WATER DUCT WITH INTERMEDIATE INTAKE. APPLICATION OF THE THEORY OF THE OPTIMIZATION AND GENERALIZATION OF THE PROBLEM

Franquet-i-Bernis, Joseph M.
Agricultural Engineer PhD, *University Rovira i Virgili*, Department of Mechanical Engineering, Tarragona, Spain.

SUMMARY/ABSTRACT

The present work deals with the study of a pressure water pipe or conduit provided with an intermediate outlet with free exit at its end is applied, also applying the theory of classical optimization and some new formulation proposed by the author. The problem is generalized by increasing the number of equidistant outlets or outlets of the conduit of equal flow, with the corresponding determination of the resulting head loss by using the Christiansen formulation, which obviates the iterative calculation process that is highly cumbersome if this number of exits is important, as it happens in sprinkler or localized high frequency irrigation. Finally, it is considered the case study of a pipeline that distributes the flow uniformly over its entire length.

Keywords: forced conduction, piezometric level, headloss, optimization, reducing coefficient, derivation, pipeline, flow, diameter.

CONDUCTO DE AGUA CON TOMA INTERMEDIA. APLICACIÓN DE LA TEORÍA DE LA OPTIMIZACIÓN Y GENERALIZACIÓN DEL PROBLEMA

RESUMEN

En el presente trabajo se aborda el estudio de una tubería o conducto de agua a presión provisto de una toma intermedia con salida libre por su extremo, aplicándose también la teoría de la optimización clásica y alguna nueva formulación propuesta por el autor. El problema se generaliza al aumentar el número de tomas o salidas equidistantes del conducto de igual caudal, con la determinación correspondiente de la pérdida de carga resultante mediante el empleo de la formulación de Christiansen, que obvia el proceso de cálculo iterativo que resulta altamente farragoso si dicho número de salidas es importante, como sucede en los riegos por aspersión o localizados de alta frecuencia. Se considera, por último, el estudio del caso de una tubería que distribuye el caudal de manera uniforme en toda su longitud.

Palabras clave: conducción forzada, nivel piezométrico, pérdida de carga, optimización, coeficiente reductor, derivación, tubería, caudal, diámetro.

CONDUITE D'EAU AVEC PRISE INTERMÉDIAIRE. APPLICATION DE LA THÉORIE DE L'OPTIMISATION ET GÉNÉRALISATION DU PROBLÈME

RÉSUMÉ

Dans le présent travail, est abordée l’étude d’une canalisation ou d’un conduit d’eau sous pression dotée d’une prise intermédiaire avec sortie libre à son extrémité, appliquant également la théorie de l’optimisation classique et une nouvelle formulation proposée par l’auteur. Le problème se généralise en augmentant le nombre de prises ou de sorties équidistantes du conduit de flux égal, avec la détermination correspondante de la perte de charge résultante grâce à l'utilisation de la formulation de Christiansen, qui évite le processus de calcul itératif qui est très fastidieux si le nombre de sorties est important, comme dans l’arrosage par aspersion ou irrigation localisée à haute fréquence. Enfin, l'étude du cas d'un pipeline distribuant le flux de manière uniforme sur toute sa longueur est considéré.

Mots clés: conduite forcée, niveau piézométrique, perte de charge, optimisation, coefficient de réduction, dérivation, conduite, débit, diamètre.

CONDUCTE D'AIGUA AMB PRESA INTERMÈDIA. APLICACIÓ DE LA TEORIA DE L'OPTIMITZACIÓ I GENERALITZACIÓ DEL PROBLEMA

RESUM

En el present treball s'aborda l'estudi d'una canonada o conducte d'aigua a pressió que disposa d’una presa d’aigua intermèdia amb sortida lliure pel seu extrem, aplicant també la teoria de l'optimització clàssica i alguna nova formulació proposada pel propi autor. El problema es generalitza en augmentar el nombre de preses o sortides equidistants del conducte d'igual cabal, amb la determinació corresponent de la pèrdua de càrrega resultant mitjançant l'ús de la formulació de Christiansen, que obvia el procés de càlcul iteratiu que resulta altament carregós si el nombre de sortides és important, com succeeix en els regs per aspersió o localitzats d'alta freqüència. Es considera, finalment, l'estudi del cas d'una canonada que distribueix el cabal de manera uniforme en tota la seva longitud.

Paraules clau: conducció forçada, nivell piezomètric, pèrdua de càrrega, optimització, coeficient reductor, derivació, canonada, cabal, diàmetre.

INDEX

Page

****

1. Pipeline with an intermediate outlet

Let pipeline AC of length L and constant diameter D, in which there is an outlet at B (Fig. 1), fitted with its stopcock, divides said length into two sections of lengths L_1 and L_2. So:

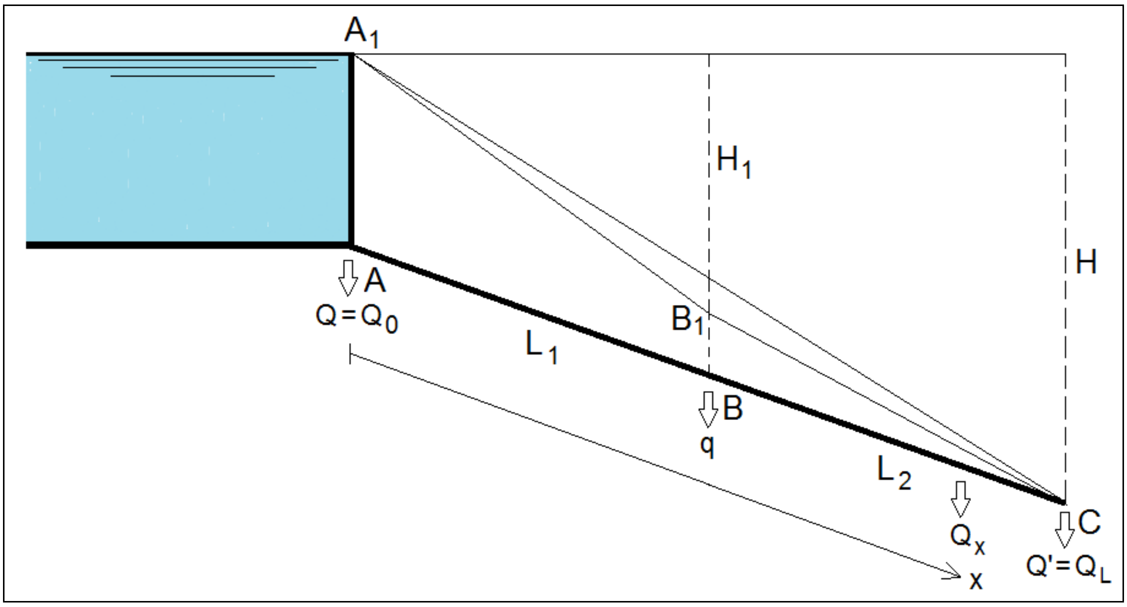

Fig. 1. Outline of the problem.

If both sections are of different internal diameters, D_1 and D_2, it would be a mixed pipeline (in series) that could be replaced, for calculation purposes, by a certain "equivalent pipe" of constant diameter De. In this sense, a pipe is said to be equivalent when the head loss, for a given flow rate, is the same as in the pipe system it replaces (Torres, 1970). In our case it would happen that:

$$\frac{L}{D_e^5} = \frac{L_1}{D_1^5} + \frac{L_2}{D_2^5}, \text{ that is: } D_e = \sqrt[5]{\frac{L}{\frac{L_1}{D_1^5} + \frac{L_2}{D_2^5}}}, \text{ considering: } L = \sum_{i=1}^{2} L_i \qquad (1)$$

This can be generalized to any number n of sections of different internal diameters of which the forced conduction in the study consists, thus,

$$D_e = \sqrt[5]{\frac{L}{\sum_{i=1}^{n} \frac{L_i}{D_i^5}}}, \text{ considering } L = \sum_{i=1}^{n} L_i .$$

It should be noted, however, that the average of the formulations proposed in our studies (Franquet, 2005) turns out to have the generic configuration:

$J = \gamma \cdot Q^2 \cdot D^{-5.3}$, with which the exponent 5.00 and degree of the root appearing in expression (1) could be modified by the value 5.30 or applied for each different case according to the specifically adopted roughness category. Indeed:

$$J = \gamma \cdot \frac{Q^2}{D^{5.3}} = \frac{\gamma \cdot \left(\frac{\pi \cdot D^2}{4}\right)^2 \cdot V^2}{D^{5.3}} = \frac{\gamma \cdot \frac{\pi^2}{16} \cdot D^4 \cdot V^2}{D^{5.3}} = \gamma' \cdot V^2 \cdot D^{-1.3}, \text{ with } \gamma' \approx 0.0013,$$

that constitutes the average generic expression, of the J (V, D) type, of the loss of continuous unit load that appears in our research.

In fact, note that according to the different formulations proposed by many authors, the mentioned exponent of the diameter in the function J (Q, D), which is given by $(2\beta_1 + \beta_2)$, being β_1 and β_2, respectively, the values of the exponents of the speed and the internal diameter in the formulation used for the calculation of the continuous unit losses of load, would adopt values oscillating around 5.00, as can be seen in Table 1 referring to some of them. Thus, table 2 shows the exponent of the diameter that should be applied, according to our own formulation, for the calculation of these continuous unit load losses according to the different categories of roughness k proposed in our studies.

Table 1. Exponent of the diameter in J (Q, D) according to different authors.

Author	***Exponent of D***
Manning-Strickler-Gaukler (1890, 1923)	5.33
Sonier, **Franquet** (2005)	5.30
Mougnie	5.25
Darcy-Weisbach (1843), Dupuit (1848), Colombo, Catani	5.00
Scobey (1930)	4.90
Hazen-Williams (1920)	4.87
Veronese-Datei	4.80
Scimemi (1951)	4.79
Cruciani-Margaritora	4.75

Source: self made.

The formulations proposed by this author for the dimensioning of forced conductions (Franquet, 2005) have the advantage that, as in free conductions, the exponent of the J is, in all cases: $\nu = \frac{1}{2} = 0.5$ (however, said exponent , in relation to speed or flow, can range from 0.5 in turbulent regime, which is the most normal, to 1.0 in laminar regime, as seen in the exercises in the previous section); also in the explicit formula of J the exponent of velocity V is 2.0, while the exponent of the internal diameter β in the explicit formula of velocity increases progressively with the degree of roughness k, from 0.6215 to 0.67725.

Table 2. Applicable D exponent value according to the roughness categories adopted.

Roughness (k)	β_1	β_2	Adjusted D exponent
1.0	**2**	**1.2430**	**5.24**
1.5	2	1.2560	5.26
2.0	**2**	**1.2691**	**5.27**
2.5	2	1.2821	5.28
3.0	**2**	**1.2952**	**5.30**
3.5	2	1.3032	5.30
4.0	**2**	**1.3112**	**5.31**
4.5	2	1.3210	5.32
5.0	**2**	**1.3308**	**5.33**
5.5	2	1.3426	5.34
6.0	**2**	**1.3545**	**5.35**

Source: self made.

The aforementioned exponent of value 5.30 corresponds exactly to the different measures of the central value or averages usually used, both classic and more typical of robust statistical methods (arithmetic, geometric or harmonic means, trimedia, median, Gastwirth median, mode) in the frequency distribution in Table 2 above.

Suppose, now, that there is a free discharge of the liquid into the atmosphere at the C-end and that the length of the line is large enough to be able to neglect accidental or singular head losses ($h_s = 0$). If the key of B did not exist or was completely closed, the piezometric line would be A_1C, and under the conditions of the problem an expense Q would come out of C, which according to the simplified formula of Darcy (1865), would be:

$$Q^2 = m' \cdot J = m' \cdot \frac{H}{L}, \text{ from where:}$$

$$Q^2 \cdot L = m' \cdot H \qquad (1')$$

and with: $m' = \frac{\pi^2 \cdot D^5}{64 \times b}$, and $b = \alpha + \frac{\beta}{D}$, being the parameters α and β dependent on the constituent material of the pipe. In the specific case to be treated, v. gr., from a plastic pipe (Franquet, 2005), the value of the continuous total loss of load at the C end of the pipeline will be given by the expression:

$$H = J \times L = 0.000743 \times \frac{V^2 \times L}{D^{1.243}}$$

If the tap located in B is opened, and a certain flow q is derived from it, the piezometric level line becomes A_1B_1C and a residual flow Q' will come out of the end C, so that the flow in the first section AB of length L_1 will be: q + Q' = = Q, and in the second section BC of length L_2 it will be Q', being able to establish the following equations (Torres, 1970),

$$(q + Q')^2 \cdot L_1 = m' \cdot H_1 \tag{2}$$

$$Q'^2 \cdot L_2 = m' (H - H_1) \tag{3}$$

Now adding equations (2) and (3), we obtain:

$$(q + Q')^2 \cdot L_1 + Q'^2 \cdot L_2 = m' \cdot H ,$$

and taking into account the expression (1') it turns out:

$$(q + Q')^2 \cdot L_1 + Q'^2 \cdot L_2 = Q^2 \cdot L ,$$

whence it follows that:

$$L\, Q'^2 + 2\, q \cdot L_1 \cdot Q' + q^2 \cdot L_1 - L \cdot Q^2 = 0 .$$

Dividing by $L = L_1 + L_2$, will be:

$$Q'^2 + \frac{2q \cdot L_1}{L} \cdot Q' + \frac{q^2 \cdot L_1}{L} - Q^2 = 0 ,$$

and solving this 2nd degree equation, it turns out:

$$Q' = \frac{-\frac{2qL_1}{L} + \sqrt{\frac{4q^2L_1^2}{L^2} - \frac{4q^2L_1}{L} + 4Q^2}}{2} = \frac{-q \cdot L_1}{L} + \sqrt{\frac{q^2 \cdot L_1^2}{L^2} + Q^2 - \frac{q^2 \cdot L_1}{L}} =$$

$$= \frac{-q \cdot L_1}{L} + \sqrt{Q^2 - \frac{L_1 \cdot L_2}{L^2} \times q^2} ,$$

and also:

$$Q' = -\frac{q \cdot L_1}{L} + Q\sqrt{1 - (q^2 / Q^2) \cdot \frac{L_1 \cdot L_2}{L^2}} \tag{4}$$

2. Application of functional optimization theory

At this point, and in view of the radicand that appears in expression (4), one might ask: **when is the quotient between the geometric mean of the distances L_1 and L_2 and their sum (total length) maximized and what is their value?** Thus, it would be a matter of maximizing the quotient:

$$\frac{G}{L_1 + L_2} = \frac{\sqrt{L_1 \cdot L_2}}{L} = \sqrt{\frac{L_1 \cdot L_2}{L^2}} \tag{5}$$

We are, therefore, faced with a classic optimization problem, consisting in the maximization with conditions of a real function of real variable, since it is a matter of maximizing a function of two positive or null variables, for which reason we will consider simplifying the maximization of the radicand from the previous expression (5): $[MAX]Z = \frac{L_1 L_2}{L^2}$, subject to the following conditioning equation: $L_1 + L_2 = L$, using the well-known method of Lagrange multipliers or operators.

The problem posed lies in obtaining the value of the variables L_1 and L_2 where the differentiable objective function Z reaches a maximum, knowing that these variables are related to each other through the expressed boundary condition or geometric equality restriction, referring to the length of the pipeline.

To do this, we form the Lagrange or auxiliary equation:

$$\Phi = \frac{L_1 L_2}{L^2} + \lambda\,(L_1 + L_2 - L).$$

- *Necessary or 1st condition degree:*

$$\left.\begin{array}{l} \Phi'_{L_1} = \dfrac{\partial \Phi}{\partial L_1} = \dfrac{L_2}{L^2} + \lambda \cdot L_1 = 0 \\ \Phi'_{L_2} = \dfrac{\partial \Phi}{\partial L_2} = \dfrac{L_1}{L^2} + \lambda \cdot L_2 = 0 \\ \Phi'_{\lambda} = \dfrac{\partial \Phi}{\partial \lambda} = L_1 + L_2 - L = 0 \end{array}\right\} \begin{array}{c} \lambda = -\dfrac{L_2}{L^2 \cdot L_1} = -\dfrac{L_1}{L^2 \cdot L_2}; \\ L^2 \cdot L_2^2 = L^2 \cdot L_1^2; \;\; \text{that is :} \\ L_1 = L_2 = \dfrac{L}{2} \end{array}$$

- *Sufficient or 2nd grade condition:*

It is true that: $\lambda = -\frac{L_2}{L^2 \cdot L_1} = (L_1 = L_2) = -\frac{1}{L^2}$; Taking into account that:

$\Phi''_{L_1^2} = \frac{\partial^2 \Phi}{\partial L_1^2} = \lambda;\;\; \Phi''_{L_1 L_2} = \Phi''_{L_2 L_1} = \frac{1}{L^2};\;\; \Phi_{L_2^2} = \lambda$; and we will have the determinant "relevant Hessian border":

$$H(L_1, L_2, \lambda) = \begin{vmatrix} \Phi''_{L_1^2} & \Phi''_{L_1L_2} & \Phi''_{L_1\lambda} \\ \Phi''_{L_1L_2} & \Phi''_{L_2^2} & \Phi''_{L_2\lambda} \\ \Phi''_{L_1\lambda} & \Phi''_{L_2\lambda} & \Phi''_{\lambda^2} \end{vmatrix} = \begin{vmatrix} -\frac{1}{L^2} & \frac{1}{L^2} & \frac{L}{2} \\ \frac{1}{L^2} & -\frac{1}{L^2} & \frac{L}{2} \\ \frac{L}{2} & \frac{L}{2} & 0 \end{vmatrix} =$$

$= \frac{L^2}{4L^2} + \frac{L^2}{4L^2} + \frac{L^2}{4L^2} + \frac{L^2}{4L^2} = \frac{4L^2}{4L^2} = 1 > 0$ (2 variables), then it is a maximum.

In short, the maximum value of the Z function will be

$$[\text{MAX Z}] = \frac{L_1L_2}{L^2} = \frac{L^2}{4L^2} = \frac{1}{4}, \text{ and also: } [\text{MAX}\left(\sqrt{\frac{L_1 \cdot L_2}{L^2}}\right)] = \sqrt{\frac{1}{4}} = \frac{1}{2}.$$

The length being constant: $L_1 + L_2 = L$, the maximum value of $\frac{L_1L_2}{L^2}$ corresponds, therefore, to: $L_1 = L_2 = \frac{L}{2}$, and for this maximum value it is:

$$\text{Maximum value of } \left(\frac{q^2}{Q^2} \cdot \frac{L_1 \cdot L_2}{L^2}\right) = \frac{q^2}{Q^2} \cdot \frac{L^2}{4L^2} = \frac{1}{4}\left(\frac{q}{Q}\right)^2.$$

So, equation (4) will look like this:

$$Q' = Q - q = -\frac{q \cdot L_1}{L} + Q\sqrt{1 - \left(\frac{q}{2Q}\right)^2} = -\frac{q}{2} + Q\sqrt{1 - \left(\frac{q}{2Q}\right)^2}, \text{ and also:}$$

$Q = \frac{q}{2} + Q\sqrt{1 - \left(\frac{q}{2Q}\right)^2}$, and taking into account that: $Q > 0$ and $q > 0$, it results:

$$Q = \frac{\sqrt{4Q^2 - q^2} + q}{2}.$$

Likewise, in equation (4), if the flow q derived from the intake B is very small compared to Q, it can be stated, in approximate terms, that:

$$Q' \approx Q - \frac{q \cdot L_1}{L} \quad (6)$$

whereby for equal head loss, the flow rate Q 'at the end of the line is Q minus a fraction of q that depends on the relative position of the intermediate tap B. Then, when said tap is located in the middle section to which we have previously referred, that is $L_1 = L_2 = \frac{L}{2}$, the total flow will take an approximate value:

$$Q \approx Q' + \frac{q}{2}.$$

This same expression will be true exactly when the tap moves to the end of pipe C, so $L_2 = 0$, $L_1 = L$ and $Q' = 0$, whereby:

$$Q' = Q - q = 0 \Rightarrow Q = q.$$

From this expression it can be deduced that the residual flow rate Q' sale that comes out from the end C of the pipe will be less the higher L_1, that is, the closer the intermediate connection is located to the end C of the line.

To get the flows q and Q′ to be approximately equal, we will:

$$q = Q - \frac{q \cdot L_1}{L}$$

whence the situation of the intermediate connection, in this case, will be defined by the distance L_1, from it, to the origin of the pipe:

$$L_1 = \frac{(Q-q)}{q} \cdot L \text{, and since } L_1 < L \text{ will be } Q - q < q \text{, or } \quad q > \frac{Q}{2},$$

and since $Q > q$, will be: $Q > q > Q/2$.

In addition, in the case of intermediate intake at B and constant or equivalent internal diameter $D = D_e$, at nodes B and C, respectively, the outflows q and Q′ are had. To find the total head loss H we will proceed in such a way that the sum of the head losses in sections L_1 and L_2 gives us this total head loss H. That is, in the proposed example of the plastic pipe, we would have:

$$H = H_1 + H_2 = J_1 \times L_1 + J_2 \times L_2 = 0.000743 \times \frac{V_1^2 \times L_1}{D^{1.243}} + 0.000743 \times \frac{V_2^2 \times L_2}{D^{1.243}} =$$
$$= \frac{0.000743}{D^{1.243}} \left(V_1^2 \times L_1 + V_2^2 \times L_2\right) \qquad (7)$$

In the case of trying to determine the H applying the formulation of Christiansen (1942), which will be seen in more detail in the next section of our work, there would be values of m = 2 (exponent of speed or flow in the formula empirical used for the determination of the continuous pressure losses) and $n_0 = 2$ equidistant outputs of flows q and Q ', such that q = Q' is fulfilled, in the respective nodes B and C, exhausting the flow in this last point or end of driving. So:

$$F = \frac{1}{1+m} + \frac{1}{2 \cdot n_0} + \frac{\sqrt{m-1}}{6 \cdot n_0^2} =$$

$$= \frac{1}{1+2} + \frac{1}{2 \cdot 2} + \frac{\sqrt{2-1}}{6 \cdot 2^2} = 0.333 + 0.250 + 0.042 = 0.625 \ ,$$

and the total head loss H would be given by:

$$H = F \times J \times L = 0.625 \times \frac{0.000743}{D^{1.243}} \times V^2 \times L \ ,$$

the resulting pressure drop in this conduit being 100 - 62.5 = 37.5% lower than that which would occur if all the flow Q were to pass through the same tube without bypasses, and the water leaving freely at the C end.

As it has been said, in all the described process it has been assumed that the pipe is of great length, therefore accidental losses have not been taken into account. In any case, if interested, the continuous losses can be increased approximately by 10% for installations with few special parts or singular points, by 15% for normal installations (if a different rating is not assumed) and by 20% for installations with a high number of these pieces. Greater precision in the calculation would require the detailed determination of the accidental pressure losses that occur in each special part, having sufficient data and making use of existing formulas or tables in this regard, which frequently express the equivalence in aggregate linear meters of pipe.

3. Generalization to a pipeline with various intermediate taps

Let us see, regarding the different values that the coefficient or exponent **m** can take in the following expression (8) that, in general, the continuous unit losses of load of a pipe under pressure or forced conduction, as a function of the flow through it circulating , respond to a potential expression of the type:

$$J = n \cdot Q^m \qquad (8)$$

being here: $n = \frac{1}{m'} = \frac{64 \times b}{\pi^2 \times D^5}$, that in the case of Darcy's simplified expression, as in those proposed by the author of this paper (Franquet, 2005), it adopts the following value: m = 2.00, as well as those of Lèvy (1899), Gaukler (1867), Weisbach (1843), Kütter (1870), Mougnie, Chèzy, Sonier, Manning-Strickler (1923) or Catani. In those of the society SOGREAH (1962), Flamant (1891) or Blasius (1913) it is m = 1.75, as well as in those of Saph and Schoder (1903); in that of Scimemi-Veronese (1925), it is m = 1.78571, in that of Hazen-Williams (1920) it is m = 1.852, in those of Biegeleisen and Bukowsky (1942) it is a value m = 1.90, as well as in that of Meyer- Peter (1931), while in the various formulations proposed by Scobey (1930) we find the values: m = 1.80, 1.90, etc., but always within the interval of existence [1.50, 2.00], and expressing them, all of them, as monomial potential formulas. On the other hand, it is common to consider m = 1.75 for drip irrigation and m = 1.80 for spray irrigation (Franquet, 2003). Only in the case of other types of fluids (petroleum, benzine, kerosene, diesel, vegetable oils) with a viscosity higher than that of water, Carothers adopts a value m = 1.50 (Forchheimer, 1935-1950).

The versatility of such a broad formulation leads us to conclude that such information includes the different styles of work successively employed throughout almost two centuries and, basically, representative of an evolution of knowledge that tends to generalize and unify, each once again, his affirmations, in the pursuit of a final synthesis not yet reached. In this same sense, we have made a research effort, both in relation to the calculation of free pipes and in the calculation of forced or pressure pipes (Franquet, 2019).

It is unavoidable, nowadays, to distinguish, according to the experimentation of Von Kàrmàn-Nikuradse and Colebrook-White, the smooth, rough and intermediate pipes, these names established not according to the texture of the wall, but according to the hydraulic behavior, by virtue of the configuration of the boundary layer that is perfectly defined in each case. Thus, it happens that the law of resistance in smooth pipes is unique, independent of its constitutive material and expressible by an analytical law of which the Blasius formula is a first approximation that has been prolonged by other researchers (Franquet, 2003).

Thus, it will turn out, in short:

$$\boxed{F = \frac{1}{1+m} + \frac{1}{2 \cdot n_0} + \frac{\sqrt{m-1}}{6 \cdot n_0^2}} \tag{9}$$

which is the approximate expression adopted by Christiansen in his study on hydraulic pipes with en-route service.

In any case, as determined in our studies (Franquet, 2019) the determination of the most correct degree of the root that appears in the numerator of the third sum or fraction of the 2nd member of the expression (9) is immediate by: $\frac{\sqrt[1/x]{2}}{6} = \frac{1}{4} = \frac{2^x}{6}$ where it is inferred that: $x = \frac{\log 1.5}{\log 2} = \frac{0.1760912}{0.30103} = 0.585$, and the degree of the root should be: 1/0.585 = 1.71, whereby the expression of Christiansen's formulation would be definitively corrected like this, providing exact values:

$$\boxed{F = \frac{1}{1+m} + \frac{1}{2 \cdot n_0} + \frac{\sqrt[1.71]{m-1}}{6 \cdot n_0^2}}, \forall\ m \ \text{such that} \ m \in [1,2,3]\ .$$

We have, therefore, the general case of a pipeline with service en route, with $\mathbf{n_0}$ derivations of constant flow, with a spacing between outlets **l** and the first derivation being at a distance $\mathbf{l_0}$ from the origin of conduction A, which may be a tertiary pipeline irrigation, a high tank or a pumping group. This is:

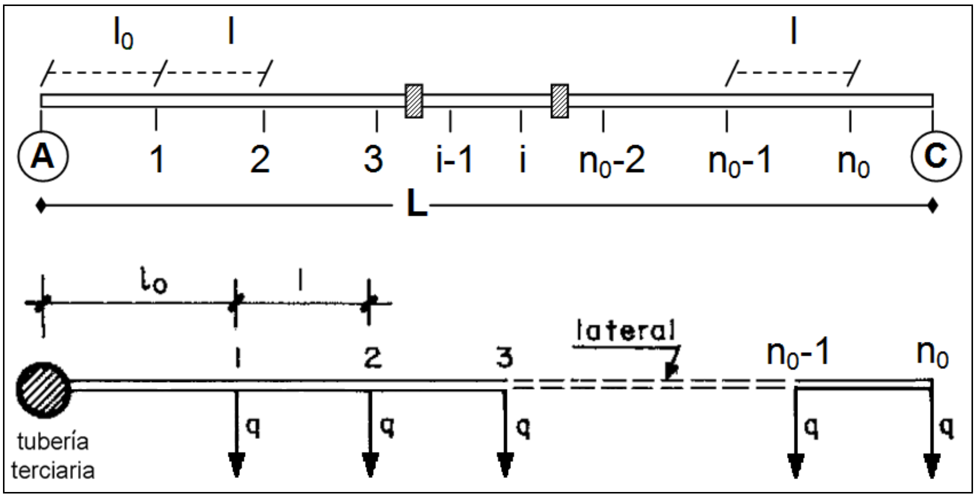

Fig. 2. Pipeline with en-route service and equidistant branches of constant flow q.

that for all l will be fulfilled such that: $l_1 = l_2 = ... = l_i = ... = l$.

Well, the output flow of A, which runs out in C, will be: $Q = n_0 \cdot q$, and the total length of the pipe, taking into account that: $l_0 = r \cdot l$, is:

$$L = l_0 + (n_0 - 1) \cdot l = (r + n_0 - 1) \cdot l\ .$$

Theoretically, in a pipe of the characteristics expressed, the reduction coefficient for outlets, applicable to the continuous pressure losses experienced by a pipe that exhausts the flow at its end C and maintains constant the flow q, the internal diameter, its roughness absolute K and the equidistance between the outputs l, with $l_0 = l$, would respond to the expression:

$$F = \frac{1}{n_0^{1+m}} \cdot \sum_{i=1}^{n_0} i^m\ ,$$

for which Christiansen (1942) obtained the function expressed in (9).

The problem that arises here constitutes a generalization of the classic problem of a simple pipe with several intermediate intakes (of non-excessive number) and constant diameter, whose resolution is usually presented by application of the well-known formula of Darcy and Bazin (1865), although it can also be apply the formulation proposed by this author (Franquet, 2005), and the prior determination of the line of piezometric levels[1].

In the case of equidistant leads and constant flow **q** for each of them, the determination of said piezometric line would be obtained by dividing the total load H into parts proportional to the sequence of real numbers: n_0^2, $(n_0-1)^2$, ..., 1.

It should also be borne in mind that this formula (9) will only be valid for the specific case in which the first exit is from the beginning of the conduction (Fig. 2) at a distance $l_0 = l$, that is, equal to the distances between the different outputs ($r = l_0/l = 1$). And in the case of the formulation proposed by this author (Franquet, 2005), with m = 2.00, Christiansen's expression will adopt the configuration:

$$F = \frac{1}{1+2} + \frac{1}{2 \cdot n_0} + \frac{\sqrt{2-1}}{6 \cdot n_0^2} = \frac{(n_0+1) \cdot (2n_0+1)}{6n_0^2} = \frac{2n_0^2 + 3n_0 + 1}{6n_0^2} \qquad (10)$$

It is obvious, on the other hand, that when the number of bypasses or outlets increases indefinitely (that is, when the flow is distributed uniformly throughout the entire forced conduction, as in the case of irrigation by exudation), the previous expression is will become:

$$\lim_{n_0 \to \infty} F = \frac{1}{1+m} \qquad (11)$$

which constitutes, in these circumstances, the minimum value to which the experimental reduction coefficient in question tends. If the residual or extremal flow rate Q' of the line is zero, and considering the normal case m = 2.00, let us see that this indicates that the loss of continuous pressure that takes place is one third of what would occur if the expense or Initial flow would flow through the entire pipeline and freely exit through the end of the pipeline (and this considering that the pipeline in question distributes a uniformly distributed expense that is obtained by adding all the expenses of the branches and dividing this sum by the total length of the pipe). This would therefore be equivalent to a Christiansen reducing coefficient of value: F = 1/3 = 0.333 that would be obtained from formulation (9) when n_0 tends to ∞ and also when m = 2.00.

Normally, in certain localized high-frequency irrigations (RLAF) such as dripping or micro-sprinkling, it will be true that m = 1.75, while when the regimen is laminar, this situation is frequent in exudation irrigation in which the

[1] The "piezometric line" corresponds to the locus of the water levels of the piezometric tubes or piezometers connected to the pipe and is the sum of the pressure and position heights or tachometric height. The difference in height between the power line and the horizontal line or load plane represents the total loss of load due to friction between any two points of the pipeline under study.

loss of load is, practically, continuously and not discretely, we will have that with: m = 1.00 it will be F = 0.500 and with m = 2.00 we have that F = 0.333, which corresponds exactly to the formulation proposed by that American author.

Let's see that, depending on the number n_0 of equal outlets or derivations of flow *q*, Christiansen's reduction coefficient F follows different paths according to the formulation used in hydraulic design, as can be seen in the following graph:

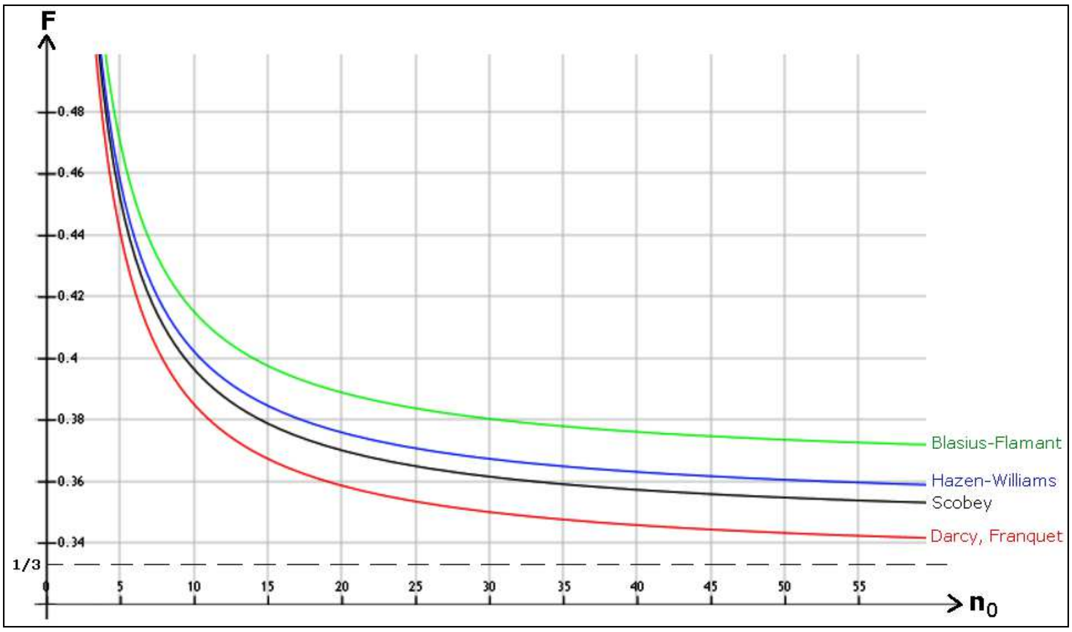

Fig. 3. Path of F according to the formulation.

The 4 functions represented in Fig. 3, respectively, are as follows:

- Blasius-Flamant, Cruciani-Margaritora (m = 1.75):

$$F = \frac{1}{2.75} + \frac{1}{2 \cdot n_0} + \frac{\sqrt{0.75}}{6 \cdot n_0^2} = 0.364 + \frac{0.5}{n_0} + \frac{0.144}{n_0^2}.$$

- Hazen-Williams, Ludin (1932), (m = 1.852):

$$F = \frac{1}{2.852} + \frac{1}{2 \cdot n_0} + \frac{\sqrt{0.852}}{6 \cdot n_0^2} = 0.351 + \frac{0.5}{n_0} + \frac{0.154}{n_0^2}.$$

- Scobey, Biegeleisen-Bukowsky, Meyer-Peter (m = 1.9):

$$F = \frac{1}{2.9} + \frac{1}{2 \cdot n_0} + \frac{\sqrt{0.9}}{6 \cdot n_0^2} = 0.345 + \frac{0.5}{n_0} + \frac{0.158}{n_0^2}.$$

- Darcy-Weisbach, Franquet, Manning-Strickler-Gaukler, Tillmann, Lèvy, Ganguillet-Kütter, Dupuit, Bazin, Mougnie, Sonier, Colombo, Catani (m = 2):

$$F = \frac{1}{3} + \frac{1}{2 \cdot n_0} + \frac{1}{6 \cdot n_0^2} = \frac{2 \cdot n_0^2 + 3 \cdot n_0 + 1}{6 \cdot n_0^2} = 0.333 + \frac{0.5}{n_0} + \frac{0.167}{n_0^2}.$$

It may be of some interest for its usefulness, in some specific cases, in addition to the above, consider other expressions such as the following:

- Scimemi (m = 1.78571):

$$F = \frac{1}{2.78571} + \frac{1}{2 \cdot n_0} + \frac{\sqrt{0.78571}}{6 \cdot n_0^2} = 0.359 + \frac{0.5}{n_0} + \frac{0.148}{n_0^2}$$

- Veronese-Datei, Stucky, Lampe (m = 1.8):

$$F = \frac{1}{2.8} + \frac{1}{2 \cdot n_0} + \frac{\sqrt{0.8}}{6 \cdot n_0^2} = 0.357 + \frac{0.5}{n_0} + \frac{0.149}{n_0^2}$$

- Wegmann-Aeryns (m = 1.856):

$$F = \frac{1}{2.856} + \frac{1}{2 \cdot n_0} + \frac{\sqrt{0.856}}{6 \cdot n_0^2} = 0.350 + \frac{0.5}{n_0} + \frac{0.154}{n_0^2}$$

- Eytelwein (1801), (m = 1.944):

$$F = \frac{1}{2.944} + \frac{1}{2 \cdot n_0} + \frac{\sqrt{0.944}}{6 \cdot n_0^2} = 0.340 + \frac{0.5}{n_0} + \frac{0.162}{n_0^2}.$$

Thus, it is observed that when $n_0 \to \infty$, the value of the Christiansen coefficient takes on values $F \in [0.333, 0.364]$ according to the empirical formula used to calculate the corresponding head losses in the water pipes. For higher viscosity liquid conduits (vegetable oils, petroleum and derivatives ...), as in the aforementioned expression by Carothers (with m = 1.50), F = 0.400 will result.

4. Generalization to a pipeline that distributes an evenly distributed flow

Be it now an AC pipe of length **L** and diameter **D**, which has its origin in a pumping group or in a water tank raised on the ground as the one in Fig. 1, with several close and uniformly spaced lateral intakes, for which are derived identical expenses.

When in a conduction of these characteristics, the number of branches is sufficiently large (typical, i. e., in localized high-frequency irrigation systems, such as micro-sprinkling, exudation and dripping in its different modalities), the calculation, with great and acceptable approximation, assuming that an expense is distributed uniformly distributed along the path, which is obtained by adding all the flows of the derivations and dividing this sum by the total length of the pipe or distance: L = AC. This flow thus obtained is called "flow per unit length" of pipe.

In these cases, the movement of the water through the pipe can be assimilated to a succession of infinitesimal uniform movements of variable law with the flow -or with the section of the line if it is not constant- due to the proximity of the changes and the small variation of the flow that takes place as a consequence of them. Although it would be necessary, for the faultless resolution of the problem, the exact knowledge of said law of flow variation, we could admit, with a good approximation, that the service on the route is uniformly distributed throughout the length of the pipe, reducing the flow by a certain quantity **q** per unit length of pipe. It would be the paradigmatic case of a tape of underground irrigation by exudation. **In other words, a flow q per unit length of the pipe is used up or consumed.**

Now using the following notation:

$$\begin{cases} \text{Q = expenditure at the origin A of the pipe.} \\ \text{q = derived expense per unit length of pipe.} \\ Q_x \text{ = expenditure available at a generic point of the pipe located at a} \\ \text{distance from the origin x.} \end{cases}$$

Obviously, it will be verified that:

$$Q_x = Q - q \cdot x \tag{12}$$

where $(q \cdot x)$ is the cost distributed on the path that runs from A to x, whereby the initial flow: $Q_0 = Q$.

Let us now express the continuous pressure loss due to friction in the pipe section from point A to x, using the formula:

$$z = n\int_0^x Q_x^2 \cdot dx = n\int_0^x (Q - q\cdot x)^2 \cdot dx = n\int_0^x (Q^2 - 2q\cdot x\cdot Q + q^2\cdot x^2)\cdot dx = n\left[\frac{q^2x^3}{3} - qQx^2 + Q^2x\right]_0^x$$

$\forall$ x such that $x \in [0,L]$.

The integration constant is null, since for x = 0, also: z = 0 $\Rightarrow$ c = 0; that is:

$$z = n(Q^2\cdot x - q\cdot Q\cdot x^2 + \frac{1}{3}q^2\cdot x^3) = n\left[(Q_x + q\cdot x)^2\cdot x - q(Q_x + q\cdot x)x^2 + \frac{1}{3}q^2\cdot x^3)\right], \text{ that is:}$$

$$z = n(Q_x^2\cdot x + Q_x\cdot q\cdot x^2 + \frac{1}{3}q^2\cdot x^3) \qquad (13)$$

which is the equation of a cubic parabola or polynomial function representative of the line of piezometric levels.

If we call Q 'the residual or extremal flow that comes out of the end C of the pipe we will have, according to equation (13), we will have the following continuous total head loss:

$$H = n(Q'^2\cdot L + Q'\cdot q\cdot L^2 + \frac{1}{3}q^2\cdot L^3) \qquad (14)$$

Now, if the end C of the pipe is a dead point, that is, if all the flow is derived and consumed along the pipeline without any residual flow reaching point C, it will obviously be that:

$Q' = Q_{x=L} = 0$, and therefore, in (12) we will have: $Q = q \cdot L$,

and substituting these values in equation (14), we will obtain:

$$H = \frac{1}{3}n\cdot q^2\cdot L^3 = \frac{1}{3}n\cdot (q\cdot L)^2\cdot L = 0.333\cdot n\cdot Q^2\cdot L \qquad (15)$$

expression that tells us that the continuous head loss that takes place is the third part of that which would occur if the cost Q traveled all the pipeline and left freely at the C end of it, as has already been stated in the epigraph previous. This would therefore be equivalent to a Christiansen reducing coefficient F = 0.333, when n_0 tends to ∞ and m = 2.00.

The previous equation (15) can also be expressed like this:

$$H = \frac{1}{3}n\cdot Q^2\cdot L = n\cdot (Q_1)^2\cdot L \text{ , from where it results:}$$

$$Q_1 = \frac{Q}{\sqrt{3}} = 0.577 \cdot Q \quad (16)$$

which means that the continuous head loss is equivalent to the one that would occur if a constant flow Q_1 circulated through the pipe and equal to 57.7% of the initial flow Q.

A simple example of the above would be the case of a watering tape[2] with self-compensating integrated drippers spaced every 20 cm of flow rate 1l/h, which would imply obtaining a unit flow rate (per unit length of pipe) of 5 l/h·m, that is, if the branch has a total length of L = 100 m, it has (Fig. 4):

$$q = \frac{Q}{L} = \frac{500\,l/h}{100\,m} = 5\,l/h{\cdot}m\,.$$

In this way, the linear equation (12) will be expressed as follows:

$Q_x = 500 - 5x$, $Q_0 = Q = 500$ l/h, with the following graphic representation:

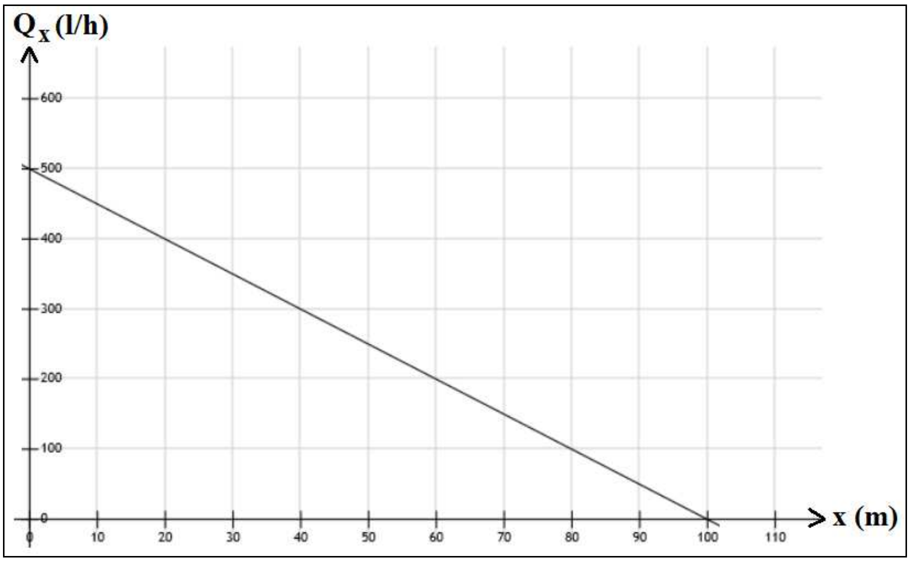

Fig. 4. Graph (Q_x - x) on an irrigation tape.

5. Duct sizing

Next, we will present the procedure used to determine the suitable diameter, so that the pipeline can distribute the cost evenly distributed in the manner indicated above.

[2] This irrigation system, which can also be provided with drippers in turbulent regime, should not be confused with the exudation tape, which constitutes another hydraulic system, without dripping and frequently used in underground irrigation, which applies water continuously through a porous tube that exudes water evenly throughout its path.

Equation (14) is equivalent to the formulation:

$$H = n(Q'^2 + Q' \cdot q \cdot L + \frac{1}{3} q^2 \cdot L^2) \cdot L = n \cdot Q_1^2 \cdot L = J_1 \cdot L \,;$$

introducing a "dummy flow" Q_1 that when circulating through the pipe in a constant way produces a continuous pressure drop H. Thus:

$$Q_1^2 = Q'^2 + Q' \cdot q \cdot L + \frac{1}{3} q^2 \cdot L^2 \tag{17}$$

but if we consider that:

$$(Q' + \frac{1}{2} q \cdot L)^2 = Q'^2 + Q' \cdot q \cdot L + \frac{1}{4} q^2 \cdot L^2 < Q_1^2$$
$$(Q' + \frac{1}{\sqrt{3}} q \cdot L)^2 = Q'^2 + \frac{2}{\sqrt{3}} Q' \cdot q \cdot L + \frac{1}{3} q^2 \cdot L^2 > Q_1^2$$

it turns out that the value of Q_1 is bounded between the limits:

$$Q' + \frac{1}{2} q \cdot L < Q_1 < Q' + \frac{1}{\sqrt{3}} q \cdot L \text{, or what is the same:}$$

$$Q' + 0.5 \cdot q \cdot L < Q_1 < Q' + 0.577 \cdot q \cdot L \,,$$

then it can be taken with enough approximation, as Q_1 value, to calculate the internal diameter of the pipe:

$$\boxed{Q_1 = Q' + 0.55\, q \cdot L} \tag{18}$$

, which is the formula that is usually used for the design of agricultural, industrial and domestic water supply networks (Torres, 1970).

Knowing the values of Q_1 and J_1 = H/L, the value of D and S is easily found. Specifically, in the previously proposed example of a plastic pipe, we will have, according to the formulation of the author of this article for pipes in service (Franquet, 2005), which:

$$Q_1 = 28.82 \times D^{2.6215} \times \sqrt{J_1} \text{ , from where: } D = \sqrt[2.6215]{\frac{Q_1}{28.82 \times \sqrt{J_1}}} \,.$$

In the same way:

$$S = \frac{\pi \times D^2}{4} = \frac{\pi}{4} \times \sqrt[1.3108]{\frac{Q_1}{28.82 \times \sqrt{J_1}}} \tag{19}$$

the previous formulas being expressed in the IS's own units[3], that is:

$$Q\ (m^3/s),\ V\ (m/s),\ D\ (m),\ L\ (m),\ H\ (m),\ S\ (m^2) \text{ and } J\ (m/m).$$

If no flow reaches point C (whereby: Q' = 0), the following will be taken, as we have shown as the value of Q_1 (16):

$$Q_1 = \frac{1}{\sqrt{3}} q \cdot L \ \ ; \text{ or what is the same:}$$

$$\boxed{Q_1 = 0.577 \cdot q \cdot L \approx 58\% \text{ de } Q} \qquad (20)$$

6. Resolution of two practical cases

6.1. Pipeline with continuous loss of flow

The following exercise, taken from Hernández and Crespo (1996), pp. 147 and next, is illustrative enough to contrast some of the concepts expressed here.

Statement:

The feed flow of a water pipe, diameter D = 10 cm and length L = 5 km, is Q_0 = 10 l/s. Due to the numerous water intakes that exist along the pipe, it can be estimated that its flow decreases uniformly by q = 6 l/h per meter of pipe length. Calculate the pressure loss along the pipeline as well as study the hydraulic regime and the convenience of choosing the most suitable material constituting it. Suppose a constant friction factor of value: f = 0.020.

Solution:

In an elementary section of pipe of length *dl* there will be a height loss, regardless of local losses, according to the general or universal Darcy-Weisbach equation, of:

$$dH_\phi = f \frac{V^2}{2g \times} \cdot \frac{1}{D} dl\,.$$

[3] The *International System of Units*, abbreviated IS, also called the *International System of Measurements*, is the heir to the old decimal metric system. One of the main characteristics of the International Measurement System is that its units are based on fundamental physical phenomena. The only exception to this is the definition of the unit of magnitude Mass, the kilogram, which is defined as the mass of an international prototype of the kilogram that is stored at the International Bureau of Weights and Measures, in Sèvres, France. SI units are the international reference for the indications of all measuring instruments, and to which they are referred through an uninterrupted chain of calibrations or comparisons.

According to the data of the previous statement, the variation of the flow along the pipeline (with en-route service) is given by:

$$Q = Q_0 - q\cdot l, \text{ being: } q = \frac{0.006}{3600} = 1.667\times10^{-6}\,m^3s^{-1}m^{-1}\ .$$

The speed of the water will be: $V = \frac{4\cdot Q}{\pi\cdot D^2} = \frac{4}{\pi\cdot D^2}\left(Q_0 - q\cdot l\right)$.

Substituting this expression in the initial equation and integrating, it results:

$$H_\phi = \frac{8f}{g\pi^2 D^5}\int_0^L \left(Q_0 - q\cdot l\right)^2 dl\ .$$

Solving the integral and substituting values, we finally obtain:

$$H_\phi = \frac{8f}{g\pi^2 D^5}\left(Q_0^2 L + q^2\frac{L^3}{3} - 2Q_0\cdot q\frac{L^2}{2}\right) = \frac{(8)(0.02)}{(9.807)\pi^2(0.1)^5}\Big[(0.01)^2(5000) +$$
$$+\left(1.667\times10^{-6}\right)^2\frac{(5000)^3}{3} - (0.01)\left(1.667\times10^{-6}\right)(5000)^2\Big] = 32.93\,\text{m.c.a.}$$

In short, the problem data is as follows:

q = 6 l/h·m = 0.006 m^3/h·m; D = 10 cm = 0.10 m; L = 5000 m.

Flow at the start of the pipe:

Q_0 = 10 l/s ≡ 36000 l/h = 0.01 m^3/s.

Flow expense along the pipeline:

Q = 5000 m × 6 l/h·m = 30000 l/h ≡ 30 m^3/h, and a residual or extrem flow:

Q' = 36000 l/h – 30000 l/h = 6000 l/h = 6 m^3/h = 0.0017 m^3/s.

Thus, according to expression (14), there will be a total head loss of:

$$H_\phi = J\times L = n\left(Q'^2\times L + Q'\times q\times L^2 + \frac{1}{3}\times q^2\times L^3\right), \text{ being:}$$

$$n = \frac{8\cdot f}{g\cdot\pi^2\times D^5} = \frac{8\times0.02}{g\times\pi^2\times0.1^5} = 165.25294\text{; and so:}$$

$$H_\phi = 165.25294\times\left(\overbrace{0.0017^2\times5000}^{0.01445}+\overbrace{0.0017\times1.667\times10^{-6}\times5000^2}^{0.0708475}+\right.$$

$$\left.+\underbrace{\frac{\left(1.667\times10^{-6}\right)^2\times5000^3}{3}}_{0.115787}\right)=33.23$$ m.w.c., which turns out to be practically identical to that obtained by the previous formulation. Considering, therefore, a total head loss of 33 m.w.c., there will be a unit head loss of:

$$J=\frac{H_\phi}{L}=\frac{33}{5000}=0.0066 \text{ m/ml.}$$

Since the flow decreases uniformly along the pipeline (road service), there would be an average flow of:

$$Q_m=\frac{Q_0+Q'}{2}=\frac{0.01+0.0017}{2}=0.00585 \text{ m}^3/\text{s},$$

which implies an average speed along it of:

$$V_m=\frac{Q_m}{S}=\frac{4\times Q_m}{\pi\times D^2}=\frac{4\times0.00585}{\pi\times0.1^2}=0.745 \text{ m/s.}$$

We now test, based on our proposals, looking for the corresponding pressure loss according to the various categories of roughness of the pipes of possible installation. For this, it will be necessary to consider that the proposed pipe is new and subjected to normal wear, which will have a relative coefficient of friction α_2 of value (Franquet, 2019):

$$\left[\alpha_2=\frac{0.50+\sqrt{D}}{0.30+\sqrt{D}}=\frac{0.50+\sqrt{0.1}}{0.30+\sqrt{0.1}}=\frac{0.816}{0.616}\approx1.3247\right],$$

(which can be verified in table 1.3 of Annex nº: 3 of the Appendix of the related manual, with a value: α_2 = 1.324555), and if it is tested with a degree of roughness of k = 1.5, it is taken for the expression of the continuous unit pressure drop proposed here (see Chap. 4, table 4.10 of the above manual) a value of:

$$J=\frac{0.000845\times V_m^2}{\alpha_2\times D^{1.256}}=\frac{0.000845\times0.745^2}{1.324555\times0.1^{1.256}}=0.0064\cong0.0066 \text{ m/m,}$$

reason why it would be a pipe of intermediate roughness between the plastic ones (k = 1) and the fiber or asbestos cement ones (k = 2). In any case, the strict commercial internal diameter of 100 mm only exists for tubes of the latter material (FIB).

Likewise, let's see that the hydraulic regime is variable with the descending speed that takes place along the tube, although for an assumed average water temperature of 20º C, with a corresponding kinematic viscosity of: $\nu = 1.0164 \times 10^{-6}$ m^2/s (1 m^2/s = 104 stokes), which is deduced from the application of our nonlinear quadratic minimum adjustment (Franquet, 2003):

$$\nu = (1.7224 - 0.0461 \cdot t + 0.0006 \cdot t^2 - 0.000003 \cdot t^3) \times 10^{-6} =$$
$$= (1.7224 - 0.922 + 0.24 - 0.024) \times 10^{-6} = 1.0164 \times 10^{-6}\ m^2/s = 1.0164\ cSt\ ,$$

and an average Reynolds number of:

$$Re = \frac{V_m \times D}{\nu} = \frac{0.745 \times 0.1}{1.0164 \times 10^{-6}} = 73298\ ,$$

which corresponds, effectively, to a smooth turbulent hydraulic regime ($4 \times 10^3 < Re < 10^5$). In such case, the following values of the coefficient of friction would be based on the formulations generally used in the case and our own (Franquet, 2019):

- According to Blasius: $f = \frac{0.3164}{Re^{0.25}} = \frac{0.3164}{73298^{0.25}} = 0.0192$
- **According to Franquet**: $f = \frac{1.32556}{(0.875 \times \ln Re - 1.495647)^2} = 0.0192$
- According to Kozeny: $f = \frac{2g}{(7.78 \times \log Re - 5.95)^2} = 0.0193$
- According to Colebrook – White: $\frac{1}{\sqrt{f}} = 2 \cdot \log(Re\sqrt{f}) - 0.8$, or better still in its explicit approach: $f = \frac{0.835}{(\log Re)^{2.38}}$ = 0.0193,

being the four previous determinations very close, by default, to the initially assumed of f = 0.020, given in the statement of the problem posed, so it is considered completely acceptable.

The study of the hydraulic regime of the pipe in question would require the calculation of the Re number in each place of it. For this, we will prepare the following table and the graph corresponding to the following 6 values:

$$\left.\begin{array}{l}
Q_0 \rightarrow 0.0100\ m^3/s \rightarrow V_0 = \dfrac{4\times 0.01}{\pi\times 0.1^2} = 1.27\ m/s \rightarrow Re = 124951 \\
Q_1 \rightarrow 0.0083\ m^3/s \rightarrow V_1 = \dfrac{4\times 0.0083}{\pi\times 0.1^2} = 1.06\ m/s \rightarrow Re = 104290
\end{array}\right\}\ \boxed{\text{R.T.I.}}$$

$$\left.\begin{array}{l}
Q_2 \rightarrow 0.0067\ m^3/s \rightarrow V_2 = \dfrac{4\times 0.0067}{\pi\times 0.1^2} = 0.85\ m/s \rightarrow Re = 83628 \\
Q_3 \rightarrow 0.0050\ m^3/s \rightarrow V_3 = \dfrac{4\times 0.005}{\pi\times 0.1^2} = 0.64\ m/s \rightarrow Re = 62967 \\
Q_4 \rightarrow 0.0033\ m^3/s \rightarrow V_4 = \dfrac{4\times 0.0033}{\pi\times 0.1^2} = 0.42\ m/s \rightarrow Re = 41322 \\
Q_5 \rightarrow 0.0017\ m^3/s \rightarrow V_5 = \dfrac{4\times 0.0017}{\pi\times 0.1^2} = 0.21\ m/s \rightarrow Re = 20661
\end{array}\right\}\ \boxed{\text{R.T.L}}$$

, being Q_5 = Q', R.T.I. (Intermediate Turbulent Regime) and R.T.L. (Smooth Turbulent Regime).

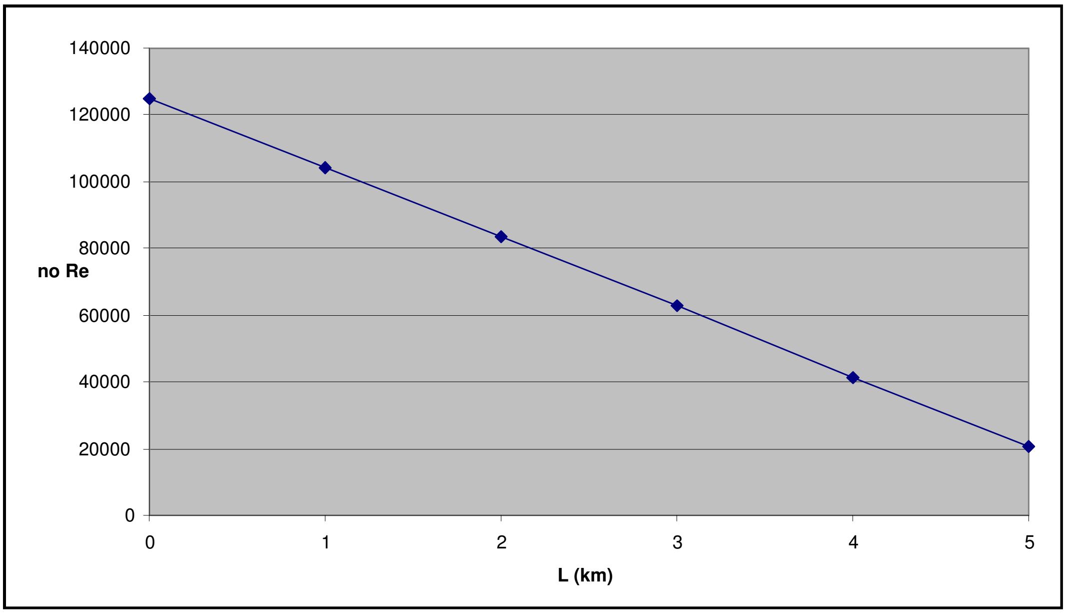

Fig. 5. Function Re = f (L).

Similarly, the three-dimensional function of the type: Re = f (t, L) could also be studied.

Let us now see that, as a consequence of the corresponding tables of reduction of the bearing capacity (see Appendix of the aforementioned manual of Franquet, tables 2.5. And 2.6. Of Annex nº: 3) for each constituent material of both types of pipe, we will have to, In the case of the asbestos cement pipe, the temporal path of the flow considering a useful life of tu = 50 years will be:

Table 3. Evolution of flow loss (asbestos cement).

t (years)	Q_0	Q'	q·l	$\Delta = Q_0 - Q_t$
0	10.00	1.67	8.33	0
5	10.00	1.67	8.33	0
10	10.00	1.67	8.33	0
15	10.00	1.67	8.33	0
20	9.42	1.57	7.85	0.58
25	8.86	1.48	7.38	1.14
30	8.30	1.38	6.92	1.70
35	7.74	1.29	6.45	2.26
40	7.17	1.20	5.97	2.83
45	6.61	1.10	5.51	3.39
50	6.05	1.01	5.04	3.95

Source: self made.

, to which the following graph corresponds:

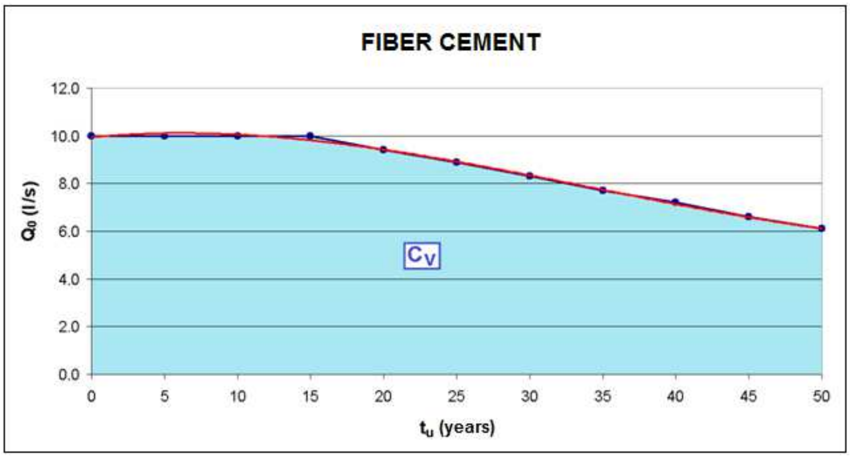

Fig. 6. Aging function and C_v (fiber-asbestos cement).

, with a volumetric capacity (Franquet, 2019) of:

$$C_{v1} \cong 10.00 \times 15 + \frac{10.00 + 6.05}{2} \times 35 = 430.875,$$

and an average flow in the analyzed period of: $Q_m = \frac{430.875}{50} = 8.6175 \, l/s.$

This implies a real volumetric capacity of this pipe of:

C_{v1} = (8.6175/ l/s × 50 years × 365.25 days/year × 24 h/day × 3600 s/h)/10^9 ≡ ≡ 13.60 hm^3 of water. The volumetric efficiency of this pipe (Franquet, 2019) will be given for the expression:

$$E_{v1} = \frac{C_{v1}}{Q_0 \times t_u} \times 100 = \frac{430.875}{10 \times 50} \times 100 = 86.18\% .$$

On the other hand, in the case of the plastic pipe, there will be:

Table 4. Evolution of flow loss (plastic).

t (years)	Q_0	Q'	q·l	$\Delta = Q_0 - Q_t$
0	10.00	1.67	8.33	0
5	10.00	1.67	8.33	0
10	10.00	1.67	8.33	0
15	10.00	1.67	8.33	0
20	10.00	1.67	8.33	0
25	9.49	1.58	7.91	0.51
30	8.89	1.48	7.41	1.11
35	8.29	1.38	6.91	1.71
40	7.69	1.28	6.41	2.31
45	7.08	1.18	5.90	2.92
50	6.48	1.08	5.40	3.52

Source: self made.

, to which the following graph corresponds:

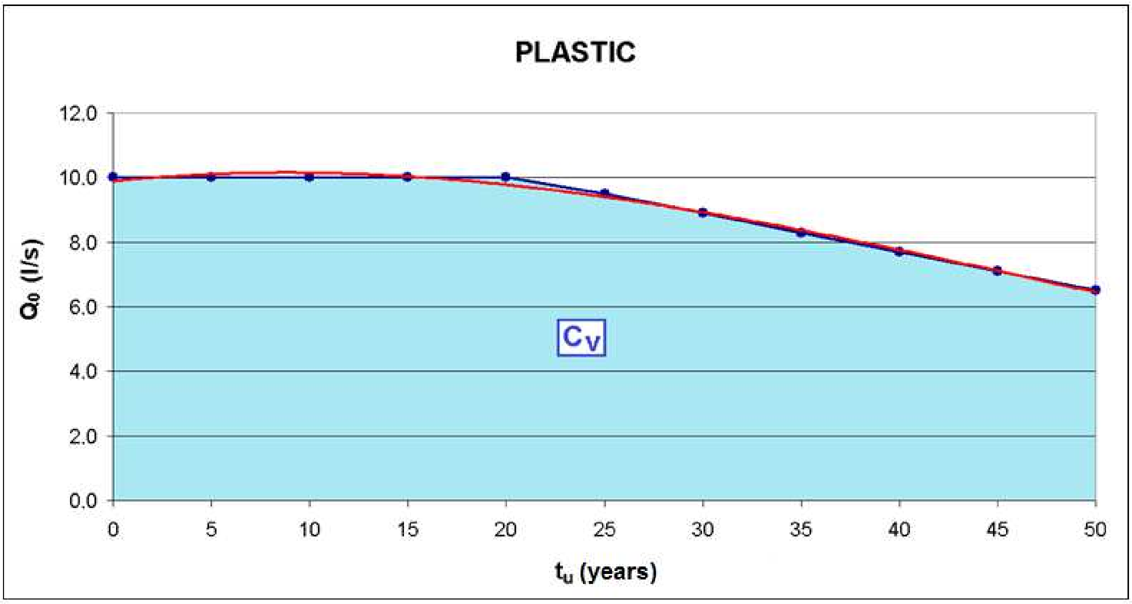

Fig. 7. Aging function and C_v (plastic).

, with a volumetric capacity of:

$$C_{v2} \cong 10.00 \times 20 + \frac{10.00+6.48}{2}\times 30 = 447.20,$$

and an average flow in the analyzed period of: $Q_m = \frac{447.2}{50} = 8.944 \, l/s.$

This implies a real volumetric capacity of this pipe of:

$$C_{v2} = (8.944/ \text{ l/s} \times 50 \text{ years} \times 365.25 \text{ days/year} \times 24 \text{ h/day} \times 3600 \text{ s/h})/10^9 \equiv$$
$$\equiv 14.11 \text{ hm}^3 \text{ of water.}$$

The volumetric efficiency of this pipe will be given by:

$$E_{v2} = \frac{C_{v2}}{Q_0 \times t_u}\times 100 = \frac{447.2}{10\times 50}\times 100 = 89.44\% \,.$$

The difference between the volumetric efficiencies of both pipes turns out to be 3.26% in favor of the plastic one.

On the other hand, the difference between the actual volumetric capacities of both aging functions will be as follows:

$$\Delta(q_1,q_2) = | C_{v1} - C_{v2} | = | 13.60 - 14.11 | = 0.51 \text{ hm}^3 = 510000 \text{ m}^3 \text{ of water.}$$

As expected, in this case the volumetric performance of the second possible pipe (plastic) is higher than that of the first (asbestos-cement) in a: $\frac{14.11-13.60}{13.60}\times 100 = 3.75\%$, which makes it somewhat more advisable for the design of the installation, although this Calculation, as also stated before, should be completed with the intervention of sanitary and economic cost factors, both in terms of initial installation and subsequent maintenance, before making a final decision.

Finally, let us see that a "concentration measure" of the hydraulic variable under study (volumetric capacity C_v) is the Gini index and the corresponding polygonal Lorenz curve, as well as the "Lorenz index", the meaning and operation of which they are specified in Franquet (2019). The lower the value of said indices, the better and more uniform the volumetric capacity of the pipe will be throughout its useful life, which is why we consider its determination highly interesting, as it is also for comparative purposes with other types of pipe. interested in installing in the project or execution phase of the work.

Based on this, and taking into account the choice made here between the choice of plastic and asbestos-cement pipes, we must prepare the following tables:

Table 5. Calculation aid (fiber-asbestos cement).

	t_i	Q_0	C_v	ΣC_v	$\frac{t_i}{\sum t}\times 100$	$p_i(\%)$	$\frac{C_v}{\sum C_v}\times 100$	$q_i(\%)$	$\lvert p_i - q_i \rvert$
	0	10.00	0	0	0	0	0	0	0
	5	10.00	50.00	50.00	10	10	11.61	11.61	1.61
	10	10.00	50.00	100.00	10	20	11.61	23.22	3.22
	15	10.00	50.00	150.00	10	30	11.61	34.83	4.83
	20	9.42	48.55	198.55	10	40	11.27	46.10	6.10
	25	8.86	45.70	244.25	10	50	10.61	56.71	6.71
	30	8.30	42.90	287.15	10	60	9.96	66.67	6.67
	35	7.74	40.10	327.25	10	70	9.31	75.98	5.98
	40	7.17	37.28	364.53	10	80	8.67	84.65	4.65
	45	6.61	34.45	398.98	10	90	8.00	92.65	2.65
	50	6.05	31.65	430.63	10	100	7.35	100	0
Σ	275	-	430.63	≅ 430.875	100 %	550	100 %	592.42	42.42

Source: self made.

Thus, according to the formulation given by Pulido (Alcaide, 1973), the value of the Gini index, in this case, will be:

$$G_1 = \frac{\sum_{i=1}^{K-1}(p_i - q_i)}{\sum_{i=1}^{K-1} p_i} = \frac{42.42}{450} = 0.0943 \equiv 9.43\ \%,$$

with its corresponding graphic representation:

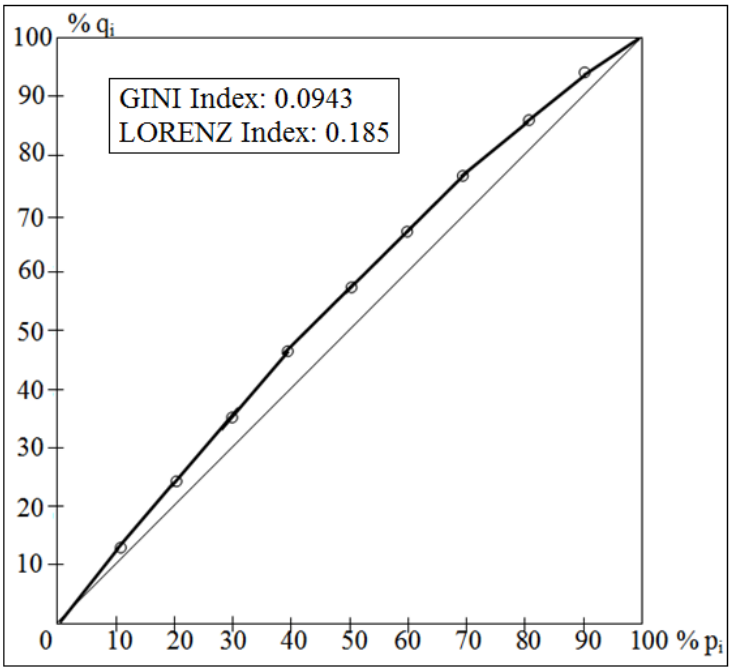

Fig. 8. Lorenz curve (fiber-asbestos cement).

Table 6. Calculation aid (plastic).

t_i	Q_0	C_v	ΣC_v	$\frac{t_i}{\sum t}\times 100$	$p_i(\%)$	$\frac{C_v}{\sum C_v}\times 100$	$q_i(\%)$	$\lvert p_i - q_i \rvert$
0	10.00	0	0	0	0	0	0	0
5	10.00	50.00	50.00	10	10	11.15	11.15	1.15
10	10.00	50.00	100.00	10	20	11.15	22.30	2.30
15	10.00	50.00	150.00	10	30	11.15	33.45	3.45
20	10.00	50.00	200.00	10	40	11.15	44.60	4.60
25	9.49	48.73	248.73	10	50	10.86	55.46	5.46
30	8.89	45.95	294.68	10	60	10.25	65.71	5.71
35	8.29	42.95	337.63	10	70	9.58	75.29	5.29
40	7.69	39.95	377.58	10	80	8.91	84.20	4.20
45	7.08	36.93	414.51	10	90	8.24	92.44	2.44
50	6.48	33.90	448.41	10	100	7.56	100	0
Σ 275	-	448.41	≅ 430.875	100 %	550	100 %	584.60	34.60

Source: self made.

Thus, according to the formulation given by Pulido (Alcaide, 1973), the value of the Gini index, in this case, will be:

$$G_2 = \frac{\sum_{i=1}^{K-1}(p_i - q_i)}{\sum_{i=1}^{K-1} p_i} = \frac{34.60}{450} = 0.0769 \equiv 7.69\ \%\text{, corresponding to the graph:}$$

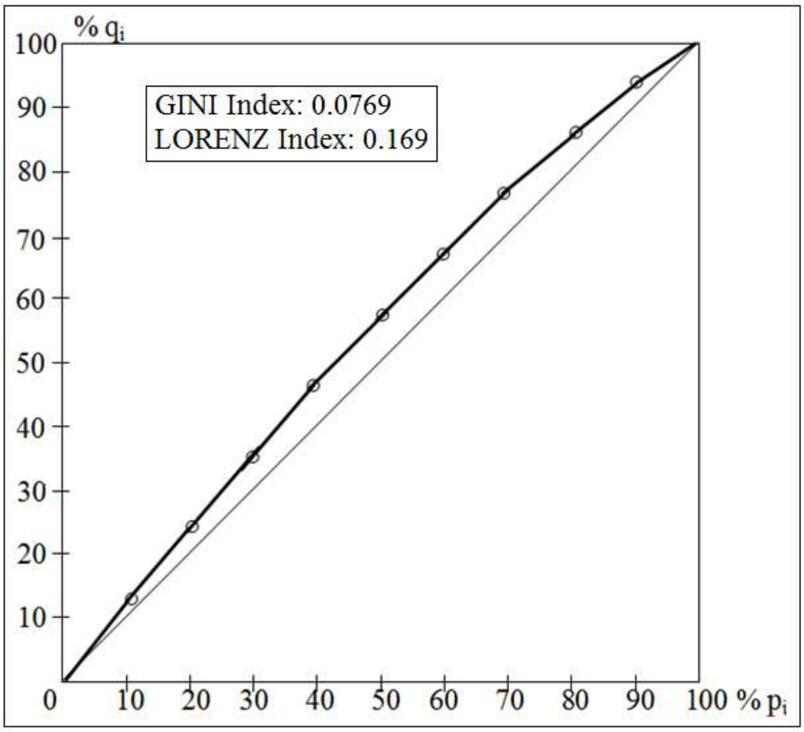

Fig. 9. Lorenz curve (plastic).

This shows the more uniform distribution of the C_v values over time of its useful life in the case of plastic pipe (as expected), since: $G_2 < G_1$.

Another index of application to the case (Franquet, 2019) is the so-called "Lorenz concentration index", which is obtained by applying the formula based on the accumulated percentages, which is commonly used in practical work in Economics, also related in the next chapter of this same work. Here, with n = 11 and q_n = 100, it will happen that:

$$L = 1 - \frac{2}{n-1} \times \frac{\sum_{i=0}^{n-1} q_i}{q_n} = 1 - \frac{2}{10} \times \frac{\sum_{i=0}^{10} q_i}{100} = 1 - \frac{\sum_{i=0}^{10} q_i}{500} .$$

The result that the application of the previous formula offers is the following, bearing in mind that it is necessary to order the values of the hydraulic variable under study (C_v) from smallest to largest, for the correct application of the formula, thus:

Table 7. Calculation assistant (Lorenz index).

	ASBESTOS CEMENT		**PLASTIC**	
	X_i	**q_i**	**X_i**	**q_i**
	0	0	0	0
	7.35	7.35	7.56	7.56
	8.00	15.35	8.24	15.80
	8.67	24.02	8.91	24.71
	9.31	33.33	9.58	34.29
	9.96	43.29	10.25	44.54
	10.61	53.90	10.86	55.40
	11.27	65.17	11.15	66.55
	11.61	76.78	11.15	77.70
	11.61	88.39	11.15	88.85
	11.61	100.00	11.15	100.00
Σ	100.00	507.58	100.00	515.40

Source: self made.

The following Lorenz indices correspond to these probability distributions, respectively:

$$\begin{cases} L_F = 1 - \dfrac{407.58}{500} = 0.185 \text{ (asbestos cement)} \\ L_P = 1 - \dfrac{415.40}{500} = 0.169 \text{ (plastic)} \end{cases}$$

whose results ($L_P < L_F$) confirm the previously made determinations on the degree of concentration of the variable C_v under study for each alternative pipeline[4].

6.2. Pipeline with discrete flow loss

The following exercise, taken from Cabrera et alt. (1996), vol. II, p. 713 et seq., We believe it is illustrative to contrast some of the concepts expressed here.

Statement:

It is requested to design, from an economic point of view, the system of three pipes in series of the following figure for the flows indicated in it, which constitutes a general pipeline, knowing that the minimum service pressure required at the end is 23 mwc (mca), assuming a constant friction factor[5] f = 0.015, common to all pipes.

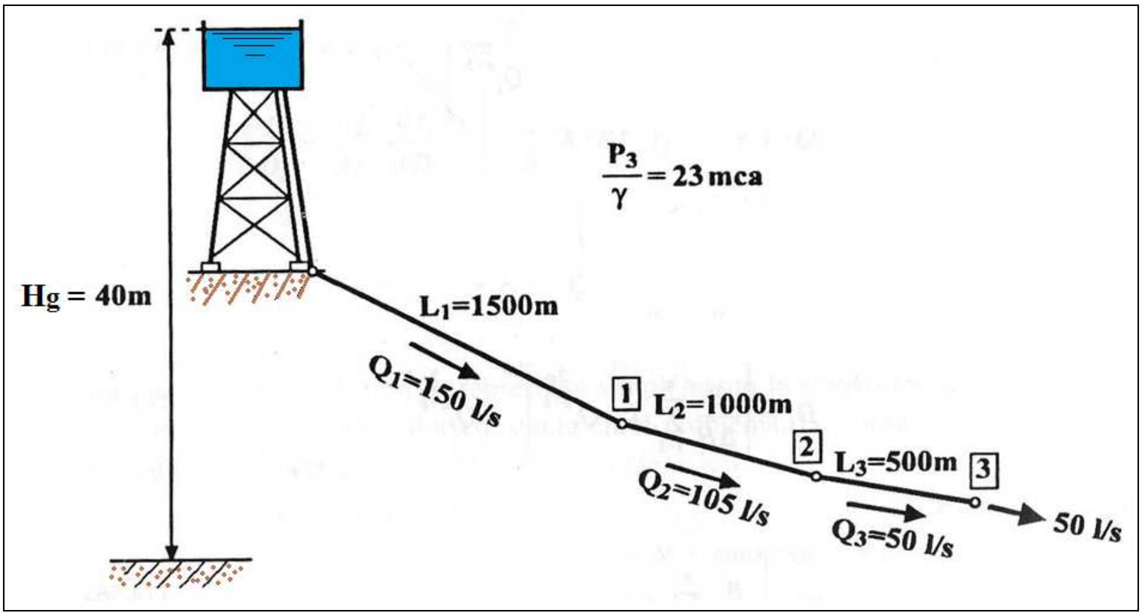

Fig. 10. Series of pipes to be dimensioned.

Solution:

First, we will separately calculate each of the factors that appear in the expression for constant K. Consider the following price table for the installed pipeline and determine the exponent a of the cost equation. The corresponding cost equation must be established for each type of pipe. Thus, the following table of unit prices of the installed PVC pipe with a ring pressure of 1 MPa (10 bar),

---

[4] In the present exercise it has been taken into account that the internal diameters of the tubes are not very sensitive to small variations in the friction factor f, in view of the Moody diagram (1947). Therefore, it is possible to take an intermediate value for all this service pipeline en route, which, according to the statement, is f = 0.020.

[5] The same can be said in this second exercise, in which a value of f = 0.015 is considered.

included in Fig. 11, has been determined for the supply of drinking water, with its corresponding layout:

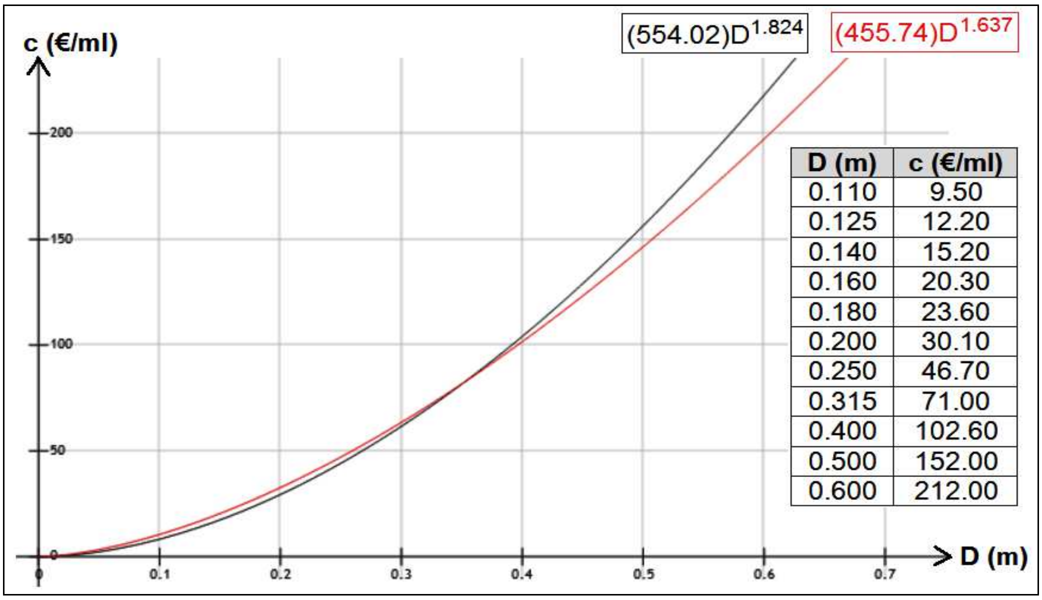

Fig. 11. Graphical representations of the functions c(D).

having obtained, in this case, the potential equation of costs for non-linear least square regression: c (€/ml) = $554.02 \times D^{1.824}$ (DN 600 mm is an average between commercial DN 560 mm and DN 630 mm) with a non-linear correlation coefficient r = 0.999432, very high, so both correlations can be considered practically perfect.

In effect, the study of another more general case has led to the following determination, which coincides closely with the previous one:

D (m)	0.150	0.175	0.200	0.250	0.300	0.350	0.400
c (ptas/ml)	3392	4352	5546	7712	10670	13152	17342
c (€/ml)	20.39	26.16	33.33	46.35	64.13	79.05	104.23

so that the cost equation adjusted for non-linear (potential) regression is:

$$c = 75829 \times D^{1.637} \text{ ptas/ml, or: } c = 455.74 \times D^{1.637} \text{ €/ml},$$

with the correlation coefficient r = 0.999427 (very high, like the previous one) from which it can also be deduced that the value: a = 1.637 > 1, so the larger diameters are relatively more expensive. Both cases have been represented in Fig. 11.

On the other hand, the loss equation has:

$$B=\frac{8\cdot f}{\pi^2 g}=\frac{8\times 0.015}{\pi^2\times 9.807}=0.00124 \quad ; \quad b=5 \quad ;$$

while the maximum allowed loss is: $\Delta H = H_g - \frac{p_3}{\gamma} = 40-23=17$ m.w.c.

Finally we obtain the summation value:

$$\frac{2\cdot a}{a+b}=\frac{2\cdot 1.64}{1.64+5}=0.5\text{, and so:}$$

$$\sum_j L_j Q_j^{2a/(a+b)}=1500\cdot 0.15^{0.5}+1000\cdot 0.105^{0.5}+500\cdot 0.05^{0.5}=1017.$$

Substituting now all this in the expression K, it turns out:

$$K=\left(\frac{B}{\Delta H}\sum_j L_j\cdot Q_j^{2a/(a+b)}\right)^{1/b}=\left(\frac{0.00124}{17}\times 1017\right)^{1/5}=0.594\,,$$

with which we can proceed to carry out the dimensioning. To the right of each calculated diameter is the final normalized trade diameter.

$$\frac{2}{a+b}=\frac{2}{1.64+5}\approx 0.30\text{, and then as: } D_j = K\times Q_j^{2/(a+b)}\text{, we have to:}$$

$$\begin{cases} D_1 = K\cdot Q_1^{0.3} = 0.594\cdot 0.15^{0.3} = 0.336\,\text{m} \Rightarrow D_1 = 350\,\text{mm.} \\ D_2 = K\cdot Q_2^{0.3} = 0.594\cdot 0.105^{0.3} = 0.302\,\text{m} \Rightarrow D_2 = 300\,\text{mm.} \\ D_3 = K\cdot Q_3^{0.3} = 0.594\cdot 0.05^{0'3} = 0.242\,\text{m} \Rightarrow D_3 = 250\,\text{mm.} \end{cases}$$

To conclude, let's check the actual pressure drop in the conduction corresponding to the standard diameters:

$$\Delta H=\sum_i R_i L_i Q_i^2 = B\sum_i L_i D_i^{-5} Q_i^2 = 0.00124\big(1500\cdot 0.35^{-5}\cdot 0.150^2 + \\ +1000\cdot 0.3^{-5}\cdot 0.105^2+500\cdot 0.25^{-5}\cdot 0.05^2\big)=15.18\text{ m.w.c.,}$$

which gives us a residual pressure of real service:

$$\frac{p_3^1}{\gamma}=H_g-\Delta H^1 = 40-15.18=24.82\text{ m.w.c.} > 23\text{ m.w.c.,}$$

which is slightly higher than required, which effectively solves the problem posed.

Another way to solve it in practice, regardless of the unit cost of the pipeline, would be the following, considering the calculation for PVC pipeline in service, with 15% of accidental pressure losses and a stamping of 6 bar. Applying the conservative formula or criterion of Weyrauch (1915) to each section, the following would happen:

$$\begin{cases} \text{Section 1} \rightarrow D_1 = 1.04 \times \sqrt{Q_1} = 1.04 \times \sqrt{0.15} = 0.403\text{ m } (400 \times 11.7\text{ mm}) \\ \text{Section 2} \rightarrow D_2 = 1.04 \times \sqrt{Q_2} = 1.04 \times \sqrt{0.105} = 0.337\text{ m } (355 \times 10.4\text{ mm}) \\ \text{Section 3} \rightarrow D_3 = 1.04 \times \sqrt{Q_3} = 1.04 \times \sqrt{0.05} = 0.233\text{ m } (250 \times 7.3\text{ mm}) \end{cases}$$

, and pipes of similar internal diameters result, although not exactly the same (as they are somewhat larger), than those deduced from the calculation carried out first. The speed of each section will be:

$$\begin{cases} V_1 = \dfrac{4 \times Q_1}{\pi \times D_1^2} = \dfrac{4 \times 0.15}{\pi \times 0.3766^2} = 1.35\text{ m/s.} \\ V_2 = \dfrac{4 \times Q_2}{\pi \times D_2^2} = \dfrac{4 \times 0.105}{\pi \times 0.3342^2} = 1.20\text{ m/s.} \\ V_3 = \dfrac{4 \times Q_3}{\pi \times D_3^2} = \dfrac{4 \times 0.05}{\pi \times 0.2354^2} = 1.15\text{ m/s.} \end{cases}$$

and then, the total head loss, according to the formulation proposed here, jointly considering the continuous and singular losses, will be:

$$\Delta H' = 1.15 \times 0.000743\ (1.35^2 \times 0.3766^{-1.243} \times 1500 + 1.2^2 \times 0.3342^{-1.243} \times 1000 +$$
$$+ 1.15^2 \times 0.2354^{-1.243} \times 500) = 1.15 \times 0.000743\ (9203 + 5624 + 3992) =$$
$$= 16.08\text{ m.w.c.,}$$

whereby at the end of the pipe there will be a service pressure of:

$H = H_g - \Delta H' = 40.00 - 16.08 = 23.92 > 23.00$ m.w.c., then it is acceptable.

The assumption - which appears in the statement - of a common friction factor in the entire pipeline of f = 0.015 is, without a doubt, a simplification of the problem (in fact, as we will see below, it is only true in the third section of the itself). Indeed, assuming an average water temperature of 20ºC, a kinematic viscosity of (Franquet, 2003):

$$\nu = (1.7224 - 0.0461 \cdot t + 0.0006 \cdot t^2 - 0.000003 \cdot t^3) \times 10^{-6} =$$
$$= (1.7224 - 0.922 + 0.24 - 0.024) \times 10^{-6} = 1.0164 \times 10^{-6}\text{ m}^2\text{/s} = 1.0164\text{ cSt .}$$

- <u>Section 1</u>: $\mathrm{Re} = \frac{V_1 \times D_1}{\nu} = \frac{1.35 \times 0.3766}{1.0164 \times 10^{-6}} = 500207$,

then it is an intermediate or transitional turbulent regime. Since here it turns out that Re < 10^6, we will apply the formula proposed here, with which:

$$f = 0.0028 + \frac{0.25 + 0.0905 \times 500207^{0.12}}{500207^{0.32}} = 0.013 .$$

- <u>Section 2</u>: $\mathrm{Re} = \frac{V_2 \times D_2}{\nu} = \frac{1.20 \times 0.3342}{1.0164 \times 10^{-6}} = 394569$,

then it is an intermediate or transitional turbulent regime. Since here it turns out that Re < 10^6, we will apply the formula proposed here, which will have a coefficient of friction of:

$$f = 0.0028 + \frac{0.25 + 0.0905 \times 394569^{0.12}}{394569^{0.32}} = 0.014 .$$

- <u>Tramo 3</u>: $\mathrm{Re} = \frac{V_3 \times D_3}{\nu} = \frac{1.15 \times 0.2354}{1.0164 \times 10^{-6}} = 266342$,

then it is an intermediate or transitional turbulent regime. Since here it turns out that Re < 10^6, we will apply the formula proposed here, which will have a coefficient of friction of:

$$f = 0.0028 + \frac{0.25 + 0.0905 \times 266342^{0.12}}{266342^{0.32}} = 0.015 .$$

7. Conclusions

The problem of studying a pressure pipe with an intermediate connection and a free outlet at its end appears frequently in treatises and in hydraulic installations. The classical function optimization theory has been applied to better understand the analysis carried out. The specific and generalized treatment when the number of exits increases indefinitely is the object of the present work, which also highlights the usefulness of the approximate function of Christiansen for the determination of the loss of continuous load in this type of conductions with en-route service, Equidistant outlets and exhaustion of flow. The work culminates with the resolution of two practical cases of pipes with intermediate intakes that are deemed illustrative to contrast some of the concepts expressed.

Bibliographic references

1) ALCAIDE, A. (1973). *Estadística Económica*. Ed. SAETA. Madrid, 370 p.

2) CABRERA, E., ESPERT, V., GARCÍA-SERRA, J. & MARTÍNEZ, F. (1996). *Ingeniería Hidráulica (Aplicada a los Sistemas de Distribución de Agua). Volumen I y II*. Unidad Docente Mecánica de Fluidos. Ed. Universidad Politécnica de Valencia. Valencia. 1168 p.

3) CHRISTIANSEN, J.E. (1942). "Irrigation by Sprinkling". *Bulletin* 670. University of California. College of Agriculture. Agricultural Experimental Station. Berkeley, California, 124 p. Recovered on line from:
https://archive.org/details/irrigationbyspri670chri

4) DARCY, H. & BAZIN, H. (1865). *Recherches hydrauliques entreprises par M. Henry Darcy continuées par M. Henri Bazin. Deuxième partie. Recherches expérimentales relatives au remous et à la propagation des ondes*. Tomo II. Paris, Imprimerie impériale.

5) FORCHHEIMER, P. (1935). *Tratado de hidráulica*. Ed. Labor, S.A. Barcelona, 1935-1950. 628 p.

6) FRANQUET BERNIS, J.M. (2003). *Cinco temas de hidrología e hidráulica*. Ed. Bibliográfica Internacional, S.L. – Universitat Internacional de Catalunya (UIC). Tortosa (Spain). 594 p.

7) FRANQUET BERNIS, J.M. (2005). *Cálculo hidráulico de las conducciones libres y forzadas (Una aproximación de los métodos estadísticos)*. Ed. Bibliográfica Internacional, S.L. – Universitat Internacional de Catalunya (UIC). Tortosa (Spain). 590 p.

8) FRANQUET BERNIS, J.M. (2019). *Dimensionamiento y distribución de las conducciones hidráulicas. Una contribución de la ingeniería matemática*. Universidad Nacional de Educación a Distancia (UNED) – Centro Asociado de Tortosa (Campus del NE). Colección CADUP-Estudios. Tortosa (Spain). 964 p.

9) HERNÁNDEZ RODRÍGUEZ, J. & CRESPO MARTÍNEZ, A. (1996). *Problemas de mecánica de fluidos y máquinas hidráulicas*. UNED. Madrid.

10) MOODY, L.F. (1947). "An approximate formula for pipe friction factors". *Trans. ASME 69(12)*, pp. 1005–1011.

11) TORRES SOTELO, J.E. (1970). *Apuntes de hidráulica general y agrícola. Primera y Segunda Parte*. Universidad Politécnica de Valencia. Escuela Técnica Superior de Ingenieros Agrónomos. Valencia.

12) WEYRAUCH, R. (1915). *Hydraulic Computation.* Stuttgart. Recovered on line from:
https://www.dora.lib4ri.ch/eawag/islandora/object/eawag%3A13205/datastream/PDF/view

****

List of Tables and Figures:

Institutional address of the author:

Joseph Maria Franquet Bernis
Agricultural Engineer PhD, Economic and Business Sciences PhD
Universidad Nacional de Educación a Distancia (UNED)
Associated Center of Tortosa (Northeast Campus)
C/ Cervantes, 17. 43500 Tortosa (Tarragona, Spain)
info@tortosa.uned.es
jfbernis@iies.es