that Rupture, the Caſe (as to the *Thames*) is ſomewhat altered. For the Weſtern Tide (between us and *France*) which was then ſtopped at this *Iſthmus*, doth now flow on (through that *Fretum*) beyond the Mouth of the *Thames*, as high as the *Dogger-ſands*; which doth therefore ſupply the *Æſtuary* of the *Thames*, which was formerly furniſhed from the Northern Sea.

And theſe ſmaller *Æſtuaries* might ſooner be ſwarved up by what every Tide lodgeth there, while thoſe greater *Æſtuaries* are but ſhortned, and become narrower, than they had formerly been.

And as to the *Thames* in particular; it ſeems very evident, if we conſider their Situation, and the Nature of their Soil, that much of the Low Grounds (in *Kent* and *Eſſex*) on both Sides of the Mouth of the *Thames* (adjacent to the Sea) had formerly been Sea, (as well as that of *Romney-marſh.*) And when the Mouth of the *Thames* was ſo much wider, no doubt but it flowed much farther than now it doth. And how far, who can tell?

It may perhaps be objected, that the ſmall Rivers now remaining, in the Bottom of theſe Valleys, which may have been ſuppoſed (in former Times) to have been *Æſtuaries*, do now run more wriggling (with more Turnings and Windings) than do theſe Valleys. But this need not at all ſeem ſtrange, when as we may daily ſee the ſame, in the Bottom of a muddy Ditch (or Water-courſe) when the Water is almoſt drained off, the Mud yet remaining ſoft: the little Water, yet remaining, will work out of itſelf (amidſt the Mud) a wriggling Paſſage (according as the Mud will more or leſs give way) much more crooked, than was ſuch Ditch when full of Water, and the like muſt needs happen in the gradual Draining of ſuch *Æſtuaries*, according as the ſoft Earth will permit. Which Crookedneſs will continue, when the Banks on both Sides do (by Degrees) grow firmer.

As to what I ſaid concerning the Iſle of *Oxney*; A low Level in that Iſle, which had for divers Years lain under Water, is now raiſed by intromitting the Tide, to a conſiderable Height above what it had formerly been; and the Channel from thence to *Rye*, is (by the Tide's paſſing in and out) become much wider and deeper than heretofore. Both which are evident, and not to be denied.

If we look in the more ancient Maps of *Kent* (older than the Year 1640) we will find, that what we call the *Iſle of Oxney*, was then but a *Peninſula*; being (by a ſmall *Iſthmus* or Neck of Land at the North-Eaſt Corner of it) continued to the reſt of the Country: And the Tide from *Rye* to that Place (which now flows ſtraight onward on the North Side of the Iſle) was there ſtopped by that *Iſthmus*, and did

I wheel

wheel about on the South Side of it: Or rather, the River *Rother* did (from the North Side of the Iſland) wheel about by the South Side (to that Eaſtern Corner) and thence (by the Channel) to *Rye*.

While Things were in this ſtate; divers Mooriſh or Marſh-lands, adjoining to the River *Rother*, were oft in Danger (upon great Rains) to be drowned. But ſo it once happened (by what Accident I know not) that this drowned Land had unexpectedly (in a Night's Time, or little more) diſcharged itſelf on another Level, ſomewhat lower than itſelf.

Upon which Indication, it was thought adviſeable (by cutting that *Iſthmus*) to allow thoſe Waters on the North Side of the Iſland a ſtraighter Paſſage towards *Rye*; and to let thoſe lower Grounds for ſome Time to lye under Water (paying the Rent of them) till ſuch Time as (by intromitting the Tide) they might be ſomewhat heightned, and then timely recover'd. In Order to which, Commiſſioners of Sewers have ever ſince from Time to Time been iſſued out for that Purpoſe; and the Work in a good Meaſure effected, though not finiſh'd.

An Account of Large Teeth *dug up in* Ireland, *by Mr.* F. Neville. n. 346. p. 367.

IX. 1.] Four large Teeth were lately found in the North of *Ireland*, about eight Miles from *Belturbet*, at a Place call'd *Magbery*, in Part of the Biſhop of *Kilmore*'s Lands, ſinking the Foundation for a Mill near the Side of a ſmall Brook, that parts the Counties of *Cavan* and *Monaghan*.

Two of them are of a larger and two of a ſmaller Sort; the largeſt is the fartheſt Tooth in the under Jaw, the other is like it, and belongs to the oppoſite Side; the leſſer Tooth I take to be the third or fourth Tooth from it, and has its fellow: Theſe are all that were found, and one of them in a Piece of the Jaw-bone, which fell to Dirt as ſoon as taken out of the Earth; there was Part of the Scull found alſo of a very large Size and Thickneſs, but as ſoon as expoſed to the Air, that mouldered away as the Jaw had done.

The Account I had, led me laſt Week to the Place where I was reſolved to make the niceſt Search I could; but the Water-wall of the Mill being built, and the Ground all incumbered with the Earth, that was thrown up, I could have little Opportunity of doing any Thing but to enquire of the Workmen the Manner of finding the Teeth, and where and how they lay. There were ſome few Pieces of Bones found, but none entire, yet by thoſe Bits that were found, one might gueſs that they were Parts of thoſe that were of a larger Size.

The Place where this Monſter lay was thus prepared; which makes me believe it had been buried, or that it had laid there ſince the Deluge. It was about four Feet under Ground, with a little Riſing above the Superficies of the Earth, which was a Plain under the Foot of a Hill, and about 30 Yards from the Brook or thereabout. The Bed whereon it lay, had been laid with Fern, with that Sort of Ruſhes here called

Sprits,

Sprits, and with Bushes intermixed. Under this was a stiff blue Clay, on which the Teeth and Bones were found: Above this was first a Mixture of yellow Clay and Sand, much of the same Colour; under that a fine white sandy Clay, which was next to the Bed: The Bed was for the most Part a Foot thick, and in some Places thicker, with a Moisture clear through it; it lay sad and close, and cut much like Turf, and would divide into Flakes, thicker or thinner as you would; and in every Layer the Seed of the Rushes was as fresh, as if new pulled, so that it was in the Height of Seed-time, that those Bones were laid there. The Branches of the Fern, in every Lay as we opened them, were very distinguishable, as were the Seeds of the Rushes and the Tops of Boughs. The whole Matter smelt very sour, as it was dug, and tracing it I found it 34 Feet long, and about 20 or 22 Feet broad.

It will be well worth Consideration, what sort of a Creature this might be, whether Human or Animal: if Human, there was some Reason for the Interment, and for that Preparation of the Bed it was laid on; if Animal, it was not worth the Trouble; if Human, it must be larger than any Giant we read of; if Animal, it could be no other than an Elephant, and we do not find, that those Creatures were ever the Product of this Climate. And considering, how long this must have laid here, I do not believe the Inhabitants then had any Curiosity or Conveniency to bring such into this Kingdom; for I suppose the best of their Ships could not carry one. Then if an Elephant, or some other Beast which must have Proportion to the Teeth, it must have laid there ever since the Flood; and if so, then the Bed, on which it lay, must be of its own making: Whence it will follow, that the Flood coming on him, while he lay in his Den, he was there drown'd, and covered with Slime or Mud, which since is turn'd into the Substance of the Earth before-mention'd. There were also a great many Nut-shells found about the Bed, perhaps those might have been on the Bushes, which composed Part of the Bed.

The two large Teeth are of equal Weight, two Pound three Quarters each: The two little Teeth are six Ounces each; but there are some of them wasted, and some of the Holders that go into the Jaw broken off.

2.] The *Four Teeth*, with some of the Fragments of the Bones that were found with them, have been brought to *Dublin*, where I have examin'd them particularly, and taken the following correct Sketches. ——Remarks *on them by* *Dr*. T. Molyneux. *ibid.* p. 370.

I am fully convinced, that they must certainly have been the *Four Grinding Teeth* in the lower Jaw of an *Elephant*: And that the many loose Fragments of those large Bones, that were found with them, must have been Remains of the same *Animal*.

 Fig.

Fig. 28. *Fig.* 28. *AA* is the larger Grinder of the under Jaw on the right Side, weighing two Pounds and three Quarters of a Pound.

b, b, b, b, b, b, b, are white, rough, indented Borders, ſeven in Number, of an irregular Shape, ariſing about the tenth of an Inch higher than the hard black ſhining Surface of the Tooth; this rough raiſed Work ſerves for the bruiſing and grinding the *Animal's* Food, the tough Grains of Rice, Leaves, Fruits, and the Boughs of Trees, and is made of ſo extreme an hard Texture, that it reſembles large knotted Threads of white *Glaſs*, laid on and cloſely faſtned to the dark Superficies of the *Tooth:* And anſwers that glaſſy Surface wherewith Nature has armed the Outſide of the *Teeth* of moſt *Animals*, to prevent their wearing from the conſtant Attrition in Chewing of their Foods.

c, c, c, c, c, is that Part of the Tooth which riſes above the Gums, and continues even now diſtinguiſh'd from the reſt of the Bone, by having its Colour of a different Shade.

d, d, d, d, d, d, d, are many ſtrong Tangs or Roots, ſeemingly united all together, by which the Tooth received its Senſe and Nouriſhment, and though it was ſo large and ponderous, by theſe it kept firmly fixt into the Jaw.

For the Mechaniſm Nature ſhews itſelf to have followed in framing the *Teeth* of this *Animal*, is no more than this: Whereas in other Creatures, ſhe has divided that bony Subſtance wherewith they chew their Food, each having its peculiar Roots to ſecure its Articulation in the Jaw-bone; ſhe has in this of ſo great Bulk, for the greater Strength, Stabiliment, and Duration of its *Teeth*, and the better to provide for a compleat Attrition of the Aliment, in order to perfect the Digeſtion ſo thoroughly, as to ſuſtain the Life of the Animal for two or three hundred Years, (as it is a common received Opinion in the *Eaſt*) ſhe has, I ſay, contrived to make the Subſtance of the *Teeth* in their Roots below, and in their upper Parts above the Gums, cloſely unite together; and coaleſcing thus, form a few large maſſy Teeth inſtead of many ſmall ones.

As for Inſtance, in *Man's Body*, that is of ſo much a leſs Size, the Number of the Teeth, (when the whole Sett is compleat) reckons to thirty-two; whereas in the large *Elephant*, the Teeth of both the Jaws amount in all but to Eight, beſides its two great Tuſks, which rather ſerve as Horns for its Defence, than Teeth to prepare its Food; and therefore I think not ſo very properly call'd Teeth.

Fig. 29. *Fig.* 29. *E, E,* is the ſmaller *Grinding Tooth* of the under Jaw on the ſame Side; its Surface covered over with the ſame white indented Work, as before deſcribed for grinding of the Food.

f, f, f, are three large Roots, that kept it firmly fixed in the Jaw-Bone.

This ſmaller Tooth weighed full ſix Ounces.

Fig.

Fig. 30. *G, G,* is the large *Grinder* of the under Jaw on the left Side, much of the Size, and Shape, and Weight, with its fellow Tooth, describ'd *Figure* 28. It ſhews its Roots and all its Parts, with the rough protuberant white Work on its upper Surface made after the ſame Contrivance, and formed after the ſame ſtrong Model as the former. Fig. 30.

And truly if one conſiders it, 'tis plain, that were not the *Teeth* of this Creature made of ſo large a Size, and withal of ſo maſſy and firm a Subſtance, 'twere abſolutely impoſſible they could reſiſt the Force, and bear all that Preſſure, wherewith thoſe vaſt Muſcles exert themſelves, that move the lower Jaw in Maſtication, in this ſo ſtrong an Animal.

Fig. 31. *H, H,* is the ſmaller *Grinding-Tooth* of the under Jaw on the ſame Side; it is leſs compleat than the ſmall Tooth deſcrib'd before in *Fig.* 29. for ſome of the Root is wanting, and Part of its outward grinding Surface is broke off at *k, k,* ſo that it weighs ſomewhat leſs; yet what remains, exactly ſhews the ſame kind of Work and Shape of the other Tooth, that anſwer'd it on the right Side. Fig. 31.

Theſe *Four Teeth* here deſcrib'd, fully compleat the Set of the Teeth, wherewith Nature has furniſhed the lower Jaw of the *Elephant*; and are anſwered by juſt as many more, formed after the ſame Manner in the upper Jaw, as Dr. *Moulins* informs us, who diſſected the *Elephant* that was burnt at *Dublin* in 1681. In its Anatomy, *p.* 40. ſpeaking of the Teeth, he aſſures, there were beſides the Tuſks, only four Teeth in each Jaw, two in every Side; and that theſe eight Teeth were all *Molares*, ſo that he had no *Inciſores*.

But notwithſtanding this, perhaps it will be ſaid, we may not haſtily conclude from hence, that our *Great Teeth* dug up in *Ireland*, muſt certainly have been the *Four Grinders* of an *Elephant*, ſince they might as well belong to ſome other large kind of *Terreſtrial* or *Marine Animal*. As for the Hint of their being *human* or *gigantick*, 'tis ſo groundleſs a Thought, and ſo contradictory to *comparative Anatomy*, and all *Natural Hiſtory*, it does not deſerve our Conſideration.

To obviate this, I take Notice firſt in general, that the differing Kinds of living Creatures, wherewith Nature has ſtock'd the World, are not more diſtinguiſhed by the Make of any Part of their Bodies from one another, than by the various Shape and Diſpoſition of their *Teeth*: And hence it is, we ſhall not find any two diſtinct *Claſſes* of *Animals*, that do exactly agree in the ſame Make and Ranging of their Teeth.

But to be more particular, I ſhall here ſet down at length, the Words of two late Authors, that purpoſely have deſcribed the *Teeth* of the *Elephant*.

Dr. *Blair*, in his Deſcription of the Teeth of this *Animal*, ſays, *Dr.* Moulins *well obſerves, that they are all* Molares, *being two Inches broad in that Part of them wherewith they grind, and ſix Inches and a half* *Vid.* Infra, V. v. p. 117.

 long

long on the Right Side, and five Inches and a half on the Left. Their Surface, though flat, is yet very unequal, for they have alternately placed, running from the Right to the Left Side, an Hollowneſs and then an Eminence; and this Eminence is ſurrounded by a rough protuberant Border. There are nine of theſe Hollowneſſes, and as many Eminences, undulated as they paint Sea Waves.

'Tis remarkable, how very exactly all this agrees with our Figures: 'Tis true, thoſe *Hollowneſſes* and *Eminences*, which he mentions to be *nine*, do not ſo nicely hit with the Number of thoſe in our *Teeth*; but this Difference proceeds from hence, that he deſcribes here the Grinders of the upper, whereas ours are the *Teeth* of the lower Jaw; though ſuch a Diſtinction as this, I am apt to think, may very well ariſe even in thoſe of the ſame Jaw, in various Animals, from ſome peculiar Diſpoſition in one from another, nay, and perhaps in the ſame Animal, at different Times, according as it happens to be older or younger.

* *Vid.* infra. V. v. p. 121.

A little farther * where he gives an Account of thoſe of the under Jaw, he ſays,

The hind Tooth of the Right Side is four Inches, and that on the Left five; the one half of their Surface, where they begin to appear above the Gums, is ſemicircular, with the forementioned Ridges and Sulci *running tranſverſly, four on the Right Side, and five on the Left; the other half* (or Tooth I ſuppoſe he means) *has five of theſe Eminences where it grinds on the Right, and four on the Left: Each of the four Teeth is ſix Inches long, and has ſix or ſeven of the forementioned Eminences, and as many Depreſſions: Theſe Teeth are the moſt firm, ſolid, and weighty Bones, of any Animal yet known.*

Mr. *Ray* in his *Synopſis Animalium Quadrupedum*, when he comes to give us the Deſcription of the *Elephant*, has the following Words: *Os pro mole Belluæ parvum, quatuor in utrâque maxillâ Dentibus molaribus ſeu Dentium molarium Maſſis inſtructum; ſiquidem* plurimi Dentes in Os ſolidum & durum *ita infixi ſunt, ut cum eo & inter ſe* unum & continuum Corpus *efficiant. Dentes hi lineas parallelas undulatas octo vel novem in ſuperficie maſſæ efficiunt; ſuntque reliquo oſſe* candidiores: *Maſſæ integræ, Dentium ſingularium modo, per Gomphoſin maxillis inſeruntur. Inciſoribus omnino caret.*

Thus Mr. *Ray* deſcribes the *Teeth* of this *Animal:* And if we compare Dr. *Blair*'s Words with his, and the Particulars of both Accounts with the Deſcription and Figures we have given of the *Teeth* dug up in *Ireland*, and obſerve how they all agree exactly, even ſo as one may ſay they tally together; I think it will amount to nothing leſs than Demonſtration, and that all our *Ideas* have been taken from one and the ſame *Natural Object*; and as they, ſo we, muſt certainly have deſcribed no other *Teeth* but thoſe of the *Elephant*.

But then perhaps it will be aſk'd, what is become of all the reſt of the *Teeth*, that were in the upper Jaw, which being as firm and ſolid Bones, as thoſe that are here preſerved, might for the ſame Reaſon have ſtill remained entire.

But

But ſince we find it otherwiſe, 'tis obvious to imagine a probable Conjecture, how this might come about. From what Mr. *Nevil* mentions, 'tis plain that the Bed where all theſe Bones were found, muſt once have been the outward Surface of the *Earth*, the *Green-Sod*, producing *Ruſhes*, *Ferns* and *Nuts*: And when the heavy Beaſt firſt fell dead upon this Spot, the Skull, with all the Bones and Teeth of the upper Jaw, being the higheſt Parts of the *Animal*, might likely fall in ſuch a Poſture, as to be expoſed ſome while above the *Earth*; though thoſe of the under Jaw firſt coming to the Ground, might make themſelves a Bed, and being covered with the Mould, remain preſerved; whilſt the upper Teeth, and moſt of the other Bones, lying expoſed to the Injuries of the Air and Weather, before they got a Covering, might rot and quickly moulder all away.

But though this be allow'd, yet ſtill a greater Difficulty remains unſolved; how this large-body'd *Animal*, a *Native* of the remote warm *Climates* of the *World*, ſhould be depoſited in this wild *Northern Iſland*, (where *Greeks* or *Romans* never had a footing) ſo many Miles from Sea, and diſtant from thoſe Places of the Iſle, where People might moſt probably reſort.

And to make the Difficulty yet greater, we muſt conſider, not only from the dark black Colour of the Teeth, contracted by their lying long under Ground, and the remarkable Alteration wrought on their bony Subſtance, which (by the mineral Steams and Exhalations it has imbib'd whilſt it was in the Earth) is now become more ſolid, hard, and ponderous, than it was naturally at firſt, (nay, in ſome Parts we find it plainly petrified) but alſo from the periſhing of all the other Bones of the Animal's Body, and from the conſiderable *Depth* of *Earth* that covered thoſe that were found: We muſt conclude from hence, that they have lain in this Place for many Centuries: I won't ſay, with Mr. *Nevil*, ever ſince the *Flood*, becauſe I can't ſuppoſe that the ſlight Texture of vegetable Subſtances, *Nuts* and the Seeds of *Ruſhes*, could poſſibly have been preſerved ſo long: But this, at leaſt, may ſafely be affirmed, that theſe Remains muſt be Cotemporaries with ſome of the remote Ages of the *World*; which carries us ſo far back into the earlieſt Times, that we can never imagine the rude Inhabitants of *Ireland*, or any of their neighbouring Countries, were Maſters of ſo much Art, in thoſe Days of Ignorance and Darkneſs, as to make Carriages by Sea ſtrong and capable, or of Curioſity and Politeneſs enough, to tranſport a Beaſt of this large Size from thoſe far diſtant Countries, where 'twas bred.

Theſe Conſiderations, grounded on other Inſtances of the like kind, make me inclined to think, this *Elephant* might not be brought hither by any Care or Induſtry of *Man*; but the Surface of this Terraqueous Globe might, in the earlieſt Ages of the *World*, after the *Deluge*, but before all Records of our oldeſt *Hiſtories*, differ widely from its preſent *Geography*, as to the Diſtribution of the *Ocean* and *Dry-land*, its

Iſlands,

Iſlands, *Continents*, and *Shores*, ſo as to allow this Beaſt, and others of its Kind, for ought I know, that may by ſome ſuch Accident hereafter be luckily diſcovered, a free and open Paſſage into this Country from the Continent.

For otherwiſe, how can we ever explain that that other vaſt large ſtately *Animal* the *Mooſe-Deer*, little inferior to the *Elephant* itſelf, could have been brought to *Ireland*, (where elſewhere I have ſhewn it formerly was common) from diſtant *North America*, even long before that Quarter of the World was known, and is the only Region, I can hear, where this great Beaſt is found at preſent.

And can we well imagine, that *Foxes*, *Otters*, *Badgers*, *Tygers*, *Wolves*, with *Linxes*, and ſuch ravenous *Animals*, as we have been told, have lately been diſcovered by the great Snows that fell this preſent Winter, in the *Iſland* of *Sardinia* and other Places, ſhould ever be imported (being uſeleſs noxious Beaſts of Prey) by the Induſtry of *Man*, to propagate in *Iſlands*?

Nay, how can we ſuppoſe, that *Birds* of ſhorteſt Flight, the various ſorts of poiſonous *Serpents*, and of offenſive *creeping Vermin*, with all the various Tribes of ſmaller *Inſects*, could poſſibly be found in *Iſlands*, unleſs they had been ſtock'd with thoſe Inhabitants, when the Intercourſe between them and the Continent was free and open?

But in whatever Manner this *Elephant* might firſt have made its way for *Ireland*, this is beyond Diſpute, that the *Bones* of *Elephants* have been diſcovered deep under Ground, in other Places, as well as this Kingdom, and thoſe too out of the Way, far diſtant from the native Countries of this Animal.

For not many Years ago, in a Hill near *Erfurt*, a Town of the *Upper Saxony* in *Germany*, ſeveral Parts of the *Skeleton* of an *Elephant* were dug up; an Account of which is given by *Tentzelius* *.

* *Vid.* ſupra, V. II. C. III. S. XXXVIII.

And I am well perſuaded, by the beſt Conſtruction I can make of thoſe imperfect and obſcure Accounts, we have in *Evert Iſbrand Iddes* curious Travels from *Muſcovy* to *China* over Land, *Chap.* 6. (which he confeſſes he only gather'd from the barbarous *Oſtiacks*, Inhabitants of that Country) concerning the vaſt *Teeth* and *Bones* and *Limbs* of **Mammuths**, as he calls them, frequently found (and diligently ſought after to make Profit of them) in the Hills and Banks of ſeveral Rivers in *Siberia*, the *Keta*, *Jenize*, *Trugan*, *Montgamſea*, and *Lena*; that they are nothing elſe, but the Remains and *Skeletons* of *Elephants* buried there, and accidentally diſcovered by the Earth's opening, and falling down, on the ſudden Thaws, after ſevere long Froſts.

Mr. *Cambden* in his *Britannia* is of Opinion, that thoſe great monſtrous *Teeth* and *Bones*, which he takes Notice to have been at ſeveral Times dug up in many Parts of *Great Britain*, muſt have been the Remains of *Elephants*; but then he thinks, they muſt be of thoſe that *Dion Caſſius* the Hiſtorian tells us the *Roman* Emperor *Claudius* brought over, when he made his Expedition into that *Iſland*. But that this truly

truly is ſo, I own is but Surmiſe as yet, and has not been fairly proved by him or any other.

What Mr. *Somner* * has publiſhed is more remarkable; he informs us, that in the Year 1668, in the Village of *Chartham* near *Canterbury*, in *England*, *digging within twelve Rods of a River, they found a Parcel of ſtrange monſtrous Bones, ſome whole, ſome broken, together with four Teeth, perfect and ſound, each weighing ſomething above half a Pound, and ſome of them almoſt as big as a Man's Fiſt. They are all Cheek Teeth or Grinders; the Earth in which they lay being like a Sea-Earth, or Fulling-Earth, with not a Stone in it.*

* *Vid.* ſupra. p. 222.

'Tis obſervable how this Account, in many of its Circumſtances, agrees with that of Mr. *Nevil*'s; as that the Teeth were all Grinders, four in Number, found with other large broken Bones, near a Brook, and in a claiey Earth, without a Stone: But then the Weight and Magnitude of our largeſt *Teeth* ſo far ſurpaſs thoſe, that were found in *England*, that theſe did not come up to a fifth Part of thoſe, which ſhews, they could not be the Teeth of the ſame Animal. I muſt confeſs, the Author does not ſo much as ſuſpect they were *Elephants Teeth*, but on the contrary, is of Opinion that they belong'd to another *Species*, the *Hippopotamus* or *River-Horſe*, a Beaſt that's yet a greater Stranger in theſe Parts of the World, than the *Elephant* itſelf; and therefore its Paſſage hither can never be accounted for, but by ſome ſuch like Suppoſition, as we have made.

Mr. *Luffkin* † differs in his Judgment from Mr. *Somner* about theſe Teeth, which he thinks muſt have been *Elephants Teeth*; as he is poſitive thoſe large Bones he deſcribes in the ſame Letter, and found near *Harwich* in *Eſſex*, certainly muſt have been.

† *Vid.* infra. p. 245.

Not having ſeen any of the Bones or Teeth concern'd in this Controverſy, either thoſe that were found in *Kent*, or thoſe in *Eſſex*, I cannot well take upon me to determine any thing in this Matter: But this at preſent I can ſafely ſay, that if the *Figures* of the *Teeth* given us by Mr. *Somner* || be genuine and well expreſt, they no way ſeem to agree either in Shape or Make, or in that particular and *Characteriſtick* Work on the grinding Superficies, with the *Teeth* of the *Elephant*; or with the Deſcription and Figures we have given, which are correct and natural.

|| *Vid.* Fig. 26, 27.

I am inclined to think, (even from theſe imperfect Hints) that if we had more correct Hiſtories and Obſervations of this kind, made in diſtant Countries, ſkilfully regiſtred, with all their inſtructive Circumſtances, they might lead us into great and momentous *Truths* relating to the *Deluge*; to the wiſe Methods of Providence, in repleniſhing all Regions of the *World* with *Animal Beings* ſoon after the *Flood*; and to the Knowledge of ſeveral important Changes, that may have happen'd on the Surface of this our *Terraqueous Globe*.

[*The Tooth* Fig. 34. *is* Nine *Inches and a* half *long, whereby the Magnitude of the others may be eſtimated.*]

This

Remarks *by Dr.*E.Halley. *ibid.* 3.] This Account of Mr. *Nevil*'s, with Mr. *Molyneux*'s Draughts of the Teeth, and his Remarks upon them, having been produced and read before the *Royal Society*, they order'd, that what Teeth they had of like ſort ſhould be look'd out and laid before them; to which Sir *Hans Sloan* was pleaſed to furniſh a yet greater Variety, out of his Collection of *Natural Rarities*. And to obviate all Doubts, there being at this Time in *Weſtminſter* the entire Skull of a large Elephant with the Teeth in it, That was likewiſe ordered to be viewed and compared with the Figures: Which done, it appeared that the Teeth in Queſtion could be no other than thoſe of an Elephant.

By this Enquiry we were likewiſe ſatisfied, that the Number of Teeth found being but four, was no Objection; it appearing that the Number of *Molares* in this Animal is not certain. *Pliny*, *Lib.* XI. c. 37. ſays expreſly, *Dentes Elephanto intus ad mandendum quatuor, præter eos qui prominent*. And in the Remains of that mighty Elephant deſcribed by *Tenzelius**, there were no more than four Teeth found. In that at *Weſtminſter* there were ſix, *viz.* one in each lower Jaw, and two in each of the upper, whereof the inner Tooth is about three Times as long as the other, and both together longer than thoſe of the under Jaw, by about an Inch; the upper ſmall Teeth being much worn by grinding. Theſe we have thought fit to repreſent by *Fig.* 32. ſhewing the rough grinding Surface of the left under Tooth, being conſiderably *concave*; and by *Fig.* 33. the ſame Roughneſs on the upper Teeth is ſhewn, having a *Convexity* tallying with the *Concavity* of the under, which is a Circumſtance not obſerved by any of thoſe that have deſcribed them.

* *Vid.* ſupra, V. II. C. III. S. XXXVII.

Fig. 32.

Fig. 33.

And altho', by the Obſervation of Mr. *Du Verney*, Dr. *Moulins*, and Dr. *Blair*, who diſſected three different Elephants, it appears, that each of them had eight *Molares*; yet from theſe it is alſo evident, that in the Diviſion of them Nature obſerves no Rule: For Dr. *Moulins* found the two Teeth, in each of the upper Jaws of that he diſſected, to be divided after a different Manner; ſo that the inner Tooth on the one Side, and the outer on the other, was bigger than its adjoining Fellow, yet not ſo as to be very unequal: And Mr. *Du Verney* and Dr. *Blair* had on both Sides the much greater Tooth outwards: Whereas the *Weſtminſter* Skull, on the contrary, has only a ſmall one outwards, and the much greater Grinder within. All which conſidered, we may with Aſſurance conclude, that this Elephant found in *Ireland* had but four Teeth in his Head when he died; and that the two greater were thoſe of the upper Jaws, and the other two thoſe of the under.

Again, by the Size of the grinding Part, we may conclude theſe to be the Teeth of a very young and ſmall Elephant; ſince they are not much above half the Length of thoſe, that are to be ſeen at *Weſtminſter*, which belonged to a Beaſt of not more than between 10 and 11 Feet high; nor much above one Third of the Length of a Foſſil Elephant's Grinder in the *Royal Society*'s Repoſitory, the which is here repreſented by *Fig.* 34. Hence it is not to be wondered at, that the

Fig. 34.

Bones

Bones of ſo young an Animal, having not acquired their Firmity, as being in a growing State, ſhould be diſſolved by long lying in the Earth, as alſo the Roots of the Teeth.

Matthew Paris in his Hiſtory aſſures us, that in his Time *Louis* IX. (afterwards St. *Louis*) King of *France*, made a Preſent of an Elephant to his Cotemporary *Henry* III. of *England*, and that in the Year 1255, after the *Engliſh* had been fourſcore Years Maſters of *Ireland*. Of this, ſays *Matthew*, *Nec credimus, quod unquam aliquis Elephas viſus eſt in* Angliâ, *præter illum*.

X. Having read Mr. *Somner*'s Account of ſtrange Bones found at *Chartham*, I think it not improper to acquaint you with ſomething like it: That in 1701, at *Wrabneſs*, a ſmall Village, ſituate in the moſt Eaſtern Parts of *Eſſex*, upon the River *Stour*, near *Harwich*, divers Bones of an extraordinary Bigneſs were found at fifteen or ſixteen Feet beneath the Surface of the Earth, in digging for Gravel to mend the Roads with, &c. the largeſt and moſt remarkable of which was procured and ſent to me by Mr. *Rich*, Miniſter of the Place.

An Account of Large Bones *found near* Colcheſter, *by Mr.* J. Luffkin n.274. p.924.

We read in *Cambden*, p. 351. that in the Time of King *Richard* II. and in the Reign of Queen *Elizabeth*, there were found in the moſt Eaſtern Promontory of *Eſſex*, at a Place call'd *Odulfineſs*, which I take to be *Walton*, large Teeth, and Bones of an extraordinary Bulk, which were eſteemed the Bones of Giants. But Mr. *Childrey* in his *Britannia Baconica*, p. 100. rather thinks them to be the Bones and Teeth of ſome Elephant, buried there by the *Romans*.

That theſe were the Bones and Teeth of ſome Elephant, I am prone to believe; firſt, becauſe they far ſurpaſs in Magnitude the Bones, &c. of the largeſt Creatures that we have at this Day in our Iſland.

Secondly, becauſe 'tis evident from *Dion Caſſius*, as quoted by Mr. *Cambden*, (ſee the *Romans* in *Britain*, and in his *Britannia*, pag. 347.) that Abundance of Elephants were brought over into *England* by the Emperor *Claudius*, in his Wars with the *Britons*; even into *Eſſex*, as appears from the ſame *Dion*, a little after in theſe Words: *Claudius* having at laſt joined *Plautius*, and took the Command of the Army, paſſed the River (meaning the *Thames*) and upon a fair Engagement with the Enemy, who were poſted there to receive him, obtained the Victory, took *Camalodunum*, &c.

Thirdly, in comparing this Bone with the Oſteology of Dr. *Moulins*, in his Anatomical Account of the Elephant burnt at *Dublin*, &c. I find it perfectly to agree to and with the *Os humeri* thereof, not only to outward Appearance or Form, but to Meaſure alſo; from which Circumſtances we may conclude, that theſe were the Bones, &c. of ſome Elephant, rather than of any other Animal.

And it does ſeemingly appear to me, that theſe Teeth and Bones mention'd by Mr. *Somner*, might have been the Teeth and Bones of ſome Elephant, rather than that of the *Hippopotamus*: and that, Firſt, in reſpect of the Place; for, as Mr. *Cambden* ſays in his *Britannia*, p. 197. ſpeaking of *Chilham* in *Kent*, of which this *Chartham* is a neighbouring Village, ſituate in the ſame Down, and on the ſame River *Stour*, that it is a current Report amongſt the Inhabitants, that *Julius Cæſar* encamp'd there, in his ſecond Expedition againſt the *Britons*; and thence it was called *Julham*, as if one ſhould ſay, *Julius* Station or Houſe.

It appears farther, *Britan.* p. 208. that *Rhutupiæ* (which whether *Richborough* or *Stoner* matters not) ſituate near the preſent *Sandwich*, was the Place of *Claudius*'s landing in *Britain*; and that through this Down was his neareſt Paſſage to the *Thames* whither he was going, is indiſputable. So that 'tis highly probable, that during the Stay, paſſing or repaſſing of theſe *Roman* Armies through theſe Downs, ſome one of their Elephants might periſh or die, and be buried there.

Secondly, By the Teeth themſelves, for if you compare the Icons given by Mr. *Somner*, with the Deſcriptions of Dr. *Moulins*, p. 40. you will find them the very ſame as to Breadth and Depth, &c. and their being *Molares*; for, ſays the Doctor, theſe eight (which were all the Elephant had, beſides the two Tuſhes) were *Molares*, for he had no *Inciſores*.

And laſtly, to ſolve that great Difficulty which obliged this Gentleman to imagine this Down to have been an *Æſtuary*, that his *Hippopotamus* might therein dig itſelf a Grave, otherwiſe how ſhould theſe Bones be found at ſuch a Depth? For who with Reaſon (ſays Mr. *Somner*) can imagine, that any Land Creature could ever have had (at firſt) ſo deep a Burial?

But 'tis eaſily explained, why theſe Bones ſhould at this Day be found at ſuch Depths, if we conſider the Alteration or Riſing of the Vallies, by the continual waſhing down of the looſe Earth or Soil by the Rains and Snows from the adjacent Hills, and by the annual Rollings of the Graſs, Sedge, &c. for Proof whereof take the following Inſtance from Dr. *Plott*'s Nat. Hiſt. of *Staffordſhire*, Chap. vi. p. 48. p. 220. ſpeaking of a Moſs, &c. wherein there was found a Lump of Coins of *Edward* IV. of *England* (ſuppoſed to be loſt in a Purſe or Cloth now rotted away) at 18 Feet deep, which being about 200 Years ſince (that is, when they were found) whoever pleaſes to compute it, will find this Moſs grew about one Foot in 11 Years, or one Inch *per Annum* and $\frac{1}{11}$ *proximè*. Divers other Inſtances of Alteration are mentioned in the ſame Hiſtory, as in Chap. 3. par. 11, 12. and Chap. 6. par. 45, 46, 47, 48, &c. Now it will be eaſily granted, that if this Moſs grew or advanced itſelf above its Surface 18 Feet in 200 Years, then this Vale or Down might advance itſelf 17 Feet in almoſt 1700 Years.

Coins, &c. *found* under Ground *in* Lincolnſhire, *by* — n. 279. p. 1156.

XI. 1.] In *July* 1701, one *Edward Lenton*, who lives with one *Philip Wolverſton* of *Fleet* in *South Holland* in *Lincolnſhire*, being about to fence in a Hay-ſtack, and digging a Grip for that Purpoſe about the Depth of half

half a Yard, struck his Spade upon a Pot, which when he broke, there was no less than 36 Pound Weight of old *Roman* Copper Coin found in it. The Pieces were found set in Rows edge-ways, one by another, and stuck so together with the Verdigrease or Rust of Copper, that many of them required a Chizel, or some such Thing, to separate them; but being separated, clean'd and brighten'd, the Heads or Figures of all, or most of them, were very fair, (some as when newly stamped) and the Inscriptions of many are very legible. The Fellow carelesly gave them away, and dispersed them up and down the Country to such as desired them. Here was amass'd together a great Variety of Coins in this Pot: They say Dr. *Hart* of *Wisbich* has a Dozen of the best Pieces; and an Apothecary at *Long Sutton* a Score of the same, the largest and most legible: And *Philip Wolverston* himself has two or three, so very large and fair, that he will not part with them. The Place where they were found, is in the midst of the vastest Flat or Level in *England*, and in a Ground that for many Ages past used to be cover'd with Water in the Winter, and over-grown with Reed in the Summer. 'Tis about a Mile and a half South *by* West from *Fleet-Church*, and about as far South *by* East from *Holbeach*. There are no Banks or Hillocks, old Works or Ruins, to be seen near it; nor any Remains or Tokens of any Thing extraordinary to have been there; (but the old Sea Bank about two or three Miles off; which *Dugdale* from a Passage in *Tacitus* believes to be cast up by the *Roman* Soldiers). But all is as flat as the Sea, and a low Country, producing a coarse flaggy Grass for the most part, round about it. The Pot, which was narrowest at the Top and Bottom, but thicker in the middle, had an Inscription about it, which, though it seemeth in some of the Shreds or Pieces to be fair at first Sight, yet is not legible, though what it may be to Men skill'd in Antiquities, I know not.

Near the River *Welland*, (about 5 or 6 Years ago) that runs thro' the Town of *Spalding* in *Lincolnshire*, at the Depth of above eight or ten Feet, there were found *Jettys*, as they call them, to keep up the old River's Bank, and the Head of a Tunnel that emptied the Land-water into the old River; and at a considerable Distance from the present River, I guess 20 or 30 Yards, there were dug up (about the like Depth) several old *Boats*; which Things shew, that anciently the River was either much wider than now it is, or ran in another Place, or both. On the other, *viz.* the North-West Side of the River, and more upwards in the Town, were digg'd up (at about the aforemention'd Depth) the Remains of old *Tan-vats* or *Pits*, a great Quantity of *Ox-horns*, and *Shoe-soals*, (of a strange Form) and I think the very Tanners Knobs, &c. which Things shew, that the Surface of the Country lay anciently much lower than now it does, and has been raised by the Sea's throwing in its Sand in the Maritime Parts (now most inhabited) and by the *Moor* or *rotted Sedge* in the fenny Parts next the high Country; the whole Level is about 50 Miles in Length, and 30 Miles over in the broadest Parts.

Boats, Oxhorns, &c.

Parts. No Record (printed or MS.) or Tradition whatfoever, (that I ever heard of) tell us when thefe Mutations here difcoverable happen'd.

One Thing farther I have to add, that lately at the laying of the prefent new Sluice or *Goat* (as they call it) at the End of *Hamorebeck*, at its Fall into *Bofton* Haven, taking up the Foundation of the old *Goat*, they met with the *Roots of Trees*, many of them iffuing from their feveral *Poles* or *Trunks*, fpread in the Ground, which when they had taken up (Roots and Earth they grew in) they met with a folid, gravelly, and ftony Soil, of the high Country Kind, (but black and difcoloured by the Change that had befallen it) upon which hard Earth they laid the Foundation of this new *Goat*: Where thefe Roots were dug up, was certainly the Surface of the old Country, the certain Depth whereof I cannot now tell, but that it was much deeper than that at *Spalding*, as the Land is there at prefent higher. The *Archimedean* Screw, or fcrew-like Trunk or Cylinder, by which the Workmen cleared themfelves of Water, was very pretty.

A Remark, *by Mr.* R. Thorefby. *ibid.*

2.] The Matter of Fact in thefe Relations, is indifputable, this worthy Perfon being an Eye-witnefs; and I take it for an experimental Confirmation of Mr. *Ray*'s Opinion, that the great Level of the Fens running through *Holland* in *Lincolnfhire*, the Ifle of *Ely* in *Cambridgefhire*, and *Marfhland* in *Norfolk*, was fometime part of the Sea, and atterated by Earth brought down by Floods from the upper Grounds, by the great Quantity of Mud there fubfiding, which by degrees raifed it up. The Form of the *Shoe* was much like thofe found with fome Urns at *Kirby Thore* in *Weftmoreland*, as defcrib'd in *Ph. Tran.* N° 158. *

* *Vid.* fupra, V. III. P. II. S. XXVI.

Of an uncommon Sinking *of the* Earth, *by Mr.* J. Sackette. n. 349. p. 469.

XII. I fhall give the beft Account I can, of what is remarkable, and known to almoft all hereabouts, concerning the preffing forward of the Cliffs, and finking of the Hills in the Neighbourhood of the Town of *Folkftone* in *Kent*. I fhall give a Sketch of the Situation of the Country, by defcribing a ftraight Road from what we call the *Mooring-Rock*, to *Tarlingham-Houfe*; the Manner of the Country, as to the Rifing and Falling, being much the fame, for about a Mile on either Hand of the Road defcribed.

Fig. 35.

A, the *Mooring-Rock*, about half way between High and Low-Water Mark. *B*, the Foot of the Cliff, 50 Yards from the Rock. *C*, the Top of the Cliff, about 6 Yards high. *C D*, a Plain of 50 Yards. *D E*, a cragged Cliff, of 60 Yards high. *E F*, a Plain above a Mile long. *F G*, an Hill of fteep Afcent, near half a Mile. *G H*, the Land from the Top of the Hill to the Houfe, near a Mile. *I*, *Tarlingham-Houfe*, lying near two Miles and a half N. N. W. from the Rock. *E G H*, a Line of Sight. *K B L*, the Shore at High-Water Mark.

The *Mooring-Rock* (though it lies furrounded with great Numbers of other Rocks) is itfelf a moft noted one, known by this Name, Time out of Mind. At this Veffels ufe to be moored, while they are loading other Rocks, which they take from hence, not only for our

own

own *Pier-Heads*, but for thoſe of *Dover-Pier*, and a very great Quantity of them were ſhipp'd in the Time of *Oliver*'s Uſurpation, and carried to *Dunkirk*, for the Service of that Harbour.

This Rock has remain'd fix'd thus, for the Memory of Man; and old Men have obſerved, that for forty Years and upwards, the Diſtance between it and the Foot of the leſſer Cliff *A B*, has been much the ſame; neither can they be much out in their Gueſs, the Diſtance being ſo ſmall. Though there ſeems nothing extraordinary in this, yet its what they take ſpecial Notice of, to their great Surprize: For they ſay, and prove by good Marks and Tokens, that the leſſer Cliff *B*, *C*, has been conſtantly falling in, inſomuch, that from Time to Time, in their Memory, near 10 Rods forward to the Land has been carried away by the Sea. From whence, as it appears that the Plain between the Top of the leſſer Cliff and the Foot of the higher *C D*, has been formerly double the Breadth that it is at preſent, ſo the Diſtance between the Rock and the Foot of the leſſer and lower Cliff *A B*, ſhould have increaſed in Proportion, and would have been double at preſent, to what it has been formerly. But this Diſtance remaining the ſame, or rather leſs, (in the Opinion of many) is what is greatly wonder'd at: Nor can it be accounted for otherwiſe, than by ſuppoſing, that the Land preſſing forward into the Sea, is waſhed away by the high Tides; and, as often as this happens, preſſes forward again. This preſſing forward of the Land into the Sea, would be incredible, were it not ſhewn to be Matter of Fact; and that not only at this one Place of Obſervation, but by the like Obſervations all along this Coaſt, as far as the Situation continues the ſame.

Now let us climb both theſe cragged Cliffs, and place our ſelves at the Top of the higher one, at the Point *E*. And here we are to obſerve, that (as old Men inform us) upward of forty Years ago, not ſo much as the Top of *Tarlingham-Houſe* could be diſcern'd, neither from hence, nor yet a good Diſtance off at Sea; but it diſcover'd it ſelf by Degrees; till at this Day, not only the whole Houſe, but a great Tract of Land below it, is plainly to be ſeen, as in the Line of Sight *E*, *G*, *H*. In this there can be no Fallacy, and we can aſcribe it to nothing leſs than the ſinking of the Hills (for their Tops could never wear away conſiderably, being always cover'd with Graſs, and never broken up by the Plough, or otherwiſe). Theſe Hills are all of Chalk, and have probably very large Caverns within, Springs of Water always flowing plentifully from the Foot of them; and I have had it obſerved to me, that upon their Tops frequent Cracks have been taken Notice of. Whatever be the Cauſe of it, 'tis not to be doubted, but that theſe Hills are greatly ſunk. And this ſinking of the Hills, the People at this Place believe, forces the Cliffs and all the Land forward into the Sea. The Cliffs conſiſt of great ragged Sand Stones, till we come to near a Yard (at ſome Places more) of the Bottom; then we meet with what they call a Slipe, *i. e.* a ſlippery ſort of Clay always wet.

Upon

Upon this Slipe, at the Bottom, they presume, that the hard stony Land above, slides forwards toward the Sea, as a Ship is launched upon tallow'd Planks.

We whose Names are under-written, do hereby testify the Truth of the Matters of Fact in this Account; *Benjamin Master*, a Jurat of the Town, aged 74. *Robert Hammond*, Sen. a Jurat of the Town, aged 77. *William Godden*, a Fisherman, aged 74. *Thomas Marsh*, a Fisherman, aged 72. *William Hall*, a Fisherman, aged 73. *James Godden*, a Fisherman, upwards of 60.

Part of a Hill *sinking down in* Ireland. *Communicated by the* Bishop *of* Clogher. n. 537. p. 267. Fig. 36.

XIII. Let *S*, *T*, *Fig*. 36. represent part of the Ridge of an Hill, gradually rising from *S* to *T*, for near half a Mile; and *S*, *T*, *W*, *U*, the North-side of the Hill, with a Declivity from *S* to *U*, and from *T* to *W*. The perpendicular Height at *X*, to the Plain of the Bottom at *Y*, 150 Feet, and the Slope Line or *Hypotenuse X*, *Y*, 630 Feet.

The Declivity is pretty uniform from *X* to *L*, and from *L* to *Y* considerably steeper: The Bank *A*, *E*, *F*, *D*, overgrown with shrubby Wood, all the Ground on the Side of the Hill being firm, green, and arable; of a mixed Soil, Clay and Gravel, but more clayey.

On *Tuesday* the 10*th* of *March*, 1712-13, in the Morning, the People observed a Crack in the Ground, like a Furrow made with a Plough, going round from *A*, by *B C*, to *D*. They imputed this to (what they call) a Thunderbolt; because there had been Thunder and Lightning on *Monday* Night. But on *Tuesday* Evening an hideous dull Noise raised their Curiosity; and they observed that the whole Space *A*, *B*, *C*, *D*, containing about three *Irish* (i. e. 4½ *English*) Acres, had been all Day in a gentle Motion: And the Noise continued all Night, occasioned by the rubbing of Bushes, tearing of Roots, rending and tumbling of Earth. The Motion ceased on *Wednesday* after Noon, when they saw the Bushes on the Bank *E F*, were removed, some standing and some overthrown, to the plain Meadow *Y y*. The green Ground above *E F*, when it came to the Top of the steep Part at *E F*, rent with hideous Chasms, ten, fifteen, or twenty Feet deep, and tumbled down in Rolls of a Yard or two thick, and ten or twenty long and broad; not unlike a smooth Water breaking over a Cataract, and tumbling in Waves below.

There was a Precipice at the Top *X x*, 65 Feet perpendicular, making the Slope-Line *X x*, 126 Feet. The Ground from *x* to *L*, was made more level, the whole perpendicular Height of *x* not exceeding the Plain of *L*, above 30 Feet; but the Ground at *L*, in the whole Line from *E* to *F*, was mounted above 20 Feet higher than the unmoved Ground on either Side at *E* and *F*; and the Height of *L*, above the Plain of *y*, is 55 Feet.

There was a Ditch *H I*, went cross the Ground, which being broken off at *o*, *o*, is removed, together with the moving Part, 34 Feet lower down than the immoveable; but, at the Bottom *y*, it is tumbled 60 Feet over the plain Meadow. The Breadth at the Bottom *a*, *b*, is 400 Feet, and at *c*, *d*, about 300.

The

The whole Face of the Precipice *X x*, is of a blue Clay, mixed with many little blue Stones. The Metal is very hard when dry; but upon any Rain ſoftens to a kind of Mortar, without the Degree of Toughneſs and Stiffneſs that is natural to Clays. It is very much like that Gravel or Sand (as they call it) which is ſomewhat of a grey marly Nature, and with which of late they ſo much improve the ploughed Land in this Country.

About *x*, there are Chaſms or Gapings full of Water, which make a Rill down the *Hiatus B, E, A*, but in no greater Quantity, than might have been expected from a Well ſunk to a leſs Depth. Though I was told, that there were Holes in the higher Mountains, that received Water under Ground; yet I can find no ſuch Thing, nor any Symptoms of a Current under Ground, either where it enters or riſes, in all the neighbouring Ground for ſome Miles.

It ſeems to me, that there has been no Vacuity under Ground to receive the ſubſiding Earth; for what the Bank *E, L, F*, is raiſed higher, and what is tumbled down to the Plain *a, b*, may very well compenſate the Subſiding at the Precipice *X, x*.

Before the Rupture, the Declivity from *X* to *L*, was not altogether uniform, but was hollower where *x* is now, than the adjacent Parts: It might have been, by the Deſcription I have from the People, 10 Feet deep in the Middle, and 100 Feet Diameter; and they have a Tradition, that this was made by a Subſiding before the Forty-one Wars, (the oldeſt *Epocha* the Country *Iriſh* know.)

It lies in the Lands of *Slat-beg*, two *Engliſh* Miles S. W. of *Clogher*, on Mr. *Mowtray*'s Eſtate.

I have enquired diligently of the Neighbours, if they found any Shocks or Indications of an Earthquake, but don't find the leaſt Appearance of any.

They impute it to the great and conſtant Rains we have had laſt Harveſt and Winter, which have ſoak'd and ſteep'd all the Ground, but cannot gueſs after what Manner they ſhould produce this Effect; for it is impoſſible any Water ſhould ſtand on the Ground, or in the Vicinity, it being all on the Declivity of the Hill.

An Account of the Sunk Iſland, *in the* Humber, *recovered from the* Sea: *Communicated by* J. Chamberlayne, *Eſq*; n. 361. p. 1114.

XIV. This Iſland goes by the Name of the *Sunk* Iſland; ſo called, I ſuppoſe, from the ſinking Marſh Ground about. It is yet within the Memory of Man ſince it began to raiſe its Head above the Ocean, there being ſeveral old People here alive who can remember when there appeared nothing of it but a waſte and barren Sand; and that only at Low-Water too; when for the Space of a few Hours it ſhewed its Head, and then was buried again till the next Tide's Retreat: Thus ſucceſſively it lived and died until the Year 1666, when it began to maintain its Ground againſt the Inſult of the Waves; about which Time it began to be reſcued wholly from future Danger, by the Care and

and Induſtry of Colonel *Gilby*, who having, as I am inform'd, a Leaſe or Gift of it from the Crown, did raiſe Banks about the riſing Grounds of it, and ſo defending it from the Encroachments of the Water, it became firm and ſolid, and in a ſhort Time afforded good Paſturage for Sheep and other Cattle. The Expences at firſt, to improve it to what it is, muſt needs have been very conſiderable; it being encompaſs'd with high Banks, and deep Canals for receiving and diſcharging the Liquid Element, which every now and then notwithſtanding threatens to repoſſeſs it, but hitherto in vain.

This Iſland is now about 9 Miles in Circumference, within the Banks, which ſeem to render it impregnable againſt all future Attacks of the Sea, and is of a very fat and fertile Soil, affords good Graſs, Corn and Hay, and is repleniſhed with numerous Flocks of Sheep, which are of a larger Size and finer Wool, than thoſe in *Holderneſs*, from which it is divided by about two Miles in Water; and from *Lincolnſhire* by about four. It is ſtor'd with vaſt Numbers of Rabbets, that ſeem innumerable, they appearing through all Parts in prodigious Swarms; their Skins are counted the fineſt in *England*, of a dark Mouſe-Colour, ſhagg'd, and ſoft as Silk.

There are alſo Cows and Horſes feeding conſtantly in the Place, with great Plenty of Wild Fowl.

The Inhabitants are not ſo numerous, there being only three Families, that live conſtantly upon the Place; however they are never too ſolitary, there being Abundance of Workmen and Labourers, that continually reſort thither, ſometimes I am told to the Number of a Hundred and upwards, for the repairing of the Banks, *&c.*

The yearly Income of the Proprietor amounts to about 800 *l.* and pays the King's Taxes to thoſe who collect for the *Eaſt-Riding*, and is uſually uplifted by thoſe of the Liberty and Townſhip of *Ottringham*, from the Marſhes of which there is a Paſſage over the Sands to the *Sunk* at Low-water. But this Cuſtom of paying the King's Ceſs to them, proceeds from the Conveniency, not Neceſſity; for it never belong'd to that, or any other Pariſh, ſo that I cannot reſolve you, in what Dioceſe this Iſland lies, unleſs it had been united to ſome neighbouring Pariſh, or converted to one of itſelf; which if effected, the Tythe of Lambs, Wool and Rabbets, *&c.* would make up a handſome Benefice. It lies nearer indeed to the Dioceſe of *York*, by at leaſt two Miles, than to that of *Lincoln*, being two Miles South of *Holderneſs*, in the River *Humber*, and four Miles North of *Lincolnſhire*.

An Account of the Sinking *of* 3 Oaks *into the* Ground: *Communicated by Mr.* P. le Neve. n. 355. p. 766.

XV. On the 23*d* of *July* 1717, near the Seat of Sir *Charles Potts* at *Manington* in *Norfolk*, in the Day Time, to the great Aſtoniſhment of thoſe that were preſent, firſt one ſingle Oak, with the Roots and Ground about it, was ſeen to ſubſide and ſink into the Earth; and not long after, at about 40 Yards Diſtance, two other Oaks, that were contiguous, ſunk after the ſame Manner into a much larger Pit, being about

33 Feet

Fig. 28.

Fig. 32.

Fig. 33.

Fig. 34.

Fig. 30.

Fig. 26.

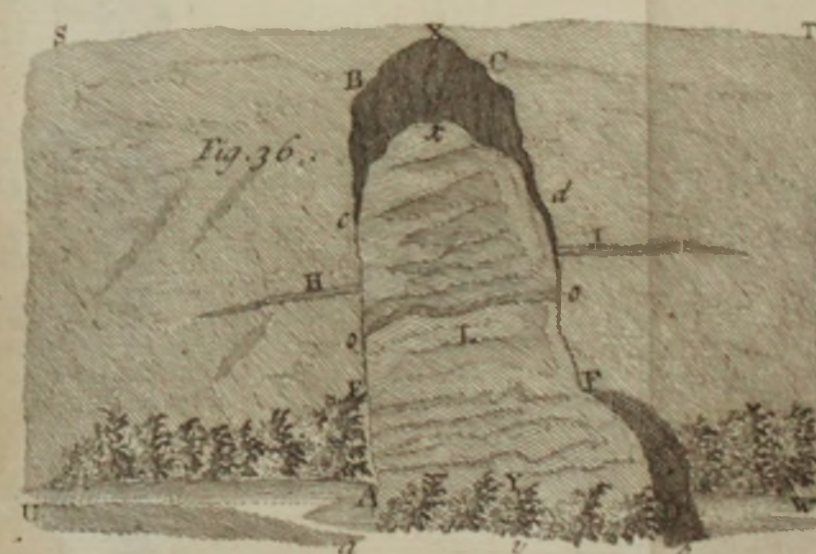
Fig. 35.

Fig. 29.

Fig. 27.

Fig. 31.

Fig. 36.



33 Feet Diameter; whereas the former is not fully 18. Theſe as they ſunk fell acroſs, ſo that obſtructing each other, only the Root of one of them reaches the Bottom, whereas the firſt ſtands perpendicular.

When the firſt Tree ſunk, it was obſerved that the Water boil'd up in the Hole; but upon the ſinking of the greater Pit, that Water drain'd off into it, from the former, which now continues dry. The depth thereof to the firm Bottom is nine Feet three Inches; and the Tree that ſtands upright in it is three Feet eight Inches in Girt, and its Trunk about eighteen Feet long, half of which is now within the Pit. In the Bottom of the greater Pit there is a Pool of Water about eight Feet Diameter; whoſe Surface is eleven Feet three Inches below the Ground, and the Trees that are in this Pit are much of the ſame length with the other, but ſomewhat ſmaller, the one being in Girt three Feet five Inches, the other but two Feet nine Inches.

The Soil, on which theſe Trees grew, is gravelly; but the Bottom is a Quick-ſand over a Clay, upon which there are Springs, which feed large Ponds adjoining to Sir *Charles Potts*'s Houſe, at about a Quarter of a Mile from theſe Holes.

The Nature of the Soil ſeems to afford us a reaſonable Conjecture at the Cauſe of this odd Accident: The Springs running over the Clay at the Bottom of a Bed of very minute Sand, ſuch as Quick-ſands uſually are, may reaſonably be ſuppoſed, in many Ages, to have waſh'd away the Sand, and to have thereby excavated a kind of ſubterraneous Lake, over which theſe Trees grew: And the Force of the Winds on their Leaves and Branches, agitating their Roots, may well have looſened the Sand under them, and occaſioned it to fall in, more frequently than elſewhere; whereby in length of Time, the thin Bed of Gravel being only left, it might become unable to ſupport its own Weight, and that of the Trees it bore. That this is not a bare Conjecture, may appear from the boiling up of the Water at firſt in the leſſer Hole, and its ſtanding in the bigger and lower. And if it ſhall be found, that it was a very windy Day, whereon this Accident happen'd, it will much add to the Probability of this Solution.

An Accident not unlike this lately happened in *Fleet-ſtreet*, *London*, by the Defect of the arched Roof of a very deep Common-Sewer. The Earth gradually falling into the Sewer, was carried away by it, ſo as not to obſtruct the Water; and the continual Tremor of the Ground, occaſioned by the conſtant paſſing of Carts and Coaches, by Degrees ſhook down the Earth, ſo as to leave a very great Cavern, the Top whereof at length grew ſo very thin, that one Day a weighty Cart having juſt paſt it, a great ſpace of the Pavement ſunk in, in the middle of the Street, not without hazard to a Coach then driving by.

XVI. 1.] There are Grounds in *Scotland*, which we call *Moſſes*, from whence the Country People dig Turf and Peats. The Surface is cover'd with a Heathy, and (as they call it) Heathery Scurf. Under that

Of the Moſſes *in* Scotland, *&c. by the Earl of* Cromertie. n. 330. p. 296.

that Scurf there is a black, moist, spungy Earth; in some Places shallower, and in some deeper; from 3 or 4 to 7 or 8 Feet deep; and in some Places, but not in many, to twice or thrice that Depth. They cut the heathy Scurf with a flat kind of a Spade, which they force Horizontally betwixt the Scurf and the spongy Earth, and turn up the Scurf in flat thin Flakes, which they call Turfs. It is over-run with the small Roots of Heath or Heather, and when dried, makes a healthy brisk Fire; but with much Ashes of a whitish, duskish, or reddish Colour; always the whiter, as it contains more of the woody Roots.

The black spongy Earth, which is under the Turf, they cut out in oblong Squares, with Iron Spades made of that Shape, about 8 or 9 Inches long, and about 4 or 5 Inches broad: And as the Men cut them up, the weaker Men, Women and Children, carry them in small Wheel-barrows, scattering them on some dry Ground, to be dried by Sun and Wind: Some become harder, some softer, according to the Nature of the Mould, or Earth; the more solid, the better Fire; and they are less esteemed, which are more spongy. And when they have cut off one Surface, of 4 or 5 Inches deep, they proceed downward to another, until at last they come to the hard Channel, unless they be stopped by Water; which also they ordinarily remove by making a Channel to some Descent, if they can; and if they cannot, there the Water stagnates.

And in such wasted Pits, where Water hinders to cut the spongy Earth to the Bottom; the Pits will be filled up again, in a good Number of Years, with new Ground of spongy Earth; which in Progress of Time, will come to the Consistence of *Peat-Moss*, as at first, and a Scurfy Heath-Turf will at last grow on the Top of it.

I have observed that Peat-Pits, which have been digged since I remember, have grown up again with new Peats; and that sometimes oftner than once in the same Pits; some Mosses growing in shorter Time than others. But I have observed also, that when they dig the Peats to the Channel, and in Places where the Water runs off, and doth not stagnate, that the Mosses did not grow, nor renew there again; which moved me to order my Tenants not to cut the Mosses to the Channel, nor in very large Openings; but rather in smaller Pits, that they may grow again more hastily: And the Event hath answer'd my Design. But Sir *Robert Adaire* has told me, That without cutting the Mosses, in the Method of Pits, but by cutting in fully to the Channel, and by laying the heathy Turf, which is cut off the Top of the Moss, on the Channel, so as to cover the Channel over, that in Progress of Time a Moss would grow there again; but not so hastily as in the Pits.

I never observed any of these Mosses, which did not stand on Plains: Albeit the heathy or heathery Turf, do over-spread the Faces and Declivities of the *Scots* Mountains for the most Part: There are many Mosses which stand very high on these Hills; sometimes not very far from

from the Top. But the Peat-Mosses are always in a Plain, though there be Descents to them, and Descents from them; yet I never observed them to stand on such a Plain, as the Water might stagnate on: And they always have a Descent to them, from some higher Grounds, whereby Water did descend to that Plain; which I take to be the Parent of Peat.

In many of these Mosses, there are found Quantities of Fir and Oak Wood; I never observed nor heard of other Woods in them. These are ordinarily found in whole Trees; but the smaller Branches are seldom found unconsumed: I have seen very many, and very great Trees, of both Kinds; but generally speaking, the Oak is always black; the Fir sometimes whiter, sometimes redder, as is observed in all Fir-Woods: But neither Fir nor Oak are found with any Bark upon them. The Fir is generally as fresh and tough, and as fit for Use, as any other old Wood is: Only the Wood of these found in Mosses, has so imbibed the Water, that it takes a long Time to dry, and fit it for Use, especially the Oak; insomuch, that when it is put into any small Work, it readily warps and changes its Figure. We never find any of the Oaks standing in the Woods, have that Blackness; so that I presume, the Blackness accrues from the Water.

There are many Places, where Woods do not now grow; albeit People endeavour to cultivate them; and yet the Mosses in those Places are well stored with this kind of under-ground Timber, both Oak and Fir, but especially Fir; such are *Orkney*, the *Lewes* (which are Isles) *Cathness*, *Tarbartness*, and the Coast of *Buchan*. But yet it would appear, that there have been Woods of old in these Places, or how else could they come to these Mosses? For a Proof of which, take the following Account:

In the Year 1651. I being then about 19 Years old, and occasionally in the Parish of *Locbbrun*, passing from a Place called *Achadiscald*, to *Gonnazd*, I went by a very high Hill, which did rise in a constant steepness from the Sea; only in less than half a Mile up from the Sea, there is a Plain about half a Mile round; and from thence the Hill rises in a constant Steepness, for more than a Mile in Ascent. This little Plain was at that Time all covered over with a firm standing Wood, which was so very old, that not only the Trees had no green Leaves, but the Bark was totally thrown off; which, the old Countrymen told me, was the universal Manner in which Fir-Woods did terminate; and that in 20 or 30 Years after, the Trees would ordinarily cast themselves up from the Root; and that they would lie in Heaps, till the People would cut them, and carry them away. They likewise shew'd me, that the outside of these standing white Trees, and for the Space of one Inch inward, was dead white Timber; but what was within that, was good solid Timber, even to the very Pith, and as full of Rosin as it could stand in the Wood.

About fifteen Years after, I came the ſame Way, and ſaw not ſo much as a Tree, or Appearance of the Root of any; but in the Place thereof, the whole Bounds, where the Wood had ſtood, was all over a plain green Ground, covered with plain green Moſs. I aſked the Country People, who were with me, what became of the Wood, and who carried it away? They told me, no body was at the Pains to carry it away; but that it being all overturn'd from the Roots by Winds, the Trees did lie ſo thick and ſwarving over one another, that the green Moſs (there, in the *Britiſh* Language called *Fog*) had overgrown the whole Timber; which, they ſaid, was occaſion'd by the Moiſture that came down from the high Hill, which was above it, and did ſtagnate upon that Plain; and they ſaid, none could paſs over it, becauſe the Scurf of the *Fog* would not ſupport them. I would needs try it; and accordingly I fell in to the Arm-Pits, but was immediately pull'd up by them. Before the Year 1699, that whole Piece of Ground was turn'd into a common Moſs; where the Country People are digging Turf and Peats. The Peats, as yet, are not of the beſt, and are ſoft and ſpongy, but grow better and better; and as I am inform'd, it does now afford good Peats.

This Matter of Fact, did diſcover the Generation of Moſſes; and whence it is, that many Moſſes are furniſh'd with ſuch Timber.

Theſe Highland Woods are ordinarily ſtored with other Kind of Timber, as Birch, Alder, Aſh, beſides Shrubs, and Thorns; yet we never find any of thoſe Woods remaining in the Moſſes.

What the Reaſon may be, That the Fir and Oak do not now grow in ſeveral Countries, where they are found ſo plentifully in the Moſſes, *Inquirendum eſt*. I ſhall only add, that in a Moſs near the Town of *Elgin* in *Murray*, though there be no River or Water, that runs into the Moſs, yet three or four Feet in the Moſs, there is a ſort of little Shell-Fiſh, reſembling Oyſters, found numerouſly in the very Body of the Peats, and the Fiſh alive with them; though no ſuch Fiſh be found in any Water near to that Moſs, nor in any adjacent River, nor in the ſtagnating Pits that are in that Moſs; but only in the very Subſtance of the Turf: Some of which were ſent to me from the Place, a little before I came from *Scotland*.

—— *on the* Same, *by Dr.* Hans Sloan. *ibid.* p. 302.

2.] What the *Earl* of *Cromertie* obſerves in the *Moſſes*, &c. in *Scotland*, I have found to be true alſo in the North of *Ireland*. I have been an Eye-witneſs there, that when the Turf-diggers have come to the Bottom, or firm Ground, by having dug out all the Earth proper to make Turf or Peat, and come to the Clay or other Soil, by draining off the Water, that then there have appeared Roots of Fir-Trees, with their Stumps ſtanding a Foot or two ſtraight upright, and their Branches ſpread out on every Side horizontally on that firm Surface; as if it had been formerly the outward Face of the Ground, and Place of their Growth. And I have obſerv'd theſe Roots to be ſometimes ſo near one another,

as

as that their Branches were, as it were, matted, grew over, and gave place to one another, as we every Day ſee in Roots of Trees, where they grow too cloſe. I ſaw once the Body of a Fir-Tree dug up ſo big, as to be judg'd fit for the main Poſt of a Wind-Mill; which was diſcover'd, as many of them (which are not found in digging Turf) are, by the Graſs, which grew over it, being in a very dry Summer of a yellowiſh Colour.

Mr. *de la Pryme* ſent me ſome of the Cones found with this Timber in the great Fens of *Lincolnſhire*, which differed in nothing from thoſe of the *Scotch* Fir, which is plentifully growing in *Scotland* at this Day, and which ſome Years ſince were judged ſo proper by ſome to afford Maſts for the Navy Royal, that ſome Perſons were ſent thither for that Purpoſe. But they were not able to bring about what they intended, by reaſon of the Difficulties in the Roads, by which they were to be conveyed to the Sea; which in *Norway* I have heard is in a great Meaſure effected by the Rivers. *Cæſar*, indeed, in his Commentaries, ſays, that the Sorts of Timber in this Iſland, are the ſame as in *France*, *præter fagum & abietem*, except Beach and Fir. The *Earl* of *Cromertie* is a ſufficient Witneſs of his Miſtake, as to one Sort of theſe Trees, and the Beaches in the *Chiltern* Countries near *London*, prove the ſame, as to the other. For the Uſes of this under-ground Timber, beſides thoſe of other Wood, it is ſplit into Pieces, and being lighted, ſupplies the Uſe of Candles. It is alſo made into Ropes, as may be ſeen in the *Muſæum* of the Royal Society, by a long piece of ſuch Rope, bought by the Honourable *Edward Southwell*, Eſq; in *Newry* Market in *Ireland*. The long ſoaking in Water having render'd the Wood of thoſe Trees fit to be made into Ropes. This ſeems to prove, that as the ſoaking of Hemp, Flax, Aloe Leaves, *&c.* in Water, diſſolves the pulpy part, and leaves the fibrous fit for making into Threads and Ropes, ſo the long ſoaking of Trees may make in Length of Time the ſame, or an analogous Change in thoſe of Wood and Timber. I have ſeen what I thought had been Pieces of Wood, not only in Clay-pits, but even in Quarries or Stone-pits, in the Blocks of Stone raiſed out of their *Strata*, or Layers; and have been aſſured by Mr. *Bellers*, that he hath ſeen large Pieces of Wood in the Stone-pits in *Glouceſterſhire*; and alſo in *Lancaſhire* there is a Moſs, or Turf-Bog, where the black ſpongy Mould, made uſe of for Peats, ſmells very ſtrong of *Bitumen*, or *Petroleum*; of the Oil of which it yields a very great Quantity by Diſtillation. And the late Sir *Edward Hannes* has told me, that near the Lord *Bleſſington*'s Houſe at *Bleſſington* in *Ireland*, there appeared a Light, where the Horſes trampled with their Feet on a certain Space of ſoft Ground: On my Deſire he procured me ſome of this Mould, which agrees exactly in its dark Colour, Lightneſs, *&c.* with Peat Earth. And on Examination of this by a Microſcope, I found the Light proceeded from many ſmall half tranſparent whitiſh live Worms, which lay in it.

The

The Blackness of the Oak, comes, in my Opinion, from the vitriolic Juices of the Earth soak'd into the Oak, which being astringent, is turn'd black by them. Ink is made of Galls, an astringent Excrescence of a sort of Oak in *Turkey*, made by an Insect there; and of green Vitriol, which is made of the Pyrites dissolved by Rain-Water, and Iron. Earth of all Sorts, and even human Calculi, and the Ashes of Vegetables, have in them Particles of Iron, in greater or lesser Quantities. The Pyrites is also very common. The Particles of Iron coming to be dissolved by this Pyrites, Subacid, or other Salts dissolved by Water, or perhaps by Water it self, and carried into these Bogs, there fastens to the Tree, soaks into it, and turns it black.

These Particles in some River Water, fastening to the Oak-Timber floated in it, give the same a darkish Colour, taken Notice of by Mr. *Pepys* in his Naval Memoirs of *England*, p. 71. where we are told by the most famous Ship-Builders of *England*, "That the best foreign "Plank for the Royal Navy, was brought either from *Dantzick*, "*Quinbarow*, (that is *Koningsberg*) or *Riga*, of the Growth of *Poland* "and *Prussia*, or from *Hamburg*; namely, that sort thereof, which is "shipped from thence of the Growth of *Bohemia*, distinguish'd by its "Colour, as being much more black than the other, and render'd so "(as is said) by its long sobbing in the Water during its Passage thi-"ther."

In the Turf Bogs of *Ireland* 14 Feet deep, are found not only the Mouse-Deers Horns, mentioned in one of the *Transactions*, but likewise their whole Skeletons, wherein the Bones bear the same Proportions to the like Bones of other Deer, as the Horns bear to their Horns. There are also found therein, Gold Chains, Pieces of Money, and Roots of Heath, several *musci*, and Branches of Trees, so soft, as to give no Resistance to the Turf-Spade: And I was told, that in cutting Turf in one, they at several Feet deep cut through what the *Irish* call a Ruskin of Butter (which was a Firkin, or Vessel, made of the Barks of Trees, used by the old *Irish* for putting up their Butter.) And I remember, that in digging the wet Dock at *Deptford*, there were found at the Bottom, about nine Feet deep, Grass Leaves, Hazel-Nuts, and Roots of Trees: And there was also found a Piece of Money, as they call'd it; which prov'd to be a Leaden Seal to some Bull of Pope *Gregory* the IX*th*, who continued Pope from the Year of our Lord 1227, to 1241.

From *Leland*, * who wrote in the Reign of King *Henry* VIII. we may learn the common Opinion in his Days, of the Cause of the Destruction of Woods, the growing of Mosses and Pools; and that, at that Time, in *Wales*, the Sense of the Inhabitants was, that the under-growing Trees found there, had formerly grown there.

* Vol. V.

"In

"In these Deyes in *Mone* where they digge Turves be founde greate Leland's *Itinerary*, *Vol.* V.
"Rootes of Trees that serve Men for Wood. For after the Trees wer
"cut doune sogging Yerth and Mosse overcoverid them, and now the p. 13.
"same Yerth parid away for Turves, the old mayne Rootes appere.

"Likewise at low Water about al the shores of both Shores of
"*Aberdein* and *Towen Merioneth* appere like Rootes of Trees.

"I saw hard by on the lift Honde a great Fenny More, owt of wich p. 67.
"the Inhabitantes therabout digge Turfs for Fier, and by the same
"Fenne is a fair *LLin*, cawllid *LLinridde* ii Miles from *Strateflur*.

"*Stratesflure* is set round about with Montanes not far distant, except p. 68.
"on the West Parte, wher *Diffrin Tyve* is. Many Hilles therabout
"hath bene well woddid, as evidently by old Rotes apperith, but
"now in them is almost no Woode.

"The Causses be these; First the Wood cutt doun was never copi-
"sid, and this hath beene a great Cause of Destruction of Wood tho-
"rough *Wales*. Secondly after cutting doun of Woddys the Gottys
"hath so bytten the young Spring that it never grew but like Shrubbes.
"Thirddely Men for the nonys destroied the great Woddis that thei
"should not harborow Theves.

"From *Whitchurch* a Mile and a half of I cam by the Pale of the p. 75.
"large Parke of *Blakmer* longging to the Erle of *Shreusbiri*, wherein is a
"very fair Place or Loge. The Park hath both redde Dere and falow.
"In the Parke (as I hard say) be iii. faire Poles, of the wich I saw
"by the Pale the largest callid *Blakein*, whereof the Park is namid.

"It is to be supposid that thes Pooles for the most part in Morisch
"Groundes, and lying sumwhat in low Groundes, dreane the moist
"Places about them, and so having no Place to issue owt stagne there.

"Sum be likelyhod have begon of Marle Pittes. For the Sandy
"Grounde of sum Parts of *Shropshire*, and especially of *Chestreshire*
"and *Lancastreshire*, will not bere Corne plentifully but it be merlyd.

"From *Blakemers* to *Byklem* in a Fosse iii. Miles of Sand hard by
"*Cholmeley*, first I saw the great Numbre of Firre-Trees, the wiche
"the Inhabitants thereby communely digge up for Fier Wood, but
"there did I se no Fyrre-Trees grouing. Oftentimes in diggin in this
"Mosse or More for Petes or Turves they finde the hole Trees of the
"first, sum short and sum veri long, without Twike or Bow, lying
"sumtime not a Foote, sumtime iii. or iiii. Foote depe in the Grounde,
"but how or when thes Trees cam doune ether be cutting or Wind
"Faulle no Manne ther can telle. The Wood of them in Burning
"savorith of Resine.

"*Morle* (in *Darbyshire*) Mr. *Lelandes* Place is buildid saving the Fun- p. 79.
"dation of Stone squarid that risith within a great Moote a vi. Foote
"above the Water, all of Tymbre after the commune sort of building
"of Houses of the Gentilmen for most of *Lancastreshire*. There is as
"much Pleasur of Orchardes of great Varite of Frute and fair made
"Walkes and Gardines as ther is in any Place of *Lancastreshire*. He
"brennith

"brennith al Turfes and Petes for the Commoditie of Moſſes and "Mores at hand. For *Chateley* Moſſe that with breking up of A-"bundance of Water yn hid did much hurt to Landes thereabout, and "Rivers with wandring Moſſe and corrupte Water, is within leſs than "a Mile of *Morle*. And yet by *Morle* as in Hegge Rowes and Grovettes "is meately good Plenti of Wood, but good Huſbandes keep hit for "a Jewell.

"Syr *John Holcroftes* Houſe within a Mile or more of *Morle* ſtood in "jeopardi with fleting of the Moſſe.

"Riding a Mile and more beyond *Morle* I ſaw on the right hond a "Place nere by of Mr. *Adderton*, and ſo a ii. Miles of to *Lidiate Moſſe*, "in the right ſide wherof my Gide ſaid that ther were Rootes of Fyrre "Wood.

p. 81. "Al *Aundernesſe* for the moſt parte in time paſt hath been ful of Wood, "and many of the Moores repleniſhid with hy Fyrre Trees.

1719 *Obſervations on the* Strata *in* Coal-Mines, &c. *by* J. Strachey, *Eſq*; n. 360. p. 968. Fig. 37.

XVII. I have made ſome Obſervations relating to the different *Strata* of Earths and Minerals found in the Coal-Mines of *Mendip* in *Somerſetſhire*. The Draught (*Fig.* 37.) you muſt ſuppoſe the Section of a Coal Country, and to take in about four Miles from the North-Weſt, to the South-Eaſt, and may be applied to the Veins of Coal, as they lie at *Faringdon Gourney*, and likewiſe at *Biſhop-Sutton*, which laſt Place is near *Stowy*, but in the Pariſh of *Chew-Magna* in *Somerſetſhire*. For Diſcovery of Coal they firſt ſearch for the *Crop*, which is really Coal, though very friable and weak, and ſometimes appears to the Day, as they term it, or elſe for the *Cliff*, which is dark or blackiſh Rock, and always keeps its regular Courſe as the Coal does, lying obliquely over it: For all Coal lies ſhelving like the Tyle of a Houſe, not Perpendicular nor Horizontal, unleſs it be broken by a Ridge, which is a parting of Clay, Stone or Rubble; as if the Veins by ſome violent Shock were disjointed and broken, ſo as to let in Rubble, &c. between them. The Obliquity or *Pitch*, as they term it, in all the Works hereabout, is about 22 Inches in a Fathom; and when it riſeth to the Land, is called the *Crop*, but in the North *Baſſeting*. In the Works near *Stowy*, and likewiſe at *Faringdon*, it riſeth to the North-Weſt, and pitcheth to the South-Eaſt; but the farther they work to the South-Weſt, the *Pitch* inclines to the South; and *è contra*, when they work towards the North-Eaſt. So likewiſe they obſerve, as they work to the South-Weſt, when they meet with a *Ridge*, it cauſeth the Coal to trap *up*; that is, being cut off by the *Ridge*, they find it over their Heads when they are through a *Ridge*: But on the contrary, when they work through a Ridge to the North-Eaſt, they ſay it *traps down*, that is, they find it under their Feet.

Coal is generally dug in Valleys or low Grounds. The Surface in theſe Parts is moſtly a red Soil, which under the firſt or ſecond Spitt degenerates into *Malm* or *Loom*, and often yields a Rock of Reddiſh

Fire-

Firestone, till you come to four, five, and many Times to twelve or fourteen Fathom depth, when by Degrees it changeth to a grey, then to a dark or blackish Rock, which they call the *Coal Clives*. These always lie shelving and regular as the Coal doth. But in these Parts they never meet with *Firestone* over the Coal, as at *Newcastle* and in *Staffordshire*. These *Clives* vary much in Hardness, in some Places being little harder than *Malm* or *Loom*, in others so hard as that they are forced to split them with Gun-powder: So likewise in Colour, the Top inclining to red or grey, but the nearer to Coal the blacker they grow; and wheresoever they meet with them they are sure to find Coal under them. But to their Disappointment 'tis not always worth the digging. The first or uppermost Vein at *Sutton* is called the *Stinking Vein*. It is hard Coal, fit for mechanick Uses, but of a sulphurous Smell. About five Fathom and half, seldom more than seven Fathom, under this lies another Vein, which from certain Lumps of Stone mix'd with it like a *Caput mortuum*, not inflammable, called *Cats-head*, they call the *Cathead Vein*. About the same Depth under this again lies the *Three Coal Vein*, so called because it's divided into three different Coals; between the first and second Coal is a Stone of a Foot, in some Places two Feet thick; but the middle and third Coal seem placed loose on each other, without any Separation of a different Matter. These three Veins before-mentioned are sometimes work'd in the same Pit: But the next Vein which I am going to mention, is generally wrought in a separate Pit; for though it lies the like Depth under the other, the *Cliff* between them is hard and subject to Water; wherefore I have represented a Pit sunk through the three upper Veins at *A*, and another Fig. 37. sunk upon the *three Coal Veins* only at *B*; and so if they sink on any of the lower Veins they go more to the North-West.

Firestone sh'd be Freestone see Coal page

Next under the *three Coal Veins* is the *Peaw Vein*, so denominated because the Coal is figured with Eyes resembling a Peacock's Tail, gilt with Gold, which Bird in this Country Dialect is called a *Peaw*. The *Cliff* also over this Vein is variegated with *Cockle-shells* and *Fern-Branches*, and this is always an Indication of this Vein, which, as I before hinted, is always searched for about 15 Fathom to the North-West of the former.

Under this again, between five and six Fathom lies the *Smith's Coal Vein*, about a Yard thick; and near the same Depth under that again, the *Shelley-Vein*: And under that a Vein of 10 Inches thick, which being little valued, has not been wrought to any Purpose.

Some say there is also another under the last, but that has not been proved within the Memory of Man. At *Faringdon* they have the same Veins, which, as I am informed, agree in all Parts with those of *Bishop-Sutton* before-mentioned. But as *Faringdon* lies four Miles South-East from *Bishop-Sutton*, so, in the regular Course, they would lie a Mile and a half deeper than those at *Sutton*. But as in Fact they are dug near the ſame Depth, it follows there muſt be a *Trap*, or ſeveral *Traps down*, which in all muſt amount to that Depth between the ſaid Works.

Between *Faringdon* and *High Littleton* the ſame Veins ſeem to retain their regular Courſe; but at *Littleton* their undermoſt and deepeſt Vein is the beſt Coal, which at *Faringdon* proves ſmall.

On the other hand, in the Pariſh of *Stanton-Drew*, to the North-Eaſt of the Coal-works at *Sutton* aforeſaid, about a Mile diſtant, and in the true Courſe with thoſe at *Sutton*, the ſame Veins are found again. But here they wind a little, and their Courſe or Drift runs almoſt North, and they dip to the Eaſt; which Winding is attributed to *Ridges*, which the Workmen have met with on both Sides, and have occaſioned them to diſcontinue the Work that way. At *Stanton* they have little of the red Earth or *Malm* on the Surface, but come immediately to an *Iron-Gritt* or *grey Tile Stone*, which is a Fore-runner of the *Coal Clives*; in all other Matters they agree with the Works near *Stowy*.

In the ſame Pariſh of *Stanton-Drew*, a little to the Eaſtward, they have another Coal-work, but the Veins are in all Reſpects different from the former. Their Drift or Courſe is to the Eleven a-Clock Sun, as they term it, they *Pitch* to the Five a-Clock Morning, and riſe to land; conſequently to the Five a-Clock Evening Sun. They have ſeveral Veins, but as yet only three are thought worth working. The uppermoſt about three Feet thick ſmall *Lime Coal*. The next is about three Fathom under it, about two Feet and an half thick, fit for culinary Uſes: The undermoſt is about the like Depth under the former, only 10 Inches thick, but good hard Coal.

At *Clutton*, about two Miles from theſe latter, in the ſame Drift, *viz.* almoſt to the South Eaſt and by South, theſe laſt Veins appear again. The Surface here is red, and ſo continues to ten, and ſometimes to fourteen Fathom, and in other Reſpects agree with the laſt mention'd Works at *Stanton-Drew*.

At *Burnet*, *Queen-Charlton*, and *Briſleton*, they have four Veins which pitch to the North nearly, and conſequently the Drift lies almoſt Eaſt and Weſt. The Surface is red Land generally to the Depth of four or five Fathom. The uppermoſt is from three to ſix Feet thick at *Briſleton*, but leſs at *Charleton* and *Burnet*. The next call'd *Pot-Vein*, is ſix Fathom under the former, eighteen Inches thick, all hard Coal. *Thirdly*, The *Trench-Vein*, ſeven Fathom under the other, which is from two Feet and a half to three Feet thick, all ſolid Coal. *Fourthly*, *Rock-Vein*, always diſtinguiſhed by a Rock of Paving-Stone, called *Penant*, lying over it, which Rock is ſometime twenty Feet thick or more, and therefore this Vein is never wrought in the ſame Pit with the former Vein, but about 200 Yards more to the South, or to Land, as they term it. It's computed ſeven Fathom under the former.

This

This is all I can say in Relation to the different Veins of Coal and Earth in the Coal-works in these Parts; wherein all agree in the oblique Situation of the Veins; and every Vein hath its *Cliff* or *Clives* lying over it, in the same oblique Manner. All of them pitch or rise about twenty-two Inches in a Fathom, and almost all have the same *Strata* of Earth, Malm, and Rock over them, but differ in respect to their Course or Drift, as also in Thickness, Goodness, and Use.

Now as Coal is here generally dug in Valleys, so the Hills which interfere between the several Works abovemention'd, seem also to observe a regular Course in the *Strata* of Stone and Earth found in their Bowels. For in these Hills (I mean those only that are dispersed between the Coal-works above-mention'd) we find on the Summits a stony Arable mixt with a spongy yellowish Earth and Clay; under which are Quarries of *Lyas*, in several Beds, to about 8 or 10 Feet deep, and six Feet under that, through yellowish *Loom*, there is a blue Clay inclinable to *Marle*, which is about a Yard thick: Under this, is another Yard of whitish *Loom*, and then a deep blue Marle, soft, fat, and soapy, six Feet thick; only at about two Feet thick it is parted by a *Marchasite* about six Inches thick.

It is to be observed, that these Beds of *Stone* and *Marle*, different from *Coal*, lie all horizontal.

XVIII. 1.] About half a Mile from *Reculver* towards *Herm*, there appear in the Cliff *Strata* of Shells in a greenish Sand; they seem to be firm, and some of them are entire, but when you go to take them from their Beds, they crumble to Powder between your Fingers; but that which is most remarkable, is, that in the lower Part of the *Strata*, where the Shells are more thickly dispersed, there lies scatter'd up and down Portions of Trunks, Roots, and Branches of Trees; the Wood is become as black as Coal, and so rotten, that large Pieces of it are easily broken with one's Fingers. I know not what Depth these may lie, the *Strata*'s Surface not appearing above two Feet from the Beach, but I judge it from the Superficies of the Top of the Cliff, about 12 Feet. I saw the Stump of one Tree standing upright, broken off about a Foot from the Ground. The Shells were of the White *Conchites*.

Of the Fossils *of* Reculver *Cliffs, &c. by Mr.* S. Gray. n. 368. p. 762.

2.] It is very likely, that the Black Wood, mention'd by Mr. *Gray*, is Oak, which has lain so long as to be turn'd of that Colour by the vitriolic Juices of the Earth, in which it has lain; as Galls and a Solution of Vitriol turn of that Colour. I never saw any Oak that had lain any Time in any Kind of Earth, where Water came to soak into it, that was not turned of that Colour: And I have seen many Trees of Black Wood of great Bigness, taken up (as well as lesser Pieces) and all of it was Oak. It looks at first taking up like Ebony, is very ponderous, but as it dries, it splits, grows friable, light, and comes to be good for little.

— *A* Remark *by Dr.* Hans Sloane, *ibid.*

Some Remarks on Foſſils, by Mr. E. Lhuyd. n. 291. p. 1566.

XIX. 1.] The State of *Foſſils* is quite different in *Eſſex*, from what it is in *Wales* and *Ireland*. In thoſe Countries the Shells are generally *Cryſtalline*, but in *Eſſex* (and ſometimes about us at *Oxford*) they are *Teſtaceous:* Which Difference is, doubtleſs, to be attributed to the Soil, and particularly to Chalk and Flint, which all thoſe Countries want, excepting a ſmall part (I know not by what Chance of *Diluvian Diſſolution*) got into the *North* part of *Ireland*. But there 'tis remarkable that their Chalk is abſolutely petrify'd: I mean, whereas the Flints are here imbodied in Chalk, they are there in a Chalk-white *Lime-ſtone*. And as chalky Countries only afford thoſe *Echinitæ* I have ſtil'd *Pileatus*, *Galeatus*, and *Cordatus*; ſo I could never find them in all my Travels, but at that Place; from whence, in the Time of *Paganiſm*, the *Druids* procur'd them, and ſold them amongſt our *Northern Britons* for Stones of miraculous Efficacy againſt Perils by Fire and Water; perſuading the Vulgar they were generated in *Cocks Knees*, as Thouſands in the *High-lands* believe at this Day. And one Fellow had the Impudence to tell me (finding me a little hard of Belief) that he himſelf had taken one (that his Maſter had ſhew'd me) out of a Cock's Knee with his own Hand.

We were ſurpriz'd here at *Oxford*, to find ſo many *Foſſils*, ſcarce diſtinguiſhable from *Sea Shells*; the Caſe being uſually otherwiſe in thoſe places I ſearched. We have indeed in theſe Parts, one or two *Foſſil Shells* of a *Teſtaceous* Subſtance, but in Colour they recede farther from thoſe of the Sea, than thoſe in *Eſſex*. I find that thoſe in *Eſſex* are ſometimes found imbedded in ſolid Stone; which takes off any Objection ſome might offer, of their being an accidental ſcattering of *Gulls*, *Crows*, &c. on the *Harwich Cliffs*.

Of Harwich *Cliff, and its Foſſils, by Mr.* S. Dale. *ibid.* p. 1568.

2.] *Harwich Cliff* is a ſort of *Promontory*, which divides *Orwel Haven* from the *Æſtuarium* contained between that and *Walton Naſe*; it is ſituate on the ſouthern Part of the Town, about a Quarter of a Mile diſtant, or not ſo much, and contains many Acres of Land. The Height of it from the Strand or Beach to the Top, where it's higheſt, is 40 or 50 Feet. At the Bottom of this Cliff, there is a *Stratum* of Clay about a Foot thick, which is ſucceeded by another of Stone for a Foot more; in this *Stratum* of Stone are imbedded divers Shells (though but thinly) as well of the *Turbinate*, as *Bivalve* Kind, and alſo Pieces of Wood and Sticks; over this, are divers *Strata* of blueiſh Clay, about the Height of twenty Foot, or more; this Clay hath *Pyrites* or *Copperas* Stones, ſticking in it, but no Shells, that I could obſerve: Above this, are likewiſe divers *Strata*, which reach to within about two Feet of the Surface, ſome of which are only of fine Sand, other ſmall Stones and Gravel, mixt with Fragments of Shells, and in others ſmall Pebbles are mixt; and it is in ſome of theſe laſt mentioned *Strata*, that the *Foſſil Shells* are imbedded, which lie promiſcuouſly together, I mean the *Bivalve* or *Turbinate*; neither do the

Strata

Strata with the Shells observe any Order in their lying, being sometimes higher, and sometimes lower in the Cliff; and sometimes 2 or 3 one above another with other *Strata* of Sand, Fragments, and Gravel, between. Above all these, is a Covering of common sandy Earth, which is about 2 Feet thick, in which, in some places are Veins of a Species of *Osteocolla*, though more tender than *Osteocolla Officinarum*, which is brought from *Germany:* This I call *Osteocolla Anglicana*, it doth incrust about small Strings, like the Fibres of the Roots of Trees, it's of divers Magnitudes, and sends forth Branches here and there, but is so tender, as not to be gotten out of the Earth in any large pieces. Whether like the *German* it appears above the Earth, I never could discover.

Before this Cliff, the Shore, as far as the ebbing of the Sea would permit my Observation, was rudely paved with Stones, divers of which are vein'd with that sort of Body, which by *Helmont* and other latter Naturalists, is called *Ludus Paracelsi:* Of these Stones the Inhabitants have a Tradition, that they are form'd by the Clay, which tumbling down from the Cliff, and being washed by the flowing of the Sea, are in a short Time converted into Stone; and Mr. *Silas Taylor* in his Manuscript *Collections of Harwich and Dovercourt*, (a Copy of which I have) thus writes concerning it. *The Washing of these* Cliffs *discovers a blueish Clay, which tumbling down upon the Shore, altho' washed by the Sea at High-water, within a short time turns into Stone: There they may be seen, some that are new fallen, as soft as the Clay in the Cliff; and others that have lain there longer, crusted over and hard, but if opened or broke, the Clay still soft in the middle; others that have lain longest petrified to the very heart, and with these the* Walls *of the* Town *are for the most part built, and the* Streets *generally are pitch'd.* How far this is Matter of Fact, I will not determine, my Stay at *Harwich* being alwaws too short for me to make Observations so critical as this *Phænomenon* doth deserve; and although I must at the same time own, that many of the Stones are washed out from the *Stratum*, at the Bottom of the Cliff; yet I have sometimes been inclined to Mr. *Taylor*'s Opinion, because he lived long upon the Spot, being *Store-keeper* of the *King's Building-yard* for many Years, and by his *Collections*, &c. seems to be a person of probity and Learning; and also, because divers of the said Stones have Cracks or Chops in them, as Clay and Earth will have by being exposed to the Sun; and there is yet [*Anno* 1702,] lying upon that Shore a Stone, in which a large pile (perhaps of Oak) such as was formerly made use of there, to preserve the Cliff from the Injuries of the Sea, doth evidently appear to be imbedded; which can owe its Situation to no other Original, than by being prest into the Superficies of the Clay, while soft, and petrifying with it, which being square, takes off an Objection, which some might make, had it been round, of its being lodged there in the general Deluge.

I am

I am not insensible, that this Manner of Petrification is not only different from the common Methods Nature uses in that Operation, but also is oppos'd by divers learned and ingenious Men; as particularly by the Reverend Mr. *John Morton* in *Oxendon* in *Northamptonshire*, whose Thoughts upon this Subject I shall transcribe from a Letter of his, to me, dated *August* 4, 1699. ——— *At* Harwich, *under the Cliff, upon the Sea-shore, there is a* Stratum *of a* Clayey-Stone, *which is cover'd here and there with ragged Stones of a closer texture, which was formerly (I conjecture) another entire* Stratum, *but is broken thus by the Tearing of the Waves. The* Clayey-Stone Stratum, *Mr.* Luffkin, *and you, were of opinion, had been formerly a softer Substance, but was daily petrified by the Sea Water. Having argued a little about it, when turning to the Cliff, I found a* Stratum *there, of the very same sort of* Clayey-Stone, *with that upon the Shore; yet the Sea Water very seldom comes up hither, unless by Storms, and at Spring-tides. I broke a little piece off, and shew'd it to you, and then you was convinc'd (I think) it was not hardned or petrified by the Sea Water, but in its natural state. And I have often met with just such sort of Stone in many of our* Stone-Pits *here, in Inland Countries. It appears to me, that the Water should have rather softned, than hardned the Stone upon the Shore, tho' by washing away the looser Clayey matter and other Earthy stuff, that is sometimes left upon it at the ebb, it might seem to be a sort of* Petrification, *and occasion this mistake.*

As to Petrifactions: *I've only observed these three sorts.* 1st, A Stony Incrustation, *upon Sticks and any thing that lies in the way, in the Petrifying Springs; the Earth in those Waters is usually intermixt with particles of Stone, that trickle down into it with the Water, and are there detain'd. Of this first sort you have doubtless many instances in* Essex, *and I think there is one at* Harwich *Cliff; tho' this in my opinion is not so properly call'd a* Petrifaction. 2dly, *The second sort is that, which is perform'd by the* Permeation *or* Insinuation *of the finer sorts of Stony* Particles, *as it is in the case of some of our Petrifying waters, (I believe) particularly that at* Knaresborough *sometimes; the Stony particles however of the* Knaresborough *Spring are very fine. And many of the* Fossil-shells *have undergone the same fate.* 3dly, *The third, which indeed is a* Petrification, *properly so call'd, is often met with on the sides of* Caves *and* Grotto's, *at* Pooly-hole *in the* Peak, *and in the* Fissures, *and* Clefts *of* Mines *and* Quarries. *Of this kind are the several sorts of* Fluors, the Lap, Stillatitii, Stalagmitæ, &c. *that we meet with in the* Fissures, *and* Hiatus's *of the Earth. These are continually growing (as they vulgarly say) that is, are receiving an additional increase of real and solid Stone, as is observed in many* Caves *in the* Peak, *&c. This I take to be perform'd in such a manner, as the* Incrustations *are*, viz. *the particles of Stone are brought along with the Water, as their* Vehicle, *and are deposited at length upon the sides of the* Cave *or* Fissure, *(this is matter of fact, that there is always a watry Stream, and usually Water trickling down upon the sides of those* Caves) *but here, the particles of Stone are extremely minute and fine, and do thereby* *by naturally concrete and join together very close; whereas in our* Incrustations *the Particles of Stone being grosser, the Stone is rough and coarse, and friable. And this I leave to your Judgment, if it be not a more reasonable* Hypothesis *than that of Dr.* Plot, *in* pag. 33. *of his* History of Oxfordshire, *viz.* That the very Body of the Water is turned into Stone as it drops down from the Rocks. *I know not indeed of any other sorts of* Petrification, *than these I have already mention'd. As to that* Hypothesis *of the* Transmutation *of a* Stratum, e. gr. *of Chalk to Clay, of Coal to common Stone, or the like, I must confess I never met with any thing in Nature which would countenance it, that is, such a* Transmutation *in the Bowels of the Earth. Nor is there any thing that proves it, that ever I have met with in any* Natural Observations. *Only some will guess and fancy such a Thing; but for making it out, I am sure I am no more able to do it, than to make the* Philosophers Stone, *whatever they are.*

A late Author is of Opinion, that *this Bed of Stones was the Foundation of the Loamy Cliff, where the Cliff has been washed away, or cut: And that they are the Production of a Vitrioline Juice, in Conjunction with the Loam*; as the common Copperas Stones are by the same Juice in a *Gravel*, and that the latter were only to be *found* where the *Cliff was gravelly*, and not *where the Loam is.* How far these Stones are the Effect of a *Vitrioline Juice*, I will not determine, but this I can affirm, that I have now by me some of the *Pyrites*, or common *Copperas Stones*, which I did pick out of the *Clayey Stratum* of this Cliff, in which they may be frequently met with. Nor do I remember at any Time to have observed these *Stones* to be *invested with* either *Gypsum* or *Trichitis*, as the same Author affirms, but with the aforesaid *Ludus Paracelsi*, and some other sorts of *Lap. Stalagmitæ* frequently.

How those Shells or Marine Bodies come to be here posited, is a Subject, which hath imployed the Heads and Pens of divers learned and ingenious Men. I shall not pretend to determine the Controversy; I shall only make some Remarks on the positive Assertion of the aforesaid Author, concerning the imbedding of these *Fossil Shells* in this Cliff, and the Alteration of the Channel; *viz. That* this *Bed of Shells, which covers the Cliff, was carried thither at the making of the Harbour or clearing of it. For the Harbour or Channel there is artificial, and of no old date, the Current having been formerly on the other side of* Landguard-Fort, *which then stood in* Essex. Against the first Part of which, altho' many Reasons might be given to prove the contrary, I shall only mention the following; and as our Author begs the Question, *How else could the Shells lie a-top* of this Cliff? So I shall also ask him, why the same *Strata* of Sand, and Fragments of Shells, with the same *Fossils* imbedded, are to be found at *Walton Ness*, on the other Side of the *Æstuarium*, which is 5 or 6 Miles broad from *Harwich*, as likewise at *Bawdsey* Cliff in *Suffolk*, which is 8 or 9 Miles distant, and in other Cliffs on that Shore, where I have met with them. A second Question may here be ask'd, How it comes to pass, that none of those *Buc-*

cinæ Heterostrophæ, (whereof such Plenty of their *Exuviæ* are in all the Cliffs hereabouts) are not now to be found in this Channel, nor the adjacent Seas? (where I have divers Times been a Fishing) for I cannot think the clearing this Harbour could have destroyed all that Species of Shell-Fish, whereof there was then such Plenty; and therefore some other Original must be allow'd them, than what this Author has assign'd. Nor can I allow the *Harbour* here to be *Artificial*, because so great a Work as this is, *viz.* the making a Channel two Miles wide, as it is in this Place, would not have been without some Record thereof in History; and besides the Earth, *&c.* which must arise by this Work, must consequently have made a much greater Hill than the Cliff ever was; and another Doubt will from hence arise, why the Workmen should bring all the Earth, *&c.* to this side the Channel, and not lay some thereof on the other, as it's plain they did not. The Ground on which *Landguard Fort* stands, as far as *Walton Colness*, which is about three Miles, is only a sandy Level or Beach, which I believe hath in Time subsided there, as may be observed at the Mouths of other great Rivers. And as to the Argument, which our Author brings of *Landguard Fort*, being accounted to stand in *Essex*, to confirm his Hypothesis of the Change of this Channel, it will be of no Force with any one, who doth but observe, that not only Parts of Parishes, but likewise of Counties, are often divided from those Parishes and Counties to which they belong, and included in others, of which I could give you many Instances, *e. gr.* a Part of *Kent* is on the *Essex* Side the *Thames*; and in *Oxfordshire* the Parishes of *Shilton* do belong to *Berkshire*, *Daylesford* to *Warwickshire*, *Compton* to *Gloucestershire*, and *Stratton-Audley* to *Buckinghamshire*, although all included in the other: And there is a Farm, which doth belong to the Parish of *Braintree*, that is separated from it at least two Miles, and many others might be given, but let these suffice. And to me a probable Reason of this Fort's being accounted in *Essex*, is, the Sands here subsiding, made at first, I suppose, an *Insula*, which being nearest to *Essex*, was accounted of that County; or 2dly, the Island so made belonging to none but the Crown, it was at the Pleasure of the *King's Officers*, to call it of which County they pleased. Nor was it *the Gentleman in Cambden*'s Ignorance (whatever this Author saith) that made him *mention* these Stones *for Petrifactions made by the Sea*; for Mr. *Taylor* in his aforesaid *Collections* did not omit the Tradition, the Inhabitants of this Town have, about the Alteration of the Mouth of this Haven, which I will transcribe in his own Word's. *It's generally believed, that* Stoure *did formerly in a straighter Current (than now it doth) discharge itself into the Sea, about* Hoasley-bay, *under the High-land of* Walton-Colness *and* Felix-Stow *in the County of* Suffolk, *betwixt which and* Landguard-Fort *are (as they are reputed) certain Remains of the old Channel, which the neighbouring Inhabitants still call* Fleets, *retaining at this Day the Tradition*

dition of the Course of the Water, and the Entrance into this Haven to have heretofore been by and through them.

And I am of Opinion that this Tradition is Matter of Fact, having before hinted what Mutations the Mouths of great Rivers daily undergo by the Lodgment of Sands, *&c.* which may be assign'd as a better Reason for this Alteration than that of our Author, *i. e.* that it was artificial; and the yearly washing of the Cliff on the *Harwich* side, doth likewise add to its Probability; it being a constant Observation, that where the Sea gaineth on one Side, it loseth on the other. And that this Level was so made, I am confirmed by the modern Removal of the Fort, more towards the Point; more Sands, I conjecture, being added after the old Fort was built: This Alteration is taken notice of by the aforesaid Mr. *Tayler* in these Words: *And altho' several now living pretend to the Remembrance of the building it,* [Landguard-Fort]; *yet we find there was an ancienter Fort thereabouts, and called by the same Name* [Anno 1553.] *which was not far distant from this modern one, a little North of it, where are yet to be seen two* Faces *and* Flankers *of a* Bastion, *the rest of it being eaten away by the Sea, but in its Place hath left upon the Shore a long row of Sand Banks.*

The Spring mentioned by Mr. *Gibson* in his *English* Edition of *Cambden*, from the aforesaid Manuscript of Mr. *Silas Tayler*, is a very small inconsiderable Thing; nor could I observe that it did petrify or incrustate either Pieces of Wood or Sticks; but I have a Piece, which I broke off from a large Pile upon that Shore, which was petrified so far as it was drove into the Earth, and the Sea-Water came; and do suspect there yet remains some others of the same. And of this sort I believe is that large Piece sent from hence, which Mr. *Tayler* mentions to be *reserved in the Repository of the Royal Society.*

I have already taken notice, that the *Fossil Shells* are imbedded in a loose *Stratum* of Sand, Gravel, *&c.* which may serve to demonstrate, that their *Matrix* is not a *Clay Bed upon the Top* of the Cliff; as likewise for another Argument, to evince, that they could not be there scattered by *Crows*, *Gulls*, and other Sea Fowls, as well as that some of them are likewise bedded in Stone at the Bottom of the Cliff; and although some few of them may be met with upon the Top of the Cliff, yet it's only where the Earth has been broken by digging Ditches, *&c.*

A *Catalogue* of the *Fossils*, found by me at this *Cliff*.

1. Buccinum fossile heterostrophum rostratum lævem maximum *Listeri* referens. 2. Buccinum fossile rostratum maximum *Listeri* referens. 3. Buccinum fossile minus ventricosum, mucrone obtuso. 4. Buccinum fossile tenue confragosum. 6. Buccinum fossile, striis prominulis marginalibus insignitum. 7. Buccino-turben fossile reticulatum minus. 8. Buccino-turben fossile sulcatum. 9. Buccino-turben fossile rostratum.

tum. 10. Buccino-turben maximum roſtratum foſſile, ſpiris intus ſtriis elatis inſignitis. 11. Cochlea foſſilis maxima umbilicata quinque ſpirarum. 12. Cochlea foſſilis umbilicata, mucrone obtuſo. 13. Nerita parva foſſilis. 14. Turbo foſſilis, ſpiris duabus ſtriis eminentibus inſignitis. 15. Pecten minor foſſilis unica aurita. 16. Auricularia maxima. 17. Pectunculus foſſilis fere circinatus ſtriis tenuibus, valvis per ginglymon connexis. 18. Pectunculus foſſilis craſſus roſtro acuto ſtriis majoribus. 19. Pectunculus foſſilis faſciis tranſverſis undantibus notatus. 20. Pectunculus vulgaris foſſilis. 21. Pectunculus foſſilis ſtriis majoribus & elatioribus. 22. Pectunculus maximus foſſilis, Liſterianum maximum referens. 23 Pectunculites maximus ſtriis latis. 24. Concha parva foſſilis, faſciis tranſverſis inſignis. 25. Concha longa foſſilis faſciata. 26. Conchites lævis maxima. 27. Conchites parva faſciata. 28. Trigonella minor, ſive vulgatior Anglica *Lithoph. Brit.* 816.

An Account of Land *and* River-Shells, *&c. found under Ground by Mr.* J. Morton. n. 305. p. 2210.

XX. Mr. *Coxe* of *Mears-Aſhby* in *Northamptonſhire* lately diſcover'd ſome *Land* and *River-Shells* under Ground, in a mooriſh Paſture in *Mears-Aſhby* Field, which Place I went afterwards to view.

It is the more remarkable, becauſe *Land* and *River-Shells* are ſo very rarely met with in digging into the Earth, in Compariſon of *Sea Shells*, and the Teeth and Bones of *Marine* Animals, which occur almoſt every where and in all Countries. The Reaſon of which is now no longer a Difficulty, theſe Bodies having been ſhewn to be all Remains of the Univerſal Deluge; and the Marine Shells being more ponderous than thoſe of the Lands and Rivers ſunk, and were lodged deeper in the Earth, and ſo were preſerved by that Means; whereas the latter being left generally upon the Surface, periſh'd, and are at this Day rarely met with.

Cauſing one to dig into the mooriſh Ground above-mentioned, we found a ſmall Number of Snail Shells of various kinds buried there. At about a Foot in depth they lay very thick; and ſinking ſtill downwards, the Number rather encreaſed till we came to the depth of about three Feet. 'Twas troubleſome to ſink deeper on Purpoſe; but we made Trials for a conſiderable Extent of Ground, *viz.* about 250 Feet in length, and 130 in breadth. Beſides, the ſame Shells were caſt up in ſeveral Places, at Diſtance, by Moles. What we principally obſerved in this Search, was, 1. A moiſt mooriſh black Earth, in ſome Places a Foot and a half, in others ſomewhat above two Feet in Thickneſs. The lower half of it is blacker and denſer than the upper half, of a bituminous Nature, and has all the Characters of Peat-Earth. Beſides Shells we found Stalks and Leaves of Graſs, and alſo of many kinds of other Vegetables, repoſited as uſual in like Bituminous Moors in other Parts of this Iſland. 2. White Earth; ſo at firſt we call'd it: But upon cloſer Inſpection it appeared to be little more than Hay half waſted. So deep as we ſunk into it, we found it every where copiouſly interſperſed with Shells.

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The finding these Shells under Ground, made it very reasonable to enquire, whether there were any of the like at this Time, living upon the Surface. I diligently search'd this Place, but could not meet with any live ones of any Kind whatever, there.

The Fossil Shells were some the *Exuviæ* of Land-Snails, the rest of River or Fresh-water Snails: Of the former, there were the three following Kinds: 1. A small *Buccinum*, of five Wreaths, the *Buccinum exiguum quinque anfractuum*, *Tit.* 7. *List. in Tractat. de Cochleis Terrestr. Angl.* A Kind observ'd by Dr. *Lister* to live in Moss upon old Garden Walls at *Estrope* in *Lincolnshire*; by my self at the Mossy Roots of old Trees in many of the *Northamptonshire* Woods, as also amongst Moss upon the boggy Sides of several standing Springs.

2. A *Cochlea* of the compressed Kind, but not so much compressed as some of them are. It has six Wreaths, and a small circular *Sinus* in the Center. This, if it is not the *Cochlea umbilicata*, &c. N. 79. *List. Hist. Conchil. Lib.* 1. has not hitherto been mention'd by any Writer; though common enough in the Woods in *Northamptonshire*: I found a great Number of them, for the Compass of Ground, inclosed in the Earth, than ever I have done in any of the Places where they naturally breed.

3. The *Cochlea citrina*, *Tit.* 3. *List. de Cocbl. Terrest. Ang.* The common strip'd Snail Shell. But most of these in the Moor are white, of the Colour of the Shells that have been a long Time dead. In some I saw faint Footsteps of their former Stripes. Most of the Shells of this Kind, were lodg'd about four Feet deep.

We met with only two different Kinds of River Shells: 1. A Perewinkle Shell of three Wreaths, generally less than the *Buccinum trium Spirar. Tit.* 24. *List. de Cochleis Fluviatil. Ang.* There was a greater Number of these buried in the Moor, than of any of the former Kinds.

2. A Perewinkle Shell of five Wreaths, much smaller, and more prominent than those of the *Buccinum longum sex Spirarum*, *Tit.* 21. *List. de Coch. Fluviat.* 'Tis otherwise very like that *Buccinum* in the Fashion of its Wreaths. It has not yet been describ'd by any Author. We find the Kind now living in one of the *Northamptonshire* Brooks, call'd the *Ise*.

The moorish Ground, wherein these Shells were buried, extends from near the Top to very near the Foot of a small Hill. Above the Moor, upon the Top, and at the Brow of the Hill, is a sandy Soil of a reddish Colour. The whole Face of the Moor is plain and even, conformable to the rest of the Hill not thus moory of the same Declination with it, and appears to be in a natural and undisturbed State, as much so as any of the Slades in the neighbouring Fields; excepting that three or four Trenches have been cut through it of late.

'Tis evident, that these Shells were left at the Deluge, when those from Sea were also reposited at Land; and not buried since by De-

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terrations from the Ground above: For then the upper parts of the Moor must have been cover'd with a reddish Sand, such as the Ground is, for the main, compos'd of: But nothing like that appears near the Shells in this Moor. Besides, here are dug up several Shells, that in all Likelihood never bred here, but are Inhabitants of a different Soil; particularly the striped Snail-shell: For these Animals have peculiar Soils, and affect particular Regions.

An Account of the Skeleton *of a large* Animal *impress'd in* Stone, *by Dr.* W. Stukeley. n. 360. p. 936.

XXI. At the Reverend Mr. *South*'s at *Elston* near *Newark* in *Nottinghamshire*, was lately discover'd a Skeleton, almost entire, of a large Animal (which I have procur'd for the *Repository* of the *Royal Society*) impress'd in a very hard Stone; it had lain Time out of Mind, at the side of a Well, where it had serv'd for a Landing-place to those that drew Water; but upon removal, the under side exhibited this unusual Form.

The Stone itself is a blue Clay-stone, the same as (and undoubtedly came from) the neighbouring Quarries of *Fulbeck*, or thereabouts, upon the Western Cliff of the long Tract of Hills, extending quite through the adjacent County of *Lincoln*.

It is a great Pity, that so considerable a Rarity should be maim'd and imperfect; but where the remaining part of the Stone is, which contain'd the upper Part and Continuation of the Skeleton, or that which was the other side, and tally'd with it, is utterly unknown. This Skeleton at first was taken to be *Human*, which upon View I am persuaded it cannot be; it seems rather to be that of a *Crocodile* or *Porpess*. There are sixteen *Vertebræ* of the Back and Loins, very plain and distinct, with their Processes and intermediate Cartilages. Nine whole or partial Ribs of the Left-side, the *Os Sacrum*, *Ilium in situ*, and two Thigh Bones displac'd a little, the Beginnings of the *Tibia* and *Fibula* of the Right Leg; on one Corner there seem to be the *Vestigia* of a Foot, with four of the five Toes, and a little way off, an entire Toe, now left perfect in the Stone: There are no less than eleven Joints of the Tail, and the Cartilages between them of a white Colour, distinguishable from the rest. We should impose upon our Senses, to question whether these be the real Reliques of an Animal; for the very Bones themselves are now to be seen as plainly, as if preserv'd in an *Egyptian* Mummy; a very little while ago, the Society had a Draught of a Crocodile, though a small one, found after the like Manner, inclos'd in Stone, from a Quarry in the Mountains of *Upper Germany*. I suppose the same Reason accounts for both, and all the rest of these Kinds of Fossils; and it is an ocular Evidence, and a great Confirmation of what I laid before the *Royal Society*, in a late Discourse, where I hinted at a Solution of some obvious and remarkable Phænomena, in the external Face of the Globe, consequent to its Formation, as set forth in the *Mosaic* Account; and of some Changes it suffer'd at the universal *Cataclysm*, and Proofs of that great *Catastrophe* of the Animal and Vegetable

Fig. 38.

getable World in Plants, Shells, and Parts of living Creatures found in Rocks and Quarries.

It is remarkable, that all the Stone Pits about the Country whence this came, abound with prodigious Quantities of Shells, and the like, and the greateſt Part of the Subſtance of the Stone is a Compoſition of them. There are many Accounts of them in the *Tranſactions*, and this Stone has many Shells of different Kinds in it. Sir *Hans Sloane* has a Fiſh Skeleton, amongſt his immenſe Treaſure of Curioſities, found near this Place, given by the Duke of *Rutland*. If we look upon a Map of the Country, and obſerve the *Lincolnſhire Alps*, how they run fifty Miles North and South, and on the Weſt-ſide are ſteep and rocky, we may ſee the Reaſon why theſe Quarries ſhould be ſo ſtuft with them; for it is juſt to conceive, that upon retiring of the Waters of the Deluge from the Superficies of this Country, into the Eaſtern Seas, theſe heavy Bodies met a full Stop, and were intercepted by this Cliff, which has retained ſuch vaſt Quantities of them ever ſince: Whilſt thoſe which fell upon common Mould are moſtly rotten, and now loſt.

Sir *Iſaac Newton*'s Doctrine of the Attraction of the Particles of Matter, according to the Quantity of its Solidity, Proximity, and Surface; eſpecially that it is infinitely greater in the Point of Contact, upon which depends its Coheſion and all the Varieties of Phyſical Action, will eaſily direct us to a Notion of Petrifaction. We learn how a proper Degree of Heat or Cold, Moiſture, Motion, Reſt and Time, promote this Principle, from the common Experiments of Chryſtallization and Freezing, even before the Fire, and in many chymical Mixtures. Whence we cannot be ignorant of Stone growing in the Quarries gradually, not by any fancied Vegetation, though there is ſomething like it in Corals, but generally by Appoſition of Parts to Parts, as is notorious in the *Fluors* of ſubterraneous Grots and Caverns. So that we have no Reaſon to doubt but what was Clay, Sand, or Earth 3000 Years ago, may now be Stone or Marble, according to the Proportion of Concurrence of ſuch mentioned Cauſes. This will perſuade us, that the now barren and rocky plains of the Countries of *Syria*, *India*, and *Arabia*, are owing to natural Cauſes, as well as an immediate Curſe of God, for the Diſobedience of its ancient Poſſeſſors his peculiar People; becauſe the ſame is obſervable of the famous Countries of *Greece* and *Africa*, warm Regions ſo renowned for Fertility in ancient Authors. Wherefore there may be ſome Likelihood in the Opinion of thoſe who think that in many Ages the whole Face of the Globe may become one great Rock. Dr. *Plott*, in his Natural Hiſtory of *Oxfordſhire*, gives an Account of a *Tumulus*, now a perfect Mount of Stone; and upon St. *Vincent*'s Rock near *Briſtol*, are Fortifications now become ſolid Cliff. I remember, about ſix Years ago, Mr. *Ralph Widdrington*, Brother to the Earl of that Name, ſhew'd me many human Bones taken from whole Skeletons, with *Britiſh* Beads, Chains, Iron

Rings,

Rings, Braſs Bitts of Bridles, and the like, which were dug up in a Quarry, near the Seat of the Family, at *Blankney*, *Lincolnſhire*; which very probably was plain Mould, when theſe old Corpſes of the *Britons* were interred; and ſince then I ſaw many human Bones and Armour, with *Roman* Coins, *Fibulæ*, *&c.* found in a Stone Pit in the Park at *Hunſtanton*, *Norfolk*; belonging to Sir *Nicholas L'Eſtrange*, in whoſe Cuſtody they now are, which were conjectured to have been buried in Earth after a Battle. From whence we may judge it a vulgar Miſtake, when in the Ruins of the old Caſtles and Walls we admire the Tenacity of the Mortar, and are apt to praiſe our Anceſtors, for an Art which we ſuppoſe now loſt; when doubtleſs the Strength of the Cement is owing to the Length of Time: And in future Ages our modern Buildings may obtain the ſame Judgment.

From all which Inſtances, I infer the ancient State of theſe Cliffs, where this Skeleton was, and Shells are daily found, intimately mixt in the Subſtance of the Stone, to have been formerly of a ſofter Conſiſtence, capable of admitting them into its Bowels, and to have immur'd them as part of itſelf; and that Earth which is now manageable by the Plough, may poſſibly in Time aſſume the ſame Denſity, at leaſt very little below the Surface; for in this very Cliff the upper *Strata* are yet Clay, growing harder as deeper. What Creature this has been, for want of a Natural Hiſtory of Skeletons, we cannot poſitively determine; but generally find the like to be amphibious or marine Animals. Why ſuch, rather than many others, ſhould chance to be thus entombed, may be thought, becauſe they were able, much longer than Terreſtrial Animals, to live in that World of Waters, even till they began to abate and fall away into their deſtin'd Receptacles; ſo that while the Bodies of the reſt ſoon periſhing, were corrupted, and their Bones ſeparated and diſperſed much earlier; this Skeleton, with others of its like, fell entire into the Fiſſures of this Bed of Clay, which has ſince turned into Stone, and made this noble Monument and pregnant Token of that general Inundation.

Of Chryſtal, *&c. by Mr.* R. Thoreſby. n. 277. p. 1071.

XXII. Dr. *J. Cay* preſented me with a *Chryſtal* (and other Natural Curioſities) which he brought from *Milan*: I ſhall give you his Deſcription of it, firſt premiſing his Arguments upon a ſort of *Spar* within a *Flint*. That within the Flint (ſays he) ſeems to differ from the reſt of its Subſtance, and ſomewhat to reſemble *Spar*: Though after all, *Spar* being nothing elſe but a *Chryſtalline* ſort of *Lime-Stone*, it differs not from *Flint* in Reality, but only in Appearance, *i. e.* in the Manner of *Concretion*: Though if the encloſed Matter had in its Nature differed from the reſt of the Stone, the Thing had not been very uncommon; it being uſual enough for Stones (eſpecially thoſe of a Globular or Oval Form) to have Coat upon Coat, and thoſe Coats ſometimes very different one from another, ſome of them ſoft, ſome hard; nay, ſometimes, after a long Space of Time, one of theſe Coats will ſhrink from

another,

another, after the Manner of a Kernel, when the Shell grows dry; and then, if the incloſed Subſtance continue ſoft and marly, they call that Stone *Geodes*; but if ſtony, it makes one of thoſe rattling Stones that are known by the Name of the *Ætites* or *Eagle-Stone.*

To confirm what I have advanced, many Inſtances may be brought, that it is no unuſual Thing for Stones to encloſe Subſtances of a very different Nature from themſelves: The *Shells* which I have ſeen in *Suſſex* Marble is one Proof; and the *Stones* found in our *Coal-Pits*, and known among the Workmen by the Name of *Cat-heads*, may ſerve for another; they are found in a particular *Stratum* near the Coal, and encloſe a *Fern*, or ſometimes *Polipody Leaf* in the middle of them; and for that Reaſon being ſtruck with a Hammer very readily break there: I think they are a ſort of *Iron-Stone*, a-kin to that which they call in *Staffordſhire Ballmine*; and Dr. *Liſter*, *Minera ferri Pilæformis*; they have it upon the Weſtern Coaſts near *Whitehaven*, and call it there by the Name of *Cat-ſcamps:* I have ſeen it too upon the *Yorkſhire* Coaſts in *Robin-Hood*'s Bay: It may be called *Lapis Mineræ ferri, Pilæformi ſimilis in cujus Meditullio unum vel plura Filicis folia repræſentantur.* I have Specimens of both Sorts. And as an Inſtance, that *one and the ſame Piece of Rock does not always ſhoot into Stone at one and the ſame Time, but firſt one Part of it, and then another*, and they too not after the ſame regular Manner: I have a Piece of *Rock Chryſtal*, where one may eaſily obſerve the *Modus Concreſcendi*, in the Middle to have differed from that of the outſide; nay ſometimes I have ſeen in the Middle of ſome tranſparent Stones, a ſmall Drop that never would take the ſolid Form of the reſt of the Stone at all.

I have received from Dr. *Cay* a Piece of an *Iron Bolt* (two Inches long) found in a *Stone* Quarry, now returned into *Iron Ore* again; this being a Property that Iron has, and no other Metal, as Dr. *Liſter* obſerves in his Journey to *Paris.* **Iron.**

I have received from *Sweden* a Piece of *Copper Ore* regularly ſhot into an *Octoedrous* Form: It has eight ſolid Triangles, and conſequently ſix angular Points; and is of the Bigneſs and Figure of the Draught, (*Fig.* 46.) it came from the Copper Groves at *Fallum*, where very many of the ſame Form were then found. **Copper-Ore.** Fig. 46.

XXIII. I ſhall give an Account of my Obſervations on ſeveral natural ſolid *Noctiluca*'s, not hitherto by any, as I know, taken notice of; (I think I may be well aſſured ſome of the *Phænomena* never were:) and ſhall firſt ſpeak a little concerning the artificial *Phoſphorus*, which is a Subject I have made a great Number of Experiments about, whereby I was naturally led to the following Remarks. *Of the* **Luminous** *Qualities of* **Amber, Diamonds, Gum-Lac,** *by Dr.* Wall. n. 314. p. 69.

Many Years ago (about the Year 1680.) Mr. *Boyle* communicated to me his Way of making the *Phoſphorus* with Urine: But his Chymiſt, Mr. *Bilgar*, was forc'd to evaporate a prodigious Quantity of Urine, to get a very little of the *Phoſphorus*; which induced me to think for ſome other

Matter,

Matter, from which more might be made than from Urine: It being then a very hot Summer, I caus'd a Piece of the dry'd Matter in the Fields, where they empty the Houses of Office, to be digg'd up, in which, when broken in the Dark, a great Number of small Particles of *Phosphorus* appear'd; but of this could be made little or no *Phosphorus*, till another Matter was added to it in Distillation.

Reflecting on the artificial *Phosphorus*, I considered whether there might not be, *in rerum naturâ*, other natural ones, besides those that Mr. *Boyle* and some others have given an Account of.

Human Urine and Dung do plentifully abound with an *Oleosum* and common Salt, so that I take the *artificial Phosphorus* to be nothing else but that Animal *Oleosum*, coagulated with the Mineral Acid of Spirit of Salt, which *Coagulum* is preserved, and not dissolved in Water, but accended by Air.

These Considerations made me conjecture that *Amber* (which I take to be a Mineral *Oleosum* coagulated with a Mineral Volatile Acid) might be a natural *Phosphorus*, so I fell to make many Experiments upon it; and at last found, that by gently rubbing a well polish'd Piece of Amber with my Hand in the Dark, which was the Head of my Cane, it produced a Light; whereupon I got a pretty large Piece of Amber, which I caused to be made long and taper, and drawing it gently through my Hand, being very dry, it afforded a considerable Light. I then used many kinds of soft Animal Substances, and found none did so well as that of Wool. And now new *Phænomena* offered themselves; for upon drawing the Piece of Amber swifty thro' the Woollen Cloth, and squeezing it pretty hard with my Hand, a prodigious Number of little Cracklings were heard, and every one of those produced a little Flash of Light, but when the Amber was drawn gently and slightly through the Cloth, it produced a Light, but no Crackling; but by holding one's Finger at a little Distance from the Amber, a large Crackling is produced, with a great Flash of Light succeeding it, and, what to me is very surprizing, upon its Eruption it strikes the Finger very sensibly, wheresoever apply'd, with a Push or Puff like Wind. The Crackling is full as loud as that of Charcoal on Fire; nay, five or six Cracklings, or more, according to the Quickness of placing the Finger, have been produced from one single Friction, Light always succeeding each of them. Now I make no Question, but upon using a longer and larger Piece of Amber, both the Cracklings and Light would be much greater, because I never yet found any Crackling from the Head of my Cane, although 'tis a pretty large one; and it seems, in some Degree, to represent Thunder and Lightning; but what to me is more strange, is, that though upon Friction with Wool in the Day-Time, the Cracklings seem to be full as many, and as large, yet by all the Trials I have made, very little Light appears, though in the darkest Room; and the best Time of making these Experiments, is when the

Sun

Sun is 18 Degrees below the Horizon; and when the Sun is so, though the Moon shines never so bright, the Light is the same as in the darkest Room, which makes me chuse to call it a *Noctiluca*.

As the artificial *Phosphorus* led me to that of Amber, so Amber directed me to that of a *Diamond*, from its being Electral as well as the other; which is also a Natural *Phosphorus*, or rather a *Noctiluca*, exceeding all others, and may, without any Exception, be called a Mineral *Phosphorus*, it being, as I think, the most pure of all *Oleosums*, coagulated with a Mineral *Acidum*; and if in the Discovery of this I have not obliged the Learned, I am in Hopes I shall all those who deal in Diamonds: Mr. *Boyle* has given the World an Account, at the latter End of his Book of *Colours*, of Mr. *Clayton*'s Diamond, and afterwards says, that some Diamonds would not shine in the Dark: but if any one else has since then made a Discovery, that all Diamonds would give Light in the Dark, they have been very unkind to the World in not letting them know it, because I am well assured that a great many People have been but too often cheated with them, which I hope to prevent. I have now by me a yellow Diamond, which I have shewn to a great many Jewellers and others, and but a few of them will allow it to be a Diamond; but by as many Trials as I have made, I think my Way of distinguishing Diamonds is so certain, that none need fear to affirm them to be so.

A Diamond, by an easy slight Friction in the Dark, with any soft animal Substance, as the Finger, Woollen, Silk, *&c.* appears in its whole Body to be Luminous; nay, if you keep rubbing for a little while, and then expose it to the Eye, 'twill remain so for some little Time: But if the Sun be 18 Degrees below the Horizon, if any one holds up a Piece of Bays or Flannel stretch'd tight between both Hands, at some Distance from the Eye, and another rubs the Bays or Flannel with a Diamond swiftly and pretty hard on the other side of it, the Light to the Eye of him that holds it, seems much more pleasant and perfect, than any other Way I have yet try'd. But what to me seems more surprising is, that a Diamond being expos'd to the open Air, in View of the Sky, gives almost the same Light of itself without rubbing, as if rubb'd in a dark Room; and if in the open Air you put your Hand or any Thing else a little over it, to hinder its Communication with the Sky, it gives no Light: I have try'd all or most of the precious Stones, but could find no such *Phænomenon* in any of them. All these Experiment were made at the latter End of *May*, and Beginning of *June*, and therefore, I cannot pretend to account for the *Phænomena* that may attend Experiments made while the Sun is on the other Side of the *Equator*.

I am well assur'd, that all or most of the Bodies which have an Electricity yield Light; for in my Opinion 'tis the Light that is in them, which is the Cause of their being Electral; yet this Electricity never shews itself without Friction; if you rub any Body that has an Electricity, and apply it near to ſome light Bodies, as particularly very thin Slices of Cork, 'twill put them into a great Agitation, and make them ſeem to the Eye, as hanging at the Body by a fine Hair.

I forbear ſpeaking of *Jet*, which ſeems to me to be a black Amber, having moſt of the Properties of Amber, but not ſo perfect and pure.

I muſt not forget to ſpeak of another Subſtance, not hitherto by any (as I know) taken Notice of to be endu'd with a luminous Quality, which is alſo another natural *Phoſphorus* or *Noctiluca*, and that is *Gum Lac*, and alſo red Sealing-Wax, which is made with *Gum Lac* and *Cinnabar*, the *Cinnabar* no Way impeding, but rather promoting its luminous Quality; for I cauſed long taper Rolls to be made up of *Lac* alone, and of pure red Sealing-Wax, both being well poliſh'd: The Sealing-Wax upon Friction, ſeems to me to emit its Crackling and Light ſooner than the *Lac*, which I impute to the *Cinnabar*'s conſtringing its Parts, tho' I think *Lac, per ſe*, has the greateſt Electricity, both having all or moſt of the Properties of Amber; and by all the Tryals I have hitherto made of *Lac* and Sealing-Wax, I find that though the Cracklings are as plentiful in the Day-time, as when the Sun is down, yet in the darkeſt Places I could diſcover but a little Appearance of Light, ſo that this deſerves the Name of a *Noctiluca* or *Phoſphorus*, as well as the others; it being no other than a vegetable *Oleoſum*, coagulated with an Animal Volatile *Acidum*. I don't know in the Animal Kingdom any Thing but Piſmires, that affords a Volatile Acid, and in the *Eaſt-Indies* there's a large kind of them that live on the Sap of certain Plants, affording both a Gum and a Colour, which Sap paſſing through the Body of thoſe Inſects or Animals, is by their Acid Spirit converted into an Animal Nature; which is the Reaſon, that with the Colour extracted from *Gum-Lac* (which *Gum-Lac* is nothing elſe but the Excrements of theſe Inſects or Animals) almoſt as good, and full as laſting, Colours are made as from *Cochineal*: I am the more confirmed herein, becauſe I know of an Artificial Way of converting Vegetable Colours into an Animal Nature very much like this, by which the Colours are made more pleaſant and permanent. After the ſame Manner the remaining Gum, which is an *Oleoſum*, being digeſted and paſſing through the Bodies of thoſe Inſects or Animals, is by their Volatile Acid converted into a Vegetable Animal *Phoſphorus* or *Noctiluca*; the Artificial *Phoſphorus* is a Mineral Animal *Phoſphorus*, whereas I take the others to be altogether Mineral.

A Treatiſe on Ambergreaſe, by *G. J. Camelli*, n. 290. p. 1591.

XXIV. According to *Avicenna*, *Ambergreaſe* is generated in the Manner of a Fungus upon Rocks and Trees in the Bottom of the Sea. Others again will have it, that Fungules grow upon the Surface of the Sea, in the ſame Manner as they grow upon the Surface of the Earth. *Hieronymus à Huerta*, in his Notes upon a Tranſlation of *Pliny* aſſerts, that there are certain Sea-Weeds or excrementitious Maſſes upon the Surface of the Sea, formed like a kind of Fungus, which have no Smell when they are firſt gathered, nor do they become fragrant till they are prepared: And that it is not a *Bitumen*, nor the Seed of the Whale,

Whale, becauſe theſe when once hardened, cannot be rendered ſoft again like the *true Ambergreaſe*. *Jonatius Alzina* in his Hiſtory of *Biſai*, ſays it is called by the *Indians Teſa Bonganſiſo*, that is, the Excrement of the Whale, becauſe it is ſometimes found in the Whale, and ſometimes vomited up by it. And he imagines that it is produced from Sea Weed, reſinous Sea Plants, the Reſin of Trees growing in the Bottom of the Sea, or even from rotten Sea Weed, or what they call Sea Ware, ſwallowed by the Whale and digeſted into a kind of Mucilage. *Nicolaus Monardes* will have it to be a kind of Bitumen flowing from Fountains in the Bottom of the Sea; and *Guilli du Wallig*. is of the ſame Opinion. *Simon Sethi* and others ſay that it ſprings up in different Places, and that there are Fountains of it as of Bitumen. *Garcias ob Orta* is of Opinion that probably there are Iſlands or Countries where Ambergreaſe may be found. *Ferdinand Lopez Caſtanneda* will have it to be the Dung of Birds of the Iſland *Maldiva*, which feed upon odoriferous Herbs. *Servatius Marel* told *Carol. Cluſius* that it was produced from a viſcid Matter found in the Stomach of the true Whale. Others again alledge, that it is ſwallowed by the Fiſh called *Azelum*, and taken out near its Back-bone. *Ferdinand Caſtrillo* ſays it is a Liquor collected about the Oeſophagus of Fiſhes, as ſome who are return'd from *Braſil* teſtify. *Fran. Combes* ſays that the Inhabitants of *Iolo* will have it to be the Excrement of the Fiſh *Guaiamina*, which is different from the Whale, and larger. * Some ſay that it is produced from an odoriferous Fruit ſwallowed by the Whale. Some call it the Seed, ſome the Liver, ſome with the *Chineſe* (who call it *Hay ano ſay*, that is, preſerving Cloaths from the Moth) the Excrement of the Whale; while ſome again will have it to be a Foam of the Sea. *Cteſias* ſays it is the Seed of the Elephant. *Fuchſius* calls it a Compoſition of Aloes Wood, Civet, Storax and Ladanum. The *Lutai Indians* according to *Combes* ſay, that it is produced from maſſy or viſcid Excreſcencies ſticking to a large Aromatick Tree in the Sea. Others, and thoſe of better Senſe, again alledge, that it is produced from the Reſin of that Tree, and this Opinion the Author himſelf goes into, adding that the crude Ambergreaſe is void of all Flavour, ſo like the Reſin of the *Pilis* (of the Almond and Piſtach Kind) and ſo eaſily inflammable, that many have been deceived by it, and ſuffered for their Heedleſſneſs. *Hieronimus à Huerta* ſays it is the Opinion of ſome, that Ambergreaſe is the Reſin or Gum of Trees, or the Reſin of the Pine-tree. *Thomas Bartholin* in the Medico-philoſophical Tranſactions of *Copenhagen* of the Year 1673, Obſ. 122. f. 306. † affirms that it has the ſame Origin with Amber, (*viz.* after he had proved Amber to be the Reſin of Trees) and makes no doubt but that there are odoriferous Trees in *America* which pour out Juices of the ſame Nature. *Tacitus* writes, that as the Trees in the Eaſt ſweat *Thus* and *Balſams* (and why not alſo *Ambergreaſe?*) ſo it was poſſible that thoſe

* *Joes Leon.* in his *Deſcription of Africa* ſays that Ambergreaſe is thrown up by the Fiſh *Ambara*. † *Odoardus Barboſa* remarks the ſame.

of the Weſt might ſweat Amber. All theſe Opinions and Conjectures being conſidered, is it not moſt probable that its Origin is derived from Trees? For that it is produced in the Manner of Funguſes is extremely doubtful; and that it is derived originally from Sea Weed, or a putrid Recrement of the Sea, I cannot eaſily believe. Neither is it probable that it is a Bitumen, or kind of Earth; and that it is made by Art, is falſe. That it is the Dung of Birds, the Seed of the Elephant, or of the Whale, or a Liquor, the Liver, Seed, or Excrement of ſome other Fiſh, appears to be fabulous. I ſhall willingly grant, however, that ſometimes *Ambergreaſe* being unfit for Nutrition (as *Odoardus Barboſa* remarks) has been vomited up by Whales, or other Fiſhes of the Whale Kind, or has been found in their Oeſophagus, or ſometimes in their Stomachs, theſe Monſters devouring whatever comes in their Way. I very much ſuſpect too, begging Pardon of thoſe who are of the contrary Opinion, that that reſinous Fluid, which the *Indians* of different Provinces call *Agacabac*, *Hagabac*, *Bintogo*, *Bintoco*, *Apitono* and *Malibabo*, *Cayancang* and *Bolotic*, and which is the very ſame with that dry and hard Reſin, which was brought to me in large Lumps from the Mountains of *Ilocos* and *Paynan*, is not the true *Ambergreaſe*. But after a violent Storm, and heavy continued Rain, overflowing the whole Country, it is hurried down into the Sea by the Impetuoſity of the Torrent, from the uninhabited inland Parts, and inacceſſible Tops of the Mountains, and being waſhed and toſſed about violently by the Waves, and ſomewhat cleared and ſoftened, by means of the Salt Water and the Force of the Sun's Rays, it is worked up and prepared in the Form that we find it, * from a Reſin either lately waſhed from the Trees, or hardened by lying ſome Time upon the Mountains; and is more or leſs pure, as it has been macerated a longer or ſhorter while as the Sea Water has been more or leſs toſſed, and expoſed to the Sun. But ſuch weighty Bodies are ſeldom thrown out upon the Shore, except after the Country has been overflowed with continued Rains, and the Sea ſtrongly agitated with furious Tempeſts. Thoſe large, ſolid, and pure Pieces of *Ambergreaſe*, free of heterogeneous Bodies, gathered upon the Shore at *Palagpag*, and brought to *Manilla* in *January* 1694, afford a ſtrong and evident Argument for this Conjecture. (Note, towards the End of *February* 1693 there was a violent Tempeſt.) They were *partly* of the Colour of *yellow Amber*, or that of *Gum Arabick*; more duſky than the Reſin ſent from *Ilocos*, which was almoſt as pellucid as Cryſtal, and very like the Reſin which was brought from *Paynan*, but leſs odorous when they were burnt, eaſily melting in the Fire, in the ſame manner as the Reſin, and almoſt of a ſtony Hardneſs, ſo that neither the Knife nor the Teeth could make any Impreſſion upon them. *Partly*, or on one Side, of a whitiſh gray Colour, variouſly cracked, and brittle indeed, but more ſolid than any *Ambergreaſe* I have ever ſeen.

* *Barboſa* ſays, that by being continually expoſed to the Sun and Moon, it becomes reſinous.

They were ſold however for genuine and the beſt Kind of *Ambergreaſe.* A certain Perſon made a Objection to this Opinion of mine, ſaying that *Ambergreaſe* could not be the Reſin of Trees, becauſe if was found in ſuch large Lumps; to which I anſwered, that the various little Lumps of Reſin of which theſe large ones are compoſed, being homogeneous and adheſive, and drove againſt one another by the Force of the Waves or Torrent, might unite and cohere firmly together ſo as to form larger Pieces. And why ſhould it be impoſſible for large Pieces of this Reſin to be found, when theſe Mountains nouriſh reſinous Trees of an incredible Height, and at *Pilis* and *Lauvan* there are found Blocks of Reſin of a hundred Feet long, and thoſe Fragments that were brought from *Ilocos* and *Paynan* plainly ſhew that the Stock they were taken from muſt have been very large. *Fran. Combes* in his Hiſtory of the Iſlands of *Mindanai*, f. 15. ſays that upon the Shore *Iolò* (called by the *Dutch Date Iſland*) there was found a Piece of Ambergreaſe larger than the Body of an Ox, but it was almoſt all ſold for common Reſin. *Franc. Colin* ſeems to mean the ſame, in his Hiſtory of the *Philippine Iſlands* f. 49. when he ſays that in the Iſland *Iolò* there was found a Piece of *Ambergreaſe* of a gray Colour, and of the very beſt Kind, weighing upwards of two hundred Pounds. And I imagine *Ignatius Alzina* in his Hiſtory of the *Byſaian Iſlands* means alſo the ſame, where he tells of a Piece of Ambergreaſe found at *Iolò*, thicker than a Man's Body, and twice as long, which on Account of its great Plenty was ſold very cheap. In the Year 1632, at the Cape of the *Holy-Ghoſt*, upon the Coaſt of *Igbabao*, there was found a Fragment of *Ambergreaſe* weighing about five and fifty Pounds, which being extremely good, was ſold for a thirty Imperial Crowns the Ounce. For an *Indian* who found it accidentally, had filled three Baſkets with it, two of which, not knowing what it was, and conſequently ignorant of the Value of it, he had carried Home to burn inſtead of Reſin (for formerly the *Byſaians* for the moſt Part made this Uſe of the *Ambergreaſe*, not diſtinguiſhing it from Reſin) and was going to do the ſame with the third, if a Gueſt he had in his Houſe, who knew a little more of the Affair, had not diſcovered by the Fragrance of the Smoak, that it was not common Reſin, but exceeding fine *Ambergreaſe.* † In *Bangabun*, an old Woman gathering of Shell Fiſh, found a Piece of *Ambergreaſe* of the Bigneſs of one's Arm, which ſhe took to be common odoriferous Reſin, (for there are a great many different Kinds of odoriferous Reſins thrown out upon the Shore, as *Batete*, which ſmells like *Ambergreaſe*, and I take it to be the ſame with what they call the black *Ambergreaſe*; becauſe when purified, it is frequently ſold for a cheaper Kind of *Ambergreaſe*; *Dairiangao*, and *Raporago*, which ſmell like *Gum Benzoin*, *Tangay* and *Samato*, with others *Samata*, ſmelling

† *Caſpar Bryam* about the Year 168c, met with a very large Lump of *Ambergreaſe* in the Sea between *Aden* and *Mecca*.

like

like liquid Amber or Balsam) set it apart for Fumigation, and had almost consumed the whole of it before she was informed what it was. *Ambergrease* is frequently very adhesive, and there are many who affirm that it is sometimes found as soft as Tar, which *Fran. Combes* seems likewise to insinuate, when he says, that sometimes it is gathered soft and recent upon the Shore, and after being kept and prepared, turns out extremely good. But the common Opinion is, which both *Hieronymus de Huerta* and *Fran. Combes* confirm, that the crude Ambergrease, or that which is new gathered, has no Sort of Fragancy. But *Joh. Botero Benes*, f. 90. an *Ambergrease* Merchant at *Ava* says, that even when it is not adulterated, it has a very fragrant Smell, and so sharp, as being held to the Nose, to set it presently a bleeding. *Alphonsus de Ovalle* in his History of *Chili* writes, that the *gray Ambergrease* has a sweet Smell, and the *black* a more pungent one, but this Difference of Smell and Colour proceeds from hence, that the black has been a shorter while in the Sea, and less exposed to the Sun than the gray. * I therefore impute the Bleaching, Hardness, and Density of *Ambergrease* to the Sun, and the Sea Water, as *Thomas Bartholine* in the Medico-philosophical Transactions at *Stockholm* in the Year 1671. *Obs.* 57. f. 113. imputes the Solidity of *Amber* to the same Causes. I have a Piece of the whitest and finest Kind of *Ambergrease* bored in different Places, with five Shell Fish, and a Bit of rotten Wood sticking in it. Which is a certain Argument, that the Goodness of it is owing to its being frequently washed, and long exposed upon the Shore.

Of the Asbestus, *and the way of making the* Incombustible Cloth, *by Seignior* J. Campini. n. 273. p. 911.

XXV. 1.] I have four sorts of the *Asbestus* in my *Musæum*. The first from *Corsica* or *Corsu*, long, of a woody Form, of half a Palm length and more, of a whitish Colour, something inclining to a reddish. The second of a silverish Lead Colour, softer and shorter, about three Inches, this was from *Sestri di Ponente* in *Liguria*. The third (which is the worst of all) is like Scales or *Laminæ* one upon another (like an Onion) of a blackish Earth Colour, with some white, black, and dark red Veins, interspersed, scarce two Parts of an Inch *Roman* long, therefore fitter for making of Paper, than spinning or weaving. The fourth sort, given me by Signior *Boccone*, found in the *Pyrenæans*, some whereof were a *Roman* Palm long; its Filaments, though longer, were yet thicker and rougher; I have heard of another sort in *Volateranis Montibus*.

Some have supposed, that the Wieks of the sepulchral Lamps of the Antients were made of this; but from Experiments I conclude it unfit for that Purpose, always finding the Wieks made of it to go out, and not contract or continue up the Oil for the Flame.

I have kept it for 3 Weeks in a Glass-house Fire, but found it unalter'd; but it would not preserve a Stick wrapt in it from the Fire; whence I conclude, the *Amiantus* loses nothing in the Fire, because it

* *Sennertus* upon Bitumen, thinks that the odorous *Ambergrease* is a Bitumen flowing from Fountains in the Sea, and when it is exposed to the Air upon the Surface of the Sea, is thickened and coagulated in the manner of Amber.

does

does not burn nor flame; but in the handling it waſtes, though not much, as I found by an exact Ballance.

As to the Manner of *Spinning* it, I have tried thus; firſt, I laid the Stone in Water (if warm, the better) for ſome Time to ſoak; then it is opened and divided with the Hands, that the Earthy Parts may fall out of it, which are whitiſh like Chalk, and hold the thready Parts together; this makes the Water thick and milky; this is repeated ſix or ſeven Times with freſh Water, where it is again opened and ſqueezed, till all the heterogeneous Parts are waſhed out, and then the Flax-like Parts are collected, and laid in a Sieve to dry.

Of the four ſorts of *Amiantus*, I found that from *Corſica* beſt, being long and ſoft; and the *Cyprian* worſt; I am in doubt, whether mine was of the beſt ſort, ſince the *Cyprian* was commended by *Pancirollus* and others. The Way of ſpinning it, diſcovered to me, was thus: Lay the *Amiantus*, cleanſed as before, between two Cards, ſuch as they Fig. 39, 40.
card Wool with, where let it be gently carded, and then clapt up in between the Cards, ſo that ſome of it may hang out at the Sides; then lay the Cards, faſt upon a Table or Bench. (*Fig.* 41.) Take a ſmall Reele, Fig. 41.
(*Fig.* 42.) made with a little Hook at the End, (*Fig.* 43.) and a Part to Fig. 42, 43, 44.
turn it by, (*Fig.* 44.) ſo that it may eaſily be turned round; this Reele is to be wound over with fine Thread; then having a ſmall Veſſel of Oyl ready, (*Fig.* 45.) with which the Fore-finger and Thumb are con- Fig. 45.
ſtantly to be kept wet, both to preſerve the Skin from the corroſive Quality of the Stone, and to render the Filaments thereof more ſoft and plyant: thus by twiſting the Thread upon the Reele about, with the *Aſbeſtus* hanging out of the Cards, ſome of it will be worked up together with it; by little and little this Thread may with Care be woven into a coarſe ſort of Cloth, and by putting it into the Fire, the Thread and Oyl will be burnt away, and the incombuſtible Cloth remain. But finding this Way of uniting the Stone with the Thread very tedious, inſtead of the Thread I put ſome Flax upon a Diſtaff, and by taking three or four Filaments of the *Aſbeſtus*, and mixing them with the Flax, I found they might eaſily be twiſted together, and the Thread thus made much more durable and ſtrong: So that there is no Need of carding, which rather breaks the Filaments, than does any Good; open only and ſeparate the Filaments after waſhing upon a Table, and take them up with the Flax, which is ſufficient. As to the making of Paper, in the waſhing the Stone, there will remain ſeveral ſhort Pieces in the Bottom of the Water, and of theſe after the common Method Paper may be made.

The beſt Way of preſerving the Cloth, or any other Thing made of the Stone, when made (for by Reaſon of its exceeding Dryneſs it is very apt to break and waſte) is to keep it always well oyled, which is the only Preſervative for it; and when the Cloth is put in the Fire, the Oyl burns off, and the Cloth comes out white and purified.

2.] In the Grounds of *Francis Gordon* of *Achindore*, in the Shire of *Aberdeen*, near the *Highlands*, on the Side of a Hill of a Heath kind of Ground, ſomewhat inclining to what we call Moſs, in a very ſmall Brook,

—— Aſbeſtus, *&c. found in* Scotland, *by Mr.* Wilſon, n. 276. p. 1000

Brook, and hard by it, in the Bounds of ten or twelve Yards, I found a great many Stones, some a Foot in length, which appeared plainly like Wood: But because I could not perceive any Footstep of Wood thereabout, neither could any of them be found, except in that very Spot of Ground, I could not be persuaded they were petrified Wood. Then I went to cut up the Ground about the Place with my Knife, where I found likewise some Pieces of the Stone, and very near the Superficies I got several Pieces of a fibrous Matter, which my Knife could not cut; this I immediately judged to be an *incombustible* Matter, as it proved afterwards, when I try'd it by the Fire. And because, so far as I then remember'd I had heard or read of it, I thought it had been always esteemed certain Filaments that came off the *Lapis Amianthos*, I resolved to observe more narrowly the Production of it.

When I found some Pieces of the Stones very hard in the Middle, and the fibrous Matter on the Outsides and Ends, I was inclined to believe that the Flax came from the Stone: But then finding several Pieces of the Flax so condensed and pressed together, that at first they appeared to be hard Stones, but being a little wet, the Filaments were easily parted from one another. Many more I got, some less and some more condensed into the Nature of a Stone; and all of it, both that which was condensed together, and what was not, was lying about an Inch within the Ground, parallel with the Surface so interwoven with the Fibres of the Roots of the Grass, that it seemed to me much more probable to believe, that the Lint turned into the Stone, than the Stone into the Lint: Especially seeing most Part of the Stones appeared so tender and brittle on the Outside, that it's hard to believe how they could turn into that tough Substance of Flax.

The Stones are of different Sorts, some are white, the Colour of the Lint, and of a very soft Substance; so they may be easily cut with a Knife without blunting it; others are much mix'd with a whitish *Talk*, but most of them are of a grayish Colour, and very hard.

As for the Production of the Flax, I think it's hard to determine in this Place; because the greatest Quantity I found of it was lying, as I said before, about an Inch at most within the Ground, parallel with the Superficies, interwoven with the Roots of the Grass, without any Root of itself, but alike at both Ends, as if it were cut with a Knife. The Ground wherein it is found is of a grayish Colour, about one Inch or two thick, under which there is a black Earth for a Foot in Depth. So that I could find nothing in the Places where most of it was got, that I could rationally conclude to produce it: But in some other Spots I found much of a *Talkish* Sand, and some Pieces of Flax near to it; as also Pieces of the Stone much whiter than the rest, and very like *Talk*; which would incline one to believe that it was produced of it. Yet there being no Appearance of any *Talk* in the other Places, where most of it was found, I can scarce conclude any Thing about the Production of it, but leave it to others.

Pliny,

The Stone is 3 Foot long, and 2 Foot 2 Inches broad.

Pliny, *Aldrovandus*, and *Olaus Wormius* make it very short, whereas some of this I found five, six, seven, and some eight Inches long.

As for the making of it into Cloth, they all conclude it very hard: *Pliny* calls it *inventu rarum, textu difficile propter brevitatem. Olaus Wormius* in his *Musæum* says, *Modus vero, quo ex eo fiunt lina, jam penitus ignoratur.* I confess indeed, it is true what *Pliny* says; yet it may be seen, by the Experiment I have shewn, in making Yarn of it, that Cloth may be made of it also, for the Difficulty is much greater in the one than the other.

3.] I receiv'd the following Account from a Gentleman in the *Highlands*, (not many Miles from *Coupar* of *Angus*) who had lately built an House of a *singular* kind of *Stone*, digg'd out of a Quarry not far from him. This Stone, after the Rubbish (which is not very deep) is done away, lies Horizontally in a Bed endu'd with parallel Fibres, with few Interstices, soft at the Beginning, and easy to be smooth'd and polish'd without any Tool, but rather with Sand, or another hard Stone of a blueish Colour, which afterwards hardens so, that it resisteth the Injuries of Air or Prejudice of Fire. When first the Quarrier began to dig it, he was at a mighty Loss; for endeavouring to cut and raise it after the ordinary Manner with Wedges, and other usual Instruments, it broke and crumbled all to Pieces: But afterwards, observing more narrowly the Duct of its Fibres (so to speak) he endeavoured to cut it with Spades lengthways; and by this Means he procured Stones as big as he had a Mind, which smoothed very easily along the Tract of their Fibres; but when cut transverse, no Means nor Methods could render them smooth, but their Surface remain'd unequal as the Extremities of a Piece of Wood. Although this Quarry has but few Interstices, yet in those it has the true *Asbestos*, of a whitish Silver Surface, consisting of several *fasciculi* with parallel Fibres, like to those of the muscular Fibres of salted Beef; easily separable from each other, pure white, till it becomes so small as the finest Flax; and so ductile, that it may be spun into the finest Thread, whereof it were easy to make the incombustible Cloth, so famous for Shrines among the Ancients. In other Places of those Interstices was likewise to be observed a reddish Substance, near to the Colour of *Sanguis Draconis*; but whether fibrous or not, I cannot say, since the Gentleman could not shew me any of it; but added, he believed it might be good for dying. I got a small Parcel of the *Asbestos* from him; and he told me, if he had known its Value, he could have preserv'd some Pounds of it. I am ready to think the second kind was fibrous too, which might make a very beautiful Cloth, being striped with the other. This whole Quarry may be said to be *Asbestos* of different Colours, the blueish being of a much coarser, and the white and red of a finer Grain.

—*on the same, by Mr.* P. Blair. n. 333. p. 434.

XXVI. *Papers Omitted.*

1. A Description of some *Shells*, brought from the *Molucca Islands*, by Mr. *Sylvanus Landon*, and Mr. *Rowleston Jacobs*, by Mr. *J. Petiver*. n. 274. p. 927.

n. 282. p. 1266. 2. A Defcription of fome *Shells*, fent from *Fort St. George*, by the Reverend Dr. *George Lewis* to Mr. *J. Petiver*, by the *fame*.

n. 299. p. 1952. 3. An Account of fome *Shells* and *Animals* fent from *Carolina* to Mr. *J. Petiver*.

n. 286. p. 1419. 4. A Defcription of fome *Corals*, and other curious *Submarines*, fent from the *Philippine Ifles*, by the Reverend *G. J. Camelli* to Mr. *J. Petiver*.

n. 301. p. 2042. 5. Catalogus *Concharum foffilium*, *Metallorum*, *Mineralium*, &c. quæ a Cl. D. *Johanne Scheuchzero* nuper accepit D. *J. Petiver*.

n. 302. p. 2082. 6. *Mineralia* quædam, *Conchylia* petrefacta, & alia *Foffilia* è *Berolino*, a Cl. *Chriftian. Maximiliano Spenero*, Doct. Med. Reg. *Pruff.* Aul. Acad. S. R. I. Cur. & Soc. Scient. Reg. *Brandenburg*. Colleg. ad D. *J. Petiver* miffa.

n. 311. p. 2397. 7. De *Conchyliis Turbinatis*, *Bivalvibus* & *Univalvibus*; item de *Mineralibus*, *Foffilibus* & *Thermis Philippenfibus*, ex MSS. R. P. *Geo. Jof. Camelli* communicavit D. *J. Petiver*.

n. 337. p. 222. 8. A fhort Account of fome *Swedifh Minerals*, &c. fent from Mr. *Angeftein*, Overfeer of the King of *Sweden*'s Mines, to Mr. *J. Petiver*.

n. 314. p. 77. 9. An *Advertifement* of a *Catalogue* of feveral Specimens of *Figured Foffils*, to be had of Mr. *Alban Thomas*.

XXVII. *An Account of a Book omitted.*

n. 291. p. 1604. Specimen *Lithographiæ Helveticæ* curiofæ, quo Lapides ex Figuratis *Helveticis* Selectiffimi Ære incifi fiftuntur & defcribuntur, a *Johanne Jacobo Scheuchzero*, M. D. *Tiguri* 1702. 8*vo*.

CHAP. IV.

Magnetics.

Of the Invention *and* Improvements *of the* Mariner's Compafs, *by Dr.* J. Wallis. n. 276. p. 1035. n. 278. p. 1106.

I. THE Doctrine of the *Magnet* has been, I believe, more improv'd by the *Englifh*, than by any other Nation; and I am of Opinion, that the *Mariner's Compafs* was originally an *Englifh* Invention; not only becaufe *England* was of old as famous for Navigation as any other Nation, long before the *Holland* Sea-Trade was in being; but (fince new Inventions commonly take their Names from the Place where the Invention itfelf is taken) from the *Name* itfelf of the Mariner's *Compafs*.

The Word *Compafs* (in Latin *Circulus Nauticus*) is an antient *Englifh* Word, for what we otherwife now call by a *French* Name a *Circle*. In *Kent*, where I was born and bred, it was commonly ufed in that Senfe, when I was a Youth; and *Minfhew* in his Dictionary takes *Circle* and *Compafs* indifferently to fignify the fame with *Circulus*: And hence it is that *Circinus* is in *Englifh* call'd a *Compafs*, (or a *Pair of Compaffes*) as being the Inftrument wherewith we defcribe a *Compafs* or *Circle*; but whether

whether *Circinus*, call'd by us a Pair of *Compaſſes*, may have ſome like Name in another Language, I do not know, nor how antiently.

I do not know that the Word *Compaſs*, or any Word like it, was ever uſed in any other Language for a Circle *Indefinitely*, or for any *other* Circle than the *Circulus Nauticus*. In *French* it is *Cercle*, *Cerchio* in *Italian*, *Circulo* in *Spaniſh*, or ſome other Word deriv'd from the *Latin Circulus*; and from hence the *Circulus Nauticus* may come to be call'd the *Mariner's Compaſs*, which Name being given it by the firſt Inventors, might give Occaſion ſor like Names in other Languages, *French*, *Italian*, *German*, &c. *Compaſs*, *Compaſſo*, *Zee-Compaſs*, &c. which Name, together with the Art, I gueſs they borrow'd from *England*.

I might urge the ſame from another Name, *Boſſolo*, *Boſſola*, &c. For as *Circulus Nauticus* is the *Mariner's Compaſs*, ſo *Pyxis Nautica* is the *Mariner's Box*, (for the *Engliſh Box* is from the *Latin Pyxis*) and *Pyxidula* (as a Diminutive from *Pyxis*) muſt be *Boxel*, or ſome Word like it, which eaſily paſſeth into the *French Buxole*, *Bouſſole*; and the *Italian Boſſola*, *Bouſſula*; which all ſeem to be from the *Engliſh Boxel* (*Pyxidula*) *a little Box*; ſoftening the ſound of the Letter *x* in *ſſ*; as in *Aleſſandro* for *Alexandro*.

All which, though it be not a direct *Demonſtration*, yet (ſince it is not agreed by whom, or where the *Compaſs* was invented) may in the Silence of Hiſtory be admitted as a probable *Conjecture*, and a plauſible *Pretence* to the Invention, till a better claim do appear; for *New Inventions* commonly take their *Names* from whence the Invention itſelf is taken.

And where Inventions creep in by Degrees, it muſt not be thought ſtrange, if it be not eaſy to ſay, who is the *Firſt Inventor*.

In the preſent Caſe, he who firſt obſerved (I know not by what Accident) that the *Magnet* hath a *Polarity*, or Inclination Northward, made the *Firſt Step* towards this Invention. This (I think) was at firſt wont to be ſhewed, by putting a Magnet into a little Boat, ſwimming on Water, when it was obſerved, that this Magnet would of itſelf ſo ſteer this little Boat, as that a certain Point in the Magnet would (if not hindered) turn toward the North. Which Point was thereupon called the Magnet's *North Pole*.

He that afterward obſerved that this Verticity, or Polarity, was communicable to a Piece of Iron or Steel, rubbed on a Magnet, added a further Step.

And he who contrived a Way to ſet a Needle or Piece of Steel ſo touched on a ſharp Pin, ſo as in the Air to move horizontally thereon, ſo as of itſelf to find out the North, and point toward it, as before the ſwimming Magnet in its Boat had done on the Water, had now diſcovered a new Experiment in Natural Philoſophy, very ſurprizing.

But this cannot yet be called *Circulus Nauticus* (or the *Mariner's Compaſs*) till they had further contrived a Way how to put a Needle thus

thus poiſed into a Box, with a Compaſs or Circle round it; ſo divided as to denote the *Azimuthal* Points of the *Horizon*, or, as they be now called, the *Points of the Compaſs*; and ſo commodiouſly to fix this Box, ſo prepared, to the Ship, as thereby to inſtruct the Mariner or Steerſman toward what Point of the Compaſs the Ship moved; that by the Help of the Rudder he might put it into ſuch a Courſe, as was proper for his Voyage. And it was now indeed *Pyxis Nautica* or *Circulus Nauticus*, (the *Mariner's Box* or *Compaſs*) but not till then. And he who firſt contrived this Application, did *compleat* this Invention of *Circulus Nauticus*. But all thoſe antecedent Diſcoveries were Steps towards it, and Parts of the Invention.

Now it is not likely that all theſe Diſcoveries were made at once, by the ſame Man, at the ſame Time, but ſucceſſively, by the joint Advice of divers inquiſitive Men, and in a conſiderable Tract of Time; yet all perhaps of the ſame Nation, and probably the *Engliſh*.

But whoever gave the firſt Hint of this Invention, certain it is, that the great *Improvements* of the Magnetic Doctrine are due to the *Engliſh*, and chiefly to thoſe about *London* and *Greſham* College. And it is fit the Memory of it ſhould be preſerved.

The Caſe is much the ſame with that of *Printing*, which we cannot reaſonably ſuppoſe to be invented all at once, nor perhaps all by the ſame Man; but rather, by the concurrent Advice of divers, and in a conſiderable Tract of Time, before it came to that Degree of Perfection which we now call *Printing*.

It might be firſt obſerved, that the Shape of a Letter, Figure, or Picture, graven on Wood or Metal, might (with help of a convenient Preparation of Oil, Ink, or coloured Liquor) be ſtamped on Paper; and, if once, then as oft as you pleaſe.

And if by ſtamping the Print on Paper, then as well by due Application of the Paper to the Print, thus prepared.

And if one, then by the ſame Reaſon to two or more, if fitly conjoined, and even to a whole Page at once; and, of that, as many Copies as we pleaſe.

But, this being admitted, it remains further to be contrived, how all theſe Prints or Stamps for a whole Page ſhall be ſo compoſed into one Frame, that the Paper may be applied to all at once.

It is then to be conſidered farther, what kind of *Ink*, (or ſomewhat inſtead of Ink) is to be applied to the Face of the Letters, thus compoſed; for common Writing-Ink will not ſerve the Turn.

Then, how the Paper ſhall be applied (with an equal Preſſure, and ſufficient) ſo as to take off juſt ſo much of that Ink, as repreſents the Face of thoſe Letters, and no more.

And after all this, it muſt be further contrived, how to erect ſuch a Structure, as what we now call a *Printing-Preſs*, and how to manage it, ſo as to anſwer all theſe Exigences: For, till all this be done, we are not arrived at what we call *Printing.* But all thoſe previous Contrivances, muſt be owned as *Parts* of the *Invention.*

And in the *Magnetic Doctrine* likewiſe. And to thoſe *previous* Diſcoveries, muſt be added, the *ſubſequent* Improvements of Magnetic Knowledge, ſince the firſt *Uſe* of the *Mariner's Compaſs.*

But whoever was the firſt Inventor of the *Mariner's Compaſs*, it is certain, that the *Doctrine* of the *Magnet* has receiv'd very great Improvements from the *Engliſh*; from *Blagrave, Gunter, Gellibrand, Gilbert, Norwood, Wright, Brigs, Foſter,* &c. and of late by Capt. *Halley* in his *Map* of the *Magnetic Variations*; which I look upon as an excellent Deſign, well contriv'd, and well executed, and which fixes the Buſineſs of the Magnetic Variation in theſe Seas for the *preſent* Time.

I think it is agreed on by all Hands, that what we call *the Variation of the Needle*, is an *Engliſh Diſcovery*, (of Mr. *Gellibrand*, if I miſtake not, one of Sir *Thomas Greſham*'s Profeſſors at *Greſham* College) about the Year 1625. that is, that the *Magnetic Needle* in its horizontal Poſition doth not retain the ſame *Declination* or *Variation* from the true North, in the ſame Place, at all Times, but doth ſucceſſively vary that Declination from Time to Time; which, though it were about that Time a *New Diſcovery*, is now admitted as an undoubted Truth.

It was about the Beginning of the Reign of King *Charles* the Firſt, that Mr. *Gellibrand* (if I have not been miſinformed) cauſed the Great *Concave Dial* to be erected in the *Privy-Garden* at *Whitehall*, with great Care to fix a true *Meridian-Line*; and with a large *Magnetic Needle*, ſhewing its *Variation* from that *Meridian* from Time to Time. And, I think it were not amiſs, if exact Obſervation were now made, whether the *Meridian* be now juſt the ſame as it was then; for it is very poſſible, that the *Pole* of the Earth may in Time ſuffer ſome little Variation (which may not readily be diſcerned) which may cauſe an Alteration of the *Meridian* Line: And this, if ſo, will be more diſcernible nearer the Pole, than farther off.

What we call the *Dipping Needle*, is admitted alſo to be an *Engliſh* Diſcovery, (I think of Mr. *Blagrave*'s) ſomewhat older than the former, that is, that the *Magnetic Needle*, beſides its *Horizontal* Direction toward the North, hath alſo a Direction of *Altitude* above the Horizon, if balanced on an Horizontal Axis; pointing, as it were, with its Northern End in our Climate to ſome Point within the Body of the Earth. Whether or no this Direction do vary from Time to Time as doth that of its Horizontal Poſition Northward, I cannot tell; nor do I know whether or no it hath been yet obſerved; nor whether or no the Southern End in other Parts of the World do *dip*, as the Northern End doth with us.

'Tis alſo an *Engliſh Obſervation*, that not only a *Magnetic Needle*, but any Piece of *Iron* (if kept long in the ſame Poſture) ſhall of itſelf contract a *Polarity.* As for Inſtance, an erect Bar in a Window, after long Continuance in that Poſition, will, if duly poiſed, be found with its upper

upper End, to point toward the North; and Southward with the other End. And if afterwards it be continued long in a contrary Pofition, it will attain a contrary Polarity.

And Mr. *Gilbert*'s Notion (of the Earth's whole Body being but one great Magnet; and, leffer Magnets being fo many *Terrella*'s, fympathizing with the whole) is *Englifh* alfo.

It hath been obferved alfo, that a magnetic Needle, if heated red-hot, will lofe its Polarity; and, if then cooled in a contrary Pofition, will acquire a contrary Polarity.

It hath alfo been obferved by our *Englifh* Mariners, (and, I think, more than once) that, upon a great Flafh of *Lightning* at Sea their Magnetic Needle hath loft its former Polarity, and contracted the contrary.

In general, the Doctrine of *Magnetifm* hath been more improved by our *Englifh* Naturalifts, than (for ought I know) by any other Nation. And, if fome one would take the Pains to give us a true Hiftory of thefe (and the like) Improvements, it would be an acceptable Work, and for the Honour of the Nation.

Magnetical *Experiments and Obfervations, by Mr.* W. Derham. n. 303. p. 2136.

II. 1.] Having lately invented an *Azimuth Compafs*, as I was preparing it for obferving the *Magnetic Variation*, I took Occafion to try divers Magnetic Experiments, and by that Means happened upon this odd *Phænomenon*.

Having touched a Piece of Wire, fo that it ftrongly tended N. and S. I was minded to fee whether it would have any Inclination to either of the Poles of the World, when turned round like a Ring, fo that the two Ends of the Wire met: And having again ftraightened it, I was furprized to find it had quite loft its *Verticity*; the Caufe of which, I prefently concluded, to be the Contact of the Northern and Southern Ends of the Wire, which I thought might fo influence one the other, as to confufe its Poles; although I confefs I had never obferved any fuch Confufion to arife upon the bare Contact of the Northern and Southern Ends of two other touched Pieces of Wire.

Upon this, I touched ftrongly the fame, and other Pieces of frefh Iron Wire, and having found them all greedily to turn N. and S. I coyled them round fo as that the Ends fhould not come near one another, and again fpeedily opened them ftraight; and found, as before, that every Piece had utterly loft its *Verticity*: Nay, the *Magnetic Virtue* was fo abfolutely deftroyed by bending the Wire, that it had not only loft its Inclination to either Pole, but the two Ends of each Wire feemed indifferent to the Poles of the *Loadftone*, *viz.* whereas before the bending, the adverfe Poles of the *Loadftone* would repel, and the fimilar Poles attract the adverfe, or fimilar Ends of the Wire; now the repulfive Virtue was quite extinguifhed, and either End would indifferently be attracted by either Pole of the Magnet; all one as if the

the Wire had been heated red-hot (which is well known to deftroy the Virtue) or never had been touched at all.

This I experimented over and over again upon Wires of different Lengths, with the fame Succefs. Only this muft be obferved, if you only bend the Wire round, fo as that it fhall fpring back into its Place, or recoil, fo as to be near the fame Straightnefs, that then no fuch, or but little of fuch, Effect will enfue. But to produce this Effect, the Wire muft be *fharply* bent, fo as that *Violence* may be exerted upon it. If it be coyled two or three times round a fmall round Stick, it will beft fucceed. And farther alfo, it is neceffary that every Part of the Wire fhould be bent, to evacuate the Magnetic Virtue: For if the Ends, or any other Part happen not to fuffer the Violence of bending, that Part fhall retain its Magnetifm: As for Inftance, if the Wire be all coyled, except half an Inch, or indeed half a tenth of an Inch at each End, every Part fo coyled fhall both lofe its Verticity, and fhall incline indifferently to either Pole of the Magnet; but the two Ends (although not able to turn the whole Wire N. and S.) fhall fly from, or tend unto the refpective Pole of the Magnet: Or if every Part of the Wire be coyled, except a fmall Bit at one End only, all that coyled Part, when extended, fhall utterly be deprived of its Magnetifm as before; and only that uncoyled Bit retain its Averfion, or Inclination to the Magnetic Poles.

From the Confideration of all which Particulars, it is very manifeft, that the Violence exerted upon the Wire by bending, doth utterly extirpate the Magnetic Virtue, or at leaft make fuch a Confufion therein, that it is as if wholly deftroyed: Which is a Cafe very odd, and never (that I have ever met with) taken Notice of before.

FurtherObfervations on the fame, by the fame. *ibid.* p. 2138.

2.] The Account which I lately gave of the *Deftruction of the Magnetic Virtue* in a touched Piece of Iron Wire, by *Bending*, or *Coyling round*, I thought had been New: But by looking over what others have written of Magnetics, I find in *Grimaldi de Lumine & Colore*, that he, and in our *Phil. Tranf.* N° 188. * that M. *de la Hire* had hit upon the fame Difcovery before me. And I am glad that I have the Authority of others on my Side, the Experiment not fucceeding in fome Tryals fince.

* *Vid.* Supra V. II. C. IV. S. IX.

The Matter of Fact was thus, and to me furprizing: I touched and coyled feveral Iron Wires, but the Effect that enfued was not fuch as I told the Society. The Verticity was indeed much weakened, but not totally deftroyed, and the Ends of the Wires would be attracted or repelled by the Poles of the Magnet; whereas I faid they ufed only to be attracted. The next Morning I tried again; and then the Magnetifm of the Wires was totally deftroyed, as I related. This Experiment I repeated divers Times, and on divers Wires this Winter, and commonly find, that, all the Day, coyling will evacuate the Magnetifm; but that it will not abfolutely do it in the Evenings. But whether it

will

will do so in Summer, or all Weathers, or whether it succeedeth thus only in different Times of the Day, I must leave to farther Tryals. I well know that the Orb of the Activity of Magnets, is larger, or less, at different Times. That noble Magnet in the Society's Repository, found in *Devonshire* by Dr. *Cotton*, is known in some Weathers (or at some Times) to keep a Key, or other Piece of Iron, suspended to another Iron, at 8, 9, or 10 Feet Distance. But at other Times, the Iron will drop down at the Distance of 3 or 4 Feet from the Magnet. Now whether at all, or how far this may reach the fore-mentioned Case, I cannot say, not having as yet sufficiently experimented the Matter.

Finding the Case thus with *Coyled*, or *Bent Wire*, I was minded to try the Event of *Twisting* of Iron Wire from End to End, after it had been well touched. The Success was, the *Verticity* was always weakened, and sometimes inverted. And when it was so, the Load-stone did accordingly commonly repel or attract, all one as if the twisting the Wire had given a new Touch the contrary Way.

But in some Wires so twisted the Verticity was wholly destroyed, or rather much confused: For I found by drawing one of the Poles of the Load-stone along near the Sides of the Wire, that in some Places it would attract, in others repel, and so attract and repel all along the Wire. Nay, I fancied in some Places, that one Side of the Wire would be attracted, the other repelled by one and the same Pole of the Load-stone.

To these odd Changes I could add divers others, which the *Twisting* produced: But these do sufficiently shew, that the Magnetic Virtue is put into great Confusion by the Violence exerted upon the Wire by *Twisting*: Which not only separateth the Fibres of the Iron (as may be seen with the Eye, especially assisted with a Microscope) but also changeth their Situation from Longways to Screw-ways.

I then try'd what would be the Issue of *Splitting* or *Cleaving* touched Wires: Particularly whether they would exert the same Effects that Magnets are said to do, when sawn in two Meridionally. Concerning which Dr. *Ridley* ‖ saith, "Cut a Piece from a Magnet-stone meridionally, and that End which was placed S. when it was whole, being severed, will turn North, although naturally at first it was the S. Point." But Mr. *Barlow* is of a contrary Mind, and saith, That the Poles of such a Piece of Magnet, when severed, will abhor the same Poles, to which it grew in the whole Magnet. But he subjoins †; "But here you must beware of an Error, which some unhappily have entangled themselves withal, who beholding the aforementioned Discord, wrongfully supposed, that if both these Magnets the greater and the less [*i. e.* the Piece cut off] were conveniently placed to swim in Water, the little one would not with his End point unto the South of the Earth as it did in the Magnet being entire, when it was a Part of the true North End, but would point contrarily. There is (saith he) no Manner of any such "Alteration,

‖ *Treatise of* Magnet. Bodies *and* Motions. Ch. 9.

† *Magnet. Advertisements.* Ch. 2.

" Alteration, but that both the great one, the little one, and all the " like, that are cut Meridionally one from another, will abſolutely " point the ſame Way which the entire one did. Only the Meridian " will be ſomewhat removed, *&c.*"

Dr. *Gilbert* is as expreſs as Mr. *Barlow.* For (*L.* 2. *c.* 5.) ſpeaking of a Magnet divided, and ſhewing how that the Parts, which in the whole Stone coaleſced, do by Separation repel one another, he ſaith, *That what was the N and S. Pole before, is ſuch ſtill. Non enim* (ſaith he) *immutatur Verticitas (quod male affirmat* B. Porta.) *Nam licet* [*Poli ſeparati*] *non conveniunt, ut alter ad alterum inclinaret; tamen uterque in idem Horizontis punctum convertuntur.*

How the Truth lieth between Dr. *Ridley* and the two latter Authors, I cannot determine, having never ſo cut a Magnet. But by the Magnetic Laws, as well as from the Authority of Dr. *Gilbert* and Mr. *Barlow*, I doubt not but the latter is the trueſt Opinion.

But in *Cleft Wires* the Caſe is very uncouth: Oftentimes the Poles are quite changed: So that what was the *North*, becometh the *South* Pole of the Wire in all Reſpects; I mean, not only turning, but alſo embracing, or avoiding the Poles of the Load-ſtone, as if it had received a new and contrary Touch. Sometimes one half of the Wire will retain its Magnetiſm, which it had before ſplitting, and the other half have it quite changed. Sometimes no Change at all will enſue, only the Magnetiſm be much weakened; as indeed it always is in all the Experiments where the Wire is ſplit. (But generally, where one of the Halves hath ſuffered Change, the other not, I have obſerved, that 'tis the thinneſt and weakeſt that hath been changed, and the thickeſt hath retained its Touch.) Sometimes where one of the ſplit Halves received an *inverted Verticity*, or ſeemeth to have no Verticity at all, one of its Ends will incline to one of the Poles of the Magnet, not according to its Touch, but in an inverted Order, and the other End be attracted indifferently by both the Poles of the Loadſtone. And in ſome Caſes, that End ſhall be attracted by one Pole, but be neither attracted nor repelled by the other; but ſtand as it were heſitating whether it had beſt fly to, or from that Pole of the Loadſtone. Only if that Pole of the Magnet be too near, then that End of the Wire will conſtantly fly thereto: As indeed it is the Nature of all Magnets and Magnetic Bodies to do, when they touch or approach very near one another, though they repelled before.

The Cauſe of theſe great Changes in touched Wire produced by *Splitting*, I have ſometimes imagined to ariſe from the Violence exerted thereon by bending. But in ſome Wires that I ſplit, or cleft with very little bending, one half hath been utterly changed, the other not.

In others that I cleft, by ſuffering the Halves to bend as much as they would, no Change hath been; and ſome have quite ſuffered Change.

Sometimes I have imagined that the Splitting the Wires in a N. or S. Poſition, or that the Beginning to ſplit at the N. or S. End of the Wire firſt, might be the Cauſe of this Contraverſion of the Poles. But Trials ſhewed there was little in any of this.

Thus I would have done with *Split* or *Cleft Wires*; but there is one Thing very ſurprizing, *viz. That the laying one, or the other Side of the Half uppermoſt*, will cauſe a great Alteration in its Tendency, or Averſion to the Poles of the Magnet (as I have ſaid). But if you lay the contrary Side of that Half uppermoſt, the ſame End ſhall be attracted by one, and repelled by the other Pole of the Magnet. In other Pieces, where the Ends are regularly attracted or repelled, only in an inverted Order (as if new touched) if it lay with the round Side uppermoſt at that Time, and be then turned upſide down, *viz.* the flat cleft Side uppermoſt, 'tis ten to one if one of the Ends be not either attracted by both the Poles, or repelled by both; or elſe attracted or repelled by one, and heſitates as to the other. For ſo it often befals.

The Cauſe of this Lubricity of the Magnetiſm, I imagined might be, becauſe the Sides or Edges of the Wire had received contrary Poles by Splitting: And conſequently were turned topſy-turvy, that what was the N. might then be the S. Edge of the Half. But I could never diſcover but that the Sides of each End, or of any other Part, were the ſame, when I held the Loadſtone to one or the other Side. Which indeed I always did in every Experiment for greater Certainty Sake.

My Hand being in, I try'd the old Experiment of touching Wires, *by rubbing them backwards and forwards* with one of the Poles of the Loadſtone, becauſe it might probably give ſome Light into the aforementioned ſtrange *Phænomena.*

Mr. *Barlow* was I think the firſt that diſcovered the Error of this Way of Touching, *viz.* That it weakeneth or much hurteth the Touch. This I try'd, and found what is ſaid not only to be true, but alſo that the Reaſon thereof is, *Becauſe the Poles of the Wire, or Needle, ſo touched, are not at the Ends, but in or near the Middle of the Wire or Needle.* Sometimes one is near the Center, the other at one or both Ends. For in ſome Wires ſo touched, both the Ends of the Wire would be attracted by one Pole of the Loadſtone, and repelled by the other: And in ſuch Caſe the repelling Pole always found a ſympathetic Part near the Center of the Wire. In others (eſpecially where a Verticity ſucceeded, as ſometimes it will do, and that pretty ſtrongly too, in ſuch a Caſe) the Verticity would be inverted, and the Ends of the Wire be attracted and repelled in a direct contrary Manner to the Natural Form. And the Reaſon of all this will be manifeſt from theſe following Experiments.

I touched a Wire from End to End with only one Pole of the Magnet: This gave ſo vigorous a Touch, that I am almoſt of Opinion, *It is the beſt Way of Touching.* The Conſequence was, the End where I began

I began always turned contrary to the Pole that touched it. I again touched the ſame Wire, and others too with the other Pole of the Magnet, from the ſame End, and then that End turned the contrary Way, *e. g.* Mark one End of a Wire for the North End, and touch that Wire, by drawing the N. Pole of the Magnet divers Times along the Wire from the N. to the S. End; this Wire, ſo touched, ſhall have a vigorous Verticity, but the North End ſhall ſtand South: But if you touch that, or another Wire, (for it is all one, becauſe the latter deſtroys the former Touch; I ſay, if you touch) by drawing the N. Pole of the Magnet from the S. to the N. End of the Wire, then this N. End will turn N. And ſo it will do the ſame, if you touch with the Southern Pole from the N. to the S.

There is one Experiment more, doth yet give farther Light into what goeth before, *viz.* I touched an Iron Wire exactly in the Middle with only one Pole of the Loadſtone, without drawing it backwards or forwards. The Event was, that in that Place that Pole of the Wire was, and the two Ends were the contrary Pole of the Wire, and were accordingly repelled or attracted by the Poles of the Loadſtone: And the Middle, and an Inch or more on each Side, was attracted by the Pole only that touched it.

If we reflect and compare the foregoing Experiments one with another, they not only illuſtrate one another, but ſeem to lay open a fair Way towards the Diſcovery of a great many of the intricate *Phænomena* of Magneticks.

Experiments concerning the Proportion *of the* Power *of the* Loadſtone *at different* Diſtances, *by Mr.* F. Hawkſbee. n. 335. p. 506.

III. 1.] Without mentioning the Difficulties that attend the making of Experiments of this Nature, I ſhall give an Account of their Succeſs.

I took a Quadrant of four Feet Radius, and having fix'd it to the Floor, in the Poſition of the Needle, whoſe South Point directed itſelf to no Degrees, I then fix'd a Board (likewiſe on the Floor) in a direct Angle from the ſame, the Graduations on which Board were 3 Inches diſtant from each other. The Needle was ſuſpended on a Point ariſing from the Center of the Quadrant, from whence were meaſured the ſeveral Stations of the Magnet. The Magnet was laid on a thin Piece of Board, under which, to one Side was nail'd a narrow Slip of Wood, to ſlide it along the Side of the foremention'd graduated Board, whereby the Stone might be always kept in the ſame Direction to the Needle. The Stone that I uſed, weighed about ſix Pounds; was rough, and of an irregular Figure; yet I could diſcover no Inconveniency in the Experiment ariſing from the ſame, it being, and acting at all Diſtances in the ſame Poſition as it is firſt plac'd on the Board: And I ſee no Reaſon to doubt, but the Proportions of its Power will be regular, and agreeable to the ſeveral Diſtances, as more than once I have obſerv'd. For when the Stone hath been differently poſited on the fore-mention'd thin Board, different Angles of the Needle would enſue at the ſame Stations, yet their Proportions one to another would be nearly the ſame. My

Meaning is this: Suppose the Stone was so plac'd, as at 3 Inches from the Needle, it would give the Needle an Angle of 90 Degrees, the Stone being continued in the same Direction, at the several Stations, the Proportions of its Power one to another would be much the same, as if the Angle of the Needle at the first Beginning made but 87, or even but 80 Degrees on the Quadrant; for upon a small Alteration of the Poles of the Stone, such Diversity of Angles will arise.

In these Experiments I made use of two Needles, one of a *Radius* of 6 Inches, the other but one Inch; which last, after many Trials, I found to be most accurate, besides the Advantage it gave in beginning the Experiment 6 Inches nearer the Stone, than the other; and from two Feet Distance from the same, it became nearly agreeable to the Angles made by the long Needle to all the farther Distances; as you will find by the following Tables, which were made with the several Needles in the same Direction of the Stone. I measured the Angles by a Silk Thread strained directly over the Needle to that Part of the Quadrant to which it was directed; which was the best Way I could contrive to come nearest the Truth.

It may be observ'd from the following Tables, that the long Needle at 9 Inches from the Stone, made somewhat a larger Angle than the short Needle at 3 Inches Distance from the same; that the short Needle at the Distance of 9 Inches, made an Angle of 9 Degrees less than the long one at the same Place. But this Odds will easily be accounted for, if we consider the Disproportions of the Needles Lengths; for the Point of the long Needle at 9 Inches, was brought within an Inch as near the Stone, as the Point of the short Needle was, when but 3 Inches distant from the same: The Point of the short Needle at 9 Inches from the Stone, was 5 Inches farther from it, than the long one at the same Station. These Disproportions being consider'd, it is no Wonder such Difference of Angles should ensue upon the Use of the several Needles near the Stone; for at two Feet, and the farther Distances, they become nearly agreeable, as I said before. When I speak of Distances from the Needle, I always mean from the Center of it.

Farther it is observable, that the Stone at 5 Feet Distance from the Needle made an Angle of 2 Degrees with one, and with the other of two and a half; yet upon the Absence of the Stone, they would return to no Degrees, as at first: Which plainly shews, that the Influence of the Stone extended farther; although Observations, at remoter Stations, could not easily be determined.

At greater Distances, and even the more remote in these Tables, the Power of the Stone is so weak, and the measuring the Angles at all Times exactly, so difficult, that 'tis well if we come sometimes within 10 or 20 Minutes of the Truth.

Expe-

Experiments by the ſhort Needle.

Diſtances of the Load-ſtone from the Needle in Inches.	The ſeveral Angles of the Needle at the ſeveral Di-ſtances.	The differences compared one with another, at the ſeveral Ob-ſervations, in Minutes.
	D	
3	87 — 00	180
6	84 — 00	330
9	78 — 30	570
12	69 — 00	735
15	56 — 45	795
18	43 — 30	630
21	33 — 00	540
24	24 — 00	360
27	18 — 00	270
30	13 — 30	150
33	11 — 00	135
36	8 — 45	105
39	7 — 00	90
42	5 — 30	60
45	4 — 30	40
48	3 — 50	30
51	3 — 20	20
54	3 — 00	15
57	2 — 45	15
60	2 — 30	00

Experiments by the long Needle.

Diſtances of the Load-ſtone from the Needle in Inches.	The ſeveral Angles of the Needle at the ſeveral Di-ſtances.	The differences compared one with another, at the ſeveral Ob-ſervations, in Minutes.
	D	
9	87 — 30	345
12	81 — 45	570
15	72 — 15	1137
18	53 — 20	1100
21	35 — 00	660
24	24 — 10	380
27	17 — 50	280
30	13 — 10	180
33	10 — 10	130
36	8 — 00	90
39	6 — 30	75
42	5 — 15	65
45	4 — 10	40
48	3 — 30	30
51	3 — 00	25
54	2 — 35	20
57	2 — 15	15
60	2 — 00	00

The Stone with which theſe Experiments were made, was of this Form, and weigh'd exactly ſix Pound, one Ounce, and a Quarter, Averdupois-Weight. Its Breadth at the North-pole was four Inches, at the South-pole five Inches; the Poles running through the Stone, in the Direction of the prick'd Line. The Length of the ſhorteſt Side was ſix Inches and a Half, and of the longeſt Side ſeven Inches and a Half. Its Thickneſs at the North-pole was one Inch and a Half, and at the South-pole one Inch.

2.] By Order of the *Royal Society*, Mr. *Hawkſbee* and myſelf made an Experiment with the great Loadſtone belonging to the Society, in order to diſcover the Law of the Magnetical Attraction, an Account of which I gave to the Society, in a Letter to Dr *Sloane*, dated *June* 25, 1712. Since that, Mr. *Hawkſbee* made another Experiment of the ſame

An Account of an Experiment *to diſcover the* Law *of the* Magnetical Attraction, *by* Dr. B. Taylor. n. 344. p. 294.

ſame Nature with a ſmaller Loadſtone; upon comparing the Numbers of that Experiment with thoſe of the other, I find the Numbers of the firſt Experiment to be very much more regular; wherefore I conclude the firſt Experiment to be the beſt. It was made in the following Manner:

We placed the Great Loadſtone belonging to the *Royal Society* ſo, that its two Poles lay in the Plane of the Horizon, and were in a Line exactly at Right Angles with the natural Direction of the Needle we made uſe of (which was that Dr. *Halley* had made to obſerve the Variations with:) And by means of a Carriage contriv'd for that purpoſe, the Stone was eaſily moved to and fro, the Poles continuing always in the ſame Line. The Needle was ſo placed, that the Center it play'd upon, was in the ſame Line with the Poles of the Stone; the North-pole being towards the Needle. We meaſur'd the Diſtances from the Center of the Needle to the Extremity of the Stone: and we found the Variations of the Needle from its natural Poſition, to be as in this Table.

Diſt. Feet.	*Variat*	
1	81	45
2	58	00
3	30	00
4	16	00
5	9	20
6	5	35
7	3	30
8	2	20
9	1	35

CHAP. V.

Agriculture. Botany.

Of the Manuring *of* Lands *by* Sea-ſhells *in* Ireland, *by the Archbiſhop of* Dublin. n. 314. p. 59.

THE Counties of *London-Derry* and *Donegal* in *Ireland*, are very mountainous, and thoſe Mountains covered with Bogs and Heath, inſomuch that there is little Arable Ground in them, except what has lately been made ſo. There are three Ways practiſed to reduce Heath and Bog to Arable Land: The firſt is, by cutting of the Scurf of the Ground, making up the Turf ſo cut in Heaps, and when the Sun has dried theſe Heaps, they are then ſet on *Fire*; when burnt as much as they can be, then thoſe Heaps are ſcattered on the Ground, and it being ploughed, it beareth Barley, Rye, or Oats, for about three Years.

The Inconveniences are, firſt, that ſuch Burning defiles the Air, cauſeth Rain and Wind, is not practical in a wet Summer, and by deſtroying the Sap of the Earth and Roots of the Graſs, and all other Vegetables, renders it uſeleſs for ſeveral Years after the third, in which it is ploughed.

The

The ſecond Way is by *Liming*; this is much better than the former, becauſe it doth not ſo much depauperate the Ground, will laſt long, and beareth better Grain, and whatever is pretended, doth not deſtroy the Graſs, if due Care be taken not to over-plough it; but then this is very dear, and Lime-ſtone is not every where to be had, and in many Places Fire is wanting.

Dung is the common Manure in all Places, and therefore I ſhall ſay nothing of it.

Marl is not uſed, that I have obſerved, in the North, but about the Sea-ſide the great Manure is *Shells:* Any one that will look into the Map, will ſee how the Bay of *London-Derry*, commonly called *Loughfoyle*, lies; towards the Eaſtern Part of it there lie ſeveral Eminencies that hardly appear at low Water; theſe are made of Shells of Sea-Fiſh of all ſorts, more particularly of Perewinkle, Cockles, Limpet, *&c.* The Country Men come with Boats at low Water, and carry Loads of theſe Shells away; they leave them in Heaps on the Shore, and there let them lie till they drain and dry, and by that Means become much lighter for Carriage; they carry them by Boats as far as the Rivers will allow them, and then in Sacks on Horſes, perhaps ſix or ſeven Miles into the Country; they allow ſometimes 40, but moſtly 80 Barrels to an Acre; they agree with boggy, heathy, clayey, wet, or ſtiff Land, but not with ſandy. They ſeem to give the Land a ſort of Ferment, as Barm doth to Bread, opening and looſening the Clods, and by that Means making Way for the Roots to penetrate, and the Moiſture to enter into the Fibres of the Roots: The Manure continues ſo long, that I could find none that could determine the Time of its enduring.

The Reaſon of its long Continuance ſeems to be this, that the Shells melt every Year a little, till they be all ſpent, which requires a conſiderable Time, whereas Lime, *&c.* operates all in a Manure at once; but it's to be obſerved, that in ſix or ſeven Years the Ground grows ſo mellow, that the Corn that grows on it becomes rank, and runs out in Straw to ſuch a length, that it can't ſupport itſelf, and then the Land muſt be ſuffered to lie a Year or two, that the Ferment may be a little quieted, and the Clods harden, and then it will bear as long again, and for ought I know and could find, it continues to do ſo with the like Intermiſſions for 20 or 30 Years.

In the Years in which the Land is not ploughed, it bears a fine Graſs, mixed with Daiſies in Abundance; and it is pleaſant to ſee a ſteep high Mountain, that a few Years before was all black with Heath, on a ſudden look white with Daiſies and Flowers.

It fines the Graſs, but makes it ſhort, tho' thick. Obſerving that this Manure produced Flowers in the Field, I made my Gardener uſe theſe Shells in my Flower-Garden, and never ſaw better Carnations, or Flowers fairer or larger than in that cold Climate; and it contributes to deſtroy Weeds, at leaſt doth not produce them ſo much as Dung; it likewiſe produces very good Potatoes at about a Foot Diſtance from Potatoes.

one

one another; and this is a Method of reducing boggy barren Land. They lay a little Dung or Straw on the Land, and ſprinkle it with Shells; ſometimes they cut the Potatoes, if large, that they may go the further, and then dig Trenches about ſix or ſeven Feet Diſtance, and throw the Earth or Soil, they take out of them, on the Potatoes, ſo as to cover them, and then fencing the Plot of Ground ſo planted, let them grow. Plant them in *April* or *May*, and they are ripe in *Auguſt*; they dig them as they have Occaſion, and let them lie till next Year, then dig them again, and ſo the third Year; every Year they by this Means go deeper in the Earth, and the laſt they dig them, then pick them out as carefully as they can, that little Seed may remain; and the fourth Year they plough the Ground, and ſow Barley, and the Produce is very good for ſome Years. Some Potatoes will remain and grow up without any Hurt to the Barley or Oats; and thoſe they dig and pick out, and the Ground remains good and arable ever after.

'Tis obſervable, that Shells do beſt in boggy Ground, where the Surface is Turf; Turf generally is nothing but the Product of Vegetables, ſuch as Graſs, Heath, *&c.* that being rotten, the Salt is waſhed away by the Water, and there remains only the earthy, and eſpecially the ſulphureous Parts of them, as appears from the Inflammability of Turf; now Shells being chiefly a Salt, it incorporates with the Sulphur of the Plants, and renders them fit for the Vegetation of new Plants.

And this appears further from this, that Shells that have been under the Salt Water are much better than ſuch as have been in the Earth, to dry at the Strands: Almoſt about the Bay of *Londonderry*, if you dig a Foot or two, it yields Shells, and whole Banks are made up of them; but theſe, tho' more intire than ſuch as are brought out of the Shell Iſland, are not ſo profitable for Manure.

I obſerved in a Place near *Newtown-Lamavady*, about two Miles from the Sea, a Bed of Shells, ſuch as lie on the Strand; the Place was cover'd with a Scurf of wet ſpouty Earth, about a Foot thick; the Country People uſed the Shells, but they were not reckon'd ſo good as thoſe that are found in the Sea or near it.

The Land about the Sea-ſide bears very indifferent Wheat, nor will the Shells do in that Particular, without ſome Dung; but I very much doubt, whether that be not due to the Ignorance of the Farmers, that generally underſtand nothing of Wheat.

Some Thouſands of Acres have been improved by the Shells, and that which formerly was not worth a Groat *per* Acre, is now worth four Shillings: They have in many Places thus improved the very Mountains, that before were very Turf Bogs. In theſe they meet with this Inconveniency, that if the Seaſon for Ploughing proves wet, their Horſes ſink ſo deep in the Soil, that they can't plough it, eſpecially after two or three Years.

They

They commonly made Lime of the Shells formerly, and ſome do ſo ſtill. I have not, that I remember, ſeen any ſuch Lime, but I underſtood that it bound very well, and I believe it is not ſo corroſive as Lime made of Stone; for I find, in the Hiſtory of *Ceylan*, that they make up their Land with Lime of Oyſter-Shells, and which I believe would be impracticable with Common Lime.

About 30 Years ago they made Lime of the Shells, and manur'd their Lands with it; but a poor Countryman, that, out of Lazineſs or Poverty, had not provided to make Lime, threw the Shells unburnt on his Land; his Crop prov'd as good as his Neighbours, and the ſecond and third Crop better, and all took the Hint, and have uſed them ſo ever ſince.

Where Shells are not to be procur'd, Sea-Rack or Sand ſupply the Want of them, but are not ſo good; Sea-Rack laſts but three Years, and Sand little longer.

'Tis certain, *Ireland* has been better inhabited than it is at preſent: Mountains that now are cover'd with Bogs, have formerly been plow'd; for when you dig five or ſix Feet deep, you diſcover a proper Soil for Vegetables, and find it plow'd into Ridges and Furrows. This is obſervable in the wild Mountains between *Ardmagh* and *Dundalk*, where the Redoubt is built, and likewiſe on the Mountains of *Altmore:* The ſame, as I am inform'd, has been obſerv'd in the Counties of *London-Derry*, and *Donegal*; a Plow was found in a very deep Bog in the latter, and a Hedge with Wattles ſtanding under a Bog that was five or ſix Feet deep above it. I have ſeen the Stump of a large Tree in a Bog, ten Feet deep, at *Caſtle-Forbes*; The trunk had been burnt, and ſome of the Cinders and Aſhes lay ſtill on the Stump. I have ſeen likewiſe large old Oaks grow on Land, that had the Remains of Ridges and Furrows: And I am told, that on the Top of a high Mountain in the North, there are yet remaining the Streets and Footſteps of a large Town; and indeed there are but few Places, which do not viſibly (when the Bog is remov'd) retain Marks of the Plow; which ſure muſt prove, that the Country was well inhabited. It's likely that the *Danes* firſt, and then the *Engliſh*, deſtroy'd the People; and the old Woods ſeem to thoſe that pretend to judge, to be about three or four hundred Years ſtanding, which was near the Time that *Courcey* and the *Engliſh* ſubdued the North of *Ireland*, and 'tis likely made Havock of the People, that remain'd after the *Danes* were beat out of *Ireland*.

II. The Burning of the Surface of the Land is ſo much practis'd in *Devonſhire*, that 'tis elſewhere known by the Name of *Devonſhiring*; but it is uſed only for bad Lands, and by worſe Huſbands, for it robs the Ground.

Of the Manuring *of* Land *by* Sea-Sand *in* Devonſhire, *by Dr. A. Bury.* n. 316 p. 142.

Salt quickens dead Land, and is uſed in the South Weſt Part of the Country, which would elſe be the barrenneſt, but is now the richeſt Part of it. They go as far as the Sea will permit them, at the loweſt Ebb, and

and take the Sand in Bags; and carry it on Horseback 14 Miles into the Country, and spread it upon the Land, and thereby improve it both for Corn and Grass. In other Parts they force their barren Land, by mingling the Earth with Lime, and casting it upon the Ground.

In this they differ, that Crude and Single *Salt*, if strew'd upon the Ground, does not improve, but corrode it; but *Lime*, tho' unmingled, betters it: But in this they agree, that they produce not Grass fit for the Scythe, but for Pasture, short and sweet, and which grows all the Winter; so that their Sheep know not either Hay or Water, nor are their highest Grounds parch'd by the Sun in the hottest Summer. The best Way is, to mix these Male and Female Salts; for the Sea Salt is too lusty and active of itself; the Lime has a more Balsamic, but gentler Salt, and regularly join'd with the other, is thereby invigorated. How to mix those two, *Glauber* thus directs: *Take* (says he) *Quick-Lime, let it slack by Time without Water; then take Salt and Water, mingle them together, and make them into Balls or Pieces, which you please; dry them as you do Bricks, then burn them for about two Hours; this Compost will enrich the poorest Land.*

Those who are much devoted to Agriculture, should chuse such a Situation for a Dwelling, as is best accommodated with Lime, Salt, and Coals; and if our Gentry understood this Husbandry, they would so far free Salt from its Tax, as it should be employ'd on Land, which is not intended to pay for it.

Observations *relating to the* Motion *of the* Sap *in* Vegetables, *by Mr.* R. Bradley. n. 349. p. 486.

III. It may in general be observ'd of *Plants*, that they are either *Terrestrial*, *Amphibious*, or *Aquatic*; and so nearly do Vegetables agree with Animals in most Points, except Local Motion and its Consequences, that from the Knowledge of the one we are reasonably led to the Discovery of the other.

Those Plants, which I call *Terrestrial*, are such as *Trees*, *Shrubs*, and *Herbs*, which grow only on the Land. These like Land-Animals have Diversities of Food, a Method of Generating, and certain Periods of Life.

Of the *Amphibious* Race, which live on Land as well as in the Waters, are the *Willows*, *Rushes*, *Mints*, &c. These are not unlike in many Respects to the *Otter*, *Tortoise*, *Frog*, &c.

The *Aquatics*, whether of *Lakes*, *Rivers*, or the *Sea*, are very numerous: These may be compar'd with the *Fish-kind*, and will not live out of their proper Elements. In *Fresh Waters* are the *Water-Lillies*, *Plantains*, &c. and in the *Sea*, *Corals*, *Fuci*, &c.

Plants seem to possess only the next Degree of Life below the most stupid *Animal*; or where Animal Life leaves off, the Vegetable Life seems to begin.

The Seasons of Motion in *Plants* are the same with those of *Animals*, which sleep during the Winter. An Artificial Heat will give Motion to either of these in the coldest Time.

The

The common Opinions relating to the *Sap*'s Motion, are as follows: First, the *Sap* does not rise by the *Pith*; because some have observ'd the Trunks of large Trees to be without that Part, and yet the same Trees have continued to put forth Fruit and Branches on their Tops. I have observ'd, that the *Pith* is not found in those Branches of a Tree, which exceed two or three Years Growth; and it is certain, that the *Pith*, which is in a Branch of this Year, will (the greatest Part of it) be distributed into those Boughs, which form themselves the next Season.

It is said by some, that the Tree does not receive its Nourishment by the *Bark*, for that Trees having lost that Part, will still continue their Growth. Others tell us, that if the *Bark* be cut away round the Trunk of a Tree, it will presently die. These various Opinions seem to have been set on Foot without extraordinary Consideration, upon the Belief, that a Tree has but one *Bark*; whereas, upon Examination with the Microscope, we find four distinct Coverings to each Branch, without the woody Parts. The two outermost *Barks* may be taken from a Tree without any great Damage, but the other two, which lie nearer the Wood, being stripp'd off, will kill the Tree.

Some affirm, that the *Sap* doth neither rise nor fall in the woody Part of the Tree, because they have not been able to discern any *Sap* to issue out of that Part when a Branch has been cut. The Microscope plainly shews us the Vessels in the Wood, through which the *Sap* riseth from the Root; but as these Tubes are not large enough to admit into them any Thing more gross than Vapour, so they have not been esteem'd to be of any great Use. The Explanation of *Figure* 47. will in some measure discover the Office of these and of such other Parts of a *Plant*, as are severally design'd for the Growth of Vegetables; but let us first enquire a little into the Nature of the *Root*.

The *Root* of a Tree is chiefly compos'd of a *Parenchyma*, more gross than that in the Stem or Body of the Tree; it has likewise Vessels and a Covering. The Root, that is, the principal Part of it, receives into it such Juices of the Earth, as are proper for it, and no other. Somewhat like a Wiek of Cotton, which having been impregnated with Oil, will only admit Oil into it. This Provision being made in the *Stomach* of the *Plant* (as I call it) chiefly in the Autumn Months, the Tree is prepared for Germination, so soon as the Earth is sufficiently warm'd, either by the Sun's Beams, or an artificial Heat, such as Horse-dung, Bran and Water, or other such like Ferments, These Heats raise into Vapour the Juices contained in the Root, and by that Means cause Vegetation.

Fig. 47. is part of the Branch of an *Apple Tree* made in *May* 1715, and cut in *April* 1716. It was cut in Figure of a half Cylinder, the length somewhat more than the Diameter, which was about a Quarter of an Inch. This being magnified with one of *Campani*'s Microscopes, discovers the following Parts, *viz.* Fig. 47.

1, 2, 3, 4, 5, 6, 7, Are Capillary Veſſels, which run longitudinally through the Branch, in the ligneous part, which was made in the Year 1715. Through theſe Tubes, the Steam riſeth from the Root, the Strength of which is well explain'd by the Engine for raiſing Water by Fire, invented by the late Captain *Savery*.

From A to B, we may view Veſſels of the ſame ſort, made at the ſame Time.

8, 9, Are Veſſels of the ſame Uſe with the former, now forming themſelves for the Uſe of the Year 1716.

By this Means the Diameter of the Branch is increas'd, and additional Nouriſhment ſuffer'd to paſs into thoſe Buds, which are to make new Branches. Theſe are made out of the fourth or innermoſt Bark, mark'd C, C.

The Mouths of the Capillary Tubes of the Years 1715, and 1716, are D, E. The Vapour which riſeth from the Root, is continued in theſe Veſſels, to the Extremities of the Branches, where it meets with Parts (not here deſcrib'd) like to *Glands*; which *Glands*, if we may ſo call them, are likewiſe found at every Knot or Joint. At theſe Places, the Vapour coming near the Air, is condenſed, and returns between the Barks, by means of its own Weight, down F, G, H, leaving in each Bark mark'd I, K, L, ſuch Juices as each of them naturally is inclin'd to ſeparate from it; till at laſt, the more oily part paſſing to the Root, may lengthen the *Fibres* thereof, as icicles are lengthened; and by its oleous Particles, preſerve them from rotting by the Wet. The parts which compoſe the ſeveral Barks, are *Parenchymous* or *Spungy*.

The firſt mark'd M, is of a cloſer Texture than the ſecond N, and the ſecond cloſer than the third O, and ſo on till theſe parenchymous parts are interwoven with the longitudinal Wood-Veſſels, where they are ſomewhat conſtrain'd, till they come to make the Pith, mark'd P. Then they are much larger than in any other part of the Tree; and by what I have obſerv'd, ſeem to contain a more finiſh'd Juice than the reſt, and may well enough be ſtiled the *Medulla*.

We may note, that when the fourth or innermoſt Bark C, has once compleated its Sap-Veſſels, and is firmly join'd to the wooden part, then the third Bark O, takes its place for the ſucceeding Year; and ſo the reſt, except that the firſt, mark'd M, ſplits and divides itſelf, to ſupply the place of the ſecond.

The following Enquiry I recommend to the Curious, *viz.* If the ſeveral Barks, having different Texture of parts, admit into each ſeparate and different Juices from the reſt; whether thoſe Juices may not be of very different Virtues; the firſt more *Aſtringent* than the others, the ſecond perhaps *Emetick*; and the third *Cathartick*.

IV. The

IV. The nice and curious Texture of the *Flower* in Plants, and its Furniture, has employ'd the Enquiries of many Learned Men. But since these Searchers into the History and Operations of Nature, seem not so successfully to have reach'd her Design in this Case, as in many others, I shall attempt to account for the Fabric and Use of these Parts, and to make the Propagation of Vegetables more intelligible, whereby the Ways of Nature will appear more harmonious, and of a piece.

Some new Observations on the Parts and Use of the Flower in Plants, by Mr. S. Moreland. n. 287. p. 1474.

It has been long ago observ'd, that there is in every particular Seed a Seminal Plant, conveniently lodg'd between the two Lobes, which constitute the Bulk of the Seed, and are design'd for the first Nourishment of this tender Plant.

Dr. *Grew* is the only Author I can find, who has observ'd that the *Farina* (or fine Powder which is at its proper Season shed out of those *Thecæ* or *Apices Seminiformes*, which grow at the Top of the *Stamina*) doth some Way perform the Office of Male Sperm. But herein I think he falls short, in that he supposes them only to drop upon the outside of the *Uterus* or *Vasculum Seminale*, and to impregnate the included Seed by some spirituous Emanations or energetical Impress.

That which is now subjected to Enquiry, is, whether it be not more proper to suppose, that the Seeds which come up in their proper *Involucra*, are at first like unimpregnated *Ova* of Animals; that this *Farina* is a *Congeries* of Seminal Plants, one of which must be convey'd into every *Ovum* before it can become prolifick; that the *Stylus* in Mr. *Ray*'s Language, the upper part of the *Pistillum* in Mr. *Tournefort*'s, is a Tube design'd to convey these Seminal Plants into their Nest in the *Ova*; that there is so vast a Provision made, because of the Odds there are, whether one of so many shall ever find its Way into, and through so narrow a Conveyance.

To make this Supposition the more credible, I shall lay down the Observations I have made upon the Situation of these *Stamina* and the *Stylus* in some few Species of Plants.

First, in the *Corona Imperialis*, where the *Uterus* or *Vasculum seminale* of the Plant stands upon the Center of the Flower, and from the Top of this ariseth the *Stylus*, the *Vasculum seminale* and *Stylus* together representing a *Pistillum*. Round this are planted six *Stamina*, upon the Ends of each of these are *Apices*, so artfully fixt, that they turn every Way with the least Wind, being in Height almost exactly equal to the *Stylus* about which they play, and which in this Plant is manifestly open at the Top, as it is hollow all the Way. To which we must add, that upon the Top of the *Stylus* there is a sort of *Tuft*, consisting of pinguid *Villi*, which I imagine to be plac'd there, to catch and detain the *Farina*, as it flies out of its *Thecæ*. From hence I suppose the Rain either washes it, or the Wind shakes it down the Tube, till it reach the *Vasculum seminale*.

In *Capri-folium*, or *Honey-suckle*, there rises a *Stylus* from the rudiments of a Berry, into which it is inserted to the Top of the *Monopetalous* Flower, from

from the Middle of which Flower, are ſent forth ſeveral *Stamina*, that ſhed their *Farina* out of the Caſes upon the Orifice of the *Stylus*, which in this Plant is villous or tufted, upon the ſame Account as the former.

In *Allium* or *common Garlick*, there ariſes a *Tricoccous Uterus*, or Seed-Veſſel, in the Center of which is inſerted a ſhort *Stylus*, not reaching ſo high as the *Apices*, which thus over-topping it, have the Opportunity of ſhedding their Globules into its Orifice more eaſily. For which Reaſon I can diſcern no Tuft upon this (as in the former) to enſure their Entrance, that being provided for by its Situation juſt under them.

I ſhall now make ſuch Reaſonings or Reflections as the foregoing Account doth ſuggeſt, and will ſupport.

Nothing can be more natural than to conclude, that where a fine *Powder* is curiouſly prepared, carefully repoſited, and ſhed abroad at a peculiar Seaſon, where there is a Tube ſo planted, as to be fit to receive it, and ſuch Care in diſpoſing this Tube, that where it doth not lie directly under the Caſes that ſhed the Powder, it hath a particular *Apparatus* at the End to enſure its Entrance: Nothing can be more genuinely deduced from any Premiſes, than from this it may, that this *Powder* or ſome of it was deſigned to enter this Tube. If theſe *Stamina* had been only excretory Ducts, as has been hitherto ſuppoſed, to ſeparate the groſſer Parts, and leave the Juice deſigned for the Nouriſhment of the Seed more reſerved, what need was there to lodge theſe Excrements in ſuch curious Repoſitories? They would have been conveyed any where, rather than where there was ſo much Danger of their dropping into the *Seed Veſſel* again, as they are here.

Again, the *Tube* over the Mouth of which they are ſhed, and into which they enter, leads always directly into the *Seed Veſſel*.

To which we muſt add, that the *Tube* always begins to die, when theſe *Thecæ* are empty'd of their Contents; if they laſt any longer, it is only whilſt the *Globules* which enter at their Orifice, may be ſuppoſed to have finiſhed their Paſſage. Now can we well expect a more convincing Proof of theſe *Tubes* being deſigned to convey theſe *Globules*, than that they wither when there are not more *Globules* to convey?

If I could ſhow that the *Ova*, or unimpregnated Seed, are ever to be obſerved without this *Seminal Plant*, the Proof would ariſe to a Demonſtration; but having not been ſo happy as to diſcern this, I recommend the Enquiry to thoſe who are Maſters of the beſt Microſcopes. Though, in the mean Time, I have made ſome Steps towards a Proof of this Sort, and have met with ſome ſuch Hints as make me not deſpair of being able to do it: For, not to inſiſt upon this, that the *Seminal Plant* always lies in that Part of the Seed, which is neareſt to the Inſertion of this *Stylus*, or ſome Propagation of it into the *Seed Veſſel*; I have diſcovered in *Beans*, and *Peaſe*, and *Phaſeoli*, juſt under one End of that we call the *Eye*, a manifeſt Perforation (diſcernible by the groſſer ſort of Magnifying Glaſſes) which leads directly to the *Seminal Plant*, and at which I ſuppoſe the *Seminal Plant* did enter; and

I am apt to think, that the Beans or Peafe which do not thrive, will be found deftitute of it.

I fhall now defcribe fome other Plants, whereby it will appear, that there is a particular Care always exercifed to convey this Powder into a *Tube*, which may convey it to the *Ova*. Now in *Leguminous* Plants, if we carefully take off the *Petala* of the Flower, we fhall difcover the *Pod* or *Siliqua* clofely covered with an involving Membrane, which about the Top feparates into many *Stamina*, each fraught with its Quantity of *Farina*, and thefe *Stamina* are clofe bound upon the *Brufh*, which is obfervable at the End of that *Tube*, which here alfo leads directly to the *Pod*: It ftands not upright indeed, but fo bended, as to make near a right Angle with it.

In *Rofes* there ftands a *Column*, confifting of many *Tubes* clofely clung together, though eafily feparable, each leading to their particular Cell, the *Stamina* in a great Number planted all round about. In *Tithymalus*, or *Spurge*, there rifes a *Tricoccous* Veffel, that whilft it is fmall and not eafily difcernible, lies at the Bottom till impregnated, but afterwards grows up, and ftands fo high upon a tall *Pedicle* of its own, as would incline one to think that there were to be no Communication betwixt this and the *Apices*, which he fees dying below.

In *Straw-berries* and *Rafp-berries*, the Hairs which grew upon the ripe Fruit (which I fuppofe may be furprizing to fome) are fo many *Tubes* leading each to their particular Seed, and therefore we may obferve, that in the firft opening of the Flower, there ftands a Ring of *Stamina* within the *Petala*, and the whole inward *Area* appears like a little Wood of thefe Hairs or Pulp, which when they have received and conveyed their *Globules*, the Seeds fwell and rife in a carneous Pulp.

I have obferved, and can deduce the Contrivance and Adminiftration of the Parts in all the Plants I have obferved, and I doubt not but others may be able to perfect what I have rudely hinted; and that from this Theory many Corollaries may be derived, which will let in Light into many Parts of Natural Philofophy. I fhall at prefent only fuggeft, that hence one would conclude, that the *Petala* of the Flower were rather defigned to fever fuperfluous Juices, from what was left to afcend in the *Stamina*, than the *Stamina* to perform this Office, either for them, or the unimpregnated *Semina*; and obferve the Analogy between *Animal* and *Vegetable* Generation, as far as was neceffary, there fhould be an Agreement between them.

The Explication of the Figures.] *Fig.* 48. reprefents a *yellow Lilly*. *A* the Top of the *Piftillum* or Tube, at which the *Seminal Plants* are fuppofed to enter, and through which they are conveyed to the unimpregnated *Seed* in the *Seed Veffel*. Fig. 48.

b, *b*, *b*, *b*, *b*, *b*, the *Apices Seminiformes*, which when they are ripe open, and fhed that *Powder* which enters the *Tube* at *A*.

C, the Place of the *Seed Veffel* at the Bottom of the *Tube*, the Seed-Veffel itfelf being concealed under the Leaf in this Draught.

Fig.

Fig. 49. *Fig.* 49. *D*, the *Siliqua* in a Flower of the *Pea-kind*.

E, the *Tube* which ariſes from the *Siliqua*, and conveys the *Plants* to it.

F, the *Membranous* Coat that involves the *Siliqua* laid open.

g, *g*, *g*, *g*, *g*, *g*, the *Apices*, which, before the Membranous Tegument is laid open, appear to riſe from its Edges, and by the *Petala* of the Flower, are kept cloſe upon the *Orifice* of the *Tube*, that they may conveniently ſhed their *Farina* into it.

Fig. 50. *Fig.* 50. A *French-Bean* repreſented ſidwiſe.

Fig. 51. *Fig.* 51. The ſame opened.

h, The Seminal Plant.

i, A Perforation, at which, 'tis ſuppoſed, the Seminal Plant firſt enter'd.

Of the Quick Propagation *of* Moldineſs, *&c. in a Melon, by Mr.* R. Bradley, n. 349. p. 490.

V. I had lately a large Melon-Fruit, which I ſplit lengthways thro' the Middle, in order to obſerve the Veſſels which compoſed the Membrane or Tunic of each Ovary; but not continuing the Work I had begun, I laid by one half of the Melon, to be examined when I might have more Leiſure.

At the End of four Days, I found ſeveral Spots of Moldineſs began to appear on the fleſhy Part of the Fruit, ſomewhat Green towards the Rind; and of a paler Colour towards the Middle of the Fruit. Theſe Spots grew larger every Hour, for the Space of five Days; at which Time the whole Fruit was quite covered.

This ſurprizing Vegetation made me curious to examine if there was any Difference between thoſe Parts which were Green and the others, beſides their Colour. The firſt being ſeen with the Microſcope, appear'd to be a *Fungus*, whoſe Cap was filled with little Seeds, to the Number of about five Hundred; which ſhed themſelves in two Minutes after they had been in the Glaſſes. (Fig. 52.)

The other Sort had many Graſs-like Leaves, among which appeared ſome Stalks with Fruit on their Top, each Plant might well enough be compared to a ſort of *Bull-Ruſh*. They had their Seed in great Quantities, which I believe were not longer than three Hours before they began to vegetate; and it was about ſix Hours more, before the Plants were wholly perfected: For, about ſeven of the Clock one Morning, I found three Plants at ſome Diſtance from any others; and about four the ſame Day, I could diſcern above five Hundred more growing in a Cluſter with them, which I ſuppoſed were Seedling Plants of that Day. The Seed of all theſe were then ripe and falling. (Fig. 53.)

When the whole Fruit had been thus cover'd with Mold for ſix Days, this vegetable Quality began to abate, and was entirely gone in two Days more. Then was the Fruit putrified, and its fleſhy Parts now yielded no more than a ſtinking Water, which began to have a gentle Motion on its Surface, that continued for two Days without any other Appearance. I found then ſeveral ſmall Maggots to move in it, which grew for the Space of ſix Days; after which they laid themſelves up (Fig. 54.)

in

in their Bags. Thus they remain'd for two Days more without Motion, and then came forth in the Shape of Flies. The Water at that Time was all gone, and there remained no more of the Fruit than the Seeds, the Vessels which composed the Tunics of the Ovaries, the outward Rind, and the Excrement of the Maggots; all which together weighed about an Ounce. So that there was lost of the first Weight of the Fruit when it was cut, about twenty Ounces. Fig. 55.

We may judge from this, and other Cases of the like Nature, how much vegetable Life is dependent on Fermentation, and animal Life on Putrefaction.

Of the Husbandry of Canary Seed. by Mr. F. Tenison. n. 337. p. 91.

VI. To prepare Land for *Canary Seed*, let it be broke up some Time in *April*, and ploughed again about *Midsummer*, and ploughed again in *August*, that by frequent Tillage the Weeds may be burnt up, and destroyed. Plough the last Time about the latter End of *February*, or the Beginning of *March*, if the Season proves dry; if not, you had best wait for a dry Season; for in such a Season only will the Ground be fit to receive the Seed. With a Hoe, (that has a Bit about the Bigness of an Onion-Hoe) you must from Time to Time carefully cut up the Weeds. If they are not kept entirely under, much of the Seed will be lost for Want of ripening. In very good Land half a Bushel of Seed will be enough to sow an Acre. It will thrive best upon a stiff Clay: It will grow upon any sort of Loamy Land, that is rich enough to bear Hemp. If you apprehend that the Land is not sufficiently strong, you will do well to allow from half a Bushel to seven Gallons of Seed to sow an Acre with.

The Seed is ripe sooner or later, according as the Spring affords you an early or late Season of sowing it. In some Summers 'tis cut in *August*, but the most usual Time is after Wheat-Harvest. When it is cut it must in most Years lie five or six Days in swarth, and then be turn'd, and lie till one Side is dryed and rotted as much as the other, which may be about four or five Days longer. The certain Number of Days cannot be fixed, because they must be more or less, according as the Weather proves fair or rainy.

The Reason of its lying so long in swarth is, that the lower Heads of the Seed (being exposed to the Air, Wind and Sun) may the better perfect their Ripeness, and the Grass and Weeds that sprung up with the Stalks be thoroughly withered, and the Ears or Heads well and sufficiently rotted, and that the Seed upon threshing may come out clean.

The Produce upon Land that is very good, is about six Quarters *per* Acre.

If the Land be but indifferent, or if the Weeds be not kept under, then from four to five Quarters upon an Acre, is as much as you can expect.

The Price of Seed is, from two Pounds to ſix Pounds *per* Quarter; but the moſt uſual Price is from forty Shillings to three Pounds.

It is difficult to threſh. So much of the Seed as, after threſhing, is beaten out (as ſoon as 'tis fanned) is to be run through a Wire-Sieve (ſuch as is uſed to ſeparate Cockle from Corn) and the Huſks of every ſifting, that will not paſs through the Sieve, are to be thrown by in a Heap to be threſhed over again.

The ordinary Price for threſhing is Five Shillings, but in ſome Years the Threſher has Six Sillings *per* Quarter.

Experiments on Vegetation, by Mr. Abr. de la Pryme. n. 281. p. 1214.

VII. Some have made Experiments of the Meliorating, Fertilizing, and Multiplying of Grain, by ſteeping them in divers Liquors. *Digby* ſomewhere mentions, a Plant of Barley all riſing from one Corn, that by Steeping and Watering with Salt Petre diſſolved in Water, brought forth 249 Stalks, and above 18000 Grains. And the laſt Edition of *Cambden* mentions a Thing very obſervable, that the Corn ſown in a Field in *Cornwall*, after a great Battle in the Civil War Time, brought forth four or five Ears on every Stalk. I have try'd ſome ſuch like Experiments on ſeveral Grains, and though the Encreaſe was not ſuch as I expected, I communicate them to you.

On the 22*d* of *March*, 1699, I laid to ſteep a Pea, a Barley-Corn, and a Wheat-Corn in Brimſtone-Water.

A Pea, a Wheat, a Barley, and an Oat-Corn, in Allum-Water, and the ſame in an old Diſſolution of Salt of Tartar, in the *Caput Mortuum* of *Sal Armoniac* diſſolved in Urine, in a Diſſolution of the Salt of Walls, in a Diſſolution of Salt-Petre, in a Diſſolution of *Noſtoc* or Star-Gelly, and in Urine.

I ſteeped them thus five Days and five Nights, and ſet them in a Garden in a good Soil, againſt a North Wall full in the Sun, on the 27*th* of the ſame Month after a rainy Night, with a Pea, a Wheat, a Barley, and an Oat Corn unſteep'd.

Upon the 10*th* of *April* following, I found that ſome were juſt come up, ſome not.

The Pea, the Barley, and the Wheat ſteeped in Brimſtone-Water came all up together.

The Pea ſteeped in Allum-Water was very big and ſwelled, but not ſo much as ſprouted, but the Barley, Wheat, and Oat above Ground.

The Pea ſteeped in the old Solution of Salt of Tartar, was half come up, the Wheat ſcarce ſprouted, but the Barley and Oat quite up.

The Pea, the Wheat, the Barley, and the Oat ſteep'd in the *Caput Mortuum* of *Sal Armoniac* diſſolv'd in Urine were all up together; as were alſo the next Row, that were ſteep'd in the Solution of Salt of Walls.

The Pea and Wheat ſteep'd in the Diſſolution of Salt-Petre were about half up, but the Barley and Oat quite up.

Thoſe which were ſteep'd in *Noſtoc*, were none of them come up, nor ſcarce ſprouted.

The Barley and Oat ſteep'd in Urine, were come up, but the Pea and Wheat ſcarce ſprouted.

And, to my Surprize, the Pea, Wheat, Barley, and Oat, that were not at all ſteep'd, were all of them as ſoon up as any of the former, except only the Wheat, which was about half up.

I ſet them all about a Finger deep in the Ground, and there was all the Time of their Growth very fine Weather.

From all which I ſuppoſe, that Allum-Water is againſt the Nature of Peas, and retards their Growth, but agrees well enough with Wheat, Barley, and Oats.

That the Solution of Salt of Tartar is not friendly to the Nature either of Peas or Wheat, but agreeable to the Nature of Oats and Barley.

That the Water of Salt-Petre had not any of the great Power or Virtue that I ſuſpected, *&c.*

And that theſe Steepings did not further any of the ſaid Grains in their Growth and Coming, but plainly retarded ſome or moſt of them.

Then I digg'd all of them up, but three Spires of Barley, which I let ſtand about a Foot and a half, or two Feet, from one another; which grew and increaſed ſo exceedingly, that they had ſixty, ſixty-five, and ſixty-ſeven Stalks a-piece, from their ſingle Grain and Root, with every one an Ear on, and about forty or ſomewhat more Corns a-piece in them; which Increaſe proceeded not ſo much perhaps from the Grain having been ſteep'd in any Liquors, as from the Fertility and Goodneſs of the Soil, and their competent Diſtance one from another. I obſerved that new Shoots continually ſtruck up from the Root; and that as in the *Eaſt* and *Weſt-Indies*, there are Trees that always bear Bloſſoms and Flowers, green and ripe Fruit at the ſame Time, ſo that here, if the invigorating Heat of the Sun had not been cool'd and weaken'd by the Approach of the Winter-Seaſon, there would have been continually new, ripe Corn, and empty Ears, on the ſame Root.

VIII. At *Sutton-Coldfield* in *Warwickſhire*, a peaty Ground near a Pool (of which it was formerly a Part) was ſown with *Turnip-Seed* on the 2*d* Day of *July* 1702. In leſs than three Days Time the Turnips were ſeen above Ground. At three Weeks End the Roots were in Bigneſs equal to Walnuts. Within leſs than five Weeks after the Sowing, the Gardener drew great Quantities of Turnips to ſell, they then being as big as large Apples. At the End of ſix Weeks, *viz.* on the 12*th* Day of *Auguſt*, a large Turnip was plucked up (though probably not ſo big as ſeveral others then growing upon the ſame Ground) which, together with its Top and long deſcending Part of the Root, weighed above two Pounds and fourteen Ounces. At the ſame Time alſo was weighed an Ounce of the ſame Sort of Turnip-Seed, that the Gardener had ſown his Ground with; and afterwards a thouſand of the Grains were counted

Of the Great *and Speedy* Vegetation *of* Turnips. *Communicated by Dr.* J. T. Deſaguliers. n. 360. p. 974.

counted ſingly out of the Ounce ſo weighed; and the reſt of the Ounce was divided into Heaps, as near as could be gueſſed, equal to the 1000 Seeds firſt ſevered and laid together: And it was found, that the whole Ounce contained above 14600 ſingle Grains; which number multiplied by 46 (*viz.* the Number of Ounces that the Turnip weighed) produced 671600, *viz.* the Number of ſingle Grains of Seed required to equal the Weight of the Turnip. From whence may be gathered, that (upon Suppoſition that the Increaſe of the Turnip was all along uniform and equal, from the Time it was ſown, till it was pluck'd up) the Grain of Seed which it ſprung from, weighing when it was ſown but $\frac{1}{14600}$ of an Ounce, was increas'd in Weight according to the following Proportions, *viz.*

In ſix Weeks Time		671600	Times its own Weight.
Every	Week	$111933\frac{1}{3}$	
	Day	$15990\frac{1}{2}$	
	Hour	$666\frac{1}{4}$	
	Minute of an Hour	11	

Some Days after another Ounce of the ſame Sort of Seed was exactly weighed, and the Grains were found to be in Number 14673.

Another Turnip of the ſame Crop was pluck'd up on the 21*ſt* of *October*, and was found to weigh above 10 Pounds and a half, which unuſual and wonderful Bulk it acquir'd (it being ſuppoſed as above that the Growth was all alike) by increaſing the Weight of the Seed it was raiſed from, 15 Times in every Minute of an Hour, from the ſowing to the drawing of it.

The Gardener neglected to thin his Turnips in due Time, elſe probably their Growth had been more conſiderable.

At another Time, in two other Sorts of Turnip-Seed, it was found by counting, that an Ounce of one Sort contained 14702 Grains; and an Ounce of the other Sort no fewer than 14905 Grains.

It is credibly reported, that of late Years Turnips have been pretty frequently found growing in ſeveral Counties of this Kingdom, that have weighed above twice as much; one of which was ſeen at *Birmingham* about the Year 1710.

Of the Culture *of* Tobacco *in* Ceylan, *by Mr.* Strachan. n. 279. p. 1164.

IX. There are two Sorts of Tobacco, both which, they call *Dunkol*, the Signification is a Smoaking-Leaf; for *Dun* is Smoak, *Kol* a Leaf, the one they call *Hingele Dunkol*, or *Singele Dunkol*, for they make no Diſtinction of *H* and *S*; the other is called *Dunkol Kapada*, which Word *Kapada* ſignifies gelding, and is derived from the *Portugueſe*; which Tobacco is very intoxicating, and much ſtronger than the former; it is the ſame Plant, the Difference is only that *Singeleſe* Tobacco has little attendance,

tendance, upon the other a great deal of Pains is taken, until it be fit for Use; and it is done after the following Manner:

They clear a little piece of Ground, in which they sow the Seed of Tobacco, as the Gardeners here sow Parsley and Coleworts; against the Time that this is ready for transplanting, they choose a piece of Ground, which they hedge about; when the Buffalo's begin to chew the Cud, they are put within this Hedge-Ground, and let stand until they have done, and this they continue Day and Night, until the Ground be sufficiently dunged; then the Ground is tilled with a Spade, in Form of a Pick-ax, such as Carpenters use when they smooth Planks, by hoeing the Ground, and turning the same, and mixing the Dung among the Earth; when they have made the Ground smooth, they remove the Plants out of the Bed, wherein they were sown, and set them in this Ground, about a Foot Distance one from another, and then they grow up almost like a Dock; when the Stem has got 15 Leaves, they cut off all the Tops of the Plants; if they desire not to have the Tobacco to be very strong, they let it grow until it have 18 or 20; if they will have it stronger, they top it when it has got 10 or 12 Leaves, not counting the 3 or 4 lowest Leaves, which are nearest the Ground, because they never grow so big and good, as those above them. Thus the Moisture of the Ground being hinder'd from wasting in more Leaves, Flowers and Seed, all the said Moisture enters into the Leaves remaining, so that these Leaves will be 4 or 5 Times larger, fuller of Fatness, Strength and Virtue, than the Tobacco which is not ordered after this Manner. Now the Moisture ascending from the Root, being constrained within the Bounds of these Leaves, forces its Way betwixt the Stem and Leaves remaining, and sends forth young Sprouts, and would grow forth in Branches, if no Care were taken to hinder. Therefore every 3 or 4 Days they go through all the Stems, and break off these Buds whenever they spring forth, and this they continue, until these Leaves be ripe (which takes as much Time as the *Singele* Tobacco does, which gets Flowers and ripe Seed, and then begins to wither and spoil, if no Use be made of it) which is known by the Thickness and Firmness.

Then before the Leaf begins to wither, and is green, they cut down the Stem together with the Leaves, and do bring them into their Houses, and lay them in a Heap; and thus the Leaves will begin to ferment, turn hot, and sweat, then when the Leaves begin to sweat, they turn the innermost outmost, that they may easily ferment; otherwise the innermost would ferment too much, spoil, and rot: Thus the longer they lie in a Heap together, the Tobacco turns the more dark of Colour. When they think it has sweat enough, they hang it asunder upon Cords, till the Leaves be dry, then they separate the Leaves from the Stalks, and lay them up in Bundles together, until they have Use for them.

Now the other Tobacco, called *Singele Dunkol*, is only sown, and then planted, and has Liberty to grow, to shoot out, flower, and seed; thus all ripen together: Then it is cut down and cast together in a Heap;

some

ſome will ferment too much, and rot, others will ferment not at all, and will remain green, although it be dry, and will have a ſmell of Hay or dry Graſs. The Soldiers, who delight to ſmoak a big Pipe full, and that frequently in one Day, do ſmoak this common ſort, ſome will mix ſome of the *Kapada* among it; the *Cingualeſes*, who ſmoak not ſo much at once, neither ſo frequently, do take a piece of the *Kapada*, and roll it together, then roll a piece of dry Leaf of the *Wattukan* Trees about this, and kindle it at one End, and ſuck at the other, until it be conſumed. Some do chew it among Betle, taking but very little at once.

Of a Tartarian Plant *called* Gin-ſeng, &c. *by Father* Jartoux, *taken from the Tenth Vol. of* Letters *of the Miſſionary* Jeſuits. n. 337 p. 237.

X. The Map of *Tartary*, which we made by Order of the Emperor of *China*, gave us an Opportunity of ſeeing the famous Plant *Gin-ſeng* or *Nin-zin*, ſo much eſteem'd in *China*, and ſo little known in *Europe*. Towards the End of *July* 1709, we arrived at a Village, not above four ſmall Leagues diſtant from the Kingdom of *Corea*, which is inhabited by thoſe *Tartars* called *Calca tatze*. One of theſe *Tartars* went and found upon the neighbouring Mountains four Plants of the *Gin-ſeng*, which he brought us entire in a Baſket. I took one of them, and deſigned it, as well as I could.

The moſt eminent Phyſicians in *China* have writ whole Volumes upon the Virtues and Qualities of this Plant, and make it an Ingredient in almoſt all Remedies which they give to their chief Nobility; for it is of too high a Price for the common People. They affirm, that is a ſovereign Remedy for all Weakneſſes occaſion'd by exceſſive Fatigues either of Body or Mind; that it diſſolves pituitous Humours; that it cures Weakneſs of the Lungs, and the Pleuriſy; that it ſtops Vomitings; that it ſtrengthens the Stomach, and helps the Appetite; that it diſperſes Fumes or Vapours; that it fortifies the Breaſt, and is a Remedy for ſhort and weak Breathing; that it ſtrengthens the Vital Spirits, and increaſes Lymph in the Blood: In ſhort, that it is good againſt Dizzineſs of the Head, and Dimneſs of Sight, and that it prolongs Life in old Age.

No Body can imagine that the *Chineſe* and *Tartars* would ſet ſo high a Value upon this Root, if it did not conſtantly produce a good Effect. Thoſe that are in Health often make uſe of it to render themſelves more vigorous and ſtrong: And I am perſuaded, that it would prove an excellent Medicine in the Hands of any *European* who underſtands Pharmacy, if he had but a ſufficient Quantity of it, to make ſuch Trials as are neceſſary, to examine the Nature of it chymically, and to apply it in a proper Quantity, according to the Nature of the Diſeaſe for which it may be beneficial.

It is certain, that it ſubtilizes, increaſes the Motion of, and warms the Blood; that it helps Digeſtion, and invigorates in a very ſenſible manner. After I had deſigned the Root, I obſerved the State of my Pulſe, and then took half of the Root, raw as it was, and unprepar'd: In an Hour after, I found my Pulſe much fuller and quicker; I had an Appetite,

tite, and found my ſelf much more vigorous, and could bear Labour much better and eaſier than before.

But I did not rely on this Trial alone, imagining that this Alteration might proceed from the Reſt that we had that Day: But four Days after, finding my ſelf ſo fatigued and weary that I could ſcarce ſit on Horſeback, a *Mandarin*, who was in Company with us, perceiving it, gave me one of theſe Roots: I took half of it immediately, and an Hour after I was not the leaſt ſenſible of any Wearineſs. I have often made Uſe of it ſince, and always with the ſame Succeſs. I have obſerved alſo, that the green Leaves, and eſpecially the fibrous Part of them chewed, would produce nearly the ſame Effect.

The *Tartars* often bring us the Leaves of *Gin-ſeng* inſtead of *Tea*; and I always find my ſelf ſo well afterwards, that I ſhould readily prefer them before the beſt Tea. Their Decoction is of a grateful Colour; and when one has taken it twice or thrice, its Taſte and Smell become very pleaſant.

As for the Root of this Plant, it is neceſſary to boil it a little more than Tea, to allow Time for extracting its Virtue; as is practiſed by the *Chineſe*, when they give it to ſick Perſons, on which Occaſion they ſeldom uſe more than the fifth Part of an Ounce of the dry'd Root. But as for thoſe that are in Health, and take it only for Prevention, or ſome ſlight Indiſpoſition, I would adviſe them not to make leſs than ten Doſes of an Ounce, and not to take of it every Day. It is prepared in this Manner: The Root is to be cut into thin Slices, and put into an Earthen Pot well glazed, and filled with about a Quarter of a Pint of Water *Paris* Meaſure: The Pot muſt be well covered, and ſet to boil over a gentle Fire; and when the Water is conſumed to the Quantity of a Cupful, a little Sugar is to be mix'd with it, and it is to be drank immediately. After this, as much more Water is to be put into the Pot upon the Remainder, and to be boiled as before, to extract all the Juice and what remains of the ſpirituous Part of the Root. Theſe two Doſes are to be taken, one in the Morning, and the other at Night.

The Places where this Root grows are between the thirty-ninth and forty-ſeventh Degree of Northern Latitude, and between the tenth and twentieth Degree of Eaſtern Longitude, reckoning from the Meridian of *Pekin*. There is there a long Tract of Mountains, which the thick Foreſts, that cover and encompaſs them, render almoſt unpaſſable. It is upon the Declivities of theſe Mountains and in theſe thick Foreſts, upon the Banks of Torrents or about the Roots of Trees, and amidſt a Thouſand other different ſorts of Plants, that the *Gin-ſeng* is to be found. It is not to be met with in Plains, Vallies, Marſhes, the Bottoms of Rivulets, or in Places too much expoſed and open. If the Foreſt take Fire and be conſumed, this Plant does not appear till two or three Years after: It alſo lies hid from the Sun as much as poſſible; which ſhews that Heat is an Enemy to it. All which makes me believe, that if it is to be found in any other Country in the World, it may be parti-

particularly in *Canada*, where the Foreſt and Mountains, according to the relation of thoſe that have lived there, very much reſemble theſe.

The Places where the *Gin-ſeng* grows, are on every Side ſeparated from the Provinces of *Quan-tong* (which in our old Maps is called *Leao-tum)* by a Barrier of wooden Stakes, which incompaſſes this whole Province, and about which Guards continually patroll, to hinder the *Chineſe* from going out and looking after this Root. Yet how vigilant ſoever they are, their Greedineſs after Gain incites the *Chineſe* to lurk about privately in theſe Deſerts, ſometimes to the Number of two or three Thouſand, at the Hazard of loſing their Liberty, and all the Fruit of their Labour, if they are taken either as they go out of, or come into, the Province.

The Emperor having a mind that the *Tartars* ſhould have the Advantage that is to be made of this Plant, rather than the *Chineſe*, gave Orders this preſent Year 1709 to ten Thouſand *Tartars*, to go and gather all that they could of the *Gin-ſeng*, upon Condition that each Perſon ſhould give his Majeſty two Ounces of the beſt, and that the reſt ſhould be paid for according to its Weight in fine Silver. It was computed, that by this Means the Emperor would get this Year about twenty Thouſand *Chineſe* Pounds by it, which would not coſt him above one fourth Part of its Value. We met by Chance with ſome of theſe *Tartars* in the midſt of thoſe frightful Deſerts: And their *Mandarins*, who were not far diſtant out of our Way, came one after another, and offer'd us Oxen for our Subſiſtence, according to the Commands they had received from the Emperor.

This Army of Herbariſts obſerved the following Order. After they had divided a certain Tract of Land among their ſeveral Companies, each Company, to the Number of an Hundred, ſpreads itſelf out in a ſtraight Line to a certain fix'd Place, every Ten of them keeping at a Diſtance from the reſt. Then they ſearched carefully for the Plant, going on leiſurely in the ſame Order; and in this Manner, in a certain Number of Days, they run over the whole Space of Ground appointed them. When the Time is expir'd, the *Mandarins*, who are encamp'd with their Tents in ſuch Places as are proper for the Subſiſtence of their Horſes, ſend to view each Troop, to give them freſh Orders, and to inform themſelves if their Number is compleat. If any one of them is wanting, as it often happens, either by wandering out of the Way, or being devoured by wild Beaſts, they look for him a Day or two, and then return again to their Labour as before.

The poor People ſuffer a great deal in this Expedition. They carry with them neither Tents nor Beds, every one being ſufficiently loaded with his Proviſion, which is only Millet parched in an Oven, upon which he muſt ſubſiſt all the Time of his Journey; ſo that they are conſtrain'd to ſleep under Trees, having only their Branches and Barks, if they can find them, for their Covering. Their *Mandarins* ſend them from time to time ſome Pieces of Beef, or ſuch Game as they happen

to

to take, which they eat very greedily and almoſt raw. In this manner theſe ten thouſand Men paſſed ſix Months of the Year; yet notwithſtanding their Fatigues, continued luſty, and ſeemed to be good Soldiers. The *Tartars*, which were our Guard, did not fare better, having only what remained of an Ox, that was killed every Day, and had firſt ſerved fifty Perſons for their Subſiſtence.

A, ſhows the Root of the Plant; which, when waſh'd, was white and a little rugged and uneven, as the Roots of other Plants generally are. Fig. 56.

B, *C*, *D*, repreſent the Length and Thickneſs of the Stalk; which is ſmooth and pretty round, of a deepiſh red Colour, except near its Beginning at *B*, where it is whiter, by Reaſon of its Nearneſs to the Ground.

D, is a ſort of Knot or Joynt, made by the ſhooting out of four Branches, which all riſe from the ſame Center, and divide from another at equal Diſtances, and at the ſame Height from the Ground. The underſide of the Branch is green, mix'd with white; the upper Part is much like the Stalk, of a deep red, inclining to the Colour of a Mulberry. Theſe two Colours gradually decreaſe and unite together on the ſides in a natural Mixture. Each Branch has five Leaves. It is remarkable, that theſe Branches ſeparate from each other at equal Diſtances, as well in Reſpect of themſelves, as of the Horizon, and make with their Leaves a circular Figure, nearly parallel to the Surface of the Ground.

Tho' I have finiſh'd the Deſign but of half of one of the Leaves at *F*, yet any one may eaſily conceive and perfect the reſt in the ſame Manner. I do not know that ever I ſaw Leaves, ſo large as theſe, that were ſo thin and fine: Their Fibres are very diſtinguiſhable; and on the upper ſide they have ſome ſmall whitiſh Hairs. The Skin between the Fibres riſes a little in the middle above the Level of the Fibres. The Colour of the Leaf is a dark green above, and a ſhining whitiſh green underneath. All the Leaves are ſerrated, or very finely indented on the Edges.

From *D*, the Center of the Branches, there riſes a ſecond Stalk *D E*, which is very ſtraight and ſmooth, and whitiſh from Bottom to Top, bearing a Bunch of round Fruit of a beautiful red Colour. This Bunch was compoſed of twenty-four Berries, two of which are here drawn, marked 9, 9. The red Skin that covers the Berry, is very thin and ſmooth: It contains within it a white ſoftiſh Pulp. As theſe Berries were double (for they are ſometimes found ſingle) each of them had two rough Stones, ſeparated from one another, of the Size and Figure of our common Lentils, excepting that the Stones have not a thin Edge like Lentils, but are almoſt every where of an equal Thickneſs. Each Berry was ſupported by a ſmooth, even, and very fine Sprig, of the Colour of thoſe of our ſmall red Cherries. All theſe Sprigs roſe from the ſame Center, and ſpreading exactly like the Rays of a Sphere, they make the Bunch of Berries, that they bear, of a circular Form. This Fruit is not good to eat. The Stone is like the Stones of other common Fruit; it is hard, and incloſes a Kernel It is always placed upon the ſame Plan or Level with the Sprig that bears the Berry. From whence it is, that the Berry is not round, but a little flat on each ſide. If it be double, there is a kind of Depreſſion or hollow Place in the middle, where the two Parts unite. It has alſo a ſmall Beard at Top, diametrically oppoſite to the Sprig on which it hangs. When the Berry is dry, there remains only a ſhrivel'd Skin that ſticks cloſe to the Stones, and is then of a dark red, or almoſt black Colour.

This Plant dies away, and ſprings again every Year. The Number of its Years may be known by the Number of Stalks it has ſhot forth, of which there always remains ſome Mark; as may be ſeen in the *Figure* by the Letters *b*, *b*, *b*, &c. From whence it appears, that the Root *A* was ſeven Years old, and that the Root *Fig.* 57. was fifteen.

As to the Flower, not having ſeen it, I can give no Deſcription of it. Some ſay that it is white and very ſmall: Others have aſſured me, that this Plant has none, and that no Body ever ſaw it. I rather believe that it is ſo ſmall, and ſo little remarkable, that they never took notice of it: And what confirms me in this Opinion, is, that thoſe that look for the *Gin-ſeng*, having Regard to, and minding only the Root, commonly neglect and throw away all the reſt of the Plant, as of no Uſe.

There are ſome Plants, which beſide the Bunch of Berries I have deſcribed, have alſo one or two Berries like the former, plac'd an Inch or an Inch and a half below the Bunch. And when this happens, they ſay, if any one takes Notice of the Point of the Compaſs that theſe Berries direct to, he can't fail of finding the Plant at ſome Places diſtant that way, or thereabouts. The Colour of the Berries, when the Plant has any, diſtinguiſhes it from all others, and makes it remarkable at firſt Sight: But it ſometimes happens that it bears none, tho' the Root be
Fig. 57. very old; as that repreſented *Fig.* 57. had no Fruit, tho' it was in its fifteenth Year.

They having ſowed the Seed in vain, without its producing any Plant, might probably give Occaſion to this Story, which is current among the *Tartars*. They ſay that a Bird eats it as ſoon as it is in the Earth, and not being able to digeſt it, it is putrified in its Stomach, and afterwards ſprings up in the Place, where it is left by the Bird with its Dung. I rather believe that the Stone remains a long Time in rhe Ground, before it ſhoots out any Root. And this Opinion of mine ſeems the more probable, becauſe there are found ſome Roots, which are not longer, and not ſo big as ones little Finger, tho' they have ſhot forth ſucceſſively, at leaſt ten Stalks in as many different Years.

Tho' the Plant I have here deſcribed had four Branches, yet there are ſome that have but two, others but three, and ſome that have five or ſeven; which laſt are the moſt beautiful: Yet every Branch has always five

five Leaves, as well as this here figur'd, unleſs the Number has been diminiſh'd by any Accident. The Height of the Plants is proportionable to their Bigneſs and the Number of their Branches. Thoſe that bear no Fruit, are commonly ſmall and very low.

The Root, the larger and more uniform it is, and the fewer ſmall Strings or Fibres it has, is always the better; on which Account, that marked *Fig.* 57. is preferable to the other. I know not for what reaſon the *Chineſe* call it *Gin ſeng*, which ſignifies *the Repreſentation* or *Form of Man:* Neither I my ſelf, nor others who have ſearched and inquired into it on purpoſe, could ever find it had any Reſemblance to the Signification of its Name; though among other Roots there may now and then be found ſome which by accident have very odd Figures. The *Tartars* with more Reaſon call it *Orbota*, which ſignifies *the chief of Plants.*

It is not true, that this Plant grows in *China*, as Father *Martini* affirms from the Authority of ſome *Chineſe* Books, which make it to grow on the Mountains of *Yong-pinfou* in the Province of *Peking.* They might eaſily be led into this Miſtake, becauſe that is the Place where it firſt arrives when it is brought from *Tartary* into *China.*

Thoſe that gather this Plant, preſerve only the Root, and bury together in ſome certain Place in the Earth, all that they can get of it, in ten or fifteen Days Time. They take Care to waſh it well, and cleanſe it with a Bruſh from all extraneous Matter; then they dip it into ſcalding Water, and prepare it in the Fume of a ſort of yellow Millet, which communicates to it part of its Colour. The Millet is put into a Veſſel with a little Water, and boils over a gentle Fire; the Roots are laid upon ſmall tranſverſe pieces of Wood over the Veſſel, and are thus prepared, being covered with a Linnen-cloth or ſome other Veſſel placed over them. They may alſo be dry'd in the Sun, or by the Fire; but then, though they retain their Virtue well enough, yet they have not that yellow Colour which the *Chineſe* ſo much admire. When the Roots are dry'd, they muſt be kept cloſe in ſome very dry Place; otherwiſe they are in Danger of corrupting, or being eaten by Worms.

[*This Plant* (Gin-ſeng) *grows to the Height of about* 18 *Inches.*]

A new Genus *of* Plants, *of which the* Gin-ſeng *is a* Species. *Communicated by Mr.* Vaillant *to Dr.* W. Sherrard. n. 354. p. 705.

XI. *Araliaſtrum* is a *Genus* of Plants, whoſe *Flower* A* is complete †, regular, polypetalous, and hermaphrodite, ſtanding on the *Ovary* B. The *Ovary*, which is crown'd by a *Calyx* cut into ſeveral Parts, becomes a Berry D, in which are, for the moſt part, two flat Seeds, like a Semicircle, which both together repreſent a ſort of a Heart. Add to this, the Stalk, which is ſingle, ending in an Umbel, of which each Ray bears but one Flower. Above the Middle of the Stalk come out ſeveral *Pedicles*, (as on that of the *Anemone*) on the Extremities of which grow ſeveral Leaves like Rays, or like an open Hand.

* *Vid.* ARALIA *Inſt. rei herb.* Tab. 154.
† Complete, *that is to ſay, that has a* Calyx.

The Species *of this* Genus *are,* [1. *Araliaſtrum Quinquefolii ſolio, majus,* Nin-zin *vocatum D.* Sarrazin. *Gin-ſeng.* Des lettres edifiantes & curieuſes, *Tom.* X. *pag.* 172.

2. *Araliaſtrum Quinquefolii folio, minus.* D. Sarrazin. *Plantula Marilandica, foliis in ſummo caule ternis, quorum unumquodque quinquefariam dividitur, circa margines ſerratis.* N. 36. Raii Hiſt. III. 658.

3. *Araliaſtrum Fragrariæ folio, minus.* D. Vaillant. *Naſturtium Marianum Anemones ſylvaticæ foliis, enneaphyllon, floribus exiguis.* Pluk. Mantiſſ. 135. Tab. 435. Fig. 7.

To ſhew wherein *Araliaſtrum* differs from *Aralia,* (from whence it takes its Name) 'tis convenient to give alſo the Character of this laſt *Genus,* ſuch as Mr. *Vaillant* eſtabliſh'd it, in his Demonſtrations of the Year 1717.

Aralia * is altogether like the *Araliaſtrum,* as to the Structure and Situation of its Flower, but its Berry conſiſts of five Seeds plac'd round an *Axis.* Moreover its Leaves are branched, almoſt like thoſe of *Angelica*; and its Stalks (which in ſome *Species* are naked, and in others have Leaves ſet alternately) bear each ſeveral Umbels at their Top, in the Form of a Bunch of Grapes.

The Species *of* Aralia *are,* [1. *Aralia caule aphyllo, radice repente.* D. Sarrazin *Chriſtophoriana Virginiana Zarzæ radicibus ſurculoſis & fungoſis, Sarſaparilla noſtratibus dicta.* Pluk. Almag. 98. Tab. 238. Fig. 5. *Zarſaparilla Virginienſibus noſtratibus dicta, lobatis umbelliferæ foliis, Americana.* Ejuſd. Almag. 396.

2. *Aralia caule folioſo lævi,* D. Sarrazin. *Aralia Canadenſis.* Inſt. rei Herb. 300.

3. *Aralia caule folioſo & hiſpido,* D. Sarrazin.

4. *Aralia arboreſcens ſpinoſa,* D. Vaillant. *Angelica arboreſcens, ſpinoſa, ſeu Arbor Indica, Fraxini folio, cortice ſpinoſo,* Raii Hiſt. II. 1798. *Chriſtophoriana arbor aculeata Virginienſis,* Pluk. Almag. 98. Tab. 20.

All the *Species* of theſe two *Genera,* except the laſt of each of them, are common in *Canada.* The Inhabitants of that Colony, and thoſe of *Virginia,* call the firſt *Species* of *Aralia* by the Name of *Sarſaparilla,* becauſe its Roots have almoſt the ſame Figure and Virtues.

Mr. *Sarrazin* writes, that he had a Patient who had been cured of an *Anaſarca,* about two Years before, by the Uſe of a Drink made of theſe Roots; and aſſures us, that the Roots of the ſecond *Species,* well boil'd and apply'd by way of *Cataplaſm,* are very excellent for the curing of old Ulcers; as alſo the Decoction of them, with which they bathe and ſyringe the Wounds. He does not at all doubt, but the Virtues of the third *Species* (which I ſhall briefly deſcribe) are the ſame with thoſe of the ſecond.

* *Vid. Inſt. rei Herb.* 300. Tab. 154.

Its

Its Roots creep, and ſend forth Stalks, which riſe commonly to the Height of a Foot and half, and ſometimes to two Feet; the bottom part of them is rough, with reddiſh, ſtiff, and prickling Hairs. Theſe Stalks are ſet from the Bottom to almoſt the Top (which are divided ſucceſſively into ſeveral naked Branches charg'd with Umbels) with branch'd alternate Leaves, almoſt like thoſe of *Podagraria hirſuta Angelicæ folio & odore* D. Vaillant; which Plant is grav'd in the ſecond Tome of *Boccone*'s *Muſæum*, by the Name of *Cerefolium rugoſo Angelicæ folio*, *Aromaticum*, Tab. 19. and in *Rivini* by that of *Myrrbis folio Podagrariæ*.

XII. Monſieur *Marchand* acquainted the Aſſembly of the *Academy* of *Sciences* in *France*, with the Diſcovery of a new *Simple*. The firſt that brought it into Reputation, was a *Portugueze Surgeon*, who having lived many Years in *Brazil*, diſcovered the Virtues of this Plant; after returning into *Portugal* with a Deſign to raiſe a great Trade with it, he ſent ſeveral *Specimens* of it every where. He called the Plant *Iquetaia*, and attributed to it no leſs Virtues than the Cure of *Apoplexies*, *Pleuriſies*, and *Intermitting Fevers*. He added one Thing, which though more particular, yet ſeemed more probable, which was, that the Leaves infus'd with *Senna*, took from it its diſagreeable Taſte and Smell, without altering any Thing of its *Purgative* Quality. The *Samples* that he ſent, were not in ſufficient Quantity to make Experiment on the Diſtempers, he ſaid it was proper for; but there was enough to try, whether they had the Virtue to correct the Taſte and Smell of *Senna*. Therefore there was infus'd two *Drachms* of it, with as much *Senna* in a *Chopine* of Water, and the Experiment confirm'd the Matter of Fact: Being deſirous to know what *Species* of Plant it was, and it being impoſſible to diſcover it by the Leaves, which the *Portugueſe Surgeon* had taken ſo much Care to cut very ſmall, Monſieur *Homberg*, who had ſome of it ſent him, perceived ſome Seeds ſwimming on the Water, in which they were infus'd; and taking up as many as he could of theſe Seeds, gave them to Monſieur *Marchand*, who ſowed them; from whence grew up a Plant, which we need not go to *Brazil* to ſeek, it grows in *Europe*; nor need we go out of *France* to find it; nay, we may have it all round *Paris*; 'tis the *Scrophularia aquatica*. To be the more certain of it, there was ſome of our *Scrophularia* ſowed on a Bed, and ſome of the other Seed on another, and there was obſerved but ſome ſmall Differences, which may be well attributed to the different Culture and Soil. There was likewiſe try'd the Virtue of our *Scrophularia*, and it was found to have the ſame Effect, in taking away the Taſte and Smell of *Senna*. Monſieur *Marchand* concluded from this Diſcovery, that it was more fit to labour to know the Remedies that are in our own Land, than to run over all the Earth in queſt of that, with much Labour and Charge, which we may have for nothing at Home, if we took the Pains to ſearch: And added, that a knowing *Botaniſt* (meaning his Father) after many long Travels

Of a New Plant *call'd* Iquetaia, *the ſame with* Scrophularia Aquatica, *by Monſ.* Marchand. n. 273. p. 1103.

Travels avowed, that there might be found in all Countries Remedies for all Diseases, and that having apply'd himself to this Matter, he had discovered a good Number of Simples commonly to be found, that had great Virtues, of which he had graved the Plants. He named among others, the *Achillea Montana Penæ*, which smoaked in a Pipe as *Tobacco*, considerably eases an *Asthma*.

Of the Attmella *and its* Lithontriptic *Virtue, by Dr.* P. Honton. n. 268. p. 790.

XIII. *Attmella*, *Acemella*, and *Hacmella*, for the Seeds are so named which were sent to me in the Year 1691 from the Island *Ceylon*, where this Plant grows and is very well known.

The Plant itself, which I reared in the Year 1692, bears Flowers on the Tops of the Stalks, composed of several smaller tubulous Flowers collected together, and forming a kind of Head, supported by a six-leaved or many-leaved Calix, very much resembling the wing-stalked *Curassa* Marigold, with the Orange Flower (of which there is a Figure in *Pluknet*'s Phytog. in *Herman*'s Par. Batavo, in the Horto Monspel. *Magnol.* and the Flora Noribergensi of *Volkamer)* but somewhat yellow. After the Flowers are shaken, the Seeds follow, which are of a greyish brown Colour, long, flat, with a double Beard at Top, to which the Flowers were contiguous. It grows up in square Stalks covered with conjoined Leaves, longer and more prickly than those of the Nettle or Dead-nettle; whence it is conjectured that this Plant is doubtless of the Thistle or Artichoke Tribe, and of the same Species with that called *Bidens* by *Cæsalpinus*, and after him by *Tournefort*, from its forked Seed. Wherefore since this Plant hitherto has no Name, I think from the Nature of the Plant the following may be given it.

Bidens Urticæ folio Lithontriptica *Zeylanica*, or the Lithontriptic Nettle-leav'd *Bidens* of *Zeylan*. For amongst all the Medicines which have been used for dissolving the *Calculus*, this Plant of late Years is become the most celebrated, both amongst our Countrymen living in that Island, and very lately it is likewise become very famous amongst ourselves.

A Soldier, who in the Year 1690 first gave an Account of this Herb to our *East-India* Company, declared that he had cured upwards of a hundred Persons of the Stone and Nephritick Complaints. And the Governor and chief Council of the *Dutch* in the Island of *Ceylon*, testify in Letters wrote that same Year, its Success on two Persons who were troubled with the Stone; for they say that these Persons voided a great deal of Sand, and broken Pieces of Stone, almost without any Pain.

Colombo, chief Surgeon to the Hospital in the above Island, in Letters which he wrote to me in the Year 1699, in order to confirm the Truth of the above, says, that he is positive there is no Medicine that has hitherto been found more effectual against the Stone and Nephritick Disorders; and he farther adds, that with very great Pains he had found out three Species of it. The first of these is covered with light green Leaves, and its Seed is of a dirty yellow Colour. The second has

Leaves

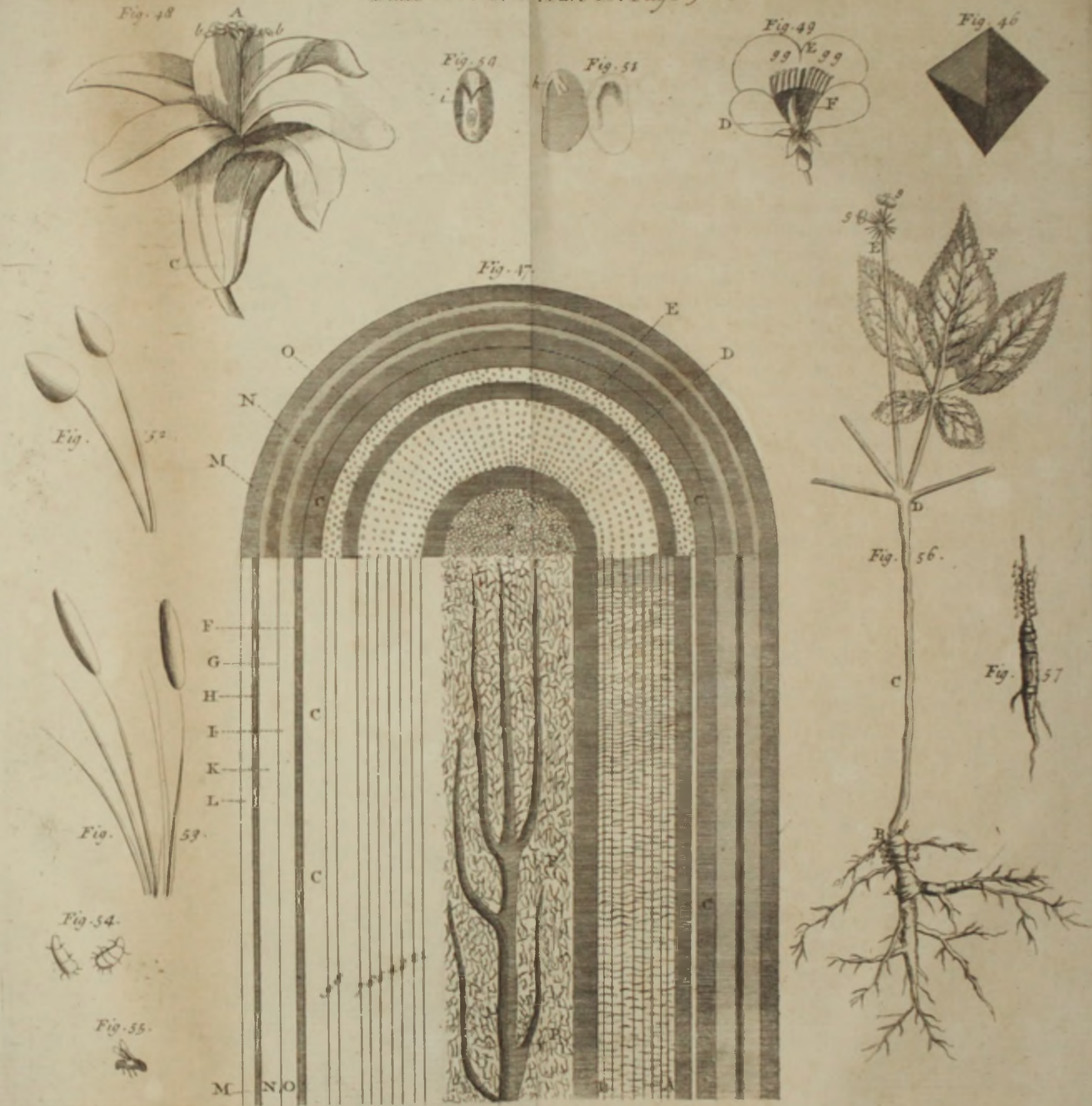
Fig. 48
Fig. 50
Fig. 51
Fig. 49
Fig. 46
Fig. 47.
Fig. 52
Fig. 56.
Fig. 57
Fig. 53
Fig. 54
Fig. 55

Leaves of a deep green Cast, and its Seed is of the same Colour with the other. The Seed of the third is black, and the Leaves are a great deal larger than those of the other two, which he says are the best. He adds, that this Plant is extremely fertile, each producing upwards of ten thousand Seeds.

They make use both of the Leaves of this Plant and the Seeds, which *Colombo* praises above all the rest of the Plant; as also of the Root, Stalks and Branches.

The Leaves being gathered before the Flowers push out, dried in the Shade, and reduced to a Powder, are given in a proper Vehicle, or infused in hot Water, and drank in the same manner as Tea. They are likewise infused in Spirit of Wine, and a Spirit is distilled from the Root, Stalks and Branches.

Another Governor of the Hospital at *Ceylon* asserts, that he had used the Flowers, Roots, Extract and Salt with good Success in the *Pleurisy*, *Colick* and *Fevers*.

Another Plant.

Mr. *Colombo* in his Letters commends likewise for their Lithontriptic Quality, the Bark of the Roots, and the Roots themselves, of a certain Herb, called by the People of *Ceylon*, *Mangul Caranda Pottu*; but what Kind of Herb this is, I know not.

Of the Jesuit's Bark, *by D.* W. Oliver. n. 290. p. 1590.

XIV. *Peru Bark* comes from a Tree of about the Bigness of a Plumb Tree, with Leaves like Ivy, but not quite so big, and are always green. The *Indians* call it *Querango*. It is gather'd in Autumn, and the Rind taken off all round, as well from the Trunk as Boughs, which grows again in four Months, as *Cork* does: The Trunk is about the Bigness of a *Man's Thigh*; it bears a Fruit not unlike a *Chesnut*, (except in its outer Rind or Shell) which is properly called *China China*, and is esteem'd by the Natives beyond the Bark taken from the Trunk or Boughs. This Account I received from an ingenious Apothecary at *Cadiz* in *Spain*, *A.D.* 1694, who had liv'd in *Peru*, and had seen it growing, and gather'd it several Times. From this History I made this Observation, that probably *China China*, or the Rind of the Fruit, was first only in Use, and the more powerful Medicine used in smaller Quantities, and that the Bark of the Tree came not into Use, till some Time after; when the Virtues of it known in *Europe*, occasioned a greater Demand for it.

A new Kind of Walnut-Tree, *&c. by Mr.* Reneaume. n. 273. p. 908.

XV. Of the *Walnut-Tree* Authors seem to have known but six *Species*, tho' I can reckon nine. They confounded (unless I am deceived) with the *common sort*, that which the Country People call *Noix Angleuses*, which one may call *Nux Juglans putamine durissimo*, which appears to me to be that which in *Hermolaus*, and in the *Historia Lugdunensis* is called *Moratiæ Moracillæ*, and which *Cæsalpin* calls *Surdæ*.

I don't see that the same Authors have distinguish'd another Species, which might be call'd *Nux Juglans fructu præcoci*, because they are sooner ripe than the others, and eaten about the Feast of St. *John en Cerneaux*, which

which has given them, amongſt the Country People, the Name of *Noix Joanneties.* As for that Species I am to treat upon, I can't find any Author that knew of it, and therefore I ſhall call it *Nux Juglans, folio eleganter diſſecto*, or *Acanthi-folia.*

The Oil which is preſſed out of the Walnut-Tree, in certain Provinces is uſed inſtead of Butter and Oil-Olive. In *Berry*, where they have very good Wool, and where they trade very much in Cattle, they have yet but very little Butter; and that little which they have, is worth nothing, and is very dear; ſo that they uſe Nut-Oil in dreſſing their Meat to eat. For this Reaſon there are an infinite Number of Walnut-Trees planted in the middle of the plough'd Lands, in ſuch ſort, that afar off one would take theſe Lands for Woods of Walnut-Trees.

The Want of theſe Trees in this Country obliges the Inhabitants to cultivate them, and they take care to nouriſh them in particular Places, as in a ſort of Nurſery, in order to plant them afreſh when they die, whether it be of Age (which is rare) or whether they decay, or that they are fell'd, for the Wood to work with.

The laſt *Autumn*, two Leagues from *Selles* in *Berry*, in the Pariſh of *Lis*, as I walked in an Orchard, looking upon ſome Plants near a Place where they bred up a vaſt Number of young Walnut-Trees, I perceiv'd in the middle a ſort of Leaf, (or Foliage) which I had never taken Notice of before. I went thither forthwith, and having examin'd it, as I knew not the Subſtance of this Leaf, I taſted it. The Taſte, Smell, Wood and Figure of the Tree, perſuaded me to believe that it was a Walnut-Tree, and I concluded that this was one, tho' I did not remember that I had ever read, or heard of any ſort like this.

This Tree is very young, and did never yet bear any Fruit, perhaps, becauſe it may be *(in a manner)* choak'd up, and that there is neither Air nor Nouriſhment enough, by reaſon of the great Number of other Walnut-Trees, which grow round about it. It is near ſix Feet high, and two Inches Diameter at the Bottom. 'Tis adorned at the Top with many Branches, and (as the Country People ſaid) was about eight or nine Years old, and that they had always found its Leaves like thoſe which I ſaw.

The *(common)* Walnut-Tree bears its Leaves by Pairs, upon a Stalk, which terminates with a like Leaf, that is ordinarily bigger than the reſt: And it has very ſeldom above three Pairs upon each Stalk.

This has ſometimes four or five Pairs, and ſometimes more, which are one while oppoſite, another while alternate, altho' its Leaves appear ſmaller than thoſe of the common Walnut-Tree, becauſe of the Cuttings or *Slaſhes.* They are neverthele ſs as big, if one minds their Circumference taken from the Extremities of theſe *Slaſhes.*

The firſt Pair, and ſometimes the ſecond, are leſs *cut* than the reſt, being ſo only upon the Circumference: but the others are cut ſo deep, that it looks as if the Nerve in the middle of the Leaf was only a Stalk;

and the *Cuts* of the Leaves are ſometimes by Pairs, ſometimes ſingle on one ſide. Theſe Leaves are ſometimes forked at the End, and ſometimes end with a Point. There are alſo ſome Places, where it looks as if the Leaf was torn on purpoſe, almoſt like the *Angelica Canadenſis, foliis quaſi præmorſis.* There are others, where it ſeems that they are double, as if the Stalk or the Nerve was winged, juſt as the winged Stems, or Trunks, or *Caules alati.* All theſe Cuts and Slaſhes are not like Indentures or Notches, but finiſh with a Round. And notwithſtanding all theſe Irregularities, they look ſo pretty, that I can't compare them better *(to any thing)* than thoſe wrought Leaves, which ſerve for Ornaments to the Painters, almoſt like thoſe which adorn the Capital of *(Columns of)* the *Corinthian* Order, or that which in Heraldry they call *the Mantles*, or that which the Botaniſts term *Acanthus* or *Branca Urſina*, which is the firſt Original of this ſort of Ornament.

Dalechamp has obſerv'd an Aereal Honey of a yellowiſh Colour upon the Leaves of a Walnut-tree, during the greateſt Heats of the Summer; which can be nothing but an Effect of the Tranſpiration of this Tree, as of all other Trees, wherein the ſame thing is to be found; as I prov'd in a Diſcourſe to the *Academy* laſt Year, in ſpeaking of the *Sycamore.*

XV. *Papers Omitted.*

1. An Account of Mr. *Sam. Brown*'s (a Phyſician at *Fort St. George*) *Third* Book of *Eaſt-India* Plants, with their Names, Virtues, Deſcriptions, *&c.* by Mr. *J. Petiver.* *n.*271.*p.*843.

2. An Account of the *Fourth* Book of the ſame. *n.*274.*p.*927.

3. ——— of the *Fifth* Book. *n.*276.*p.*1007.

4. ——— of the *Sixth* Book. *n.*277.*p.*1055.

5. ——— of the *Seventh* Book. *n.*282.*p.*1251.

6. ——— of the *Eighth* Book. *n.*299 *p.*1952.

7. *G. J. Camelli* de *Plantis Philippenſibus Scandentibus Tractatus*, ſent to Mr. *Petiver.* To which is added, a *Catalogue* of *Herbs* he formerly ſent him, the *Deſigns* of which are *deſcribed* by Mr. *J. Ray* in the *Appendix* to his *Third* Volume of *Plants.* *n.*293 *p.*1707.

8. *G. J. Camelli* de Plantis *Philippenſibus* Scandentibus, Pars *Secunda.* *n.*294 *p.*1763.

9. ——— Pars *Tertia.* *n.*295.*p.*1809.

10. ——— Pars *Quarta.* *n.*296.*p.*1816.

11. An Account of ſeveral Rare *Plants* lately obſerved in ſeveral curious Gardens about *London*, and particularly the *Company* of *Apothecaries* Phyſic Garden at *Chelſea*, by Mr. *J. Petiver*, in Seven Tracts. *n.*332.*p.*375. *n.*333 *p.*416. *n.*337.*p.*33, 177. *n.*343.*p.*229. *n.*344.*p.*269. *n.*346.*p.*353.

XVI. *Accounts of Books Omitted.*

n.210.p.2442. 1. The whole *Art* of *Husbandry*, by *John Mortimer*, Esq; 8*vo.*

n.285.p.1411. 2. *Gazophylacii Naturæ & Artis* Decas. In quâ *Animalia* Quadrupeda, Aves, Pisces, Reptilia, Insecta, Vegetabilia; item *Fossilia*, Corpora marina, & Stirpes Minerales, è Terrâ eruta; *Lapides* figura insignes, &c. *Descriptionibus* brevibus & *Iconibus* illustrantur. Hisce annexa est Supellex *Antiquaria*, Numismata, Gemmæ excisæ & Sculpturæ, Opera *Figulina*, Lucernæ, Urnæ, Instrumenta varia, Inscriptiones, Busta, reliquaq; ad Rem *Priscam* spectantia; item Machinæ, Effigies clarorum *Virorum*, omniaq; Arte producta. A *Jacobo Petiver*, R. S. S.

n.331.p.344. 3. *Gazophylacii Naturæ & Artis*, &c. Vol. I. in V. Decadibus, per *J. Petiver.*

n.306.p.2253. 4. *Samuelis Dale*, *Pharmacologiæ* sive *Manuductionis* ad Materiam Medicam *Supplementum*; Medicamenta *Officinalia* complectens: Ut & Notas *Generum* Characteristicas, *Specierum* Synonyma, Differentias, & Vires. Cum duplici Indice, generali Altero nominum & synonymorum præcipuorum; Altero Anglico-Latino, in Gratiam Tyronum. 12*mo.* *Lond.* 1705.

n.325.p.35. 5. Index *Plantarum* Horti *Lugduno-Batavi*, per *Hermannum Boerhaven.* Lugd. Bat. 1710. 8*vo.*

n.345.p.350. 6. *Ludovici Ferdinandi Marsilii* Dissertatio de *Generatione Fungorum.* Romæ 1710. 8*vo.*

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INDEX

OF ALL

The MATTERS contain'd in these Two VOLUMES.

NOTE, The *Numeral Letters* distinguish the *Volumes*, ii *signifies* the *Second Part* in each VOLUME, and the small *Figures* signify the *Pages*.

B.

C.

Of

N.

O.

P.

THE END.

PHILOSOPHICAL TRANSACTIONS

VOL. IV

1700-1720